Rochester Center for

Economic Research

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Working Paper No. 244 September 1990

University of Rochester

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March 1990 revised: September 1990

The authors gratefully acknowledge support from the National Science Foundation. We have received valuable comments on this paper from Mario Crucini, Michael Dotsey, Teresa Garcia—Mila, Marvin Goodfriend, and participants in the Federal Reserve Bank of Richmond's economics workshop. An earlier version of this paper, "Multipliers in Equilibrium Business cycle Models," was prepared for the Second World Basque Congress, September 1987, held in San Sebastian, Spain. We thank Carmen Gallestegui for encouraging the presentation of our early research on this topic.

<u>Abstract</u>

FISCAL POLICY IN GENERAL EQUILIBRIUM

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This paper studies the effects of fiscal policy within a quantitative general equilibrium model. We find that the basic neoclassical model has important dynamic interactions of capital and labor in response to policy disturbances, and that these interactions alter standard neoclassical predictions about the equilibrium effects of fiscal policy. Our main findings are that (i) there is likely to be a long run multiplier associated with changes in government purchases, (ii) permanent changes in government purchases induce larger effects than temporary changes, (iii) the financing decision associated with changes in government purchases is quantitatively much more important than the direct resource cost of changes in government purchases, and (iv) public investment policies have dramatic effects on output and on private investment.

1. Introduction

This paper studies the equilibrium effects of fiscal policy within a quantitative version of the basic neoclassical model of macroeconomic activity. We focus on four central questions:

- (1) What are the macroeconomic effects of permanent changes in the level of government purchases? Is there a multiplier effect, i.e., does an increase in government purchases bring about a greater than one-for-one increase in output?
- (2) How do the effects of temporary changes in government purchases differ from the effects of permanent changes in purchases?
- (3) How important is the financing decision in determining the effect of changes in government purchases?
- (4) Do the effects of purchases which augment the public capital stock or yield utility directly to individuals differ importantly from the effects of purchases which do not have these effects?

The macroeconomic literature on "the equilibrium approach to fiscal policy," summarized by David Aschauer [1988] and Robert Barro [1989], is also motivated by these questions. We share with this literature an emphasis on the supply side responses of labor and capital to shifts in government demand and tax rates. The central new finding of our analysis is that the basic neoclassical model has important dynamic interactions of capital and labor in response to policy disturbances. The presence of these strong interaction effects alters standard neoclassical predictions about the equilibrium effects of fiscal policy disturbances on macroeconomic activity.

Our answers to the four questions posed above may be briefly summarized as follows. First, we find that permanent changes in government purchases have important effects on macroeconomic activity. There is likely to be a multiplier effect in the long run; if labor supply is highly elastic, one may even exist in the short run. Each of these multipliers depends in an

Second, we find that permanent changes in government purchases are associated with larger output effects than temporary changes in purchases, which is contrary to the suggestions of Barro [1981a] and Robert Hall [1980]. Third, we find that the financing decision is quantitatively much more important than the direct resource cost of government purchases. For example, we find that output falls in response to higher government purchases when these are financed by general income taxes. Fourth, we find that the macroeconomic effects of government purchases depend importantly on whether these directly affect private marginal utility or marginal product schedules. If government purchases are directly valued by consumers, the macroeconomic consequences depend crucially on whether these services are complements or substitutes with privately provided consumption goods. If government capital augments the productivity of private capital and labor, public investment policies can have dramatic effects on output and private investment.

The organization of the the paper is as follows. Section 2 briefly summarizes the major features of U.S. fiscal policy that we take as background to our analysis. Section 3 provides a brief discussion of our model and our methods of policy analysis.

Next, we turn to our four policy questions. Throughout, we use the research of "the equilibrium approach to fiscal policy" as a reference point. For each of our policy questions, we review prior empirical findings, discuss their consistency with our results, and indicate directions for additional empirical research. Section 4 investigates the macroeconomic consequences of permanent shifts in basic government purchases, defined as those which have no effect on private marginal product or marginal utility schedules. Section 5 details the conditions under which a multiplier results from permanent

changes in basic government purchases. Section 6 compares the macroeconomic effects of temporary and permanent changes in government purchases. Section 7 investigates the macroeconomic implications of alternative methods of public finance for basic government purchases. Section 8 studies the effects of government purchases that have direct implications for private marginal products or marginal utilities. Section 9 concludes the paper.

2. Fiscal Policy in the United States, 1930-1985

As illustrated in Figure 1-A, the past fifty years have witnessed a permanent increase in the fraction of gross national product absorbed by government. This fraction increased from 10 percent of GNP in 1930 to 20 percent in 1985. There also have been large temporary movements in government purchases, as stressed by Barro [1981a]. At one point during World War II, for example, government purchases exceeded 45 percent of GNP.

The expanding influence of government over this period has generated increases in taxation. Figure 1-B plots two comprehensive measures of taxation: (i) total government receipts as a fraction of gross national product, and (ii) the average marginal personal income tax rate series produced by Barro and Chaipat Sahasakul [1986]. These measures of taxation rose substantially between 1930 and 1985, reaching a level in excess of 30 percent. Of the total increase in taxes during this period, roughly half reflects an increase in tax-financed government purchases. The remainder corresponds to tax-financed transfer payments, which have also grown substantially in the years since World War II.

In addition to an increasing volume of government purchases of goods and services, there have also been important changes in the composition of government purchases. Figure 2 illustrates the altered composition of

government purchases over the post World War II period, during which the government absorbed roughly one-fifth of national product. Military purchases have declined as a fraction of total government purchases, with some increases during the Korean and Vietnam Wars as well as the Reagan rearmament. Public net investment rose sharply during the 1960s and fell sharply during the 1970s: it reached a high of about 9 percent of total spending in 1966 and a low of about one percent in 1982. Figure 2 shows that there have also been substantial and persistent changes in publicly provided consumption services, i.e., government purchases that are not military purchases or public investment.

3. Modeling Fiscal Policy in General Equilibrium

To study the macroeconomic consequences of changes in fiscal policy, we construct a version of the basic neoclassical model that permits a variety of public interventions.

3.1 A Dynamic Competitive Model

The model that we employ is as follows:

Structural Elements

Our model's basic structural elements are preferences, technologies and resource constraints for both private and public agents, together with rules governing public finance.

Preferences: Our representative agent has preferences over sequences of consumption and leisure, summarized by the lifetime utility function:

(3.1)
$$U = \sum_{t=1}^{\infty} \beta^{(t-1)} [u(C_t, L_t, C_t^G)].$$

Utility depends in a standard way on privately-provided consumption services $\mathbf{C_t}$ and leisure $\mathbf{L_t}$. In addition, as in Aschauer [1985] and Roger Kormendi

[1983], we incorporate utility-enhancing services provided by the government, denoted $C_{\mathbf{t}}^{\mathbf{G}}$. The momentary utility function takes the form:

(3.2)
$$u(C_t, L_t, C_t^G) = \frac{1}{1-\sigma} \left[\Psi(C_t, C_t^G) \nu(L_t) - 1 \right]^{1-\sigma},$$

where Ψ is a constant-returns "aggregator" of publicly and privately provided consumption services and $\nu(L)$ is a positive and increasing function, whose specification controls the labor supply elasticity in the model. For most of our analysis, we further specialize momentary utility:

(3.3)
$$u(C_t, L_t, C_t^G) = log(\Psi(C_t, C_t^G)) + \theta_L log(L_t),$$
 which is a form commonly employed in equilibrium macroeconomics.

Production Technology: Output at date t is the result of private capital, public capital, and labor applied to a Cobb-Douglas production function:

(3.4)
$$Y_t = F(K_t, K_t^G, N_t) = A K_t^{\theta_K} N_t^{\theta_N} K_t^G \theta_G$$
, where K_t is the private capital stock, K_t^G is the stock of publicly-provided capital, N_t is the quantity of labor input. In (3.4), both capital stocks are predetermined at date t. We also assume that there are constant returns to scale over privately provided inputs: $\theta_N + \theta_K = 1$.

Accumulation Technology. Private capital evolves according to:

(3.5)
$$K_{t+1} = [(1-\delta_K)K_t + I_t],$$

where I_t is gross investment and δ_K is the rate of depreciation of capital. Public capital similarly evolves according to:

(3.6)
$$K_{t+1}^{G} = [(1-\delta_{K})K_{t}^{G} + I_{t}^{G}],$$

where It denotes government investment.

