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Abstract

This paper studies cyclical fluctuations in hours/worker, employment, real wages and the duration of unemployment. It characterizes equilibrium allocations for a two sector economy where aggregate shocks affect sectors differentially and workers are restricted so that they can only be in one sector at any given time and changing sectors requires a spell of unemployment. The paper also demonstrates that for a certain class of non-concave programming problems, standard results continue to hold: The unique solution to the problem is given by the unique solution to the first order conditions.

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SECTION 1

INTRODUCTION

One of the main tasks of the labour market is to carry out adjustments in response to changes in the underlying economic environment. There are a variety of forms which this adjustment may take: we may for example observe changes in any or all of employment, hours of work, effort, retirement age, wages, and turnover. Most aggregate theories of the labour market assume that adjustment in total hours of work takes place in a single dimension: either employment or hours per worker (see for example, Lucas and Rapping [6], Jovanovic [3], Lucas and Prescott [5], Kydland and Prescott [4]). This paper considers cyclical fluctuations in employment, hours per worker, the duration of unemployment, and real wages. In particular it addresses the following stylized facts.

(i) Employment and hours/worker decrease simultaneously during recessions (although the former lags the latter).

(ii) Duration of unemployment is countercyclical.

(iii) Fluctuations in real wages and hours/worker are more highly correlated than either real wages and employment or real wages and total hours.

The distinction between changes in these variables is apparently quite significant. If the only cyclical impact in labour markets was, say, a 10% reduction in everyone's hours of work, public policy related to the labour market would be very different. Also, many institutional questions depend critically upon these distinctions: Reductions in hours of work do not qualify an individual for unemployment compensation in the U.S. (or Canada, although it does in some European countries, e.g. West Germany). Recently there has been attention focussed on the issue of the impact of a reduction in the standard workweek in industry on the unemployment rate.

The model considered here is an alternative to that in which workers face a fixed set-up in cost of working. The disadvantage of those models is that with homogeneous agents adjustment in hours/worker and employment never occur simultaneously: Hours/worker decline to a certain critical value and then remain fixed while further reductions take the place in employment.

However, just as was true in the case of a fixed set-up cost, non-convexities will play a critical role in generating the results of the paper. As a result of this, in addition to studying fluctuations in the above-mentioned variables, it is necessary to develop some results of a technical nature, which although apparently have standard looking conclusions, do not follow from standard results because of the non-convexity.

The model considered is similar to that used by Diamond [1] to analyze welfare implications of changes in hours/worker and changes in the sectoral composition of employment.

Finally, it should be mentioned that the presence of non-convexities will force us to use a continuum of agents in order to get exact existence of an equilibrium. Throughout the paper Lebesgue measure will be used and sets of measure zero will be ignored.

SECTION 2

SOME EMPIRICAL EVIDENCE

In this section we present some stylized facts concerning cyclical fluctuations in labour markets. First we consider fluctuations in employment, hours/worker and real wages. We consider three different sources: ten industries in durable manufacturing for the U.S., nine industries in non-durable manufacturing for the U.S., and data for the manufacturing sector in ten industrial countries. The analysis of data was very unsophisticated; it simply involved taking the series of yearly percentage changes in each case and calculating the standard deviations for these series. The results of this exercise are contained in Tables 1-3. For each of these cases we then calculated correlation coefficients for the various series of standard deviations. These results are presented in Table 4. The following pattern emerged. In all cases, fluctuations in hours/worker and wages were more highly correlated than were fluctuations in employment and wages. Not surprisingly, the correlation between the sum of fluctuations in employment and hours/worker (which is approximately the fluctuation in total hours) and real wages lies in between these two values. The fourth column of Table 4 examines correlations between the relative fluctuations in employment to hours/worker and real wages. Given the above findings, the correlation was as expected negative. It appears however, that this correlation is not as strong as that between hours/worker and real wages.

We now consider some limited evidence on the nature of sectoral flows over the business cycle. Table 5 presents data from a study carried out by Vroman [10]. It should be noted that 1964-66 was a cyclical upswing and 1969-71 was a cyclical downturn. Also, the categories industry dropouts and industry entrants refer respectively to individuals who obtain employment outside of the given industry and as individuals who were previously employed outside of the given industry. The table gives the size of the group as a fraction of employment in the industry. Although the evidence is limited in scope, it suggests the following:

(i) During a downturn there is an increase in the flow of workers from durable manufacturing into retail.

(ii) During an upturn there is an increase in the flow of workers from retail into durable manufacturing.

(iii) There is no apparent movement in intra-industry movement.

SECTION 3

THE ENVIRONMENT

The economy last for two periods. There is a continuum of identical agents with names in [0,1]. There are three commodities: output, capital, and labour. Output is produced in two distinct sectors and is subject to a stochastic shock. A given shock determines the production possibilities in both sectors for both periods. Let $f_{ij}^k(K,L)$ be the production function for sector k in period j assuming state i has occurred, where f_{ij}^k: $R_+ \times R_+ \rightarrow R_+$ for k=1,2, j=1,2, i=1,2,...,N. K is input of capital and L is input of manhours of labour. For reasons that will become clear later, it is convenient to assume that

$$f_{i1}^2(K,L) = 0 \text{ for all } K,L \geq 0.$$

It will be assumed that the rest of the $f_{ij}^k(K,L)$ satisfy the following:

(i) homogeneity of degree one in (K,L)

(ii) weakly concave in (K,L) jointly, strictly concave in each of K and L individually for a fixed value of the other argument

(iii) twice continuously differentiable in both arguments

(iv) $f_{ij}^k(0,0) = 0$, $\lim_{L \to 0} \frac{\partial f_{ij}^k(K,L)}{\partial L} = \infty$,

$$\lim_{L \to \infty} \frac{\partial f_{ij}^k(K,L)}{\partial L} = 0, \text{ for all } K > 0, \text{ and}$$

$$\frac{\partial f^k_{ij}(K,L)(K,L)}{\partial K} > 0, \ \frac{\partial f^k_{ij}(K,L)}{\partial L} > 0, \text{ all } K,L \geq 0.$$

The probability distribution of the technology shocks is given by:

$$q_i = \text{prob[state of nature i occurs]},$$

where $0 < q_i \leq 1$, $i=1,\ldots,N$.

Capital is sector specific and cannot be accumulated or transformed and does not depreciate. Each agent (or worker) is endowed with one unit of each type of capital in both periods of each state of nature. Each agent is also endowed with one unit of time in both periods of each state of nature. Any fraction of this unit of time may be supplied as labour with the following restrictions:

(i) labour cannot be supplied in both sectors simultaneously.

