Migration Between Home Country and Diaspora: An Economic Analysis

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MIGRATION BETWEEN HOME COUNTRY AND DIASPORA:
AN ECONOMIC ANALYSIS*

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ABSTRACT

This paper investigates the distribution of a population group between a
home country and diaspora, given sequential decision-making regarding
migration at the individual level. The home country is attractive to the
members of the group, yet their presence there requires a fixed amount of
public spending (e.g. on defence). The per-capita tax burden depends then on
the size of the domestic population, reflecting a case of "fiscal
externality". This results in an inefficient distribution of the group
between the home country and the diaspora. Encouraging immigration to the
home country is an interest of not only of those individuals who are
currently in the home country but also of those residing in the diaspora.
However, only when the burden of public spending in the home country is large
enough do the latter volunteer to bear part of it.

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*Tel Aviv University.
1. Introduction

This paper investigates the distribution of a population group between a home country and diaspora, given sequential decision-making regarding migration at the individual level. The home country is attractive to the members of the group, yet their presence there requires a fixed amount of public spending (e.g., on defence). The per-capita tax burden depends then on the size of the domestic population, reflecting a case of "fiscal externality" (see Flatters, Henderson and Mieszkowski (1974) and Boadway and Wildasin (1984)), which results in an inefficient distribution of the group between the home country and abroad. Encouraging immigration is naturally an interest of those individuals who are currently in the home country. But, what perhaps is more interesting is whether the diaspora residents have an incentive to encourage immigration to the home country by volunteering to bear at least part of the burden there. These issues are investigated in this paper.

This work is motivated by the case of Israel and its link with the Jewish diaspora. However, the framework of this analysis may also apply to other groups which are distributed between a home country and abroad, while having some sort of externality associated with the domestic population size. Paradoxically, conditions similar to those described here may be relevant to the Palestinians if they have their own state next to Israel. They too have a significant and strongly attached
diaspora, and a similar "fiscal externality" may exist if they perceive a threat from the neighboring Israel. Another example may be Mexico and its U.S. diaspora. This case, however, may be modelled with congestion in the home country, that is, with a negative externality.

The paper proceeds as follows. Section two presents the setup of the model and Section three addresses the sequential decision-making about migration. The individual who is currently residing in the home country has to decide whether to stay there at least one more period of time, or to emigrate. The individual who is currently in the diaspora has to decide whether to stay there or to immigrate to the home country. These individual decisions are made taking the domestic population and the domestic tax burden as given.

Section four analyzes the distribution of the population group between the home country and the diaspora under laissez-faire conditions. Then, Section five addresses the socially optimal distribution, given a Utilitarian social welfare function. Because of the "fiscal externality" associated with individual decisions, the optimal distribution allocates more individuals to the home country than the laissez-faire equilibrium does.

It also follows from this model that the individual welfare of each member of the group, both in the home country and abroad, increases with the domestic population size. For the domestic residents this welfare improvement follows directly from the fact that the burden of public spending on each one declines as more individuals share it. For the
diaspora resident this welfare improvement follows from the possibility that he or she may want to immigrate to the home country some time in the future. This issue is followed up in Section six by discussing whether it is in the interest of the diaspora residents, as a group, to encourage immigration to the home country by bearing part of the burden of public spending there. It is shown that if this burden is large enough the answer is positive. In other words, when the domestic public spending is sufficiently large, the diaspora residents as a group may be better off by creating an institution that commits them to donate part of their income to the domestic residents. The optimal transfer from the diaspora residents' point of view is also analyzed.

Section seven summarizes the paper, mentioning issues related to the present framework which were not addressed here.

2. The Setup of the Model

Consider a population group of size \( \hat{N} \), of whom \( N \) live in the home country and \( \hat{N} - N \) live in the diaspora. Each individual lives for ever and is endowed with one unit of labor per period of time. The analysis concentrates on the stationary state with constant population distribution over time. However, the identity of those residing in the two locations changes over time according to the individual migration decisions.

At the beginning of each period the individual decides whether to stay where he or she is for an additional period of time, or to
emigrate. The decisions to immigrate to or to emigrate from the home country depend on the net incomes at home and abroad and on migration costs.

The gross per-capita income abroad is the constant $k$. In the home country the gross per-capita income is $w_0 + z_t$, where $w_0$ is a common constant component and $z_t$ is an idiosyncratic stochastic component. The random productivity shift $z_t$ is identically and independently distributed, both across individuals and over time. The distribution is uniform over the interval $[-\sigma, \sigma]$. The length of each period of time can be imagined as of several years. Under this interpretation of a period the assumption of i.i.d. shocks, made for analytical convenience, is more realistic. It is assumed that $k > w_0$, so that the diaspora offers, on average, higher income. Yet, there are realizations of $z$ such that the individual's domestic income exceeds $k$, i.e., $w_0 + \sigma > k$. It is assumed that there is full current information about $z_t$ to all individuals in the group. Hence, an individual who is currently abroad becomes aware of his or her opportunities in the home country as they appear.$^1$

The members of the group are characterized by associating a non-pecuniary income to the residence in the home country. That is, the term $w_0$ is conceived as involving a non-pecuniary component, $w_0^n$, in addition to a pecuniary component, $w_0^p$. The first component is a
subjective flow of utility such that \( w_0^p + w_o^n > k \), while \( w_0^p + \sigma < k \). Therefore, individuals who do not perceive \( w_o^n \) would never choose to live in the home country and hence do not participate in the group.