Government Purchases. Aggregate government purchases are denoted by $\mathbf{G}_t = \mathbf{G}_t^B + \mathbf{I}_t^G + \mathbf{C}_t^G, \text{ where } \mathbf{G}_t^B \text{ denotes "basic purchases"} \text{—those that absorb resources without shifting private marginal product or utility schedules.}$

Resource Constraints. In each period, the representative agent faces two resource constraints: (i) the sum of time devoted to work and leisure cannot exceed his endowment of time, and (ii) total uses of goods (for consumption and investment) cannot exceed disposable income:

(3.7)
$$L_t + N_t \le 1$$

(3.8)
$$C_t + I_t \le (1-\tau)Y_t + TR_t$$
,

where, $\tau_{\rm t}$ denotes the tax rate on output (or, equivalently, the uniform tax rate on labor and capital income) and TR_t denotes transfer payments. Finally, the economy-wide resource constraint is given by:

$$(3.9) C_t + I_t + G_t \leq Y_t.$$

Below, when we refer to the resource cost of government purchases, we mean that consumption or investment must fall when government purchases increase.

Public Finance Rules. The flow government budget constraint is:

$$\tau_{\mathbf{t}} Y_{\mathbf{t}} = G_{\mathbf{t}} + TR_{\mathbf{t}}.$$

In the analysis below, we explore the importance of the financing decision by considering the use of either changes in the tax rate, $\tau_{\rm t}$, or changes in transfer payments, TR to finance a given increase in total purchases. We do not explicitly consider financing by debt issue, since Barro's [1974] arguments show that changes in transfer payments are equivalent to debt financing when the sequence of tax rates, $\{\tau_{\rm t}\}$, is held fixed.

Aspects of Macroeconomic Equilibrium

Competitive equilibrium in our economy is a time path of quantities and prices consistent with (3.1)-(3.10) and private efficiency conditions.

Private Efficiency Conditions. Our representative consumer selects consumption, leisure and investment choices in a dynamically efficient

manner, equating the marginal utility of date t consumption to its opportunity cost; the marginal utility of leisure to the value of foregone earnings; and the opportunity cost of investment to its future returns. We assume that each agent acts competitively, treating the tax rate and transfer payments as exogenous to their choice of consumption and investment, even though (3.10) must hold in the aggregate by appropriate adjustment of taxes or transfers. Formally, the efficiency conditions are:

$$(3.11a) \quad D_1 u(C_t, L_t, C_t^g) = \lambda_t$$

(3.11b)
$$D_2^{u(C_t,L_t,C_t^g)} = \lambda_t^{(1-\tau_t)D_2^{F(K_t,N_t,K_t^g)}}$$

(3.11c)
$$\beta \lambda_{t+1} [(1-\tau_{t+1})D_1 F(K_{t+1}, N_{t+1}, K_{t+1}^g) + 1-\delta_k] = \lambda_t.$$

In these conditions, $\lambda_{\rm t}$ represents the shadow value of private consumption at date t, i.e., the Lagrange multiplier on (3.8) in a discrete dynamic optimization problem. Taken together, these equations can be used to answer our four questions about the dynamic effects of fiscal policy.

Long Run Positions: In the long run, our economy progresses to a steady state position in which the values of all variables are constant through time. In every steady state of the neoclassical model, the "supply side" dictates that relative prices and "great ratios" are independent of the level of labor input. The standard demonstration of this result—in the absence of government purchases or taxes—is as follows: the ratio of capital to labor, which we denote κ , is implicitly determined by

$$[D_1F(\kappa,1) + 1-\delta] = 1 + r,$$

where r is the steady state rate of return, determined by the rate of time preference. Given this capital-labor ratio, the supply side determines relative prices: the real wage rate (w = $D_2F(\kappa,1)$) and the real rate

 $(q = D_1F(\kappa,1))$. It also determines the great ratios: the average product of labor $\alpha \equiv Y/N = F(\kappa,1)$; the shares of capital and labor in national income, $s_K = qK/Y = \kappa D_1F(\kappa,1)/\alpha$ and $s_N = 1-s_K$; and the allocation of national product between consumption and investment, $s_I = (I/K)(K/N)/(Y/N) = (I/K)\kappa/\alpha$ and $s_C = 1-s_I$. Given these relative prices and great ratios, the scale of labor input is then determined by preferences, i.e., by the efficiency conditions (3.11a,b). Below, we discuss how government affects this long run situation.

3.2 Methods of Policy Analysis

Our investigation involves two types of policy analysis: (i) steady state analysis; and (ii) analysis of dynamics near an initial steady state.

Steady State Analysis

The long run effects of public policies can be studied using the equations that describe the steady state. It is useful to separate the two ways in which public policies affect the steady state.

Resource Costs and Benefits: Government purchases generally influence the net resources available to the society along the resource constraint: basic government purchases financed by lump sum taxation exert only a resource cost. Hence, in studying basic purchases in sections 4 and 5 below, the focus will be on labor supply since all relative prices and great ratios are unaffected by changes in basic purchases. The equilibrium supply of labor is given by from the long run resource constraint, $C + G = \alpha(1 - s_I)N$, together with the consumption-leisure tradeoff implicitly described by (3.11a,b). Below, we first present some analytical results on the long run output effects of basic purchases. We then provide a quantitative sensitivity analysis.

Substitution Effects of Taxes and Purchases: The policies studied in sections 6-8 below generally affect steady state relative prices and great ratios, complicating the study of their long run effects. For example, permanent changes in the tax rate τ or the stock of public capital $\mathbf{K}^{\mathbf{G}}$ alter the steady state capital labor ratio, determined by $[(1-\tau)(\mathbf{K}^{\mathbf{G}})_{\mathbf{A}}\theta_{\mathbf{K}}(\kappa)] + 1-\delta] = 1+r$. Hence, rather than providing rather inelegant analytical formulae, we numerically compute the shifts in the steady state arising from small perturbations in purchases that have tax, utility or productivity consequences.

Dynamic Analysis

For studying the dynamic response of macroeconomic quantities and prices to various shifts in government activity, we similarly employ numerical simulation. In this paper, we adopt the parameter values used in the baseline example of Robert King, Charles Plosser and Sergio Rebelo [1988a]; we also use the numerical techniques described in that paper to compute the model's dynamic equilibrium. The full set of baseline parameter values used is given in Table 1, which also provides a convenient review of notation.

4. Macroeconomic Effects of Permanent Government Purchases

In the past fifty years the U.S. economy has experienced a permanent increase in the fraction of output purchased by the government. We therefore begin by studying the effects of permanent increase in purchases.

4.1 The Dynamic Response to Permanent Government Purchases

We consider an unanticipated, permanent increase in government purchases, taken to be 1% of initial output, which we call a commodity unit. Figure 3

shows the dynamic response of price and quantity variables. In this figure and others below, the changes in consumption, investment, and output are expressed in commodity units. The changes in all other variables are expressed as percentage deviations from initial steady state values. These quantitative model results highlight a more general principle: there is an important dynamic interaction of labor and capital input as the economy responds to permanent shifts in government purchases. In the long run, to which the economy rapidly converges, there will be higher capital and labor input, but the capital-labor ratio is unchanged. In the long run, as shown in Figure 3, Y rises by about 1.1 units, C falls by .2, and I rises by .3. Labor input-measured as a percentage of initial stationary level-rises by slightly over 1 percent in the long run. To build up the long run capital stock, however, there must be a temporary investment boom: on impact (date t=1) investment rise by .4 units, with output rising by .8 units and consumption falling falls by .6 units. This short-run overshooting of investment is the handiwork of the model's "investment accelerator": given an initial stock of capital, higher future labor input makes it desirable to expand investment.

The relative price implications of the permanent increase in purchases are shown in Panel B of Figure 3. In response to the increase in labor supply, the real wage (w) declines dramatically in the short run. The rental rate on capital (q) correspondingly increases dramatically on impact, since a predetermined capital stock is cooperating with more units of labor. Despite the fact that the increase in government purchases is permanent, these factor price movements are temporary since the accumulation of capital ultimately restores the original capital/labor and wage/rental ratios at the new higher level of labor input. The final two subpanels of Figure 3 trace out the

effects of ΔG on the path of one period interest rates and the impact effect on the term structure. Along the transition path, the one period real interest is high but declining: at early dates, its high value encourages labor supply and postponement of consumption, enabling the "investment boom" to occur. The term structure shifts at date t=1 to reflect this expected future path of short rates: since the short rate is declining over time, the near end of the term structure increases the most sharply.

4.2 Implications for Empirical Research

Estimates of the output and real interest rate effects of permanent components of government purchases have been one focus of empirical research in the "equilibrium approach to fiscal policy" (as in Barro [1981a] and [1981b]). Using standard regression analysis, with all variables in levels, Barro estimates that a measure of permanent government purchases has an output effect of about .5 and with little detectable effect on the real interest rate. This set of findings accords with predictions obtained by Aschauer [1988] for a variable—labor economy without capital, and by Barro [1989], for a fixed—labor economy with capital accumulation. Each of those alternative models implies that unanticipated changes in permanent government purchases do not affect the real rate in the short run or in the long run.