(ii) labour can only be supplied in sector two in period two and requires that the worker be idle in period one and suffer a psychic cost m associated with locating and adjusting to a new job.

The nature of the above restrictions is such that it will be useful to distinguish formally between labour and capital supplied in sector one and labour and capital supplied in sector two. In any given state then, there will be ten commodities: labour in two sectors in each period, capital in two sectors in each period, and output in each period. We will use the following system to index commodities in each state of nature:

Commodity 1 = output in period one

commodity 2 = output in period two

commodity 3 = labour in sector one in period one

commodity 4 = labour in sector two in period one

commodity 5 = labour in sector one in period two

commodity 6 = labour in sector two in period one

commodity 7 = capital in sector one in period one

commodity 8 = capital in sector two in period one

commodity 9 = capital in sector one in period two

commodity 10 = capital in sector two in period two

We can now proceed to define the environment more precisely, beginning with consumption sets. First define

$$\overline{X} = \{x \in R^{10}: x_1 \geq 0,\ x_2 \geq 0,\ -1 \leq x_i \leq 0,\ i=3,4,\ldots,10,$$
$$x_4 = 0,\ x_5 \cdot x_6 = 0,\ x_3 \cdot x_6 = 0\}$$

The restriction $x_4 = 0$ implies labour cannot be supplied to sector two in period one; $x_5 \cdot x_6 = 0$ implies that labour cannot be supplied in both sectors in period 2; $x_3 \cdot x_6 = 0$ implies that a worker must be idle in period one if labour is supplied to sector two in period two.

The set $\overline{X}$ is non-convex. As illustrated in other work [7], in such cases it proves to be desirable to expand the consumption set to include lotteries as a means of convexifying consumption possibilities.[1] In order to accomplish this we define the following sets:

$X^1 = \{x \in \overline{X}: x_6 = 0\}.$

$X^2 = \{x \in \overline{X}: x_6 \neq 0\}$

Now define the set X which indicates the consumption possibilities for each state of nature by:

$$X = X^1 \times X^2 \times [0,1].$$

The interpretation is that a worker chooses an allocation contingent upon remaining in sector one, an allocation contingent upon moving to sector two, and a probability of remaining in sector one. Note that X is convex. An element $x \in X$ will be written

$$x = (x^1, x^2, \phi) \text{ where } x^i \in X^i, \ i=1,2, \quad \in [0,1]$$

and

$$x^i = (x_1^i, \ldots, x_{10}^i).$$

We now describe preferences. Define a function

$\overline{U}: \overline{X} \to R$ by

$$\overline{U}(x_1, \ldots, x_{10}) = \begin{cases} x_1 + x_2 + v(x_3) + v(x_5), \text{ if } x_6 = 0 \\ x_1 + x_2 + v(0) + v(x_6) - m \quad \text{if } x_6 \neq 0 \end{cases}$$

where $m \geq 0$ and $v: [-1,0] \to R$ satisfies

(i) $v(\cdot)$ is twice continuously differentiable

(ii) $v'(\cdot) > 0$, $v''(\cdot) < 0$, on $[-1,0]$

(iii) $\lim_{s \to 0} v'(s) = 0$, $\lim_{s \to -1} v'(s) = +\infty$

Now, each agent has preferences over X specified by the Von-Neumann

Morgenstern utility function

$$U: X \to R$$

specified by:

$$U(x^1,x^2,\phi) = \phi\bar{U}(x^1) + (1-\phi)\bar{U}(x^2) \ .$$

Using the same indexing system we can define technology sets by:

$$Y_i = \{y \in R^{10}: y_1 \geq 0,\ y_2 \geq 0,\ y_j \leq 0,\ j=3,4,\ldots,10,$$

$$y_1 \leq f_{i1}^1(-y_3,-y_7),\ y_2 \leq f_{i2}^1(-y_5,-y_9) + f_{i2}^2(-y_6,-y_{10}) \ .$$

The economy described above is completely described by the following list:

$$\mathcal{E} = (X,v(\cdot),\ m,\ \{q_i,i=1,\ldots,N\},\ \{Y_i,i=1,\ldots,N\})$$

SECTION 4

REMARKS ABOUT THE MODEL

In this section we discuss the intended interpretation of the model and the nature of some of the assumptions.

The basic situation envisaged is as follows. When an economy enters a cyclical downturn the sectoral distribution of consumption changes drastically. Demand for industrial production declines substantially relative to demand for services. However, the timing and severity of a particular downturn is subject to uncertainty. The model presented does not produce this phenomena endogenously: It simply studies the impact of this phenomena on the operation of the labour market.

It should be noted that output of both sectors is treated as a single commodity (alternatively, they are different commodities which consumers view as perfect substitutes). Instead of the model used here we could have considered one where there are two consumption goods and technologies are nonstochastic but preferences are stochastic and the marginal utilities of the two consumption goods are subject to shocks. Intuitively, the two approaches should produce similar results.

There is also some asymmetry in the treatment of product and labour markets, in the sense that redistributing output among consumers is very different from redistributing workers among firms. On one hand this is simply an abstraction since the main goal of the current study is to focus on the labour market. On the other hand, this asymmetry probably exists in the real world to a significant degree.

The nature of the uncertainty in the model is such that after the initial realization of the state there is no additional uncertainty. Hence there is no stochastic development over time. This more general case is treated in an infinite horizon model in [9].

The linearity of utility in consumption is important for some of the results derived here. The case of concave utility is treated in [8].

In the model as presented there are no workers originally located in sector two. This is done for technical reasons to allow a representative agent approach to be used. However several alternatives could have been used. If agents were assigned randomly to sectors then there could be workers initially in sector two and the representative agent approach could still be used. Alternatively, if the production function in sector two is linear then we can allow for the existence of a group of agents initially located in sector two who are not permitted to work in sector one. Linearity of production means that interaction between the two groups can be ignored in equilibrium. Clearly either of these alternatives would improve the model cosmetically, however, for the purposes considered here the substance is not sufficiently altered to necessarily warrant consideration of one of these alternatives over the model in the previous section.

SECTION 5

EQUILIBRIUM AND OPTIMALITY

In this section we prove existence and optimality of equilibrium for $\mathcal{E}$ and show that equilibrium is the unique solution to the first order conditions for a certain non-concave programming problem. We begin with some basic definitions:

Definition: An allocation for $\mathcal{E}$ is a list (x,y) where $x: [0,1] \rightarrow R^{10N}$ is measurable and $y \in R^{10N}$.

