Business Cycles and the Asset Structure of Foreign Trade

Baxter, Marianne and Mario Crucini

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Marianne Baxter

and

Mario Crucini

Rochester Center for Economic Research
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1. Introduction

The primary economic function of financial markets is to permit individuals to smooth consumption in the face of fluctuations in income. On a national level, domestic financial markets provide opportunities for intertemporal trade between citizens who face different expected income profiles, and risk-sharing between individuals who are subject to idiosyncratic income risks. On an international level, financial markets provide a potentially important avenue for individuals to smooth consumption in the face of country-specific fluctuations such as shifts in productivity, taxes or government expenditure. The extent of financial linkages among countries may therefore be a central determinant of the amplitude, persistence, and international transmission of business cycles.

This paper explores the implications for open economy business cycles of restricting international trade in financial assets. The key restriction that we impose is that domestic residents must hold all risky claims to domestic output, trading only noncontingent bonds on the international asset markets. This assumption is traditional in models of small open economies, and also in Friedman's [1957] and subsequent partial equilibrium models which embody the permanent income hypothesis. We build a quantitative general equilibrium model of interacting economies subject to random shocks to productivity; interest rates and asset prices are thus determined endogenously. As a benchmark for evaluating the implications of restricted asset trade, we compare the predictions of this model to those of the complete markets model in our earlier study (Baxter and Crucini [1993]). We find that, when there are important differences between the two models, these differences can be traced primarily to differential wealth effects of shocks under alternative asset structures.

This paper is related to several recent contributions to the literature on open economy business cycles. This literature is divided into partial equilibrium analyses of small open economies (e.g., Cardia [1991], Finn [1989], and Mendoza [1991]), and
general equilibrium analyses of a world comprising two national economies (Backus, Kehoe and Kydland [1992], Baxter and Crucini [1993], and Stockman and Tesar [1991]). In the partial equilibrium analyses, asset markets are highly restricted; at most, individuals are assumed to be able to trade noncontingent debt with the rest of the world. In the general equilibrium analyses, asset markets are assumed to be complete in the sense that the dynamic equilibria of these models display complete risk-pooling.

Both branches of the literature have serious shortcomings. The partial equilibrium approach postulates an exogenous interest rate process, shutting down any possibility of discussing the determinants of the world interest rate, and making the analysis sensitive to the stochastic process specified for this key variable. The small open economy approach also prohibits the study of business cycle linkages among non-infinitesimal economies: the countries that comprise the OECD, for example. The existing general equilibrium analyses, on the other hand, have been conducted under the assumption that all risks are fully pooled internationally—including risk deriving from fluctuations in labor income, and shocks to government expenditure and tax rates. The complete-markets assumption was initially maintained for two reasons. First, it was a natural benchmark. Second, competitive and optimal allocations coincide under complete markets, so that the competitive equilibrium can be computed using straightforward extensions of methods for solving optimal decision problems which had been developed for the closed-economy literature (specifically, the linear approximation methods of Kydland and Prescott [1982]). But, for reasons discussed more fully below, many have been skeptical about the relevance of the complete-markets paradigm.

This paper thus studies dynamic general equilibrium models of the world economy with restricted international trade in financial assets. It consequently features endogenous interest rate determination, without incorporating the assumption that all risks are fully pooled internationally. The dynamic system describing the behavior of this suboptimal world economy is computed using log-linear approximation of the
system of Euler equations which implicitly characterize the competitive equilibrium. Because the dynamic behavior of both optimal and suboptimal economies are defined by the appropriate system of Euler equations, approximation of the suboptimal model's dynamic behavior via the "Euler equation approach" involves no increase in conceptual complexity. In the two-country model studied in this paper, the effect of introducing market incompleteness is simply to add one variable to the state vector, which is one country's level of international indebtedness.

As noted above, there is widespread skepticism concerning the validity of the standard, complete markets assumption: this skepticism has arisen for at least two reasons. First, there are no existing internationally-traded assets which are explicitly contingent on realizations of many types of uncertainty (variations in national tax rates, for example). Whether existing assets effectively act to hedge this type of risk is a more subtle question, and whether the risks are empirically important is also open to debate (see the recent contribution by Cole and Obstfeld [1991]). Second, many of the implications of these complete markets models are strongly at variance with the stylized facts of international business cycles; many of these implications plausibly stem from the extreme assumptions concerning risk-pooling. Specifically: one-sector, complete-markets models generically predict international consumption correlations that are too high, relative to the data, and cross-country correlations of investment, labor input, and output that are too low. Notably, the one-sector models of Backus, Kehoe, and Kydland [1992] and Baxter and Crucini [1993] predict near-perfect correlation of consumption movements across countries. In the data, we find that cross-country consumption correlations are typically very weak, and are not even always positive (see the Appendix to Baxter and Crucini [1993]).

Further, these models have trouble generating positive comovement of investment, labor, and output because of two reinforcing factors. First, these one-sector equilibrium models all possess a version of the neoclassical "accelerator" mechanism by which
investment responds rapidly and strongly to changes in the return to capital. In a multi-country setting with shocks that are partially country-specific in nature, this translates into a strong tendency for negative international comovement of investment. There is a simple economic reason for this: with one final good in the world economy, capital owners' primary concern is to locate their capital in the most productive location. Second, complete risk pooling implies that the equilibrium quantities of consumption and labor input in each country are those that would be observed under optimal labor income insurance. Although these quantities may be supported as equilibria under a variety of financial market structures (i.e., there are many ways to "decentralize" the equilibrium) it is useful to think about the equilibrium as if this insurance were explicitly utilized.

The optimal risk-sharing arrangements involve the following state-contingent responses to productivity shocks. First, those individuals who receive a favorable productivity shock work harder. Second, the optimal insurance character of the equilibrium requires that they transfer part of the proceeds to individuals living in the less productive country. Individuals who live in the less productive location work less hard, but their consumption increases because of the international transfer of goods (i.e., the insurance payments). Because of these two reinforcing factors, complete-markets models must be driven by shocks that are highly correlated across countries if they are to be able to replicate the observed tendency for national outputs and investments to move together. However, even in this case the complete markets model predicts that labor inputs are negatively correlated across countries.

Finally, is hard to imagine that optimal labor insurance would be sustainable (i.e., enforceable) in an international context: the country receiving the favorable shock would, ex post, not wish to pay the "insurance benefits" to the other country. These considerations motivated us to study the link between the international character of business cycles and the structure of international asset markets.
The paper is structured as follows. Section 2 describes the model economy and discusses aspects of the solution procedure that differ from the prior, complete markets analyses. Section 3 begins with a review of previous work on estimating the stochastic process for productivity, and presents some new results. Taken together, these results suggest that productivity shocks are highly persistent, are correlated across countries, and may contain unit roots. Therefore, we compared the predictions of the complete markets model to the predictions of the model with restricted asset markets under two alternative parameterizations of the productivity process: (i) a trend–stationary process with innovations that are correlated across countries and with international transmission of shocks; and (ii) a difference–stationary process without transmission but with correlated innovations. We find that the empirical implications of the models are very sensitive to the specification of the stochastic process for productivity. If productivity follows a trend–stationary process with highly persistent shocks and international transmission, the business cycle implications of the incomplete markets economy are very similar to those of the complete markets economy. However, if productivity follows a random walk without transmission, the implications of the alternative models are quite different. Section 4 explores the economic forces behind the differential response under alternative asset structures by studying the dynamic response to a productivity shock originating in one country. Using King's [1990] method for decomposing consumption and labor responses into wealth and substitution effects, we find that differences across asset structures—when they exist—can be traced primarily to differential wealth effects. Section 5 briefly summarizes the paper's main results and discusses avenues for future research.

2. The Model

The basic structure of this model, in terms of preferences and technology, is identical to the structure in Baxter and Crucini [1993]. The main difference arising
from restrictions on asset trade appears in the flow constraints (budget constraints), which differ across the two models. Foreign country variables are denoted by stars, and all variables are in national per capita terms.

