Multinational Entry, Expropriation and Contract Compliance Under a Boycott Threat

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Abstract

We study the problem of multinational investment in a host country where the only threat to counter contract violation by the host govt. is a total boycott by future entrants. If the MNC investment is amortized over the lifetime of the project, then, for each project lifetime there is range of discount factors for the host government under which the contracts will not be violated. Otherwise there will be no entry. It is shown that for countries with a rate of time preference outside the compliance zone, a self-enforcing contract that specifies an amortization period smaller than the project life ensures compliance and entry.

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1. **INTRODUCTION:**

A Multinational corporation (MNC) investing in a host country faces two types of expropriation threats once its plant and equipment are in place. First, its investment project may be nationalized; and second, its tax agreement may be renegotiated. The absence of legal or other credible devices to enforce contracts that leads to this time inconsistency problem [Eaton and Gersovitz (1983)] where the host country has an incentive to renege on its contractual obligations once the MNC has committed itself, led Bronfrenbrenner (1954-55) to conclude that economic "'neo-isolationism'" could be the only reasonable option for developed nations. But the sword has two edges: the absence of mechanisms whereby a host country can credibly precommit to maintain its contractual obligations means that it may fail to attract MNC investment and thus fail to make even the gains that accrue from contract compliance.

Yet MNC investment across borders proceeds. Since the MNC, inspite of various legislations and international agreements [See Thomas and Worrall (1990) and thereferences cited there] cannot reasonably expect to recoup its losses is the host government reneges on its obligations, it will enter only if its contract is self enforcing
i.e., the time inconsistency-problem has been removed. The host country on the other hand, because of its inability to precommit credibly to contract compliance, will accept such a contract as this enables it to ensure the second best option. This is the crux of the recent literature on the subject.

Marjit (1990) shows that an investment sharing contract is sufficient to neutralize the threat of nationalization in the case of a one shot entry game. Magee (1977) suggests that MNC's will introduce complex production processes which the host country cannot operate to for-stall nationalization. But if complex processes are also more capital intensive, then the introduction of such processes worsens the time inconsistency problem since the host government's incentive to expropriate increases. It is this idea that lead Eaton and Gersovitz (1984) to conclude that MNC's will keep the level of investment low to prevent nationalization. Bond and Samuelson (1989) investigate the problem of tax rate negotiation in a model where the entrant firm operates over two periods. They assume that commitments to the first period tax rates are credible, but the rate for the second period may be renegotiated unilaterally by the
host country once the MNC commits itself by investing in the first period. Their focus is on the desirability of commitment not to renegotiate tax rates by the host country if such commitment is possible. They find that if there is bargaining without commitment the MNC's investment will be low; whereas if bargaining is with commitment, but future taxes on capital use are distrotionary, the MNC will underinvest. Thomas and Worral (1990) investigate the same problem. They argue that the one shot model is not a reasonable vehicle for studying the problem. Since any tax contract in such a model can not be self enforcing. In their model, the MNC's investment is distributed over an infinite sequence of periods and the contract to be agreed upon stipulates the quantity of investment and the tax rate in each period. They establish a characterisation of the time structure of investment where there is underinvestment in the early periods. Brander and Spencer (1987) employ a totally different approach. If MNC investment generates employment then the host government may face a trade off between tax revenue and employment. It may then keep tax rates low in order to induce higher MNC output which in turn will generate higher employment. The crucial element,
then is to use a device that makes the contract self enforceable. In this paper we investigate the role of boycott by future potential entrants as a deterrent against contract violation. The idea is not new [see Bond and Samuelson (1989)]. But we believe that it merits more careful scrutiny. The intuitive idea is simple enough: if the host government violates its contract with an MNC, then it scares off all future entrants. Is this fear sufficient to guarantee compliance? To focus on this issue, we construct a model that is shown of most 'real world' details. In our model, identical MNCs invest an amount that is exogenously fixed, entering the host country in a sequence of periods. The MNC's project life may exceed one period, so that at any point a number of MNC's of different entry vintage may co-exist. It is assumed that the host country cannot operate the equipment on its own and that this equipment has a zero resale value. The last assumption was made to eliminate the threat of nationalization from the realm of discourse. The host country and the MNC agree on a contract that allows the latter to keep an exogenously given reservation level of profit after amortization. Our model has no uncertainty and all information is common knowledge. Hence, informational incompleteness or asymmetries play no role in the model.
Once an MNC commits itself by setting up its plant and equipment, it is open to the threat of expropriation of its reservation level of profits. There is, however, no threat that the amortization deductions will be expropriated since the MNC will leave in that case and the government will not earn anything.

