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Comments Welcome

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Abstract

This paper considers workers’ on-the-job search behavior as another microfoundation for efficiency wage theory. If intensive job search efforts of employees harm productivity, firms may have an incentive to pay an efficiency wage premium to reduce workers’ on-the-job search intensities. Adding a labor supply-search decision into an efficiency wage framework is more than cosmetic. Our model has different policy implications from previous efficiency wage models. The industrial policies designed to subsidize employment in higher wage jobs are not effective when the search-labor supply decision is explicitly accounted for. An employment subsidy to high wage sectors encourages workers’ job search intensities and rent seeking behavior rather than employment.

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I. Introduction

The presence of unemployment raises the question of why firms do not cut wages to take advantage of an excess supply of labor. Institutional impediments such as minimum wage laws and unionization may provide part of the answer, but a convincing rationale that a wage reduction could be unprofitable is provided by the literature on efficiency wage models. If workers' productivity is affected by the wage the firm pays, a wage reduction could lower productivity more than a firm's wage bill and the firm may end up with lower profits. Akerlof and Yellen (1986) and Katz (1986) classified efficiency wage models by the mechanism through which the wage affects productivity. Several such mechanisms have been proposed in the literature, including shirking (Shapiro and Stiglitz (1984)), adverse selection (Weiss (1980)), labor turnover (Salop (1979)) and sociological factors (Akerlof (1982)).

The purpose of this paper is to examine the implications of workers' on-the-job search as a mechanism through which wages can affect productivity. If intensive job search efforts of employees harm productivity, firms may have an incentive to pay an efficiency wage premium to reduce workers' on-the-job search intensities. The presence of a job search decision introduces an explicit link between effective labor supply and the prevailing

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1 Our model is not immune to the primary criticism of efficiency wage theory -- more sophisticated contract with performance bonding or seniority wage system can solve the problem in an efficient manner. The usual disclaimers why the contract market might fail are assumed in our paper: Shapiro and Stiglitz (1985) argue that employment fees in the presence of shirking problems may exacerbate the adverse selection problem. Akerlof and Katz (1988) argue that a seniority wage system is not a perfect substitute for the contracts with up-front bonds.
wage distribution which is missing in previous efficiency wage models. This link causes the policy implications of our model to differ from what have been considered standard policy implications of efficiency wage theories.

For example, it is well known that efficiency wage models provide a justification for strategic trade and industrial policies which subsidize sectors with higher wage jobs (Bulow and Summers (1986), Greenwald and Stiglitz (1988) and Katz and Summers (1989)). By subsidizing "winners" — high wage, good jobs — the government can cause the transfer of workers from low to high productivity jobs and thereby increase total output. In our model, however, an employment subsidy to high wage sectors encourages workers' job search incentives, and firms have to pay a higher efficiency wage premium to reduce the increased job search incentives. In a case analyzed in section II, this higher efficiency wage premium completely offsets the positive effect of the subsidy on employment. The intuition behind our result can be easily understood by comparing the Shapiro and Stiglitz (1984) shirking model with ours. In their model, employment subsidies can increase employment by shifting the labor demand curve outward with an invariant effective labor supply (non-shirking condition) curve.

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2 As far as we are aware of, Mortensen (1989) is the first approach which introduces an efficiency wage premium in a search framework. His model derives an efficiency wage premium using the shirking model of Shapiro and Stiglitz (1984), whereas an efficiency wage premium is due to workers' on-the-job search incentives in our model. As discussed in section III, these two models have drastically different policy implications.

3 Katz and Summers (1989, pp 257-60) discuss some perverse effects of industrial policy and conclude that they are of no practical importance compared to welfare gains from the expansion of the primary (efficiency wage) sector. Our implication is different from theirs in that the expansion of the primary sector might not be large: Both the positive gains and the perverse effects discussed in their paper could be small.
However, if we consider the search decision explicitly, the outward shift of the labor demand curve can be offset by a matching inward shift of the no-search condition curve, leaving the employment level intact.

The rest of the paper is organized as follows. Section II describes our model and its welfare properties with a constant returns to scale technology. Section III discusses the ineffectiveness of employment subsidies. Section IV concludes. In an appendix, a model with a decreasing returns to scale technology and imperfectly competitive labor market is analyzed.