Preferences: Individuals consume two goods: a produced consumption good, C, and leisure, L. They maximize expected lifetime utility, given by:

\[ E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma} [C_t^\theta L_t^{1-\theta}]^{1-\sigma} \quad \text{home country;} \] (1)

\[ E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma} [C_t^* L_t^{*1-\theta}]^{1-\sigma} \quad \text{foreign country.} \] (2)

In each country, individuals are subject to the constraint that hours worked in the marketplace plus hours of leisure cannot exceed the time endowment, normalized to one unit:

\[ 1 - L_t - N_t \geq 0 \quad \text{home country;} \] (3)

\[ 1 - L_t^* - N_t^* \geq 0 \quad \text{foreign country.} \] (4)

Technology: Production functions exhibit constant returns to scale. Production of the single final good requires the input of both labor and capital. Capital used in production in a specific country is not necessarily owned by residents of that country. Thus \( K_t \) represents capital in place in the home country, not necessarily capital owned by residents of the home country. Labor is internationally immobile. Letting \( N_t \) denote labor employed in the home country, these production functions are given by:

\[ Y_t = A_t K_t^{1-\alpha}(X_t N_t)^{\alpha} \quad \text{home country;} \] (5)

\[ Y_t^* = A_t^* K_t^{1-\alpha^*}(X_t^* N_t^*)^{\alpha^*} \quad \text{foreign country} \] (6)

In these production functions, the variables \( X_t \) and \( X_t^* \) represent the level of purely labor-augmenting technical change in the home and foreign countries, respectively, and each grows at a common, constant gross rate: \( \gamma = X_{t+1} / X_t = X_{t+1}^* / X_t^* \). The
variables $A_t$ and $A^*_t$ represent the stochastic component of the productivity variable, and are assumed to follow a vector Markov process.

New capital goods are internationally mobile, subject to costs of adjustment governed by the function $\phi(I/K)$, with $\phi>0$, $\phi'>0$, $\phi''<0$. Capital accumulates over time according to:

$$K_{t+1} = (1-\delta)K_t + \phi(I_t/K_t)K_t$$  \hspace{1cm} \text{home country; (7)}

$$K^*_{t+1} = (1-\delta)K^*_t + \phi(I^*_t/K^*_t)K^*_t$$  \hspace{1cm} \text{foreign country. (8)}

2.1 A two country general equilibrium model with complete markets

The first model that we shall study is also, in many ways, the simplest. There are two countries in the world, and individuals in the two countries are free to trade any state-contingent asset they wish. Thus, in equilibrium, individuals will bear no idiosyncratic risk.

Resource constraints: Since the consumption/investment good is internationally mobile, there is a single world resource constraint for this good. Letting $\pi$ denote the fraction of the world population residing in the home country, the world resource constraint is:

$$\pi[Y_t - C_t - I_t] + (1-\pi)[Y^*_t - C^*_t - I^*_t] \geq 0.$$  \hspace{1cm} (9)

Model solution: The equilibrium of this economy consists of a set of functions describing the behavior of endogenous variables such as consumption, saving, investment, etc., as functions of the exogenous shocks to the model (i.e., the productivity shocks). Before solving our model, we transform it to remove deterministic trend components; this is accomplished by dividing all home country variables by $X_t$, and all foreign country variables by $X^*_t$. Lowercase letters are used below to denote transformed variables. Note that labor and leisure cannot have deterministic trends; otherwise, the "time constraints" (3) and (4) would eventually be
violated. These variables continue to be represented by uppercase letters. Finally, the rate of time preference for the transformed (world) economy is \( \beta(\gamma) \theta(1-\sigma) \).

It can easily be shown that the second welfare theorem applies in our economy, i.e., competitive equilibrium and Pareto optimum will coincide. Thus a straightforward way to compute the equilibrium for the economy is to solve the following Lagrangian problem:

\[
\mathcal{L} = \sum_{t=0}^{T} \beta_t \left\{ [\pi u(c_t, L_t) + (1-\pi)v(c_t^*, L_t^*)] 
+ \pi w_t (1-L_t-N_t) + (1-\pi)w_t^* (1-L_t^*-N_t^*) 
+ \pi \lambda_t [\gamma k_{t+1} - (1-\delta)k_t - \phi(i_t/k_t)k_t] 
+ (1-\pi)\lambda_t^* [\gamma k_{t+1}^* - (1-\delta)k_t^* - \phi(i_t^*/k_t^*)k_t^*] 
+ \pi \zeta_t (k_t - ks_t) + (1-\pi)\zeta_t^*(k_t^* - ks_t^*) 
+ p_t [\pi(A_t F(ks_t,N_t) - c_t - i_t) + (1-\pi)(A_t^* F(ks_t^*,N_t^*) - c_t^* - i_t^*)] \right\} 
\] (10)

In programming this model, we found it useful to distinguish between the capital stock in a particular location (k and k*) and capital services used in production (ks and ks*), since the multipliers on the constraints in (10) have the following natural interpretations as (utility–denominated) shadow prices:

- \( w_t, w_t^* \): wage rate
- \( \lambda_t, \lambda_t^* \): price of existing capital
- \( \zeta_t, \zeta_t^* \): value marginal product of capital
- \( p_t \): price of the final good (price of new capital).

Letting \( D \) to denote the total derivative of a function of a single variable, and letting \( D_j \) denote the partial derivative of a function with respect to its jth argument, the first–order necessary conditions for this Lagrangian problem are:
\[(c_t) \quad D_1 u(c_t, L_t) - p_t = 0 \quad (11)\]
\[(L_t) \quad D_2 u(c_t, L_t) - w_t = 0 \quad (12)\]
\[(ks_t) \quad p_t A_t D_1 F(ks_t, N_t) - \zeta_t = 0 \quad (13)\]
\[(N_t) \quad p_t A_t D_2 F(ks_t, N_t) - w_t = 0 \quad (14)\]
\[(i_t) \quad \lambda_t D \phi(i_t/k_t) - p_t = 0 \quad (15)\]
\[(\zeta_t) \quad k_t - ks_t = 0 \quad (16)\]
\[(w_t) \quad 1 - L_t - N_t = 0 \quad (17)\]
\[(\lambda_t) \quad \gamma k_{t+1} - (1-\delta)k_t - \phi(i_t/k_t)k_t = 0 \quad (18)\]
\[(k_{t+1}) \quad E_t u'(i_{t+1}/k_{t+1})\beta \lambda_{t+1} + \beta E_t \zeta_{t+1} - \gamma \lambda_t = 0 \quad (19)\]
\[(p_t) \quad \pi[A_t F(ks_t, N_t) - c_t - i_t] + (1-\pi)[A^* F(ks^*_t, N^*_t) - c^*_{t+1} - i^*_t] = 0 \quad (20)\]
\[E_0 \lim_{t \to \infty} \beta^t \lambda_t k_{t+1} = 0 \quad (21)\]

for all \(t \geq 0\), where \(\mu(z) \equiv [\phi(z) - z \phi(z)] + (1-\delta)\). In addition, there are foreign-country analogs to equations (11)-(19) and (21). These Euler equations, derived from an optimum problem as specified in the Lagrangian (10), also describe the equilibrium of a decentralized economy in which atomistic consumers interact with atomistic, competitive firms.

It is well known that the system of equations that implicitly defines the equilibrium of the one-sector closed-economy model does not have an analytic solution, except in a small number of special cases. A variety of numerical methods have recently been developed for obtaining approximate solutions to a particular nonlinear equilibrium problem: see the summary paper by Taylor and Uhlig (1990) and the papers cited therein. One method which has been shown to work well for the closed-economy neoclassical model is log-linear approximation of the equilibrium decision rules that solve the Euler equations. The point around which the approximation is taken is the model's initial deterministic steady state. The resulting linear system is solved by application of standard linear systems theory. This method
is described in detail in King, Plosser, and Rebelo [1987], and is the method we use in this paper.

2.2 A Partial Equilibrium Model of a Small Open Economy

This sub-section describes a model of an open economy that is assumed to be too small to affect the world interest rate. This is a partial equilibrium model because the small open economy optimizes in the face of an exogenous process for the world interest rate. Solving the small open economy model is a useful first step toward constructing a general equilibrium model with restricted asset trade.

*Flow budget constraint:* In this model, the bonds take the form of real discount bonds. We let \( r_t \) denote the exogenously–given world rate of return on risk–free securities, and let \( R_t = (1+r_t)^{-1} \) denote the price per unit of one-period discount bonds purchased in period \( t \). \( B_{t+1} \) denotes the quantity of bonds purchased in period \( t \) (maturing in \( t+1 \)). Following our earlier convention of letting lowercase letters refer to the transformed economy, we let \( b_t \equiv B_t/X_t \) denote the value of bonds in the transformed economy. Then the flow budget constraint for the small open economy in period \( t \) is:

\[
\gamma R_t b_{t+1} + c_t + i_t \leq y_t + b_t. \quad (22)
\]

The Lagrangian for the partial equilibrium, small open economy problem is:

\[
L = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ u(c_t, L_t) + \omega_t (1 - N_t - L_t) + \lambda_t [\gamma k_{t+1} - \phi(i_t/k_t)k_t + (1-\delta)k_t] + \zeta_t [k_t - k_s] + p_t [b_t + A_t F(k_s, N_t) - c_t - i_t - R_t \gamma b_{t+1}] \right\}.
\]
The interpretation of the Lagrange multipliers for this problem is similar to the equilibrium problem discussed above: \( \omega_t \) is the (utility-denominated) wage rate; \( \lambda_t \) is the value of a unit of installed capital, \( \zeta_t \) is the value marginal product of capital, and \( p_t \) is the value of an additional unit of the consumption good.

