A Strong Incompatibility Between Efficiency and Equity in Non-Convex Economies

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Working Paper No. 426
August 1996

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July 16, 1996

Abstract

In allocation problems of perfectly divisible goods, we study the equity property of 'no-domination', according to which no agent can receive strictly more of all goods than any other agent. We prove that no-domination is incompatible with Pareto efficiency, as soon as preferences are allowed to be non-convex.

*Chargé de Recherche F.N.R.S. at Facultés Universitaires Notre-Dame de la Paix. This paper was prepared when I was visiting the Department of Economics, Duke University. I would like to thank Hervé Moulin for stimulating discussions. Financial support from the Wallis Institute of Political Economy, University of Rochester, is gratefully acknowledged. Finally, I am also grateful to Tarık Kara, Maria-Teresa Naranjo, James Schummer and especially William Thomson for their comments and suggestions.

1 Introduction

It is undoubtedly one of the most intuitive equity criteria in allocation problems where goods are perfectly divisible and preferences are monotonic that no agent gets strictly more of all goods than any other agent. Let us call it the no-domination criterion (it has been formally defined e. g. in Thomson [3], Thomson and Varian [5] and Moulin and Thomson [1]).

In this paper, we consider division economies where a fixed social endowment has to be divided among agents characterized by their preferences, and we show that no-domination and Pareto efficiency may be in conflict, if continuous, strictly monotonic, differentiable but non-convex preferences are admissible. Specifically, we prove the following statement:

Theorem 1 *There exist 2-good 3-agent division economies with continuous, strictly monotonic and differentiable preferences where there is domination at every Pareto efficient allocation.*

This conflict between two very simple and natural properties is a new and striking example of the impossibility to generalize a number of classical results from convex to nonconvex economies[1]. It also sheds some light on the implicit consequences of assuming convex preferences.

Real situations where preferences exhibit non-convexities can be easily found (think of preferences over quantities of for instance different beverages, textiles, etc... which are not profitably mixed).

In a concluding section, we discuss the assumptions made in theorem 1 about the number of goods, the number of agents and the kind of economies covered by the result. Moreover, we briefly comment on the consequences of the impossibility result we present here for the theory of equitable allocation.

2 Proof of the theorem

The proof below is by way of an example. Actually, it does not require any sophisticated mathematical tool. As a matter of fact, the structure of the Pareto set in 3-agent non-convex economies is not well known. In the economy we construct, however, the Pareto set has a simple configuration (e.g. all Pareto efficient allocations are supported by the same price vector) precisely because preferences are not convex. Then, there are three Pareto

[1] Recall that when a competitive equilibrium operated from equal division of the social endowment exists, the requirements of no-domination and Pareto efficiency are clearly compatible.

efficient and Pareto indifferent allocations v, w, and y exhibiting a cycle of domination: at v, agent 1's bundle is dominated by agent 2's bundle, at w, agent 2's bundle is dominated by agent 3's bundle, and at y, agent 3's bundle is dominated by agent 1's bundle. Moreover, as soon as one or two agents enjoy a higher welfare level than at v, w, or y, the domination of one of these agents' bundle over an other agent's bundle increases too.

Proof of Theorem 1

Step 1: Notation and terminology

In order to construct the example, it will prove useful to use the following terminology. First of all, we fix $\Omega = (42, 50)$, denoting the amounts of good to divide. Let $N = \{1, 2, 3\}$ be the set of agents. Let R_i, P_i, I_i, $i \in N$, stand for agent i's weak preference, strict preference and indifference relation respectively over points in $\mathbb{R}^2_+$. Let $I(R_i)$ denote agent i's class of indifference curves. Let $Z \subset \mathbb{R}^{2\times 3}_+$ denote all feasible divisions of Ω among the three agents. If $z = (z_1, z_2, z_3) \in Z$, then z_i is called i's share at z. Let $A \subseteq \mathbb{R}^2_+$, A closed, $i \in N$, be given. Then, $m(R_i, A) \subset \mathbb{R}^2_+$ denotes the set of preferred points in A by agent i, that is,

$$m(R_i, A) = \{a \in A : \forall b \in A, a\, R_i\, b\}$$

Let $j \in N$, and $z_0 \in \mathbb{R}^2_+$ be given. Then $C(R_j, z_0) \subseteq Z$ denotes the set of feasible divisions such that agent j is at least as well off as at z_0, that is

