Moral Hazard, Imperfect Risk-Sharing, and the Behavior of Asset Returns

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This is a revision of an earlier paper with the title "Moral Hazard, Imperfect Risk-Sharing, and the Equity Premium." I received helpful comments from colleagues at Rochester, and from participants at the NBER Summer Institute session on Credit Markets and Economic Activity.
Abstract

This paper examines the implications of imperfect risk-sharing for the behavior of asset returns. The first part of the paper motivates the imperfect risk-sharing in a static general equilibrium model with moral hazard. A two-period model is then developed, and parameters are chosen to mimic as closely as possible the level and variability of equity and risk-free asset returns, as well as the growth rate and variability of per capita consumption. Consideration of risk-free rate variability turns out to be crucial in tying down the model's implications. While the model appears unable to fit the facts precisely, it comes considerably closer than previous efforts: For example, the model is consistent with an equity premium in the 3.5 to 4 percent range. For CES utility the range of parameters that best fit the aggregate data implies that individuals behave as if they face very severe idiosyncratic risk.
MORAL HAZARD, IMPERFECT RISK-SHARING, AND THE BEHAVIOR OF ASSET RETURNS

Recent research in macroeconomics and finance has focused on what might be called "macro-anomalies" in financial markets—for example, excess volatility in stock market prices (e.g. Shiller, 1981), or the size of the premium on equity returns (Mehra and Prescott, 1985). In contrast to the much larger and older body of research on financial anomalies, which examines contradictions to comparatively weak notions of market efficiency, the macro-anomalies literature tends to test narrowly specified asset pricing models such as representative agent exchange economies with complete markets. The failures of these simple models have sparked a broad search for theories that are more consistent with the facts, a search that has extended to models with irrational or quasi-rational agents (e.g. DeLong et al., 1987), agents with non-expected utility preferences (Epstein and Zin, 1987, Weil, 1988), and models with missing markets (Mankiw, 1986).

This paper considers a model that incorporates market imperfections into asset market equilibrium, but that maintains the assumptions of rationality and optimizing behavior. The starting point is a simple incentive problem adapted from the principal-agent literature. A risk-averse agent supplies unobservable labor (or effort) to produce output. Actual output is a random variable, with expected output increasing in labor. If effort were observable, there could be complete risk-sharing (i.e. consumption would be perfectly correlated across agents), and each agent's effort choice would be socially optimal. With unobservable effort, however, complete risk-sharing will be suboptimal. In order to induce the optimal level of effort, each agent's consumption will depend in part on his own output. Thus agents will bear idiosyncratic risk, and the variance of a representative agent's consumption will be larger than the variance of per capita consumption.
The question the paper seeks to answer is how the behavior of equilibrium asset returns may be affected by such incomplete risk-sharing. Specifically, can the model simultaneously rationalize the observed levels and variability of returns on equity and (nearly) risk-free assets? While several authors (e.g. Mankiw, 1986, Ben-Zvi and Sussman, 1988) have alleged that explanations along the lines pursued in this paper are capable of rationalizing the equity premium, they have not necessarily considered other important implications of their models, particularly with respect to return variability.

Indeed a closer examination frequently leads to the replacement of one puzzle or anomaly with another. For example, Weil (1988) finds that by considering alternative preferences he can increase the predicted equity premium (slightly), but then finds that the low level of the risk-free rate emerges as a new puzzle. The results in this paper suggest that models with idiosyncratic risk can rationalize the levels of average equity and risk-free rates of return, but to do so they either require extreme assumptions about the probability distribution of individual consumption, or they predict much greater rate variability than is observed empirically. If we do not accept the extreme amount of idiosyncratic risk needed to rationalize the data, then the anomaly in the context of this model becomes: Either the equity premium is too large, or ex ante real rates are too smooth (or both!).

The plan of the paper is as follows: Section 1 provides microfoundations for imperfect risk-sharing. It describes a simple static general equilibrium with aggregate and idiosyncratic risk, the latter being a consequence of unobservable work effort. Section 2 shows that imperfect risk-sharing leads to the possibility of a greater premium on equity returns than in the corresponding model with observable effort. Sections 3 and an Appendix describe extensions to dynamic settings that allow calibration of the model to data from the U.S. economy. Section 4 concludes.
1. Microfoundations

This section develops a simple static model of an economy with a countably infinite number of agents and unobservable work effort. The agents are ex ante identical. Each is endowed with a productive opportunity and one unit of "capital", with stochastic output per unit of input. There is also an aggregate state variable $\theta \in \Theta$ that affects the distribution of each agent's output conditional on work effort. This allows output to be correlated across agents so that there is both systematic and non-systematic risk. Agents can observe $\theta$ prior to their choice of work effort.\footnote{Similar results obtain under the assumption that the aggregate state variable is not ex ante observable. The main difference is that work effort would not be state-contingent.}

