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Do Open Market Operations Matter?
Theory and Evidence from the Second Bank of the United States

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Do Open Market Operations Matter?
Evidence from the Second Bank of the United States

An important tenet of corporate finance is that under certain conditions the composition of a firm's financial structure is irrelevant for determining the firm's value. That such an irrelevance result might also hold more generally for the consolidated balance sheet of a country's treasury and central bank was suggested by Wallace (1981). His work showed that open market rearrangements of the central bank's portfolio might be irrelevant if these rearrangements were accomplished with fiscal policy "held constant" in a particular sense. Irrelevance, in Wallace's setting, included the possibility that both equilibrium resource allocations and the time path of the price level might be independent of the government's portfolio. The result indicates that the direct money supply--price level link so beloved by generations of economists need not hold; a wide variety of monetary arrangements can be compatible with the same equilibrium price level path.

In this paper we demonstrate that such an irrelevance result can, and apparently did, hold in an economy with a privately owned central bank. We develop an overlapping generations model in the style of Wallace (1981) which incorporates a central bank with publicly traded equity. A unique feature of our model is that excess earnings generated by the bank's portfolio rearrangements are distributed to private agents through dividends and/or changes in the price of shares in the bank. This standard corporate behavior provides the "rebate" channel required to hold "fiscal policy constant" in Wallace's sense. We then derive the conditions under which irrelevance of open market operations holds and find them fewer in number and historically more plausible than those found in other models in the literature.

Our model is designed to capture the features of actual central banking as it existed in the United States (and elsewhere) in the early 19th century. Further, our model makes explicit predictions about the impact of innovations to the bank's financial structure on the economy and on the price of bank shares. We demonstrate that from 1823 to 1832 the portfolio decisions of the Second Bank were irrelevant for the behavior of the price level and had the
predicted effect on the bank's market value. To do so we employ general economic data of the
time, data from the operations of the Second Bank of the United States and the innovation
accounting provided by a vector autoregression. Because the Second Bank of the United
States was the country's central bank at that time, this provides a concrete example of monetary
irrelevance for actual central bank open market operations. In addition, we show that the
income velocity of money was highly variable and also rapidly trending during this period,
which was one of substantial economic stability. The contrasts with the Friedman and
Schwartz [1963] finding that the relative instability of velocity and economic volatility are
related, and is consistent with the irrelevance result that the model provides.

There are several important contributions that arise from our work. First, the initial
irrelevance proposition of Wallace involved a set of very restrictive assumptions. While these
assumptions were significantly relaxed by Chamley and Polemarcharkis (1984) and by Sargent
and Smith (1987), the conditions under which irrelevance has been theoretically demonstrated
to occur are quite limited. Our model provides additional cases in which irrelevance will arise,
thereby expanding the domain in which these "Modigliani-Miller" theorems can be applied to
government finance. Moreover, as we argue below, these cases are of substantial historical
relevance.

A second, and perhaps more intriguing, contribution of this research is our
demonstration that such irrelevance of open market operations does in fact arise. While several
authors [Sargent (1982); Riley and McCusker (1983); Smith (1985a,b,1988)] have noted
episodes in which the money supply and the price level have moved divergently, such
movements need not reflect the actions of the central bank (or of more general government
monetary operations). To our knowledge, our work is the first to demonstrate with actual
central bank data an irrelevance result involving the bank's portfolio decision and economic
variables. The existence of such a result calls into question various traditional assertions about
monetary policy and price level behavior.
Finally, our work also yields important insights into both the operations of this early central bank and into the behavior of the economy in a period heretofore little studied. Our focus on the macroeconomic impact of the bank provides an in-depth look at the inter-relations of economic variables in the early 19th century. This paper thus complements the study of Highfield, O'Hara and Wood (1991) who investigate behavioral issues relating to the actions of the Second Bank at that time. Our results here suggest that the nature of the early 19th century economy precluded any significant effects associated with manipulation of the Bank's portfolio alone, and suggest that much of the behavior of the economy was exogenously driven. Given the fledgling state of economic development in the U.S. at that time, such "small open economy" results are not unreasonable.

The paper is organized as follows. In the next section we develop a stochastic overlapping generations model that explicitly incorporates a privately owned (equity-issuing) central bank. In Section 3, we describe an equilibrium in our model, and derive our irrelevance of open market operations proposition. To make the implications of our result more intuitive, we relate our irrelevance theorem to those of Wallace (1981) and Chamley and Polemarchakis (1984). In Section 4, we then test the model by examining the effects of portfolio actions by the Second Bank of the United States using vector autoregression (VAR) analysis and by examining general economic data. We examine several alternative explanations for our empirical findings in Section 5, and Section 6 is a conclusion.

2. The Model

In this section, we develop an overlapping generations model with bank liabilities, as well as an array of financial assets. The model closely follows that of Wallace (1981) and we employ his notation wherever possible. Our analysis differs from Wallace's in that we consider an economy in which the central bank is privately owned, and has actively traded shares. We note that while we follow Wallace in placing some structure on the economy (for instance, we assume a single good, pure exchange economy), little of this structure is essential
to the analysis. Following Wallace, we also consider an economy where no asset is dominated
in rate of return, although at some cost in added complexity, this too can be included.
Examples illustrating how to do so appear in Sargent and Smith (1987a,b).

We consider a discrete time economy, with time indexed by $t = 1, 2, \ldots$. At date $t$,
$H(t)$ two-period lived agents are born, with agents indexed by $h = 1, 2, \ldots, H(t)$. These agents
consume a single, non-storable good in each period. In addition, there is some randomness in
the economy, so at each date one of $I$ possible states occurs; states are indexed by $i = 1, 2, \ldots, I$. We let $f_i \in [0, 1]$ be the probability that state $i$ occurs at any date, so the state of the economy
is an iid random variable.

Young agents are born at $t$ after the realization of the time $t$ state, and we denote the time
$t$ consumption of agent $h$ by $c^h_t(t)$. However, agent $h$'s old age consumption may be state
contingent, and we denote it by $c^h_i(t + 1)$. Agent $h$ has preferences given by the utility function
$u^h[c^h_t(t), c^h_i(t + 1)]$; each of the functions $u^h$ is assumed to have standard properties. Finally,
agent $h$ at $t$ has a young period before-tax endowment of $y^h_t(t)$, and an after-tax endowment of
$w^h_t(t)$. These are assumed not to be state contingent. Old period endowments are permitted to
be state contingent; agent $h$'s old age before (after) tax endowment is $y^h_i(t + 1)[w^h_i(t + 1)]$ in
state $i$. Notice that taxation here is lump-sum in nature.

Trading

Agents in our model can trade a wide range of financial claims. These financial claims
are as follows.

(i) Agent $h$ can borrow or lend abroad in amount $b^h(t)$. $b^h(t)$ is measured in real
terms, and represents the possibility of trading bills of exchange abroad. This
possibility was an important feature in the economy we consider. The one
period gross return on bills of exchange from $t$ to $t + 1$ is denoted $x(t+1)$, and is
exogenous to the economy. We thus are de facto considering a small open
economy, which is again the empirically relevant situation. We also allow
$x(t+1)$ to be (potentially) random, so that $x(t+1) \in \{x_1, x_2, \ldots, x_I\}$, and
prob$[x(t+1) = x_i] = f_i$.

(ii) Agent $h$ can hold currency in the nominal amount $m^h(t)$. $p_i(t)$ denotes the (state
contingent) time $t$ price level.
(iii) Agents can borrow from or lend to banks. Let $\ell^h(t)$ be net borrowing from banks in real terms (negative borrowing denotes a deposit). The state contingent gross return from $t$ to $t+1$ is denoted $R_i(t+1)$.

(iv) Agents can trade state contingent claims. Let $d^h_i(t)$ be the quantity of the good $h$ promised to deliver at $t+1$ if state $i$ occurs. The price of one unit to be delivered at $t+1$ if $i$ occurs is denoted by $s_i(t)$.

(v) Agents can purchase shares in the central bank. Let $z^h(t)$ denote the quantity of shares in the bank purchased by agent $h$. The price of a share is $q_i(t)$ (in real terms). The bank pays a state contingent dividend of $\pi_i(t)$ at $t$. If $h$ holds $z^h(t)$ shares, he receives $z^h(t)\pi_i(t+1)$ at $t+1$, and then sells his shares.