$$C(R_j, z_0) = \{z = (z_1, z_2, z_3) \in Z \mid z_j\, R_j\, z_0\}$$

Let $a_1, \ldots, a_m \in \mathbb{R}^2_{++}$ and $\epsilon \in (0, \frac{1}{8})$ be such that $a_{i-1,1} < a_{i,1} - \epsilon$ and $a_{i-1,2} > a_{i,2} + \epsilon$, for all $i \in \{2, \ldots, m\}$. Then, $ST(a_1, \ldots, a_m)$ is a continuous, strictly monotonic and differentiable indifference curve satisfying the two following properties[2]:

$$ST(a_1, \ldots, a_m) \cap U(a_1, \ldots, a_m) = \{a_1, \ldots, a_m\} \tag{1}$$

where $U(a_1, \ldots, a_m) = \{x \in \mathbb{R}^2_+ \mid \exists i \in \{1, \ldots, m\} \text{ s.t. } x \leq a_i\}$, and

$$ST(a_1, \ldots, a_m) \cap L(a_1, \ldots, a_m) = \{a_1, \ldots, a_m\} \tag{2}$$

where $L(a_1, \ldots, a_m) = \mathbb{R}^2_+ \backslash \{x \in \mathbb{R}^2_+ \mid \exists i \in \{1, \ldots, m\}, \lambda \in [-1, 1] \text{ s. t. } x \gg (a_{i,1} + \lambda\epsilon, a_{i,2} - \lambda\epsilon)\}$, and, by convention, $a_{0,2} = \Omega_2$ and $a_{m+1,1} = \Omega_1$.

In the economy we construct, all elements of $I(R_i)$, $i \in N$, are "staircase" indifference curves $ST(a_1, \ldots, a_m)$, where $\frac{a_{1,1}}{a_{1,2}}, \ldots, \frac{a_{m,1}}{a_{m,2}}$ denote the

[2]Vector inequalities: $\forall x, x' \in \mathbb{R}^2_+$, $x \gg x' \Leftrightarrow x_l > x'_l \forall l \in \{1, 2\}$, and $x \geq x' \Leftrightarrow x \neq x'$ and $x_l \geq x'_l \forall l \in \{1, 2\}$.

proportions in which this agent consumes the two goods, when her/his marginal rate of substitution is equal to -1. The way an indifference curve $ST(a_1, \ldots, a_m)$ is constructed is illustrated in figure 1 for $m = 2$.

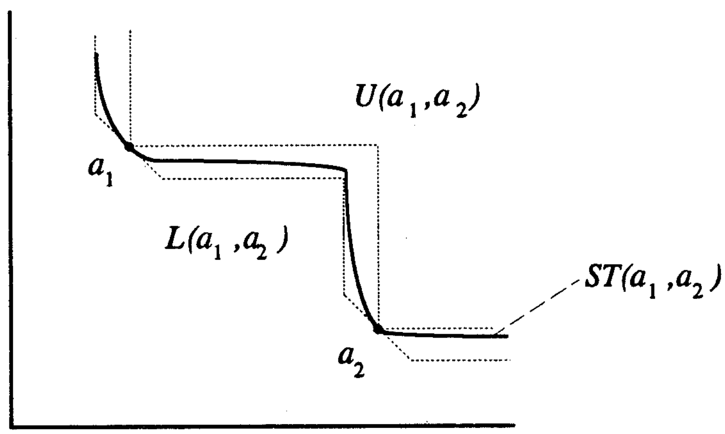

Figure 1

Finally, let $H(a_1, \ldots, a_m) \subset \mathbb{R}^2_+$ denote the lower comprehensive hull of $(a_1, \ldots, a_m)$, and $H_\epsilon(a_1, \ldots, a_m) \subset \mathbb{R}^2_+$ the lower comprehensive hull of small linear segments centered around the points $a_1, \ldots, a_m$ and with a slope equal to -1. Formally,

$$H(a_1, \ldots, a_m) = \left\{x \in \mathbb{R}^2_+ \mid \exists i \in \{1, \ldots, m\} \text{ s.t. } x \leq a_i\right\}$$

$$H_\epsilon(a_1, \ldots, a_m) = \left\{x \in \mathbb{R}^2_+ \mid \exists i \in \{1, \ldots, m\}, \lambda \in [-1, 1] \text{ s.t. } x \leq (a_{i,1} + \lambda\epsilon, a_{i,2} - \lambda\epsilon)\right\}$$