By contrast, our model predicts a sizable short run increase in the real interest rate when there is an unanticipated permanent increase. Thus, we are led to inquire about the consistency of this implication with Barro's empirical findings. Since the empirical specification employed links the level of the real interest rate to the level of permanent government purchases, our interpretation is that the results are principally long run findings. That is, given the trend in the share of government in Figure 1,

the least squares estimator will seek to match the trend in the dependent variable (real interest rate) with the trend in the explanatory variable (government purchases). The implication of our model is that there should, in fact, be no long run relation between trends in government purchases and the long run real interest rate, since the latter is pinned down by time preference.

In order to detect the short run interest rate effect predicted by our model, one would need to extract unanticipated changes in the permanent component of government purchases. This high real rate should be accompanied by an increase in investment.

5. Multipliers in Equilibrium Macroeconomic Models

Within the basic neoclassical model, there is a potential for multiplier effects of permanent government purchases on output in both the short and long run. Both multipliers arise from the dynamic interaction of labor and capital supply, but the mechanisms differ in their details. The long run multiplier proximately results from the effects of higher long run labor input on the steady state capital stock. The short run multiplier results from a version of the investment accelerator, in which increases in the long run capital stock exert a dramatic short run influence on labor input.

5.1 Static Analysis

Although multipliers cannot be generated in static settings, as in Bailey [1962] and Barro [1984], it is useful to begin with a discussion of the effects of permanent government purchases in such a setup. Consider an increase in government purchases in the amount ΔG , financed by lump sum taxes, starting from a position of no government purchases or taxes. For

simplicity, we set the marginal product of labor or real wage, w, independently of the level of labor input. ⁸ Then, since the consumer experiences a reduction equal to ΔG in his income, he reduces both consumption and leisure so long as neither is an inferior good. With consumption and leisure both declining, there is an increase in output but no multiplier: $0<\Delta Y/\Delta G<1$.

More specifically, the consumer's budget constraint requires that purchases of goods and leisure not exceed "full income," which includes both the value of the (unit) time endowment and nonwage income, Π :

$$(5.1) wL + C \leq Y^{f} = w + \Pi.$$

Denote the full income elasticity of leisure demand as η_L and the leisure expenditure share in full income as $s_L^f = wL/Y^f$. Then the influence of a small change in full income on leisure is $w\Delta L/\Delta Y^f = s_L^f \eta_L$. Higher government purchases simply decrease full income by the same amount, so that

(5.2)
$$\frac{\Delta Y}{\Delta G} = w \frac{\Delta N}{\Delta G} = -w \frac{\Delta L}{\Delta G} = s_L^f \eta_L.$$

To determine the quantitative output effect of a government purchase, we require values for η_L and s_L^f . A natural starting point is $\eta_C = \eta_L = 1$, as implied by (3.3). Table 1 shows that N = .2 for the postwar U.S. Leisure expenditure is quite a large fraction of full income: $s_L^f = wL/Y^f = L = .8$, thus $\Delta Y/\Delta G = .8$. Alternative assumptions about the wage elasticity of leisure demand with respect can alter this conclusion. Since the preference specification (3.2) requires that leisure is invariant to common growth in full income and the real wage, it follows that the wage elasticity of leisure demand ϵ_L is just equal to $-\eta_L$. Hence, a higher magnitude of the leisure demand elasticity with respect to the real wage implies that $\Delta Y/\Delta G$ also increases.

5.2 The Amplification Effect of Capital

Why does the dynamic model produce a long run multiplier when the static model does not? In both models, labor supply increases and consumption falls in response to the permanent increase in government purchases. But in the dynamic model, the increase in labor supply raises the marginal product of capital, which stimulates capital accumulation. This "amplification effect" of endogenous capital supply means that there may be a multiplier.

Yet, the expansion of capital input also has implications for full income that act to counter this amplification mechanism. As individuals become wealthier with a higher capital stock, there is a tendency for them to work less and consume more. Full income includes capital income net of gross investment. In the steady state, this component of nonwage income is proportional to labor input, so that $Y^f = w + qK + I - G = w + \alpha(s_K - s_I)N - G$, where N is steady state labor input. Using this definition along with the equations above, we find:

(5.3)
$$\frac{\Delta Y}{\Delta G} = \frac{(s_L^f \eta_L/s_N)}{1 + (s_K - s_I)(s_L^f \eta_L/s_N)}.$$

The numerator of this expression combines the direct labor supply effect, as in (5.2) above, with the amplification effect of capital accumulation (since $\alpha/w = 1/s_N$). The denominator reflects the mitigating influence of accumulation—via net income from capital (qK-I)—on labor supply.

How sensitive is the long run multiplier result to parameter values? 10 Table 2 shows results for some alternative sets of parameter values drawn, for the most part, from other recent studies. Generally, multipliers do exist, with an upper bound being provided by indivisible labor economy, which effectively has infinite intertemporal substitutability of leisure (see

Richard Rogerson [1988]). With a low labor supply elasticity, as in the studies of male labor supply surveyed by John Pencavel [1986], there is no multiplier.

5.4 The Short Run Multiplier

The interaction of capital and labor supply also is crucial to the existence of a short-run multiplier. To see why, imagine the macroeconomic response to a permanent increase in government purchases if investment is exogenously held fixed at the level necessary to maintain the pre-existing capital stock. Then, as in Barro and King's [1984] analysis of a point-in-time production economy, permanently higher government purchases imply lower consumption and higher labor input, at levels that are constant through time. Hence, the real interest rate is constant at the rate of time preference.

Yet, with the constant returns to scale production function of the neoclassical model, higher labor input implies that the marginal product of capital is also higher. In fact, the capital stock would need to respond proportionately to the rise in labor input in the impact period, if the rate of return on capital is to be equated to the rate of time preference. It is this large investment demand shift at unchanged interest rates which yields the potential for a multiplier effect at date t=1.

In terms of generating a multiplier, it is important how much intertemporal substitution is present in labor supply, since this feature will determine the extent to which the shift in investment demand yields additional output. If the labor supply elasticity is maximized, as in the indivisible labor economy studied by Rao Aiyagari, Lawrence Christiano and Martin Eichenbaum [1990], then there is a short-run multiplier, $\Delta Y/\Delta G|_{t=1}$ =

1.09. On the other hand, the Pencavel [1986] estimates of male labor supply imply that $\Delta Y/\Delta G|_{t=1}$ = 0.31.

5.5 Comparison with A Keynesian Model's Multiplier

Many Keynesian macroeconomic models imply substantial effects of government purchases. For example, in their classic study of multipliers in Klein's [1950] model I, Theil and Boot [1962] report that a purely temporary increase in government purchases has an output effect $\Delta Y/\Delta G|_{t=1}=1.8$. For the next two years the output effect is positive but declining; it reaches a value of -.8 after six years and then continues to follow an oscillatory trajectory. This Keynesian model thus has strikingly different implications from the neoclassical model studied here.

6. Temporary Versus Permanent Movements In Government Purchases

In U.S. history, wartime periods have been associated with temporarily high levels of government purchases and temporary increases in aggregate output. To investigate the effects of temporary changes in government purchases, we consider an increase in basic purchases which lasts for T years. We first study a four year disturbance, which we think of as a four year war. Second, we study how the impact effect, $\Delta Y/\Delta G|_{t=1}$, changes with the duration of the disturbance.

Turning to Figure 4, we see that the dynamic response to the four year war is broken into two phases. First, during the war years, there are reduced opportunities for private consumption, leisure, and investment due to the increased government absorption of resources. Second, after the war has ended and government purchases have returned to their steady state level, investment is above its long run level as the economy works to rebuild the

capital stock. Private consumption and leisure are correspondingly low along this transition path. Labor input increases more in the initial phase—when government spending is high—than it does along the transition path. Higher labor input is the dominant factor in the output increase during the war years. In contrast to the case of a permanent change in purchases, the temporary war generates a decline in investment on impact, and further declines over the next three years.

In terms of relative price implications, the wage rate (w) and the rental rate (q) move in opposite directions during the initial period of high government spending and also in the subsequent transition phase. This accords with the quantity movements, since labor input is high and capital input is low during both phases. Looking across the term structure in the impact period (the first period of the war), we find that short and long term interest rates rise, but with the short end of the term structure moving more. This pattern of interest rate responses is similar to that observed for permanent changes in basic purchases (Figure 3 above). Comparing the output effects of temporary versus permanent changes in purchases (Figures 3 and 4) we see that the impact effect on output is smaller for the temporary change: .56% for the temporary change, versus .86% for the permanent changes.