The interpretation is as follows: For each $t \in [0,1]$ $x(t)$ gives a complete description of agent t's state contingent allocation. We will write $x(t) = (x_1(t),\ldots,x_N(t))$ where in turn each of the $x_i(t)$ can be written as

$$x_i(t) = (x_i^1(t),x_i^2(t),\phi_i(t)).$$

Definition: A price system for $\mathcal{E}$ is a vector $p \in S_+^{10N}$ where S_+^{10N} is the subset of the unit simplex in R^{10N} for which all components are non-negative.

We will write:

$$p = (p_1,\ldots,p_N)$$

where each of the p_i can in turn be written as:

$$p_i = (p_{i1},\ldots,p_{i10}).$$

We can now define equilibrium.

<u>Definition</u>: An equilibrium for $\mathcal{E}$ is a list (x,y.p) where (x,y) is an allocation for $\mathcal{E}$ and p is a price system for $\mathcal{E}$, such that:

(i) For each $t \in [0,1]$ $x(t)$ is a solution to:

$$\max_{x_1,\ldots,x_N} \sum_{i=1}^{N} q_i U(x_i)$$

$$\text{s.t.} \quad x_i \in X \qquad i=1,\ldots,N$$

$$\sum_{i=1}^{N} p_i(\phi_i^2 x_i^1 + (1-\phi_i)x_i^2) \leq 0$$

(ii) for each $i=1,\ldots,N$ y_i is a solution to:

$$\max_{y} py$$

$$\text{s.t. } y \in Y_i$$

(iii) $\int_0^1 [\phi_i(t)x_i^1(t) + (1-\phi_i(t))x_i^2(t)]dt \leqq y_i \qquad i=1\ \ldots,N.$

The interpretation of the above conditions is straightforward. Condition (i) is utility maximization, condition (ii) is profit maximization, and condition (iii) is market clearing.[2]

We will be most interested in equilibria in which all agents receive identical allocations. Formally we define:

Definiition: A symmetric equilibrium (SE) for $\mathcal{E}$ is a list (x,y,p) such that (x,y,p) is an equilibrium for $\mathcal{E}$ and x(t) is constant.

It is possible to show as in [8] that for any non-symmetric equilibrium there is a symmetric equilibrium with the same aggregate properties, however we will not demonstrate that here.

In the remainder of this section we wish to show that a unique SE exists and that it can be characterized by the FOC of a particular programming problem. Consider the following problem:

$$\text{(P-1)} \qquad \max_{\substack{x_1,\ldots,x_N \\ y_1,\ldots,y_N}} \sum_{i=1}^{N} q_i U(x_i)$$

$$\text{s.t.} \quad x_i \in X \quad i=1,\ldots,N$$

$$y_i \in Y_i \quad i=1,\ldots,N$$

$$\phi_i x_i^1 + (1-\phi_i)x_i^2 \leq y_i, \quad i=1,\ldots,N.$$

Under "usual" circumstances we would show that any SE is a solution to this problem, that this problem has a unique solution and that this solution can be supported as an equilibrium. However, the above programming problem is not concave and hence standard results do not necessarily apply. There are two possible directions to pursue. As shown in [10] if we are willing to put conditions on the third derivatives of certain functions then standard results can be applied. Alternatively, and the method which will be used here it can be shown using the methods originated by Holmstrom [2] that the standard conclusions can be obtained without additional assumptions.

The following proposition, nonetheless is still immediate, since it does not depend on convexity. As the proof is standard it is omitted.

Proposition 1: *If* (x,y,p) *is a* SE *for* $\mathcal{E}$ *then* (x,y) *is a solution to* (P-1).

The following proposition is proved in the appendix:

Proposition 2: *Problem* (P-1) *has a unique solution* (x^*,y^*). *Furthermore,* (x^*,y^*) *is the unique solution to the first order conditions for problem* (P-1) *and there exist prices* p^* *such that* (x^*,y^*,p^*) *is a* SE *for* $\mathcal{E}$.

Note that propositions one and two together imply the existence of a unique SE for $\mathcal{E}$.

SECTION 6

SOME RESULTS

The goal of this section is to demonstrate that equilibrium allocations for $\mathcal{E}$ are consistent with certain observations. In this section we will parametrize the economy $\mathcal{E}$ in the following manner. Let f(K,L) be a production function satisfying the same conditions as were listed for $f^1_{i1}(K,L)$, $f^1_{i2}(K,L)$ and $f^2_{i2}(K,L)$. Let λ_{ki}, i=1,...,N, k=1,2 be positive real numbers. Then define

$$f^1_{ij}(K,L) \equiv \lambda_{1i} f(K,L), \ i=1,\dots,N, \ j=1,2.$$

$$f^2_{i2}(K,L) \equiv \lambda_{2i} f(K,L), \ i=1,\dots,N$$

The first condition implies that technology is constant across time within each state of nature in sector one.

We now characterize equilibrium allocations for this specification. The following is immediate:

Lemma 1: *If* (x^*,y^*,p^*) *is a* SE *for* $\mathcal{E}$ *then:*

$$y^*_{i3} = y^*_{i5}, \ i=1,\dots,N \text{ and } x^{*1}_{i3}(t) = x^{*1}_{i3}(t) = x^{*1}_{i5}(t), \ i=1,\dots,N, \quad t \in [0,1]$$

Proof: This follows immediately from the fact that once ϕ_i, i=1,...,N are fixed problem (P-1) is a strictly concave programming problem and that $f^1_{i1}(K,L)$ and $f^1_{i2}(K,L)$ are identical. //

It is then sufficient to study the properties of the following problem:

$$\text{(P-6)} \qquad \max_{\substack{c_1,c_2,\phi\\h_1,h_2}} \quad c_1 + c_2 + \phi 2v(-h_1) + (1-\phi)[v(-h_2) + v(0) - m]$$

$$\text{s.t.} \qquad 0 \le h_i \le 1 \quad i=1,2.$$

$$0 \le \phi \le 1$$

$$0 \le c_1 \le \lambda_1 f(\phi h_1)$$

$$0 \le c_2 \le \lambda_1 f(\phi h_1) + \lambda_2 f(1-\phi)h_2).$$

Note that subscripts denoting states have been deleted, and we have used $f(\cdot)$ to denote $f(1,\cdot)$. We can now prove the following:

Proposition 3: *Let* $h^*_1(\lambda_1,\lambda_2,m)$, $h^*_2(\lambda_1,\lambda_2,m)$ *and* $\phi^*(\lambda_1,\lambda_2,m)$ *be the functions denoting solutions to problem* (P-6) *for strictly positive values of the arguments. Then*

(i) h^*_1 *is increasing in* λ_1,λ_2, *decreasing in* m.