II. The Model

We consider an economy consisting of a continuum of spatially separated islands. Each island is comprised of firms which use labor to produce an identical product with a constant returns to scale technology. Firms on the same island have identical labor productivity $\mu$ which is a random draw from a distribution function $G(\mu)$, where $G \in C^1$ and $G(0) = 0$. Each island has an uncertain but finite lifetime described by an exponential distribution with an exogenous death rate $\delta$. Islands that disappear are immediately replaced by new islands whose productivity is a random draw from the same distribution $G(\mu)$. By normalizing the measure of the set of all islands to be one, $G(\mu)$ also represents the distribution of productivity across all islands in the economy.

Information on wages is imperfect in the sense that each worker only knows the prevailing wage on the island where he is currently hired. Information about the wages prevailing on other islands are available only through job search. Workers do know that differences in wages across islands exist and this knowledge motivates workers to search for jobs with higher
wages. Workers participate in job search by paying a fixed cost $c$. *On-the-job* search is possible. Each job searcher receives a wage offer from one island with probability $\lambda dt$ during a short time period of $dt$. If a worker rejects the offer, he receives his current wage $w$ if he is employed, or $b$ if he is unemployed, where $b$ can be interpreted as unemployment insurance, value of leisure, or preferably the wage rate in a secondary sector where he can always find a job.

II.1. Workers

The expected return from search depends, among other factors, on workers' belief about the wage distribution function, $F(w)$. Rational expectation equilibrium requires that workers' belief about $F(w)$ coincides with the actual equilibrium wage distribution, which will be endogenously determined in the model. A Worker who has a current wage offer $w$ chooses a search strategy to maximize the expected present value of income, $V(w)$. In a continuous time framework, $V(w)$ satisfies the following equation:

$$
\rho V(w) = \max_{s \in (0,1)} \left\{ \max(w,b) - cs + \lambda s \int \{\max[V(z),V(w)] - V(w)\} \, dF(z) \\
+ \delta[V(b)-V(w)] \right\}, 
$$

(1)

where $\rho$ denotes the discount rate, $s$ is a search intensity, $c$ is a fixed per unit search cost, $\lambda$ is the offer arrival rate, $\delta$ is the job death rate, and $b$ is the wage rate in the secondary sector. We restrict the intensity of search to be either zero (no search) or one for simplicity. The above equation is of the form "a required return from an asset is the sum of dividends and expected capital gains or losses."
The solution of this maximization problem can be summarized as follows:

Since the cost of search is the same whether employed or not, a worker accepts any offer above \( b \) and continues to search if necessary. Since \( V(w) \) is monotonically increasing when \( w \geq b \), he will switch jobs whenever a new wage offer is higher than the current wage. The optimal search strategy also satisfies the reservation property in that a worker stops on-the-job search if his current wage is higher than the search reservation wage, \( R \). The search reservation wage \( R \) is determined at the level where the expected gain from search is equal to the fixed cost of on-the-job search\(^4\):

\[
\lambda \int_R^\infty [V(z) - V(R)]dF(z) = \frac{\lambda}{\rho + \delta} \int_R^\infty [z - R]dF(z) = c. \tag{2}
\]

We assume that \( R > b \) to guarantee that unemployed people participate in the search. This assumption is equivalent to the condition:

\[
\frac{\lambda}{\rho + \delta} \int_b^\infty [z - b]dF(z) > c. \tag{3}
\]

In summary, a worker does not participate in job search if \( w \geq R \). He searches while employed if \( R > w \geq b \). If \( w < b \), he searches while unemployed.