Each agent $i$ chooses work effort $\ell(i) \geq 0$ for a production technology that yields stochastic output $y_i$. There are two possible realizations of $y_i$, denoted $g$ and $b$, with $g > b$. The probability distribution is

\[
(1) \quad y_i = \begin{cases} 
  g & \text{prob. } p(\ell; \theta) \\
  b & \text{prob. } 1-p(\ell; \theta)
\end{cases}
\]

where $p: \mathbb{R}^+ \times \Theta \to [0,1]$ is strictly increasing in $\ell$. The function $p(\ell, \theta)$ is also assumed to be strictly concave and twice differentiable, and satisfies $0 < p(\ell, \theta) < 1$ for $\ell > 0$. The Inada conditions $\frac{\partial p}{\partial \ell} \big|_{\ell=0} = \infty$ and $\frac{\partial^2 p}{\partial \ell^2} \big|_{\ell=\infty} = 0 \ \forall \theta$ can guarantee an interior solution for $\ell$ if that is necessary. The binomial distribution is just a simplifying assumption that is unnecessary for the qualitative results, as should be clear below (see Cole, 1987).

The state variable $\theta$ is assumed for simplicity also to have a binomial distribution on $\{0,1\}$. Consequently by the law of large numbers, aggregate output $Y$ (contingent on identical work effort choices) also has a binomial distribution:

\[
(2) \quad Y = \begin{cases} 
  G & \theta=1 \ (\text{prob. } \pi) \\
  B & \theta=0 \ (\text{prob. } 1-\pi)
\end{cases}
\]
where $G = p_1g + (1-p_1)b$, $B = p_0g + (1-p_0)b$, with $p_1$ and $p_0$ referring to $p(\ell(1); \theta=1)$ and $p(\ell(0); \theta=0)$. Thus $E(Y) = E(y) = \pi G + (1-\pi)B$, while $\sigma^2_Y = \pi(1-\pi)(G-B)$, $\sigma^2_\bar{Y} = \bar{p}(1-\bar{p})(g-b)$, where $\bar{p} = \pi p_1 + (1-\pi)p_0$.

We can set up a social planner's problem to see how the tradeoff between risk-sharing and work effort is solved. Since both the $g$ and $b$ outcomes have positive probability for any $\ell$ regardless of $\theta$, there is no room for non-pecuniary penalties or "grim" punishment strategies. There is just a simple tradeoff between risk-sharing and the incentive to work. The planner chooses sharing rules so as to maximize the expected utility of the representative agent. The rules can be contingent on observables, which include the individual's output and aggregate output. In general the optimal arrangement is to have the individual's consumption depend on his own output and on the aggregate, that is:

$$(3) \quad c^i = f(y^i, Y),$$

with $f$ increasing in both of its arguments. In the $2 \times 2$ state case considered here, we can describe the optimal arrangement without loss of generality in terms of state-contingent linear sharing rules:

$$(4) \quad c^i_\theta = \alpha_\theta y^i + (1-\alpha_\theta)Y \quad \theta = 1,0.$$

Thus the social planner's problem is to choose $\alpha \equiv (\alpha_1, \alpha_0)$ to maximize

$$(5) \quad E[u(c, \ell)] = \pi[p_1u(\alpha_1g+(1-\alpha_1)G) + (1-p_1)u(\alpha_1b+(1-\alpha_1)G) - v(\ell_1)] + (1-\pi)[p_0u(\alpha_0g+(1-\alpha_0)B) + (1-p_0)u(\alpha_0b+(1-\alpha_0)B) - v(\ell_0)]$$

subject to
(6) \( G = p_1 g + (1-p_1) b, \ B = p_0 g + (1-p_0) b \)

(7) \( p_1 = p_1(\ell_1), \ p_0 = p_0(\ell_0) \)

(8) \( \ell_0^* \in \arg \max_{\ell > 0} p_0 u(\alpha_0 g + (1-\alpha_0) G) + (1-p_0) u(\alpha_0 b + (1-\alpha_0) G) - v(\ell) \quad \theta = 1,0 \)

where subscripts 1 and 0 indicate values conditional on that realization of \( \theta \).

Note that the social planner takes account of \( \alpha \)'s effect on \( \ell \) (and thus on \( G, B, p_1, \) and \( p_0 \)) whereas each agent takes \( \alpha, B, \) and \( G \) as given in deciding how much to work. Thus if \( \alpha_0 = 0 \) then agents' consumption levels would be independent of work effort in state \( \theta \), so each would choose \( \ell_0 = 0 \). The first-order conditions for an agent's choices of \( \ell \) are

(9) \( p_1(\ell)[u(\alpha_1 g + (1-\alpha_1) G) - u(\alpha_1 b + (1-\alpha_1) G)] \leq v'(\ell_1) \)

