Efficiency Bound Calculations for a Time Series Model with Conditional Heteroskedasticity

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INTRODUCTION

The purpose of this note is to present an algorithm for calculating an efficiency bound among a class of estimators for a continuous time financial economics model examined by Grossman, Melino, and Shiller (1987), Hall (1987), and Hansen and Singleton (1987) and the martingale taxation model examined by Barro (1981). In these continuous time models there are two-period conditional moment restrictions that result from time averaging the underlying continuous time processes and then sampling these averages. These conditional moment restrictions can be used to estimate an unknown parameter vector using the generalized method of moments (GMM) procedure set out by Hansen (1982). There is a vast array of GMM estimators that can be used to estimate the parameter vector. Hence it is of interest to compare the performance of the alternative GMM estimators. We apply a method developed by Hansen, Heaton, Ogaki (1988) to calculate a greatest lower bound for the asymptotic covariance matrices of the alternative GMM estimators, i.e. an efficiency bound.¹

In the continuous time models cited above, there is a martingale restriction. When only time-averaged discrete time data is available, the martingale restriction results in a two-period conditional moment restriction. This restriction leads to an econometric disturbance. Under certain assumptions this disturbance has a first-order autocorrelation which is known a priori as Working (1960) pointed out. This knowledge implies another two-period conditional moment restriction. The second moment restriction induces conditional heteroskedasticity in the disturbance. We provide an algorithm to calculate the efficiency bound for GMM estimators

¹In this paper we follow the notation of Hansen et al. (1988) as closely as possible.
when both of these conditional moment restrictions are used. Hansen and Singleton (1987) calculated the efficiency bound using our algorithm. They also calculated the efficiency bound when only the first moment restriction is used. The algorithm they used for this case exploits the analysis in an example of Hansen (1985) which assumes conditional homoskedasticity. The calculations of Hansen and Singleton showed that there is a notable efficiency gain from the imposition of the second moment restriction for their model.

**Calculating Efficiency Bounds**

Consider a q-dimensional random vector Gaussian process \( \{x_t : -\infty < t < \infty \} \). Let \( \{w_t : -\infty < t < \infty \} \) denote a q-dimensional random vector process that is fundamental for \( x_t \) in the sense of linear prediction theory. In other words, the entries of \( w_t \) are orthonormal, the process \( \{w_t : -\infty < t < \infty \} \) is serially uncorrelated, and \( w_t \) and \( x_t \) are informationally equivalent from the vantage point of linear least squares prediction [e.g. see Rozanov (1967)].

We can view that the stochastic process \( w_t \) is generated from a random variable \( w \) on a probability space \( (\Omega, A, Pr) \) in conjunction with a transformation \( S \) mapping \( \Omega \) onto \( \Omega \) in the following manner (see Breiman [1968] pp. 106-120). The stochastic process \( \{w_t : -\infty < t < \infty \} \) is constructed from the random vector \( w \sim \mathcal{N}_q \) via \( w_t(\omega) = w[S^t(\omega)] \) for \( -\infty < t < \infty \). Here \( S^t \) is interpreted as the transformation \( S \) applied \( t \) times for positive values of \( t \). In addition we can take the transformation \( S \) so that \( S \) is one-to-one, measurable, measure-preserving, and ergodic and \( S^{-1} \) is measurable. Since \( S \) is one-to-one \( w[S^t(\omega)] \) is also well-defined for negative values of \( t \). Suppose that \( h \) is a random vector on \( \Omega \). Then \( h \) in conjunction with \( S \) generates a stochastic process via \( h_t(\omega) = h[S^t(\omega)] \). Throughout this note we find it convenient to index stochastic processes constructed in this
fashion.

For notational convenience, we define \( y' = [x', x_{-1}'] \). Let \( B_t \) be the
sigma algebra generated by \( y_t, y_{t-1}, \ldots \). Since \( x \) is Gaussian, best linear
predictors coincide with conditional expectations and \( w \) is normally
distributed and independent of \( B_{-1} \).