An important characteristic of the home country is the existence of a fixed public expenditure of size \( G \). This expenditure can be conceived as dictated by given defense needs. The home country has a government, whose role is to finance \( G \) by lump-sum taxes on the domestic residents. Hence, the average net income there is \( w = w_o - G/N \).

The cost of emigration from the home country is denoted by \( m_1 \) and the cost of immigration to it by \( m_2 \). It is assumed that these costs are not "too large"--in a sense to be made precise in the next section--so as not to inhibit all migration.

The preferences of the representative individual in the group are described by the utility function

\[
(1) \quad E \sum_0^\infty \beta^t y_t,
\]

where \( 0 < \beta < 1 \) is a subjective discount factor, and

\[
y_t = w + z_t', \quad \text{if the individual is in the home country and stays one more period},
\]

\[
= k - m_1', \quad \text{if the individual is in the home country and emigrates this period},
\]

\[
= k, \quad \text{if the individual is abroad and stays there one more period},
\]

\[
= w + z_t' - m_2', \quad \text{if the individual is abroad and immigrates this period}.
\]
Finally, it is assumed that the real rate of interest in the international capital market is $1/(1-\beta)$. This implies, under the linear preferences specified above, that it will not be beneficial to save or to borrow.

3. The Individual's Problem: Solution and Comparative Statics

The problem has the same structure of sequential decision making over two alternatives as in job search models. Every period, the alternatives are to stay where the individual currently is, or to migrate. The search literature has been surveyed by Mortensen (1986) and an analysis of sequential decisions about migration has recently been carried out by McCall and McCall (1987). The setup in this section shares with the latter study the sequential structure, but it is designed to facilitate the analysis of equilibrium in the presence of a fiscal externality in the following sections.

From the individual's point of view the domestic population size and hence net-domestic income are given. The problem of an individual who lives in the home country at the beginning of period $t$, and whose current domestic income is $w + z_t$, can be represented by the choice of $V(z_t)$ satisfying

\begin{equation}
V(z_t) = \text{Max} \{w + z_t + \beta E_t V(z_{t+1}), \ k - m_1 + \beta E_t W(z_{t+1})\},
\end{equation}
where \( E_t W(z_{t+1}) \) is the expected value of being abroad next period. The individual emigrates if current foreign income plus the expected value of being abroad, net of emigration costs, is higher than current domestic income plus the expected value next period of staying in the home country.

The problem of the individual who resides abroad at the beginning of period \( t \), and whose realization of productivity in the home country is \( z_t \), can likewise be expressed as

\[
W(z_t) = \max (w + z_t - m_z + \beta E_t V(z_{t+1}), k + \beta E_t W(z_{t+1})).
\]

In this case the immigration costs apply, and they are deducted from domestic income for the evaluation of the migration decision. Notice that when the migration costs are zero the expressions for \( V(z_t) \) and \( W(z_t) \) coincide. In this case the location at the beginning of the period is unimportant: individuals decide where to locate themselves in the current period only by comparing \( k \) to \( w + z_t \).

Figure 1 here

The solution to these two problems is represented diagrammatically in Figure 1. The diagram uses the fact that \( z_t \) is i.i.d., which implies that \( EV \) and \( EW \) are constant over time. The solution is characterized by the pair \( \bar{z} \) and \( \bar{z} \). The level \( \bar{z} \) is the productivity below which
Figure 1
individuals currently in the home country decide to emigrate, and \( \tilde{z} \) is the productivity above which individuals currently in the diaspora decide to immigrate. The individuals for which the realization of \( z_t \) is between those two values, stay where they are. The reservation levels \( z \) and \( \tilde{z} \) correspond to the situations where the individual is indifferent regarding the choice of staying or migrating. These conditions are:

\[
(4) \quad w + z + \beta EV = k - m_1 + \beta EW
\]

\[
(5) \quad w + \tilde{z} - m_2 + \beta EV = k + \beta EW.
\]

From (4) and (5) it follows that

\[
(6) \quad \tilde{z} - z = m_1 + m_2.
\]

This condition says that when two individuals, one in the home country and the other abroad, are indifferent regarding whether to stay or to migrate, the increase in income of both to be gained by switching places is equal to the migration costs in both directions.