II.2 Firms

The output of a firm depends on the productivity, \( \mu \), and the work effort of its employees, \( e \). For simplicity, we assume that work effort \( e \) is a function of the on-the-job search intensity, \( s \), such that \( e = 1 - s \). Then, under the assumption of a constant returns to scale technology, the profit of a firm can be written as,

\[\text{Profit} = \frac{\mu}{\rho + \delta} \int_0^\infty [e - f(s)]dF(s).\]

\(^4\) For a derivation, see Mortensen (1986, p890-91).
\[ \pi = [ (1 - s)\mu - w ] L, \]  
\[ (4) \]

where \( L \) is the number of workers employed. We assume that the labor market on each island is perfectly competitive so that the equilibrium profit is equal to zero. Since workers continue to search (i.e., \( s = 1 \)) as long as their wage is less than the search reservation wage \( R \), it is obvious from (4) that firms with \( \mu \) less than \( R \) should expect negative profits if they operate, and therefore, they would not hire any workers. Only the firms with \( \mu \) higher than \( R \) can pay more than the search reservation wage and employ workers by paying wage \( w = \mu \). Since all operating firms pay wages higher than \( R \), only unemployed workers search in our model even though on-the-job search is possible.\(^5\)

In summary, given the distribution of labor productivity \( G(\mu) \) and the firms' belief about the workers' reservation wage \( R \), the wage distribution generated by the firms' employment decision, \( F(w;R) \), is:

\[ F(w;R) = \begin{cases} 
G(R) & \text{if } w < R, \\
G(w) & \text{if } w \geq R.
\end{cases} \]  
\[ (5) \]

II.3 The Market Equilibrium

The Nash equilibrium in this model can be characterized as follows. Taking a wage distribution \( F(w) \) as given, each worker chooses a search reservation wage \( R \) given by equation (2). In equilibrium, the distribution \( F(w) \) should also be consistent with the wage distribution generated by the firms' employment decision -- the distribution \( F(w;R) \) in (5).

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\(^5\) Note that without efficiency wage considerations, only firms with \( \mu \) less than \( b \), not \( R \), would stop producing.
Proposition 1: There exists a unique Nash equilibrium.

Proof: Since $G$ is exogenously given, $F(w,R)$ can be uniquely determined by the equilibrium reservation wage. Thus, it suffices to show the existence of a unique reservation wage, $R_m$, which satisfies (2) and (5) simultaneously. Using (5), we can rewrite equation (2) as

$$h(R) = \frac{\lambda}{\rho + \delta} \int_R^\infty (z-R)dG(z) - c = 0. \quad (6)$$

Since $h(R)$ is monotonically decreasing in $R$ with $\lim_{R \to \infty} h(R) = -c < 0$ and $\lim_{R \to 0} h(R) > 0$, there exists a unique solution for $h(R) = 0$. Moreover, it follows from (3) that $h(b) > 0$ and, therefore, $R_m > b$. Q.E.D.

II.4 Welfare Analysis

In this section, we demonstrate that the market equilibrium in our model is not Pareto optimal. To set up the central planning problem, let us define $P(w,R)$ as the steady state probability that the wage income of a representative worker is less than or equal to $w$ when the search reservation wage is $R$. Then, $P(w,R)$ can be described as follows:\footnote{The derivation is as follows. First, consider the case when $R \geq b$. Since a minimum wage of $b$ is guaranteed in the secondary sector, $P(w,R) = 0$ if $w < b$. When $w = b$, the inflow of workers into the group with wage less than or equal to $w$ consists of workers whose wages were higher than $w$ but employment opportunities have ended because of job death at the rate of $\delta$. The outflow of workers consists of workers who find jobs with wages higher than $w$ through job search. Thus, the instantaneous change in $P(w,R)$ is: if $b \leq w \leq R$,

$$dP(w,R)/dt = \delta[1-P(w,R)] - P(w,R)\lambda[1-G(w)] = \delta[1-P(R,R)] - P(R,R)\lambda[1-G(R)],$$

and if $R < w$, $dP(w,R)/dt = \delta[1-P(w,R)] - P(R,R)\lambda[1-G(w)]$. $P(w,R)$ can be obtained by setting $dP(w,R)/dt = 0$.