Many of the first-order necessary conditions for this problem are exactly the same as the corresponding efficiency conditions for the complete-markets economy described above. Specifically, the first-order conditions for \( c_t, L_t, ks_t, N_t, i_t, \zeta_t, \omega_t, \lambda_t, \) and \( k_{t+1} \) are given by equations (11) to (19) above. There are two first-order conditions that are different: these are the efficiency conditions for \( b_{t+1} \) and for \( p_t \), which are as follows:

\[
(b_{t+1}) \quad \beta E_t p_{t+1} - \gamma p_t R_t = 0 \\
(p_t) \quad b_t + A_t F(ks_t, N_t) - c_t - i_t - R_{t}\gamma b_{t+1} = 0
\]

Thus the steady state real interest rate is determined by equation (23): \( R\gamma = \beta \). \(^2\)

The conditions (11)–(17) determine the vector of controls \([c_t, L_t, ks_t, N_t, i_t, \zeta_t, q_t, \omega_t]\) as functions of the controlled states \((b_t, k_t)\); the corresponding costates \((p_t, \lambda_t)\); and the exogenous variables \((A_t, R_t)\). The four equations (18), (19), (23), and (24) define the fundamental state–costate difference equation.

**Model Parameterization:** By permitting asset accumulation, we have introduced a new parameter: the steady state level of assets relative to output. In closed economies and in multi-country models with fully pooled equilibria, each country's holdings of assets which are in zero net supply must be constant along any equilibrium path. In general, however, whenever there are multiple countries and incomplete risk pooling, the steady state level of asset holdings will not be invariant to shocks to the world economy. This introduces an additional degree of freedom into the parameterization. In our applications, we use this degree of freedom to specify \( \theta_b = b/y \): the initial steady state bond–to–output ratio of the home country.
2.3 General Equilibrium in a Two Country World Trading Goods and Bonds

The model differs from the general equilibrium economy described in section 2.1 above in that this world economy is restricted to trade only goods and non-contingent real debt. In world general equilibrium, each of the countries faces the problem described in Section 2.2, but in general equilibrium the interest rate process \( (R_t) \) is endogenously determined. As before, let \( \pi \) be the share of home country in the world economy. Then, bond market clearing requires that:

\[
\pi b_t + (1-\pi)b^*_t = 0
\]  

(25)

since the bonds are in zero net supply in the world economy. Combining equation (25) with aggregate financial asset accumulation equation

\[
\pi R_t b_{t+1} + (1-\pi)R_t b^*_{t+1} \leq 
\pi \left\{ b_t + A_t^*F(ks_t^*,N_t^*) - c_t - i_t \right\} + (1-\pi) \left\{ b^*_t + A^*_tF(ks^*_t,N_t^*) - c^*_t - i^*_t \right\}
\]

implies goods–market clearing (due to Walras' Law):

\[
\pi (A_t^*F(ks_t^*,N_t^*)-i_t-c_t) + (1-\pi)(A^*_tF(ks^*_t,N^*_t)-c^*_t-i^*_t) \geq 0
\]  

(26)

Modifications of efficiency conditions: In this incomplete markets setting, we nevertheless have the same equilibrium condition for \( p_t \) as in the complete–markets model of section 2.1:

\[
(p_t) \quad \pi[A_t^*F(ks_t^*,N_t^*) - c_t - i_t] + (1-\pi)[A^*_tF(ks^*_t,N^*_t) - c^*_t - i^*_t] = 0
\]  

(27)

In addition, we have one additional pair of state and co-state equations which are the efficiency conditions for \( b^*_{t+1} \) and \( p^*_t \). These are given by (28) and (29) below:

\[
(b^*_{t+1}) \quad E_t(p^*_t+1/p^*_t) = E_t(p_{t+1}/p_t)
\]  

(28)

\[
(p^*) \quad b^*_t + A_t^*F(ks^*_t,N^*_t) - c^*_t - i^*_t - \gamma R_t b^*_{t+1} = 0
\]  

(29)

where (28) uses the fact that \( R_t = \beta E_t(p_{t+1}/\gamma p_t) \).
Model solution: The key issue is how to use the information contained in (25) and (26) to compute the world general equilibrium. The procedure we use is as follows. First, we drop one of the asset accumulation equations since (25) implies that, in a two–country world, only one of the asset stocks are independent. In this two–country setting, we let this be the foreign country's asset stock. Second, we treat the home country's shadow price (p) as an additional control variable. That is: we add the efficiency condition (26) to the system of equations \{(11)–(19), (23)–(24)\} for both countries. This augmented system determines the world control vector as a function of the world state vector \([k_t, k_t^*, b_t^*]\); the world costate vector \([\lambda_t, \lambda_t^*, p_t^*]\); and the exogenous variables \([A_t, A_t^*]\). Third, we impose the equilibrium condition that \(R_t = \bar{\beta} E_t(p_{t+1}/\gamma p_t)\). That is, we replace \(R_t\) with the expression \(\bar{\beta} E_t(p_{t+1}/p_t)\) in the accumulation equations for \(b_t^*\). This three–step procedure yields a dynamic system with that can be linearized and solved in the standard manner.

3. Implications for Business Cycles

In this section we examine the implications for the character of international business cycles of restricting the portfolio of internationally tradable assets. We compare two asset structures: (i) the complete–markets structure employed in the equilibrium business cycle research program; and (ii) a restricted structure in which the only traded asset is noncontingent bonds, as assumed in traditional small open economy models and in Friedman's [1957] and subsequent partial equilibrium models embodying the permanent income hypothesis.

Previous research in the international real business cycle literature has found that aspects of the models' implications for business cycles are sensitive to specification of the exogenous process driving the model, and we find that this is the case here as well. Therefore we briefly review the findings of this prior literature and present some new empirical evidence on international productivity.
3.1 Measuring productivity

Following the work of Solow [1957], it has become commonplace to measure disembodied productivity ($A_t$ and $A^*_t$; in our notation) as a residual from a Cobb–Douglas production function. In the notation of our model, the "Solow residuals" would be measured (using (5) and (6)) as:

$$\log(A_t) \equiv \log(y_t) - (1-\alpha)\log(k_t) - \alpha \log(N_t)$$

$$\log(A^*_t) \equiv \log(y^*_t) - (1-\alpha^*)\log(k^*_t) - \alpha^*\log(N^*_t) .$$

Measurement of the Solow residual therefore requires measures of output, capital input, labor input, and factor shares. For the United States, measures of all these variables are available, although there naturally is substantial disagreement concerning the accuracy of these measures. For other countries, in many cases, the necessary data are not readily available. Backus, Kehoe, and Kydland (BKK) [1992] used output data and employment data to construct estimates of Solow residuals for the U.S., Canada, and an aggregate of six European countries. These measures omit the term involving capital input. However, Costello [1993] has shown that the stochastic properties of Solow residuals are sensitive to the measure of capital used to construct the residuals. Further, the mismeasurement of labor input by using employment in place of total hours worked is a potentially serious problem: Burdett and Wright [1989] show that for many European countries, more of the variance in total labor input is explained by hours variation than by employment variation. Despite the measurement problems, however, the BKK estimates provide a valuable starting point, and we briefly review their findings here.

Backus, Kehoe, and Kydland [1992] modeled the productivity shock process as the following vector–autoregressive process:
\[
\begin{bmatrix}
\log A_t^* \\
\log A_t^*
\end{bmatrix} =
\begin{bmatrix}
\rho & \nu \\
\nu^* & \rho^*
\end{bmatrix}
\begin{bmatrix}
\log A_{t-1}^*
\log A_{t-1}
\end{bmatrix} +
\begin{bmatrix}
\epsilon_t \\
\epsilon_t^*
\end{bmatrix}.
\]

with \( E(\epsilon) = E(\epsilon^*) = 0 \) and \( E(\epsilon^2) = \sigma^2_\epsilon, \) \( E((\epsilon^*)^2) = \sigma^2_{\epsilon^*}, \) and \( E(\epsilon_t, \epsilon_t^*) = \psi \) for all \( t. \) Backus, Kehoe, and Kydland [1992] estimate (30) for (i) the U.S. versus Canada, and (ii) the U.S. versus an aggregate of six European countries. Their estimates are given below; standard errors are in parentheses.