**Individual Behavior**

The budget constraint of young agent $h$ at $t$ is given by:

\begin{equation}
\begin{aligned}
c^h_i(t) + \frac{m^h_i(t)}{p(t)} + z^h(t)q(t) &= w^h_i(t) + b^h(t) + \ell^h(t) + \sum_i s_i(t)d^h_i(t) \\
\end{aligned}
\end{equation}

Similarly, the budget constraint of agent $h$ when old is given by (if state $i$ occurs):

\begin{equation}
\begin{aligned}
c^h_o(t+1) &= w^h_o(t+1) - R_i(t+1)\ell^h_i(t) - x_i(t+1)b^h(t) \\
&+ \frac{p(t)}{p_i(t+1)} \frac{m^h_i(t)}{p(t)} - d^h_i(t) + z^h(t)[q_i(t+1) + \pi_i(t+1)]; \quad i = 1, \ldots, I. \\
\end{aligned}
\end{equation}

If we multiply both sides of equation (2) by $s_i(t)$, sum the result over $i$, and then substitute for $\sum s_i(t)d^h_i(t)$ in equation (1) we obtain:

\begin{equation}
\begin{aligned}
c^h(t) + \sum_i s_i(t)c^h_i(t+1) &= w^h(t) + \sum_i s_i(t)w^h_i(t+1) \\
&+ \left[1 - \sum_i s_i(t)R_i(t+1)\right]\ell^h(t) + \left[1 - \sum_i s_i(t)x_i(t+1)\right]b^h(t) \\
&+ \left[\sum_i s_i(t)\frac{p(t)}{p_i(t+1)} - 1\right]\frac{m^h(t)}{p(t)} + \left[\sum_i s_i(t)\left[q_i(t+1) + \pi_i(t+1)\right] - q(t)\right]z^h(t) \\
\end{aligned}
\end{equation}
Agent \( h \) then chooses \( c_i^h(t), \{c_{ii}^h(t+1)\}_i, \ell^h(t), b^h(t), z^h(t), \) and \( m^h(t) / p(t) \) to maximize \( \sum f_i u^h[c_i^h(t), c_{ii}^h(t+1)] \) subject to (3).

**No Arbitrage**

A number of "no-arbitrage" conditions must be satisfied for the agent's problem to have a non-trivial solution. If \( b^h(t), \ell^h(t), m^h(t), \) and \( z^h(t) \) can take on any sign, then an absence of arbitrage opportunities requires that equilibrium prices, interest rates, and dividends satisfy:

\[
\begin{align*}
(4) & \quad \sum_i s_i(t) R_i(t+1) = 1 \\
(5) & \quad \sum_i s_i(t) x_i(t+1) = 1 \\
(6) & \quad \sum_i s_i(t) \frac{p(t)}{p_i(t+1)} = 1 \\
(7) & \quad \sum_i s_i(t) [q_i(t+1) + \pi_i(t+1)] = q(t)
\end{align*}
\]

Equations (4), (5), and (7) require that asset prices be efficient, while (6) requires that money not be dominated in rate of return.

**Bank Behavior**

The bank issues notes in amount \( N(t) \) (in dollars), makes loans (net of deposits) with a real value of \( L(t) \), and incurs net foreign borrowings of \( B(t) \) (in real terms). The bank does not (by assumption) trade state contingent claims.

Recalling that \( \pi_i(t) \) is dividend payments by the bank if state \( i \) occurs at \( t \), the bank's time \( t \) budget constraint is

\[
\begin{align*}
(8) & \quad \pi_i(t) = B(t) - L(t) + \frac{N(t) - N(t-1)}{p(t)} + R_i(t)L(t-1) - x_i(t)B(t-1).
\end{align*}
\]

Note that (8) allows for retained earnings on the part of the bank.
The Government

The government may issue money separately from the central bank. We let \( M(t) \) denote the nominal stock of "non-bank money" at \( t \).\(^1\) In addition, the government has an expenditure level of \( G(t) \) at \( t \), and levies lump-sum taxes on agent \( h \) of \( y^h_t(t) - w^h_t(t) \) when young, and of \( y^h_{t+1}(t+1) - w^h_{t+1}(t+1) \) when old. Throughout we assume that \( \{M(t)\} \) is a fixed sequence, and that taxes and government expenditure are held constant. This assumption is motivated by the economy and time period we consider, in which government activity in these dimensions was relatively limited.

Equilibrium

An equilibrium is a sequence of prices \( \{s_i(t), \ p_i(t), \ R_i(t), \ q_i(t)\}, \ i = 1, \ldots, I; \) a sequence of allocations \( \{c^h_t(t), \ c^h_{t+1}(t+1)\}, \ h = 1, \ldots, H(t), i = 1, \ldots, I; \) a set of portfolio choices for agent \( h \) at each date \( \{b^h(t), \ x^h(t), \ d^h_t(t), \ m^h_t(t), \ z^h_t(t)\}, \ h = 1, \ldots, H(t), i = 1, \ldots, I; \) a sequence of policies for the bank \( \{\pi_i(t), \ B(t), \ L(t), \ N(t)\}, \ i = 1, \ldots, I, \) which satisfy (8); and a sequence of government policies \( \{M(t), \ y^h_t(t) - w^h_t(t), \ y^h_{t+1}(t+1) - w^h_{t+1}(t+1)\}, \ h = 1, \ldots, H(t), i = 1, \ldots, I; \) such that

(i) \quad (4) - (7) are satisfied.

(ii) \quad \( c^h_t(t), \ c^h_{t+1}(t+1) \) maximize \( \sum_i f^h_i \left[c^h_t(t), \ c^h_{t+1}(t+1)\right] \) subject to (3).

(iii) \quad the following market clearing conditions are satisfied:

\[
\begin{align*}
G(t) + \sum_{h} c^h_t(t) + \sum_{h} c^h_{t-1,i}(t) = \sum_{h} y^h_t(t) + & \sum_{h} y^h_{t-1,i}(t) \\
+ & \sum_{h} b^h_t(t) + B(t) - x_i(t) \left[ B(t-1) + \sum_{h} b^h(t-1) \right]; \quad i = 1, \ldots, I, \ \forall t,
\end{align*}
\]

\(^1\)In the economy we examine, \( M(t) \) might correspond to money issued by the mint.
(contingent claims market clearing)

\[ \sum_{h} d_i^h(t) = 0; \quad \forall i, t \]

(loan market clearing)

\[ L(t) = \sum_{h} \ell_i^h(t); \quad \forall t \]

/share market clearing)

\[ \sum_{h} Z_i^h(t) = 1; \quad \forall t \]

(money market clearing)

\[ \sum_{h} m_i^h(t) = N(t) + M(t); \quad \forall t. \]

Satisfaction of the government's budget constraint is implied by these conditions and Walras' Law.

3. **Irrelevance of Open Market Operations**

We now state conditions under which rearrangements of the central bank's balance sheet are irrelevant for the set of equilibrium allocations and relative prices. Having done this, we will then specialize our conditions further to indicate when rearrangements of the bank's balance sheet will also be irrelevant for (that is, have no effects on) the stochastic processes governing equilibrium interest rates and price levels.

**A General Irrelevance Theorem**

Suppose that for a given sequence of policy choices for the bank, \((\hat{L}(t), \hat{B}(t), \hat{N}(t), \hat{\pi}_i(t))\), an equilibrium (denoted the "\(\wedge\)" equilibrium) exists. We seek a new equilibrium, denoted the "\(\_\)" equilibrium, which satisfies
\begin{align}
\dot{c}_{i}^{h}(t) &= \ddot{c}_{i}^{h}(t) \\
\dot{c}_{i}^{h}(t+1) &= \ddot{c}_{i}^{h}(t+1) \\
\tilde{s}_{i}(t) &= \hat{s}_{i}(t)
\end{align}

i.e., an equilibrium under which consumption and state contingent claim prices are unaffected.

We now sketch the construction of this equilibrium.

**Step 1.** Subtract (9) for the (\wedge) equilibrium from (9) for the (\dashv) equilibrium, and impose (14), (15), and \( \hat{G}(t) = \bar{G}(t) \) to obtain

\begin{align}
\sum_{h} \left[ \hat{b}_{i}^{h}(t) - \ddot{b}_{i}^{h}(t) \right] + \bar{B}(t) - \hat{B}(t) &= x_{i}(t) \left[ \bar{B}(t-1) - \hat{B}(t-1) \right] \\
&\quad + x_{i}(t) \sum_{h} \left[ \hat{b}_{i}^{h}(t-1) - \ddot{b}_{i}^{h}(t-1) \right]
\end{align}

Equation (17) must hold \( \forall i,t \). Moreover, since \( \bar{B}(0) \equiv \hat{B}(0) \) and \( \hat{b}^{h}(0) \equiv \ddot{b}^{h}(0) \) must hold (these quantities are given as initial conditions), we can apply repeated substitutions to (17) to obtain the following:

\begin{equation}
\sum_{h} \left[ \hat{b}_{i}^{h}(t) - \ddot{b}_{i}^{h}(t) \right] = \bar{B}(t) - \hat{B}(t) \quad \forall t.
\end{equation}

Equation (18) simply states that any increase in foreign borrowing on the part of the bank must be offset by an equal decrease in foreign borrowing by the public.