We begin by establishing that the threat of future boycott by MNC's has no effect if the sequence of periods over which entry may occur is finite and known. To make the threat meaningful, we assume that the sequence is infinite. We find that in our model, the critical elements that ensure compliance under a boycott threat are the size of the host country's discount factor and the length of the amortization period.

We begin by assuming that each MNC amortizes its investment over the entire life of its project. This is necessary to make the expropriation threat on all MNCs existing in any period credible. Our primary finding is that for each project lifetime (since MNC's are identical, they have the same project lifetime) there is an interval (connected) of values of the host country's discount factor for which the contract is enforcable for all MNC's [Proposition 4].
If the discount rate is not within the specified range, no entry occurs and the host country is forced to accept the third best situation of zero earnings. Proposition - 5 shows that this interval contracts as the length of the project lifetime rises. That is, MNC's with long project lifetimes will enter only if the host country places a high enough value on future incomes. What makes Proposition 5 interesting is that it provides a way in which countries with low discount factors can ensure the second best alternative when faced with projects of relatively long life-spans. If the country allows the MNC's to amortize over a period shorter than their project lifetime, then the maximal number of "hostages" in any period falls. This means that the relative gain from compliance (as compared to cheating the currently existing MNCs) increases and therefore the range of discount factors that ensure compliance increases. Hence, given any discount factor, a suitably chosen amortization period guarantees compliance. The host country would of course, agree to this if the only other alternative is the third best outcome. We push this argument further to show that of there is uncertainty about the host country's discount rate, but the range of its variability is known, a sufficiently
small amortization period can be found that will make MNC contracts self-enforcing and therefore ensure the second best outcome for the host government.

The paper is organized as follows. In Section 2, we study the case where the number of entry periods is finite. Section 3 develops the infinite period case Section 4 concludes. All proofs, except the one for Proposition 1, are relegated to an appendix.

2. THE FINITE HORIZON CASE

In this section we study the problem under simple analytical scenarios. The objective here is to expose what we believe are the essential issues involved. Consider first the case of a host country that negotiates with a Multinational Corporation (MNC) to install, finance and operate a project of fixed investment value (I) in its territory. Suppose the contract involves an agreement on the sharing of the net revenue (S), i.e., the revenue after the deduction of operating costs and depreciation, whereby the MNC gets its reservation profit (\( \pi \)) and the rest is extracted by the government in the form of a lump sum tax (T). To keep the story simple, let us also suppose that: a) the host country is unable to operate the project on its
own; and b) the MNC amortizes its investment over the life of the project. No MNC would agree to enter into such a contract. For suppose an agreement is reached and the MNC installs its plant in the host country. The host country's government can then unilaterally raise the lumpsum tax and expropriate all of the MNC's profit. As long as the MNC is able to recoup its investment, it will stay, making the threat credible. The upshot of all this is that if the potential MNC entrant has any financial stake in the project, then collusion in the sense that contractual agreements will be honoured, is not possible. In this case, collusion is only possible if the MNC enters as a consultant operator without any financial stage. Why then do MNCs enter into agreements with host governments that involve financial commitments on their part? The reason presumably is that MNC's believe that the host government may have incentives to honour their contractual agreements. The intuition behind this idea is simple. Since the government by breaking its contractual agreements with a particular MNC may scare off future potential entrants, it would stand to lose the stream of tax returns from the future entrants. In that case, collusion may be a good strategy for the host
government. In the rest of this paper, we explore this idea more carefully.

The basic problem that we are trying to address may be stated thus: can the fear that violation of a contract today will lead to a total boycott by all potential future entrants, ensure contract compliance? To study this, we extend out one shot model to include future entrants in a very obvious way. Suppose the host government negotiates with a finite stream of potential entrants who enter at the rate of one per period. Each MNC is of the type described in our single entrant story. Further, it is common knowledge that if the host government cheats on its contract with one MNC, then all further entrants avoid the host country. The following proposition demonstrates that contract compliance cannot be guarantees in this case.