**U.S.**

\[
\begin{bmatrix}
\log A_t \\
\log A_t^*
\end{bmatrix} =
\begin{bmatrix}
0.989 & 0.000 \\
0.060 & 0.093
\end{bmatrix}
\begin{bmatrix}
\log A_{t-1} \\
\log A_{t-1}^*
\end{bmatrix} +
\begin{bmatrix}
\epsilon_t \\
\epsilon_t^*
\end{bmatrix}
\]

\( \rho(\epsilon_t, \epsilon_t^*) = .434. \)

**Canada**

\[
\begin{bmatrix}
\log A_t \\
\log A_t^*
\end{bmatrix} =
\begin{bmatrix}
0.131 & 0.796 \\
0.052 & 0.079
\end{bmatrix}
\begin{bmatrix}
\log A_{t-1} \\
\log A_{t-1}^*
\end{bmatrix} +
\begin{bmatrix}
\epsilon_t \\
\epsilon_t^*
\end{bmatrix}
\]

\( \rho(\epsilon_t, \epsilon_t^*) = .258. \)

We see from these estimates that shocks to productivity are highly persistent, and that there is some evidence of transmission of shocks from one country to another (\( \nu, \nu^* > 0 \)). Further, the innovations to productivity are positively correlated across countries.

Because the BKK estimates indicate that shocks to the productivity process are highly persistent and that the transmission or "spillover" parameters are positive but carry large standard errors, we investigated the hypothesis that the Solow residuals follow a random walk without spillovers, but with possibly correlated innovations. Table 1, panel A reports the results of the \( J(p,q) \) test for a unit root and zero transmission in each of the three Solow residual series generated by Backus, Kehoe, and
Kydland (Canada, Europe, and the U.S.). In each case, we fail to reject the null hypothesis at conventional significance levels.

A natural next question is whether there is a cointegrating relationship between the Solow residual time series. Table 1, Panel B reports the results of tests for cointegration: there is evidence of cointegration between the U.S. and Canada, but the evidence for cointegration is weaker for the U.S. versus Europe. Based on these results, we estimated a vector-error-correction model for the U.S. versus Canada, and a standard VAR in first-differences for the U.S. versus Europe; the results are in Table 1, panel C. There appears to be no significant international transmission of shocks, with the possible exception of transmission from the U.S. to Canada. Based on these estimates, we feel that we cannot reject the hypothesis that productivity in each country follows a random walk with drift, without transmission, but with innovations that are positively correlated across countries. In the remainder of the paper, we therefore examine the business cycle implications of alternative asset structures under two assumptions concerning the stochastic process for productivity: (i) BKK's "symmetrized parameterization," which is characterized by trend-stationary shocks with correlated innovations and substantial international transmission ("spillovers"); and (ii) a random walk process for productivity without spillovers, but with correlated innovations.

3.2 Trend stationary productivity with spillovers

We begin by comparing the cyclic behavior of the complete markets economy to that of the bond economy when the stochastic process for productivity is given by the BKK "symmetrized parameterization" of the relationship between the U.S. and Europe under which $\rho=\rho^*=0.906$, $\nu=\nu^*=0.088$, and $\psi=0.258$. Under this parameterization, innovations to productivity are positively correlated across countries ($\psi>0$) and shocks that originate in one country "spill over" to the other country at the rate of 8.8% per
quarter \((\nu=0.088)\). We set the innovation variances equal to one.\(^4\) The world is assumed to comprise two equally-sized countries, and there are small costs of adjustment in investment. The parameters of preferences and technology are the same as in Baxter and Crucini [1993]: \(\sigma=2; \alpha=.58; \beta=.988; \gamma=1.004; \delta=.025\). The parameters of the adjustment cost process are set as follows: (i) \(\phi\) is set so that the steady state value of Tobin's "q" \((1/\phi')\) is one (i.e., the model with adjustment costs has the same deterministic steady state as the model without adjustment costs); and (ii) the elasticity of the investment–capital ratio with respect to Tobin's "q," is
\[
\eta = -(\phi' / \phi'')^2(i/k) = 15.
\]
(See Baxter and Crucini [1993] for additional discussion regarding parameter choice, and sensitivity analyses for some critical parameters).

Table 2 presents summary business cycle statistics for eight OECD countries; the data has been filtered with the Hodrick and Prescott [1980] filter. The central stylized facts of business cycles are similar across countries: (i) there is a tendency for consumption to be less volatile than output while investment is more volatile; (ii) output movements are highly persistent; and (iii) consumption and investment are both highly correlated with fluctuations in output. The lower panel of Table 2 shows that both outputs and consumptions have a tendency to covary positively across countries, but international consumption correlations tend to be lower than international output correlations. Finally, labor market data for the U.S. shows that (i) labor input is less volatile than output, as is average labor productivity; (ii) labor input is highly correlated with output, as is the average product of labor, and (iii) average labor productivity and the level of labor input are roughly uncorrelated.\(^5\)

Table 3 compares the response of the complete markets economy to the economy which is restricted to financial trade in bonds alone when both economies are driven by the trend–stationary productivity shock with spillovers. The statistics reported in this table are the model's population moments for Hodrick–Prescott [1980] filtered time series.\(^6\) Surprisingly, the differences between the business cycle implications of these
two (apparently) very dissimilar models are really quite minor. Compared with the complete markets economy, the bond economy displays similar volatility of output, consumption, investment, labor input, the wage rate, and the net export ratio. In the bond economy, however, bond holdings as a fraction of output are about three times as volatile as output. (Bond holdings have zero variance in the complete markets economy; a well-known characteristic of fully-pooled equilibria is that asset holdings need not fluctuate.) In terms of persistence, the two models are essentially indistinguishable except, once again, for the behavior of bond holdings.

Turning to the contemporaneous correlation of macroeconomic aggregates with output, we see that the bond economy generally predicts higher correlations of most variables with same-country output, although the numerical differences are very small. The bond economy predicts higher international correlations of output, investment, and labor input, and smaller international correlations between consumption and wage rates. The within-country correlation between saving and investment is slightly lower in the bond economy compared with the complete markets economy. This might seem surprising, since one's intuition is that closing asset markets, thus forcing individuals to bear more country-specific risk, would act to increase within-country saving-investment correlations. However, this "basic saving measure" (defined as output minus consumption) need not be a good measure of true saving in an open economy, as discussed by Obstfeld [1986] and Stockman and Svensson [1987].

Finally, the asset structure is of minor importance for the predicted correlation between the wage rate and the level of labor input. The complete markets economy predicts a correlation of 0.66, which is about the same as the predictions of the closed-economy real business cycle model calibrated to U.S. data (e.g., the model in Baxter and King [1991] predicts a correlation of 0.65). Restricting asset trade to bonds alone has the effect of slightly increasing the predicted correlation, to a level of 0.69. (In the U.S. data, this correlation is −0.04.)
How do these models do overall in terms of generating empirically accurate predictions? As discussed by Backus, Kehoe, and Kydland [1992] and Baxter and Crucini [1993], the complete markets economy does reasonably well in matching the stylized facts concerning volatility and persistence of macro aggregates. Much more problematic are the complete markets model's implications for cross-country correlations of output, consumption, investment, and labor input. Specifically, this model has difficulty generating positive output comovement (and correspondingly positive comovement of investments and labor inputs across countries). Further, the model predicts a level of cross-country consumption correlation that is much too high relative to the data.

Because individuals are subject to idiosyncratic (nation-specific) risk in the bond economy, in equilibrium this economy will display nation-specific fluctuations in consumption. Thus we expect that the international correlation between consumptions should be lower in the bond economy, and it is—but not much lower. The complete markets economy predicts an international consumption correlation of .95, while the bond economy predicts a correlation of .92. (Table 2 shows that the empirical correlations range from 0.11 to 0.65.)

Similarly, the absence of insurance against labor income risk in the bond economy is important for the cross-country correlation of labor inputs. In the complete markets economy, the response to a positive productivity shock in one country generates an increase in labor input in the productive country, and a tendency for a decline in labor input in the relatively unproductive country. Because of the optimal insurance character of the complete markets equilibrium, workers in the productive country agree to "share" some of the additional output generated by the increase in productivity and labor input, in exchange for similar "sharing" when the other country receives a positive productivity shock. In the bond economy, individuals can only smooth consumption across time (by buying or selling bonds); they cannot smooth consumption
across different "states of nature" because of the absence of contingent securities. This reduces the tendency for labor input to decline in the temporarily unproductive location (we will discuss the details of these mechanisms further in Section 4). But again, while we see this effect in somewhat higher international correlations between output, investment, and labor input, the effect is not strong enough to make the bond economy a good description of the international data along these dimensions.