To solve for the reservation levels it is necessary to calculate the expected values of being in the home country, \( EV \), and abroad, \( EW \). Applying the expectation operator to the \( V \) and \( W \) equations in (2) and (3) yields:
(7) \[ EV = [k - m_1 + \beta EW] \frac{z+\sigma}{2\sigma} + \frac{w + \beta EV}{2\sigma} - \frac{\sigma}{z} \int \frac{z}{2\sigma} dz \]

(8) \[ EW = [k + \beta EW] \frac{z+\sigma}{2\sigma} + \frac{w - m_2 + \beta EV}{2\sigma} + \frac{\sigma}{z} \int \frac{z}{2\sigma} dz \]

The system of four equations, (4), (5), (7) and (8), can now be solved to determine the four unknowns \( z \), \( \hat{z} \), \( EV \), and \( EW \). The solution is

(9) \[ EV = \frac{w + (\sigma + z)^2}{4\sigma}/(1-\beta) \]

(10) \[ EW = \frac{k + (\sigma - \hat{z})^2}{4\sigma}/(1-\beta) \]

(11) \[ \hat{z} = \frac{k - w + (1-\beta)(m_2 - m_1)/2}{A} + \frac{(m_1 + m_2)}{2} \]

(12) \[ \bar{z} = \hat{z} - \frac{(m_1 + m_2)}{2} = \frac{k - w + (1-\beta)(m_2 - m_1)/2}{A} - \frac{(m_1 + m_2)}{2} \]

where \( A = 1-\beta(m_1 + m_2)/2\sigma \).

Observe that in order for immigration to occur, it should hold that \( \hat{z} < \sigma \). Otherwise, income in the home country can never be high enough to attract an individual from abroad. The corresponding condition for emigration is \( z > -\sigma \). If this condition is not satisfied income in the home country can never be low enough to persuade an individual to
emigrate. These two inequalities and equation (6) place the following restriction on the migration costs, \( m_1 \) and \( m_2 \):

\[
(13) \quad m_1 + m_2 < 2\sigma.
\]

Unambiguous comparative statics results are obtained under the additional assumption

\[
(14) \quad m_1 - m_2 < 2(k - w)/(1-\beta).
\]

This restriction is plausible if the home country is smaller and less developed, such that the set-up cost of an immigrant is larger there than abroad. In this case the condition is \( m_1 - m_2 < 0 \), of which (14) is a weaker requirement. All the results would go through, therefore, when \( m_1 = m_2 \).

Equations (9)-(14) can now be used to evaluate the effects of changing the different parameters on \( z \), \( \tilde{z} \), EV and EW. It is evident that both the reservation floor for emigration, \( z \), and the reservation ceiling for immigration, \( \tilde{z} \), increase with \( k \) and decrease with \( w \). This reflects the fact that the incentives to emigrate increases with foreign income, \( k \), and decreases with domestic income, \( w \). The opposite holds regarding the incentives to immigrate.

Consider the effects of an increase in \( \sigma \), which represents a mean preserving spread of domestic productivities. It follows from (11)-(14)
that both reservation levels decrease with \( \sigma \), making the home country more attractive. The intuitive explanation of this effect is the following. A larger spread of \( z \) provides the individual with some better and some worse opportunities at home. However, the individual is not compelled to take the worse ones, and, therefore, he or she is certainly better off. More specifically, the values of \( z \) below the initial \( \bar{z} \) do not affect the individual's decisions since in any case they favor emigration.

An increase in either migration cost \( m_1 \) or \( m_2 \), tends to increase \( \bar{z} \) and decrease \( z \) making migration less likely. Intuitively, the immigration cost \( m_2 \) has a direct negative effect on the attractiveness of the home country relative to staying abroad, and hence \( \bar{z} \) increases with \( m_2 \). Additionally, it also has an indirect negative effect on the incentive to emigrate from the home country because in future periods the individual may want to return, facing at that stage higher moving costs. This explains why \( z \) goes down with \( m_2 \). Similarly, \( m_1 \) discourages emigration and, indirectly, immigration, and thus it affects the reservation levels in the same way.

The effects of the different parameters on the expected utilities, \( \text{EV} \) and \( \text{EW} \), can be derived from (9) and (10) and the effects on \( z \) and \( \bar{z} \). A summary of the comparative statics results is given in Table 1.
Table 1
Comparative Statics: The Effects of the Parameters on the Reservation Levels and Expected Utilities

<table>
<thead>
<tr>
<th></th>
<th>( z )</th>
<th>( \dot{z} )</th>
<th>( EV )</th>
<th>( EW )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w )</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>( k )</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>( m_1 )</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( m_2 )</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

Another parameter of interest is the time-discount factor \( \beta \). Because of the underlying assumption about the international capital market, a change in \( \beta \) corresponds to a parallel change in the world real interest rate. If all individuals in the world are identical, except for the way they perceive \( w_{0}^{n} \), a comparative statics exercise on \( \beta \) would then correspond to changing the time preferences of all individuals.

Given the interpretation above, consider the effect of an increase in \( \beta \) on \( \dot{z} \) and \( z \). For simplicity assume here that \( m_1 = m_2 \). From (11) and (12), and the assumption that \( k > w_{0} \), it follows that both reservation levels increase. Hence, caring more about the future leads to a higher propensity to emigrate and a lower propensity to immigrate. The intuitive explanation for this effect is that individuals who stay in the home country, and those who come to it in
a given period, do that because they currently enjoy a high level of \( z \). The future, however, looks different. The expected value of a future \( z \) is zero, and hence the expected income differential, \( k-w_0 \), is favorable to the diaspora. Hence, the decision to stay in the home country or to immigrate to it entails a sacrifice of income in the future to enjoy a presently high consumption level. A higher \( \beta \) implies a heavier weight ascribed to the expected sacrifice in the future, and hence it leads to higher values of \( z \) required to stay in the home country or to immigrate to it.