(10) \( p_0(\ell_0)[u(\alpha_0 g + (1-\alpha_0) B) - u(\alpha_0 b + (1-\alpha_0) B)] \leq v'(\ell_0) \),

with a strict inequality implying \( \ell_0^* = 0 \) in either case. These condition make clear that \( \alpha_0 = 0 \) cannot be the optimum unless \( \ell_0^* = 0 \). It can also be shown that under suitable regularity conditions both \( \alpha_1 \) and \( \alpha_0 \) are positive and less than one, but are not generally equal. Conditions (9) and (10) imply values of \( \alpha_1 \) and \( \alpha_0 \) contingent on desired levels of work effort \( \ell_1 \) and \( \ell_0 \). Clearly the greater the desired level of work effort in a given state, the greater must be the spread between consumption in the \( g \) and \( b \) outcomes, and hence the larger the value of \( \alpha \). If effort were unproductive in one state then \( \alpha \) could equal 0 for that state.

As an example, suppose \( u(c) = \log(c) \), \( v(\ell) = \ell^{1+\delta}/(1+\delta) \), and \( p(\ell, \theta) = s(\theta) k\ell/(1+k\ell) \), where \( 0 \leq s(\theta) \leq 1, \ k \geq 0 \). This specification for \( p \) satisfies all of the requirements, though it does not satisfy the Inada condition at \( \ell = 0 \). Note that as
\( t \) goes to infinity, \( p \) goes to \( s(\theta) \). The marginal expected product of \( t \) is \( (g-b)sk/(1+kt)^2 \). This example does not yield closed form solutions, but numerical results are provided in Table 1 for various values of \( k \) and \( \delta \) under the assumption that \( s(1)=1, s(0)=0.75 \). Although it is hard to say what parameters are realistic, the relationship between the endogenous variables and \( s, k \) and \( \delta \) seems reasonable. Note that the marginal disutility of work effort is decreasing in \( \delta \) at the equilibrium values of \( t \). Thus \( t \) increases with \( \delta \), while \( \alpha \) does not need to be as high (there is less of an incentive problem at any given level of \( t < 1 \)). All three quantities—\( t, \alpha, \) and \( p \)—are "procyclical". Also, making \( k \) as well as \( s \) contingent on \( \theta \) could generate a richer variety of cyclical patterns.

[Table 1 here]

The basic intuition underlying the model is that risk-sharing results in a free-rider problem: The more risk-sharing that takes place, the less incentive for any one individual to provide work effort. Even though agents bear some risk under complete risk-sharing because of the presence of aggregate disturbances, it always pays to make their consumption somewhat more positively correlated with their own output so as to induce the optimal (second best) amount of work effort.

2. Equilibrium Asset Returns

We can describe the optimal contingent consumption allocations in terms of agents’ portfolios. The expected return on the "market" \( E(R_M) \) in this case will just be \( E(Y) \). Suppose agents hold a state-contingent share in their own project of \( \alpha_0 \), with the rest of their claims in some combination of the market and a risk-free asset. Then agent i’s return (consumption) will be
\begin{equation}
\begin{split}
c_i &= \begin{cases} 
\alpha_i y_i + (1-\alpha_i)[sG + (1-s)\rho] & \theta=1 \\
\alpha_0 y_i + (1-\alpha_0)[sB + (1-s)\rho] & \theta=0
\end{cases}
\end{split}
\end{equation}

where \( \rho \) is the certain return on the risk-free asset. The risk-free asset is in zero net supply, so in equilibrium we must have \( s = 0 \).

The next step is to determine equilibrium values of the risk premium as a function of \( \alpha \). To do this we can condition on \( \alpha \) and the corresponding level of work effort that comes from the planner's problem in the previous section. We need to find the value for \( \rho \) such that the expected utility-maximizing choice for \( s \) is zero. Suppose utility has the constant elasticity form \( u(c) = c^{1-\gamma}/(1-\gamma) \), and let \( z_\theta = \alpha_0 y_i + (1-\alpha_0)[sY + (1-s)\rho] \). Then expected utility as a function of \( s \) is

\begin{equation}
E(z^{1-\gamma}/(1-\gamma) = \\
\pi \left[ \frac{P_1}{1-\gamma} [\alpha_g + (1-\alpha_i)[sG + (1-s)\rho]]^{1-\gamma} + \frac{1-P_1}{1-\gamma} [\alpha_i b+(1-\alpha_i)[sG+(1-s)\rho]]^{1-\gamma} \right] \\
(1-\pi) \left[ \frac{P_0}{1-\gamma} [\alpha_0 g + (1-\alpha_0)[sB + (1-s)\rho]]^{1-\gamma} + \frac{1-P_0}{1-\gamma} [\alpha_0 b+(1-\alpha_0)[sB+(1-s)\rho]]^{1-\gamma} \right]
\end{equation}