Suppose that a particular linear combination of \( x \) is a two-period-ahead
forecast error. More precisely, \( E(u | B_{-2}) = 0 \) where \( u = [1 \beta_0 0]x \) and \( \beta_0 \)
is a scalar parameter to be estimated. In addition,

\[
u = [1 \beta_0 0]x = \nu_0 \cdot w + \nu_1 \cdot w_{-1}
\]

where \( \nu_0 \) and \( \nu_1 \) are \( q \)-dimensional vectors of real numbers. A second
restriction is that \( E(u \ u_{-1} | B_{-2})/E(u^2 | B_{-2}) = \nu_0 \cdot \nu_1 / (\nu_0 \cdot \nu_0 + \nu_1 \cdot \nu_1) = \rho \),
or equivalently that

\[
E(\rho u^2 - u \ u_{-1} | B_{-2}) = 0
\]

where \( \rho \) is .25. The law of motion for the process \( x \) can be quite
complicated and may depend on a large (or even infinite) number of auxiliary
parameters. There is only one parameter of interest, namely \( \beta_0 \), and
accordingly we focus on efficiency bounds for that parameter.

Define the disturbance vector

\[
e = \begin{bmatrix}
u \\
\rho u^2 - u u_{-1}
\end{bmatrix},
\]

and a function

\[
\phi(y, \beta) = \begin{bmatrix}
[1 \beta 0]x \\
\rho \cdot (1 \beta 0 \cdot x)^2 - (1 \beta 0 \cdot x) \cdot (1 \beta 0 \cdot x_{-1})
\end{bmatrix}.
\]

The conditional moment restrictions take the form \( E(e | B_{-2}) = 0 \) where \( e = \phi(y, \beta_0) \) is the disturbance vector. Notice that the model is nonlinear in
both the parameters and the variables even though the underlying process \( x \) is Gaussian. The entries of \( e \) are either normally distributed random variables or products of normally distributed random variables. Hence these entries have moments of all orders.

We construct estimators of \( \beta_0 \) as follows: Let \( z \) be an \( n \times k \) matrix of random variables that are measurable with respect to the time \(-2\) information set \( B_{-2} \). In addition, suppose that \( E|z|^{\eta} \) is finite for some \( \eta \); then \( E(z'e) = 0 \). We refer to a consistent estimator \((b_t: T \geq 1)\) of \( \beta_0 \) for which
\[
(1/T)^{1/2} \sum_{t=1}^{T} [z' (y_t, b_t)] : T \geq 1 \]

is \( o_p(1) \) (converges in probability to zero) as a GMM estimator with index \( z \). Such estimator can also be viewed as an \( H \) estimator and as an instrumental variables estimator where \( z' \) is a matrix of instrumental variables.

Since there is great flexibility in the selection of \( z \) we have at our disposal a rich class of estimators of \( \beta_0 \). We find it convenient to introduce an index set for a family of such estimators: Let \( B^r \) be the subsigma algebra of \( B_{-2} \) that is generated by \( x, x_{-1}, \ldots, x_{-r} \), and

\[
Z^r = \{z: z \text{ is an } n \times k \text{ matrix of random variables that are measurable with respect to } B^r \text{ and } E(|z|^{\eta}) < \infty \}. 
\]

Indexes in the space \( Z^r \) are constructed using functions of the current and \( r \) lags of the vector \( x \). The specification of the finite lag \( r \) is both arbitrary and inconvenient. For this reason, we consider the larger index set \( Z = \bigcup_{r=1}^{\infty} Z^r \). This space is a linear subspace of \( L^\eta(Pr) \) but is not necessarily closed.

Let \( d = \partial \phi(y, \beta_0)/\partial \beta, B^* = B_{-2}, \) and \( d^* = E(d|B^*) \). The matrix of random variables \( d \) is given by
d = \left[ \begin{array}{c}
[0 1 0]x \\
2\rho([1 \beta_0 0]x)([0 1 0]x) - ([1 \beta_0 0]x)([0 1 0]x_{-1}) \\
- ([0 1 0]x)([1 \beta_0 0]x_{-1})
\end{array} \right].

The entries of both d and e are either normally distributed random variables or products of normally distributed random variables. Hence these entries have moments of all orders. Let \( \eta \) be any positive number greater than 2, then Assumption 4 of Hansen et al. (1988) is satisfied.

As discussed in Hansen et al. (1988), a GMM estimator \( b_z \) indexed by z in \( Z \) has a limiting normal distribution with asymptotic covariance matrix:

\[
\text{Cov}(z) = [E(z'd^*)]^{-1}\sum_{\tau=1}^{\tau=\infty} [E(z'd^*)]'^{-1}
\]

(5) as long as \( E(z'd^*) \) is nonsingular. When \( E(z'd^*) \) is singular, we define \( \text{Cov}(z) = \infty \). Relation (5) gives a mapping \( \text{Cov} \) from the index set \( Z \) into the collection, PSD, of \( k \times k \) positive semidefinite matrices augmented by the point infinity. The set PSD can be partially ordered as follows: The inequality \( c \preceq c^* \) is satisfied for \( c \) and \( c^* \) in PSD if \( c^* - c \) is in PSD. Let LB be the subset of PSD containing all matrices \( c \) that satisfy \( c \preceq \text{Cov}(z) \) for all \( z \) in \( Z \). The efficiency bound \( \text{Inf}(Z) \) is defined to be the maximal element of LB assuming such a maximal element exists.