In summary, with the BKK parameterization of the productivity process, we find that restricting international trade in financial assets to noncontingent bonds alone has very minor effects on the model's predictions for the business cycle behavior of the key macroeconomic aggregates. In particular, restricting asset markets helps only slightly in remedying the chief empirical failings of the one-sector international equilibrium business cycle model, which are (i) predicted international correlations of consumptions that are too high, relative to the data, (ii) a tendency to predict very low international correlations of output, labor input, and investment, and (iii) too-high predicted correlations between labor input and wage rates.

3.3 Random-walk productivity without spillovers

As shown in Section 3.1, we cannot reject the statistical hypothesis that the logs of total factor productivity (log $A_t$ and log $A_t^*$) follow random walk processes without spillovers but with correlated innovations. This section therefore examines the implications of this process for the behavior of the complete markets economy and the bond economy. Table 4 presents the two models' predictions for the central business cycle statistics, under the assumption that $\rho=\rho^*=1$, $\nu=\nu^*=0$ (as in Table 3, these are HP filtered population moments). All other parameters, including the contemporaneous correlation of the shocks and the innovation variances, are the same as in Table 3.

It is immediately evident from Table 4 that there are important differences between the complete markets economy and the bond economy. In contrast to the
results for the BKK parameterization, reported in Table 3, market structure matters a great deal when shocks are permanent and there is no transmission. First, the levels of output volatility, investment volatility, and labor input are substantially higher in the complete markets economy, compared with the bond economy; in fact investment and labor input are about twice as volatile in the complete markets economy. Recall that one effect of the complete risk-pooling in the complete markets economy is a strong increase in labor input in response to positive productivity shocks; some of the additional product generated is sent to citizens of the nonproductive location. Because of the complementarity of labor input and capital input, the stronger labor response in the complete markets setting is accompanied by a stronger investment response and, consequently, a stronger output response.

The most striking differences between the models appear when we look at the international correlations of output and consumption. As noted above, a well-known failing of the complete markets model is its robust prediction of too high an international consumption correlation, combined with a too-low prediction for the international correlation of outputs. In section 3.2, we saw that the bond economy shares this flaw when shocks to total factor productivity are trend-stationary and subject to spillovers. When the shocks are purely permanent, as in Table 4, the complete markets economy continues to exhibit this counterfactual pair of predictions; in fact, the predictions for output correlations are even worse (i.e., even more strongly negative). But when the bond economy is subject to purely permanent shocks, this model predicts a substantial, positive international output correlation (0.54) and a negative consumption correlation (-0.28)! (While this configuration of correlations is unusual in the data, Baxter and Crucini [1993] did find this pattern of positive output correlations and negative consumption correlations for four country pairs.)

With random walk productivity, the two asset structures also differ importantly in their implications for the cyclic behavior of labor input. The bond economy predicts a
weak (0.19) contemporaneous correlation of labor with output compared to the prediction of 0.93 for the complete markets economy. However, despite the fact that asset market restrictions have increased output correlations to an empirically reasonable level, the bond economy continues to underpredict international comovement of labor input and investment. With the random walk specification, both asset structures predict a negative correlation between the net export ratio and output, which is characteristic of most OECD countries (although not the United States). Under this parameterization, the complete markets model continues to predict high saving–investment correlations but the bond economy does not. In fact, the predicted correlation of 0.04 in the bond economy is much lower than saving–investment correlations typically found in the data for this measure of saving. (As noted earlier, however, this "basic saving" measure may not be an accurate measure of true saving in the economy.) Finally, the complete markets model predicts a substantial positive correlation between productivity and labor input, while the bond economy generates a strongly negative correlation. In the data, these variables are roughly uncorrelated.

Evidently, the assumption that shocks are permanent and do not spill over has, in the context of the bond economy, more than fixed the problems of (i) overpredicting consumption correlations, (ii) underpredicting output correlations, and (iii) overpredicting the correlation between productivity and labor input. Because of the high degree of uncertainty associated with parameter estimates for the productivity process means that the data very likely would not reject the hypotheses that (i) productivity contains both temporary and permanent components, and (ii) spillovers are positive. Thus there may be a plausible parameterization of productivity which, combined with a restricted asset structure, would lead to accurate model predictions along these three key dimensions. In the concluding section of this paper, therefore, we stress the need for continued work on measurement of international productivity.
4. Dynamic Response to a Productivity Shock

The preceding section explored the implications for the summary statistics of business cycles of alternative assumptions concerning (i) the stochastic process for productivity shocks and (ii) market structure. The chief findings of that section were that restricting financial trade to noncontingent bonds alone had minor effects on the business cycle statistics when productivity was assumed to follow a trend stationary process exhibiting high persistence with substantial international transmission of shocks. However, when productivity contained a unit root, the restrictions on asset trade had important effects.

In order to explore the economic mechanisms behind these differential responses, this section studies the impulse responses of the alternative models when driven by the trend–stationary (BKK) process of section 3.2 versus random walk shocks, as in section 3.3. Throughout, we study the response of the world economy to a 1% increase in total factor productivity which originates in the home country: \( \hat{A}_t = 0.01 \).

4.1 Random walk productivity

In many ways the responses to purely permanent shocks are easier to understand, so we start with this case. Figure 1 plots the responses of aggregate quantity variables in the two countries, and Figure 2 plots the responses of real wages, real interest rates, and bond holdings. In both figures, stars denote the response of the bond economy, and open circles denote the response of the complete markets economy.

Figure 1 shows that, under both asset structures, home country output, consumption, and investment increase in response to the shock, while foreign country investment falls. However, labor market behavior across countries is sensitive to the asset structure, as is the comparative behavior (across countries) of consumption and output. First, under complete markets, labor input increases in the home country and falls in the foreign country; the reverse is true in the bond economy. Second, under
complete markets, consumptions move together across countries while outputs move in opposite directions. In the bond economy, by contrast, consumptions move in opposite directions while output rises in both countries (at least for the first few periods).

Figure 2 shows that the real interest rate implications of the shock are virtually identical under the two asset structures; and that the positive productivity shock causes the home country wage rate to rise under both structures, reflecting the positive effect of the shock on labor productivity. However, the foreign country wage rate rises on impact in the complete markets economy but falls in the bond economy, mirroring the labor responses. Finally, with asset trade restricted to bonds alone, the foreign country accumulates bonds in response to the productivity shock in the home country, (there is no change in asset holdings in the complete markets economy). We have already seen that, in the bond economy, the foreign country responds by decreasing consumption and increasing labor input; thus they must be accumulating bonds over time. When adjustment to the shock is complete, the foreign country will work less and consume less than in the pre-shock steady state; however, a higher share of this consumption will be financed by the interest generated by the increased stock of debt accumulated over the transition path.

Figures 1 and 2 illustrate that the the within-country and cross-country responses of consumption and labor are sensitive to the asset structure. In order to gain additional insight into the reasons why consumption and labor responses differ across asset structures, we employ King's [1990] "Hicksian" method for decomposing the consumption and labor supply responses into (i) a wealth effect, (ii) a real interest rate effect, and (iii) a wage effect. The wealth effect is computed as follows. First, compute the discounted present value of the change in utility caused by the altered time path of consumption and leisure (in response to the shock). Next, compute the constant consumption and leisure profiles that yield the same change in utility, using initial steady-state wages and interest rates. The real interest rate effect is that part
of the response due to alterations in the interest rate alone, holding fixed wealth and wage rates at their initial steady state levels; the wage effect is computed in a similar fashion. These effects are plotted in Figure 3.

Beginning with the wealth effect on home country consumption, we find that the positive productivity shock has a positive wealth effect in the bond economy, but has a negative wealth effect under complete markets. The positive wealth shock in the bond economy is easy to understand — the positive productivity shock means that more output can be obtained using the same level of inputs. In the bond economy, these inputs are completely domestically-owned. Because individuals value both consumption and leisure, the natural response to a positive wealth shock (holding fixed all prices) is to consume more and work less; we see that the wealth effect on labor input is in fact negative in the bond economy (Figure 3-B).

Why is the home country wealth effect on consumption negative under complete markets? Recall that, under complete markets, the response to a location-specific positive productivity shock is for individuals living in the productive location to increase labor supply, taking advantage of the increase in productivity, while transferring some of the proceeds of the increased labor input to individuals living in less productive locations. Although home country consumption rises in response to the shock, home country leisure falls so much that home country discounted utility actually falls in response to the shock. Thus, the home country suffers a negative wealth effect.