Step 2: Construction of the preferences
Let us begin with an informal description of, say, R_2, depicted in figure 2. Agent 2's indifference curves in the lower contour set of $(12, 20)$ (or $(15, 15)$), are obtained by south-west translation of her/his indifference curve through $(12, 20)$ and $(15, 15)$[3], whereas indifference curves in the upper contour set of $(12, 20)$ (or $(15, 15)$) are almost obtained by a north-east translation but

[3]For $\lambda \geq 12$, the arguments of $ST(\cdot, \cdot)$ do no longer fit in with the definition. Indifference curves are then easily obtained by simply keeping the projection of the curve on $\mathbb{R}^2_+$. A similar remark holds for R_1 and R_3.

the two points of each indifference curve at which the marginal rates of substitution are equal to -1 do not move to the north-east at the same speed: the point corresponding to $(15, 15)$ moves a little bit faster than the point corresponding to $(12, 20)$, and the "relative speed" is given by the ratio $\frac{102}{100}$. Agent 1's and agent 3's indifference curves are constructed in a similar way. Finally note that each one of agent 3's indifference curves has three points at which the marginal rates of substitution are equal to -1.

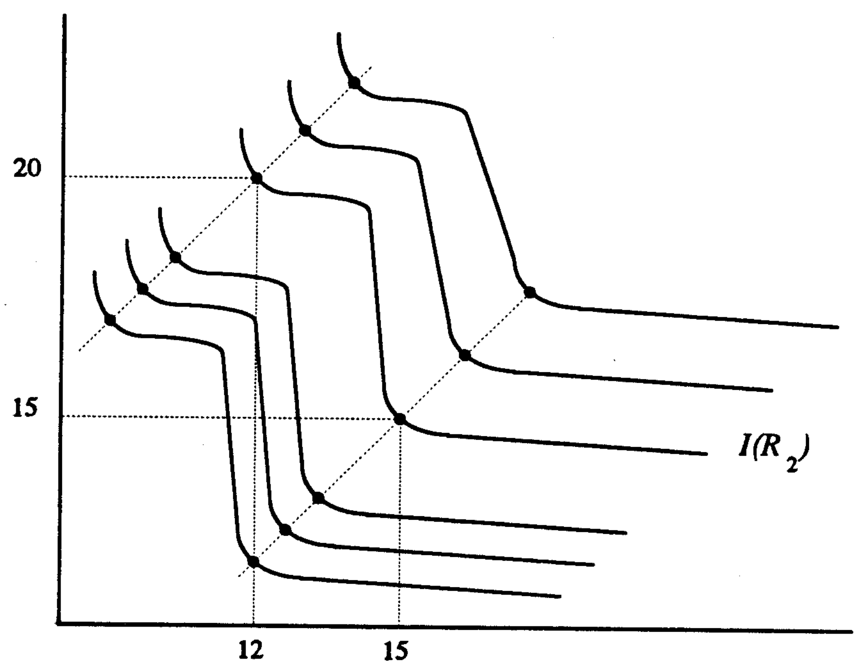

Figure 2

The following conditions are sufficient to define preferences R_i over her/his shares in Z.

<u>Conditions satisfied by R_1:</u> $\forall \lambda \in \mathbb{R}_+$,

$$ST\Big((11,19) - (\lambda,\lambda), (14,18) - (\lambda,\lambda)\Big) \in I(R_1)$$

$$ST\Big((11,19) + \frac{103}{100}(\lambda,\lambda), (14,18) + (\lambda,\lambda)\Big) \in I(R_1)$$

Conditions satisfied by R_2: $\forall\, \lambda \in \mathbb{R}_+$,

$$ST\Big((12,20)-(\lambda,\lambda),(15,15)-(\lambda,\lambda)\Big) \in I(R_2)$$

$$ST\Big((12,20)+(\lambda,\lambda),(15,15)+\frac{102}{100}(\lambda,\lambda)\Big) \in I(R_2)$$

