Indivisible Labor, Lotteries, and Equilibrium

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Abstract

This paper considers environments where labour supply is indivisible. It shows that a straightforward application of standard equilibrium theory can result in suboptimal equilibrium allocations. It demonstrates that this feature can be corrected for by including a particular class of lotteries to the consumption set. Existence, uniqueness and optimality are demonstrated for this environment. The paper also discusses why this may help in understanding aggregate fluctuations in the labour market.

SECTION 1

INTRODUCTION

One modelling strategy that has become increasingly popular over the years has been to construct well specified environments and to analyze the allocations generated by competitive equilibrium. The motivation for focussing attention on this particular set of allocations derives from the properties which this set of allocations display, e.g. the relationships between Pareto optimal allocations and equilibrium allocations; between core allocations and equilibrium allocations and incentive and informational properties of the competitive mechanism (see Hurwicz [6,7]). Also, in many cases, assuming homogeneity of agents implies that equilibrium allocations can be characterized as the solution to a programming problem.

The main point of this paper, is that in certain non-convex environments, the competitive mechanism, narrowly interpreted, does not possess such nice properties. In particular, it does not generate optimal allocations. However, in the context of a specific class of environments it is demonstrated that the economy can be modified by adding a certain type of lottery to the consumption set, and that competitive equilibrium can be defined in a straightforward manner so as to preserve many of the usual properties.

The class of economies to be examined is those where labour supply is indivisible.

In addition to this it is also demonstrated that in the case where utility is separable between consumption and leisure the equilibrium allocations in the extended economy produce the same aggregate properties as an economy where there is no indivisibility and utility is linear in leisure. This result is important to macroeconomists studying equilibrium models of the labour market in that it can account for the discrepancy between estimates of the elasticity of labour supply based on individual data and values of elasticity required by theoretical models to reconcile fluctuations in wages and total hours of labour.

In order to achieve existence of equilibrium it will be necessary to assume that there is a non-atomic measure of agents (see Aumann [2], Mas-Colell [9]). Lebesque measure will be used throughout the sets of measure zero will also be ignored throughout the analysis.

SECTION 2

ENVIRONMENT

The economy consists of a continuum of identical agents, with names in the interval [0,1]. There are three commodities: labour, capital and output. All activity takes place in a single time period. Capital and labour can be used to produce output but the technology for doing so is subject to a stochastic shock. It will be assumed that the shock can take on only one of two values. Let

$$f_i(K,L): R_+ \times R_+ \to R_+$$

be the production function if the shock takes on the value i, i=1,2. It is assumed that $f_i(K,L)$ satisfies

(i) continuity in K and L

(ii) strictly monotone in both arguments

(iii) homogeneous of degree one in K and L

(iv) jointly concave in K and L, strictly concave in each of K and L separately

(c) $f_i(0,0) = 0$

for each i=1,2. The probability distribution governing the technology shock is specified by

$$q_i = \text{prob[shock is i]}.$$

for each $i=1,2$.

Each agent (or worker) is endowed with one unit of time and one unit of capital in each state of nature. Time is indivisible: Either the entire unit is supplied as labour or none of it is supplied as labour. All workers have an identical Von-Neumann-Morgenstern utility function specified by

$$V(c,\ell,k): R \times \{-1,0\} \times R \to R$$

where c is consumption of output, ℓ is supply of labour, and k is supply of capital. It is assumed that $V(c,\ell,k)$ is of the form

$$V(c,\ell,k) = u(c) + m\ell$$

where the following hold:

(i) $u(c)$ is continuous, strictly monotone increasing and strictly concave.

(ii) m is strictly positive.

It will prove convenient to represent the economy described above using contingent commodities. In each of the two states of nature there will be three commodities. In each state we will use the following convention:

commodity 1 = output

commodity 2 = labour

commodity 3 = capital

We now define consumption and production sets. In each state a worker will receive a bundle in the set

$$X = \{x \in R^3 \colon x_1 \geq 0,\ x_2 \in \{-1,0\},\ -1 \leq x_3 \leq 0\}.$$

Similarly, in each state i a production vector is chosen from the set:

$$Y_i = \{y \in R^3 \colon y_1 \geq 0,\ y_2 \leq 0,\ y_3 \leq 0,$$
$$y_1 \leq f_i(-y_2,-y_3)\}.$$

The economy $\mathcal{E}$ is completely specified by the list:

$$\mathcal{E} = (X, u(\cdot), m, f_1(\cdot,\cdot), f_2(\cdot,\cdot), q_1, q_2).$$

SECTION 3

EQUILIBRIUM AND OPTIMALITY IN $\mathcal{E}$

In this section we define the notion of equilibrium for $\mathcal{E}$ and show that equilibrium allocations may fail to be optimal. We first present some standard concepts.