**Step 2.** We observe that, since \( \tilde{s}_{i}(t) = \hat{s}_{i}(t) \), \( \tilde{w}_{i}^{h}(t) = \hat{w}_{i}^{h}(t) \), and \( \tilde{w}_{i}^{h}(t+1) = \hat{w}_{i}^{h}(t+1) \) hold \( \forall h, i, t \), the set of utility maximizing consumption allocations is the same in the (\wedge) and the (\dashv) equilibria.
**Step 3.** Sum equation (2) over \( h \) and use \( \sum_h d^h(t) = 0 \) to get

\[
\sum_h c^h(t+1) = \sum_h w^h(t+1) - R^h(t+1) \sum_h p^h(t) - x^h(t+1) \sum_h b^h(t) \\
+ \frac{p(t)}{p(t+1)} \sum_h \frac{m^h(t)}{p(t)} + \sum_h z^h(t) [q^h(t+1) + \pi^h(t+1)] .
\]

(19)

Now substitute (13), (12), and (11) into (19) to get

\[
\sum_h c^h(t+1) = \sum_h w^h(t+1) - R^h(t+1) L(t) - x^h(t+1) \sum_h b^h(t) \\
+ \frac{p(t)}{p(t+1)} \left[ \frac{M(t) + N(t)}{p(t)} \right] + q^h(t+1) + \pi^h(t+1) .
\]

(19')

Now subtract (19') for the \(^\wedge\) equilibrium from (19') for the \(^-\) equilibrium and use (14), (15), and (18) to obtain

\[
x^i(t+1) \left[ \hat{B}(t) - \hat{B}(t) \right] = -R^i(t+1) L(t) + \hat{R}^i(t+1) \hat{L}(t) \\
+ \frac{\bar{p}(t)}{\bar{p}(t+1)} \frac{N(t)}{\hat{p}(t)} - \frac{\hat{p}(t)}{\hat{p}(t+1)} \frac{\hat{N}(t)}{\hat{p}(t)} + \frac{M(t)}{\hat{p}(t+1)} - \frac{\hat{M}(t)}{\hat{p}(t+1)} \\
+ \bar{q}^i(t+1) - \hat{q}^i(t+1) + \bar{\pi}^i(t+1) - \hat{\pi}^i(t+1) .
\]

(20)

Equation (20) states that the additional cash flow in period 2 associated with a change in the bank's foreign borrowing, domestic lending or note issuance must be reflected by an equal change in the sum of dividends and market value of the bank.

Furthermore, if we subtract (8) for the \(^\wedge\) equilibrium from (8) for the \(^-\) equilibrium, we have that
\[
\pi(t+1) - \pi_t(t+1) = \bar{B}(t+1) - \hat{B}(t+1) - \bar{L}(t+1) + \hat{L}(t+1) + \frac{\bar{N}(t+1)}{\bar{p}(t+1)}
\]

(21) \[
-\frac{\bar{N}(t+1)}{\bar{p}(t+1)} - \frac{\bar{N}(t)}{\bar{p}(t)} \frac{\bar{p}(t)}{\bar{p}(t+1)} + \frac{\hat{N}(t)}{\hat{p}(t)} \frac{\hat{p}(t)}{\hat{p}^0(t+1)} + \bar{R}_i(t+1) \bar{L}(t) - \hat{R}_i(t+1) \hat{L}(t) - \bar{x}_i(t+1) [\bar{B}(t) - \hat{B}(t)]
\]

Substituting (21) into (20) gives

\[
-\left[\bar{q}_i(t+1) - \bar{q}_i(t+1)\right] = \bar{B}(t+1) - \hat{B}(t+1) - \bar{L}(t+1) + \hat{L}(t+1) + \bar{N}(t+1) + M(t)
\]

(22) \[
+ \frac{\bar{N}(t+1) + M(t)}{\bar{p}(t+1)} - \frac{\hat{N}(t+1) + M(t)}{\hat{p}(t+1)}.
\]

Equation (22) states that any changes in the bank's portfolio must be immediately reflected in the contemporaneous market value of the bank.

Based on these observations, it is straightforward to prove the following result.

**Theorem.** Consider an original set of government and central bank policies 
\[
\{\hat{G}(t), \hat{M}(t), \hat{y}_i(t) - \hat{w}_i(t), \hat{y}_h(t+1) - \hat{w}_h(t+1)\text{ and } \{\hat{\pi}_i(t), \hat{B}(t), \hat{L}(t), \hat{N}(t)\} \text{ which support an equilibrium (the "\^\text{\textquotedbl}" equilibrium). Then there is an alternate equilibrium (the "\-'\text{\textquotedbl}" equilibrium) satisfying (14)–(16), } \bar{G}(t) = \hat{G}(t), \bar{M}(t) = \hat{M}(t), \bar{w}_i(t) = \hat{w}_i(t), \text{ and } \bar{w}_h(t+1) = \hat{w}_h(t+1) \forall h,i,t, \text{ if } \{\pi_i(t), \bar{B}(t), \bar{L}(t), \bar{N}(t), \bar{q}_i(t), \bar{R}_i(t), \bar{p}_i(t)\} \text{ satisfies (18), (20), and (22) } \forall i,t.
\]

Thus, (18), (20), and (22) are necessary and sufficient for "irrelevance" if government policy is held constant.

The theorem just stated gives conditions under which rearrangements of the central bank’s portfolio are irrelevant for the set of equilibrium allocations and relative prices. We now state some more specialized versions of the theorem that allow central bank portfolio arrangements to be irrelevant along other dimensions as well. The first two versions

**Version I (Wallace)**

This version of the theorem states conditions under which changes in the bank’s portfolio also have no effect on real interest rates, share prices of the bank or the price level. To obtain this version, impose that, $\forall i,t$,

\[
\hat{R}_i(t) = \bar{R}_i(t) \\
\hat{q}_i(t) = \bar{q}_i(t) \\
\hat{p}_i(t) = \bar{p}_i(t)
\]

Then from (22),

\[
\frac{\hat{N}(t+1) - \hat{N}(t+1)}{\hat{p}(t+1)} = \bar{L}(t+1) - \hat{L}(t+1) - \left[\hat{B}(t+1) - \hat{B}(t+1)\right]
\]

Equation (23) asserts that note issues must be backed by equal net increases in assets.

Furthermore, from (20),

\[
\hat{\pi}_i(t+1) - \hat{\pi}_i(t+1) = \hat{R}_i(t+1) \left[\bar{L}(t) - \hat{L}(t)\right] \\
- x_i(t+1) \left[\bar{B}(t) - \hat{B}(t)\right] - \left[\frac{\hat{p}(t)}{\hat{p}_i(t+1)}\right] \left[\frac{\hat{N}(t) - \hat{N}(t)}{\hat{p}(t)}\right]
\]

Substituting (23) into (24) gives

\[
\hat{\pi}_i(t+1) - \hat{\pi}_i(t+1) = \left[\hat{R}_i(t+1) - \frac{\hat{p}(t)}{\hat{p}_i(t+1)}\right] \left[\bar{L}(t) - \hat{L}(t)\right] \\
- \left[x_i(t+1) - \frac{\hat{p}(t)}{\hat{p}_i(t+1)}\right] \left[\bar{B}(t) - \hat{B}(t)\right].
\]
Equation (25) requires all excess earnings generated to be rebated to stock holders as dividends; i.e., for the price level and stock price prices to be unaltered, changes in the bank portfolio cannot change the stream of earnings that the bank retains.\(^2\)

Version I of the theorem establishes conditions under which open market rearrangements of the central bank's portfolio have no effects on either real or nominal quantities. Relative to Wallace's (1981) theorem, we note that our version I applies more generally. In particular, Wallace's theorem (which applies to a wholly government-owned central bank) requires that any excess earnings generated by the bank's portfolio rearrangement be rebated to private agents via lump-sum transfers. These are rarely observed in practice. Moreover, Wallace's lump-sum rebates must be made to individuals in a very precise way that preserves an initial income distribution. In our model, these rebates take the form of dividends, which are commonly observed, and the bank need not concern itself with who receives these dividend payments. Thus, in the presence of a privately owned central bank, irrelevance of open market operations requires fewer conditions to be satisfied.