(ii) h^*_2 *is increasing in* λ_1,λ_2,m.

(iii) ϕ^* *is increasing in* λ_1,m, *decreasing in* λ_2.

(iv) $h^*_1\cdot\phi^*$ *is increasing in* m.

Proof: The constraints in c_1, c_2 will clearly be binding. In appendix one it was demonstrated that the solution to (P-6) will be interior. Hence we have the following conditions:

$$(6.1) \qquad \lambda_1 f'(\phi h_1) = v'(-h_1)$$

$$(6.2) \qquad \lambda_2 f'((1-\phi)h_2) = v'(-h_2)$$

$$(6.3) \qquad 2\lambda_1 f'(\phi h_1)h_1 + \lambda_2 f'((1-\phi)h_2) + 2v(-h_1) - v(-h_2) - v(0) + m = 0.$$

Substituting (6.1) and (6.2) into (6.3) gives:

$$(6.4) \qquad 2v'(-h_1)h_1 + 2v(-h_1) = v'(-h_2)h_2 + v(-h_2) + v(0) - m.$$

Note that the left hand side is increasing in h_1, and the right hand side is increasing in h_2 and decreasing in k. It follows that given h_2,m we can write

$$h_2 = r(h_1, m)$$

where r is increasing in both arguments. Using this to substitute into (6.2) we have the following system of two equations:

$$\lambda_1 f'(\phi h_1) = v'(-h_1)$$

$$\lambda_2 f'((1-\phi)r(h_1,m)) = v'(-r(h_1,m)).$$

Differentiating with respect to λ_1, using primes on h_1 and ϕ to denote derivatives with respect to λ_1 and subscripts on r to denote partial derivatives, we have:

$$(6.5) \qquad f'(\phi h_1) + \lambda_1 f''(\phi h_1)[\phi h_1' + h_1\phi'] = -v''(-h_1)h_1'$$

$$(6.6) \qquad f''((1-\phi)r(h_1,m))[(1-\phi)r_1 h_1' - r\phi'] = -v''(-r)r_1 h_1'$$

Solving (6.6) for ϕ' and substituting into (6.5) gives

$$f'(\phi h_1) + \lambda_1 f''(\phi h_1)[\phi h_1' + h_1\{(1-\phi)r_1 h_1' + \frac{v''(-r)r_1 h_1'}{f''((1-\phi)r)}\}]$$
$$= -v''(-h_1)h_1'$$

This gives:

$$\{\lambda_1 f''(\phi h_1)[\phi + h_1(1-\phi)r_1 + \frac{v''(-r)r_1}{f''((1-\phi)r)}] + v''(-h_1)\}h_1' = -f'(\phi h_1)$$

It follows that $h_1' > 0$. Hence, $h_2' > 0$. Also (6.6) implies that $\phi' > 0$. The other results are obtained similarly. //

The importance of proposition three is the following. $\mathcal{E}$ is now completely characterized by a list of the form $(\lambda_{11},\ldots,\lambda_{1N},\lambda_{21},\ldots,\lambda_{2N},m)$. Consider the specification given by:

$$\lambda_{11} > \lambda_{12} > \cdots > \lambda_{1N},\ \lambda_{21} = \lambda_{22} = \cdots = \lambda_{2N}.$$

Then, from proposition three we have that hours/worker decreases in both sectors and unemployment increases as the index of the state of nature increases. It also follows that we can perturb $\lambda_{21},\ldots,\lambda_{2N}$ such that $\lambda_{21} > \lambda_{22} > \cdots > \lambda_{2N}$, without affecting this result. Hence, even though both sectors become "worse off" as the index of the state of nature increases, the optimal allocation involves hours/worker decreasing and unemployment increasing as the index increases.

Clearly, there are two factors at work here. One is the "average" level of opportunities in the economy and the other is the dispersion of opportunities in the economy. Hours/worker responds essentially to the first factor, as evidenced by the fact that h_1 and h_2 respond similarly to changes in λ_1 and λ_2. (Also note that if sector two were removed there would be no dispersion; only hours/worker would fluctuate.) Unemployment, however, reacts essentially to the second factor, as evidenced by the fact that ϕ responds oppositely to changes in λ_1 and λ_2. It should be noted that the differential response in hours/worker and employment is due entirely to the assumption that workers cannot be employed in both sectors simultaneously.

Now consider two specifications of ε such that both are identical except for the values of m. If we consider variables concerning sector one, then proposition three implies that the economy with the smaller value of m will have higher unemployment, higher hours/worker, a higher real wage and lower output. Since the model only corresponds to the downswing of the business cycle, this suggests that if countries are parametrized only by values of m we should expect to see magnitudes of fluctuations in real wages and hours/worker positively correlated with each other and negatively correlated with the magnitudes of fluctuations in employment and output. If technological shocks are not constant then this pattern may be disturbed.

A slightly more robust prediction is that as long as $v(\cdot)$ does not change across the sample points, there should be a strong correlation between the magnitudes of fluctuations in real wages and hours/worker. This arises from the fact that if w_i is the wage in sector one in state i and h_i is the hours/worker is sector one in state i then the following must hold:

$$w_i = v'(-h_i).$$

For countries with very different wage levels this may not imply the above correlation depending upon the curvature of $v(\cdot)$. If $v(\cdot)$ is quadratic, then the above correlation necessarily holds.

SECTION 7

AN EXTENSION TO INCLUDE DURATION OF UNEMPLOYMENT

Without going through all of the details, in this section we point out how the model of the previous sections can be extended to allow for the duration of unemployment being variable. In the previous sections it was assumed that a worker who changed sectors was necessarily unemployed for one period, at a psychic cost of m. We now assume that search intensity can also be varied. In particular, an unemployed worker may be unemployed either one period or both periods. If he or she wants to become employed at the end of one period with probability p he or she undergoes a psychic cost $m(p) + m_0$, where:

(i) $m(p)$ is twice continuously differentiable

(ii) $m'(p) > 0$, $m''(p) > 0$, $p > 0$; $m_0 > 0$

(iii) $\lim_{p \to 0} m'(p) = 0$, $\lim_{p \to 1} m'(p) = \infty$.