We calculate the efficiency bound in four steps. In the first step we obtain the conditional forward moving-average representation for e given in Lemma 4.1 of Hansen et al. (1988). Applying the lemma we know that e can be represented as \( e = \lambda_0^0 e_0^+ + \lambda_1^1 e_1^+ \), where \( \lambda_0^0 \) and \( \lambda_1^1 \) are \( 2 \times 2 \) matrix of random variables that are measurable with respect to \( B^* \) and have finite second moments, \( \lambda_0^0 \) is nonsingular almost surely, and \( E(e_0^+ e_j^+ | B^*) = 0 \) for all \( j \neq 0 \).

It turns out that e is conditionally heteroskedastic. In particular, calculating from the definition of e and relation (1),
\[ E(\epsilon \epsilon' | B^*) = \begin{bmatrix} \frac{\mu_{11}}{\chi_{11} \cdot w^{-2}} \\
 \frac{\chi_{11} \cdot w^{-2}}{\mu_{12} + \mu_{11}(\nu_1 \cdot w^{-2})^2} \\
 \frac{\chi_{11} \cdot w^{-3}}{\mu_{12} + \mu_{11}(\nu_1 \cdot w^{-2})^2} \end{bmatrix} \text{, and} \ \ (6) \]

\[ E(\epsilon \epsilon'_{-1} | B^*) = \begin{bmatrix} \frac{\mu_{21}}{\chi_{12} \cdot w^{-2} + \chi_{21} \cdot w^{-3}} \\
 \frac{\mu_{22} + \chi_{22}(\nu_1 \cdot w^{-2})^2 + \mu_{21}(\nu_1 \cdot w^{-2})(\nu_0 \cdot w^{-2})}{\mu_{22} + \chi_{22}(\nu_1 \cdot w^{-2})^2 + \mu_{21}(\nu_1 \cdot w^{-2})(\nu_0 \cdot w^{-2})} \\
 \frac{\chi_{21} \cdot w^{-2}}{\mu_{22} + \chi_{22}(\nu_1 \cdot w^{-2})^2 + \mu_{21}(\nu_1 \cdot w^{-2})(\nu_0 \cdot w^{-2})} \end{bmatrix} \text{.} \ \ (7) \]

The nonrandom \( \mu \)'s and \( \chi \)'s are functions of \( \nu \) and \( \nu' \): \( \mu_{11} = (\nu_0 \cdot \nu_1) + (\nu_1 \cdot \nu_1) \), \( \chi_{11} = -[(\nu_0 \cdot \nu_0) + (\nu_1 \cdot \nu_1)] \nu_1 \), \( \mu_{12} = (\nu_0 \cdot \nu_0)^2 - (\nu_1 \cdot \nu_0)^2 + (\nu_0 \cdot \nu_0)(\nu_1 \cdot \nu_1) \), \( \mu_{21} = \nu_1 \cdot \nu_0 \), \( \chi_{12} = 2\rho(\nu_0 \cdot \nu_1)(\nu_1 - \nu_0) \), \( \chi_{21} = (\nu_0 \cdot \nu_1) \nu_1 \), \( \mu_{22} = 2\rho[\rho(\nu_1 \cdot \nu_0)^2 - (\nu_1 \cdot \nu_0)(\nu_0 \cdot \nu_0)] \), and \( \chi_{22} = -2\rho(\nu_1 \cdot \nu_0) \).

Note that the second component of \( \epsilon \) introduces conditional heteroskedasticity. As a consequence, \( \lambda^0 \) and \( \lambda^1 \) depend on conditioning information. Suppose that \( \lambda^0 \) and \( \lambda^1 \) have the form

\[ \lambda^0 = \begin{bmatrix} \xi_{11} \\
\alpha_{11} \cdot w^{-2} \xi_{12} \end{bmatrix} \text{ and} \ \ (8) \]

\[ \lambda^1 = \begin{bmatrix} \xi_{21} \\
\alpha_{21} \cdot w^{-2} + \alpha_{22} \cdot w^{-3} \xi_{22} \end{bmatrix} \text{.} \ \ (9) \]

where the \( \xi \)'s and the \( \alpha \)'s are nonrandom. We require \(|\xi_{21}| < |\xi_{11}| \) and \(|\xi_{22}| < |\xi_{12}| \) so that \( \lambda^0 e^0_0 + \lambda^1 e^1_1 \) has the same informational content as \( \epsilon \). We can calculate the conditional autocovariances of \( \epsilon = \lambda^0 e^0_0 + \lambda^1 e^1_1 \) as a function of the parameters of \( \lambda^0 \) and \( \lambda^1 \) in equations (8) and (9). Matching these autocovariances with the conditional autocovariances calculated by the definition of \( \epsilon \) and relation (1),