Second, suppose $R < b$. In this case, no worker, whether employed or not, has an incentive to search. Since every job match will eventually be separated, every worker will remain unemployed, and there will be no search}
\[ P(w, R) = \begin{cases} 
0 & \text{if } w < b, \\
\frac{\delta}{\delta + \lambda - \lambda G(R)} & \text{if } b \leq w < R, \quad \text{when } R \geq b, \\
\frac{\delta + \lambda G(w) - \lambda G(R)}{\delta + \lambda - \lambda G(R)} & \text{if } w \geq R 
\end{cases} \quad (7a) \\
\]

\[ P(w, R) = \begin{cases} 
0 & \text{if } w < b, \\
1 & \text{if } w \geq b. 
\end{cases} \quad (7b) \\
\]

The central planning problem is to maximize the expected value of the representative worker in the steady state by choosing an optimal search reservation wage \( R_s \):

\[
\text{Max } W(R) = \text{Max } R \left[ V(b)P(R, R) + \int_R^\infty V(w)dP(w, R) \right] \\
= \text{Max } R \left[ (b-c)P(R, R) + \int_R^\infty w \ dP(w, R) \right]. \quad (8)
\]

Since workers are risk neutral, the value maximization problem in the steady state is equivalent to the problem of steady-state income (output) maximization.

**Proposition 2**: Let \( R_s \) and \( R_m \) be the optimal search reservation wage and the search reservation wage in the market economy, respectively. Then \( b < R_s < \)

\[
\text{in the steady state.}
\]

By assuming that the central planner maximizes the expected value in the steady state, we are implicitly assuming that workers do not discount the future (\( \rho = 0 \)). To simplify the analysis and to avoid solving a dynamic social welfare problem, \( \rho = 0 \) is assumed in the following analysis.
Proof: Differentiating $W(R)$ with respect to $R$, we get

$$\frac{\partial W(R)}{\partial R} = \frac{\lambda \frac{\partial G(R)}{\partial R}}{[\delta + \lambda - \lambda G(R)]^2} k(R),$$  

where

$$k(R) = \lambda \int_{R}^{\infty} (w-R)dG(w) - \delta (R-b+c).$$  

Since $\frac{\partial G(R)}{\partial R} > 0$, $\frac{\partial W(R)}{\partial R} = 0$ if and only if $k(R) = 0$. $k(R)$ is monotonically decreasing in $R$ with $k(b) > 0$ and $k(R_m) = -\delta (R_m-b) < 0$ according to (6) and (10). As a result, there exists a unique solution, $R_s$, which satisfies $\frac{\partial W(R_s)}{\partial R} = 0$ and $b < R_s < R_m$. The last claim in the proposition follows from the fact that the probability of unemployment in the steady state, $P(R,R)$, is increasing in $R$ as can be seen from (7a). Q.E.D.

It would be useful to explain informally why the market search reservation wage and unemployment rate are higher than the socially optimum levels. Higher search intensities as signaled by a higher reservation wage have positive as well as negative effects on steady state welfare. The higher the reservation wage is, the more favorable the wage distribution becomes for values of $w \geq R$. However, the higher the reservation wage is, the larger is the number of firms which stop their production, and the the probability of unemployment increases. Since on-the-job search is possible, the private cost of search consists only of the fixed cost of search, $c$, not the foregone

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8 The steady-state proportion of workers with wages higher than $w$ is $1 - P(w,R) = \lambda[1-G(w)]/[\delta + \lambda - \lambda G(R)]$ when $w \geq R$, which is monotonically increasing in $R$. 

9
wages while searching. Therefore, workers equalizes the positive gains from search to the fixed cost of search, ignoring the negative effect of the reservation wage on the steady state employment. Since the social planner takes into account this negative effect on employment, \( \bar{R} \) is less than \( R_m \) and there is excess unemployment in the market economy. Note that the excess unemployment is due to the efficiency wage consideration alone; Without the efficiency wage consideration in our model, the negative effect on unemployment would not exist and the market economy would be Pareto optimal.\(^{10}\)

Whether this excess unemployment is involuntary is a semantic issue. Unemployment is involuntary in our model in the sense that unemployed workers have an incentive to underbid the minimum wage level in the economy. For example, if an unemployed worker can access a firm paying the wage rate \( R_m \), he would want to underbid the wage and replace an incumbent worker. However, the firm would not be persuaded since it correctly anticipates that the new worker will do on-the-job search if wage is lower than \( R_m \).