Conditions satisfied by R_3: $\forall\, \lambda \in \mathbb{R}_+$,

$$ST\Big((13,17)-(\lambda,\lambda),(16,16)-(\lambda,\lambda),(19,11)-(\lambda,\lambda)\Big) \in I(R_3)$$

$$ST\Big((13,17)+\frac{104}{100}(\lambda,\lambda),(16,16)+(\lambda,\lambda),(19,11)+\frac{101}{100}(\lambda,\lambda)\Big) \in I(R_3)$$

Step 3: Proof of the existence of three Pareto efficient and Pareto equivalent divisions v, w, and y exhibiting a cycle of domination

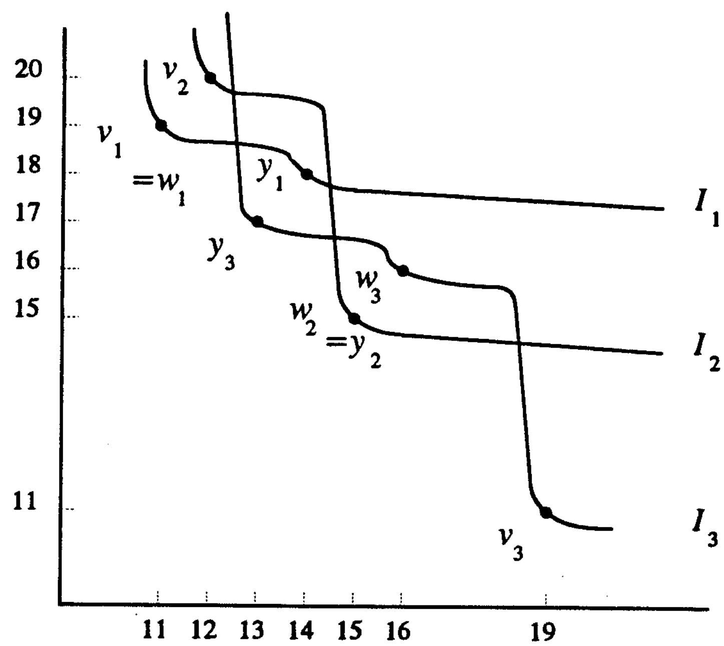

Figure 3

Consider the divisions $v, w, y \in Z$ defined by

$$v = (v_1, v_2, v_3) = ((11,19),(12,20),(19,11))$$
$$w = (w_1, w_2, w_3) = ((11,19),(15,15),(16,16))$$
$$y = (y_1, y_2, y_3) = ((14,18),(15,15),(13,17))$$

Observe that $v_2 \gg v_1, w_3 \gg w_2$ and $y_1 \gg y_3$. Divisions v, w, y and the corresponding indifference curves are depicted in figure 3.

Claim 1 *Divisions v, w, y are Pareto efficient for the economy (R_1, R_2, R_3, Ω), and no other allocation is Pareto indifferent to v, w, y.*

Proof of the claim: Given the terminology we have introduced above, a division $z \in Z$ is (strongly) Pareto efficient for (R_1, R_2, R_3, Ω) if

$$z_i \in m\left(R_i, \left(\cap_{j \neq i} C(R_j, z_j)\right)_i\right), \forall i \in N. \tag{3}$$

We simply check equation (3) for each agent. From (2), we obtain

$$(C(R_2,(12,20)) \cap C(R_3,(13,17)))_1 \subset H_{2\epsilon}((8,24),(11,19),(14,18),(17,13))$$

and from (2) again

$$\{(11,19),(14,18)\} = m\left(R_1, H_{2\epsilon}((8,24),(11,19),(14,18),(17,13))\right)$$

Second,

$$(C(R_1,(11,19)) \cap C(R_3,(13,17)))_2 \subset H_{2\epsilon}((9,21),(12,20),(15,15),(18,14))$$

and

$$\{(12,20),(15,15)\} = m\left(R_2, H_{2\epsilon}((9,21),(12,20),(15,15),(18,14))\right)$$

Finally,

$$(C(R_1,(11,19)) \cap C(R_2,(12,20)))_3 \subset H_{2\epsilon}((13,17),(16,16),(19,11))$$

and

$$\{(13,17),(16,16),(19,11)\} = m\left(R_3, H_{2\epsilon}((13,17),(16,16),(19,11))\right)$$

which completes the proof. □

Step 4: A common property of all the Pareto efficient divisions for the economy (R_1, R_2, R_3, Ω)

Claim 2 *All Pareto efficient divisions $z = (z_1, z_2, z_3)$ for the economy (R_1, R_2, R_3, Ω) are "translates" of one of the divisions v, w, y, that is, can be written $z_i = v_i + \delta_i e$ for all $i \in N$, or $z_i = w_i + \delta_i e$ for all $i \in N$, or $z_i = y_i + \delta_i e$ for all $i \in N$, with $e = (1,1)$ and $\delta_1 + \delta_2 + \delta_3 = 0$.*

Proof of the claim: Given (2) and the low value of ϵ, the way we constructed the preferences guarantees that for all $z = (z_1, z_2, z_3) \in Z$, we have $(C(R_i, z_i) \cap C(R_j, z_j))_k \subset H_{2\epsilon}(a_1, \ldots, a_m)$, $i \neq j \neq k \neq i \in N$, for some $a_1, \ldots, a_m \in \mathbb{R}^2_+$. Therefore, $z_k \notin int(H_{2\epsilon}(a_1, \ldots, a_m))$ only if y is a translate of one of the divisions v, w, y in the above sense. □

Note that a consequence of the claim is that the individual rates of substitution at any Pareto efficient allocation are equal to -1.

Step 5: *Characterization of the Pareto set when one and only one agent is strictly better off than at v, w or y*

We consider first the Pareto efficient divisions where only agent 3 is strictly better off than at v, w or y. Let the divisions v', w', y' be defined by: $\lambda_1, \lambda_2, \lambda_3 \in \mathbb{R}_+$ and

$$\begin{aligned} v' &= (v'_1, v'_2, v'_3) = (v_1, v_2, v_3) + \frac{101}{100}(-\lambda_1, -\lambda_2, \lambda_3) \\ w' &= (w'_1, w'_2, w'_3) = (w_1, w_2, w_3) + (-\lambda_1, -\lambda_2, \lambda_3) \\ y' &= (y'_1, y'_2, y'_3) = (y_1, y_2, y_3) + \frac{104}{100}(-\lambda_1, -\lambda_2, \lambda_3) \end{aligned}$$

where $\lambda_1 + \lambda_2 = \lambda_3$.

We observe that $v'_3 \, I_3 \, w'_3 \, I_3 \, y'_3$, whereas $w'_1 \, R_1 \, m(R_1, \{v'_1, y'_1\})$ and $w'_2 \, R_2 \, m(R_2, \{v'_2, y'_2\})$ with at least one strict preference. As a result, v', y' cannot be Pareto efficient. Moreover, by claim 2 and a similar argument as for claim 1, we obtain that w' is Pareto efficient and no other division is Pareto equivalent to w'.

In conclusion, a strict increase in agent 3's welfare results in a larger domination of her/his bundle over agent 2's bundle.

In the same way, we can prove that a strict increase in agent 1's (resp. agent 2's) welfare results in a larger domination of her/his bundle over agent 3's (resp. agent 1's) bundle.

Step 6: *Characterization of the Pareto set when two agents are striclty better off than at v, w or y*

We consider first the Pareto efficient divisions where both agents 1 and 3 are strictly better off than at v, w or y. Let the allocations v'', w'', y'' be defined by: $\lambda_1, \lambda_3 \in \mathbb{R}_+$ and

$$\begin{aligned} v'' &= (v''_1, v''_2, v''_3) \\ &= (v_1, v_2, v_3) + \left(\frac{103}{100}\lambda_1, -(\frac{103}{100}\lambda_1 + \frac{101}{100}\lambda_3), \frac{101}{100}\lambda_3\right) \\ w'' &= (w''_1, w''_2, w''_3) = (w_1, w_2, w_3) + \left(\frac{103}{100}\lambda_1, -(\frac{103}{100}\lambda_1 + \lambda_3), \lambda_3\right) \end{aligned}$$

$$y'' = (y''_1, y''_2, y''_3) = (y_1, y_2, y_3) + \left(\lambda_1, -(\lambda_1 + \frac{104}{100}\lambda_3), \frac{104}{100}\lambda_3\right)$$