Evaluation of the first-order condition at \( s = 0 \) leads to the following expression for the equilibrium risk-free rate \( \rho^* \):

\begin{equation}
\rho^*(\alpha) = E(z^{-\gamma}Y)/E(z^{-\gamma})
\end{equation}

The expected return on the market is simply

\begin{equation}
E(R_M) = E(Y) = \pi G + (1-\pi)B,
\end{equation}
with a standard deviation \( \sigma_M = [\pi(1-\pi)]^{0.5}(G-B) \). From (13) and (14) we get that the equity premium is

\[
E(R_M - \rho^*) = E(Y) - E[z^{-\gamma}Y]/E[z^{-\gamma}]
\]
\[
= -\text{Cov}(z^{-\gamma}, Y)/E[z^{-\gamma}]
\]

This has the standard interpretation: The more correlated is a risky asset's return with marginal utility, the lower is its premium relative to the return on a risk–free asset.

The next step is to examine the quantitative effects of idiosyncratic risk on the behavior of asset returns. In order to have a model that generates endogenous risk–free and equity rates of return, and to incorporate dynamic features of the data such as serial correlation of consumption growth, it is necessary to go at least to a two–period model. This is done in the next section.

3. Multi–Period Analysis

The static model considered in Section 2 is instructive, but is not very useful for comparing the predictions of the model to real world data. The expected return on equity is exogenous, and the effects of serial correlation and other inherently dynamic factors cannot be examined. This section of the paper extends the analysis to a two–period setting. For simplicity it is assumed that idiosyncratic risk is present only in the second period. An appendix considers an infinite horizon overlapping generations model.

Now suppose agents live for two periods. Each is endowed with a unit of human (i.e. non–consumable, non–producible) capital, and in the first period produces a random exogenously determined quantity of the perishable consumption good. For
simplicity it is assumed that output is the same for all agents in the first period. At the end of the first period, each agent decides how much to consume and how to allocate his portfolio, which consists of claims on the production of his and the other agent’s output. He then works and produces output in the second period as described in the previous section. The endogenously determined price of capital and risk-free rate of return ensure that in equilibrium each agent consumes his entire output in the first period and has zero demand for the risk-free asset in the second period.

Upon learning the output level in the first period (denoted $y_1$), agent $i$ solves the following problem:

$$
\max \limits_{c_1^i, c_2^i} E\left\{ \frac{1}{1-\gamma} [c_1^{1-\gamma} + \beta c_2^{1-\gamma}] \right\}
$$

subject to

$$
c_2^i = (Q + y_1 - c_1^i)(\alpha_0 y_2^i/Q + (1-\alpha_0)(s Y_2/Q + (1-s) \rho)]
$$

where $Q$ is the value of a full share of an agent’s human capital.

In equilibrium $Q$ and $\rho$ must be such that at the optimum $y_1^i = c_1^i$ and $s=1$. We can solve for the equilibrium values as was done in the previous section by substituting the equilibrium conditions into the first-order conditions for the maximization problem, and then solving for $Q$ and $\rho$. Let $Y_2$ denote second period per capita output, and let $z_2 = \alpha_0 y_2^i + (1-\alpha_0)(s Y_2 + (1-s) \rho Q]$. We have

\[^{2}\text{Idiosyncratic risk in first-period output leads to an asymmetric distribution of wealth going in to the second period, which complicates the model without affecting the main results. This is the reason for not considering a model with infinitely lived agents.}\]
\begin{align}
Q^*(y_t) &= \beta y_t \gamma E_t(z_t^{-\gamma}) \\
\rho^*(y_t) &= E_t(z_t^{-\gamma} Y_2)/Q^*(y_t)E_t(z_t^{-\gamma})
\end{align}

where \(E_t\) denotes the expectation conditional on period 1 information, and \(z_2\) is evaluated at \(s=1\). Thus the risk-free rate differs from that found earlier only in that it is divided by the price of capital. The price of equity is the two-period analog to the price derived in Lucas (1978). If output in period 2 were uncorrelated with output in period 1, \(Q^\ast\) would be increasing in \(y_t\). High current output induces agents to want to purchase capital rather than consume, which increases the equilibrium price of capital and lowers its expected return. The risk-free rate would move in the same direction as the expected return to capital in response to current output.

The expected return on the market portfolio conditional on period 1 information now becomes

\begin{equation}
E_t(R_M) = E_t(Y_2)/Q^*
\end{equation}

while the conditional equity premium is

\begin{equation}
E_t(R_M - \rho^*) = [E_t(Y_2) - E_t(z_t^{-\gamma} Y_2)/E_t(z_t^{-\gamma})]/Q^*
\end{equation}

\[
= \frac{-\text{Cov}(z_t^{-\gamma}, Y_2)}{E(z_t^{-\gamma})Q^*}.
\]

where the covariance and the expectations are conditional on \(y_t\) and \(Q^\ast\).