III. Industrial Policy

Since the marginal productivities in the efficiency wage sector are higher than that in the secondary sector, \( b \), a central planner has an

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9 If on-the-job search is not possible, the private cost of the higher reservation wage includes the foregone wages while searching in addition to the fixed cost of search. Therefore, its negative effect on unemployment is internalized and the market reservation wage is socially optimal. On the other hand, if search is a purely "rent seeking" behavior, i.e., search does not allocate more workers to jobs with higher productivity, the positive social gain would not exist even though the private returns from finding a higher wage would still be positive. In that case, socially optimal reservation wage would be the wage rate in the secondary sector, \( b \). The model with a decreasing returns to scale in the appendix has this feature.

10 In contrast to Shapiro and Stiglitz (1984), our economy is Pareto inefficient despite the assumption of a constant returns to scale technology.
incentive to transfer workers from the secondary sector to the efficiency wage sector. This explains the intention of an industrial policy which suggests an employment subsidy to the efficiency wage sector (Shapiro and Stiglitz (1984), Bulow and Summers (1986), and Katz and Summers (1989)). However, in this section, we show that the suggested industrial policy is ineffective when workers’ search decisions are explicitly accounted for.

Suppose that the government pays a lump sum subsidy $q$ per employment in the efficiency wage sector. The subsidy is financed by a lump sum tax $\tau$ on all workers in the economy. Then, since labor market is perfectly competitive, the wage offered by a firm with productivity $\mu$ will be $\mu + q$. Let $F_q$ be the wage distribution under the subsidy regime. Then, the workers’ problem becomes

$$
\rho V(w) = \max_{s \in \{0,1\}} \left\{ \max(w,b) - cs + \lambda s \int_q \{ \max[V(z),V(w)] - V(w) \} dF(z) + \delta[V(b)-V(w)] - \tau \right\}.
$$

The workers’ search reservation wage, $R_q$, satisfies

$$
\lambda \int_q [V(z)-V(R_q)]dF(z) = \frac{\lambda}{\rho + \delta} \int_q [z-R_q]dF(z) = c. \quad (12)
$$

Following the same procedure in section II.2, the wage distribution generated

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11 The ineffectiveness of the industrial policy in our model does not depend on the assumption of perfectly competitive labor market which implies that the wage increases as much as the subsidy. The model in the appendix shows the ineffectiveness of the policy when the subsidies are shared by firms and workers through a bargaining process.
by firms' employment decision can be derived to be;

\[
F_q(w; R_q) = \begin{cases} 
  G(R_q - q) & \text{if } w < R_q \\
  G(w - q) & \text{if } w \geq R_q
\end{cases}
\]  

(13)

Proposition 3 shows the relationship between the market equilibrium reservation wages with and without the industrial policy \( R_q \) and \( R_m \), respectively. Proposition 4 shows that the industrial policy is ineffective.

**Proposition 3:** \( R_q = R_m + q \).

**Proof:** The existence and the uniqueness of \( R_q \) can be proved following the same procedure used in the proof of Proposition 1. To prove the above equality, it suffices to show that \( R_q = R_m + q \) satisfies (12) and (13) simultaneously. By substituting \( R_m + q \) for \( R_q \) in (12), we can demonstrate that

\[
\frac{\lambda}{\rho + \delta} \int_{R_q}^{\infty} [z - R] dF_q(z) = \frac{\lambda}{\rho + \delta} \int_{R_m + q}^{\infty} [z - R_m - q] dG(z - q) = \frac{\lambda}{\rho + \delta} \int_{R_m}^{\infty} [w - R_m] dG(w) = c.
\]

The last equality follows from (6). Q.E.D.

**Proposition 4:** The Welfare level in the market equilibrium with the industrial policy \( W_q \) is the same as that without the policy \( W_m \).