4. The Home-Country Population under Laissez-Faire

Given the behavior of each individual in the group, this section discusses the stationary distribution of the population \( \bar{N} \) between the home country, \( N \), and the diaspora, \( \bar{N} - N \), under laissez-faire conditions. From the previous section, the probability that an individual abroad decides to immigrate is \( (\sigma - \bar{z})/2\sigma \). Since the relative productivities \( z \) are independent across individuals, \( (\sigma - \bar{z})/2\sigma \) is also the proportion of the population abroad who decides to immigrate. Similarly, \( (\sigma + \bar{z})/2\sigma \) is the proportion of the domestic population who prefer to emigrate. The stationary home population is the value of \( N \) which generates a balanced in and out migration, i.e.,

\[
N(\sigma + \bar{z}) = (\bar{N} - N)(\sigma - \bar{z}).
\]

From this condition and (6) the stationary home population can be expressed as
(15) \[ N = \tilde{N}(\sigma - \tilde{z})/B, \]

where \( B = 2\sigma - m_1 - m_2 > 0 \) (from (13)).

4a. The Domestic Population Size when \( G = 0 \).

To analyze the determination of \( N \), assume, as a preliminary step, that there is no public spending in the home country. In this case domestic income is exogenous at the level \( w_0 \), and therefore the corresponding level of \( \tilde{z} \)--defined as \( \tilde{z}_0 \)--is fully determined by (11) in terms of the exogenous parameters. The domestic population size is then

(15') \[ N_0 = \tilde{N}(\sigma - \tilde{z}_0)/B. \]

From (15') and (11) the effects of the different parameters on the stationary home country population can be calculated. The results are summarized in Table 2.

Table 2
Comparative Statics: The Effects of the Parameters on \( N \) (\( G = 0 \))

<table>
<thead>
<tr>
<th>w</th>
<th>k</th>
<th>( \sigma )</th>
<th>( m_1 )</th>
<th>( m_2 )</th>
<th>( \tilde{N} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>-</td>
<td>+</td>
<td>?</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>
The positive effect of domestic income and the negative effect of foreign income on \( N \) are straightforward, since the relative attractiveness of the home country increases with \( w \) and decreases with \( k \).

As seen in Section three, the dispersion of domestic incomes, \( \sigma \), tends to discourage emigration and encourage immigration by decreasing both \( \tilde{z} \) and \( z \), resulting in a larger domestic population. The spread \( \sigma \) has also a direct effect on \( N \) for any given reservation level. The latter effect is ambiguous, but, it does not offset the impact of \( \sigma \) working through \( \tilde{z} \).

The immigration cost \( m_2 \) discourages immigration directly and emigration indirectly--through its effects on the reservation levels--as shown in the previous section. It turns out that the direct effect dominates the indirect one, resulting in a reduction of \( N \). With regard to the emigration cost \( m_1 \), the effect on \( N \) is ambiguous. It is possible to show, however, that when the emigration costs tend to zero, the sign of \( \partial N / \partial m_1 \) is positive or negative as \( \sigma \) \( > \) \( (1+\beta)(k-w)/(1-\beta) \). Given \( \sigma \), this implies that the lower \( \beta \), i.e. the stronger the discounting of the future, the more likely is that \( N \) increases with \( m_1 \). In this case, the direct negative effect of \( m_1 \) on emigration becomes stronger than the indirect negative effect on immigration. The latter weakens with heavier discounting of the future because prospective immigrants are less sensitive to the possibility of future emigration.

4.b. The Domestic Population Size when \( G > 0 \).

Consider now the case of positive public spending, which is financed by a head tax levied on the domestic population. Hence, net-income in the home
country is \( w = w_0 - G/N \). In this situation the reservation level \( \tilde{z} \), which depends on \( w \), is a function of \( N \). Consequently, equation (15) becomes an implicit equation in \( N \). Intuitively, the domestic population size determines the burden of public spending on domestic residents, which in turn, through the migration decisions, affect \( N \). The right-hand-side of (15) can now be written as

\[
(16) \quad \tilde{N}(\sigma - \tilde{z})/B = f(N; G, w_0, \sigma, \beta, k, m_1, m_2).
\]

The solution for the population size is an \( N^* \) which satisfies \( N^* = f(N^*; \ldots) \). We turn now to determine and characterize this solution. Notice that, using the expression for \( \tilde{z} \) in (11), and rearranging, the function \( f(\cdot) \) can be written as

\[
(17) \quad f(N; \ldots) = N_0 - \frac{G\tilde{N}/AB}{N}.
\]