It can be shown that all of the previous results concerning existence, and uniqueness of symmetric equilibria, and the first order condition approach continue to hold. Hence, we proceed directly to consider the following problem:

$$\text{(P-7)} \quad \max_{\substack{c_1,c_2,\phi \\ h_1,h_2,p}} \quad c_1 + c_2 + \phi 2v(-h_1) + (1-\phi)[pv(-h_2) + (2-p)v(0) - m(p) - m_o]$$

$$\text{s.t.} \quad 0 \le h_i \le 1 \quad i=1,2$$

$$0 \le \phi \le 1$$

$$0 \le p \le 1$$

$$0 \le c_1 \le \lambda_1 f(\phi h_1)$$

$$0 \le c_2 \le \lambda_1 f(\phi h_1) + \lambda_2 f((1-\phi)ph_2) .$$

Proposition 4: *Let* $h_1^*(\lambda_1,\lambda_2,m_o)$, $h_2^*(\lambda_1,\lambda_2,m_o)$, $\phi^*(\lambda_1,\lambda_2,m_o)$, $p^*(\lambda_1,\lambda_2,m_o)$ *be the functions giving solutions to problem* (P-7) *for positive values of the arguments. Then*

(i) h_1^* *is increasing in* λ_1,λ_2, *decreasing in* m_o

(ii) h_2^* *is increasing in* λ_1,λ_2, *and* m_o

(iii) ϕ^* *is increasing in* λ_1, m_o, *decreasing in* λ_2.

(iv) p^* *is increasing in* λ_1,λ_2,m_o.

Proof: The first order conditions for (P-7) are:

$$\text{(7.1)} \qquad \lambda_1 f'(\ h_1) = v(-h_1)$$

$$\text{(7.2)} \qquad \lambda_2 f'((1-\phi)ph_2) = v'(-h_2)$$

$$\text{(7.3)} \qquad 2v(-h_1) + 2\lambda_1 f'(\phi h_1)h_1 - pv(-h_2) - p\lambda_2 f'((1-\phi)ph_2)h_2 - (2-p)v(0) + m(p) + m_o = 0$$

$$\text{(7.4)} \qquad \lambda_2 f'((1-\phi)ph_2)h_2 + v\ (-h_2) - v(0) - m'(p) = 0.$$

Comgining (7.1), (7.2) and (7.3) gives:

(7.5) $$2v'(-h_1)h_1 + 2v(-h_1) = pv'(-h_2)h_2 + pv(-h_2) + (2-p)v(0) - m(p) - m_o.$$

Combining (7.2) and (7.4) gives:

(7.6) $$v'(-h_2)h_2 + v(-h_2) = v(0) + m'(p).$$

The above equation implies that we can write

$$h_2 = s(p)$$

where s(p) is increasing.

Substituting (7.6) into (7.5) gives:

(7.7) $$2v'(-h_1)h_1 + 2v(-h_1) = 2v(0) + pm'(p) - m(p) - m_o.$$

Since the right hand side is increasing in p, decreasing in m_o it follows that we can write:

$$h_1 = r(p,m_o)$$

where r is increasing in p and decreasing in m_o.

We can now consider the following two equations:

$$\lambda_1 f'(\phi r(p,m_0)) = v'(-r(p,m_0))$$

$$\lambda_2 f'((1-\phi)s(p)p) = v'(-s(p)).$$

Differentiating with respect to λ_1, letting primes on ϕ and p indicate derivatives with respect to λ and using subscripts to denote partial derivatives of r, we have:

(7.8) $$f'(\phi r) + \lambda_1 f''(\phi r)[\phi' r + \phi r_1 p'] = -v''(-r)r_1 p'$$

(7.9) $$\lambda_2 f''((1-\phi)sp)[s'p'(1-\phi)p + p'(1-\phi)s - \phi' sp] = -v''(-s)s'p'$$

Substitution then leads to $p' > 0$, $\phi' > 0$, hence $h_1' > 0$ and $h_2' > 0$. Similar procedures give the other results.

Most of the results here coincide with those derived in the last section. The major improvement is that if $\lambda_{11} < {}_{12} < \cdots \lambda_{1N}$, $\lambda_{21} = \lambda_{22} = \cdots \lambda_{2N}$ it now follows that the average duration of unemployment will also be increasing in the index of the state of nature.

SECTION 8

DISCUSSION OF THE RESULTS

The results derived in the last section account for several empirical regularities pointed out earlier in the paper. In this sense the model has been relatively successful.

However, the model has one very unattractive feature. The problem lies in the fact that the model (or any of the alternatives discussed at the end of section four) predicts an increase in employment in sector two during a cyclical downturn. If hours/worker in sector two declines enough it may be the case that total work performed decreases despite the increase in employment. Several comments are in order. First, it should be recalled that the evidence contained in section two demonstrates that many workers who lose jobs in industry during cyclical downturns do in fact find employment in other sectors. Second, the structure of the model employed may lend itself more readily to explain the nature of fluctuations in sector one (e.g. manufacturing) than those in sector two (e.g. services). Most workers in the service sector do not face fluctuations in working hours. Also, the service sector tends to be characterized by relatively high turnover and many temporary and/or part-time employment opportunities. This suggests that over a period of several months the division of hours among workers may be very different in services than in manufacturing. The division in services may involve workers holding several jobs of short duration with intermittent spells of unemployment rather than a single job with reduced hours. Hence, there may be a large difference between the number of individuals employed at a specific point during a six-month period versus the number who held employment at some point in the six month period. Let $\bar{e}$ be the average level of employment over a given period and e be the number of people employed at some point over the same period. Then, if the above discussion is valid we would predict that $e/\bar{e}$ for the service sector is countercyclical. This data is not available for the service sector, however it is available for the aggregate economy. It is displayed in Table 6. This data shows that there is some tendency for $e/\bar{e}$ to be countercyclical, as it reaches peaks in 1955, 1958, 1963, 1971, and 1975.

CONCLUSION

The model presented above illustrates how the particular form of non-convexity combined with aggregate shocks which affect sectors differentially can be combined to analyze several facets of the dynamic adjustment process carried out by labour markets.