**Proof:** Let \( P(w, R_q) \) be the counterpart of \( P(w, R_m) \) in section II.4 under the subsidy policy. Then, \( P_q \) can be described as follows when \( R_q = R_m + q \):

\[
P_q(w, R_q) = \begin{cases} 
  0 & \text{if } w < b, \\
  P(R_m, R_m) & \text{if } b \leq w < R_q, \\
  P(w-q, R_m) & \text{if } w \geq R_q.
\end{cases}
\]  

(14)

Then, the welfare level under the subsidy is
\[ W_q = \left[ (b-c-\tau)P(R_q, R_q) + \int_{R_q}^{\infty} (w-\tau)dP(w, R_q) \right]. \tag{15} \]

Balanced budget requires that \( \tau \) should be set so that tax revenue equals the subsidy payment. Thus,

\[ \tau = q \left[ 1 - P \left( R, R \right) \right]. \]

Substituting this expressions for \( \tau \), and replacing \( R_q \) by \( R_m + q \) in equations (14) and (15), we obtain

\[ W_q = \left[ (b-c)P(R_m, R_m) + \int_{R_m}^{\infty} w dP(w, R_m) \right] = W_m \text{ for } q \geq 0. \quad \text{Q.E.D.} \]

The reason why the suggested policy fails to improve social welfare should be clear. If the workers' reservation wage, \( R \), is invariant to the subsidy policy, some firms, which previously could not afford to pay the wage rate \( R \), are now able to operate because of the wage subsidy. However, in the case analyzed in our model, the search reservation wage increases by the exact amount of the subsidy, yielding no new incentive to operate for the firms which did not hire workers before. The equilibrium unemployment rate is unaffected by the subsidy policy. This strong result of complete ineffectiveness of the policy depends on our assumptions about the search technology and effort function, which are not especially weak. It is easy to demonstrate, however, that workers' "rent seeking" behavior induced by the subsidy will partially offset the positive effect of the subsidy on employment, with weaker assumptions about search technology and the effort function. The size and significance of this "rent seeking" behavior is clearly an empirical question which we leave to future research. As an extension, a model with a decreasing returns to scale technology and imperfectly competitive labor market is analyzed in the appendix. The
qualitative results of the previous sections are confirmed in the extension.

IV. Conclusion

This paper considers workers' on-the-job search behavior as another microfoundation for efficiency wage theory. If intensive job search efforts of employees harm productivity, firms may have an incentive to pay an efficiency wage premium to reduce workers' on-the-job search intensities. Adding a search decision into an efficiency wage framework is more than cosmetic. Our model has different policy implications from the previous efficiency wage models. We show that the industrial policy which subsidizes higher wage jobs is not necessarily effective when the search decision is explicitly accounted for. An employment subsidy to high wage sectors encourages workers' job search intensities rather than employment. Our argument is independent of other perverse effects from "rent seeking" behavior of employers and the anti-egalitarian consequences of the industrial policy.

The failure of a subsidy to improve welfare does not necessarily imply that market economies with efficiency wages are constrained Pareto efficient. Greenwald and Stiglitz (1986, 1988) show that market economies with imperfect information and incomplete markets are, in general, not constrained Pareto efficient. If we introduce heterogeneous goods or heterogeneous agents in our model, a central planner may have several tools to reduce search reservation wages and increase employment even with the same information structure. One example may be a tax on a complementary good to search activity. However, the main result of this paper is that the industrial policy which subsidizes the efficiency wage sector is not one of these tools.
APPENDIX

The previous model is extended to include a decreasing returns to scale technology and imperfectly competitive labor market.

A.1 The Model

Consider an economy consisting of a continuum of firms across different locations. We assume that each firm, when operating, employs only one worker. This amounts to assuming an extreme form of decreasing returns. As in the previous model, a job in each firm has an uncertain but finite lifetime and those jobs that die are replaced by others elsewhere. The measure of firms and the measure of workers are normalized to one for simplicity. Productivity of each firm is a random draw from the distribution function \( G(\mu) \).

In each period, firms are either vacant or filled. Vacant firms advertise vacancy at zero cost. Workers participate in job search by paying a fixed cost of \( c \). Each job searcher can sample at most one vacancy and the probability of a job searcher contacting a vacancy during a short time period of \( \Delta t \) is \( \lambda \Delta t \). Each vacant firm can contact at most one worker per period. Due to these restrictions on search technology, wage determination in this economy is a bilateral bargaining problem. Without introducing an explicit bargaining process, we assume that the following output sharing rule is used to set wages\(^{12} \):

\[
    w = \alpha \mu, \quad 0 < \alpha \leq 1
\]  \hspace{1cm} (A1)