Equations (15), (16) and (17) imply that the stationary domestic population size is determined by the quadratic equation:

\[
(18) \quad N^2 - \frac{N}{N_0} N + \frac{G\tilde{N}/AB}{N} = 0,
\]

of which the two solutions are:
\[
N_1 = \frac{N_o + \sqrt{N_o^2 - 4GN/AB}}{2}
\]

and

\[
N_2 = \frac{N_o - \sqrt{N_o^2 - 4GN/AB}}{2}.
\]

Real roots require that \( G \) is not too large, i.e. it should satisfy

\[
G \leq \frac{N_o^2AB}{4N}.
\]

A strict inequality in (20) yields two distinct real roots, while an equality implies the unique solution \( N_o/2 \). The level of \( G \) corresponding to the latter case will be defined as

\[
\tilde{G} = \frac{N_o^2AB}{4N}.
\]

**Figure 2 here**

A diagrammatic exposition of the solution is presented in Figure 2. The function \( f(.) \) (equation (16)) is portrayed, using the fact that it is an increasing and concave function of \( N \), and that \( f(.) = 0 \) for \( N = GN/AB > 0 \). Four values of \( G \) are used in the figure, 0, \( G' \), \( \tilde{G} \) and \( G'' \), satisfying \( 0 < G' < \tilde{G} < G'' \). No real solution exists for \( G'' \), while \( \tilde{G} \) yields the unique solution \( N_o/2 \). The level \( G' \) produces two solutions, bounded from above by \( N_o \), which is the population size when \( G = 0 \).
Figure 2: Equilibrium Home Country Population as a Function of Domestic Public Expenditure.

\[ N_1 = (N_0 + \sqrt{N_0^2 - 4G'\bar{N}/AB})/2 \]

\[ N_2 = (N_0 - \sqrt{N_0^2 - 4G'\bar{N}/AB})/2 \]
In the following discussion it is assumed that \( G \) is not too large, i.e., it satisfies \( 0 < G \leq \tilde{G} \). Because the economic implications of the lower solution (\( N_2 \) in Figure 2) are implausible, this solution will not be discussed further. Therefore we concentrate on the higher solution (\( N_1 \) in figure 2).

Given that \( f(.) \) is decreasing in \( G \) (equation (17)), and that as \( G \) approaches \( \tilde{G} \) \( N \) converges to \( N_o/2 \), if follows that \( N \geq N_o/2 \) as \( G \leq \tilde{G} \). Therefore, using (17) and (20):

\[
\frac{\delta f(.)}{\delta N} = - (\tilde{N}/B) \frac{\delta \tilde{z}}{\delta w} \frac{dw}{dN} = \frac{\tilde{N}G}{ABN^2} \leq 1, \quad \text{as } G \leq \tilde{G}.
\]

The implications of (21) can be observed in Figure 2, where the slope of \( f(.) \) at \( N_1 \) is less than one, and the slope at \( N_o/2 \) is equal to one.

The effects of the various parameters on the domestic population size can now be analyzed by differentiating (15) to obtain:

\[
\frac{dN}{dx} = \left[ 1 + \frac{\tilde{N}}{B} \frac{\delta \tilde{z}}{\delta w} \frac{dw}{dN} \right]^{-1} \sum_x \frac{\delta f(N; \ldots)}{\delta x} = \left[ 1 - \frac{\tilde{N}G}{ABN^2} \right]^{-1} \sum_x \frac{\delta f(N; \ldots)}{\delta x},
\]

where \( x = m_1, m_2, w_0, k, \sigma, G \), and the expression on the right hand side represent their effects on \( N \) under \( G=0 \), appearing in Table 2 (except for \( G \)).

Equation (21) implies that the expression in brackets on the right-hand-side of (22) is positive and strictly less than one for \( G < \tilde{G} \). Hence, changes in the above parameters affect \( N \) in the same direction as in
Table 2. Furthermore, the effects are magnified through a "multiplier". The reason of this magnification effect is that any change inducing individuals to emigrate, or not to immigrate, causes a higher burden of public spending on domestic residents.

Table 2 does not include the effect of $G$ on the domestic population size. This effect is illustrated in Figure 2, where the levels of public spending $0 < G' < \hat{C}$ correspond to $N_0 > N_1 > N_0/2$. The maximum level of public spending that is consistent with a positive domestic population size is $\hat{C}$.

5. Social Optimum

In this section, the laissez-faire solution is compared to the allocation following from a social optimization. Consider the problem of a planner who maximizes a Utilitarian social welfare function for the entire population group $\bar{N}$, by dictating to each individual when to migrate. Formally, the planner's problem is to choose $(\bar{z}_t, \bar{z}_t')_0$ so as to maximize

$$
E \sum_0^\infty \beta^t Y_t,
$$

(23)

where $Y_t$ is total consumption of the group at time $t$ (the summation of $y_t$, as defined under equation (1), over all individuals), subject to the population evolution equation

$$
N_t = N_{t-1} + M_{2t} - M_{1t}.
$$

(24)
where $M_{1t}$ and $M_{2t}$ are emigration and immigration in period $t$, respectively. These migration flows can be expressed as

\begin{align}
M_{2t} & = (\bar{N} - N_{t-1})(\sigma - \bar{z}_t)/2\sigma, \\
M_{1t} & = N_{t-1}(\sigma + z_t)/2\sigma.
\end{align}