In addition to the wealth effect, the productivity shock also induces substitution effects associated with (i) alterations in the time profile of real interest rates (the intertemporal price of consumption and leisure), and (ii) alterations in the time profile for the real wage rate. Since the real interest rate response is virtually identical under the two asset structures (see Figure 2), the substitution effect stemming from this channel is also virtually the same across the two cases. The substitution effect on consumption arising from the increase in the wage profile is positive in both cases.
(although the difference across cases is minor), reflecting the fact that wages rise in response to the shock under both asset structures. Thus in the home country, the differential consumption response under the alternative asset structures is almost entirely due to differences in the size of the wealth effect.

Similar arguments explain the responses of home country labor supply (Figure 3–B). Under complete markets, the wealth effect on labor is positive; i.e., the negative wealth effect induces an increase in labor input. With financial trade restricted to bonds alone, the productivity shock implies a positive wealth shock, thus labor input falls. As with home country consumption, the discount rate effects are nearly identical across the two market structures: the increase in current real interest rates (an increase in the price of current leisure, relative to future leisure) leads to an increase in labor supply from this channel. The wage effect on labor input is positive in the bond economy, but negative in the complete markets economy. As with the consumption response, the biggest difference between the labor response across asset structures lies in the wealth effects. Because the wealth effects are of different sign under the alternative market structures, we find that labor input rises on impact in the complete markets setting, but falls on impact in the bond economy.

In the foreign country it is also the case that the dominant differences across the asset structures lie in the wealth effects. Because foreign country residents do not own productive factors located in the home country, and because there is no international transmission of the productivity shock, there is a zero wealth effect of the shock on consumption and on labor supply. Under complete markets, however, there is a positive wealth effect on consumption and a negative wealth effect on labor supply. With optimal labor insurance, the efficient arrangement calls for the less-productive country to "take a paid vacation," working less and consuming more. Under complete markets, the strength of the wealth effect in depressing labor input is sufficient to counteract positive substitution effects from the increase in the real interest rate and
the increase in the real wage rate. Thus, on impact, foreign labor input rises in response to the shock in the bond economy, but falls in the complete markets economy.

4.2 Trend–stationary shocks with spillovers

Figures 4–6 plot the dynamic response to an innovation in home country productivity when productivity follows the trend–stationary process with spillovers specified in Section 3. We have already seen, in Table 3, that the summary statistics of business cycles are largely invariant to the asset structure under this parameterization of the productivity process. The dynamic responses detailed in Figures 4–6 give a similar impression: the responses of the quantity variables (Figure 4) and prices and interest rates (Figure 5) show very similar responses under the alternative asset structures. The only significant difference is that, in the bond economy, assets are decumulated in the foreign country in response to the home country productivity shock, whereas there is no change in asset holdings in the complete markets economy.

The Hicksian decompositions of the consumption and labor responses plotted in Figure 6 confirm the general impression that, with the BKK productivity process, there is little practical difference between the two asset structures. Recall that, with random walk productivity shocks, the primary difference across asset structures was due to differential wealth effects. In Figure 6, we see that the wealth effects on the two countries of the productivity shock are first of all small, and second, are virtually indistinguishable across the two asset structures. The wealth effect of a temporary shock will always be smaller than the wealth effect of a permanent shock of the same size, so that this in itself is not surprising. The fact that the wealth effects are almost identical across asset structures is more surprising, and this is largely due to the fact that the productivity shock is transmitted across countries over time via the "spillover" parameter, $\nu$. In fact, 8.8% of the shock is transmitted each quarter, and apparently this is rapid enough so that the wealth effects of the shock are nearly identical across
countries even when asset trade is restricted to bonds alone (the wealth effect is identical across countries under complete markets). As before, the real interest rate effects are virtually identical across countries and across asset structures.

Figure 6-B is useful in understanding the forces behind negative international comovement of labor input under this parameterization. Although the wealth effects and real interest rate effects of the shocks are approximately the same across countries, the wage effects are quite different. The wage effect on home country labor is positive, reflecting the higher productivity due to the shock itself, combined with a rapid run-up in investment (see Figure 4). In the foreign country, there is no immediate effect on productivity, although individuals in that country realize that productivity will increase in the future due to the "spillovers." That is, labor productivity in the foreign country is low, on impact, compared with its expected future value. Intertemporal substitution considerations mean that foreign country residents are induced to increase current leisure with the expectation of lower future leisure when the spillover effect brings increased productivity to the foreign country.

Once we understand the importance of the international transmission of the shock (the "spillover") for the wealth effects on labor and consumption, it is easy to understand why foreign country residents decumulate bonds in response to the shock. As noted above, these permanent-income consumers know that the favorable shock will be coming to their country in a few quarters—rapidly enough that the positive wealth effect is nearly as large as in the originating economy. But on impact, productivity in the foreign country is low relative to its expected future level. Thus individuals respond on impact to the positive wealth shock by decreasing current labor supply and increasing current consumption, financing part of current consumption from the proceeds of bond sales.

In summary, we find that with trend-stationary shocks and spillovers, the absence of risk-sharing arrangements stemming from asset market restrictions is not important
for the character of international business cycles. With this parameterization of the productivity shock process, nearly all of the fluctuations in productivity are common across countries, i.e., there is little scope for risk-sharing in the first place, although there is a role for intertemporal trade since productivity arrives in the non-originating country with a lag. Since a real discount bond is the ideal instrument for undertaking intertemporal trade, the restriction of asset markets to bonds alone has little effect on equilibrium outcomes.

5. Conclusions

This paper explored the importance of financial market linkages for the character of international business cycles; our main findings are as follows. First, we found that restricting asset trade to noncontingent bonds alone does not necessarily alter in an important way the predictions of the standard, complete markets model. If the international productivity process is trend stationary with substantial international "spillovers" of productivity shocks, the two models are essentially indistinguishable. If, however, productivity in each country follows a random walk without spillovers but with correlated innovations, restricting asset trade alters the predictions of the model along several important dimensions. In particular, under this parameterization, the complete markets model predicts low cross-country output correlations and near-perfect consumption correlations; the bond economy conversely predicts high output correlations and low consumption correlations. This finding is important, since the complete markets model has been heavily criticized for its counterfactual prediction of near-perfect international consumption correlations for a wide range of parameterizations. With random walk shocks, restricting asset markets brings the consumption correlation down substantially.

Second, we found that the major differences in the macroeconomic response to shocks under the alternative asset structures are due almost entirely to differential
wealth effects. In particular, we found strong differences across asset structures when productivity follows a random walk. In this case, under complete markets, individuals receiving a favorable productivity shock experience a negative wealth effect, because the optimal insurance character of equilibrium requires them to increase labor supply while transferring a large proportion of the additional output to residents of the other country. In the bond economy, however, individuals own all the risky claims to their country's output. Thus individuals receiving a favorable productivity shock experience a positive wealth effect, which induces them to increase consumption by more than in the complete markets economy and, more importantly, causes them to decrease labor input.

Although the dominant source of differences across asset structures was found to lie in wealth effects, consumption and leisure are strongly affected in both asset structures by interest rate effects and wage effects. Thus the general equilibrium structure, in which interest rates and asset prices are determined endogenously, is important for understanding the way in which economies respond to exogenous shocks. This consideration is important even if the economy in question is "small" in the sense of having a small share of world product. As shown in Baxter and Crucini [1993], and as confirmed by the analyses of Backus, Kehoe, and Kydland [1992] and the present paper, for this type of model to have sensible business cycle implications, it is necessary that productivity have a substantial component that is common across countries. Even though shocks to the small open economy may not have any direct effect on world interest rates, the commonness of shocks means that movements in the world interest rate are correlated with shocks in the small economy. This implies that the traditional assumption that the small open economy faces a fixed interest rate is not empirically defensible. Further, as shown in section 4 of this paper, the interest rate effects on consumption and labor input are quantitatively important, even when asset trade is restricted to noncontingent bonds. Thus, even with incomplete financial markets,
problems involving small open economies should nevertheless be studied in a general equilibrium framework.

Finally, this paper makes a contribution to the technical literature on solving dynamic models by providing an example of how Euler equation methods can extended in a straightforward fashion to the study of suboptimal dynamic economies. In particular, this paper illustrates how one can compute equilibrium in models in which departures from optimality arise due to constraints on individuals' opportunities for risk-pooling, without suffering a significant increase in conceptual or computational complexity. Because the equilibrium behavior of dynamic models can always be characterized as the solution to a system of Euler equations (augmented by market clearing conditions and equilibrium "consistency conditions"), the same approach can be used to study other models in which departures from optimality arise for a number of possible reasons, such as market incompleteness, imperfect information, or departures from competitive equilibrium due to monopolistic behavior.