To get unconditional expected returns we need to move back one step in time
and specify the probability distribution of period 1 output. One natural assumption to make is that \( y_1 \) has the same distribution as \( Y_2 \). The unconditional expected returns on the market and the risk-free rate are

\[
(22) \quad E(R_M) = E(Y_2/Q^*)
\]

\[
(23) \quad E(\rho^*) = E(E_t[\gamma Y_2]/Q^*E_t[\gamma])
\]

If there is serial correlation in output these expressions will not have any obvious relation to those found in the previous section.

The next step is to specify the process for aggregate output. In keeping with the growth rate interpretation in the previous section, suppose we are measuring output relative to some initial level \( Y_0 = 1 \). Aggregate output in the first period \( Y_1 \) (which is the same as individual output \( y_1 \)) is equal to \( G \) with probability \( \pi \) and \( B \) with probability \( 1-\pi \) as before. Second period output is not independent of \( Y_1 \), though. Following Mehra and Prescott, output growth is assumed to be a Markov process in the sense that \( Y_{t+1} = Y_t \cdot X_{t+1} \), \( t=0,1 \), where

\[
(24) \quad X_2 = \begin{cases} 
G & \Pr. \quad \varphi(G, X_1) \\
B & \Pr. \quad \varphi(B, X_1)=1-\varphi(G, X_1)
\end{cases}
\]

Positive serial correlation is implied if \( \varphi(G,G) > \pi, \varphi(B,B) > 1-\pi \). With this specification calculation of the expected equity premium is straightforward.

It turns out, however, that introducing two periods in this way does not affect the results very much. For example, calculations with \( \gamma=3, \pi=0.5 \), \( \varphi(G,G) = \varphi(B,B) = 0.72 \) (implying first-order serial correlation of output growth equal to 0.44), and \( \beta=0.96 \) yielded the following: An equity premium of 0.28 percent with
complete risk-sharing ($\alpha=0$), and premia ranging from 0.21 percent up to 1.29 percent for $(p_0,p_1)$ combinations considered in Table 2. These are slightly lower but qualitatively similar to the earlier results.

The two-period model does, however, yield predictions of the risk-free and equity returns separately, as well as some indication of rate variability. Results from these simulations are given in Table 2 for various parameter settings, with $\beta$ set at 0.96. The following general conclusions can be drawn from the table. First, consideration of risk-free rate variability$^3$ indicates overwhelmingly that we should limit the focus to values of $\gamma < 5$. Second, for parameters with reasonable rate variability, the only specifications that produce equity premia on the order of 1 percent or higher, and rate levels of the approximately the right magnitude are those with very extreme implications for the behavior of individual consumption. Specifically it is those specifications that imply some possibility of near zero consumption in the worst state ($y=b$, $Y=B$). In other words, to get large equity premia, high values of $\gamma$ imply too much rate variability, while low values require extreme idiosyncratic risk in individual consumption.

[Table 2 here]

The reason higher values of $\gamma$ imply more rate volatility in equilibrium is that $\gamma$, in addition to being the coefficient of relative risk aversion, is also the inverse of the intertemporal elasticity of substitution. High values of $\gamma$ imply relative

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$^3$The measure of variability of the risk-free rate is the theoretical standard deviation with respect to uncertain first-period output. With a given set of parameters the risk-free rate can take on two values depending on the realization of first-period output. Thus the measure of rate variability is the standard deviation assuming first-period output is high or low with probability one-half each. While this does not correspond exactly to observed variability of rates, simulations with more elaborate dynamic models as described in the Appendix yielded similar results. For equity returns the measure also takes into account ex post variability.
unwillingness to substitute consumption in the future for consumption today. Thus fluctuations in current output require greater fluctuations in interest rates the larger is $\gamma$.

The next step was to conduct a more thorough (though not completely exhaustive) grid search over $(\beta, \gamma, p_0, p_1, \alpha)$, using results from the earlier simulations for guidance as to what regions of the parameter space to look. The desired criteria were as follows:

1. An average risk–free rate of less than 4 percent.
2. An average expected return on equity greater than 3.5 percent.
3. A standard deviation of the risk–free rate of between 2.5 and 9 percent.
4. An average equity premium of at least 3 percent.

These are minimal criteria that come from examination of historical realized rates of return on the stock market and short–term government securities. As reported by Mehra and Prescott, the historical average return on equity from 1889 to 1978 was 6.98 percent with a standard deviation of 16.54 percent, while the average return on "relatively" risk–free assets was 0.80 percent with a standard deviation of 5.67 percent. Variability of equity returns is not part of the list of criteria because the observed standard deviation is affected by things like leverage that are not allowed for in the model.⁴

The reason large equity premia seem to require some probability of idiosyncratic "catastrophe" is that the premium is determined largely by the effect of equity holdings on consumption at the low end of the probability distribution. A large equity premium arises when the holding of equity potentially brings one much nearer