Table 3 explores the influence of the duration of spending on the impact effect on output, $\Delta Y/\Delta G|_{t=1}$. The impact effect of temporary purchases is smaller, the shorter is the duration of the spending disturbance. As the duration of the disturbances increases, the impact effect for the benchmark labor supply elasticity asymptotically approaches the value shown in Figure 3. Note that the disturbance must be very persistent—lasting nine years or

more—for a short run multiplier to arise, even if there is high intertemporal substitution in leisure.

The relative magnitude of the output effects of permanent and temporary shifts in basic purchases displayed here is contrary to predictions made by Barro [1981] and Hall [1980]. Barro and Hall noted that permanent and temporary changes differ in two ways. First, there is a larger wealth effect associated with more permanent purchases. Second, increased persistence of government spending limits opportunities for intertemporal substitution. Barro and Hall assumed—incorrectly in our model's context—that the substitution effect is quantitatively more important than the wealth effect. They consequently argued that temporary changes in government purchases should exert larger output effects than permanent changes.

Our quantitative analysis shows that increased persistence of government purchases leads to larger impact effects on output and consumption. The analysis of Barro and King [1984] suggests that the result that permanent changes have larger effects will hold in any neoclassical model in which preferences are time separable. The argument hinges on two characteristics of the basic neoclassical model. First, consumption obeys the permanent income hypothesis, and will fall by more, when there is a more persistent shock to basic purchases. (Basic purchases can be thought of as a negative technology shock of an additive form.) Second, consumption and labor obey an intratemporal efficiency condition requiring that the utility—denominated value of the marginal product of labor equals its utility cost in terms of foregone leisure. Because consumption falls by more with more persistent shocks, this means that effort must also rise by more. Since capital is predetermined when the shock occurs, the multiplier on output is necessarily larger the more persistent is the shock to purchases.

6.2 Implications for Empirical Research

Empirical research to date has estimated the output and interest rate effects of temporary components of government purchases, which are dominated statistically by wartime movements in military spending. Informal estimates by Barro [1984, page 313] of wartime output effects imply $\Delta Y/\Delta G = .6$ and his own formal statistical estimates yield an output effect of .85 with a confidence interval of .5 to 1.2. Barro [1987] provides evidence of a positive relationship between long-term interest rates and temporary military purchases, using over two hundred years of data on the United Kingdom.

One important focus of the empirical literature to date has been comparison of the effects of permanent and temporary movements in government purchases on macroeconomic activity. Comparison of the implications of permanent and temporary government purchases in our Figures 3 and 4 reveal that neither output nor interest rate effects are markedly different in the impact period, although the output effect is smaller for the temporary case and does depend on the duration of the episode (Table 3).

Our results indicate that a major implication of the basic neoclassical model is that there should be minimal output and consumption effects of a single-period increase in government purchases, which is exactly the opposite prediction to that made earlier by Barro [1981a] and Hall [1980]. In this regard, our results suggest that one avenue for approximating such a purely temporary change is to examine the end of an interval of temporarily high government purchases: in year 4 of Figure 4, there is substantial crowding out of investment by government purchases and minimal output response. For example, it may be fruitful to conduct empirical studies of the ends of wars so as to isolate the macroeconomic effects of temporary government purchases.

7. Taxation and the Macroeconomic Effects of Government Purchases

This section examines the macroeconomic effects of government purchases under a stylized version of the Gramm-Rudman amendment. We require that tax revenues match expenditures on a period-by-period basis, holding constant the path of transfer payments: $\tau_{\rm t} = [{\rm G_t} + {\rm TR_t}]/{\rm Y_t}$. First, we study permanent changes in government purchases, focusing solely on steady state effects. Second, we study temporary increases in purchases, again within the context of a four-year "war." We find that the tax financing requirement radically alters the output effects of both temporary and permanent government purchases, leading output to decline rather than increase. As before, we consider an increase, ΔG , that represents one percent of initial output.

7.1 Permanent Increases in Government Purchases

With an increase in government purchases, there must be a rise in the tax rate to satisfy the government budget constraint. Because the rise in the tax rate reduces individuals' incentives to work and to invest, thus reducing the tax base, tax rates must increase by more than $\Delta G/Y$. There is a "supply side multiplier" at work, under which increases in purchases and tax rates induce declines in output that in turn require additional increases in tax rates.

To calculate the size of this "supply side multiplier," it is easiest to begin by holding fixed labor input. Combining the production function (3.4); the steady state equilibrium condition for capital accumulation; and the fiscal constraint (3.10) yields:

(7.1)
$$\Delta Y = -\left[\frac{\theta_{K}}{\theta_{N} - \tau}\right] \Delta G < 0$$

so long as $\theta_{\rm N}$ - au > 0, as is the case for the United States.

We study the magnitude of $\Delta Y/\Delta G$ starting from an initial position in which τ = (G/Y) = .2; this corresponds to the "benchmark" case in Table 2. Output falls by more than the increase in government purchases: $\Delta Y/\Delta G$ = -1.10. This contrasts with the earlier multiplier of 1.16 reported in Table 2 for a change in purchases financed by lump-sum taxation.

To balance the budget, the necessary tax change is

(7.2)
$$\Delta \tau = (1-\tau) \left[\frac{\theta_{N}}{\theta_{N} - \tau} \right] (\Delta G/Y),$$

with τ = (G/Y) = .2, the implied increase is $\Delta \tau$ = 1.22(Δ G/Y).

To this point, we have ignored variation in labor input. In fact, if preferences imply an exact offset of the income and substitution effects of technical shifts, as we have assumed, there will be no equilibrium variation in labor if we start from a position with zero transfers: TR = 0. As noted by Andrew Abel and Olivier Blanchard [1983], among others, such an increase in taxes works like a permanent, total factor augmenting technical shift from the standpoint of the private sector: since permanent technical shifts do not affect steady state labor, neither do permanent tax financed shifts in government purchases. Thus, this result is independent of labor supply elasticities, other than the restrictions imposed by steady state growth.

On the other hand, if we start from a position in which transfers are present, there generally will be equilibrium changes in labor input. This makes the output adjustment more complicated than that given in expression (7.1), since there are additional terms reflecting the interaction of output, labor input and capital accumulation. For example, if we set the initial level of (TR/Y) at 10%, we find that $\Delta Y/\Delta G = -2.54$ using a specification

that adds labor supply effects to (7.1). This more dramatic output adjustment reflects the fact that the increase in the tax rate reduces labor input, i.e., it shrinks the "tax base." Tax rates must therefore rise more, the more elastic is the labor supply response to changes in taxes. The tax rate change with variable labor supply is $\Delta \tau = 1.76$.

7.2 Temporary Increases in Government Purchases

In this section we explore how the economy responds to a temporary change in government purchases—specifically the four year war considered previously in Section 6—when current expenditures must be financed by current general income taxes. This financing requirement implies that the period of temporarily high government purchases becomes an interval of temporarily high taxes. Since high taxes imply temporarily low after—tax factor rewards, there is thus a strong incentive to intertemporally substitute work effort away from the wartime period, and also to curtail investment during this period.

Figure 5 displays the dynamic response of the economy to the tax-financed war. In each panel, the stars indicate the result obtained with a point-in-time balanced budget while the circles indicate the result obtained in section 6 under lump sum taxation. In contrast to our earlier findings, there is now a decline in output and work effort during the four war years, reflecting the dominance of substitution effects induced by higher taxes.

Our analysis shows that the tax-financing requirement imposes tax distortions precisely when society must reduce consumption, leisure and investment due to temporarily high government purchases. This particularly poor timing of tax distortions would be avoided by the smoothing of taxation

over time—as discussed by Barro [1979] and Sahasakul [1986]—and the related use of public debt for financing temporarily high purchases.

7.3 Implications for Empirical Research

The tax-financing requirement implies major differences in the response of the macroeconomy to both permanent and temporary changes in government purchases, relative to the case of lump-sum taxation. In our model, the public finance decision is quantitatively far more important than the issue of the duration of government purchases discussed earlier. Yet much literature in "the equilibrium approach to fiscal policy" has concentrated on the latter. Future empirical work should control for the public finance decision, perhaps by including comprehensive measures of the tax rate as an explanatory variable in regressions. This practice is likely to be especially important for studying permanent movements in government purchases, since there have also been permanent, associated changes in U.S. tax rates (Sahasakul [1986]).