**Version II** (Chamley-Polemachakis)

This version of the theorem allows the price level to be affected by bank portfolio changes, but not the bank's dividend stream. To obtain it, we impose \(\forall i, t,\)

\[
\begin{align*}
\hat{R}_i(t) &= \bar{R}_i(t) \\
\hat{q}_i(t) &= \bar{q}_i(t) \\
\hat{\pi}_i(t) &= \bar{\pi}_i(t)
\end{align*}
\]

Then, from (22),

\[
\frac{\bar{N}(t) + M(t)}{\bar{P}(t)} = \frac{\hat{N}(t) + M(t)}{\hat{P}(t)} + \bar{L}(t) - \hat{L}(t) - [\bar{B}(t) - \hat{B}(t)]
\]

\(^2\)It is easy to show that (4) - (7) are satisfied by the "-" equilibrium.
while (21) becomes

\[
\frac{\bar{N}(t)}{\bar{p}(t)} - \frac{\bar{p}(t)}{\bar{p}(t+1)} = \frac{\bar{N}(t)}{\bar{p}(t)} \frac{\hat{p}(t)}{\hat{p}(t+1)} + \hat{R}_i(t+1) \left[ \overline{L}(t) - \hat{L}(t) \right] - \chi(t+1) \left[ \overline{B}(t) - \hat{B}(t) \right]
\]

(27)

\[
+ \left[ \overline{B}(t+1) - \hat{B}(t+1) \right] - \left[ \overline{L}(t+1) - \hat{L}(t+1) \right] + \frac{\bar{N}(t+1)}{\bar{p}(t+1)} - \frac{\hat{N}(t+1)}{\hat{p}(t+1)}
\]

\[
= \frac{\hat{N}(t)}{\hat{p}(t)} \frac{\hat{p}(t)}{\hat{p}(t+1)} + \hat{R}_i(t+1) \left[ \overline{L}(t) - \hat{L}(t) \right] - \chi(t+1) \left[ \overline{B}(t) - \hat{B}(t) \right]
\]

[The latter equality follows from (26).] Now substitute (26) into (27) to get

\[
\frac{\bar{p}(t)}{\bar{p}(t+1)} = \frac{\hat{p}(t)}{\hat{p}(t+1)} \left( \frac{\hat{N}(t) + M(t)}{\hat{p}(t)} \right)
\]

\[
\left[ \overline{L}(t) - \hat{L}(t) \right] + \frac{\hat{N}(t) + M(t)}{\hat{p}(t)} \left[ \overline{L}(t) - \hat{L}(t) - \overline{B}(t) + \hat{B}(t) \right]
\]

(28)

\[
+ \hat{R}_i(t+1) \left[ \overline{L}(t) - \hat{L}(t) \right] + \frac{\hat{N}(t) + M(t)}{\hat{p}(t)} \left[ \overline{L}(t) - \hat{L}(t) - \overline{B}(t) + \hat{B}(t) \right]
\]

\[
- \chi(t+1) \left[ \overline{B}(t) - \hat{B}(t) \right] \left[ \overline{L}(t) - \hat{L}(t) - \overline{B}(t) + \hat{B}(t) \right]
\]

Equation (28) implies that the real return on bank notes in the (−) equilibrium must be a weighted average of the previous real returns on notes, loans, and foreign investments.

**Version III** (Highfield – O'Hara – Smith)

Interestingly, changes in the portfolio of a privately owned central bank can be irrelevant for allocations, relative prices, interest rates, and the price level, even if the bank directly rebates none of the excess earnings generated by the portfolio change. Chamley and Polemarchakis' theorem shows how rebates of such earnings can occur indirectly via changes
in the distribution of returns on real balances. However, even if the stochastic process

governing the price level is unchanged, when the central bank has actively traded shares such
rebates can effectively occur through changes in the price of these shares. We now illustrate
this possibility.

Imposing, \( \forall i, t, \)

\[
\hat{R}_i(t) = \bar{R}_i(t) \\
\hat{\pi}_i(t) = \bar{\pi}_i(t) \\
\hat{p}_i(t) = \bar{p}_i(t)
\]

Then (22) becomes

\[
\bar{q}_i(t+1) - \hat{q}_i(t+1) = L(t+1) - \hat{L}(t+1) - \left[ B(t+1) - \hat{B}(t+1) \right] - \left[ \frac{N(t+1) - \hat{N}(t+1)}{\hat{p}(t+1)} \right]
\]

Equation (29) asserts that the change in market value equals the net change in the value of
assets – liabilities.

From (21),

\[
L(t+1) - \hat{L}(t+1) - \left[ B(t+1) - \hat{B}(t+1) \right] - \left[ \frac{N(t+1) - \hat{N}(t+1)}{\hat{p}(t+1)} \right] = - \left[ \frac{N(t) - \hat{N}(t)}{\hat{p}(t)} \right] \left[ \frac{\hat{p}(t)}{\hat{p}(t+1)} \right] + \hat{R}_i(t+1) \left[ L(t) - \hat{L}(t) \right] - x_i(t+1) \left[ B(t) - \hat{B}(t) \right]
\]

Substituting (29) into (30) gives

\[
\bar{q}_i(t+1) - \hat{q}_i(t+1) = \hat{R}_i(t+1) \left[ L(t) - \hat{L}(t) \right] - x_i(t+1) \left[ B(t) - \hat{B}(t) \right] - \left[ \frac{\hat{p}(t)}{\hat{p}(t+1)} \right] \left[ \frac{N(t) - \hat{N}(t)}{\hat{p}(t)} \right]
\]
Equation (31) asserts that the change in the share price equals the change in earnings on assets less returns paid on liabilities.\(^3\)

4. **Monetary Irrelevance and the Second Bank of the United States**

The model in the previous section demonstrates the conditions under which open market operations will be irrelevant. As is customary in such models, irrelevance obtains because the monetary authority (or government) rebates any "excess earnings" to the public. These rebates essentially offset any changes in the earnings of the central bank which arise from alternative portfolio configurations, and hence separate profit effects from those resulting from portfolio rearrangements.

In the Wallace (1981) model, these rebates require a careful arrangement of lump sum transfers from the government to the public, while Chamley–Polemarchakis (1984) accomplish such transfers via changes in the inflation tax. Certainly the former is rarely observed, and the latter would be difficult to confirm in practice. By contrast, in our model, the earnings arising from open market operations translate into profits for the bank, which are simply rebated to their stockholders in the form of either dividends, or changes in the bank's share values. Such transfers preserve the "rebate" property in the more natural, and common, context of standard

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\(^3\)Obviously, (4) - (6) are satisfied by the "-" equilibrium. Also, (7) is satisfied iff

\[
\sum_i \hat{s}_i(t) [\hat{q}_i(t+1) - \hat{q}_i(t+1)] = \bar{q}(t) - \hat{q}(t)
\]

holds. From (31),

\[
\sum_i \hat{s}_i(t) [\hat{q}_i(t+1) - \hat{q}_i(t+1)] = \left[\bar{L}(t) - \hat{L}(t)\right] \sum_i \hat{s}_i(t) \hat{R}_i(t+1) \\
- \left[\bar{B}(t) - \hat{B}(t)\right] \sum_i \hat{s}_i(t) x_i(t+1) - \left[\bar{N}(t) - \hat{N}(t)\right] \sum_i \hat{s}_i(t) \hat{\tilde{p}}(t) \hat{\tilde{p}}(t+1) \\
= \bar{L}(t) - \hat{L}(t) \left[\bar{B}(t) - \hat{B}(t)\right] - \frac{\bar{N}(t) - \hat{N}(t)}{\hat{\tilde{p}}(t)} \hat{\tilde{q}}(t) - \hat{q}(t),
\]

where the latter equality is (29). Thus, (7) holds as well.
corporate behavior. Perhaps more important, such a dividend-paying central bank corresponds to the structure of the Second Bank of the United States, which operated as the country's central bank in the early 19th century.

In this section we empirically examine the behavior of the Second Bank of the United States and the U.S. economy, and contrast this behavior with the results of the previous section. Implicitly we also contrast this behavior with the implications of models that have a more "monetarist" flavor. While the Second Bank operated as a central bank from 1816 until 1836, both the beginning and end of its existence were characterized by great organizational instability. Beginning with Nicholas Biddle's ascendancy to the presidency in 1823, however, the Bank enjoyed a stable period of operations until 1832 or 1833. (The Federal government withdrew its deposits from the bank in 1833.) We thus focus on this period, in which many observers credit the Bank with exhibiting at least some of the behavior typically associated with modern central banking.

Because the structure and operations of the Bank do differ from those of either modern central—or even commercial—banks, we begin with a brief overview of the Bank. [For a more detailed but still concise description see Highfield, O'Hara, and Wood (1991). More expansive descriptions appear in Catteral (1903), Dewey (1910), and Smith (1953).] We then proceed to a formal empirical analysis of the effects of rearrangements in the bank's balance sheet. Because of data limitations, we then supplement the latter with some additional, but less formal, empirical evidence on monetary irrelevance.

A. An Overview of the Second Bank

The Second Bank was a quasi-public institution chartered by Congress to act as a fiscal agent for the U.S. government. The Bank was the largest corporation in the U.S., was privately owned, and its shares were actively traded in financial markets. As the nation's

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4It is also often asserted that the Bank was chartered "in the expectation that it would be able to force the state banks to resume specie payments" (Schur, 1960, p.119) or specie convertibility of their notes, which had been suspended during the War of 1812. See also Dewey (1910, p.158) or Van Fenstemaker and Filer (1986, p.29).
largest bank, it operated branches in 18 cities, and in 1830 the bank was responsible for about 20% of total bank loans and bank notes in circulation, and it held about 33% of total bank deposits and bank specie reserves. [Smith (1953), p. 234]. In its role as fiscal agent, the Bank collected all taxes and disbursed all transfer payments. The Bank also participated in foreign financial markets, both borrowing and lending as part of its operations.

The Bank issued its own notes, usually referred to as circulation. These notes, along with those issued by state banks, formed the bulk of the money supply. The notes of the Bank "circulated widely," [Smith (1953), p. 48] and were receivable by the government for duties and taxes.

Each branch of the Bank issued its own notes, and after 1818 notes were redeemable only at the branch of issue. This permitted small discounts (in the range of 0.25 to 0.5%) to emerge on the notes of some branches. However, the Bank was obliged to take the notes of any of its branches at face value from the government, which limited the scope for more substantial discounts. In general the Bank's "notes were frequently preferred to gold" [Smith (1953), p. 236-7], and even in major cities, notes were used far more heavily than bank deposits as a means of payment. [Smith (1953), p.62].

The largest component of the Bank's assets were loans, which included both domestic discounts and bills of exchange, and foreign lending. The Bank adhered largely to a "real bills doctrine" policy in domestic lending; loans were made only "for short dates, and only on good commercial paper" [Catteral (1903), p. 98-9]. In addition, the Bank's directors generally required maturing bills to be paid off, rather than rolled over. [Knodell (1991), p. 13]. The interest rate charged on loans was typically the legal limit of 6%, with some exceptions in 1830-31. [Smith (1953), p. 56]. Finally, the Bank also held large positions in specie and state bank notes. These, along with the deposits the Bank held at state banks, constituted the

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6It is easy to show that the results of section 3 continue to hold if $\hat{R}_t(1)$ is fixed by government policy, as it was in this case.
Bank's reserves. The Bank paid dividends semi-annually, with dividends set by the Bank's directors. After 1819 the annual dividend on Bank stock was about 7% [Smith (1953), p. 40].

Figure 1 illustrates the size and composition of the Bank's assets. Over the period depicted, the Bank's assets grew from 50 million to approximately 80 million dollars. As the figure indicates, this asset growth largely occurred in domestic lending, although speculative holding was also growing but variable over the period. Foreign lending was never a large component of the Bank's operations, and was in fact totally absent over large periods of time.

The Bank's liabilities are depicted in Figure 2. Perhaps most noteworthy is the large, and nearly constant, equity component of the Bank's liabilities. Unlike modern commercial banks, the Second Bank's funding arose primarily from its capital, with deposits playing a secondary role. As the graph indicates, however, the Bank did increase its leverage ratio over the period, accommodating its increasing asset size with increases in both its note issues and its deposits. Foreign borrowing did not play an important funding role for the Bank.

The equity position of the Bank is of particular importance for our analysis. The bank's equity capital arose from an initial subscription (or offering) of 350,000 shares. From available balance sheet data, it does not appear that the Bank issued any additional shares over our sample period. The Bank did earn profits over this period, however, so that the Bank had the ability to increase equity from retained earnings. The constancy of the bank's equity component suggests that such a policy was not generally pursued. As Figure 3 illustrates, the Bank's profits were largely returned to stockholders via dividend payments. This rebating policy accords well with the irrelevancy conditions stated in Section 3.

At times during our sample period, and particularly in the later years, it does appear that some of the Bank's profits were retained. Our model allows for such retention, but requires that the retained earnings be reflected in the Bank's market value. Figure 4 depicts the relationship between the Bank's market value and its net equity position. The Bank's market value is a monthly figure derived by using 350,000 as the number of outstanding shares and multiplying by stock price quotations collected from the mid-month issues of the New York
Shipping and Commercial List for our sample period. The Bank's net equity value is a similar monthly figure, and was derived from the monthly Consolidated Balance Sheets and the "General Statements of the Bank of the United States."

Version III of our irrelevance theorem requires that, when Bank dividend payments are held constant (as they evidently were from 1829-33), changes in the Bank's balance sheet that affect profitability be reflected in changes in the market value of Bank shares. Figure 4 indicates such a correspondence between the net equity and the market value of the Bank. Together figures 3 and 4 suggest that "excess earnings" generated by rearrangements of the Second Bank's portfolio were rebated (in some form) to stockholders, as required for the irrelevance result. We, therefore, proceed to investigate the empirical consequences of such rearrangements.

B. The Second Bank and the Economy: 1823-1832

Figure 5 depicts the monthly circulation figures for the Second Bank, as stated on its balance sheets. As the figure illustrates, the Bank's note issues expanded dramatically over the period; from slightly over $4 million in 1823 to over $21 million in 1831. Figure 6 depicts the Bank's net foreign borrowing. While not large in magnitude, this borrowing was evidently quite variable, and was generally increasing over our time period. As is apparent from figure 1, the general expansion in the Bank's liabilities financed a sizable increase in the Bank's domestic lending. Thus, the Bank was engaged in what might today be characterized as expansionary monetary policy.  

This policy did not have the effects on currency values that one typically associates with monetary expansion, however. Figure 7 depicts the behavior of several regional wholesale price indices, which were generally declining over the period. In the face of apparently very expansionary actions by the central bank, "beginning in 1826 and lasting through 1832 the

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7The Bank itself did not necessarily perceive things this way. For instance Biddle did not view bank note expansions as inflationary [Smith (1953), p. 12], and expressed at various times some adherence to the real bills doctrine. (Timberlake (1978), p. 40).
prices of American produced goods were extraordinarily stable ... Probably never since 1789
had the United States had a dollar which was sounder or more stable ..." [Smith (1953),
pi. 76]. This stability is further reflected in Figure 8, which depicts the movement of the
dollar-pound exchange rate. Evidently this rate was little different in 1832 than in 1823.

Such behavior is very much consistent with the irrelevance propositions described in
section 3, especially versions I and III. Those propositions describe conditions (which, as we
have argued, seem to be satisfied) under which rearrangements of the Second Bank's portfolio
would have no consequences for the price level or currency values. This is exactly what we
observe in the data. We now pursue this more formally, and then return to argue that this
observed behavior is inconsistent with standard monetarist assertions, as embodied for instance
in Friedman (1956), Friedman and Schwartz (1963), or the analyses of Lucas (1980, 1984).

C. **A VAR Analysis**

We now formally analyze our irrelevance results using a five variable vector
autoregression (VAR) with a constant term. Students of this period of economic history are
well aware of the limited data available. For example, there are no good GNP or interest rate
series, and many extant series (such as the total money supply) are only annual figures. Such
data problems generally preclude analyses employing standard techniques. Our analysis uses
monthly data of the time. While more complete data would certainly be desirable, our empirical
work here described does provide a cogent and innovative analysis of this period of the
American economy.

All five of the variables in the VAR are suggested by our model. The first three are
quantities representing the composition of the Second Bank's balance sheet. They are the
Bank's net private domestic lending (i.e., private loans less deposits), denoted NPDL, the
Bank's notes in circulation (CIRC) and the Bank's total (net) foreign borrowing (TFB). All
three of these variables are derived from the monthly consolidated financial statements of the

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8See also Thorpe (1926).
Bank.\textsuperscript{9} We are particularly interested in how, or whether, exogenous changes in these variables affect the general price level of the economy and the market value of the Bank itself. The Bank's market value (MVAL) is represented by the number of outstanding Bank shares multiplied by the market price of a share as quoted in the Mid Month issue of \textit{The New York Shipping and Commercial List}, and the general price level is represented by a wholesale price index (WP). For WP we employed several different monthly price indices (see Figure 7): we orient most of our discussion around the New York price series of Smith and Cole (1935, p. 138). The consequences of considering other available price series are discussed below.