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to zero consumption than holding the risk-free asset, i.e. when the effect on marginal utility from holding equity rather than the risk-free asset is large in some states of the world. For parameters that satisfy the other criteria (1–3) this only occurs when there is some chance of an individual's consumption falling by 95 percent or more. Perhaps empirical analysis of micro data can shed some light on whether this is plausible, but as followers of "peso problems" are no doubt aware, it is difficult on the basis of finite data sets to reject hypotheses that depend on the presence of low probability events.\textsuperscript{5} So if we are to accept the idiosyncratic risk explanation of the behavior of asset returns, we must maintain the somewhat untestable hypothesis that asset returns behave "as if" agents (1) believe that they face at least a small possibility of severe drops in consumption; and (2) believe that such a possibility is made worse (or more likely) by the holding of equity.\textsuperscript{6}

The reason larger equity premia are associated with smaller values of $\gamma$ is that for nearly logarithmic utility the equilibrium rates of return are very insensitive to the specification of idiosyncratic risk. Therefore it is possible to increase idiosyncratic risk up to the point that a large equity premium is predicted without sending the equilibrium average levels of the returns out of bounds. In fact, the equity premium is essentially unbounded for logarithmic utility because risk does not affect the equilibrium equity return. Thus as idiosyncratic risk increases, the risk-free rate falls, implying a larger equity risk premium. The predicted standard deviation of the risk-free rate under logarithmic utility, however, seems invariably to

\textsuperscript{5}Shmuel Ben–Zvi and Oren Sussman refer to Hall and Mishkin's (1982) finding that the standard deviation of individual permanent income is about 10 percent of average permanent income in their study of PSID data. Although this is obviously consistent with the possibility of greater than 90 percent drops in consumption from one year to the next, nonetheless it suggests that the assumption of idiosyncratic risk of such a huge magnitude should be regarded with suspicion.

\textsuperscript{6}The extreme skewness of the distribution of idiosyncratic risk does not play an important role. It is just a consequence of the assumption of binomial distributions. Ben–Zvi and Sussman (1988) found in a similar model that moving from a binomial to a lognormal distribution did not affect the results very much.
be less than 1.5 percent, which is substantially lower than the 5.67 percent observed historically. While some of that differential could be attributed to the fact that the assets used to proxy for the risk-free asset are not really risk-free, it seems unlikely—given the historical behavior of inflation in the U.S.—that the small amount of risk associated with short-term nominally denominated securities could account for such a difference.

Higher values of $\gamma$ imply larger equity premia and greater variability of the risk-free rate for a given amount of risk, but consideration of the levels of the returns does constrain the extent of idiosyncratic risk. In fact the predicted average returns can be extremely sensitive to small changes in idiosyncratic risk, the more so the higher the value of $\gamma$. As we will see, to get the level of the risk-free rate below 4 percent and the variability between 2.5 and 9 percent seems to require $\gamma$ to be between about 1.1 and 1.6.

Table 3 presents the outcome of this search. These results demonstrate that with no a priori restrictions on idiosyncratic risk the model can come fairly close to matching the data. No parameters were found, however, that literally produced the historical values referred to above. Nonetheless it seems reasonable to conclude that idiosyncratic risk helps considerably in the task of simultaneously rationalizing the level and variability of equity and risk-free rates of return.

[Table 3 here]

Note that most of the cases shown in the table have $\beta$ somewhat lower than is conventionally assumed. The reason for this is that rates of return are larger for lower $\beta$ at any given level of idiosyncratic risk, while they tend to decrease for a given $\beta$ as idiosyncratic risk increases (at least for $\gamma > 1$). Thus a smaller value of $\beta$ permits more idiosyncratic risk to be consistent with a given average rate of return.
There are small modifications of the model that would undoubtedly imply less extreme behavior of idiosyncratic risk. For example, instead of assuming single-period utility is $c^{1-\gamma}/(1-\gamma)$ we could assume something of the form $(c-c^*)^{1-\gamma}/(1-\gamma)$, where $c^* > 0$ is some subsistence level of consumption. This would reduce the severity of the potential drops in consumption needed to bring about the same kinds of equilibrium asset return behavior found above.

4. Conclusions

A number of authors have suggested that consideration of idiosyncratic risk can help to explain asset return anomalies. This paper has considered a specific model of idiosyncratic consumption risk and developed its implications for equilibrium asset returns. A calibration effort that took into account not just the equity premium but also the average level and variability of the return on equity and on the risk-free asset indicated that to reconcile the data and the model requires assuming that individuals behave as if they bear severe idiosyncratic risk, i.e. at least a small probability of consumption falling to near subsistence level. The importance of considering rate variability as part of the analysis was also demonstrated, since ignoring it allows one to generate high equity premia without such extreme idiosyncratic risk by letting the risk aversion coefficient be in the 5 to 10 range. Allowing for rate variability clearly restricts the risk-aversion coefficient to be in the 1.1 to 1.6 range.