In summary, we find that the asset structure of foreign trade can be important for the character of international business cycles, but that many model predictions are very sensitive to the parameterization of the productivity process. Thus an important avenue for future research is the continuation of the work begun by Backus, Kehoe, and Kydland [1992] and Costello [1993] on measuring international productivity.
References


Endnotes

1 Cole [1988] studied the implications of financial structure for business cycles in a two-period model with production. Many of our findings are qualitatively similar to his results. More recently, the relationship between asset markets and real activity has been studied by Conze, Lasry, and Sheinkman [1990], and Kollman [1990], and Backus [1991]. Their models differ from the one developed here; Kollman's model is the closest to ours. Kollman finds, as we do, that restrictions on asset markets leads to lower international correlations of consumption (Conze, et al. obtain this prediction as well). Kollman also explores the implications of additive productivity shocks as well as the traditional multiplicative shocks. He finds that the problem of negative international comovement is less severe with additive shocks.

2 We assume that \( \bar{\beta} \) is the same for all countries in the world, so that (23) holds for each country, and \( R\gamma = \bar{\beta} \) is also a world general equilibrium condition. This requirement also guarantees that no country grows arbitrarily wealthy over time in the deterministic version of the model.

3 All test statistics used in this analysis are discussed in Park [1990].

4 Since our log-linear solution algorithm generates decision rules that display certainty equivalence, only the scale of volatility changes as we change the innovation variances. Relative volatilities, such as the standard deviation of consumption divided by the standard deviation of output, are invariant in this setup to the size of the shock variance.

5 Labor market statistics for other countries are omitted since accurate measures of total labor input are not readily available, as discussed in Section 3.1 above.

6 The population moments for the filtered time series are computed using the rational polynomial version of the Hodrick–Prescott filter applicable to an infinite sample of data, as discussed in King and Rebelo [1993]. Even though the incomplete markets model implies that there is a unit root component to each country's real quantity variables, the HP filter contains four differences in the numerator of the rational polynomial, so that population moments for the filtered series are well-defined.

7 In our model, the wage rate equals the average product of labor.
TABLE 1

Statistical Properties of International Solow Residuals

Panel A: Park and Choi J(p,q) Test for Unit Root

(The null hypothesis is a unit root: the hypothesis is rejected if the test statistic is smaller than the critical value)

<table>
<thead>
<tr>
<th>Measure of Solow Residual (time period)</th>
<th>J(1,2)</th>
<th>Test Statistic J(1,3)</th>
<th>J(1,4)</th>
<th>J(1,5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. (1965:3–1988:3)</td>
<td>0.124</td>
<td>0.645</td>
<td>0.699</td>
<td>0.745</td>
</tr>
<tr>
<td>Canada (1965:3–1988:3)</td>
<td>0.343</td>
<td>1.346</td>
<td>1.948</td>
<td>3.461</td>
</tr>
<tr>
<td>U.S. (1970:2–1986:4)</td>
<td>0.010</td>
<td>0.255</td>
<td>0.275</td>
<td>0.309</td>
</tr>
<tr>
<td>Europe (1970:2–1986:4)</td>
<td>0.740</td>
<td>0.946</td>
<td>0.967</td>
<td>1.179</td>
</tr>
</tbody>
</table>

critical values:

<table>
<thead>
<tr>
<th></th>
<th>1%</th>
<th>5%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>J(1,2)</td>
<td>8.6e-5</td>
<td>0.011</td>
<td>0.055</td>
</tr>
<tr>
<td>J(1,3)</td>
<td>0.055</td>
<td>0.160</td>
<td>0.295</td>
</tr>
<tr>
<td>J(1,4)</td>
<td>0.290</td>
<td>0.452</td>
<td></td>
</tr>
</tbody>
</table>

Panel B: Tests for Cointegration

We used Park's canonical cointegrating regression to estimate $\alpha_1$ such that $A_t - \alpha A_t^* = \epsilon_t$, a stationary random variable. Next, we used Park's H(p,q) test for stochastic cointegration; p-values are given in the table below. In each case, the U.S. is the unstarred variable (i.e., $\alpha$ is the coefficient on Canada or Europe).

<table>
<thead>
<tr>
<th></th>
<th>$\hat{\alpha}$</th>
<th>se($\hat{\alpha}$)</th>
<th>p-values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$H(1,2)$</td>
<td>$H(1,3)$</td>
<td>$H(1,4)$</td>
</tr>
<tr>
<td>U.S. – Canada</td>
<td>0.580</td>
<td>0.061</td>
<td>0.313</td>
</tr>
<tr>
<td>U.S. – Europe</td>
<td>0.603</td>
<td>0.041</td>
<td>0.046</td>
</tr>
</tbody>
</table>
Table 1, cont’d.

Panel C: Estimates of stochastic processes for Solow residuals

$\Delta$ denotes the first difference of a variable, i.e., $\Delta A_t = A_t - A_{t-1}$; as before the U.S. is the unstarred country. Standard errors are in parentheses.

**U.S.–Canada:**

\[
\begin{align*}
\Delta A_t &= 0.003 + 0.113 \Delta A_{t-1} + 0.048 \Delta A^*_{t-1} - 0.074 (A_{t-1} - A^*_{t-1}) + u_t \\
\Delta A^*_t &= 0.005 + 0.283 \Delta A_{t-1} + 0.035 \Delta A^*_{t-1} + 0.021 (A_{t-1} - A^*_{t-1}) + u^*_t \\
\hat{\sigma}_u^2 &= 8.38e-3; \quad \hat{\sigma}_{u^*}^2 = 9.34e-3; \quad \hat{\rho}(u,u^*) = 0.392.
\end{align*}
\]

**U.S.–Europe:** (error-correction term omitted due to lack of cointegration)

\[
\begin{align*}
\Delta A_t &= 0.002 + 0.003 \Delta A_{t-1} + 0.193 \Delta A^*_{t-1} + u_t \\
\Delta A^*_t &= 0.005 + 0.196 \Delta A_{t-1} - 0.076 \Delta A^*_{t-1} + u^*_t \\
\hat{\sigma}_u^2 &= 9.07e-3; \quad \hat{\sigma}_{u^*}^2 = 7.95e-3; \quad \hat{\rho}(u,u^*) = 0.228.
\end{align*}
\]

**Notes:**

1. All the tests reported in this table are discussed in Park [1990].
### TABLE 2
Business Cycle Statistics for 8 OECD Countries

<table>
<thead>
<tr>
<th>Country</th>
<th>$\frac{\sigma_c}{\sigma_y}$</th>
<th>$\frac{\sigma_i}{\sigma_y}$</th>
<th>$\frac{\sigma_{nx}}{\sigma_y}$</th>
<th>$\rho_y$</th>
<th>$\rho(c,y)$</th>
<th>$\rho(i,y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>0.69</td>
<td>2.17</td>
<td>1.46</td>
<td>0.67</td>
<td>0.62</td>
<td>0.55</td>
</tr>
<tr>
<td>Canada</td>
<td>0.88</td>
<td>2.83</td>
<td>0.83</td>
<td>0.79</td>
<td>0.72</td>
<td>0.62</td>
</tr>
<tr>
<td>France</td>
<td>0.89</td>
<td>1.92</td>
<td>0.81</td>
<td>0.79</td>
<td>0.58</td>
<td>0.45</td>
</tr>
<tr>
<td>Germany</td>
<td>0.70</td>
<td>3.40</td>
<td>0.88</td>
<td>0.71</td>
<td>0.64</td>
<td>0.80</td>
</tr>
<tr>
<td>Italy</td>
<td>0.82</td>
<td>2.49</td>
<td>1.76</td>
<td>0.78</td>
<td>0.70</td>
<td>0.80</td>
</tr>
<tr>
<td>Japan</td>
<td>1.12</td>
<td>2.31</td>
<td>0.93</td>
<td>0.74</td>
<td>0.47</td>
<td>0.60</td>
</tr>
<tr>
<td>Switzerland</td>
<td>0.77</td>
<td>2.88</td>
<td>1.50</td>
<td>0.70</td>
<td>0.74</td>
<td>0.73</td>
</tr>
<tr>
<td>U.S.A.</td>
<td>0.67</td>
<td>3.00</td>
<td>0.41</td>
<td>0.84</td>
<td>0.88</td>
<td>0.90</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Country</th>
<th>correlation with same U.S. variable</th>
<th>Additional labor market statistics for the USA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>output</td>
<td>cons'n.</td>
</tr>
<tr>
<td>Australia</td>
<td>0.24</td>
<td>0.11</td>
</tr>
<tr>
<td>Canada</td>
<td>0.77</td>
<td>0.65</td>
</tr>
<tr>
<td>France</td>
<td>0.50</td>
<td>0.28</td>
</tr>
<tr>
<td>Germany</td>
<td>0.44</td>
<td>0.45</td>
</tr>
<tr>
<td>Italy</td>
<td>0.47</td>
<td>0.23</td>
</tr>
<tr>
<td>Japan</td>
<td>0.42</td>
<td>0.41</td>
</tr>
<tr>
<td>Switzerland</td>
<td>0.28</td>
<td>0.22</td>
</tr>
<tr>
<td>U.S.A.</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>
Notes to Table 2:

With the exception of the U.S. labor market statistics, all statistics are taken from Baxter and Crucini [1991]. The data is from the International Financial Statistics, and is quarterly postwar data, with coverage varying by country. This is the same database used by Backus, Kehoe, and Kydland [1992] who graciously provided us with their data.