6
\[(\xi_{11})^2 + (\xi_{21})^2 = \nu_0 \cdot \nu_0 + \nu_1 \cdot \nu_1, \tag{10}\]
\[\xi_{11} \xi_{21} = \nu_0 \cdot \nu_1, \tag{11}\]
\[\alpha_{11} = - \nu_1 (\nu_1 \cdot \nu_0) / \xi_{21}, \tag{12}\]
\[\alpha_{21} = (\nu_1 \cdot \nu_0)(2 \nu_1 - \nu_0) / \xi_{11}, \tag{13}\]
\[\alpha_{22} = - \nu_1 (\nu_1 \cdot \nu_0) / \xi_{11}, \tag{14}\]
\[(\xi_{12})^2 + (\xi_{22})^2 = (\nu_0 \cdot \nu_0)^2 + (\nu_0 \cdot \nu_0)(\nu_1 \cdot \nu_1) - (\nu_1 \cdot \nu_0)^2 \tag{15}\]
\[- (\xi_{21})^2(2 \nu_1 - \nu_0)(2 \nu_1 - \nu_0)\]

and
\[\xi_{12} \xi_{22} = 2 \rho^2 (\nu_1 \cdot \nu_0)^2 - 2 \rho (\nu_1 \cdot \nu_0)(\nu_0 \cdot \nu_0). \tag{16}\]

Equations (10), (11), (15) and (16) can be solved for the parameters \(\xi_{11}, \xi_{12}, \xi_{21},\) and \(\xi_{22}\) subject to the restrictions that \(|\xi_{21}| < |\xi_{11}|\) and \(|\xi_{22}| < |\xi_{12}|\). Equations (12)-(14) then determine \(\alpha_{11}, \alpha_{21},\) and \(\alpha_{22}\). Thus for these values of the parameters \(\lambda^0\) and \(\lambda^1\) in (8) and (9) give a conditional forward representation.

In the second step we calculate the operators \(\Psi\) and \(\Psi^-\) defined by Hansen et al. (1988). Let \(\psi^0 = (\lambda^0)^{-1}\) and \(\psi^j = - (\lambda^{-1})^{-1} \sum_{\tau=1}^{\tau} (\lambda^{\tau})^{-1} (\psi^{\tau+j})^{-1}\) for \(j \geq 1\) where \(\lambda^j = 0\) for \(j \geq 2\). Let \(D^+\) be the set of all \(n \times k\) matrices of random variables that are measurable with respect to \(B^+\). Let \(D\) be the subset of \(D^+\) for which \(E[(\psi^j)z_j | B^+]\) is well defined for each \(j\) and \(\Psi(z) = \sum_{j=0}^{\infty} E[(\psi^j)z_j | B^+]\) converges in \(L^2(Pr)\). Similarly let \(D^-\) be the subset of \(D^+\) for which \(\Psi^-(z) = \sum_{j=0}^{\infty} (\psi^j)z_j^{-j}\) converges in \(L^2(Pr)\). Then \(D\) and \(D^-\) are the domains of the operators \(\Psi\) and \(\Psi^-\), respectively. Using (8) and (9), \(\psi^0\) and \(\psi^j\) are given by:

\[
\psi^0 = \begin{bmatrix}
1/\xi_{11} & 0 \\
(\nu_1 \cdot \nu_2) / \xi_{12} & 1/\xi_{22}
\end{bmatrix}, \quad \text{and} \tag{17}
\]

\[
\psi_j = \begin{bmatrix}
\frac{(\varphi_1)^j}{\xi_{11}} & 0 \\
-\sum_{\tau=1}^{j} \frac{(\varphi_1)^{j-\tau}(\varphi_2)^{\tau-1}(\alpha_{21} \cdot \nu_{-\tau-1})}{(\xi_{11} \xi_{12})} & \frac{(\varphi_2)^j}{\xi_{22}} \\
+ \frac{(\varphi_2)^j(\nu_{1} \cdot \nu_{-2})}{\xi_{12}} & \frac{(\varphi_2)^j}{\xi_{22}}
\end{bmatrix}
\]  

(18)

for \( j = 1, 2, 3, \ldots \) where \( \varphi_1 = -\xi_{21}/\xi_{11} \) and \( \varphi_2 = -\xi_{22}/\xi_{12} \). Since \(|\varphi_1| < 1\) and \(|\varphi_2| < 1\), and the entries of \( w \) and \( d \) have moments of all orders, it follows that \( E(d|B^*) - d^* \) is in \( D \) and Assumption 8 of Hansen et al. is satisfied. In addition \( \Psi(d^*) \) has moments of all orders which implies that \( \Psi(d^*) \) is in \( D^- \). It follows from Corollary 4.1 of Hansen et al. that \( \text{Inf}(Z) = 1/|E[\Psi(d^*'), \Psi(d^*)]| \).

In the third step we calculate \( \Psi(d^*) \) which requires evaluating expectations of \( \psi_j d_j \) conditioned on \( B^* \) for \( j = 0, 1, 2, \ldots \). To perform these calculations requires more information about the process \( x \) than has been assumed so far. This computation is straightforward when \( x \) has a state space representation:

\[
Y = A Y_{-1} + C w \quad \text{and} \quad x = H Y,
\]  

(19)

where \( Y \) is a \( p \) by 1 state vector, \( A \) is a \( p \) by \( p \) matrix with eigenvalues that have moduli that are less one, and \( C \) and \( H \) are \( p \) by \( q \) matrices of real numbers. In this case \( \Psi(d^*) \) is given by:

\[
\Psi(d^*) = \begin{bmatrix}
Q \cdot Y_{-2} \\
R_1 + w_{-2}' R_2 Y_{-2} + Y_{-2}' R_3 Y_{-2}
\end{bmatrix}
\]  

(20)

where \( Q \) is a \( p \)-dimensional vector of real numbers, \( R_1 \) is a scalar real number, and \( R_2 \) and \( R_3 \) are \( p \) by \( p \) matrices of real numbers. The definitions for these parameters are as follows. Let \( H_1 = [1 \; \beta \; 0] H \) and \( H_2 = [0 \; 1 \; 0] H \).
Then

\[ Q = H_2 (I - \varphi_i \lambda_i)^{-1} A^2 / \xi_{11} \]

\[ R_1 = \nu_2 \varphi_i C A^3 (I - \varphi_i \lambda_i)^{-1} H_2 / \xi_{12} - \alpha_2 C A^{2} (I - \varphi_i \lambda_i)^{-1} H_2 / [\xi_{12} (1 - \varphi_i)] \]

\[ + \left\{ 2 \rho H_1 C \lambda_i H_2 '+ \sum_{j=0}^{\infty} \varphi_j A^{j+1} C \lambda_i A^{j+1} H_2 ' - H_2 \sum_{j=0}^{\infty} \varphi_j A^{j+1} C \lambda_i A^{j+1} H_2 ' \right\} / [\xi_{12} (1 - \varphi_i)], \]

\[ R_2 = \nu_1 H_2 A^2 / \xi_{12}, \text{ and} \]

\[ R_3 = \sum_{i=0}^{\infty} \varphi_i \left\{ 2 \rho A^{2+i} H_2 ' H_1 A^{2+i} - A^{2+i} H_2 ' H_1 A^{2+i} - A^{2+i} H_2 ' H_1 A^{2+i} \right\} / \xi_{12}. \]

Note that the first entry of \( \Psi(d^*) \) is a normally distributed random vector while the second entry is the translated sum of two quadratic forms of normally distributed random vectors.

In the final step we calculate \( E[\Psi(d^*)' \Psi(d^*)] \). This involves computing up to fourth moments of \( Y \) and \( w \) using formulas for normally distributed random vectors. The efficiency bound is then given by \( 1/E[\Psi(d^*)' \Psi(d^*)] \). Thus the efficiency bound turns out to be insensitive to the choice of \( \eta \), though the magnitude of \( \eta \) affects the size of the index set \( Z \).

It is also possible to estimate \( \beta \) using just the first conditional moment restriction and ignoring relation (2). There is a corresponding loss in asymptotic efficiency since the resulting efficiency bound is the reciprocal of the second moment of the first entry of \( \Psi(d^*) \). As discussed in the Introduction, Hansen and Sigleton (1987) found that this loss is substantial for their model.
References


Breiman, L. (1968), Probability, Reading, Massachusetts: Addison-Wesley.


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