Proposition 1

If $I > 0$ and $\pi > 0$ for each MNC, then collusion is not sustainable for a finite sequence of MNC entrants. With a well defined last period.
Proof: Consider any finite sequence of periods indexed by 1, 2, ..., n. Since I > 0 and π > 0 for the MNC entering in period n, the government will violate its contract in period n. So, MNC the potential entrant in period n will not enter. Hence the last period with a potential entrant is n-1. Repeated use of this argument proves the proposition.

Remark 1: The proposition does not involve any restrictions on the lifetimes of the MNC's projects.

Remark 2: The restriction that only one MNC enters per period was introduced purely for simplifying the proof. Introducing the possibility of multiple entrants per period does not change the result.

Proposition 1 basically implies that if the threat of boycott is to have any effect, there cannot be a well defined last period beyond which there is no entry by MNCs. Suppose that this last condition holds. Will that by itself guarantee compliance? We take up this question in the next section.
3. **The Infinite Horizon Case**

For the sake of expositional clarity, let us assume that there is a single entrant in period \( t \) called \( MNC_t \). It should be noted that as in the case of proposition 1 (see Remark 2) the introduction of multiple entrants per period does not affect our qualitative results in any way.

Suppose now that there is an infinite sequence of identical potential entrants, one entering in each period. At the beginning of time period \( t \), the government offers a contract of the type described in the last section to \( MNC_t \). The MNC has the option of rejecting it or accepting it. If it accepts then it begins production in period \( t \). Can contract compliance by ensured in this case? Consider the case where each MNC project lives for one period. Let \( \delta \), the government's discount rate, be common knowledge. The following proposition asserts that for a range of values of \( \delta \), the government will comply with its contractual agreements and will cheat otherwise.

**Proposition 2**

Suppose that there is an infinite sequence of one period lived identical potential entrants, such that \( S > T \geq 0 \).
(i) If \( S > \frac{T}{1-\delta} \) then the government will cheat the first MNC that enters.

(ii) If \( S \leq \frac{T}{1-\delta} \), then the government will never cheat.

Proof: (See Appendix)

It follows that collusion will occur if and only if

\[ \delta \in \left[ \frac{S-T}{S}, 1 \right) . \]

The problem here is that in this case there is no overhang of past MNC investment. If such a building of hostages could occur, then the host government's incentive to cheat may increases over time. The analogy here is with a problem in the theory of sovereign debt. As the debt burden increases, the incentive to repudite increases even though it is known that repudiation will lead to a cut-off of all future debt. In order to appreciate the essential nature of the problem, we look at another extreme case: each MNC project has an infinite life. Proposition 3 asserts that there will be no collusion in this case.

**Proposition 3.**

Suppose there is an infinite sequence of identical infinitely
lived potential entrants. For any $S$ and $T$ with $S > T \geq 0$ and $\delta \in [0,1)$, collusion will not occur.

(Proof: See Appendix)

The intuition of the proof is simple. Each period that the government defers cheating, it adds a new MNC to its bunch of 'hostages' whose future stream of profits it can expropriate. At some period $k < \infty$ depending on $\delta$, the present value of the stream of net revenues of the existing MNCs exceeds the present value of returns from deferring cheating any further. Since, $S$, $T$ and $\delta$ are common knowledge, everyone knows that the host government will cheat MNC (and of course all the other existing MNCs) by unilaterally increasing the lump sum tax. Since, $k < \infty$, we are back in the world of Proposition 1. Hence the result.

Proposition 2 and 3 may lead one to guess that as $n$ increases from one, the range of $\alpha$ values for which collusion will occur shrinks. The next two propositions demonstrate that this intuition is indeed correct.

**Proposition 4:**

Suppose there is an infinite sequence of identical potential
behind this is fairly simple. At the beginning of each period the host government calculates the difference in the present values of the payoffs from cheating and from contract compliance. In each period, including and following $n$, the number of MNCs that are in operation is equal to $n$. If amortization is over the entire lifetime of the project, these MNCs are hostages. By reducing the amortization period some of the firms of the earliest existing vintages are no longer hostages and they have no financial stake left in their projects. This reduces the payoff from cheating in the current period while leaving the stream of payoffs from compliance unaltered. Hence, the range of values over which compliance is preferred to violation expands.