It is interesting to compare these findings with those of Rietz (1988), who points out that in the Mehra–Prescott model if one posits the presence of a low probability "crash" state the equity premium increases significantly. Mehra and Prescott (1988) respond by pointing out that, among other things, to rationalize the observed equity premium requires allowing for implausibly large (and historically unprecedented) drops in aggregate consumption. It is also probably the case that
Rietz's assumptions imply much greater risk-free rate variability than has been observed historically. In this model, aggregate consumption per capita can be very well-behaved (i.e. essentially the same distribution assumed by Mehra and Prescott, with no possibility of large unprecedented declines), yet because of imperfect risk-sharing, the equity premium can be significantly larger than with complete information, even after constraining the variability of the risk-free rate to be in line with historical behavior.

A more complete resolution of the puzzles surrounding asset return behavior will undoubtedly require more than one simple modification of the the representative agent model. Thus it would be useful to consider idiosyncratic risk along with other extensions of the Mehra–Prescott model that have been investigated. For example, Weil's (1988) use of non-time-separable preferences to separate the risk aversion and intertemporal substitution parameters predicted a somewhat larger equity premium, but it also implied that the risk-free rate is "too low". It is possible that idiosyncratic risk (which tends to lower the equilibrium risk-free rate) could combine with other modifications such as Weil's to resolve the remaining puzzles. It may also be useful to consider other types of market imperfections. The imperfection considered in this paper is purely intratemporal, whereas it may be that limitations on some types of intertemporal trade may also help to explain the data.
References


Appendix: An Overlapping Generations Economy

This appendix describes an extension to a more fully dynamic context by specifying an overlapping generations structure. Suppose each generation has an infinite number of two-period-lived agents. At the beginning of each time period agents currently alive buy and sell equity shares (claims on their outputs) and a risk-free asset. For simplicity it is assumed that the equity shares are only one-period claims (i.e. they have no value after the dividend is paid). At the end of each time period members of the younger generation receive an identical, exogenous, stochastic endowment $W_t$ of the consumption good, while the members of the older generation realize the stochastic outputs $y_t^i$ from productive activity. Because of moral hazard agents in the older generation bear idiosyncratic risk as in the two-period model by holding more than an infinitesimal stake in their own project. Aggregate per capita output of the older generation is denoted $Y_t$. All output is perishable, so equilibrium requires zero aggregate demand for the risk-free asset and aggregate consumption demand equal to aggregate output.

I assume as in Section 3 that contingent on work effort the older generation's individual output realizations $\{y_t\}$ are the composition of a Markov Process in a single aggregate state variable $\theta_t$ and a stochastic disturbance as in (1). I also assume for simplicity that $W_t = Y_t$. Even so there may be intergenerational trade in shares—not for the purpose of diversification, since by the assumption of perfect correlation there are no benefits from that, but because one generation may be net holders of the risk-free asset.

At the end of the period the each member of the older generation will consume his end-of-period wealth, which consists simply of the dividends from his equity and

\footnote{Otherwise shares of the younger generations output would have value at the end of the period and would have to be accounted for. While this assumption makes equity different from equity as we know it, the equilibrium asset returns should not be affected.}
risk-free asset holdings. The young have consumption and saving decisions in that they may buy or sell second-period shares in exchange for current consumption, but as before the price of their shares in equilibrium will be such that no such trades take place. They can only trade amongst themselves, since the old have nothing to offer but consumption and have no interest in shares that pay dividends after they are gone.\textsuperscript{8}

We can start with the problem of a representative member of the older generation at the beginning of period $t$, before $\theta_t$ is known. He knows he will have to hold a stake $\alpha_t(\theta_t)$ in his own firm, and can invest in a risk-free asset that has a price of $P_t$ and a certain dividend of one unit of the consumption good at the end of the period. His wealth equals the value of one full equity share, which is denoted by $Q_t$. His problem is

\begin{equation}
(A.1) \quad \max_{s_t^2} \text{Eu}(\alpha_0 y_t^1 + (1-\alpha_0)[s_t^2 Y_t + (1-s_t^2)(Q_t/P_t)])
\end{equation}

where $s_t^2$ is the share of his discretionary wealth that he puts into the market, and the expectation is taken with respect to $Y_t$. Suppose that $Y_t$ is distributed as in (2). Then the first order condition for this problem is

\begin{equation}
(A.2) \quad E\{(Y_t-\rho_t)u'(\alpha_0 y_t + (1-\alpha_0)[s_t^2 Y_t + (1-s_t^2)Q_t \rho_t])\} = 0.
\end{equation}

where $\rho_t = 1/P_t$. This gives a solution for $s_t^2$ as a function of $\rho_t$, $Q_t$, $\alpha_0$, and the distributional parameters.

The young, on the other hand, do not face idiosyncratic risk. They solve the

\textsuperscript{8}Claims on subsequent generations' endowments or output are assumed unenforceable. Thus the current young generation cannot give consumption to the current old generation in return for a claim on the endowment of next period's younger generation. Issues relating to dynamic efficiency are not of interest in the context of this paper.
two-period problem

\[(A.3) \quad \max_{s_t^1, s_{t+1}^2, c_t^1, c_{t+1}^2} E\{u(c_t^1) + \beta u(c_{t+1}^2)\}\]

subject to

\[(A.4) \quad c_{t+1}^2 = (Q_{t+1} + s_t^1 Y_{t+1} + (1-s_t^1)\rho_t Q_t - c_t^1)\times\]
\[\quad \left(\alpha_0 y_{t+1}^2 / Q_{t+1} + (1-\alpha_0) [s_{t+1}^2 Y_{t+1}^2 / Q_{t+1} + (1-s_{t+1}^2)\rho_{t+1}]ight)\]

The young agent foresees that in equilibrium \(Q_{t+1}\) will be such that he will consume \(Y_t\), so the corresponding condition turns out to be

\[(A.5) \quad E\{Y_t - \rho_t u'(s_t^1 Y_t + (1-s_t^1)\rho_t Q_t)\} = 0.\]

Equations (A.2) and (A.5), together with the market-clearing condition

\[(A.6) \quad s_t^1 + (1-\alpha_0)s_t^2 = 2-\alpha_0\]

determine \(\rho_t, s_t^1, s_t^2\) as functions of \(Q_t\) and other parameters. \(Q_t\) itself is determined by the condition that \(c_t^1 = Y_t\), which turns out to be:

\[(A.7) \quad (s_t^1 Y_t + (1-s_t^1)\rho_t Q_t)^{-\gamma} = \frac{\beta}{Q_{t+1}} E\{[\alpha_0 y_{t+1}^2 + (1-\alpha_0) [s_{t+1}^2 Y_{t+1}^2 + (1-s_{t+1}^2)\rho_{t+1} Q_{t+1}]]^{1-\gamma}\},\]

which is similar to the formula described in the earlier case except that now there may be non-trivial holdings of the risk-free asset. Consequently we have the result
that even with an i.i.d. output process $Q_{t+1}$ is generally a function of $Q_t$ and $Y_t$, and therefore depends on the entire history of $Y_t$. Nonetheless, Monte Carlo simulations of the model showed that quantities such as average rates of return are very robust to initial conditions and different "random" samples.

Despite the fact that half of the population in any period is not subject to idiosyncratic risk, this model yielded qualitatively similar results to the two-period model. The reason that the presence of such agents does not wipe out the effects of idiosyncratic risk in the older generation is that in equilibrium the young accommodate by bearing more systematic risk. The old offset some of the additional risk they bear by holding some of the risk-free asset, while the young hold more than their share of the market. This additional leverage means that their consumption as well as that of the old is riskier than aggregate per capita consumption.
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This table shows theoretical asset return behavior under complete risk-sharing (α=0) and incomplete risk-sharing for a value of α such that the average return on the market is greater than 4 percent, the average risk-free rate is less than 4 percent, for several p₀ and p₁ combinations. No entry means no α was found that fit the criteria. The parameter β was set at 0.96. σ_c is the standard deviation of consumption, and %Δc is the pct. decline in consumption in the worst outcome.
Table 3: Grid Search Results

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This table displays examples of theoretical asset return behavior with parameter values for which $R_M > 3.5\%$, $-0.5\% < \bar{\rho} < 4\%$, $2.5\% < \sigma(\bar{\rho}) < 9\%$, and $R_M - \bar{\rho} > 3\%$. 
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<table>
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<tr>
<th>WP#113</th>
<th>THE ECONOMETRIC ANALYSIS OF MODELS WITH RISK TERMS</th>
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<td></td>
<td>by Adrian Pagan and Aman Ullah, December 1987</td>
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<td>PROGRAM TARGETING OPTIONS AND THE ELDERLY</td>
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<td>by Eric Hanushek and Roberton Williams, December 1987</td>
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<td>WP#115</td>
<td>BARGAINING SOLUTIONS AND STABILITY OF GROUPS</td>
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<td>MONOTONIC ALLOCATION MECHANISMS</td>
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<td>INTEREST ON RESERVES AND SUNSPOT EQUILIBRIA: FRIEDMAN'S PROPOSAL RECONSIDERED</td>
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<td>by Bruce D. Smith, February 1988</td>
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<td>INTERNATIONAL FINANCIAL INTERMEDIATION AND AGGREGATE FLUCTUATIONS UNDER ALTERNATIVE EXCHANGE RATE REGIMES</td>
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<td>by Jeremy Greenwood and Stephen D. Williamson, February 1988</td>
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<td>FINANCIAL DEREGULATION, MONETARY POLICY, AND CENTRAL BANKING</td>
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<td>by Valerie R. Bencivenga and Bruce D. Smith, March 1988</td>
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