A VAR can be viewed as an unrestricted reduced form corresponding to a quite general structural model. As has been well documented by Cooley and Leroy (1985), the ability to provide meaningful economic interpretations to a VAR depends on being able to identify unambiguously the dynamic responses of these variables to structural disturbances. In particular, to capture the response of a variable \(j\) to an independent change in another variable \(i\), we must be able to rule out a contemporaneous feedback from variable \(j\) to variable \(i\). The simplest assumption we can make to satisfy these requirements is that our VAR is the unrestricted reduced form for a recursive triangular model (a Wold causal chain). This assumption allows us to compute the structural impulse response function as the product of the VAR impulse reaction function and the unique Cholesky factorization of the variance-covariance matrix of the VAR disturbances.

In our model, the exact nature of this factorization is not immediately obvious since simple correlation tests do not allow us to rule out any obvious contemporaneous linkages. The nature of banking and the economy at this time, however, suggest that a natural ordering would be NPDL, CIRC, TFB, MVAL, WP. Such a relation views credit (which in our case is net private domestic lending) as arising to meet the needs of commerce. This "real bills" view [which was certainly held by Biddle; see Timberlake (1978), p. 40] suggests that

circulation and foreign borrowing then arise to fund lending, with changes in stock prices and wholesale prices following. We also ran our VAR with other causal orderings, but obtained little difference in the results.

Estimation and interpretation of our VAR requires a number of other issues to be addressed as well. Many, for example Doan, Litterman, and Sims (1984), and Sims (1986, 1987) have found that imposition of prior information on the coefficients, through the use of Bayesian prior distributions, is extremely useful both in increasing the forecast accuracy of the VAR and in achieving more precise estimates of the impulse response weights. The Bayesian approach employed here is similar to that employed by Litterman [1986] and Highfield, O'Hara, and Wood [1991] and involves setting the prior mean of the coefficient on the first lag of a variable in its own equation equal to one, and the prior means on all other lags and variables from all other equations equal to zero. The standard deviations of the prior distributions on the lag coefficients are determined up to an unknown scale factor by the formula

\[ S_{ijk} = \tau k^{-\lambda} / S_j \]

where \( S_{ijk} \) is the standard deviation of the coefficient on the \( k^{th} \) lag of the \( j^{th} \) variable in the \( i^{th} \) equation and \( S_j \) is the standard error of a univariate autoregression on variable \( i \) (included to adjust for measurement scale differences). \( \tau \) represents the "overall tightness" of the prior distribution about its mean and \( \lambda \) is used to control the extent to which the prior distributions become tighter around zero for longer lags. Finally, the prior distribution on the constant term is taken to be diffuse. The prior specification for each equation in the VAR is thus a random walk with unknown drift.

In our model, testing of alternative specifications revealed that a decay parameter of 1.5, and a tightness parameter of .2 (relative tightness of 1.0) provided the best fit. A related issue concerns the appropriate lag structure for our VAR. We tested various lag specifications
and found a 12-lag structure to be most appropriate. Experimentation with both seasonally adjusted and non-seasonally adjusted data also led to little difference in results, so our analysis reports only the results for non-seasonally adjusted variables.

The discussion above goes to the question of providing meaningful economic interpretation to the impulse responses that are computed from the VAR coefficients and the variance-covariance matrix of the VAR residuals. Runkle [1987] has pointed out that, quite apart from the question of economic interpretation, there has been little attention given to the econometric significance of these impulse response weights and the precision with which they can be estimated. Rather than take our computed impulse response weights at face value, we employ Monte Carlo methods to form Bayesian posterior distributions for the weights and draw our inferences from these distributions.

The impulse responses from our estimated VAR are presented in Figure 9. The impulse response graphs in each row display the median response of one variable at t+i, i=1,..., 47, to an independent shock to each variable in the system at t. The error bounds are the fifth and ninety-fifth percentiles of the posterior distribution of the impulse response function, and are derived from 1001 Monte Carlo draws from this distribution. The impulse response function provides a means to evaluate the impact of the bank's portfolio decisions (i.e., its open market operations) on the economy. For details of this method, see Highfield, O'Hara and Wood (1991).

Perhaps the first thing to note is that the impact of the bank's lending behavior (NPDL) on the other variables is negligible. A shock to lending at time 0 has a small, negative lagged effect on circulation, but this effect is not distinguishable from zero. Similarly, there is a small negative but insignificant effect on total foreign borrowing. Insignificant as these effects are, they contrast with the virtually indiscernible effects of lending on market value and the price level. The lack of price level effects resulting from a change in net lending is, of course, directly supportive of our irrelevance result. The lack of any significant consequences for the

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10 More discussion of these specification issues can be found in Highfield, O'Hara, and Wood [1991].
Bank's market value are similarly supportive if any resulting "excess earnings" are rebated via changes in dividends.\footnote{Even if the dividend stream is left unchanged, the Bank's market value can change over time in a complicated way which depends on returns on various assets, and the entire history of the innovations. Thus, an absence of any significant effect on the market value variable is consistent with the analysis of section 3.}

The impulse responses resulting from a circulation shock are somewhat more interesting. A positive shock to circulation results in an (initially) significant increase in net lending, a permanently (and significantly) higher note circulation, and a significant short term decline in foreign borrowing. Interestingly, the Bank's market value rises significantly. This is as would be expected on the basis of equation (31): if an increase in circulation results in increased lending and reduced foreign borrowing in a way that enhances the profitability of Bank operations, the Bank's market value should rise (if dividend payments are held fixed). This is what we observe. We also observe no statistically significant effect on the price level, in accordance with versions I and III of our theorem. For future reference, we note that the point estimates suggest that the price level falls following an increase in the Bank's circulation.

The impulse response patterns to an innovation in total foreign borrowing also indicate a lack of price level consequences, although such an innovation does have a short-run expansionary effect on Bank lending.

A positive innovation in the wholesale price index does have a significant positive effect on the Bank's net lending position. While this has no direct bearing on our analysis, it is of some interest. A prominent early 19th century theory of business cycles held that cycles were expectations-driven: optimism led to speculation and rising prices which were supported by expansions of bank lending.\footnote{For heuristic discussions or descriptions of these theories, see White (1984) or Schwartz (1987). Schlesinger (1953), Hammond (1957), White (1984), and Schwartz (1987) discuss the political and policy-related importance of such theories. Smith (1988b) presents a formal model consistent with the relevant heuristic arguments.} The eventual return of pessimism would then reverse the expansion. Our lending results suggest that the Second Bank did increase lending in response to higher prices, as this theory would require. Finally, a rise in prices reduces the Bank's market value, as we would expect for a net nominal creditor.
All of these results have been obtained using the wholesale price index for New York City compiled by Smith and Cole (1935). However, as indicated in Figure 7, there are four other price indices available for this period. While all of the indices exhibit generally similar qualitative behavior, they do not behave identically. Therefore, we ran our VAR separately for each of the price series. Using data for New York, Philadelphia, and Charleston (eastern seaboard cities) we obtained virtually identical results, while the New Orleans (and to a lesser extent, the Cincinnati) data did result in some differences, which we now describe.

Figure 10 presents the impulse response functions obtained using the New Orleans wholesale price series. Evidently, changes in the net lending position of the Bank continue to have no price level effects. Interestingly, increases in the Bank's circulation appear to reduce the price level in a way that is now marginally significant. While this is not consistent with our irrelevance theorem, it is a result even less consistent with more monetarist models. We return to a discussion of what might account for this finding in the conclusion. Finally, a price level shock now has few consequences. This is not particularly surprising, since the Bank probably would have responded most strongly to events in financial centers like Philadelphia and New York.

One other specification issue concerns the formulation of the VAR in nominal rather than real terms. More specifically, the irrelevance proposition in section 3 concerned the real net lending and foreign borrowing positions of the central bank, and the real value of its shares. (Of course if \( \tilde{p}(t) = \hat{p}(t) \) \( \forall i, t \), it is of no consequence whether we focus on real or nominal changes.) As a check on our results, we also computed responses to innovations in real net private lending and total foreign borrowing by the Bank, as well as the Bank's real market value. These response functions are depicted in Figure 11. What this figure represents is the consequence of a dual innovation in the price level and one other variable: for instance a positive shock to NPDLC coupled with a negative shock to prices, scaled to incorporate the correlation of the two variables. (Circulation is left as a nominal variable.) The impulse response functions in Figure 11 are substantially the same as those in Figure 9, suggesting that
a specification in nominal as opposed to real terms does not affect our results. Of course, this is not especially surprising in view of the small price level effects already observed.