Total consumption is given by

\begin{align}
Y_t & = (N_{t-1} - M_{1t})[w_o + (\sigma + z_t)/2] \\
& + (\bar{N} - N_{t-1} - M_{2t})k \\
& + M_{1t}(k - m_1) \\
& + M_{2t}[w_o + (\sigma + \bar{z}_t)/2 - m_2] - G.
\end{align}

The first term on the right hand side of (27) is the income of those who stay in the home country. It is composed of $w_o$ and the average of their $z_t$ values (those with $z_t < \bar{z}_t$ emigrate). The second term is the income of those who stay abroad. The third term is the income of emigrants less emigration costs and the forth is the income of immigrants less immigration costs. The income of immigrants includes only those with $z_t > \bar{z}_t$ because those abroad with lower $z_t$ stay there. Finally, $G$ is the burden of public spending in the home country. Since $G$ is unaffected by the population distribution, it will clearly not alter the optimal reservation levels.
The first-order conditions of this problem are presented in Appendix B. Evaluating these conditions at the stationary state, yields expressions for \( \hat{z} \) and \( \hat{z} \) which are identical to those in equations (11) and (12) for \( G=0 \). Hence the optimal domestic population is \( N=N_0 \). In the case of \( G=0 \) there is no "fiscal externality" and, therefore, the laissez-faire distribution of \( \hat{N} \) between the home country and the diaspora is socially optimal.

The two solutions differ, however, when \( G \) is positive. The social optimum is unaffected by the level of public spending, but, as shown in Section four, the home population under laissez-faire declines with \( G \). Hence, the optimal financing of \( G \) should be to levy an equal lump-sum tax of \( G/N \) on both domestic and foreign members of \( \hat{N} \), such that the income differential entering the determination of \( \hat{z} \) and \( \hat{z} \) remains unaffected by \( G \).

The optimal solution is a theoretical benchmark that has little practical relevance. It requires taxation power over members of the group who reside in foreign countries. A question of more practical interest is whether the diaspora members of the group will be willing to contribute to the financing of \( G \) in the home country. This is the subject of the next section.

6. Transfers from Abroad

Abstracting from the free rider problem, we turn now to consider the question whether the diaspora residents would be better off if they, as a group, contribute to the home country. This question can be interpreted as whether it is in the interest of the foreign members of the group to have an institution which commits them to help the home country. If the answer is
positive, related issues are how much they would like to contribute, (i.e., what share of $G$ to finance), and whether it is possible to achieve the first-best with such donations.

The problem can be formalized as follows. In the steady state, let $\alpha \geq 0$ be the part of the first-best share $G/N$ that each diaspora resident contributes. Accordingly,

\begin{equation}
\tilde{k} = k - \alpha G/N
\end{equation}

is the net-income abroad, and

\begin{equation}
\tilde{w} = \omega_0 - G/N + \frac{N - N}{N} \alpha G/N = \omega_0 - \alpha G/N - (1 - \alpha) G/N
\end{equation}

is the net non-stochastic component of the home country income, which is increased by the transfers per home country resident. In the second form of $\tilde{w}$, the burden on domestic residents is divided into the part borne by all members of the group, and the part financed only by domestic residents.

The problem considered is to determine $\alpha$ which solves

\begin{equation}
\max_{\alpha \geq 0} EW(\alpha; G),
\end{equation}

where

\begin{equation}
EW(\alpha; G) = \frac{[\tilde{k} + (\sigma - \bar{z})^2/4\sigma]/(1 - \beta)}{1 - \beta},
\end{equation}
(31) \[ \ddot{z} = (\ddot{\kappa} - \ddot{w} + (1-\beta)(m_2 - m_1)/2)/A + (m_1 + m_2)/2, \]

and \( \ddot{w} \) and \( \ddot{\kappa} \) are defined in (28) and (29), subject to the determination of the stationary population in equation (15). That is, the problem is to find the optimal fraction \( \alpha^* \), given optimal individual behavior and equilibrium population distribution, so as to maximize the average stationary utility level of a diaspora resident. Notice that \( EW \) is both the expected future value of being abroad and the average current value. A positive \( \alpha^* \) will indicate the desirability, from the point of view of the diaspora residents, of contributing to the home country.

The first-order conditions for a solution to this problem are

(32) \[ (1 - \alpha^*)\bar{G}/(ABN^2) - D \leq 0, \]

(32') \[ [(1 - \alpha^*)\bar{G}/(ABN^2) - D]\alpha^* = 0, \]

where \( D = \frac{(m_1 + m_2) - \beta(m_1 + m_2)}{2\sigma - \beta(m_1 + m_2)}, \) and \( 0 < D < 1 \). These inequalities follow from (14) and from \( 0 < \beta < 1 \). (The derivation of (32) and (32') is relegated to Appendix B).