The argument can be driven further to derive further insight. Suppose that the host country's discount factor is not known with certainty. This is quite plausible, especially in a model like ours where the horizon is infinite. Governments change. In our case the government is characterized by its discount factor. Hence, the discount factor may vary over time. Theorem 5 guarantees that even in this case a contract clause can be found that ensures compliance over time. Suppose it is known that $\delta \in [\delta_1, \delta_2]$ with $\delta_1 \geq \frac{s - I}{s}$. Then, by
entrants each with a lifetime of n periods. For any S and T with
\( S > T \geq 0, \exists \delta \in [\delta n, 1) \) such that collusion with occur where \( \delta n \in (0, 1) \). No entry will occur otherwise.

**Proposition 5**

If \( S > T > 0 \) then \( \delta n \) is monotonically increasing in \( n \).

[ For proofs of these propositions see the appendix ]

Proposition 5 says that as the project life increases, the range of \( \delta \) values for which entry will occur shrinks. But this conclusion hinges on the assumption that each MNC amortizes its investment over its entire lifetime. Suppose now that the host country's discount rate \( \delta \) is smaller than \( \delta n \), where \( n \) is the project life. Then, propositions 4 and 5 imply that the potential entrants can ensure contract compliance by bargaining for an amortization period \( m \leq n \) such that \( \delta \) lies in the range \([\delta m, 1)\). Once the loan has been amortized, the MNC has no financial stake in the project and its profit earnings thereafter are purely from its service as a consultant-cum-operator. From the governments point of view, letting the MNC stay after its investment has been amortized, is the best option as this yields the stream of tax revenues for the remaining life of the project. The intuition
choosing an amortization period that ensures that $\delta_1$ belongs to the compliance zone, compliance can be guaranteed. The condition is clearly sufficient, but need not be necessary.

CONCLUDING REMARKS:

On concern in this paper has been with the problem of devising self enforcing contracts in the context of the multinational entry problem. As we noted in the introduction, the need for self enforcing contracts emanates from the absence of outside mechanisms for enforcing contracts. What may be questioned is the assertion that there are little or no means by which contracts can be enforced from outside. A host country that reneges may be faced with consequences running from credit cut-offs to economic and political blockades. And historically, this argument is not without merit. Some studies [see Sigmund (1980)] suggest that for various reasons, the frequency with which these threats have been carried out may have diminished in the recent past. We have explored the role of a particular threat, the boycott by future entrants. In our case the boycott is total: if a country reneges, no one enters thereafter. An alternative to this could be
that the boycott is not total, but lasts for a given number of periods depending on the magnitude of the violation. Specifically the penalty length could vary directly with the proportion of the surplus of the existing MNCs that is expropriated.

Questions may also be raised about the selection of an infinite horizon. One way to avoid this may be to work with a similar model where the terminal period is uncertain. Alternatively, one could work with a finite horizon model with asymmetric information where only the host government knows its discount rate. Reputational considerations would then come into play with the host government using signals to build its reputation.

Our goal has been to formulate the problem in the simplest possible mould so as to isolate the effect of the boycott threat on contract compliance. We feel that it is an issue that needs to be tackled and laid to rest before proceeding further.
Proof of Proposition 3

We show first that there is some period $t \geq \infty$, such that at $t$, the host government has no incentive to defer cheating any further. At time $t$, there are $t$ multinationals that are operating. Hence, the present value of cheating today is:

$$P(\ t; \infty \ ) = t \cdot \frac{S}{1-\delta} \quad (1)$$

If the host government cheats $i$ periods after $t$, the present value of the payoff is:

$$P(\ t+i; \infty \ ) = T [ t + \delta(t+1) + \ldots + \delta^{i-1}(t+i-1) ]$$

$$+ \delta^i (t+i) \cdot \frac{S}{1-\delta} \quad (2)$$

The first term in equation (2) gives the present value of the tax stream that the government will receive till the period before cheating occurs. The second expression gives the present value of cheating the $t+i$ multinationals that will be in operation in period $t+i$.