In summary, the picture that emerges from the VAR analysis is quite supportive of the irrelevance proposition in section 3. Changes in the central bank portfolio have little impact on prices, and the impact they do have (focusing on point estimates) seem to have the "wrong sign" from the standpoint of more standard monetarist models. Some central bank portfolio rearrangements do affect the Bank's market value, but they seem to do so in a way that is consistent with version III of our irrelevance result.

While our results are generally consistent with the implications of a Modigliani-Miller Theorem for open market operations, we do find some regional differences within the U.S. economy. In particular, we found that prices in New Orleans responded to central bank actions somewhat more strongly (but again, with a surprising sign) than did prices in New York, Philadelphia, or Charleston. Of course, during the time period we consider New Orleans and Cincinnati were "western" cities, and their slightly different behavior may simply be explained by the less developed state of western financial markets. This is a point we return to below.

5. **Alternative Explanations**

We have shown that the data from the period 1823-32 are consistent with the proposition that rearrangements of the central bank's portfolio can be irrelevant for equilibrium allocations, relative prices, and nominal price levels. Of course, there are several alternative explanations for this observation, some of which we now consider. Data limitations (specifically, a lack of data at monthly or quarterly intervals) precludes a formal investigation of these explanations, but for all of them annual data can be brought to bear.

A. **Offsetting Changes in Other Components of the Money Supply**

One possible explanation for our findings is that an increase (or decrease) in the quantity of the Second Bank's liabilities simply led to an offsetting change in other components
of the money supply (which we do not observe on a monthly basis). Then changes in the Second Bank's balance sheet would have had no consequences for the (appropriately measured) money supply, which would explain why we observe no price level effects. There are several candidates for such offsetting changes; we consider each in turn.

1. **Specie Flows**

During the period we consider, the U.S. was a small open economy, and bank liabilities were largely convertible into specie. Thus, one might suspect that, with a pegged exchange rate, the U.S. money supply was determined by the necessity of maintaining this rate, and any changes in the Second Bank's balance sheet would have been largely offset by specie flows. However, evidence suggests that this was not the case.

Table 1 presents Temin's (1969, p. 81) estimates of specie inflows over the relevant time period. Apparently, during this period of rapid expansion in the Second Bank's liabilities, the U.S. was importing rather than exporting specie.\(^\text{13}\) Such behavior is also corroborated by Hepburn's (1924, p. 130) estimates, which we present for comparison. Thus, specie flows were not offsetting the expansion of the Second Bank's liabilities, but rather served to augment the (broadly defined) money supply.\(^\text{14, 15}\)

Finally, we consider one last possibility in this regard: perhaps the expansion in Second Bank liabilities simply displaced specie in circulation, with the specie flowing into banks rather than abroad. Again, this would result in no net change in the money supply, and

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\(^{13}\)Smith (1985a, b, 1988) examines money and price movements using colonial data. He also finds that, in many instances, the domestic paper money stock and the specie stock moved in the same, rather than in opposite directions.

\(^{14}\)It should be noted that most of the specie stock in the U.S. resided in banks, and hence constituted reserves rather than circulation. Temin (1969, p. 188) notes that, "according to the Comptroller of the Currency and the Secretary of the Treasury, there was only about seven or eight million dollars of specie outside banks in the first half of the 1830's." He continues, "was the United States almost exclusively using paper money at that time? Informed contemporaries certainly thought so ... The estimates in [Temin's] Table A.2 are in firm agreement with the views of informed contemporaries" (pp. 188-189). Given the small amount of specie that should be counted as part of the money supply, offsetting specie flows were a logical impossibility.

\(^{15}\)For a further discussion of the failure of the "price-specie flow mechanism" to operate during the period under consideration, see Davis and Hughes (1960).
account for the observed absence of price level effects. But if this were the case, the reserve-deposit ratio should have increased substantially over the period. This did not occur, as is apparent from Table 2.

2. **State Bank Liabilities**

Another possibility is that changes in the liabilities of the Second Bank simply displaced the liabilities of other banks, resulting in no net change in the money supply. This also turns out not to be the case. Table 2 reports the total money supply over our period, as reported by Temin (1969), inclusive of all bank notes, deposits, and specie outside banks. Evidently, the money supply, defined in this manner, expanded rapidly along with the expansion in Second Bank liabilities.

Contemporaries would largely have felt that deposits were not a part of the money supply. Since Temin does not report notes and deposits separately, for simplicity we report Hepburn's (1924) estimates of state and U.S. bank note circulation, plus specie stock. Hepburn's total circulation figures, which include the entire specie stock, indicate that both total circulation and the stock of bank notes approximately doubled over the period 1823-32, with the specie stock and the stock of bank notes growing at the same average rate. Hepburn also estimates a 51% increase in per capita circulation over the sample. Thus, all evidence indicates that the increase observed in Second Bank liabilities did not simply displace the liabilities of other banks. This is as expected, since an expansion in central bank liabilities would expand the reserves of other banks, and hence permit them to expand loans as well. And indeed, Smith (1953, p. 234) indicates that loans by state banks and the Second Bank generally moved in concert.

3. **Federal Debt**

A last point of note is that there was a substantial reduction in outstanding federal government debt from 1816 to 1835, as the federal government ran sustained surpluses
[especially over the period 1825-1836; see Hughes and Rosenberg (1963), p. 488]. This might have "released" private saving to absorb the increase in Second Bank liabilities. We now offer two comments on this possibility.

First, over much of our period (especially in the late 1820s and early 1830s), expansions in outstanding state debt "probably ... balanced the return flow of United States funded debt ..." [Smith (1953), p. 145]. Thus, movements in the outstanding total government debt could not easily account for our observations, which are probably most dramatic in the late 1820s and early 1830s. Second, even if they did not, a contraction in government debt along with an expansion of central bank liabilities would constitute a de facto open market operation, whether undertaken intentionally or not. Our results assert that this should not have price level consequences, while monetarist models [see Lucas (1984) for a specific proposition] would imply that it should. Thus, even if there were a reduction in total debt, this would not vitiate our results.

B. Stable Money Demand Functions

An alternative explanation of our findings is as follows: changes in the money supply may have represented passive responses (as they might if the Bank was following a real bills regime, or if the U.S. money supply was endogenous) to changes in some other economic conditions, possibly changes in interest rates or real activity. Then the empirical observations we have examined could be consistent with the underlying existence of a stable money demand function of a monetarist variety.

While monthly or quarterly observations on interest rates and real activity are lacking, (and hence preclude a formal analysis of this point) sufficient annual data exist to evaluate this argument. Table 4 presents Temin's (1969) annual money supply series, and Berry's (1988) annual series on real GNP and the GNP deflator. Evidently, from 1823 to 1832, there was roughly an 86% percent increase in real balances. This fairly large increase in the real money
supply coincided with a period of substantial real growth;\textsuperscript{16} according to Berry (1988) real GNP grew almost 52\% over this same period.

Table 4 also presents velocity figures. Over the period in question, velocity fell at an average annual rate of just over 2\%.\textsuperscript{17} This is twice as large as the 1\% average annual decline in velocity that Friedman and Schwartz (1963) found over the period 1869-1960. Moreover, velocity was also quite variable over our period. In the 91 year sample presented by Friedman and Schwartz, there are only 13 years in which velocity changed by more than 10\% (in absolute value), and of these, all but two involve either wartime episodes or years in which there were bank panics.

By contrast, from 1823 to 1832, three of ten years saw a change in velocity (with an absolute value) in excess of 10\%, with the largest change in velocity (between 1831 and 1832) exceeding 21\%. Since the largest year-to-year velocity change in the Friedman-Schwartz sample is 17\%, this suggests that velocity in this period was highly variable by historical standards. Interestingly, the period we examine was a relatively stable one, both in terms of price level and real GNP behavior.\textsuperscript{18} There were no wars or bank panics,\textsuperscript{19} and no periods of bank suspension of convertibility.

What could account for the large observed velocity movements? Figure 12 depicts movements in long-term nominal interest rates, which bear out Temin's (1969, p. 83) assertion that "the long-term rate scarcely moved at all."\textsuperscript{20} Thus, interest rate movements seem an unlikely candidate to explain the large observed movements in velocity. In view of the

\textsuperscript{16}See also Smith (1953), David (1967), and Engerman and Gallman (1982).

\textsuperscript{17}While this is a sizable long-term decline in velocity, it is not unusually large in U.S. historical experience. As Friedman and Schwartz (1963) note, velocity declined at an average rate of 3\% per year from 1879-97.