<u>Definition</u>: An allocation for $\mathcal{E}$ is a list (x,y) where $x\colon [0,1] \to X \times X$ is measurable and $y \in Y_1 \times Y_2$.

The interpretation is that for each $t \in [0,1]$, $x(t) = (x_1(t), x_2(t))$ where in turn $x_i(t) = (x_{i1}(t), x_{i2}(t), x_{i3}(t))$ is the bundle worker t receives if the state of nature is i, i=1,2. Similarly $y = (y_1, y_2)$ where $y_i = (y_{i1}, y_{i2}, y_{i3})$ is the production vector in state i.

<u>Definition</u>: A price system for $\mathcal{E}$ is a vector $p \in S_+^6$ where S_+^6 is the segment of the six dimensional unit simplex on which all coordinates are non-negative.

We will write $p = (p_1, p_2)$ where $p_i = (p_{i1}, p_{i2}, p_{i3})$. We are now ready to define the standard notion of competitive equilibrium.

<u>Definition</u>: A competitive equilibrium for $\mathcal{E}$ is a list (x,y,p) where (x,y) is an allocation for $\mathcal{E}$, p is a price system for $\mathcal{E}$ and

(i) for each $t \in [0,1]$ $x(t)$ is a solution to

$$\max_{x_1, x_2} \sum_{i=1}^{2} q_i [u(x_{i1}) - m x_{i2}]$$

$$\text{s.t. } x_i \in X \quad i=1,2$$

$$p_1 x_1 + p_2 x_2 \leq 0$$

(ii) for each $i=1,2$ y_i is a solution to:

$$\max_{y} p_i y$$

$$\text{s.t. } y \in Y_i$$

(iii) $\left(\int_0^1 x_1(t)dt, \int_0^1 x_2(t)dt\right) = (y_1, y_2)$.

Conditions (i) - (iii) simply represent utility maximization, profit maximization and market clearing respectively.

It is possible to show that a competitive equilibrium exists for $\mathcal{E}$ however this is not central to our discussion. For our purposes the interesting feature of $\mathcal{E}$ that distinguishes it from purely neoclassical economies is the indivisibility in labour supply. One of the major implications of this feature is that it is possible for identical agents to receive different allocations in equilibrium. It is straightforward to show that if consumption and technology sets are convex and preferences are strictly convex, then in equilibrium identical agents receive identical allocations. This result follows from the fact that budget sets are convex and hence if two distinct points each in the budget set give equal utilities then there necessarily exists a third point also in the budget set providing a greater level of utility.

In the economy $\mathcal{E}$ being considered here the consumption set is not convex and hence this argument no longer holds. As the following example demonstrates, neither does the result.

Example 1. Consider the following specification for $\mathcal{E}$:

$$f_1(K,L) = f_2(K,L) = \lambda K^{\alpha} L^{1-\alpha}.$$

$$u(c) = c^{\frac{1}{2}}$$

where $\alpha = .960$

$\lambda = 1.080$

$q_1 = q_2 = \frac{1}{2}$

$m = .050$

It is relatively straightforward to verify that the following is an equilibrium:

$$x_1(t) = \begin{cases} (1.0505,-1,-1) & t \in [0,.4115] \\ (1.0505,0,-1) & t \in (.4115,.8230] \\ (1,0,-1) & t \in (.8233,1] \end{cases}$$

$$x_2(t) = \begin{cases} (1.0505,0,-1) & t \in [0,.4115] \\ (1.0505,-1,-1) & t \in (.4115,.8230] \\ (1,0,-1) & t \in (.8230,1] \end{cases}$$

$$y_1 = y_2 = (1.0416,-.4115,-1)$$

$$p_1 = p_2 = (.2379,.0242,.2379)$$

This example is perhaps somewhat anomolous in that the two states of nature are identical, however it is still sufficient to illustrate the points we are interested in.