Statistics for U.S. labor markets were taken from Baxter and King [1991]; the original data source was Citibase. The data is quarterly data from 1955:1–1990:3. In this table, "N" denotes labor input (hours worked), and "prod" denotes productivity computed as output per manhour.

All data has been detrended using the Hodrick–Prescott [1980] filter.
Table 3
Trend Stationary Shocks

(1) results for complete markets economy
(2) results for economy trading noncontingent bonds and goods only

<table>
<thead>
<tr>
<th></th>
<th>Standard Deviation</th>
<th>Relative Std. Dev.</th>
<th>Persistence</th>
<th>corr w/y, lag 0</th>
<th>Other Correlations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(1)</td>
<td>(2)</td>
<td>(1)</td>
</tr>
<tr>
<td>y</td>
<td>2.01</td>
<td>1.99</td>
<td>1.00</td>
<td>1.00</td>
<td>0.75</td>
</tr>
<tr>
<td>c</td>
<td>0.97</td>
<td>0.98</td>
<td>0.48</td>
<td>0.49</td>
<td>0.81</td>
</tr>
<tr>
<td>i</td>
<td>3.72</td>
<td>3.55</td>
<td>1.85</td>
<td>1.79</td>
<td>0.73</td>
</tr>
<tr>
<td>N</td>
<td>1.07</td>
<td>1.02</td>
<td>0.53</td>
<td>0.51</td>
<td>0.73</td>
</tr>
<tr>
<td>w</td>
<td>1.13</td>
<td>1.14</td>
<td>0.56</td>
<td>0.57</td>
<td>0.80</td>
</tr>
<tr>
<td>nx/y</td>
<td>0.57</td>
<td>0.59</td>
<td>0.29</td>
<td>0.30</td>
<td>0.80</td>
</tr>
<tr>
<td>b</td>
<td>0.00</td>
<td>3.22</td>
<td>0.00</td>
<td>1.62</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>Standard Deviation</td>
<td>Relative Std. Dev.</td>
<td>Persistence</td>
<td>corr w/y, lag 0</td>
<td>Other Correlations</td>
</tr>
<tr>
<td>-------</td>
<td>-------------------</td>
<td>-------------------</td>
<td>-------------</td>
<td>-----------------</td>
<td>--------------------</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(1)</td>
<td>(2)</td>
<td>(1)</td>
</tr>
<tr>
<td>$y$</td>
<td>2.58</td>
<td>1.59</td>
<td>1.00</td>
<td>1.00</td>
<td>$y, y^*$</td>
</tr>
<tr>
<td>$c$</td>
<td>1.03</td>
<td>1.67</td>
<td>0.40</td>
<td>1.05</td>
<td>$c, c^*$</td>
</tr>
<tr>
<td>$i$</td>
<td>11.84</td>
<td>4.74</td>
<td>4.60</td>
<td>2.98</td>
<td>$i, i^*$</td>
</tr>
<tr>
<td>$N$</td>
<td>1.58</td>
<td>0.71</td>
<td>0.61</td>
<td>0.45</td>
<td>$N, N^*$</td>
</tr>
<tr>
<td>$w$</td>
<td>1.25</td>
<td>1.62</td>
<td>0.48</td>
<td>1.02</td>
<td>$w, w^*$</td>
</tr>
<tr>
<td>$nx/y$</td>
<td>2.39</td>
<td>1.61</td>
<td>0.93</td>
<td>1.01</td>
<td>$s, i$</td>
</tr>
<tr>
<td>$b$</td>
<td>0.00</td>
<td>8.18</td>
<td>0.00</td>
<td>5.13</td>
<td>$w, N$</td>
</tr>
</tbody>
</table>

(1) results for complete markets economy

(2) results for economy trading noncontingent bonds and goods only
FIGURE 1

Random walk productivity without spillovers:
Quantity responses to productivity shock in home country
FIGURE 2

Random walk productivity without spillovers:
Price and interest rate responses to productivity shock in HC
FIGURE 3–A

Random walk productivity without spillovers:
Hicksian decomposition of consumption

HC WEALTH EFFECT

* bond economy
 o complete markets

quarters

goods units

1

0

-1

10 20 30 40

HC REAL INTEREST RATE EFFECT

quarters

goods units

1

0

-1

10 20 30 40

HC WAGE EFFECT

quarters

goods units

1

0

-1

10 20 30 40

HC TOTAL CONSUMPTION EFFECT

quarters

goods units

1

0

-1

10 20 30 40

FC WEALTH EFFECT

quarters

goods units

1

0

-1

10 20 30 40

FC REAL INTEREST RATE EFFECT

quarters

goods units

1

0

-1

10 20 30 40

FC WAGE EFFECT

quarters

goods units

1

0

-1

10 20 30 40

FC TOTAL CONSUMPTION EFFECT

quarters

goods units

1

0

-1

10 20 30 40
FIGURE 3-B

Random walk productivity without spillovers:
Hicksian decomposition of labor

HC WEALTH EFFECT
- bond economy
- complete markets

percent

quarters 10 20 30 40

FC WEALTH EFFECT

percent

quarters 10 20 30 40

HC REAL INTEREST RATE EFFECT

percent

quarters 10 20 30 40

FC REAL INTEREST RATE EFFECT

percent

quarters 10 20 30 40

HC WAGE EFFECT

percent

quarters 10 20 30 40

FC WAGE EFFECT

percent

quarters 10 20 30 40

HC TOTAL LABOR EFFECT

percent

quarters 10 20 30 40

FC TOTAL LABOR EFFECT

percent

quarters 10 20 30 40
FIGURE 4

Trend–stationary productivity with spillovers:
Quantity responses to productivity shock in home country

HC output: * = bond, o = CM

FC output: * = bond, o = CM

HC consumption: * = bond, o = CM

FC consumption: * = bond, o = CM

HC investment: * = bond, o = CM

FC investment: * = bond, o = CM

HC labor: * = bond, o = CM

FC labor: * = bond, o = CM
FIGURE 5

Trend–stationary productivity with spillovers:
Price and interest rate responses to productivity shock in HC

HC real wage: *=bond, o=CM

percent

0 10 20 30 40
quarters

real interest rate: *=bond, o=CM

percent

0 10 20 30 40
quarters

FC real wage: *=bond, o=CM

percent

0 10 20 30 40
quarters

FC bond holdings: *=bond, o=CM

percent

-6 -4 -2 0
quarters
FIGURE 6-A

Trend-stationary productivity with spillovers:
Hicksian decomposition of consumption

HC WEALTH EFFECT

FC WEALTH EFFECT

* bond economy
o complete markets

HC REAL INTEREST RATE EFFECT

FC REAL INTEREST RATE EFFECT

HC WAGE EFFECT

FC WAGE EFFECT

HC TOTAL CONSUMPTION EFFECT

FC TOTAL CONSUMPTION EFFECT
FIGURE 6-B

Trend-stationary productivity with spillovers:
Hicksian decomposition of labor

HC WEALTH EFFECT
- bond economy
- complete markets

percent
-1
0
0.5
1
quarters
10
20
30
40

FC WEALTH EFFECT
percent
-1
0
0.5
1
quarters
10
20
30
40

HC REAL INTEREST RATE EFFECT
percent
-1
0
0.5
1
quarters
10
20
30
40

FC REAL INTEREST RATE EFFECT
percent
-1
0
0.5
1
quarters
10
20
30
40

HC WAGE EFFECT
percent
-1
0
0.5
1
quarters
10
20
30
40

FC WAGE EFFECT
percent
-1
0
0.5
1
quarters
10
20
30
40

HC TOTAL LABOR EFFECT
percent
-1
0
0.5
1
quarters
10
20
30
40

FC TOTAL LABOR EFFECT
percent
-1
0
0.5
1
quarters
10
20
30
40