\textsuperscript{18}On the former point see Smith (1953), on the latter see Engerman and Gallman (1982).

\textsuperscript{19}There was a bank panic in England in 1825, but this was avoided in the U.S. Also, 1825 is not one of the years with a relatively large velocity change.

\textsuperscript{20}According to Temin (1969, p. 83), "the short-term interest rate fluctuated wildly," but it reached extreme values "only in or near financial panics," of which there are none in our sample.
apparently small movements in the price level, changes in the expected inflation rate are also an unlikely candidate to explain such velocity behavior.

A last possibility is measurement error. Certainly there is good reason to be cautious of the data in Table 4. However, Temin's money supply figures are largely comparable to the other money stock series (such as Hepburn's given in Table 3). Similarly, Berry's GNP deflator moves much like the Smith-Cole (1935) New York wholesale price index, which in turn behaves much like the other regional price indices presented in Figure 7. And, for reasons discussed by Engerman and Gallman (1982, p. 16), Berry's real GNP series is much more likely to have overstated than to have understated real growth over the period. Thus, the observed average annual decline in velocity of 2% is probably a conservative estimate, and it seems safe to conclude that there were very large velocity changes. These occurred in the face of stable (long-term) nominal interest rates, and of real per capita income growth which, according to Engerman and Gallman (1982, p. 17) occurred "at a rate which came to seem low by the standards of subsequent experience." This variability in velocity seems difficult to reconcile with the presence of a stable underlying money demand function.21

6. Conclusions

Wallace (1981) and Chamley and Polemarchakis (1984) develop Modigliani–Miller Theorems for open market operations. These theorems describe when rearrangements of the balance sheets of (wholly government owned) central banks will be irrelevant. We have extended their results to the case of a privately owned central bank whose shares are actively traded. This is of importance for two reasons: privately owned central banks are commonly observed historically, and the conditions for irrelevance of open market operations are far more

21It might be noted that the largest velocity movements in our sample occurred between 1830-31. It might be proposed that this was a point of imminent instability, since Jackson vetoed renewal of the Second Bank charter in 1832. However, it was far from apparent that this would happen in 1831, or even if it had been, that this meant the effective end of the Second Bank. The veto was a major political issue in the 1832 election. Moreover, an event like the veto, which might be taken as a signal of a period of reduced banking system stability, is the kind of occurrence which usually triggers a rise in velocity (as in 1832) rather than a decline. Thus, we doubt that the large decline in velocity in 1831 was in result of anticipation of the Bank War.
likely to be satisfied for such banks. In particular, in order for open market operations to be irrelevant, any excess earnings that they generate must be rebated to the public. For a privately owned central bank this can be accomplished simply by paying dividends, or by changes in the price of the bank’s shares. For government owned central banks such rebates must occur through non-distorting taxation, which requires the rebates to be carefully constructed. Such rebates tend to be observed only rarely in practice. Thus, historical periods with privately owned central banks represent especially good opportunities for empirical investigation of Modigliani-Miller Theorems.

We have undertaken such an investigation, using observations involving the Second Bank of the U.S.. We have found substantial empirical support for the predictions of a Modigliani-Miller Theorem for open market operations. We have also argued that these findings would be difficult to reconcile with more monetarist models, like that of Lucas (1984), which predict that open market operations should be equivalent to adding zeros to the existing money supply.

To the extent that we observe any departures from the predictions of the Modigliani–Miller Theorem, these involved an expansion of the money supply causing a decline in the price level (and this was for New Orleans only).\footnote{Smith (1985a) noted that this is often observed in data from the American colonial period.} What might account for such a finding? While our answer must remain conjectural, one possibility is that an increase in intermediation (associated with an expansion of Second Bank liabilities) reduced transactions costs in the economy. The resulting resource saving would constitute a deflationary force, as pointed out by Bryant and Wallace (1979). Such an explanation seems particularly plausible for the relatively undeveloped financial markets of the south and west. A further investigation of this possibility would be an interesting topic for future research.

We conclude by suggesting other applications of our analysis in the context of the Second Bank, and by offering a caveat. First, much of the literature on the Second Bank concerns the causes and consequences of the "Bank War," which began with Jackson's veto of
the renewal of the Bank's charter, and continued with a major contraction of credit by the Bank often characterized as a "scorched earth policy." Economic analyses of the Bank War, however, have found that it was relatively unimportant economically [Temin (1969)]. Meerman (1963) noted that the price level affects of "Biddle's contraction" were mild and "paradoxical" in their timing, and that the contraction had little effect on the total money stock. These findings are entirely consistent with our arguments: the rearrangements of the Bank's portfolio involved in "Biddle's Contraction" would have been economically irrelevant. Furthermore, Temin (1969, p. 91) argued that the Bank's policies were largely irrelevant to the sustained inflation of the mid 1830s: this would again be consistent with our analysis. Finally, Sushka (1976) estimated money demand functions over the period 1823-59, and found them to be characterized by substantial instability. This is exactly what one would predict in a period of significant central bank expansions and contractions on the basis of a Modigliani-Miller Theorem.

The demise of the Second Bank ushered in the era of free banking. During this episode private banks could issue notes backed by various state bonds. Much recent research has focused on the issue of whether the value of notes issued by these banks essentially reflected the value of the banks' portfolios [Rolnick and Weber (1983, 1984, 1988); Gorton (1992)]. An affirmative finding [such as provided by Rolnick and Weber] supports the existence of a relationship between the value of money and the value of its "backing" which is in the spirit of Modigliani-Miller theorems. A more explicit attempt to connect our results with those of the literature on free banking would be an interesting topic for further investigation.

As a concluding caveat, we wish to emphasize what our analysis does not assert. While we have argued that portfolio rearrangements by the Bank were (approximately) irrelevant, this does not mean that the Bank itself was irrelevant. For one thing, the Bank was largely successful in inducing state banks to maintain specie convertibility of their notes (which was not always the case either before or after the Bank's existence). Second, as argued by Knodell (1990), the Bank successfully pegged the discount rate on inland bills of exchange.
As both Knodell (1990) and Fraas (1974) recognize, those activities by the Bank effectively replaced a system of internally flexible exchange rates with a system of internally fixed rates. Nothing in our analysis suggests that this was irrelevant.
References


Figure 1: Assets of the Second Bank of the United States. Monthly, 1821-1832.

Source: General Statement of the Bank of the United States.
Figure 2: Liabilities of the Second Bank of the United States. Monthly, 1821-1832.

Source: General Statement of the Bank of the United States.
Figure 3: Semi-annual dividends and net profits of the Second Bank of the United States, 1821-1832.

Source: Catterall (1902), App. V., p. 504.
Figure 4: Net Equity and Market Value of the Second Bank of the United States. Monthly, 1821-1832.

Source: General Statement of the Bank of the United States.

The New York Shipping and Commercial List.
Figure 5: Circulation of the Second Bank of the United States. Monthly, 1821-1832.

Source: General Statement of the Bank of the United States.
Figure 6: Net foreign lending of the Second Bank of the United States. Monthly, 1821-1832.

Source: General Statement of the Bank of the United States.
Figure 7: Wholesale price indices. Monthly, 1821-1832.


Figure 8: Exchange rate—monthly quotations at Boston of sixty-day bills of exchange on London—percentage premium or discount relative to a par of $4.49 and 4/9 per pound sterling.

Figure 9: Impulse response function +/- 2 std. dev. error bounds from 500 Monte Carlo draws from a BVAR with 12 lags, $\lambda = 1.5, \tau = .2$, orth. order NPDl, CIRC, TFB, MVAL, WP-NY.
Figure 10: Impulse response function +/-2 std. dev. error bounds from 500 Monte Carlo draws from a BVAR with 12 lags, $\lambda = 1.5$, $\tau = .2$, orth. order NPDL, CIRC, TFB, MVAL, WP-NO.
**Figure 11:** Impulse response function +/-2 std. dev. error bounds from 500 Monte Carlo draws from a BVAR with 12 lags, $\lambda = 1.5$, $\tau = .2$, orth. order NPDL, CIRC, TFB, MVAL, WP–NY.

The RNPDL, RTFB and RMVAL shocks were accomplished, for example, by a linear combination of a NPDL and a WP–NY shock. The correlation between MVAL and WP–NY shocks is approximately .2062. The RMVAL shock is a one-standard-deviation MVAL shock coupled with a negative .2062 standard deviation WP–NY shock.
**Figure 12**: U.S. interest rates on long term debt. Annually, 1821-1832.

Source: Homer (1963), Table 38.
Table 1: Specie Stocks

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Table 2

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Source: Hepburn (1924), p. 129.
### Table 4

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