The strong negative influence of tax-financed government purchases on output, discussed in section 7.1 above, principally involves the depressing effect of income taxation on capital formation. If we start from a position of no transfer payments, however, such income taxation does not affect labor supply. The difference in long run capital and labor supply elasticities suggests that future theoretical and empirical analyses would profit by distinguishing between capital and labor income taxation 13.

8. Productive and Utility-Enhancing Government Purchases

This section studies the effects of government purchases which affect private decisions by shifting marginal product or marginal utility schedules.

8.1 Public Investment

In light of the historical evidence of section 2, we focus on permanent changes in public investment. First, we look at the long run effects of an increase in public investment under alternative assumptions about the productivity of public capital. Second, we trace out the dynamic implications of this shift using a specific value of the productivity parameter for government capital.

To begin, recall that government capital works like a productivity shift from the standpoint of the private determination of capital and labor input: $F(K_t, K_t^G, N_t X_t) = (K_t^G)^{\theta_G} \begin{bmatrix} K_t & N_t \\ N_t \end{bmatrix}. \text{ Holding government capital fixed,}$ government investment works like basic government purchases (or a production function shift which does not affect private marginal product schedules).

If we treat private capital and labor as unresponsive to government capital, then steady-state net output, Y-I^G, is maximized when I^G/Y = $\theta_{\rm G}$. ¹⁴ With this rate of public investment, a marginal increase in I^G leaves net output unchanged, so we describe this as the case of zero net resource use.

Long Run Effects of Public Investment: The response of output to a sustained increase in public investment depends on (i) the direct effect of higher public capital, holding fixed private capital and labor inputs; and (ii) a supply side effect due to the response of private capital and labor. Table 4 reports the magnitudes of these effects for a range of values of the productivity parameter $\theta_{\rm G}$, under the assumption that the share of public investment, ${\bf s}_{\rm I}^{\rm G}$, is equal to .05. When public capital is unproductive, ${\boldsymbol \theta}_{\rm G}$ =0, the first row of the table replicates the results for basic government purchases obtained in section 4 above. Subsequent rows report results for higher values of ${\boldsymbol \theta}_{\rm G}$. As we increase the productivity parameter, there are

larger direct effects, as reported in the second column of the table. The direct effect is unity in the case of zero net resource use ($\theta_{\rm G}$ =s $_{\rm I}^{\rm G}$ =.05).

The results of Table 4 show that the supply side responses of capital and labor are key determinants of the magnitude of how important public investment is for output. With fixed labor and endogenous capital accumulation, (column 3), the output effect is uniformly 1.72 times the direct effect, since requiring that private capital expands to the point where its marginal product is equal to an unchanged return implies that output expands by $1/(1-\theta_{\rm K})=1.72$ with $\theta_{\rm K}$ = .42. Thus the endogenous response of private capital is quantitatively less important the direct effects on public capital on output. 16 With variable labor input, however, output effects attributable to the supply side responses of labor and capital can substantially exceed the direct productivity effects. For example, when $\theta_{\rm G} = {\bf s}_{\rm I}^{\rm G}$, the output effect with variable capital and labor is 2.6 times the direct effect. Table 4 also indicates that permanent increases in public investment induce long run increases in private consumption and investment, so long as public capital is even slightly productive. Again using $heta_{ extsf{G}}$ as a specific example, we find that the unit increase in public investment raises private consumption by about two thirds of a unit ($\Delta \text{C}/\Delta \text{I}^\text{G}$ = .66).

Our largest value of $\theta_{\rm G}$ corresponds to the highest estimate obtained by Aschauer [1989a] ¹⁷, yielding results that are dramatic in two respects. First, the direct output effect is eight times the change in public investment. Second, in contrast to results with smaller $\theta_{\rm G}$, there is little difference between the output effects with fixed labor and variable labor. Presumably, this reflects a wealth effect of highly productive public capital on the demand for leisure.

Short Run Effects of Public Investment: An unexpected, permanent increase in the flow of public investment introduces three forces which operate on the economy along its transition to the new steady state. First, there is a permanent increase in governmental absorption of resources, as with the basic government purchase that we studied above. Second, as the stock of public capital accumulates over time, it directly yields an increased flow of output. Third, the marginal product schedules for private labor and capital are shifting as a result of the rising stock of public capital, stimulating alterations in labor and private capital.

Figure 6 presents results for the case of zero net resource use. The first graph in Panel A shows direct resource cost of public investment, and the second shows the direct productivity effect of public capital, i.e., the output response holding fixed private factors. The difference between the two timepaths measures the decline in resources available for private consumption and investment: this loss equals the full shift in public investment on impact; is one half at six years; and is negligible after 20 years.

Panel B of Figure 6 plots quantity responses to the increase in public investment. Stars denote the case of productive investment; circles denote a reference case in which we eliminate the implications of government capital for private marginal product schedules (retaining the resource cost of higher public investment and the direct productivity effects of higher government capital). Comparison of these two timepaths permits an understanding of the important role played by alterations in labor and private capital in determining the economy's dynamic response in the same way that we used Table 4 decompositions to look at the implications for steady state responses. In the first year of the star economy, private investment increases, since its

marginal product has shifted up due to increases in labor input and the stock of public capital. But in the reference case, private investment declines slightly, since the economy is subject to a highly persistent but still temporary drain on the resources for private consumption and investment. In later years, as the public capital stock increases, its implications for private marginal product stimulates additional supply of both labor and capital in the star economy. The gap between the two economies rapidly expands as one transits to a new steady state with higher private investment, consumption and labor input while the other moves to the original one with unchanged private investment, consumption and labor input.

8.2. Public Provision of Consumption Services

The economic effects of public provision of consumption services depend on how these influence the marginal utility schedules for privately purchased consumption goods and leisure. We begin with the approach of Kormendi [1983], Barro [1984], and Aschauer [1985] and then consider alternatives.

The Standard Approach: Suppose that we can linearly aggregate private consumption C_t and publicly provided consumption services C_t^G to obtain a composite consumption good C_t^* , as follows: $C_t^* = C_t + \psi C_t^G$, with $\psi \ge 0$. Composite consumption is valued according to $u(C_t^*, L_t)$. We assume that public consumption services are financed by lump sum taxes, so that the individual's flow budget constraint becomes $C_t^* + I_t \le Y_t + \psi C_t^G + TR_t = Y_t - (1-\psi)C_t^G$. Under these conditions, the effects of an increase in public consumption are exactly the same as for an increase in basic purchases with two adjustments. First, we must rescale the responses since the equivalent basic purchase would be $G_t^B = (1-\psi)C_t^G$. Using estimates of ψ in the range produced by Aschauer [1985] and Kormendi [1983], $1/4 < \psi < 1/2$, means that the equivalent

change in basic purchases is only one half to three quarters of the change in publicly provided consumption. This would make a long run multiplier very unlikely, since $\Delta Y/\Delta C^G = (1-\psi) \ \Delta Y/\Delta C^B$ would be less than one for all of the values reported in Table 2. Second, we must alter consumption in Figure 3, which is C_t^* , to obtain private consumption using $C_t = C_t^* - \psi C_t^G$. For example with $\psi = 1/2$, privately provided consumption would fall by about three-quarters of the increase in C^G increase in the short run, and by about six-tenths in the long run. Hence, publicly provided consumption services differ from basic purchases as follows: (i) the output, investment and labor input responses are smaller, and (ii) private consumption is crowded out to a greater extent.

An Alternative Specification: Very different results obtain if publicly provided consumption goods raise the marginal utility schedule for privately provided consumption goods. We specialize utility as follows: $u(C_t^*, L_t) = log(C_t^*) + log(\nu(L_t))$ with $C_t^* = \Psi(C_t, C_t^G)$, but we do not assume that the Ψ function aggregates the goods linearly. Denote by ξ the elasticity of substitution of C for C G in Ψ . Then C and C G are Hicks-Allen substitutes (complements) if ξ is greater than (less than) one. Since the specification used above was an example of substitutes (in fact, an infinite elasticity of substitution), we consider here the case of complements, with an elasticity of substitution of .5. 18 Figure 7 shows the effects of a permanent increase in publicly provided government purchases when ξ = .5, so that there is a positive effect of $C_{\mathbf{t}}^{G}$ on the marginal utility of $C_{\mathbf{t}}$. In the long run, the higher marginal utility of private consumption causes an increase in labor input as individual trade leisure for the more highly valued private consumption good. Increased labor input induces an increase in long run levels of investment, capital and output. In this example, the increase in

C^G has a long run multiplier in excess of 1.5. In the short run, consumption and leisure are foregone to build up the capital stock, as the economy moves to the new steady state. Thus, labor input "overshoots" its steady state level and consumption initially declines, as shown in Figure 7.

8.3 Implications for Empirical Research

Our analysis of the effects of public investment supports Aschauer's [1989] view that variations in publicly provided capital have important macroeconomic effects. In particular, the decline in public investment, as seen in Figure 2, could potentially account for the recent decline in private factor productivity. Using a productivity parameter for government capital only one-fourth as large as that estimated by Aschauer for nonmilitary public capital, we find a long run multipler of about four. If we use Aschauer's estimate, we find that a long run multiplier of about thirteen. The sensitivity of our results to this parameter means that further effort should be devoted to obtaining more precise estimates of this parameter. Second, further empirical work would be useful in clarifying (i) the extent to which public capital augments labor versus capital productivity, (ii) which sectors of the economy are most sensitive to variation in public capital, and (iii) which types of public capital are most important for private productivity (Aschauer [1989] provide some information on this subject).

In the area of publicly provided consumption services, our experiments indicate the importance of isolating the elasticity of substitution between private and publicly provided consumption services. If publicly provided consumption services are to exert strong positive effects on macroeconomic activity, two conditions will need to be met: (i) public and private consumption services will need to be complementary; and (ii) the linkages

discussed in section 5—between labor input, investment, and output—will be a central part of the mechanism. Evidence on these two components can, in principle, be investigated separately.

9. Conclusions

In this paper, we have explored four central fiscal policy topics within the context of a quantitative equilibrium model. We found that:

- (1) Permanent changes in basic government purchases could exert important effects on macroeconomic activity, stimulating output and investment, possibly producing multipliers in the long run and the short run.
- (2) Temporary changes in government purchases, by contrast, were far less important for output, labor input, and consumption, essentially because the neoclassical model provides excellent mechanisms for smoothing temporary shocks. This smoothing occurs via a nearly complete crowding out of private investment when there is a temporary increase in government purchases.
- (3) The financing of government purchases proved to have a dramatic influence on their macroeconomic impact: output fell in response to increases in government purchases financed by increases in a comprehensive income tax, reversing the pattern observed with lump sum taxation.
- (4) The composition of government purchases matters a great deal for macroeconomic activity. The crucial issue is how particular categories of government purchases directly affect private marginal product and marginal utility schedule.

In each case, our analysis uncovered empirical predictions of the basic neoclassical model that have not previously been investigated in the literature on the equilibrium approach to fiscal policy. In particular, we found surprising the important role played by the public finance decision in

determining the pace and pattern of economic activity. We plan a detailed exploration of the links between taxation and macroeconomic activity in our future work.

Footnotes

- Throughout our discussion, we use the phrase "basic neoclassical model" to indicate a model containing capital accumulation along the lines of Robert Solow [1956], augmented to include optimal intertemporal consumption decisions, as in David Cass [1965] and Tjalling Koopmans [1965], and the labor-leisure margin, as in Finn Kydland and Edward Prescott [1980, 1982].
- The restriction on the form of the momentary utility function, (3.2), is motivated by the observation that the postwar U.S. economy has displayed only small changes in average hours worked in the face of major secular growth in real wages and real incomes. If the two consumption goods are required to move together, then (3.2) insures that trend growth in wages and income will leave hours invariant. We also require that momentary utility is concave, which restricts the range of values for σ once $\nu(L)$ has been specified.
- See, for example, Prescott [1986] and Plosser [1989]. This utility function arises as a limiting case if we drive σ toward unity and choose $\nu({\rm L})$ to be a power function, with parameter $\theta_{\rm L}$.
- We abstract here from those factors that explain trend growth in the economy, such as technical progress. However, in our quantitative analysis, we assume the production function incorporates labor—augmenting technical progress at the constant gross rate $\gamma_{\rm X}$, which is chosen to match the average growth of U.S. output.
- See King, Plosser and Rebelo [1988a, section 2.4] for a discussion of the construction of the Lagrangian and derivation of first order conditions.
- In the economy described by equations (3.1)-(3.11), the ratio I/K is δ . In our quantitative analysis, we interpret this as a stationary transformation of a growing economy, which necessitates modifying the accumulation equations (3.5,6) to the form $K_{t+1} = [(1-\delta_K)K_t + I_t]/\gamma_K$, implying that the ratio (I/K) contains both net and gross investment in the steady state, I/K = γ_X -1+ δ . This modification is quantitatively important.
- The manner in which we solve this suboptimal equilibrium system is described in King, Plosser and Rebelo [1988b, section 4]. Essentially, this procedure is a log-linear version of the "Euler equation approach" to computing suboptimal dynamic equilibria (see Marianne Baxter [1988] for a general discussion of this approach).

- The assumption that the real wage does not depend on labor input is the natural one for two reasons. First, the results reported in the text are an upper bound to the $\Delta Y/\Delta G$ value if there are consumption-leisure substitutions associated with a diminishing marginal product of labor. Second, the steady state of the neoclassical model has a real wage that does not depend on the level of labor input.
- A short proof of this condition proceeds as follows. For any efficient pair of consumption and leisure decisions, it must be the case that $D_2u(C,L)/D_1u(C,L) = w$. From the budget constraint, we know that $C + wL = Y^f$. Hence, with the specified utility function, it follows that $L + \nu(L)/D\nu(L) = Y^f/w$, which implicitly determines the demand for leisure. But since Y^f and w enter only in a ratio form, the leisure demand must have price and income elasticities that sum to zero.
- To include steady state taxation into the formula (5.3), we need to make the following revisions. First, steady state full income is adjusted as follows: Y = (1- $\tau_{\rm W}$)w + (1- $\tau_{\rm Q}$)qK + TR I = (1- $\tau_{\rm W}$)w + qK + $\tau_{\rm W}$ w I G, using the steady state government budget constraint, TR + G = $\tau_{\rm W}$ N + $\tau_{\rm Q}$ qK. Notice that $\tau_{\rm Q}$ does not alter this measure except as it affects quantities while $\tau_{\rm W}$ has a direct effect, since it influences the valuation of the leisure endowment. Second, the share of leisure expenditure in full income becomes (1- $\tau_{\rm W}$) w L /Y , which may be shown to be (1- $\tau_{\rm W}$) (L/N) s_N / [(1- $\tau_{\rm W}$) (s_N/N) + $\tau_{\rm W}$ s_N + s_K s_I s_G]. Hence, the multiplier is adjusted to:

$$(5.3)' \qquad \frac{\Delta Y}{\Delta G} = \frac{ \left[\eta_L s_L^f \right] / \left[s_N (1 - \tau_W) \right] }{ 1 + \left[\eta_L s_L^f \right] \left[s_K - s_I + \tau_W s_N \right] / \left[s_N (1 - \tau_W) \right] } .$$

This formula produces the results in Table 2.

- Wynne [1989, chapter 1] considers the basic neoclassical model's response to temporary government absorption of goods in a simulated version of the World War II experience. He argues that it is empirically important to distinguish between government absorption of goods and labor, a distinction which we do not pursue here.
- Working independently, but using the sort of argument sketched here, Aiyagari, Christiano and Eichenbaum [1990] formally prove that permanent increases in government purchases have larger output effects than temporary ones, under the standard assumption that both consumption and leisure are normal goods. Barro [1989, section 5.2.3] also provides an intuitive discussion of why this must be the case.

- Research by Anatoli Braun [1989] indicates that this distinction is potentially important for the post-war U.S.
- The derivation of this result is as follows. For the Cobb-Douglas production function (3.5), the marginal product of government capital is $\theta_{\rm G}({\rm Y_t}/{\rm K_t^G}). \quad \text{Maximization of output net of public investment,} \\ {\rm Y_t}-(\gamma_{\rm X}+\delta_{\rm K}-1){\rm K_t^G}, \ \text{requires that the marginal product be equated to} \\ (\gamma_{\rm X}+\delta_{\rm K}-1). \ \text{Using the steady state relationship between capital and} \\ \text{investment, } {\rm I}_{\rm t}^{\rm G}=(\gamma_{\rm X}+\delta_{\rm K}-1){\rm K_t^G}, \ \text{leads to the expression reported in the text.}$
- The results correspond to those obtained in section 5 above as follows: the first two columns of results for $\Delta Y/\Delta I^G$ show that (i) there is no direct output effect; and (ii) there is no output effect with variable capital and fixed labor, since basic purchases do not shift the marginal productivity schedule for capital. With endogenous labor input, we have results that accord with Figure 4 and Table 2: output increases by more than the increase in government purchases, consumption falls and private investment rises.
- The fact that the influence of capital accumulation is numerically smaller than the direct effect is similar to well known results on the long run effects of shifts in technology (since that is what public investment represents from the standpoint of the owners of capital) in models with Cobb-Douglas production technologies. In particular, the elasticity of the marginal product of capital with respect to capital is θ_K-1 . Thus, the elasticity of the long run demand for capital with respect to its rental price is $-1/(1-\theta_K)$, which has absolute value less than two whenever $\theta_K<1/2$.
- In his Table 1, panel A, equations (1.1) and (1.2), Aschauer [1989a] reports estimates that correspond to $\theta_{\rm G}$ =.39.
- The boundary case of unit elasticity of substitution, ξ =1, is easy to work out and of some interest: the answer is exactly given in our treatment of basic government purchases in sections 4 and 6, with all macroeconomic quantities responding in the manner discussed there. No adjustments of any kind are necessary, since publicly provided consumption services do not affect marginal valuations of private consumption or leisure. With ξ =1, Ψ is (at least locally) Cobb-Douglas and $\log(C_t^*)$ is simply a weighted sum of $\log(C_t)$ and $\log(C_t^G)$.

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Table 1 Notation and Parameter Selections

Benchmark Model with Basic Government Purchases:

A. Preferences

momentary utility function:

$$u(C,L) = log(C_t) + \theta_L log(L_t)$$

 $\theta_{\rm L}$ chosen so L = .8 and N = 1-L = .2

lifetime utility function:

$$U = \sum_{t} \beta^{t} u(C_{t}, L_{t})$$

 $oldsymbol{eta}$ chosen so steady state real rate is .065

B. Production Function

production function:

$$Y_t = K_t^{\theta_K} N_t^{\theta_N}$$

 $\boldsymbol{\theta}_{\text{N}}, \boldsymbol{\theta}_{\text{K}}$ chosen to match U.S. factor

share data: θ_{N} = .58, θ_{K} = .42

accumulation of private capital:

$$K_{t+1} - K_t = I_t - \delta_K K_t$$

$$\delta_{\rm K} = .10$$

Government

government share of gross national product:

 $s_G = G/Y = .20$ chosen to match historical experience in postwar period.

uniform tax rate on labor and capital income:

chosen to have no transfers in steady state $\tau = .20$

Table 1, Cont'd.

Models with Productivity and Utility Augmenting Government Purchases

A. Public Investment Model (I_t^G)

share of public investment:

$$\mathbf{s}_{1}^{G} = \mathbf{I}^{G}/Y = .05$$

production function:

$$\mathbf{Y}_{\mathsf{t}} = \mathbf{K}_{\mathsf{t}}^{\theta_{\mathsf{K}}} \mathbf{N}_{\mathsf{t}}^{\theta_{\mathsf{N}}} \mathbf{K}_{\mathsf{t}}^{\mathsf{G}}^{\theta_{\mathsf{G}}}$$

 $\theta_{\rm G}$ varied; $\theta_{\rm G}$ = $s_{\rm I}^{\rm G}$ = .05 used as benchmark.

accumulation of public capital:

$$K_{t+1}^{G} - K_{t}^{G} = I_{t}^{G} - \delta_{K} K_{t}^{G}$$

$$\delta_{\rm K}$$
 = .10

B. Public Provision of Consumption (C_t^G)

share of public consumption:

$$\mathbf{s}_{C}^{G} = C^{G}/Y = .05$$

elasticity of substitution between C and C^G : ξ varied $(\infty$, .5)

Notation

Aggregate quantities

Y = gross national product

K (K^G) = private (public) capital stocks

I (I^G) = private (public) investment

C (CG) = privately (publicly) provided consumption services

L (N) = leisure (work) allocations: N + L = 1

 $G(G^B)$ = total (basic) government purchases of goods and services

Identities

$$C + I + G = Y$$

$$G = G^B + I^G + C^G$$

Shares of national product:

$$s_{I}(s_{I}^{G}) = I/Y (I^{G}/Y)$$

$$s_{C}(s_{C}^{G}) = C/Y (C^{G}/Y)$$

$$s_{G} = G/Y$$

Other notation:

$$w = pre-tax wage rate:$$
 $w = D_2F(K,N,K^g)$

r = real interest rate

$$\alpha$$
 = average product of labor in steady state Y/N

$$A = \text{nonwage income in steady state}$$
 $A = qK + TR - I$

$$Y^f$$
 = full income in steady state $Y^f = 'w + A$

$$s_{N}$$
 = steady state share of labor income $s_{N} = w N/Y$

Table 2
Sensitivity of the Multiplier Effect to Alterations in Model Parameters

Alteration:	$(\Delta Y/\Delta G)$	$\mathtt{s}_{\mathrm{L}}^{\mathbf{f}}$	$\eta_{_{ m L}}$	
no tax: benchmark: indivisible labor: panel data labor supply: lower depreciation rate: lower real interest rate: existing transfers: higher steady state hours: higher labor's share:	1.20 1.16 1.37 0.49 1.12 1.29 1.10 1.01	0.78 0.78 0.78 0.78 0.77 0.80 0.74 0.64 0.78	1.00 1.00 1.29 0.33 1.00 1.00 1.00	

Explanatory Notes:

The first line of the table reports results from equation (4.3) of the text, which is applicable without taxes. Subsequent lines report results for equation (5.3)' of footnote 10, which is a modification for an economy with steady state taxation. The benchmark multiplier corresponds to Figure 3 and uses the parameter values of Table 1.

From the benchmark values of Table 1, we alter the parameters as follows. First, we conduct two experiments with labor supply elasticities discussed in more detail in King, Plosser and Rebelo [1988a]. In the indivisible labor economy, the utility function is effectively linear in hours (see Hansen [1985] and Rogerson [1988]). In the panel data labor supply economy, we choose $\nu(L)$ so as to give a compensated labor supply elasticity representative of the Pencavel [1986] survey. Following Woodford and Rotemberg [1989], we next lower the depreciation rate to δ =.06. Following Rouwenhorst [1989], we next lower the steady state real interest rate to 3%. The "existing transfers" experiment sets the level of τ = .30, with TR/Y=.10 and G/Y = .20. Finally, we experiment with two alternative parameter values taken from Kydland and Prescott [1980]: we raise the steady state fraction of hours worked to 1/3 and we raise labor's share to 2/3.

Table 3

Duration of Government Purchases and the Impact Multiplier on Output

Duration	$\Delta Y/\Delta G _{t=1}$				
in years	Benchmark	Panel Data	Indivisible Labor		
1	.20	0.06	0.28		
$\overset{1}{2}$.36	0.11	0.49		
3	.47	0.14	0.64		
	.56	0.17	0.76		
4	.63	0.20	0.84		
5		0.22	0.91		
6	.69	0.22	0.95		
7	.73		0.99		
8	.76	0.25	1.01		
9	.78	0.26			
10	.80	0.27	1.03		
20	.86	0.30	1.09		
∞	.86	0.31	1.09		

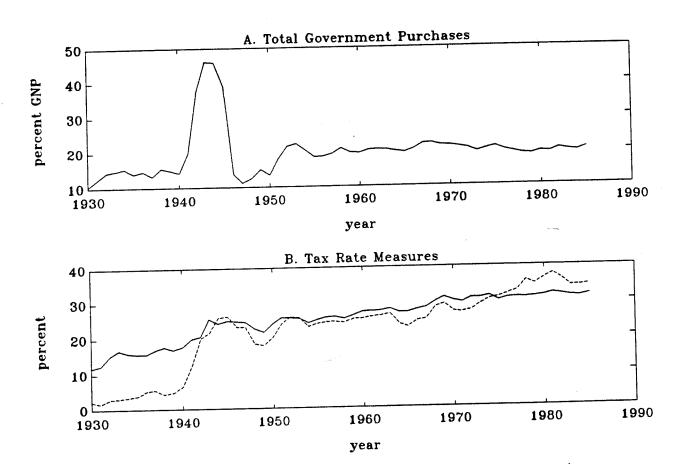
Notes: Each row of the table gives the impact multiplier value $\Delta Y/\Delta G \Big|_{t=1}$ if the change in basic spending is presumed to last the number of years specified in the first column. There are three alternative labor supply elasticities, discussed in detail in Table 2.

Table 4
Long Run Effects of Public Investment

	direct	K adj	K,N adj		
$ heta_{ extsf{G}}$	$\Delta Y/\Delta I^G$	$\Delta Y/\Delta I^G$	$\Delta Y/\Delta I^G$	$\Delta \text{C}/\Delta \text{I}^{\text{G}}$	$\Delta I/\Delta I^{G}$
0 0.01 0.03 0.05 0.10 0.20 0.40	0 0.20 0.50 1.00 2.00 4.00 8.00	0 0.34 0.86 1.72 3.45 6.90 13.79	1.16 1.45 1.90 2.64 4.12 7.09 13.02	-0.15 0.01 0.25 0.66 1.48 3.11 6.37	0.31 0.44 0.64 0.98 1.65 2.98 5.65

Notes: The column marked "direct effect" gives output effects holding fixed private factors of production. The column marked "K adj" gives output effects with labor fixed and private capital adjusting. The final three columns are output, consumption and private investment effects with both private factors adjusting.

FIGURE 1
U.S. Purchases and Tax Rates, 1930-1985



- --- Ratio of total government receipts to gross national product
- --- Average marginal tax rate from Barro and Sahaskul [1986]

FIGURE 2
Composition of Total Government Purchases, 1950-1985

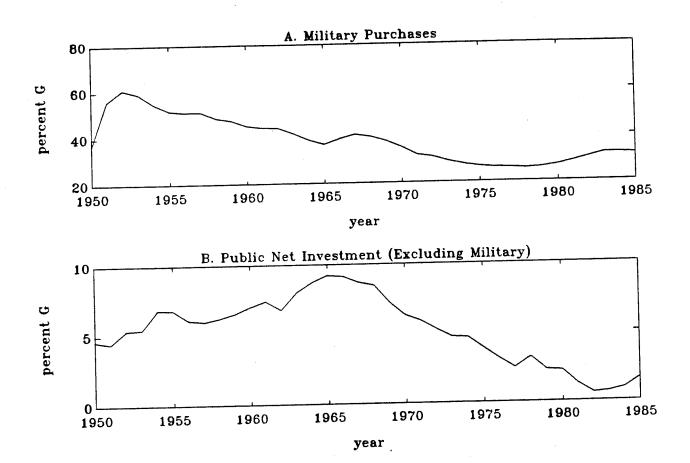
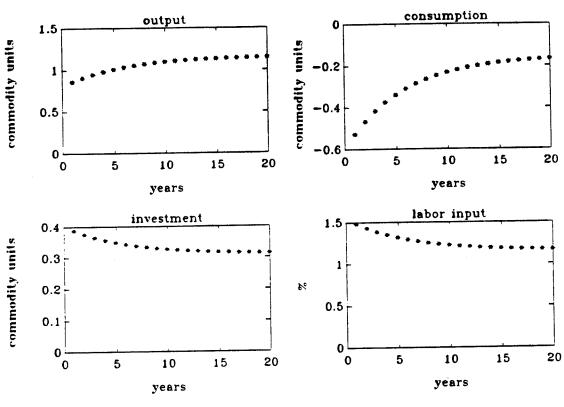


FIGURE 3

Macroeconomic Effects of A Permanent Increase
in Basic Government Purchases
A. Real Quantities



B. Relative Prices

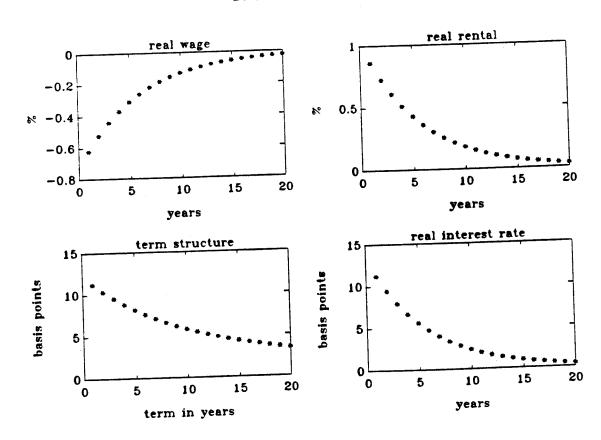


FIGURE 4

Macroeconomic Effects of A Temporary Increase
in Basic Government Purchases

A. Real Quantities

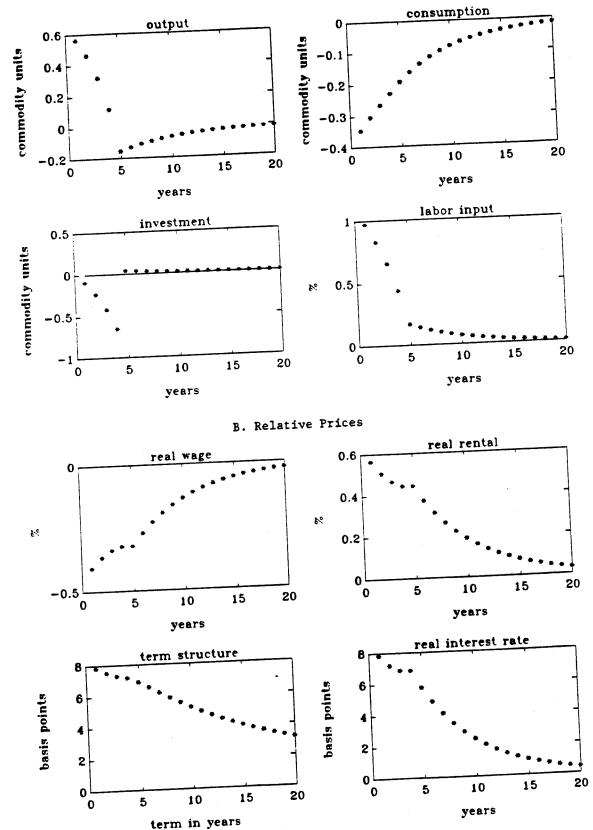
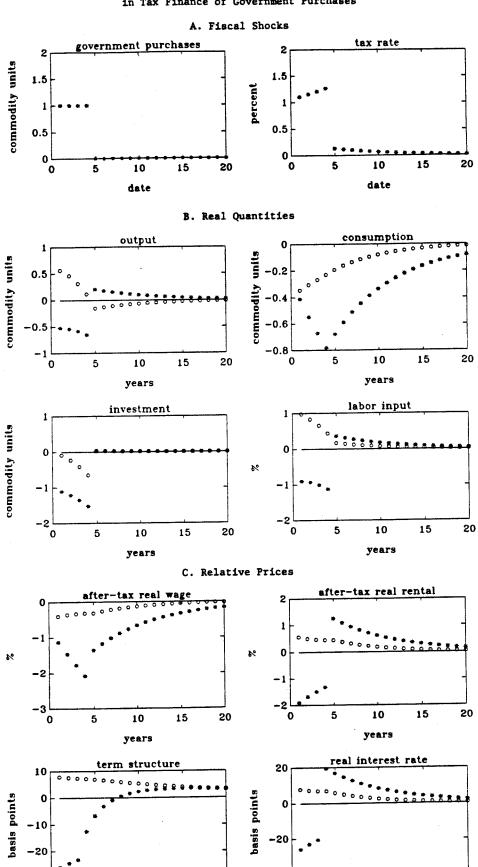


FIGURE 5

Macroeconomic Effects of A Temporary Increase
in Tax Finance of Government Purchases



Denotes responses to government purchases financed with income tax

5

10

years

15

20

Denotes responses to government purchases financed with lump sum

15

10

term in years

5

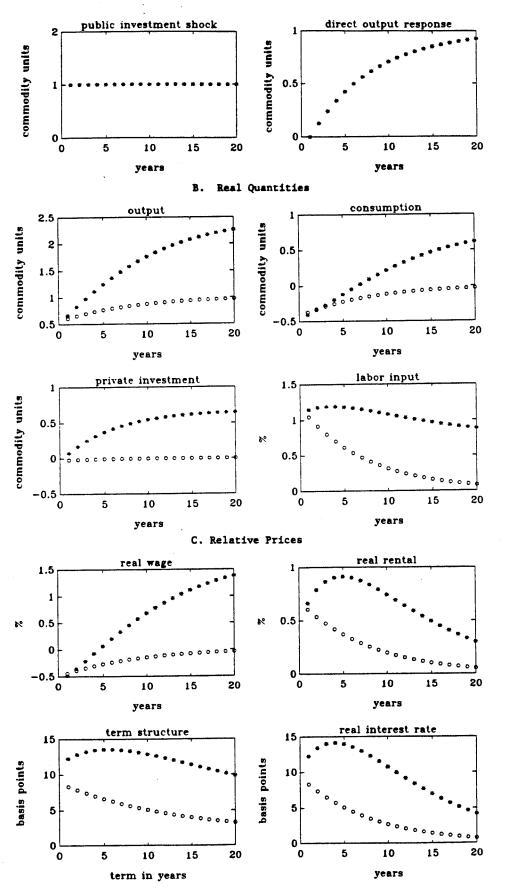
20

-30

0

FIGURE 6
Macroeconomic Effects of A Permanent Increase
in Productive Public Investment

A. Direct Resource Effects



- Denotes responses without shifts in marginal product schedules
- * Denotes responses with shifts in marginal product schedules

FIGURE 7

Macroeconomic Effects of a Permanent Increase

in Publicly Provided Consumption Services