Lemma 1: $P(t+i;\infty)$ is monotonically declining in $i$ if

$$t > \frac{\delta}{1-\delta} \times \frac{S}{T-1}. \quad -1.$$

Proof of Lemma 1:

$$P(\ t+i+1; \infty \ ) - P(\ t+i; \infty \ ) < 0$$
if and only if
\[ \delta^i \left[ -\frac{S}{I-I_0} \cdot \delta (t+i+1) - \frac{S}{I-I_0} (t+i) + T (t+i) \right] < 0 \]
[ from (2) ]

For any \( i < \infty \) and \( \delta \neq 0 \), this is true if and only if
\[ t + i > -\frac{\delta}{I-I_0} - \frac{S}{S-I} \]
(3)

But the left hand side of equation (3) increases as \( i \) increases, so that inequality (3) holds for all \( i \) if:
\[ t + 1 > -\frac{\delta}{I-I_0} - \frac{S}{S-I} \]

That is, if
\[ t > -\frac{\delta}{I-I_0} - \frac{S}{S-I} - 1 \]
(4)

**Lemma 2** \( P(t; \infty) > P(t + 1; \infty) \) if and only if:
\[ t > -\frac{\delta}{I-I_0} - \frac{S}{S-I} \]

**Proof of Lemma 2**

\[ P(t; \infty) - P(t + 1; \infty) > 0 \]
i.e. if and only if
\[ t - \frac{S}{I-I_0} \delta (t + 1) - \frac{S}{I-I_0} + Tt \]
i.e. if and only if
\[ t > -\frac{\delta}{I-I_0} - \frac{S}{S-I} \]
From Lemma (1) and (2), it follows that cheating in the current period is the best option if
\[ t \geq \text{Max} \left[ \frac{\delta}{1-\delta} \cdot \frac{S}{S-T}, \frac{\delta}{1-\delta} \cdot \frac{S}{S-T} - 1 \right] \]
\[ = \frac{\delta}{1-\delta} - \frac{S}{S-T} \]

Which is defined \( \forall \delta \in (0,1) \). Since the values of \( S, T \) and \( \delta \) are common knowledge, it is common knowledge that if entry occurs sequentially, the host government will cheat in period \( t^* \) where,
\[ t^* = \min \left\{ t \in \{1,2,3,\ldots\} \mid t \geq \frac{\delta}{1-\delta} - \frac{S}{S-T} \right\}. \]

Since \( t^* \) is finite, we are back in the world of Proposition 1, which proves the theorem.

**Proof of Proposition 4**

The objective is to find necessary and sufficient conditions under which the government will not cheat. At any \( t \), the present value of cheating today is:
\[ P(t; n) = \begin{cases} -\frac{S}{I-\delta} \left[ t - \frac{\delta^{n-t+1}(1-\delta^t)}{1-\delta} \right], & n \geq t \\ -\frac{S}{I-\delta} \left[ n - \frac{\delta}{1-\delta} (1-\delta^n) \right], & n < t \end{cases} \]
The present value of waiting for $i$ periods before cheating is:

$$
P(t+i;n) = \begin{cases} 
  T^n \delta^{i-j-1}(t+j-1) + \frac{S}{1-\delta} \frac{\delta^{n-t-i+1}(1-\delta^{t+1})}{1-\delta} & n \geq t \\
  T^n \frac{(1-\delta^t)}{1-\delta} + \delta^i \frac{S}{1-\delta} \left[ n - \frac{S}{1-\delta} (1-\delta^n) \right] & n < t 
\end{cases}$$

The payoff from never cheating is:

$$P(\infty;n) = \frac{Tn}{t-\delta}$$

Proof: (Sufficiency)

Given $n$, we want to see if there is a range of values for $\delta$ such that at each $t$, the government will defer cheating by one period. For any $\delta$ in this range, the decision to cheat will be postponed forever. Suppose first that $t \leq n$. Cheating now is no better than waiting for one period if and only if:

$$P(t;n) \leq P(t+1;n)$$

i.e.,

$$\frac{S}{t-\delta} \left[ t - \frac{\delta^{n-t+1}(1-\delta^{t+1})}{1-\delta} \right] \leq Tt + \frac{S}{1-\delta} \delta \left[ t+1 - \frac{\delta^{n-t}(1-\delta^{t+1})}{1-\delta} \right]$$

i.e.,

$$\frac{\delta}{t-\delta} \frac{(1-\delta^n)}{t} \geq \frac{S}{S-1} (1)$$
For that for each \( \delta \in [0,1) \), the left hand side of (1) falls as \( t \) increases to \( n \), the minimum value being

\[
\frac{\delta}{1-\delta} \left( \frac{1-\delta^n}{n} \right).
\]