One of these is that this economy is one in which there is no heterogeneity of risk types yet in equilibrium there is some risk-sharing taking place. More precisely, if we don't allow agents to transfer wealth across states of nature (i.e. in defining equilibrium we write the budget constraint in two parts as $p_1x_1 \leq 0$ and $p_2x_2 \leq 0$) then we obtain a different equilibrium allocation. It is also true that if we change the environment so that $q_1 = 3/4$, $q_2 = 1/4$ then the equilibrium allocation changes despite the fact that both states are identical. These peculiarities seem to arise directly from a third observation: The equilibrium allocation is not optimal. To see this consider holding a lottery of the following form: Everyone consumes 1.0416 units of output in each state, all capital is supplied in each state, and in each state a set of workers of measure .4115 is chosen randomly from the set of workers. The expected utility of such a lottery is given by:

$$(1.0416)^{\frac{1}{2}} - (.4115)(.05) = 1.0000$$

while the equilibrium allocation displayed above gives an expected utility of

$$(1.0505)^{\frac{1}{2}} - (.5)(.05) = .9999$$

to each agent. Hence, the above mentioned allocation dominates the equilibrium allocation.

This result holds despite the fact that as commonly stated, the first welfare theorem should apply to this example (see for example Debreu (3), chapter six). The solution to this apparent inconsistency lies in the fact that the lottery described previously does not correspond to any point in the consumption set X. In the remainder of this paper we show how the consumption set can be modified so that such lotteries can be attained as competitive allocations.

SECTION 4

EQUILIBRIUM WITH LOTTERIES

In this section we redefine the environment and present a corresponding notion of competitive equilibrium.

All objects will remain the same except for the consumption sets and preferences. Define

$$X_1 = \{x \in X: x_2 = -1\}$$
$$X_2 = \{x \in X: x_2 = 0\}$$
$$\overline{X} = X_1 \times X_2 \times [0,1]$$

The interpretation of this consumption set is as follows: The set X_1 represents allocations where the individual is working, the set X_2 represents allocations where the individual is not working. The third element, a number in the interval [0,1] represents the probability that the individual works. A point $x \in \overline{X}$ will be written as

$$x = (x^1, x^2, \phi)$$

where $x^1 \in X_1$, $x^2 \in X_2$, $\phi \in [0,1]$, and $x^i = (x_1^i, x_2^i, x_3^i)$.

Now define a function

$$\overline{V}: \overline{X} \to R$$

by

$$\overline{V}(x^1, x^2, \phi) = \phi V(x^1) + (1-\phi) V(x^2)$$

where V is as defined previously. The economy $\overline{E}$ is now completely described by:

$$\overline{E}: (\overline{X}, u(\cdot), m, f_1(\cdot,\cdot), f_2(\cdot,\cdot), q_1, q_2)$$

Allocations and price systems for $\overline{E}$ are defined as follows:

Definition: An allocation for $\overline{E}$ is a list (x,y) where $x: [0,1] \to \overline{X} \times \overline{X}$ is measurable and $y \in Y_1 \times Y_2$.

Definition. A price system for $\overline{E}$ is a vector $p \in S^6_+$.

We are now ready to define a competitive equilibrium for $\overline{E}$.

Definition: A competitive equilibrium for $\overline{E}$ is a list (x,p) where x is an allocation for $\overline{E}$, p is a price system for $\overline{E}$ and

(i) for each $t \in [0,1]$ $x(t)$ is a solution to

$$\max_{x_1, x_2, \phi} \; q_1\overline{V}(x_1) + q_2\overline{V}(x_2)$$

$$\text{s.t.} \quad x_1 \in \overline{X},\ x_2 \in \overline{X}$$

$$\sum_{i=1}^{2} (\phi_i p_i x_i^1 + (1-\phi_i) p_i x_i^2) \leq 0$$

(ii) for each $i=1,2$ y_i is a solution to

$$\max_{y} \; p_i y$$

$$\text{s.t.} \quad y \in Y_i$$

$$\text{(iii)} \quad \int_0^1 \phi_i(t)\, x_t^1(t)dt + \int_0^1 (1-\phi_i(t))x_i^2(t)dt = y_i, \quad i=1,2.$$

The interpretation of the above three conditions is standard but a few points are worth mentioning. Note that the value of a bundle of goods which involves a lottery is determined by taking the expected value of the bundles which comprise the lottery. Condition (iii) is the market clearing condition and takes advantage of the fact that there is a continuum of consumers.[1]

It will prove useful to consider some properties of the maximization problem faced by individuals. The problem can be written more transparently as:

$$\text{(P1)} \quad \underset{\substack{c_{ij},\phi_i,k_i \\ i,j=1,2}}{\text{Maximize}} \quad q_1\{\phi_1[u(c_{11})-m] + (1-\phi_1)u(c_{12})\} + q_2\{\phi_2[u(c_{21})-m] + (1-\phi_2)u(c_{22})\}$$

s.t. $\quad c_{ij} \geq 0 \quad i,j = 1,2.$

$0 \leq \phi_i \leq 1 \quad i=1,2.$

$0 \leq k_i \leq 1 \quad i=1,2.$

$$\phi_1 p_1 c_{11} + (1-\phi_1)p_1 c_{12} + \phi_2 p_2 c_{21} + (1-\phi_2)p_2 c_{22}$$

$$\leq w_1\phi_1 + w_2\phi_2 + r_1 k_1 + r_2 k_2$$

The variables are as follows: c_{i1} is consumption of output in state i if the individual works and c_{i2} is consumption in state i if the individual doesn't work· k_i is the capital supplied in state i,

p_i is the price of output in state i, w_i is the wage in state i and r_i is the rental price of capital in state i. We now examine some properties of this problem.

Lemma 1: *If* $(c_{ij}, k_i, \phi_i,\ i,j=1,2)$ *is a solution to* P1 *and* $\phi_i \in (0,1)$ *then* $c_{i1} = c_{i2}$ *for* i=1,2.

Proof: Without loss of generality, suppose by way of contradiction that $(c_{ij}, k_i, \phi_i,\ i,j=1,2)$ is a solution to P1, $\phi_1 \in (0,1)$, and $c_{11} \neq c_{12}$. Define

$$c_1^* = \phi_1 c_{11} + (1-\phi_1)c_{12}$$

Then, by strict concavity of $u(\cdot)$ it follows that

$$\phi_1(c_{11}) + (1-\phi)u(c_{12}) < u(c_1^*).$$

The allocation $(c_{ij}^*, k_i, \phi_i,\ i,j=1,2)$ where

$$c_{1j}^* = c_1^* \quad j=1,2$$

$$c_{2j}^* = c_{2j} \quad j=1,2,$$

is affordable and thus dominates $(c_{ij}, k_i, \phi_i,\ i,j=1,2)$ which is a contradiction. Q.E.D.

Note that if $\phi_i \notin (0,1)$ then there is no harm in assuming $c_{i1} = c_{i2}$, and hence we can consider the following problem

$$\underset{\substack{c_1,c_2,\phi_1,\phi_2\\k_1,k_2}}{\text{Maximize}} \quad q_1\{u(c_1)-\phi_1 m\} + q_2\{u(c_2)-\phi_2 m\}$$

$$\text{s.t.} \quad 0 \leq \phi_i \leq 1 \quad i=1,2$$

$$c_i \geq 0 \quad i=1,2$$

$$0 \leq k_i \leq 1 \quad i=1,2$$

$$p_1c_1 + p_2c_2 \leq \phi_1 w_1 + \phi_2 w_2 + k_1 r_1 + k_2 r_2.$$

Since $u(\cdot)$ is monotone, if all prices are strictly positive it follows that k_1 and k_2 will both be equal to one and the budget constraint will hold with equality. In this case the problem can be written as

$$\text{(P2)} \quad \underset{c_1,c_2,\phi_1}{\text{Maximize}} \quad q_1\{u(c_1)-\phi_1 m\} + q_2\{u(c_2) - \frac{[(\sum_{i=1}^{2} p_ic_i-r_i)-\phi_1 w_1]m}{w_2}\}$$

$$\text{s.t.} \quad c_i \geq 0 \quad i=1,2$$

$$0 \leq \phi_1 \leq 1$$

The following is immediate.

<u>Lemma 2</u>: *Problem* (P2) *has a solution. Moreover* c_1 *and* c_2 *are uniquely determined, and*

(i) *if* $q_2w_1 - q_1w_2 > 0$ *then* $\phi_1 = 1$

(ii) *if* $q_2w_1 - q_1w_2 < 0$ *then* $\phi_1 = 0$

(iii) *if* $q_2w_1 - q_1w_2 = 0$ *then* ϕ_1 *can take on any value in* $[0,1]$.

Proof: A solution exists by applying the Weierstrass theorem to the problem written as (P1). The uniqueness of c_1 and c_2 follows from the strict concavity of the objective. Finally, the conditions on ϕ_1 arise from the fact that the objective is linear in ϕ_1. Q.E.D.

We now examine the properties of equilibria in which all agents do not receive the same bundle. Note that an equilibrium price system necessarily has all prices strictly positive, hence lemma 2 will apply.

Proposition 1: *Suppose* (x,y,p) *is a competitive equilibrium for* $\bar{E}$ *such that* $x(t)$ *is not constant. Then there exists another equilibrium* $(\bar{x},y,p)$ *such that* $\bar{x}(t)$ *is constant and*

$$x_i^j(t) = \bar{x}_i^j(t), \quad i,j=1,2 \quad t \in [0,1]$$

$$\bar{\phi}_i(t) = \int_0^1 \phi_i(s)ds \quad t \in [0,1].$$

Proof: Since $x(t)$ is not constant we know from Lemma 2 that $q_2w_1 - q_1w_2 = 0$ must hold. Since consumption is necessarily constant across individuals (from lemma 2) it follows that $\bar{x}$ satisfies condition (i) of equilibrium. The constancy of $x_1^j(t)$ across t for each j implies that condition (iii) will also be satisfied. Since y is unchanged condition (ii) is also satisfied. This completes the proof. Q.E.D.

The importance of this proposition is it implies that for each equilibrium in which agents receive different bundles there exists another equilibrium which possesses the same aggregate properties but in which all agents receive the same bundle. We now show that there exists an (essentially) unique equilibrium of this form by connecting equilibrium allocations with allocations generated by a concave programming problem. Consider the following problem:

$$\text{(P4)}\quad \underset{\substack{c_i,k_i,\phi_i \\ K_i,L_i \\ i=1,2}}{\text{Maximize}} \quad q_1[u(c_1)-\phi_1 m] + q_2[u(c_2)-\phi_2 m]$$

$$\begin{aligned} \text{s.t.} \quad & c_i \geq 0 \quad i=1,2 \\ & 0 \leq k_i \leq 1, \ i=1,2 \\ & 0 \leq \phi_i \leq 1 \quad i=1,2 \\ & c_i \leq f_i(K_i,L_i) \quad i=1,2 \\ & 0 \leq K_i \leq k_i \quad i=1,2 \\ & 0 \leq L_i \leq \phi_i \quad i=1,2. \end{aligned}$$

Lemma 3: *There exists a unique solution to* P4.

Proof: This follows directly from the Weierstraas theorem and strict concavity. Q.E.D.

Let $(c_i^*, k_i^*, {}_i^*, K_i^*, L_i^*, i=1,2)$ be the unique solution to (P4). Then there exist non-negative numbers $(p_i^*, w_i^*, r_i^*, i=1,2)$ such that $(c_i^*, k_i^*, \phi_i^*, K_i^*, L_i^*, i=1,2)$ is a solution to

$$\max_{\substack{c_i,k_i,\phi_i \\ L_i,K_i,\ i=1,2}} \quad q_1[u(c_1)-\phi_1 m] + q_2[u(c_2)-\phi_2 m]$$
$$+ w_1^*[\phi_1-L_1] + w_2^*[\phi_2-L_2]$$
$$+ r_1^*[k_1-K_1] + r_2^*[k_2-K_2]$$
$$+ p_1^*[f_1(K_1,L_1)-c_1] + p_2^*[f_2(K_2,L_2)-c_2]$$

s.t.

$$c_i \geq 0 \qquad i=1,2$$
$$0 \leq k_i \leq 1 \qquad k=1,2$$
$$0 \leq \phi_i \leq 1 \qquad i=1,2$$
$$K_i \geq 0 \qquad i=1,2$$
$$L_1 \geq 0 \qquad i=1,2$$

The following two propositions are standard and thus proofs are not provided here. First, however we provide a mapping between the two sets of notation used previously:

$$x_i^{*1} = (c_i^*,-1,k_i^*) \qquad i=1,2.$$

$$x_1^{*2} = (c_i^*,0,k_i^*) \qquad i=1,2.$$

$$p^* = ((p_1^*,w_1^*,r_1^*),(p_2^*,w_2^*,r_2^*))$$

$$y_i^* = (f_i(K_i^*,L_i^*),\ -L_i^*,\ -K_i^*) \quad i=1,2.$$

The use of the letter p in two different contexts, although potentially confusing should not cause any problems.

Proposition 2: *As defined above,* (x^*,y^*,p^*) *is a competitive equilibrium for* $\bar{E}$.

Proposition 3: *If* (x^*,y^*,p^*) *is a competitive equilibrium for* $\bar{E}$ *and* $x^*(t)$ *is constant then* (x^*,y^*) *is a solution to* (P4).

These two propositions plus lemma 3 imply the existence of a unique equilibrium allocation for which x(t) is constant. By proposition 1 this implies that aggregate behaviour is uniquely determined in equilibrium. Note that prices are not necessarily uniquely determined in equilibrium. This could be achieved by assuming that all functions are differentiable.

Finally, we note some comparisons between the equilibria for E and equilibria for $\bar{E}$. One peculiar feature of the example presented in the last section was that risk sharing agreements existed despite the fact that all agents are identical. This does not occur in equilibria for $\bar{E}$. One would obtain the same solution as for (P4) even if the problem were broken up so as to solve for the allocations in each state separately. This follows immediately from the fact that the objective is separable in the two sets of variables and that the constraints are independent.

Lastly, although it is not straightforward to compare equilibrium allocations for E with equilibrium allocations for $\bar{E}$ it is possible to show that the solution to (P4) is at least as good as any equilibrium allocation for E.

Proposition 4: *Let* (x,y,p) *be an equilibrium for* $\mathcal{E}$. *If* $(\bar{x},\bar{y},\bar{p})$ *is an equilibrium for* $\bar{E}$ *then for each* $t \in [0,1]$:

$$q_1V(x_1(t)) + q_2V(x_2(t)) \leq q_1\bar{V}(\bar{x}_1(t)) + q_2\bar{V}(\bar{x}_2(t)).$$

Proof: Let (x,y,p) be an equilibrium for $\mathcal{E}$. Define another allocation (x^*,y) where $x^* \in \bar{X} \times \bar{X}$ by:

$$x^{*j}_{i1}(t) = \int_0^1 x_{i1}(s)ds \quad i,j=1,2,\ t \in [0,1]$$

$$x^{*j}_{13}(t) = \int_0^1 x_{i3}(s)ds \quad i,j=1,2 \quad t \in [0,1]$$

$$\phi^*_i(t) = -\int_0^1 x_{i2}(s)ds \quad i=1,2 \quad t \in [0,1].$$

Since (x,y,p) is an equilibrium it follows that

$$q_1V(x_1(t)) + q_2V(x_2(t)) = \lambda, \text{ for all } t \in [0,1]$$

If we treat labour input as a continuous variable then $q_1V(x_1(t)) + q_2V(x_2(t))$ is weakly concave in $x_1(t)$ and $x_2(t)$. Hence by Jensen's inequality it follows that

$$\lambda = q_1V(x_1(t)) + q_2V(x_2(t)) = \int_0^1 [q_1V(x_1(t)) + q_2V(x_2(t))]dt$$

$$\leq q\ V(\int_0^1 x_1(t)dt) + q_2V(\int_0^1 x_2(t)dt)$$

$$= q_1\bar{V}(x^*_1(t)) + q_2\bar{V}(x^*_2(t))$$

Since the solution to (P4) cannot be dominated by (x^*,y) and any equilibrium $(\bar{x},\bar{y},\bar{p})$ for $\bar{E}$ will result in the same utility level for each agent as the solution to (P4) provides, the proof is complete. Q.E.D.

SECTION 5

DISCUSSION

One of the main implications of the preceding sections has been that a straightforward application of Arrow-Debreu theory to a non-convex economy may result in equilibrium allocations which are suboptimal. However, it was also shown, in the context of a particular form of non-convexity that the economy can be convexified in a rather straightforward manner so that in the convexified economy equilibrium exists and produces optimal allocations.

The non-convexity in this economy was of a special type-- essentially there were "missing points" in the consumption set. Workers could supply 0 units of labour or 1 unit of labour but none of the values in between these two.

Constructing the economy $\bar{E}$ basically amounts to filling in these points. (In fact some additional points are added also but they turn out to be of no consequence.) Technically, the allocations determined by equilibria for $\bar{E}$ are identical to allocations which would be obtained for E if we simply allow labour to be supplied in any amount and let the utility function be linear in labour supplied. What differs, of course, is the interpretation.

Although the economy considered in this paper was purposefully simplified in order to make the conclusions clear, it is very similar to economies which have been used extensively in the implicit contract literature (see e.g. Azariadis (3)). One central difference is that those economies typically include a risk neutral agent. It would not be difficult to incorporate such an addition into the present framework. One of the implications of the analysis carried out here is that allocations generated by implicit contracts can be viewed as equilibrium allocations for an appropriately defined economy. Viewed in this way, implicit contract theory does not present an alternative to equilibrium theory. (This comment does not apply to environments in which there is private information, as very little is known about how to attain optimal contracts as equilibrium allocations in such environments. See, however, Prescott and Townsend [10]). Contractual restrictions imposed by some authors can be incorporated by modifying the consumption sets. For example, the case of no severance payments can be handled by setting:

$$X_2 = \{x \in X: x_1=0, x_2=0\}$$

and defining

$$\overline{X} = X_1 \times X_2 \times [0,1].$$

Note that $\overline{X}$ is still convex.

The same methodology used here should also apply to environments like Holmstrom's (5). In that paper he used competition to motivate a concept of equilibrium which apparently is not very competitive. He goes on to prove that "contractual equilibria" are different from competitive equilibria. The present analysis implies that such claims seem to lack a strong foundation.

It seems that understanding will be enhanced more by examining the consequences of a given concept of equilibrium for different classes of environments rather than considering various equilibrium notions for a given class of environments. What the analysis here has suggested is that achieving the same notion of equilibrium may require a slight modification of the underlying economy in many cases.

A few additional points should be noted. First, if utility were linear in consumption, then there would be no gains realized by introducing lotteries into the consumption set. Alternatively, there is no loss in introducing them either. Their main advantage in this case is that they still allow for the existence of a symmetric equilibrium, which facilitates characterization of equilibrium allocations.

Although there is no generic form of non-convexity of the type considered here (intuitively of the missing-point type mentioned earlier) it seems clear that the analysis used here will give similar results in other specific contexts.

SECTION 6

SOME COMMENTS ON AGGREGATE FLUCTUATIONS

A central problem which has appeared in attempts to produce equilibrium models of aggregate fluctuations in the labour market has been the elasticity of supply required by these models to account for the observed relative magnitude of fluctuations in total hours of work and real wages. (For example, see Altanji and Ashenfelter [1].) Also of the measures selected in Kydland and Prescott [8], this was the only dimension in which their model failed to account for the observed fluctuations to any reasonable degree. The model presented in this paper is related to this issue. An alternative specification of the utility function could have been:

$$U(c,h) = u(c) + v(h), \text{ where } c \geq 0, \ h \in \{0,-1\}$$

where the restriction on h comes from assuming labour supply is indivisible. As stated in the last section, the equilibrium allocations for this economy are identical in the aggregate to those that would be obtained with the following specification:

$$U(c,h) = u(c) + kh \quad c \geq 0, \ h \in [-1.0].$$

This specification can produce large fluctuations in aggregate hours of labour supplied without any fluctuation in wages. The point that is important here is that aggregate fluctuations have this property even when $\nu(\cdot)$ is not linear if the indivisibility is present. Hence, micro studies which determine properties of $\nu(\cdot)$ may be of little relevance in predicting aggregate fluctuations in the presence of institutional or other factors which cause labour supply to be indivisible.

SECTION 7

CONCLUSION

This paper has demonstrated the problems associated with applying the concept of competitive equilibrium in a narrow fashion to environments which are non-convex. It has also shown how these problems can be avoided with a suitable extension of the environment and equilibrium concept. In later work the same principle will be used in showing how certain classes of non-convex environments produce equilibrium allocations with desirable properties.

Footnotes

[1] This definition of equilibrium implicitly raises some troublesome questions involving a continuum of independent random variables. The reader is referred to the manuscript "The Law of Large Numbers with a Continuum of IID Random Variables", an unpublished manuscript by K. Judd for a justification.

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