This wage setting rule has the flavor of an implicit wage contract since firms bear all income risk (Hart(1983)). Firms are equally owned by workers

\(^{12}\) The assumption that \( \alpha \) is a constant for all \( \mu \) can be relaxed by the assumption that \( \alpha \) is increasing in \( \mu \).
and profits, if any, are equally distributed to workers as dividends.\footnote{Before discussing each agent's optimization problem, a few remarks on the search technology in this paper might be helpful for clarity. Job search in this model is a "rent seeking" behavior for the following reasons. First, since the number of jobs available is fixed under our extreme form of diminishing returns to scale technology, search does not allocate more workers to jobs with higher productivity. Secondly, since productivity is firm specific and not match specific, there is no social gain from having one worker rather than another in a particular firm, but private returns to the searcher from finding a higher wage still exist. Also, to highlight the effect of the efficiency wage premium, the possibility of a congestion externality of search is precluded by assuming that job applicants only receive information about jobs which are in fact vacant. (Diamond (1981), Albrecht and Jovanovic (1986), and Mortensen (1986))}

A.2 The Market Equilibrium

Workers adopt exactly the same search strategy as described in section II. Given workers' choice of reservation wage, the productivity, and the wage setting rule, each firm decides whether to operate or not in order to maximize its profit. The profit of a firm is

\[ \pi = (1 - s) \mu - w = (1 - s - \alpha) \mu, \]  

(A2)

where \( \mu \) is the productivity of the firm and \( 1 - s \) is the work effort of its employee. Since \( \alpha > 0 \), the firm expects negative profit as long as \( s = 1 \). As a result, all firms whose productivities are lower than \( R/\alpha \) will not operate because their wages are not high enough to prevent job search by their employees.

One may object to the above argument on the grounds that a firm whose \( \mu \) is slightly below \( R/\alpha \) has an incentive to increase \( \alpha \) to offer the wage \( R \). By doing so, it can still expect a positive profit. However, if one considers a reputation effect on the relative bargaining power, raising \( \alpha \) may not be profitable in the long run. For example, assume the presence of a ratchet effect, i.e., \( \alpha \) cannot be lowered once it is increased. (Freixas, Guesnerie,
and Tirole (1985)) Then, in our model, a manager has to weigh the trade-off between the current profit gain from raising $\alpha$ and the future profit loss from higher wage payments. The manager cares for his future profits sufficiently, he will not raise the labor share, $\alpha$.

In order to derive the Nash equilibrium wage distribution, $F(w)$, let $G_1(\mu,R)$ denote the steady-state fraction of vacant firms with productivity less than or equal to $\mu$, given that all workers choose $R$ as their reservation wage. Let $G_2(\mu,R)$ denote the complement of $G_1$ so that

$$G(\mu) = G_1(\mu,R) + G_2(\mu,R), \quad \text{for all } (\mu,R). \quad (A3)$$

Since no firms hire workers unless their $\mu$ is high enough to support wages above or equal to $R$,

$$G_1(\mu,R) = G(\mu), \quad \text{for all } \mu < R/\alpha. \quad (A4)$$

In the steady state, the flow into and out of vacant firms must be balanced. The new flow into vacant firms is $\delta G_2(\mu,R)$ since the transition from filled firms to vacant ones occurs only when the job match dissolves. Meanwhile, only firms with productivity higher than $R/\alpha$ will hire workers, and the instantaneous flow of workers who are informed about a job vacancy is $\lambda$. Hence, the product of $\lambda$ and $G_1(\mu,R) - G_1(R/\alpha,R)$ is the job filling rate among the firms whose productivity is less than $\mu$. Hence, the steady state condition requires

$$\lambda [G_1(\mu,R) - G_1(R/\alpha,R)] = \delta G_2(\mu,R), \quad \text{for } \mu \geq R/\alpha. \quad (A5)$$

Then, from (A3) and (A5),

$$G_1(\mu,R) = \frac{\delta G(\mu) + \lambda G(R/\alpha)}{\delta + \lambda}, \quad \text{for } \mu \geq R/\alpha. \quad (A6)$$