Now suppose, \( n < t \). Cheating now is no better than deferring for one period if and only if

\[
P(t; n) \leq P(t+1; n)
\]

i.e.,

\[
\frac{S}{I-\delta} \left[ n - \frac{\delta}{I-\delta} (1 - \delta^n) \right] \leq \frac{Tn}{I-\delta} (1 - \delta) + \frac{\delta S}{I-\delta} \left[ n - \frac{\delta}{I-\delta} (1 - \delta^n) \right]
\]

i.e.,

\[
\frac{\delta}{1-\delta} \left( \frac{1-\delta^n}{n} \right) \geq \frac{S-I}{S}
\]

(2)

Hence, for any \( \delta \) such that,

\[
\text{Min} \left\{ \text{Min}_{t \in \{1, \ldots, n\}} \left[ \frac{t}{I-\delta} \left( \frac{1-\delta^n}{t} \right), \frac{\delta}{I-\delta} \left( \frac{1-\delta^n}{n} \right) \right] \right\} \geq \frac{S-I}{S}
\]

(3)

Waiting for one period is at least as good as cheating today for all \( t \in \{1, 2, \ldots \} \). But the left hand side of (3) is

\[
\frac{\delta}{I-\delta} \left( \frac{1-\delta^n}{n} \right)
\]
Hence condition (3) reduces to
\[
-\frac{\sigma}{T-\rho} \left( \frac{1-\sigma^n}{n} \right) \geq \frac{S-T}{S} \quad (4)
\]
where the left hand side is defined for \( \sigma \in [0, 1] \). But for \( \sigma \in [0, 1] \), (4) is equivalent to \( \sigma + \sigma^2 + \ldots + \sigma^n \geq n \frac{S-T}{S} \) \( (5) \)
The left hand side of inequality (5) approaches \( n \) as \( \sigma \) approaches 1 from the left. Define the function:
\[
h(\sigma; n) = \begin{cases} 
\sigma + \sigma^2 + \ldots + \sigma^n - n - \frac{S-T}{S}, & \sigma \in (0, 1) \\
n \frac{T}{S} & \sigma = 1 
\end{cases}
\]
The function \( h(\sigma; n) \) is continuous in \( \sigma \) in the interval \([0, 1]\) and differentiable in its interior, with \( h_\sigma(\sigma; n) > 0 \) for all \( \sigma \in (0, 1) \). Also \( h(0; n) = -n \frac{S-T}{S} \) and
\[
h(1; n) = n \frac{T}{S}
\]
Since, \( S > T > 0 \), there is a unique \( \sigma = \sigma_n \) such that for all \( \sigma \geq \sigma_n \), inequality (4) is satisfied. \( \blacksquare \)

To prove the necessity part, we want to show that for any \( \sigma < \sigma_n \), there is some \( t \) at which cheating in the current period is
optimal. We begin by proving two Lemmas.

**Lemma 1** For any $t > n$, $P(t + i + 1; n) < P(t + i; n)$ if and only if
\[
\frac{d}{I-I-I} \frac{(1-d^n)}{n} < \frac{S-I}{S}
\]
for all $i \in \{1, 2, 3, \ldots\}$, and $\delta \in (0, 1)$.

**Proof** Given any $t > n$ and $i \in \{1, 2, \ldots\}$,

\[
P(t + i + 1; n) - P(t + i; n) < 0
\]
iff
\[
\frac{Tn}{I-I-I} (1-d^i + 1) - \frac{Tn}{I-I-I} (1-d^i) + \frac{d^{i+1}}{I-I-I} S \left[ n - \frac{S}{I-I-I} (1-d^n) \right]
\]
\[
- \frac{d^i}{I-I-I} S \left[ n - \frac{S}{I-I-I} (1-d^n) \right] < 0
\]
i.e.,
\[
\frac{Tn}{I-I-I} [1-d^{i+1} - 1 + d^i] + S \frac{d^i}{I-I-I} \left[ n - \frac{S}{I-I-I} (1-d^n) \right] \times
\]
\[
(1-\delta) < 0
\]
i.e.,
\[
\frac{Tn}{I-I-I} (1-\delta) - \frac{S}{I-I-I} \left[ n - \frac{S}{I-I-I} (1-d^n) \right] (1-\delta) < 0
\]
[Since, $\delta \in (0, 1)$]

i.e.,
\[
\frac{d}{I-I-I} \frac{(1-d^n)}{n} < \frac{S-I}{S}
\]