APPENDIX

PROOF OF PROPOSITION TWO

The proof will proceed in a number of stages. First note that problem (P-1) can be split into N separate problems, each of the following form for some i:

$$\text{(P-2)} \qquad \max_{x,y} U(x)$$
$$\text{s.t.} \quad x \in X$$
$$y \in Y_i$$
$$\phi x^1 + (1-\phi)x^2 \leq y$$

Hence, we now consider only problems of this form. For our purposes it will prove simpler to work with the above problem restated using more suggestive notation.

$$\text{(P-3)} \qquad \max_{\substack{c_{ij},h_{ij},k^{\ell}_{ij} \\ H_{ij},K_{ij},}} \phi[c_{11} + c_{12} + v(-h_{11}) + v(-h_{12})]$$
$$+ (1-\phi)[c_{21} + c_{22} + v(0) + v(-h_{22})-m]$$

$$\text{s.t.} \qquad 0 \leq h_{ij} \leq 1 \quad (i,j) = (1,1),(1,2),(2,1)$$
$$0 \leq c_{ij}, \; i,j = 1,2$$
$$0 \leq \phi \leq 1$$
$$0 \leq k^{\ell}_{ij} \leq 1, \; i,j, \; = 1,2$$
$$0 \leq K_{ij} \quad i,j = 1,2$$
$$0 \leq H_{ij} \quad i,j = 1,2$$

$$\phi c_{11} + (1-\phi)c_{21} \le f_{11}(K_{11},H_{11})$$

$$\phi c_{12} + (1-\phi)c_{22} \le f_{12}(K_{12},H_{12}) + f_{22}(K_{22},H_{22})$$

$$K_{ij} \le \phi k^i_{1j} + (1-\phi)k^i_{2j}$$

$$H_{ij} \le \phi h_{ij} \qquad j=1,2$$

$$H_{22} \le (1-\phi)h_{22}.$$

The variables are straightforward: c_{ij}, h_{ij} are consumption and hours in period j contingent upon being in sector i; k^{ℓ}_{ij} is similarly defined with ℓ denoting which type of capital; H_{ij}, K_{ij} are hours and capital used as inputs in period j in sector i; f_{ij} is the production function for sector i in period j. Note that the state of nature index has been suppressed.

Lemma 1: A solution to (P-3) exists.

Proof: This follows directly from the Weierstras theorem. //

It is clear that any solution to (P-3) will satisfy $k^{\ell}_{ij} = 1 = K_{ij}$, $i,j,\ell = 1,2$ and that the upper constraints on consumption will be binding. It follows that we can consider the following problem:

$$\text{(P-4)} \quad \max_{h_{ij},H_{ij},} \quad f_{11}(1,H_{11}) + f_{12}(1,H_{12}) + f_{22}(1,H_{22}) + \phi[v(-h_{11}) + v(-h_{12})] + (1-\phi)[v(-h_{22}) + v(0)-m]$$

$$\text{s.t.} \quad 0 \le h_{ij} \le 1 \quad (i,j) = (1.1),(1.2),(2.2)$$

$$0 \le \phi \le 1$$

$$0 \le H_{1j} \le \phi h_{ij} \quad j=1,2$$

$$0 \le H_{22} \le (1-\phi)h_{22}$$

Define $f_{ij}(\cdot) \equiv f_{ij}(1,\cdot)$, for $(i,j) = (1.1),(1.2),(2.2)$

Now fix $\mu_1 > 0$, $\mu_2 > 0$, $\mu_3 > 0$ and consider the following problem:

$$\text{(P-5)} \quad \max_{H_{ij},h_{ij},\phi} \quad f_{11}(H_{11}) + f_{12}(H_{12}) + f_{22}(H_{22}) + \phi[v(-h_{11})+v(-h_{12})] + (1-\phi)[v(-h_{22}) - v(0) - m] + \mu_1[\phi h_{11}-H_{11}] + \mu_2[\phi h_{12}-H_{12}] + \mu_3[(1-\phi)h_{22} - H_{22}]$$

$$\text{s.t.} \quad 0 \le \phi \le 1$$

$$0 \le h_{ij} \le 1 \quad i,j = 1,2$$

$$0 \le H_{ij} \le 2 \quad i,j = 1,2$$

We now proceed to study this problem in some detail.

<u>Lemma 2</u>: Problem (P-5) has a solution.

<u>Proof</u>: Follows directly from Weierstras theorem. //

Lemma 3: Problem (P-5) has (essentially) unique solutions for h_{11}, h_{12}, H_{11}, H_{12}, H_{22}, where essentially unique includes the possibility that if $\phi = 0$ or $\phi = 1$ then certain variables are irrelevant.

Proof: Let $(h_{11}, h_{12}, h_{22}, H_{11}, H_{12}, H_{22},)$ be a solution to (P-5). Then the following conditions must be satisfied:

(1) $\phi v'(-h_{11}) + r_{11} \geqq \phi\mu_1$, equality if $h_{11} \neq 0$

(2) $\phi v'(-h_{12}) + r_{12} \geqq \phi\mu_2$, equality if $h_{12} \neq 0$

(3) $(1-\phi)v'(-h_{22}) + r_{22} \geqq \phi\mu_3$, equality if $h_{22} \neq 0$

(4) $f'_{11}(H_{11}) - s_{11} \leqq \mu_1$, equality if $H_{11} \neq 0$

(5) $f'_{12}(H_{12}) - s_{12} \leqq \mu_2$, equality if $H_{12} \neq 0$

(6) $f'_{22}(H_{22}) - s_{22} \leqq \mu_3$, equality if $H_{22} \neq 0$

(7) $r_{ij} \geq 0$, $s_{ij} \geq 0 \quad i,j = 1,2$

(8) $(1-h_{ij})r_{ij} = 0$, $(2-H_{ij})s_{ij} = 0$, $i,j = 1,2$.

Consider condition (1). Assume that $\phi \neq 0$. Then we have

$$v'(-h_{11}) + \frac{r_{11}}{\phi} \geqq \mu_1$$

Now, if $h_{11} = 0$, then $r_{11} = 0$, and the above becomes $0 \geqq \mu_1$ which is a contradiction. Hence $h_{11} \neq 0$. Alternatively, $h_{11} \neq 1$, since as $h_{11} \to 1$ the LHS of (1) approaches positive infinity and hence

$$v'(h_{11}) = r_{11}/\phi + \mu_1$$

cannot hold for r_{11} finite. Hence, $h_{11} \neq 1$ and thus $r_{11} = 0$. It follows that h_{11} is the unique solution to

$$v'(-h_{11}) = \mu_1$$

If $\phi = 0$ then the value of h_{11} is irrelevant and hence we can still take h_{11} to satisfy

$$v'(-h_{11}) = \mu_1$$

Similar arguments hold for the other cases. //

In the proof of lemma three it was observed that the unique solutions for h_{11} and H_{11} depended only upon μ_1, those for h_{12} and H_{12} depended only on μ_2, and those for h_{22} and H_{22} depended only upon μ_3. Let $h^*_{11}(\mu_1)$, $h^*_{12}(\mu_2)$, $h^*_{22}(\mu_3)$, $H^*_{11}(\mu_1)$, $H^*_{12}(\mu_2)$, $H^*_{22}(\mu_3)$ be functions denoting these values. It follows from the proof that the following conditions are satisfied:

$$\lim_{\mu \to 0} h^*_{ij}(\mu) = 0$$

$$\lim_{\mu \to \infty} h^*_{ij}(\mu) = 1$$

and each $h^*_{ij}(\cdot)$ is strictly increasing. Also:

$$\lim_{\mu \to 0} H^*_{ij}(\mu) = 2$$

$$\lim_{\mu \to \infty} H^*_{ij}(\mu) = 0$$

and if $H^*_{ij}(\mu) < 2$ then H_{ij} is strictly decreasing. If $H^*_{ij}(\overline{\mu}) = 2$ then $H^*_{ij}(\mu) = 2$ for all $\mu \leq \overline{\mu}$.

We have established that the solutions for h_{11}, h_{12}, h_{22}, H_{12}, H_{22} are essentially unique. Now, note that given these values the objective is linear in ϕ. Hence, the following is immediate:

Lemma 4: Define $g(\mu_1,\mu_2,\mu_3)$ by

$$g(\mu_1,\mu_2,\mu_3) = v(-h^*_{11}(\mu_1)) + v(-h^*_{12}(\mu_2)) - v(-h^*_{22}(\mu_3)) - v(0) + m + \mu_1 h^*_{11}(\mu_1) + \mu_2 h^*_{12}(\mu_2) - \mu_3 h^*_{22}(\mu_3) .$$

Then,

(i) $g(\mu_1,\mu_2,\mu_3) > 0$ implies $\phi = 1$

(ii) $g(\mu_1,\mu_2,\mu_3) < 0$ implies $\phi = 0$

(iii) $g(\mu_1,\mu_2,\mu_3) = 0$ implies $\phi \in [0,1]$.

The following will be useful:

Lemma 5: $g(\mu_1,\mu_2,\mu_3)$ is increasing in μ_1,μ_2 and decreasing in μ_3.

Proof: Differentiate g with respect to μ_1 to get

$$g_1(\mu_1,\mu_2,\mu_3) = -v'(-h^*_{11}(\mu_1))h^{*\prime}_{11}(\mu_1) + \mu_1 h^{*\prime}_{11}(\mu_1) + h^*_{11}(\mu_1)$$

But, by definition, $v'(-h^*_{11}(\mu_1)) = \mu_1$, hence the above reduces to

$$g_1(\mu_1,\mu_2,\mu_3) = h^*_{11}(\mu_1) > 0.$$

Similar arguments work in the other two cases. //

Now define a correspondence $\phi^*(\mu_1,\mu_2,\mu_3)$ by:

$$\phi^*(\mu_1,\mu_2,\mu_3) = \begin{cases} \{1\} \text{ if } g(\mu_1,\mu_2,\mu_3) > 0 \\ \{t: t\in[0,1]\} \text{ if } g(\mu_1,\mu_2,\mu_3) = 0 \\ \{0\} \text{ if } g(\mu_1,\mu_2,\mu_3) < 0 \end{cases}$$

Note that ϕ^* is increasing in μ_1,μ_2 and decreasing in μ_3 in the following sense: If $n \in \phi^*(\mu_1,\mu_2,\mu_3)$ and $\bar{n} \in \phi^*(\bar{\mu}_1,\mu_2,\mu_3)$, and $\bar{\mu}_1 \geq \mu_1$, then $\bar{n} \geq n$. The following also holds:

(i) For each (μ_2,μ_3) there exists $\bar{\mu}_1 > 0$ such that $\mu_1 > \bar{\mu}_1$ implies $\phi^*(\mu_1,\mu_2,\mu_3) = \{1\}$

(ii) For each (μ_1,μ_3) there exists $\bar{\mu}_2 > 0$ such that $\mu_2 > \bar{\mu}_2$ implies $\phi^*(\mu_1,\mu_2,\mu_3) = \{1\}$

(iii) For each (μ_1,μ_2) there exists $\bar{\mu}_3 > 0$ such that $\mu_3 > \bar{\mu}_3$ implies $\phi^*(\mu_1,\mu_2,\mu_3) = \{0\}$

We can now prove the following:

<u>Lemma 6</u>: There exist unique positive values of μ_1,μ_2,μ_3 with the property that:

$$0 \in \phi^*(\mu_1,\mu_2,\mu_3)h^*_{11}(\mu_1) - H^*_{11}(\mu_1)$$
$$0 \in \phi^*(\mu_1,\mu_2,\mu_3)h^*_{12}(\mu_2) - H^*_{12}(\mu_2)$$
$$0 \in \phi^*(\mu_1,\mu_2,\mu_3)h^*_{22}(\mu_3) - H^*_{22}(\mu_3)$$

<u>Proof</u>: Fix $\mu_2 > 0$. Then, it follows from the properties of $h^*_{11}, h^*_{12}, H^*_{11}$ and H^*_{12} that there exists a unique positive value of μ_1 such that

$$\frac{H^*_{11}(\mu_1)}{h^*_{11}(\mu_1)} = \frac{H^*_{12}(\mu_2)}{h^*_{12}(\mu_2)}$$

Denote this value of μ_1 by $\mu_1^*(\mu_2)$. It follows that μ_1^* is continuous and strictly increasing. Also, $\lim_{\mu_2 \to 0} \mu_1^*(\mu_2) = 0$.

Since none of the H^*_{ij} ever takes on the value zero for positive values of the μ_i it follows that any (μ_1,μ_2,μ_3) with the desired property will have $\phi^*(\mu_1,\mu_2,\mu_3) \neq \{0\}$ and $\phi^*(\mu_1,\mu_2,\mu_3) \neq \{1\}$. Hence we can restrict our search to those (μ_1,μ_2,μ_3) such that $g(\mu_1,\mu_2,\mu_3) = 0$. Hence, consider the following relation:

$$g(\mu_1^*(\mu_2),\mu_2,\mu_3) = 0.$$

We then have the following intermediate result:

Lemma 1: For every $\mu_2 > 0$ there exists a value of $\mu_3 > 0$, $\mu_3 = \mu_3^*(\mu_2)$ such that $g(\mu_1^*(\mu_2),\mu_2,\mu_3) = 0$. Furthermore, μ_3^* is continuous, strictly increasing, and $\lim_{\mu_2 \to \infty} \mu_3^*(\mu_2) = \infty$.

Proof: From the definition of g it follows that $\lim_{\substack{\mu_2 \to 0 \\ \mu_3 \to 0}} g(\mu_1^*(\mu_2),\mu_2,\mu_3) = k > 0$. The result then follows from the fact that $g(\mu_1^*(\mu_2),\mu_2,\mu_3)$ is increasing in μ_2 and that for any $\mu_2 > 0$

$$\lim_{\mu_3 \to \infty} g(\mu_1^*(\mu_2),\mu_2,\mu_3) = -\infty.$$

This completes the proof of lemma 7. Continuing with the proof of lemma 6, consider the following function:

$$\frac{H^*_{21}(\mu_2)}{h^*_{21}(\mu_2)} + \frac{H^*_{22}(\mu^*_3(\mu_2))}{h^*_{22}(\mu^*_3(\mu_2))} - 1$$

This function is continuous, is strictly decreasing, approaches plus infinity as μ_2 tends to zero and approaches minus one as μ_2 tends to plus infinity. It follows that there is a unique value of μ_2 such that the above function is zero. Call this value $\hat{\mu}_2$. Define $\hat{\mu}_3 = \mu^*_3(\hat{\mu}_2)$, and $\hat{\mu}_1 = \mu^*_1(\hat{\mu}_2)$. Then it follows that $(\hat{\mu}_1, \hat{\mu}_2, \hat{\mu}_3)$ is the unique point with the desired property. This completes the proof. //

From this lemma it follows that problem (P-3) has a unique solution for all variables except consumption, and that this solution is the unique allocation which satisfies the standard first order conditions.

It is now a straightforward matter to show that this allocation can be obtained as a competitive equilibrium. We use $(\hat{\mu}_1, \hat{\mu}_2, \hat{\mu}_3)$ as wages, $\frac{\partial f_{11}}{\partial K}(1, H^*_{11}(\hat{\mu}_1))$, $\frac{\partial f_{12}}{\partial K}(1, H^*_{12}(\hat{\mu}_2))$, and $\frac{\partial f_{22}}{\partial K}(1, H^*_{22}, (\hat{\mu}_3))$ as rental prices of capital, and let the price of output be one in both periods. It is then necessary to prove that the solution to the consumer's problem is characterized as the unique solution to the first order conditions, since this problem is not a concave programming problem. This exercise is virtually identical to the one which we covered above and hence is omitted.

Finally, it is straightforward to apply these results to the case of problem (P-1). This completes the proof of proposition two. //

Table 1: Non-Durable Manufacturing U.S. 1955-1979[1]

	σ_e	σ_h	σ_w
Food and Kindred Products	1.51	.50	1.59
Tobacco Manufacturers	3.05	1.49	2.10
Textile Mill Products	3.79	1.78	1.97
Apparel and Other Textile Products	3.43	1.42	2.18
Paper and Allied Products	3.25	1.23	1.68
Printing and Publishing	2.29	.94	2.01
Chemicals and Allied Products	2.76	.65	1.83
Rubber and Misc. Plastics	6.60	2.18	2.14
Leather and Leather Products	3.66	1.56	1.93

[1]Source: Employment and Training Report of the President 1981.

Table 2: Durable Manufacturing U.S. 1950-1980[1]

	σ_e	σ_h	σ_w
Lumber and Wood Products	6.6	1.4	2.2
Furniture and Fixtures	5.7	1.8	2.0
Stone, Clay and Glass Products	4.8	1.2	2.2
Primary Metals	7.2	2.8	2.5
Fabricated Metal Products	6.4	1.4	2.1
Machinery Except Electrical	7.6	2.2	2.0
Electric and Electronic Equipment	8.5	1.2	1.8
Motor Vehicles and Equipment	12.5	3.6	3.2
Aircraft and Parts	14.2	1.6	1.7
Instruments and Related Products	6.2	1.5	1.9

[1]Source: Employment and Training Report of the President 1981.

Table 3: Manufacturing, Various Countries 1960-1979[0]

	σ_e	σ_h	σ_w[1]
Canada	2.88	.73	1.42
United States	3.51	1.09	1.27
Japan	3.09	1.72	3.83
Austria[2]	2.62	2.10	3.23
Denmark	3.91	2.78	2.60
France	1.46	.93	1.33
Germany	3.30	1.70	2.28
Norway	1.98	1.12	2.02
Sweden[3]	3.08	3.20	3.77
United Kingdom	1.96	1.18	2.75

[0]Source: OECD Historical Statistics 1966-1979, and BLS Handbook 1978.

[1]Wage series is 1960-1977.

[2]Only covers 1971-1979.

[3]Covers 1968-1979.

Table 4: Correlation Coefficients

	Pew	Phw	P(e+h)w	p(e/h)w
Non-Durable Man.	.56	.68	.61	-.37
Durable Man.	.19	.83	.50	-.60
Cross-Country[1]	.21	.89	.66	-.71
Cross-Country[2]	.20	.77	.58	-.66

[1]Without Japan.

[2]With Japan.

Table 5: Sectoral Flows

	64-66	69-71
	Durable Manufacturing	
Industry Dropouts	.14	.22
Industry Entrants	.24	.16
Intra-Ind. Movers	.12	.11
	Retail Trade	
Industry Dropouts	.30	.27
Industry Entrants	.17	.21
Intra-Ind. Movers	.17	.18

Table 6: Relationship Between Average Employed and Number of People Employed During a Period

Year	e	$\bar{e}$	$e/\bar{e}$
1950	68.9	58.9	1.170
1951	70.0	59.7	1.186
1952	70.5	60.3	1.169
1953	70.7	61.2	1.155
1954	71.8	60.1	1.195
1955	75.4	62.2	1.212
1956	75.9	63.8	1.190
1957	77.7	64.1	1.212
1958	77.1	63.0	1.224
1958	78.2	64.6	1.211
1960	80.6	65.8	1.225
1961	80.3	65.7	1.222
1962	82.1	66.7	1.231
1963	83.2	67.8	1.227
1964	85.1	69.3	1.228
1965	86.2	71.1	1.212
1966	88.6/86.3	73.0	1.214/1.182
1967	88.2	74.4	1.185
1968	90.2	75.9	1.188
1969	92.5	77.9	1.187
1970	93.6	78.6	1.191
1971	95.0	79.1	1.201
1972	97.0	81.7	1.187
1973	100.2	84.4	1.187
1974	100.7/101.5	85.9	1.172/1.18
1975	101.2	84.8	1.193
1976	104.2	87.5	1.191
1977	107.1	90.5	1.183

Source: BLS Handbook, 1978.

Footnotes

[1]The fact that utility is linear in consumption means that including lotteries does not improve welfare. Their only role here is to allow agents to receive identical allocations in equilibrium.

[2]This definition of equilibrium implicitly raises some troublesome questions involving a continuum of independent random variables. The reader is referred to the manuscript "The Law of Large Numbers with a Continuum of IID Random Variables", an unpublished manuscript by K. Judd for a justification.

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