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14 If we modify our model so that the productivity of each firm is subject to random shocks, the ratchet effect becomes stronger and more plausible.
Since the wage distribution that workers perceive is the conditional distribution of wages over vacant jobs, it can be derived from (A4) and (A6) as:

\[
F(w; R) = \begin{cases} 
\frac{(\delta + \lambda)G(R/\alpha)}{\delta + \lambda G(R/\alpha)} & \text{if } w < R, \\
\frac{\delta G(w/\alpha) + \lambda G(R/\alpha)}{\delta + \lambda G(R/\alpha)} & \text{if } w \geq R.
\end{cases}
\]  

(A7)

The Nash equilibrium in the economy is characterized by \( R \) and \( F(w; R) \) which simultaneously satisfy (A7) and (2) in section II.1.

**Proposition A1:** There exists a unique Nash equilibrium.

**Proposition A2:** Let \( R_s \) and \( R_m \) be the optimal search reservation wage and the search reservation wage in the market economy, respectively. Then \( b = R_s < R_m \); i.e., there are excess search incentives and unemployment in the market economy.

Since the proofs involve the same procedures used in section II.3 and II.4, they are omitted but available by request. In proposition A2, it is clear why the socially optimal search reservation wage is equal to \( b \). As in the previous model, a higher search reservation wage has a negative effect of reducing the number of operating firms. However, contrary to our previous model, job search in this model is a purely "rent seeking" behavior; a higher search reservation wage does not have a positive effect of making the steady state wage distribution favorable for values \( w \geq R \geq b \). Therefore, it is in

\[ \text{See footnote 9.} \]
the social planner's interest to make all firms in the efficiency wage sector operative \((R_s = b)\) as long as their productivities are higher than the productivity in the secondary sector. Since private returns to the searcher from finding a higher wage still exist, \(R_m\) is higher than \(b\).

### A.3 Industrial Policy

Consider an employment subsidy \(q\) in the efficiency wage sector, financed by a lump sum tax \(\tau\). Assume that the subsidy affects the wage setting rule in (A1) as follows:

\[
w = \alpha u + \beta q, \quad 0 < \alpha, \beta < 1,
\]

(A8)

where \(\beta\) is the workers' share of the subsidy. Since the subsidy can change the relative bargaining power of the two parties, \(\alpha\) and \(\beta\) are not necessarily the same.

Given the wage offer distribution, \(F_q(w)\), the workers' search reservation wage satisfies the condition (12) in section II.4. Following the same procedure used in the derivation of (A7), the wage distribution generated by firms' employment decision is obtained as:

\[
F_q(w; R_q) = \begin{cases} 
\frac{(\delta + \lambda)G^{R_q - \beta q}}{\alpha} & \text{if } w < R_q, \\
\frac{\delta G^{\nu - \beta q}}{\alpha} + \lambda G^{R_q - \beta q} & \text{if } w \geq R_q,
\end{cases}
\]

(A9)

The Nash equilibrium under the subsidy is characterized by \(R_q\) and \(F_q(w; R_q)\) which simultaneously satisfy (A10) and (12). Proposition A3 shows

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16 Unlike the constant returns case, the subsidy can be financed by a profit tax or a lump sum tax with the same policy implications.
the relationship between the unique market equilibrium reservation wages with and without the industrial policy ($R_q$ and $R_m$). Proposition A4 shows that the industrial policy is ineffective. The proofs are omitted but available by request.

**Proposition A3:** $R_q = R_m + \beta q$.

**Proposition A4:** The Welfare level in market equilibrium with the industrial policy is the same as that without the policy.

As in the previous model, the subsidy increases the workers' search reservation wage and, as a result, does not provide firms with new incentives for hiring. The equilibrium unemployment rate is the same irrespective of the subsidy policy. However, the above analysis did not consider the possibility of new entries of firms into the subsidized sector. When $\beta < 1$, expected profit of the subsidized industry increases by $(1-\beta)q$, providing more incentives for entry. One can consider that the above analysis is for a small subsidy which does not significantly increase profits over the fixed cost of entry. Or, preferably, one can argue that $\beta = 1$ in the long run since entry will eliminate excess profits due to a subsidy. When $\beta = 1$, an employment subsidy does not increase incentives for entry and the above analysis is applicable.
References