Observe that when \( G = 0 \), it follows from (32') and \( D > 0 \) that \( \alpha^* = 0 \). Because of continuity, this will also be true for some range of positive values of \( G \). Therefore, for that range, the corner solution \( \alpha^* = 0 \) is obtained.
However, there is other range of $G$ for which $\alpha^*$ is positive. This can be seen as follows. If $\alpha^* = 0$, the left hand side of (32) is increasing with $G$, both directly and indirectly by reducing $N$. As $G$ approaches the value $\tilde{G}$ the first term on the left-hand-side of (32) approaches 1 (from equation (21)). Hence, given that $D < 1$, there must exist some level $\hat{G}$--smaller than $\tilde{G}$--such that for larger values, the first term on the left-hand-side of (32) exceeds $D$ when $\alpha^* = 0$. Therefore, for $\hat{G} < G < \tilde{G}$, $\alpha^*$ must be positive. Summing up,

$$G < \hat{G} \rightarrow \alpha^* = 0,$$

and

$$\hat{G} < G \leq \tilde{G} \rightarrow \alpha^* > 0.$$

In words, when the burden of the public expenditure in the home country is sufficiently small, no contribution is offered by the diaspora residents. But, when this burden is sufficiently large (but not too large), the diaspora residents as a group volunteer to bear some of the burden.

It was shown in Section four that, when the present mechanism of transfers does not operate, the effect of the level of $G$ on the domestic population is negative. We turn now to analyze this effect when transfers are endogenous and $\alpha^*$ is positive, that is, when $G$ is in the interval $(\hat{G}, \tilde{G})$. This issue is closely related to the effect of $G$ on optimal transfers. These two issues can be analyzed in a simple way using condition (32'), for $\alpha^* > 0$, which can also be written as $(1-\alpha^*)G/N = \text{constant} \times N$. From this equality it follows that the net income differential $\bar{\kappa} - \bar{\omega}$, from equations (28) and (29),
is independent of $G$. Consequently, the expressions for $\tilde{z}$ and $N$ in (11), or (31), and (15) also become independent of $G$. In other words, when the optimal transfers are positive the population size is unaffected by increases in the burden of public spending. This result stands in sharp contrast with the previous negative effect of $G$ on $N$ observed in Section four.

The difference in the results has to do with the positive response of the optimal transfer to the level of $G$. To see this effect in the present context, notice that when $N$ is constant, (32) implies that also $(1-\alpha^*)G$ is constant. Hence, in the range where an interior solution applies, i.e., $(\tilde{G}, \tilde{G})$, the effect of public spending on $\alpha^*$ is positive. The story is, therefore, that a heavier burden of public spending motivates larger transfers from the diaspora residents, which are enough to leave the foreign/domestic income differential unchanged. Consequently, the motivation to emigrate, and not to immigrate, as $G$ increases disappears.

Finally, observe from (32') that since $D > 0$ the optimal share $\alpha^*$ must be strictly less than one. Hence, diaspora residents will not contribute sufficiently to support the first-best allocation. Only if the moving costs $m_1$ and $m_2$ are zero, and hence $D = 0$, the optimal share will be equal to one. In this case, however, there is no difference between $EW$ and $EV$, and to maximize $EW$ is equivalent to maximize $EV$ or a combination of both. The maximal value of $EW$ corresponds in this case to the first-best, which is achieved when $\alpha^* = 1$. 
7. Concluding comments

Because of the inefficient population distribution under laissez-faire the present framework generates a notion of "Zionism", in the sense that increasing the fraction of the population group residing in the home country improves the social welfare of the group.

It also holds in this model that each member of the group, both in the home country and abroad, is a "Zionist", where the term applies to individuals whose own welfare increases with the domestic population size. Equations (9)-(12) in Section three imply that the welfare of domestic and diaspora residents increases with \( N \). This characteristic of the model is particularly interesting with respect to diaspora residents. Their utility level increases because of the option they have to become a home country resident in the future. The lower the per-capita burden of public spending in the home country, the more valuable that option is. However, "Zionism" on the part of diaspora residents does not imply necessarily that they will be willing to transfer part of their income to the home country in order to encourage immigration to it. As shown in Section six, only if the burden of public spending is sufficiently large, the benefit to the average diaspora resident from the increase in the home country population exceeds the cost of generating such an increase. In that case the diaspora residents, as a group, may benefit by having an institution that commits them to donate part of their income to the homeland.

The analysis of the optimal transfers from the diaspora was carried out under the assumption that domestic residents as a group, or the government of
the home country, do not conduct any policy that affects the population size. Suppose that the domestic government takes the amount of foreign transfers as given, and considers, for example, taxing emigration or subsidizing immigration. First, would such policies be beneficial from the point of view of the average domestic resident? Second, how would such policies affect the willingness of diaspora residents to contribute? The latter question refers to strategic behavior.

A formal analysis of these issues is beyond the scope of this paper. However, with respect to the first question, some numerical exercises were carried to evaluate the effects of different domestic policies. Using a set of values appearing reasonable for the Israeli case, it was found that the imposition of a small tax on emigration—to finance part of the public burden—increases the population size and the average welfare of domestic residents. Furthermore, the average welfare of foreign residents also increases with this policy of the domestic government. The results are different in the case of a small immigration subsidy, financed by lump-sum taxes. It does increase the domestic population but reduces welfare at home. However, average welfare of diaspora residents increases, such that total welfare goes up.
FOOTNOTES

1. A stochastic component could be added also to foreign income. However, the main results would not be affected by this modification.

2. The details of the calculations are available from the authors upon request.

3. Since, for $G'$, $N_1 > N_o/2$ (see figure 2), it follows from (20) that

$$\tilde{\text{NG}}/\text{ABN}^2 < 4\tilde{\text{NG}}/\text{ABN}_o^2 < 1.$$
REFERENCES


APPENDIX A: THE SOCIAL OPTIMUM

The first-order conditions for maximizing (23), with \( y_t \) defined in (27), subject to (24), (25) and (26) are

\[(A1) \quad z_t: \quad \beta^t(z_t + m_1) + \lambda_t = 0,\]

\[(A2) \quad \ddot{z}_t: \quad \beta^t(\ddot{z}_t - m_2) + \lambda_t = 0,\]

\[(A3) \quad N_t: \quad \beta^t(w_o - k) + \beta^{t+1}[(\sigma^2 - z_{t+1}^2)/4\sigma - (\sigma^2 - \ddot{z}_{t+1}^2)/4\sigma - (\sigma + z_{t+1})m_1/2\sigma + (\sigma - \ddot{z}_{t+1})m_2/2\sigma] - \lambda_t + \lambda_{t+1}(\ddot{z}_{t+1} - z_{t+1}) = 0\]

where \( \lambda_t \) is the shadow price of (24).

(A1) and (A2) imply (6) of the text. Substituting (A1), evaluated at \( t \) and \( t+1 \), and (6) into (A3), and assuming a convergence to a stationary state we obtain, after a few manipulations:

\[(A4) \quad 0 = w_o - k - m_2 + \ddot{z} + \beta[(m_1 + m_2)^2/4\sigma + (m_2 - m_1)/2 - \ddot{z}(m_1 + m_2)/2\sigma] = \ddot{z}[1 - \beta(m_1 + m_2)/2\sigma] + w_o - k + \beta(m_2 - m_1)/2\]
\[ + \beta (m_1 + m_2)^2 / 4 \sigma \]
\[- m_2 \]
\[- (m_2 - m_1) / 2 + (m_2 - m_2) / 2. \]

Rearranging obtains

\[(A5) \quad \ddot{z} = [k - w_0 + (1 - \beta)(m_2 - m_1)/2]/A + (m_1 + m_2)/2, \]

which is (11) of the text.
APPENDIX B: EVALUATION OF $\partial \tilde{E}/\partial \alpha$

Substituting $\tilde{w}$ and $\tilde{k}$, as defined in (28) and (29), for $w$ and $k$, respectively, in (10) and (11) and differentiating (10), (11), and (15) with respect to $\alpha$ yields:

\begin{equation}
\partial \tilde{E}/\partial \alpha = \left( \partial \tilde{k}/\partial \alpha - \left( (\sigma - \tilde{z})/(2\sigma) \right) \partial \tilde{z}/\partial \alpha \right)(1-\beta), \tag{B1}
\end{equation}

\begin{equation}
\partial \tilde{z}/\partial \alpha = \left( 1/A \right) \partial (\tilde{k} - \tilde{w})/\partial \alpha = -\left[ G/(NA) \right] \left( 1+\left( 1-\alpha \right)/N \right) \partial N/\partial \alpha, \tag{B2}
\end{equation}

\begin{equation}
\partial N/\partial \alpha = -\left( \tilde{N}/B \right) \partial \tilde{z}/\partial \alpha. \tag{B3}
\end{equation}

(B2) and (B3) imply

\begin{equation}
\partial \tilde{z}/\partial \alpha = \left[ G/(NA) \right] \left[ 1 - (1-\alpha)\tilde{N}G/(ABN^2) \right]. \tag{B4}
\end{equation}

Differentiating (28) and substituting the result, (15), and (B4) into (B1) yields:

\begin{equation}
\text{Sign } \partial \tilde{E}/\partial \alpha = \\
\text{Sign}( -1 + \left[ B/(2\sigma A) \right]/\left[ 1 - (1-\alpha)\tilde{N}G/(ABN^2) \right] = \\
\text{Sign}(1 - (1-\alpha)\tilde{N}G/(ABN^2) - [(2\sigma A-B)/(2\sigma A)]).
\end{equation}

The last equality follows from the fact that $0 < G < \tilde{G}$, which implies that the denominator of the expression in the second line is positive.
Now, using the definitions $A = 1 - \beta(m_1 + m_2)/2\sigma$ and $B = 2\sigma - m_1 - m_2$ in (B5) it reduces to

\[(B6) \quad \text{Sign } \partial EW/\partial \alpha = \text{Sign } [(1 - \alpha)\tilde{N}G/\tilde{A}N^2 - D],\]

where $D = [m_1 + m_2 - \beta(m_1 + m_2)]/[2\sigma - \beta(m_1 + m_2)] < 1$, by (14).

(B6) implies (32) and (32') of the text.