**Lemma 2** For any $t > n$, cheating now is the best strategy if and only if:
\[
\frac{d}{I-I-I} \frac{(1-d^n)}{n} < \frac{S-I}{S}
\]
[with $\delta \in (0, 1)$]
Proof: By Lemma 1, \( P(t + i; n) \) is monotonically declining if and only if
\[
\frac{-\delta}{I-\delta} \cdot \left(1 - \frac{\delta^n}{n}\right) < \frac{S-T}{S}
\]
Also, from the proof of the sufficiency part of Proposition 4,
\( P(t; n) > P(t + 1; n) \) if and only if
\[
\frac{-\delta}{I-\delta} \cdot \left(1 - \frac{\delta^n}{n}\right) < \frac{S-T}{S}
\]
Hence, \( P(t; n) > P(t + i; n) \) for all \( i \in \{1, 2, \ldots\} \)
if and only if
\[
\frac{-\delta}{I-\delta} \cdot \left(1 - \frac{\delta^n}{n}\right) < \frac{S-T}{S}
\]
Proof of Proposition 4 (Necessity)
Let \( \delta < \delta_n \). Since \( \delta_n \) is the unique solution of
\[
\frac{-\delta}{I-\delta} \cdot \left(1 - \frac{\delta^n}{n}\right) = \frac{S-T}{S}
\]
For any \( \delta < \delta_n \), it is true that
\[
\frac{-\delta}{I-\delta} \cdot \left(1 - \frac{\delta^n}{n}\right) < \frac{S-T}{S}
\]
(see the last part of the proof of the sufficiency of Proposition 4).
But by Lemma 2, this is precisely the condition under which cheating in the current period is the best strategy for the government for all \( t > n \). Hence if \( \delta < \delta_n \), for any finite \( t_0 > n \), the best strategy
of the government is to cheat in the current period. Since \( t_0 \) is finite, Proposition 1 will apply, and there will be no entry.

**Proof of Proposition 5**

We want to show that \( \delta_n < \delta_{n+1} \)

Suppose not, i.e., \( \delta_n \geq \delta_{n+1} \)

Since 
\[
\delta_{n+1} + \delta_{n+1}^2 + \ldots + \delta_{n+1}^n - (n+1) \left( \frac{S-T}{S} \right) = 0
\]

\[
\delta_{n+1} + \delta_{n+1}^2 + \ldots + \delta_{n+1}^n - n \cdot \frac{S-T}{S} - \left( \frac{S-T}{S} \right)^n = 0
\]

i.e., 
\[
\left( \delta_{n+1} + \delta_{n+1}^2 + \ldots + \delta_{n+1}^n \right) - \left( \delta_n + \delta_n^2 + \ldots + \delta_n^n \right) = \frac{S-T}{S}
\]

i.e. 
\[
\left( \delta_{n+1} - \delta_n \right) + \left( \delta_{n+1}^2 - \delta_n^2 \right) + \ldots + \left( \delta_{n+1}^n - \delta_n^n \right)
\]

\[
+ \frac{\delta_{n+1}^n}{n+1} = \frac{S-T}{S}
\]

\[
\delta_n \geq \delta_{n+1} \Rightarrow (\delta_{n+1} - \delta_n) + (\delta_{n+1}^2 - \delta_n^2) + \ldots + (\delta_{n+1}^n - \delta_n^n) \leq 0 \Rightarrow \delta_{n+1}^n \leq \frac{S-T}{S} \quad (1)
\]

Consider \( n = 1 \). By definition:

\[
\delta_2 + \delta_2^2 = 2 \cdot \frac{S-T}{S} \quad (2)
\]

From (1), \( \delta_2^2 \geq \frac{S-T}{S} \Rightarrow \delta_2 \leq \frac{S-T}{S} \quad (3)
\]

(1) and (3) \( \Rightarrow \delta_2 \leq \delta_2^2 \Rightarrow \delta_2 \leq 1 \)

Which contradicts the assumption that \( \delta_2 \in (0, 1) \).
REFERENCES:


