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ABSTRACT

International differences in the cost of production of a key intermediate product can mean that a domestic firm is dependent on imports from a foreign vertically integrated supplier when the two firms compete in the domestic market for the final product. This paper considers the incentives of both the foreign supplier and its government to export the intermediate product to the higher cost domestic firm. At the extreme, there are no exports of the input and vertical foreclosure occurs. The difference in profit margins from the export of the intermediate and final products, supply conditions for the input in the domestic country and a domestic tariff on final product imports are all shown to be important. Whether there is Cournot or Bertrand competition at the final output stage, it is never optimal to tax the exports of one product while subsidizing the exports of the other.

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1. Introduction

Countries that are dependent on imports of a key intermediate product or raw material from a dominant world supplier are often concerned about the price and the availability of imports. A notable current example involves the computer industry. Japanese suppliers (with the help of the Japanese government) recently restricted the exports of one megabyte DRAM semiconductors, substantially raising their price\(^1\). These suppliers control about 80% of the market for semiconductors and the higher prices and shortage in supply have forced U.S. producers of computers to curtail production and increase prices. Vertically integrated Japanese firms such as Toshiba and N.E.C. have benefitted both from increased profits in the export market for semiconductors and from the improvement in their competitive position in the market for final computers.

As this example indicates, the price and availability of imported supplies can depend on both public and private incentives in the exporting country. In this paper, we first examine the private incentives for a vertically integrated firm to export an intermediate product to a higher cost rival, lowering its rival's costs. The exporting firm may choose vertical foreclosure, thus fully cutting off supplies. We then consider the public interests of the exporting country: does the exporting country gain by encouraging the export of the intermediate product, or alternatively, might 'government foreclosure' occur? The government is in a position to affect the quantity of exports of both the input and the final product by an appropriate choice of export tax and subsidy policies.

We consider the most extreme form of dependence on a vertically integrated supplier by assuming that a single vertically integrated firm controls the exports of both the input and the final product. Differences in production costs are
assumed to arise as a consequence of international differences in endowments and technologies. If the rival firm has access to the input, either through its own production or through imports, then, for most of the paper, we assume that sales of the homogeneous final product are determined by Cournot competition. The implications of Bertrand behaviour with differentiated final products are briefly considered in a later section. If the rival firm in the importing country has no independent source of supply, vertical foreclosure gives the exporting firm a monopoly of the final product market.

The exporting firm is assumed able to act first by committing to an export strategy (price or quantity) for the input prior to the decision of the high cost firm as to its own level of production of the input and to the resolution of the Cournot (or Bertrand) game for the final product. The credibility of a commitment to the quantity of exported supplies is supported by the idea that it takes time to export the input and that these supplies must be available to the rival at the time of production of the final good. In choosing its export strategy, the exporting firm takes into account its rival's reaction including the possibility that the rival will alter its own level of production of the input. This has the advantage that the vertical supply (or foreclosure) decision is made with a full understanding of its consequences.

The profitability of vertical supply as an export strategy is heavily influenced by production conditions for the input in the importing country. Two main aspects of local supply conditions are shown to be important: the absolute quantity of supplies and the response of these supplies to the price charged for the input by the exporting firm. By imposing a tariff on final product imports, the importing country can have a major influence on this decision. In particular, if the rival firm cannot produce the input and there is Cournot competition at
the final stage, even a small import tariff will shift the equilibrium from vertical foreclosure to vertical supply.

The role for government policy in the exporting country arises from the inability of the exporting firm to commit to its level of exports of the final product prior to the rival's output decision. As originally shown by Spencer and Brander (1983), the exporting country gains from a subsidy to exports in a simple Cournot duopoly setting: the optimal subsidy increases exports to what would have been the Stackelberg leader level of output in the absence of the subsidy. Consideration of the export market for the input does, however, add some significant new features.

The sign of the difference in profit margins from the export of the input and the final product is shown to play a crucial role in the determination of optimal policy. If this difference in profit margins is positive (as a consequence of an import tariff on the final product) and if there is Cournot competition at the final stage, optimal government policy serves to amplify this difference, thus encouraging an increase in the extent of vertical supply. Perhaps surprisingly, this policy is achieved by a tax, not a subsidy, on the exports of the input, together with a (larger) tax on the exports of the final product. Conversely, if the difference in profit margins is negative, optimal policy calls for a subsidy to both exports, making the difference in profit margins even more negative and discouraging vertical supply.

As might be expected from Bulow, Geanakoplos and Klemperer (1985) and Eaton and Grossman (1986), optimal policy by the exporting country is fundamentally affected by whether the final products produced by each firm are strategic substitutes or complements. Optimal policies may have opposite signs if the final products are strategic complements (as in our Bertrand analysis) rather
than strategic substitutes (as in our Cournot analysis). However, whether there is Bertrand or Cournot competition at the final output stage, the 'double tax' or 'double subsidy' result remains: it is never optimal for the exporting country to tax the exports of one product while subsidizing the exports of the other. Moreover, the underlying conditions in the importing country that determine the profitability of vertical supply are basically unaffected.

Vertical foreclosure has been an important issue in the antitrust literature and in industrial organization. The idea that vertical foreclosure can increase profits is related to the idea, developed by Salop and Scheffman (1983) and (1987), that a dominant firm can gain by increasing the costs of its rivals even at some expense to itself. Two very interesting recent papers, Salinger (1988) and Ordover, Saloner and Salop (1988) deal directly with the issue of vertical foreclosure. In both these papers, upstream producers of the input have identical and constant costs and vertical merger results in a full cutting off of supplies to downstream firms. In contrast, the present model demonstrates that asymmetries in costs can make vertical supply profitable for a dominant firm\(^2\). If local supplies of the input in the importing country are sufficiently elastic, then vertical supply by the low cost firm is always an equilibrium strategy.

This paper is also related to the international trade literature concerning policy towards exports of vertically related products in a perfectly competitive framework, (see for example Kemp (1966), Jones (1967) and Jones and Spencer (1989)). Finally, this paper draws on the literature concerning trade policy under imperfect competition. Of special relevance are Dixit (1984), Eaton and Grossman (1986), Grossman and Dixit (1986), Venables (1985) and Brander and Spencer (1985).

Section 2 of the paper contains the basic model and the Cournot equilibrium
for the final product is described in Section 3. Section 4 is concerned with the conditions for vertical supply of the input and Section 5 deals with optimal trade policies for the exporting country when there is Cournot competition at the final stage. The implications of Bertrand competition in the final product market are considered in Section 6 and Section 7 contains concluding remarks.

2. The Model

Firm 1, in country 1 exports a final product to country 2 in competition with a higher cost rival, firm 2, located in country 2. Firm 1 is vertically integrated and also (potentially) exports its lower cost intermediate product to country 2, reducing its rival's costs. Firm 2 has the option of augmenting these imported supplies by producing the input itself. Although we present the analysis as if firm 2 is vertically integrated, this is not necessary. The input could be produced by a perfectly competitive industry in country 2.

Technological relationships are simplified by assuming that one unit of the intermediate product is required to produce one unit of the final product and that there are no other factors of production. Firm 1 produces the input (and the final good) at a constant marginal cost $c_1$, whereas firm 2 can produce its own supplies of the input only at a higher and increasing marginal cost. This means that $c_2 > c_1$ where $c_2$ denotes firm 2's marginal cost of production of the first unit of the input. Although brief consideration is given to the possibility that firm 2's marginal cost is constant at $c_2$, our general assumption is that firm 2's marginal cost is strictly increasing.

The subgame perfect equilibrium incorporates two stages of decision. In stage 1, firm 1 commits to the price $r$ that it will charge its rival for the input. Equivalently, firm 1 could commit to the quantity $x$ of its exports of the input at this stage. The quantity of exports $x$ and the price $r$ charged for these
exports are related simply by the requirement that the demand for x by firm 2 in stage 2 equals the supply at price r. Unless otherwise stated, the stage 2 outputs of the final product are determined by Cournot (quantity Nash) competition. (We consider Bertrand (price Nash) competition in Section 6). Firm 2 produces the final product using the cost minimizing combination of imported supplies and its own production x_2 of the input in stage 2.

Country 1 is assumed to commit to its policies, a specific subsidy s to final product exports and a specific tax v on exports of the input, at stage 0, prior to firm 1's choice of the price and quantity of its exports of the input at stage 1. The subsidy s and the tax v may be either positive or negative. Country 2 may commit to a specific tariff t on imports of the final product at the same stage^5.

3. **The Final Goods Market**

In considering the market for the final product in country 2, we abstract from the possibility that the final product is also sold in country 1. If the two markets are segmented, this involves no loss of generality. The price p of the final good in country 2 is given by the inverse demand curve p = p(Y) where p'(Y) < 0 and Y = y_1 + y_2 represents aggregate output. The quantities y_1 and y_2 of the final product are produced by firms 1 and 2 respectively. The total profit of firm 1 from the export of y_1 and the input x is,

\[ \pi^1 = (p(Y) - (t-s+c_1))y_1 + (r-v-c_1)x. \]  

(3.1)

Firm 2 purchases x at price r and produces its own supplies x_2 at a total cost C^2(x_2), so that its profit from the production of y_2 is,

\[ \pi^2 = p(Y)y_2 - rx - C^2(x_2), \]  

(3.2)

where C^2_{xx}(x_2) > 0. We first consider firm 2's choice between using its own or imported supplies of the input. Firm 2 produces x_2 ≥ 0 so as to maximize (3.2) for given levels of y_1, y_2 and r. Substituting x = y_2 - x_2 into (3.2), at the profit
maximum, \( x_2 \) satisfies:

\[
r - C_2^* (x_2) \leq 0 \quad (= 0 \text{ if } x_2 > 0)
\]  
(3.3)

If the marginal cost of production of the first unit of the input exceeds the import price \( r \), then firm 2 sets \( x_2 = 0 \) and produces using imported supplies only. If firm 2 produces the input, (3.3) implicitly defines the supply of the input as an increasing function of \( r \): \( x_2 = x^2(r) \) where \( x^2_r = 1/C^2_{xx} > 0 \).

At the stage 2 Cournot equilibrium for the final good, firm 1 sets its output \( y_1 \) to maximize (3.1), given \( y_2 \) and the prior committed values of \( r, t, s, \) and \( v \). Similarly, firm 2 chooses \( y_2 \) to maximize (3.2), given \( y_1, r, t, s \) and \( v \). The first order conditions are

\[
\pi_1^1(y_1, y_2, r, t-s, v) = p + y_1 p' - (t-s+c_1) = 0
\]  
(3.4)

\[
\pi_2^2(y_1, y_2, r) = p + y_2 p' - r = 0
\]  
(3.5)

Solving (3.4) and (3.5) simultaneously, we obtain the Cournot equilibrium levels of output as functions of \( r \), and \( t-s \):

\[
y_1 = y^1(r, t-s) \quad \text{and} \quad y_2 = y^2(r, t-s)
\]  
(3.6)

Country 1's export tax \( v \) does not appear directly in (3.6) because of the definition of \( r \) as the price actually paid by firm 2 for imported supplies. However, since the tax \( v \) reduces the amount that firm 1 nets for its exports of the input, it affects the equilibrium by changing the price charged for the input.

We make the assumption that own marginal profit declines with an increase in the output of the other firm. That is,

\[
\pi_{12} = p' + y_1 p'' < 0 \quad \text{and} \quad \pi_{21} = p' + y_2 p'' < 0
\]  
(3.7)

so that the reaction functions in output space have negative slopes, or equivalently, that the outputs produced by each firm are strategic substitutes. This assumption ensures that the second order conditions for profit maximization hold and that the Cournot equilibrium is unique: \( \pi_{11} < 0, \pi_{22} < 0 \) and \( H = \pi_{11} \pi_{22}^2 \)
\[-\pi_{12}^1\pi_{21}^2 > 0.\] As Bulow, Geanakoplos and Klemperer (1985) show, the outputs may be strategic complements under some demand conditions.

From total differentiation of (3.4) and (3.5), and from (3.7), the comparative static effects of an increase in the input price on \(y_1\) and \(y_2\) are:

\[y_1^1(r,t-s) = -\pi_{12}^1/H > 0 \quad \text{and} \quad y_2^2(r,t-s) = -\pi_{11}^2/H < 0\]  \hspace{1cm} (3.8)

Similarly, the responses of \(y_1\) and \(y_2\) to changes in the export subsidy \(s\) and import tariff \(t\) that firm 1 pays on its final product exports are:

\[y_1^1 = -y_1^1(r,t-s) = -\pi_{22}^1/H > 0 \quad \text{and} \quad y_2^2 = -y_2^2(r,t-s) = -\pi_{21}^2/H < 0.\]  \hspace{1cm} (3.9)

Also, from (3.8) and (3.9), industry output is decreasing in the input price, increasing in \(s\) and decreasing in \(t\):

\[Y_1(r,t-s) = p'/H < 0 \quad \text{and} \quad Y_s = -Y_t(r,t-s) = -p'/H > 0\]  \hspace{1cm} (3.10)

Of importance in what follows is the sign of \(\pi_{21}^1(y_1,y_2,r)\): the effect of an increase in firm 2's output on firm 1's profits holding \(y_1\) and \(r\) fixed. Since firm 2's production of the input is held constant with \(r\), the increase in \(y_2\) is associated with an equal increase in firm 1's exports of the input. Let \(M(r,t-s,v)\) equal \(\pi_{21}^1(y_1,y_2,r)\) where \(y_1\) and \(y_2\) are given by (3.6), then, from (3.1) and (3.4),

\[M(r,t-s,v) = r-v-c_1+y_1p' = (r-v-c_1) - (p-t+s-c_1).\]  \hspace{1cm} (3.11)

As shown by (3.11), \(M(r,t-s,v)\) is just the difference in profit margins from the export of the intermediate and final products. If this difference in profit margins is positive, firm 1's profits are increased by an increase in its rival's output for a given value of \(r\). From (3.7) and (3.10), an increase in the price of the input increases the price of the final product, but by less than one unit, ensuring that

\[M_1(r,t-s,v) = 1-p'Y_r = p'(2p'+Y_p)/H > 0.\]  \hspace{1cm} (3.12)

4. The Export Market for the Input: Vertical Foreclosure or Vertical Supply

Firm 2's derived demand for imported supplies is firm 2's output of the final good at the Cournot equilibrium less its own production of the input:
\[ x(r, t-s) = y^2(r, t-s) - x^2(r) \]  

where \( x_r = y_r^2 - x_r^2 < 0 \) from (3.8) and (3.3). Vertical foreclosure occurs if firm 1 charges a prohibitive price for the input, denoted \( r^p \), at which firm 2's demand for imported supplies is reduced to zero.

Setting \( x(r^p, t-s) = 0 \) implicitly defines\(^8\) the foreclosure price as a function of \( t-s \) where

\[ r^p = r^p_r(t-s) = -y^2_r/x_r < 0 \]

An increase in the export subsidy \( s \) and a reduction in the import tariff \( t \) both increase firm 1's exports of the final product, decreasing firm 2's output and the price \( r^p \) at which it ceases to use imported supplies.

We now consider the firm 1's stage 1 choice of the price \( r \) to charge its rival for the input. Taking into account the second stage relationships, firm 1's profit is a function (denoted \( \pi^E \) where \( E \) stand for the exporting firm) of the input price \( r \) as well as trade taxes and subsidies, \( t \), \( s \), and \( v \) set in stage 0:

\[ \pi^1 = \pi^E(r, t-s, v) = (p-t+s-c_1)y^1(r, t-s) + (r-v-c_1)x(r, t-s) \]

In stage 1, firm 1 sets the price \( r \) to maximize profit subject to \( x \geq 0^7 \).

At a vertical supply equilibrium, from (4.3) using (3.4),

\[ \pi^E_r(r, t-s, v) = (r-v-c_1)x_r + x + y_r^1p'y_r^2 = 0 \]  

Condition (4.4) defines the price charged for the input as a function \( r(t-s, v) \) of the government policies \( t, s \) and \( v \) set previously in stage 0. The first two terms of (4.4) represent the direct effect of an increase in \( r \) on firm 1's profits from the export of the input. The third (positive) term captures the 'strategic effect' of \( r \) on the profits earned from final product exports\(^8\). Since \( y_r^1 > 0 \) and \( y_r < 0 \), an increase in \( r \) increases both the volume and price of final product exports. Vertical foreclosure occurs if this strategic effect is sufficiently large.

A main concern is to determine the effect of conditions in the importing
country on the vertical supply or foreclosure decision. For this analysis, we assume that \( s = v = 0 \). The results then form a base from which to examine optimal policy by the exporting country. Firm 1 engages in vertical supply if and only if a reduction in \( r \) below the foreclosure price increases its overall profits: i.e. if and only if, from (3.11), (4.1) and (4.4) with \( x = 0 \),

\[
\pi_1^P(r^P, t, 0) = M(r^P, t, 0)y_1^2 - (r^P - c_1)x_1^2 < 0. \tag{4.5}
\]

From (4.5), firm 1 will always supply its rival if \( M(r^P, t, 0) > 0 \) at the foreclosure price: that is if it earns a higher profit margin from the export of the first unit of the input than from its final product exports. From (3.11), an increase in the rival's output then increases firm 1's profits holding the rival's own production of the input fixed. Setting \( y_2 = x^2(r^P) \) in (3.5),

\[
M(r^P, t, 0) = r^Pp + t = x^2(r^P)p' + t. \tag{4.6}
\]

From (4.6), \( M(r^P, t, 0) \) can be strictly positive only if country 2 imposes a tariff on final product imports.

To examine the effects of supply conditions, suppose, first, that it is prohibitively expensive to produce the input in country 2. With \( x^2(r^P) = 0 \), the foreclosure price \( r^P \) is equal to \( p \); firm 2 will enter as a producer of the final product only if the price charged for the input is below the price \( p \) of the final product. From (4.6), with no tariff on final product imports, \( M(r^P, 0, 0) = 0 \) and, from (4.5), firm 1 forecloses. This should not be surprising: with no production of the input in country 2, vertical foreclosure prevents the entry of firm 2, giving firm 1 a monopoly of the market for the final product. However, from (4.6), any small import tariff will make the difference in profit margins \( M(r^P, t, 0) = t > 0 \), which is sufficient for vertical supply. A tariff on final product imports gives the exporting firm an incentive to get 'under' the tariff wall by supplying the good produced at a lower stage of production⁹. These results are reported in
Proposition 1.

Proposition 1 (Assume s=v=0). Suppose that it is prohibitively expensive to produce the input in country 2.

(i) In the absence of a tariff, firm 1 will vertically foreclose.

(ii) A small tariff on imports of the final product will induce firm 1 to supply its rival with the input.***

Suppose now that local production of the input is profitable at the foreclosure price. In addition to the tariff, two aspects of local production conditions now prove to be important for the vertical supply decision: the total quantity \( x^2(r^F) \) of supplies available and the responsiveness of these supplies at the foreclosure price. If a fixed quantity of the input is available in the importing country, then vertical foreclosure no longer prevents the rival firm’s entry as a competitor in the market for the final product. This consideration might lead one to expect that firm 1 would now have a greater incentive to supply its rival. In fact, the reverse in the case: the local availability of a fixed quantity of the input tends to move the equilibrium towards vertical foreclosure.

With greater production of its own supplies at the foreclosure price, firm 2 is willing to pay less for the first unit of imported supplies and the difference in profit margins \( M(r^F,t,0) \) falls. As shown in proposition 2, if we assume linear demand and supply so as to abstract from secondary changes in the magnitude of responses \( y^2_t \) and \( x^2_t \), this translates into a lower profit from vertical supply. A tariff on imports of the final good still tends to induce vertical supply by improving the relative profitability of exports of the intermediate product, but a large tariff may now be required.

Proposition 2

(i) Under linear demand and supply conditions, an exogenous increase in firm 2's
production of the input at the foreclosure price increases firm 1's incentive for vertical foreclosure.

(ii) A sufficiently large tariff on imports of the final product will induce vertical supply. Under linear demand and supply conditions, any small increase in the tariff increases firm 1's incentive for vertical supply.

**Proof**: (i) Let \( x^2 = x^2(r, \alpha) \) where \( x^2_\alpha > 0 \) and \( \alpha \) is a shift parameter. Setting \( x(r^p, t, \alpha) = 0 \) defines \( r^p = r^p(t, \alpha) \) with \( r^p_\alpha = x^2_t/x_\alpha < 0 \). From (3.12), \( dM(r^p, t, v)/d\alpha = M_x r^p_\alpha < 0 \). If \( p''(Y) = x^2_{rr} = 0 \), then, from (4.5), \( d\pi^E_t(r^p, t, 0)/d\alpha = (dM/d\alpha)y^2_t - x^2_t r^p_\alpha > 0 \) increasing firm 1's incentive to foreclose. (ii) If \( x^2 = 0 \), from Proposition 1(ii), any \( t > 0 \) induces vertical supply. If \( \varepsilon_x > 0 \), from (4.6), \( M(r^p, t, 0) \geq 0 \) is sufficient for vertical supply. From (3.10), (3.12) and (4.2), \( dM(r^p, t, 0)/dt = M_x r^p_t 1-p'Y_t > 0 \). Let \( t^* \) denote the prohibitive tariff at which \( y_1(r^p, t^*) = 0 \). Then, from (3.4), \( p-t^*-c_1 = 0 \) and \( M(r^p, t^*, 0) = r^p-c_1 > 0 \). Hence, there is vertical supply at some \( t < t^* \). If \( p''(Y) = x^2_{rr} = 0 \), from (4.5), \( d\pi^E_t(r^p, t, 0)/dt = (dM/dt)y^2_t - x^2_t r^p_\alpha < 0 \) increasing the marginal profitability of vertical supply.***

To examine the implication of the responsiveness of supplies for the vertical supply decision, let \( \varepsilon_x = r x^2_t / x_2 \geq 0 \) represent the elasticity of supply of \( x_2 \) in country 2. Greater responsiveness is measured by an exogenous increase in \( \varepsilon_x \) at \( r^p \), maintaining \( r^p \) constant. There is thus no change in the quantity of firm 2's supply of the input or in final outputs at the foreclosure price. The increase in \( \varepsilon_x \) can be represented geometrically by a clockwise rotation in the supply curve for the input at the foreclosure price.

**Proposition 3** An exogenous increase in \( \varepsilon_x \) at the foreclosure price increases firm 1's incentive for vertical supply. Firm 1 chooses vertical supply if \( \varepsilon_x \) is sufficiently large.

**Proof**: From (4.5), holding \( r^p \) constant, \( d\pi^E_t(r^p(t), t, 0)/d\varepsilon_x = -x^2(r^p)(r^p-c_1)/r^p < 0 \).
A sufficiently large $\epsilon_r$ will make $\Pi_2^*(r^P,t,0) < 0$, inducing vertical supply.***

Why does a greater responsiveness of supplies in country 2 tend to move the equilibrium towards vertical supply? When local supplies of the input are responsive to price, a reduction in the price of imported supplies causes firm 2 to cut back on its own production, substituting imports for its own production of the input. Each unit of imports is then associated with a less than one unit increase in firm 2's output of the final product. Although firm 1's profits may be reduced by an increase in firm 2's output holding local supplies fixed, as $\epsilon_r$ is increased, the cut-back in firm 2's own production of the input eventually becomes sufficiently large that vertical supply is profitable. Expressing this in an alternative way: at a higher elasticity of local supply, a given reduction in the input price enables firm 1 to achieve a greater increase in its exports of the input, for the same increase in firm 2's output of the final product.

At the extreme, the elasticity of local supply of the input is infinite when firm 2's marginal cost $c_2$ is constant. The foreclosure price $r^P$ is then equal to $c_2$ provided that firm 2 preferentially uses its own supplies when imported supplies cost the same. In this situation, firm 1 can always gain (relative to vertical foreclosure) by exporting the input at a price $r$ just below $c_2$ so as just to deter the entry of firm 2 as a producer of the input. Firm 1 then earns positive profits from the export of the input, but vertical supply has no effect on firm 2's output of the final product so that firm 1's profit from final product exports is unchanged. Under some conditions, Firm 1 can further increase its profit by lowering its price for the input to the internal solution where the first order condition (4.4) is satisfied. In either case, vertical supply is the optimal strategy for firm 1.

The underlying conditions behind the vertical supply or foreclosure decision
are illustrated in Figure 1. The curve FF illustrates the boundary at which vertical foreclosure just occurs when \( \varepsilon_r \) and the tariff on final product imports are varied. The region of vertical foreclosure is shown by the shaded area on or below FF, whereas the area strictly above FF represents the region of vertical supply. Both an increase in \( \varepsilon_r \) and a higher tariff tend to move the equilibrium towards vertical supply. If demand and supply are not too non linear, a small increase in \( \varepsilon_r \) can be offset by a small decrease in the tariff, making FF negatively sloped.

![Figure 1](image)

Along FF in Figure 1, \( r^p \) satisfies \( \pi_r^2(r^p, t, 0) = 0 \) and, from (4.5),

\[
t = (p-r^p) - (r^p-c_1)\varepsilon_r/\eta_r,
\]

where \( \eta_r = -ry^2/y_2 > 0 \) represents the elasticity (with respect to an increase in \( r \)) of firm 2's derived demand for the input. If the tariff exceeds the price spread \( p-r^p \) at which the boundary FF intersects the vertical axis of Figure 1, then \( M(r^p, t, 0) = r^p-p+t \) is strictly positive, ensuring that the equilibrium is in
the region of vertical supply. Also, an increase in the quantity of supplies $x^2(r^p)$ increases the size of the price spread $p-r^p = -x^2(r^p)p'$. This shifts up the point at which FF intersects the vertical axis, thus increasing the area of vertical foreclosure. If the input is not produced in country 2, then $p-r^p = 0$, and FF reduces to a point at the origin. Firm 1 will then foreclose when the tariff is zero (at the origin), but a small tariff induces vertical supply.

We have shown that the sign of the profit margin condition $M$ evaluated at the foreclosure price is important for firm 1's decision to supply its rival. At a vertical supply equilibrium, the difference in profit margins is given by $M(r(t,0),t,0) - r(t,0) - p + t$. Proposition 4 considers the sign of $M(r(t,0),t,0)$, which subsequently proves to be significant for optimal policy by the exporting country.

**Proposition 4**

(i) The difference in profit margins at a vertical supply equilibrium is strictly positive if $\varepsilon_\tau$ is sufficiently small: $M(r(t,0),t,0) > 0$ if $\varepsilon_\tau < rx/(r-c_1)x_2$. Conversely, $M(r(t,0),t,0) < 0$ if $\varepsilon_\tau \geq rx/(r-c_1)x_2$.

(ii) If $M(r^p,t,0) \leq 0$ at foreclosure and $\varepsilon_\tau$ is sufficiently large to induce vertical supply, then $M(r(t,0),t,0) < 0$.

**Proof:** (i) Rearranging (4.4) using (3.4) and (4.1), at a vertical supply equilibrium,

$$\pi^*_\tau(r,t,0) = M(r(t,0),t,0)y^*_\tau + x - (r-c_1)x^*_\tau = 0.$$  \hspace{1cm} (4.8)

From (4.8), $M(r(t,0),t,0) > 0$ if and only if $x - (r(t,0)-c_1)x^*_2 > 0$. Rearrangement of this last expression yields the stated elasticity conditions. (ii) Since $M_\varepsilon > 0$ from (3.12) and $r(t,0) < r^p$ when there is vertical supply, it follows that $M(r(t,0),t,0) < M(r^p,t,0) \leq 0$.

Proposition 4(i) implies that the exporting firm earns a higher profit margin from the export of the input than the final product at a vertical supply equilibrium if the quantity of independent supplies of the input is not very
responsive to price. This equilibrium can arise only if the difference in profit margins $M(r^P, t, 0)$ is strictly positive at the foreclosure price (as a consequence of a positive tariff on final product imports). If $M(r^P, t, 0)$ is negative, Proposition 4(ii) implies that $M(r(t, 0), t, 0) < 0$ when there is vertical supply.

5. Optimal Export Policy

This section is concerned with optimal policy by the exporting country. We consider a subsidy $s$ on final good exports as well as a tax $v$ on exports of the input. The welfare or objective function in country $1$ is:

$$W(t-s, v) = \pi^P(r(t-s, v)) - sy_1(r(t-s)) + vx(r(t-s)).$$

(5.1)

Thus the objective of country $1$ is the same as the objective of firm $1$ in the absence of tax and subsidy payments. The role for government in this situation arises from the inability of the firm to commit to its level of exports of the final product in stage 1 so as to be a 'Stackelberg leader' in both export markets. By credibly committing to export policies in stage zero, the government can do what the firm itself is unable to do.

**Proposition 5** Optimal policy by the exporting country adjusts exports to the levels that would have occurred at $s=v=0$ if the exporting firm could commit to the quantity of its exports of the final product as well as to the export price of the input at stage 1*** (See the Appendix for the proof).

In choosing its strategy towards exports, the exporting firm is constrained by the requirement that $x \geq 0$. If this constraint is binding, the equilibrium reduces to the familiar one in which the Stackelberg leader expands its exports of the final product above the Cournot level and the input is not exported. The export market for the input becomes significant when this constraint is not binding. From (Al) in the Appendix, exports of the final product then satisfy,

$$d\pi^{1s}/dy_1 = (p+y_1p'-t-c_1) + (r-c_1+y_1p')dy_2/dy_1 = 0,$$

(5.2)
where \( \frac{dy_2}{dy_1} \) is the slope of firm 2's reaction function and the input price \( r \) is set optimally. From the second term of (5.2), the leader firm recognizes that a reduction in its rival's output increases its profits from final product exports (as in the standard Stackelberg model), but that, in addition, its profits from the export of the input fall.

From (3.4) and (5.2), at the Cournot equilibrium with \( s = v = 0 \), \( \frac{d\pi^{12}}{dy_1} = M(r, t, 0) \frac{dy_2}{dy_1} \) where \( \frac{dy_2}{dy_1} < 0 \) from our assumption that the products are strategic substitutes. If the difference in profit margins \( M(r, t, 0) \) is positive, the leader firm reduces its final product exports below the Cournot equilibrium level. From (3.11), firm 1's profits are then increased by an increase in firm 2's output: the additional profits from the export of the input more than offset the lower profits from final product exports. As we shall show, these are just the conditions under which country 1 has an incentive to tax, not to subsidize, final product exports. Conversely, if \( M(r, t, 0) < 0 \), the leader firm expands its exports above the Cournot equilibrium level. In setting its Cournot equilibrium level of exports, firm 1 takes the rival's output \( y_2 \) as given. Since \( y_2 \) falls in response to an increase in \( y_1 \), firm 1 is 'too aggressive' in increasing \( y_1 \) when \( M(r, t, 0) > 0 \), and 'insufficiently aggressive' when \( M(r, t, 0) < 0 \).

**Export Taxes or Export Subsidies**

The exporting country sets \( s \) and \( v \) to maximize welfare as in (5.1). As shown in Appendix A, the optimal values of \( s \) and \( v \) satisfy the conditions (5.3), (5.4) and (5.5) that follow.

As in the previous Stackelberg analysis, if the constraint \( x \geq 0 \) is binding, then exports of the input are zero and the equilibrium reduces to the familiar single market case. The subsidy on final product exports then satisfies,

\[
W_s = y_1 p' \frac{dy_2}{ds} - sd_1/ds = 0
\]  
(5.3)
where \( dy_1/ds > 0 \). If firm 2 enters as a producer of the final product, then \( dy_2/ds < 0 \) and, from (5.3), the optimal subsidy is positive. This is just the Spencer and Brander (1983) result that an export subsidy increases domestic welfare when there is Cournot competition between a foreign and a domestic firm. With no exports of the input, a change in the tax \( v \) applied to exports of the input has no effect.

In the region of vertical supply, the optimal export tax \( v \) on the input satisfies, \( W_v = (vx_r - sy_r^1)r_v = 0 \). At the optimum, we can define,

\[
v(s) = sy_r^1/x_r.
\] (5.4)

From (5.4), if \( s \) equals zero, then \( v \) equals zero. Since the exporting firm is able to commit to the price charged for the input in stage 1, the gain from government intervention in the export market for the input arises only as a consequence of intervention in the export market for the final product.

When \( v \) is set optimally at \( v(s) \), the optimal value of \( s \) satisfies

\[
dW(s,v(s))/ds = (r(t-s,v)-p+t)y_s^2 - sY_s = 0.
\] (5.5)

where the 'social difference' in profit margins \( r(t-s,v)-p+t \) reduces to the private difference in profit margins \( M(r(t,0)) \) prior to policy by the exporting country. It follows that whether a subsidy or tax on final goods exports increases welfare depends on the sign of \( M(r(t,0)) \). Proposition 6 sets out country 1's jointly optimal policy towards both exports if there is initial vertical supply.

**Proposition 6** It is optimal for the exporting country;

(i) to tax the exports of both products if \( M(r(t,0),t,0) > 0 \).

(ii) to subsidize the exports of both products if \( M(r(t,0),t,0) < 0 \).

(iii) not to intervene if \( M(r(t,0),t,0) = 0 \).

**Proof:** From (5.5) at \( s=v=0 \), \( dW/ds < 0 \) if \( M(r(t,0)) > 0 \) and vice versa. \( dW/ds = 0 \) if \( M(r(t,0)) = 0 \). From (5.4), we have \( v > 0 \) if \( s < 0 \) and vice versa.***
As Proposition 6 shows, active commercial policy requires a subsidy to both exports or a tax to both exports. It is never optimal to subsidize exports of the final product and to tax exports of the input or vice versa. We know from Proposition 4(i) that \( M(r(t,0), t, 0) > 0 \) if \( \varepsilon_r \) is not too large and there is vertical supply as a consequence of a tariff set by country 2. From (5.2), a Stackelberg leader would then restrict its exports of the final product below the Cournot equilibrium level so as to gain additional profits from the export of the input. From Proposition 6(i), country 1 gains by taxing both exports in this case. High values of \( \varepsilon_r \) are associated with the subsidization of both exports.

To explain the need to tax (or subsidize) both exports, first consider that if firm 1 initially earns a higher profit margin from the export of the input than the final product, then, at any given price for the input, a tax on final product exports serves to switch sales to the more profitable export market for the input. Such a tax, however, creates a wedge between the exporting firm's objective function and welfare in country 1. Firm 1 sets the price for the input fully taking into account the effect of this price on its profits from final goods exports. If these latter exports are taxed, firm 1 sets the input price below the optimal level so as to reduce its exports of the final product and the total tax paid. This distortion can be corrected if country 1 also imposes a tax on exports of the input. At the optimal value of this tax, from (5.4), firm 1 cannot affect the net tax revenue that it pays to country 1 by altering the price that it charges for the input. A similar but opposite argument applies if the difference in profit margins from the export of the intermediate and final products is initially positive.

Since \( s \) is larger than \( v \) in absolute value (from (5.4) and \( |y^2_r/x_r| < 1 \)), the direction in which commercial policy aims to switch trade flows is generally
indicated by the sign of $s$. There is some ambiguity in the response of exports to a change in $s$ when $v$ is set optimally, but under linear demand and supply conditions, an increase in $s$, maintaining $v = v(s)$, causes a net expansion in exports of the final product and a reduction in exports of the input\textsuperscript{12}. If $s$ is positive, then both exports are subsidized, but sales of the final good are given relatively more encouragement. Since exports of the final good are subsidized if and only if firm 1 earns a lower profit margin on intermediate than final product exports, it follows that the net effect of government intervention is to widen or amplify the initial difference in profit margins. With linear demand and supply, optimal policy then serves to expand the export of the good with the higher profit margin and to contract the export of the good with the lower profit margin.

**Government Foreclosure or Supply**

We now turn to the question as to whether 'government foreclosure' could increase welfare in the exporting country. In other words, if the conditions for vertical supply obtain in the absence of policy by the exporting country, does optimal policy simply serve to adjust the levels of exports within the region of vertical supply, or can a more radical shift from vertical supply to vertical foreclosure be expected?

We link the two branches of policy by defining a critical subsidy $s^F$ on final product exports at which vertical foreclosure just occurs: i.e. $\pi^F(r^F,t-s^F,v(s^F)) = 0$ where the export tax on the input is set optimally at $v(s^F)$. As we show, under linear demand and supply conditions, a small increase in $s$ above $s^F$ maintains vertical foreclosure, whereas a small reduction in $s$ below $s^F$ induces vertical supply (see the proof of Proposition 7)\textsuperscript{13}. The profit margin difference $M(r^F,t,0)$ at the foreclosure price was shown to be important for firm 1's
vertical foreclosure or supply decision. It is also important for the equivalent decision by the exporting country.

**Proposition 7**: (Assume \( p''(Y) = 0 \) and \( x_{rr}^2 = 0 \))

(i) If \( M(r^p, t, 0) > 0 \), then there is vertical supply at \( s = v = 0 \), and optimal policy by the exporting country maintains vertical supply.

(ii) If \( M(r^p, t, 0) < 0 \) and there is vertical supply at \( s = v = 0 \), optimal policy by the exporting country will induce vertical foreclosure when the subsidy \( s^f \), needed to induce foreclosure, is sufficiently small.

(iii) If there is vertical foreclosure at \( s = v = 0 \), then optimal policy by the exporting country maintains vertical foreclosure.

The proof of Proposition 7 is provided in the Appendix.***

From (4.6), the difference in profit margins at foreclosure can be positive only as a consequence of the tariff \( t \) set by the importing country. In this case, from Proposition 7(i), government foreclosure will not occur: the exporting country does not overturn the initial vertical supply decision. Thus, by setting a sufficiently large tariff to make \( M(r^p, t, 0) > 0 \), the importing country can rely on a continuation of supply, even if the exporting country responds by imposing its optimal policies. Indeed, if the input is not produced in the importing country, from Propositions 4(i) and 6, the exporting country has an incentive to tax final product exports, which tends to increase the extent of vertical supply. More generally, it is possible that policy by the exporting country will reduce exports of the input, but not sufficiently to induce vertical foreclosure.

If \( M(r^p, t, 0) < 0 \), from Proposition 3, firm 1 will supply its rival if supplies produced in the importing country are sufficiently responsive to price. From Proposition 7(ii), government foreclosure is a then a possibility if the necessary subsidy \( s^f \) is small. The exporting country may subsidize the exports of the final
product to such an extent that it is no longer profitable for firm 1 to export the input. If firm 1 chooses vertical foreclosure initially, then, from Proposition 7(iii), it is always in the exporting country's interest to maintain foreclosure.

6. Bertrand Behaviour

In considering Bertrand behaviour, we assume that each firm produces a differentiated final product. Firm 2 can then earn positive profits despite a higher marginal cost of production. The demand functions for the final substitute products are represented by \( y_1 = q^1(p_1, p_2) \) and \( y_2 = q^2(p_1, p_2) \) where the outputs \( y_1 \) and \( y_2 \) have prices \( p_1 \) and \( p_2 \) respectively. Own price effects \( q^1_1 \) and \( q^2_2 \) are assumed to be negative and cross price effects \( q^1_2 \) and \( q^2_1 \) positive. Firm 1's profit is given by (3.1) where \( p(Y) \) is replaced by \( p_1 \) and \( x = q^2(p_1, p_2) - x^2(r) \). The first order condition for the second stage choice of \( p_1 \) by firm 1 given \( p_2 \) and \( r \), \( t-s \), and \( v \) (where the superscript \( B \) stands for Bertrand) is

\[
\pi^B_1(p_1, p_2, r, t-s, v) = y_1 + (p_1 - t+s-c_1)q^1_1 + (r-v-c_1)q^2_2 = 0. \tag{6.1}
\]

The presence of the export market for the intermediate product has a fundamental effect on firm 1's choice of the price \( p_1 \) for its final product under Bertrand competition. As shown by the positive third term of (6.1), firm 1 recognizes that, for a given value of \( p_2 \), an increase in \( p_1 \) will increase its profits from the export of the input by raising its rival's level of production of the final good.

The second stage choice of \( p_2 \) by firm 2 satisfies the standard Bertrand first order condition: \( \pi^B_2 = y_2 + (p_2 - r)q^2_2 = 0 \). The products are assumed to be strategic complements, \( \pi^B_{12} > 0 \) and \( \pi^B_{21} > 0 \), so that the reaction functions in price space have positive slopes. We assume the second order and uniqueness conditions hold.

To relate the Bertrand results to those that would be obtained with Cournot
competition for the final differentiated products, it is useful to define the
inverse demand functions \( p_1 = p^1(y_1, y_2) \) and \( p_2 = p^2(y_1, y_2) \). These functions have
partial derivatives (from total differentiation of the direct demand functions),
\[
p_1^1 = q_2^3/Q < 0, \quad p_1^2 = -q_3^2/Q < 0, \quad p_1^2 = -q_2^2/Q < 0 \quad \text{and} \quad p_2^1 = q_1^1/Q < 0 \quad (6.2)
\]
where \( Q = q_1^1q_2^2 - q_1^2q_2^1 > 0 \). By totally differentiating \( \pi^1(y_1, y_2, r) \) with respect to
\( p_1 \), holding \( p_2 \) fixed, (6.1) can be written in the form,
\[
\pi^1_{12}(p_1, p_2, r, t-s, v) = \pi^1_1(y_1, y_2, r, t-s, v)q_1^1 + Mq_1^2 = 0, \quad (6.3)
\]
where \( M = \pi^1_2(y_1, y_2, r) = r-v-c_1+y_1p_2^1 \) is the effect of an increase in the rival’s
output on firm 1’s profit holding \( y_1 \) and \( r \) fixed. As before, \( M \) can be expressed
in terms of the ‘difference in profit margins’. Substituting for \( y_1 \) from (6.1) and
using (6.2), at the Bertrand equilibrium, \( M = MB(r, t-s, v) = q_1^1p_1^1[(r-v-c_1)-(p_1-t+s-
-c_1)p_2^1/p_1^1] \). Similarly, at the Cournot equilibrium, setting \( \pi^1_1(y_1, y_2) = 0 \) to obtain
\( y_1, M = MC(r, t-s, v) = (r-v-c_1)-(p_1-t+s-c_1)p_2^1/p_1^1 \), which is the actual difference in
profit margins when the products are homogeneous. Since \( MB \) and \( MC \) are
evaluated at different prices and outputs, it is possible that they differ in
sign\(^{14}\).

We next examine the incentives for vertical supply when there is Bertrand
competition at the final stage. Taking the total derivative of \( \pi^1(y_1, y_2, r) \) with
respect to \( r \) and imposing (6.3), firm 1 sets \( x > 0 \) when \( s=v=0 \) if, at \( r^P \),
\[
\frac{d\pi^1_{12}}{dr} = MB(r^P, t, 0)[y_2^2 - (q_2^2/q_1^1)y_1^2] - (r^P - c_1)x_r^2 < 0. \quad (6.4)
\]
From (6.4), there is no change in the factors underlying the vertical supply
decision. From \( p_1^2/p_2^2 = -q_1^2/q_2^1 \) and (6.2), we obtain \( y_2^2 - (q_2^2/q_1^1)y_1^2 = (dp_2/dr)/p_2^2 < 0 \)
so that the condition \( MB(r^P, t, 0) > 0 \) is sufficient (but not necessary) for
vertical supply. By similar reasoning as in the Cournot case, both an import
tariff on the final good set by country 2 and an increase in \( \epsilon_r \) tend to increase
the region of vertical supply. Moreover, an increase in \( x^2(r^P) \) again reduces the
foreclosure price tending to make vertical supply less profitable.

The policy incentives for the exporting country can again be seen most easily by examining the decisions that would be made by the exporting firm if it were a Stackelberg leader in both markets. Rearranging (5.2) using (6.3) (with \( p' \) replaced by \( p_2' \)), at a vertical supply equilibrium, \( y_1 \) satisfies,

\[
d\pi^{1*}/dy_1 = \pi^{1*}_1(p_1, p_2, r, t, 0)/q_1^1 + (r-c_1 + y_1p_2')(dy_2/dy_1-q_2^2/q_1^1) = 0. \tag{6.5}
\]

The term \( q_2^2/q_1^1 \) represents firm 1's Bertrand conjectural variation in output space: it is the ratio of the change in outputs from an increase in \( p_1 \) holding \( p_2 \) fixed. Since the products are assumed to be strategic complements, the 'conjectured' reduction in firm 2's output from an increase in \( y_1 \) exceeds the actual reduction in output making \( dy_2/dy_1-q_2^2/q_1^1 > 0 \).

At the Bertrand equilibrium at \( s-v=0 \), from (6.1) and (6.5), \( d\pi^{1*}/dy_1 = M^B(r, t, 0)(dy_2/dy_1-q_2^2/q_1^1) \). If \( M^B(r, t, 0) > 0 \) then \( d\pi^{1*}/dy_1 > 0 \): profits increase if final product exports are increased above the Bertrand equilibrium level. Conversely, if \( M^B(r, t, 0) < 0 \), the exporting firm gains if exports of the final product are restricted below the Bertrand level. In setting its Bertrand equilibrium price \( p_1 \), firm 1 takes the rival's price \( p_2 \) as given. If \( M^B(r, t, 0) > 0 \), an increase in \( y_2 \), or, equivalently, a reduction in \( p_2 \), increases firm 1's profits from exports of intermediates by more than it reduces profits from final product exports. Since \( p_2 \) falls in response to a reduction in \( p_1 \), firm 1 is then 'not sufficiently aggressive' in reducing \( p_1 \). Conversely, when \( M^B(r, t, 0) > 0 \), firm 1 is "too aggressive" in reducing \( p_1 \).

At a vertical supply equilibrium (as shown in the Appendix), the optimal subsidy \( s \) on final product exports and the optimal export tax \( v \) on the input is

\[
s = (r-c_1 + y_1p_2')(dy_2/dy_1-q_2^2/q_1^1)/z \quad \text{and} \quad v(s) = sy_2^2/x_r, \tag{6.6}
\]

where \( z = [1-(q_2^2/q_1^1)(y_2^2/x_r)] > 0 \). From (6.6), \( v(s) \) is identical to (5.4) in the
Cournot case. It follows that if the exporting country subsidizes final exports, then it also subsidizes exports of the input and vice versa. Whether there is Bertrand or Cournot competition at the final stage, it is never optimal to tax the exports of one product while subsidizing the exports of the other.

As (6.6) shows, the signs of optimal policies (in the Bertrand case) are reversed if the final products are strategic substitutes (making \( dy_2/dy_1 - q_2^2/q_1 < 0 \)) rather than strategic complements. Moreover, if there is Cournot competition (with strategic substitutes) rather than Bertrand competition (with strategic complements) at the final stage, then, from (6.5) and (5.2), the nature of optimal policy is also reversed provided \( M^B \) and \( M^C \) have the same sign. Eaton and Grossman (1986) show that policy incentives by the exporting country are always reversed when there is a single market, but this is not necessarily the case here.

7. **Conclusion**

Many large manufacturing firms have secured their access to important intermediate inputs by integrating backwards so as to produce the input within the corporation. If the input can be produced more cheaply in one country than another, then vertical integration can give firms in one country a cost advantage relative to foreign rivals. This leads to the question as to whether high cost manufacturers need be concerned about dependence on imports of a key input from a country that is also a major exporter of the final manufactured product.

This paper first addresses this issue by examining the conditions under which a low cost vertically integrated manufacturer will export an intermediate product, lowering the costs of a rival producer of the final product. We show that, for either Cournot or Bertrand competition at the final output stage, the incentive for vertical supply is increased both by a greater responsiveness of supplies in the importing country and (more surprisingly) by a reduction in the
quantity of supplies available at foreclosure. Also, an importing country can induce vertical supply by imposing a sufficiently large tariff on final product imports.

Secondly, we consider the implications of optimal government policy by the exporting country. When there is Cournot competition for the homogeneous final products, policy by the exporting country tends to reinforce existing private incentives as measured by the difference between the profit margins from the export of the input and the final product. If this difference in profit margins is initially positive, then optimal policy tends to increase the extent of vertical supply. In the reverse case, optimal policy tends to move the equilibrium towards foreclosure. It does not pay the government to overturn a private decision to foreclose, but it may move to impose foreclosure when the private firm wishes to supply its rival with the input.

It is a familiar result from the analysis of duopoly behaviour for a single product that a Cournot firm sets output levels too low, and a Bertrand firm too high, relative to a firm that can behave as a Stackelberg leader. In our model, optimal policy by the exporting country encourages the exporting firm to expand sales in the market with the higher profit margin in the Cournot case and to de-emphasise sales in such a market in the Bertrand case. Since the specific nature of optimal policy depends on the nature of competition at the final stage, we would not expect these results to be directly useful for practical policy by the exporting country and our analysis is not intended for this purpose. In a world of imperfect information and imperfect governments, activist policy often provides an avenue for socially wasteful rent seeking. It nevertheless seems a worthwhile objective to gain some understanding of the factors that are important determinants of vertical supply.
Overall, our results indicate that taking into account both private and public incentives, in a broad class of cases a high cost firm need not be concerned about full vertical foreclosure. Also, it seems reasonable to conjecture that if there were more than one exporting firm, competition between exporters would result in a further increase in vertical supply. However, even if there is vertical supply, the price of imported supplies will generally be high. High cost firms will not fully overcome their initial cost disadvantage in the market for the final product by importing supplies from a low cost foreign rival.

Another direction in which the results could be generalized would be to consider the potential for bargaining between the low cost and high cost firm concerning the price and quantity of exports of the intermediate product. If country 2 imposes a tariff on final product imports, the joint profit maximizing solution is for firm 1 to export the input only, giving the local firm in country 2 a monopoly of the market for the final product. However, this solution would require non linear pricing and may be difficult to enforce. Merger between the two firms would seem to be a better means of achieving this outcome, but it may be ruled out by antitrust policy. Also, if the possibility is admitted that there may be more than one rival firm in the importing country, it may not be possible to monopolize the industry fully, thus making the merger solution less attractive.
APPENDIX

We first set out the conditions for the 'Stackelberg' equilibrium and then derive the conditions (5.3), (5.4) and (5.5) of the text characterizing optimal policy by the exporting country. We next combine the two sets of conditions to prove Proposition 5. Subsequently, we prove Proposition 7 and finally we derive the optimal export policies with Bertrand competition at the final stage.

**Stackelberg Equilibrium**

Suppose that the exporting firm is able to commit to both $y_1$ and $r$ in stage 1, and firm 2 is a follower, setting its output in stage 2. From (3.5), $y_2$ is defined as a function $f^2(y_1,r)$ with partial derivatives $f^2_x = dy_2/dy_1 = -\pi^{22}/\pi^{22} < 0$ and $f^2_r = 1/\pi^{22} < 0$. The values of $y_1$ and $r$ are chosen to maximize profits subject to $x = f^2(y_1,r)-x^2(r) \geq 0$. Let $L(y_1,r,\theta) = \pi^1 + \theta x$ where $\theta \geq 0$ is the Lagrange multiplier. At $s=v=0$, $y_1$ and $r$ respectively satisfy:

$$L_1(y_1,r,\theta) = p+y_1p'-t-c_1 + (r-c_1+y_1p'+\theta)f^2_x = 0 \quad (A1)$$

$$L_2(y_1,r,\theta) = (r-c_1+y_1p'+\theta)f^2_r -(r-c_1+\theta)x^2_r + x = 0 \quad (A2)$$

where $x = 0$ if $\theta > 0$.

**Optimal Policy by the Exporting Country**

The exporting country sets $s$ and $v$ to maximize (5.1), subject to the constraint $x(r,t,s) \geq 0$. Let $L^* = W(t-s,v) + \mu x(r,t,s)$ where $\mu \geq 0$ represents the Lagrange multiplier. At the maximum, $s$ and $v$ satisfy the first order conditions:

$$L^*_s = W_s + \mu dx/ds = 0, \quad L^*_v = W_v = 0 \quad \text{and} \quad L^*_\mu = x \geq 0 \quad (= 0 \text{ if } \mu > 0). \quad (A3)$$

If $\mu > 0$, then $x = 0$ and $dx/ds = x_r f^2_s + y^2_s = 0$, so that $L^*_s = W_s$ for $\mu \geq 0$. We assume that the second order conditions, $W_{ss} < 0$, $W_{vv} < 0$, and $W_{ss}W_{vv} - (W_{sv})^2 > 0$ are satisfied. These conditions hold if $p''(Y) = 0$ and $x^2_{rr} = 0$.

From (A3) and (5.1), the optimal values of $s$ and $v$ satisfy,
where \(dr/ds = r_s(t-s,v)\) and \(dr/dv = r_v(t-s,v)\) if \(r < r^p\). At \(r^p\), \(dr/ds = r^p_s\) and \(dr/dv = 0\). Since \(\pi^s = 0\) when \(r_v > 0\), the term \(\pi^s_dr/dv = 0\). From (4.3) and (3.4),

\[
\pi^s_e(r,t-s,v) = (r-v-c_1)y^2_s + y_1 + y_1p'y^2_s \quad \text{and} \quad \pi^s_v(r,t-s,v) = -x < 0.
\]  (A6)

If \(\mu > 0\), then \(r = r^p\) and using (A6), \(\pi^s_e(r^p,t-s,v) = (r-v-c_1)x_r + y_1p'y^2_r\) and \(dx/ds = 0\), (A4) becomes \(W_s = y_1p'dy_1/ds-sdy_1/ds = 0\) which is (5.3) of the text. Since \(dx/ds = dy_2/ds-x^2_r r^p_s = 0\), we have \(dy_2/ds = x^2_r r^p_s < 0\) if \(x^2_v(r^p) > 0\). If \(\mu = 0\), then \(x \geq 0\), (which includes the region of vertical supply) and, using (A6), (A5) reduces to \(W_v = (vx_r-sy^2_r)r_v = 0\) so that \(v(s) = sy^2_r/x_r\) as in (5.4) of the text. Using \(\pi^s = 0\), (A4), (A5) and (A6), \(dW(s,v(s))/ds = (r-c_1+y_1p')y^2_s-xy^2_r = 0\), which, using (3.4) and (5.4), reduces to (5.5) of the text.***

Proof of Proposition 5:

Suppose that the constraint \(x \geq 0\) is not binding so that \(\theta = 0\). Substituting the optimal value of \(s = (r-c_1+y_1p')y^2_s/y^1_s\) into (3.4), and using \(y^2_s/y^1_s = f^2_1\), it follows that the first order condition (3.4) is equal to (A1) proving that \(y_1\) is the same in both cases. Similarly, from (5.4), (5.5), and using \(y^2_r = f^2_1y^1_r+f^1_1\), (4.4) reduces to (A2): the input price \(r\) is the same in both cases. If the constraint \(x \geq 0\) is binding, then \(\theta > 0\) and from (A2) and (A1), \(L_1(y_1,r,\theta = (p+y_1p'-t-c_1)-y_1p'x^2_r f^2_1/(f^2_1-x^2_r) = 0\). From \(dx/ds = f^2_1dy_1/ds+(f^2_1-x^2_r) r^p_s = 0\) and \(dy_2/ds = x^2_r r^p_s\), it follows that (A1) is equal to (3.4) where \(s\) is given by (5.3).***

Proof of Proposition 7:

At \(s = s^F\), setting \(x = 0\) in (4.4), and using (3.4) and (4.1),

\[
\pi^s_e(r^p,t-s^F,v(s^F)) = M(r^p,t-s^F,v(s^F))y^2_r - (r-v(s^F)-c_1)x^2_r = 0.
\]  (A7)

If \(p'' = 0\), and \(s\) is increased above \(s^F\) holding \(v = v(s^F)\) fixed, then, from \(\pi^s_{rr} < 0\), \(M_s = -(1+p'Y_s) < 0\) and \(r^p_s < 0\), we have \(d\pi^s_r/ds = \pi^s_{rr} r^p_s M_s y^2_r > 0\) and vertical
foreclosure is maintained. If \( s \) is reduced below \( s^F \), using \( v(s) \) as in (5.4) and \( Y_s = Y_r \), we obtain \( d\pi^F_r/\text{ds} = \pi^F_{rr}r^p - Y_r(1-p'y_r^2) > 0 \): a small reduction in \( s \) below \( s^F \) induces vertical supply. This implies that if there is vertical supply at \( s_v=0 \), then \( s^F > 0 \), whereas \( s^F \leq 0 \) if \( x = 0 \) initially.

(i) From (4.5), \( M(r^P,t,0) > 0 \) is sufficient for \( x > 0 \) at \( s_v=0 \) so that \( s^F > 0 \). From (4.2), \( Y_s = -Y_r = -1/3p' \) and \( y_r^2 = -y_s^2 \) for \( p'' = 0 \), we obtain \( d(r^P-p+t)/\text{ds} = x_r^2/3x_r > 0 \). If \( s^F > 0 \) and \( M(r^P,t,0) \geq 0 \), it follows that \( r^P(t-s^F)-p+t > 0 \). Hence, from (5.5), \( dW/\text{ds} < 0 \) at \( s = s^F > 0 \): a reduction in \( s \) below \( s^F \) increases welfare. (ii) From (5.5), \( dW/\text{ds} \geq 0 \) at \( s^F > 0 \) if \( 0 < s^F \leq (r^P(t-s^F)-p+t)y_s^2/Y_s \). Since \( r^P(t-s^F)-p+t > 0 \) when \( M(r^P,t,0) \geq 0 \) (see (i) above), we have \( M(r^P,t,0) < 0 \). From (A7) using \( v(s^F) = s^Fy_1^2/x_r \), we obtain \( r^P(t-s^F)-p+t = [s^Fy_1^2+(r^P-c_1)x_r^2]/y_r^2 \). Hence \( dW/\text{ds} > 0 \) at \( s^F > 0 \) if \( 0 < s^F \leq \Gamma x_r \) where \( \Gamma = [(r^P-c_1)x_r^2/r^P]/y_r^2(Y_s - Y_r^2y_s^2) > 0 \).

(iii) If \( x = 0 \) at \( s_v=0 \), then \( M(r^P,t,0) \leq 0 \) from (4.4). Since \( s^F \leq 0 \) and \( d(r^P-p+t)/\text{ds} > 0 \) (see (i) above), it follows that \( r^P(t-s^F)-p+t \leq 0 \). From (5.5), we then have \( dW/\text{ds} \geq 0 \) at \( s = s^F \): it is optimal to maintain vertical foreclosure.

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Bertrand Behaviour: derivation of \( s \) and \( v \) as in (6.6)

From (6.1), \( \pi^B_1(p_1,p_2,r,t-s,v) = \pi^B_1(p_1,p_2,r,t,0) + sq_1vq_1^2 = 0 \). Assuming vertical supply at the optimum, \( \theta = 0 \), and from (6.5),

\[
dr/\text{dy}_1 = -[s-v(q_1^2/q_1^2)] + (r-c_1+v_1p_2)[dy_2/\text{dy}_1-q_2^2/q_1^2] = 0 \tag{A8}
\]

From (6.4), at a vertical supply equilibrium, \( d\pi^B/\text{dr} = (r-v-c_1+v_1p_2)[y_1^2-(q_1^2/q_1^2)y_1^2] \)

- \( (r-v-c_1)x_r^2+x = 0 \). Hence, from (A2) (with \( p' \) replaced by \( p_2^2 \)), and using \( y_r^2 = (dy_2/\text{dy}_1)y_1^1+f_r^2 \), at the optimal values of \( s \) and \( v \),

\[
dr/\text{dr} = -(r-c_1+v_1p_2)[dy_2/\text{dy}_1-q_2^2/q_1^2]y_1^2 + v[x_r^2-(q_1^2/q_1^2)y_1^2] = 0 \tag{A9}
\]

From (A8), (A9) reduces to \( d\pi^B/\text{dr} = vx_r^1sy_1^1 = 0 \), which implies that \( v(s) = sy_1^1/x_r \). The optimal value of \( s \) given in (6.6) then follows from (A8).
FOOTNOTES

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1. This action was facilitated by a U.S. anti-dumping action against certain computer chips from Japan. However, the price increase for DRAM computer chips was more than required by the trigger price anti-dumping measure. The U.S. policy encouraging the Japanese to restrict the export of computer chips is hard to defend on the basis of the analysis developed in this paper.

2. In Quirmbach (1986), vertical supply (or partial forward vertical integration) by an upstream monopolist can be an equilibrium strategy due to diminishing returns in the monopolists's downstream subsidiary. In Katz (1987), downstream firms can integrate backwards to produce the input. The alternative cost of producing the input affects the price charged for the input by an upstream monopolist, but the monopolist does not produce the final product. Greenhut and Ohta (1979), consider vertical integration by a subset of oligopolists, but vertical supply is not an issue. See Tirole (1988) for a discussion of the literature.

3. With substitutability between inputs, an increase in the price of imported supplies would cause the rival firm to substitute away from the higher priced input. In our model, the rival firm substitutes away from higher priced imports by producing its own supplies so that a relaxation of the fixed proportions
assumption should not change the general nature of the results.

4. We follow the usual practice of relating the output of each firm to the vector of marginal costs. If firm 1 commits to its exports $x$ rather than the price $r$ at stage 1, our existing analysis with respect to $r$ would apply, but, in addition, we require the relationship of $x$ to the market price $r$.

5. The tariff $t$ is not chosen optimally. In Spencer and Jones (1989), we use a similar model to consider optimal policy by the importing country.

6. The quantity of exports of the input reduces smoothly to zero as $r$ increases to $r^p$ so that there is no discontinuity in $x(r, t-s)$ at $r^p$.

7. To obtain the conditions for a maximum, define $L = \pi^x + \mu x(r, t-s, v)$ where $\mu$ represents the Lagrange multiplier. The first order conditions are then: $L_r = \pi^x_r - \mu x_r = 0$ and $L_\mu = x \geq 0$, $\mu \geq 0$, $L_\mu \cdot \mu = 0$. We assume that $\pi^x$ is strictly concave for all $r \leq r^p$, ensuring that $\pi^x$ achieves a global maximum whenever the first order conditions are satisfied. If demand and supply are linear ($p''(Y) = 0$ and $x_{rr}^2 = 0$), then $\pi_{rr}^x = y_r^2(2-p'Y_r)-2x_r^2 < 0$ except at $r = c_2$ where $\pi_{rr}^x$ is continuous but not differentiable. However, $\pi^x$ remains strictly concave in $r$ at $c_2$, since $x_r^2 = 0$ for $r \leq c_2$ and $x_r^2 < 0$ for $r \geq c_2$, making the left hand derivative $\pi_{rr}^x$ less negative than the right hand derivative.

8. Bulow, Geanakoplos and Klemperer (1985) have a general determination of the sign of such 'strategic' terms based on whether products are strategic substitutes or complements and whether there are joint economies or diseconomies across markets. In our model there are joint diseconomies across markets, since an increase in the level of exports of the input reduces the marginal profitability of exports of the final product.
9. As Spencer and Jones (1989) show, under linear demand conditions, the tariff on final product imports is also an effective tool in reducing the price charged for imported supplies in the region of vertical supply.

10. This result is related to a Katz and Shapiro (1985) proposition concerning the licensing of a superior technology to a rival Cournot duopolist under constant returns to scale. Licencing occurs because the per unit royalty charge can be set so as to leave the rival's marginal cost unaffected (as in our model).

11. Setting $x_2 = 0$ and $x = y_2$ in (4.4), firm 1 sets $r$ just below $c_2$ if (4.4) is strictly positive at this price: the constraint ($r < c_2$) is then binding. If $t$ is large, the internal solution at which (4.4) equals zero may be optimal.

12. An increase in $s$, where $v = v(s)$, affects $y_1$ and $x$ partly through its effect on $r$. From (4.4), $r_v = x_2/y_{rr}^2 > 0$. However $r_s$ is ambiguous in sign; an increase in $s$ reduces firm 2's demand for the input ($y_s^2 < 0$), but $r$ does not necessarily fall. There is a similar ambiguity in the pricing response of a monopolist to an increase in demand. If $p''(Y) = 0$, from $y_s^1 = -2y_s^2$, $y_r^2 = 2/3p'$ and (4.4), we have $\pi_{rs} = y_s^2 + p'y_sy_r^2 = -y_s^2/3 > 0$. Hence, $r_s = -\pi_{rs}/\pi_{rr} = y_s^2/3\pi_{rr} > 0$. From (5.4) and $y_r^1 = -y_s^2$, we obtain $dr/ds = r_s + r_vv'(s) = -2y_s^2/3\pi_{rr} < 0$. Using $x_r^2 = 0$ and $\pi_{rr} = x_r + y_s^2(1-p'Y_r) - x_r^2 < 0$, it can then be shown that $dy_1/ds > 0$ and $dx/ds < 0$.

13. The welfare function is continuous and differentiable at $s^F$. At $s^F$, using $\pi_r^F(r^F, t-s^F, v(s^F)) = 0$, and $dW/dr^F = 0$, $dW/ds$ for $s \geq s^F$ as given by (5.3) is equal to $dW/ds$ for $s \leq s^F$ as in (5.5).

14. At the Cournot equilibrium, from (6.3) and $\pi_1^1(y_1^*, y_2^*) = 0$, $\pi_1^B = M^Cq_1^2$. Also, using $\pi_2^2 = 0$, $\pi_2^{2B} - \pi_2^2q_2^1 < 0$ since $\pi_1^2 = y_2p_1^2 < 0$. Hence if $M^C = 0$, in the Bertrand case, firm 2 would reduce $p_2$ below $p_2^*$ so that normally $M^B$ is not zero.

15. From total differentiation of the direct demand functions, using (6.2), we obtain $dy_2/dy_1 - q_2^1/q_1^1 = Q(dp_2/dp_1)/q_1^1(dy_1/dp_1) > 0$ when $dp_2/dp_1 > 0$. 

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