We observe that $v''_1 \, I_1 \, w''_1 \, I_1 \, y''_1$ and $v''_3 \, I_3 \, w''_3 \, I_3 \, y''_3$, whereas $w''_2 \, R_2 \, v''_2$ and $y''_2 \, R_2 \, v''_2$ with at least one strict preference. As a result, v'' is not Pareto efficient. On the other hand $w''_2 \, I_2 \, y''_2 \iff \frac{103}{100}\lambda_1 + \lambda_3 = \lambda_1 + \frac{104}{100}\lambda_3$, that is, $\lambda_1 = \frac{4}{3}\lambda_3$. Therefore, either $\lambda_1 < \frac{4}{3}\lambda_3$, and then, by claim 2 and a similar argument as in claim 1, we obtain that w'' is Pareto efficient and no other division is Pareto equivalent to w'', but the domination of agent 3's bundle over agent 2's bundle increases, or $\lambda_1 > \frac{4}{3}\lambda_3$, and then, by claim 2 and a similar argument as in claim 1, we obtain that y'' is Pareto efficient and no other division is Pareto equivalent to y'', but the domination of agent 1's bundle over agent 3's bundle increases, or finally $\lambda_1 = \frac{4}{3}\lambda_3$, and we have two Pareto efficient Pareto equivalent divisions w'' and y'' where $w''_3 \gg w''_2$ or $y''_1 \gg y''_3$, and there is still some domination. By exactly the same argument, we can show that:
- if (y_1, y_2, y_3) is Pareto efficient and $v_1 \, P_1 \, y_1$, $y_2 \, P_2 \, v_2$, $y_3 \, P_3 \, v_3$, then $\lambda_2 \geq \frac{1}{2}\lambda_3$ implies $y_2 \gg y_1$, whereas $\lambda_2 \leq \frac{1}{2}\lambda_3$ implies $y_3 \gg y_2$.
- if (y_1, y_2, y_3) is Pareto efficient and $y_1 \, P_1 \, v_1$, $y_2 \, P_2 \, v_2$, $v_3 \, P_3 \, y_3$, then $\lambda_2 \geq \frac{3}{2}\lambda_1$ implies $y_2 \gg y_1$, whereas $\lambda_2 \leq \frac{3}{2}\lambda_1$ implies $y_1 \gg y_3$.

In conclusion, there is domination at every Pareto efficient division of the economy (R_1, R_2, R_3, Ω).

End of the proof of Theorem 1

3 Concluding remarks

1) It is clear that the example in theorem 1 can be adapted to economies with more than 2 goods[4]. For instance, we can replace goods 1 and 2 with composite goods made out of several goods which all the agents wish to consume in fixed proportions (the same proportion for all) when the individual marginal rates of substitution are all equal to -1.

2) The result can not be stated with 2 agents only. In this case indeed, we can always construct a Pareto efficient allocation where there is no domination. A formal proof of this statement is given in the appendix.

3) The example in theorem 1 can be modified so as to prove that, in production economies, as soon as there is some non-convexity in the production set (even if the returns to scale are decreasing), we can design 2-agent economies with convex preferences where there is domination between two

[4] If there is only one good, then no-domination requires to divide it equally, and the resulting division is Pareto efficient.

bundles at every Pareto efficient allocation. Actually, Thomson ([4], chapter 4), modifying an example given by Vohra [7], already showed the incompatibility between the two requirements in the case of a quasi-convex production function.

4) The no-domination criterion we have studied in this paper is a weaker requirement than the celebrated no-envy requirement. An allocation is said to satisfy no-envy if no agent strictly prefers the bundle assigned to another agent to her/his own bundle. No-envy is a central concept in equity theory. It is compatible with Pareto efficiency in a wide range of models, but was proven by Varian to be incompatible with Pareto efficiency in non-convex division economies (see Varian [6]). The theorem presented above is much stronger than Varian's result.

On the other hand, our result stands in sharp contrast with the general compatibility between Pareto efficiency and another central equity concept, that is, egalitarian equivalence, proposed by Pazner and Schmeidler [2]. An allocation is egalitarian equivalent if there exists a reference bundle to which all agents deem their assigned bundle equivalent (giving the reference bundle to every agent is in general not possible). The existence of Pareto efficient and egalitarian equivalent allocations is guaranteed as soon as preferences are continuous and strictly monotonic, and does not require any convexity assumption (cfr Pazner and Schmeidler [2]). Egalitarian equivalence has been criticized precisely because, when combined with Pareto efficiency, it is incompatible with no-domination. There exist indeed convex economies where there is domination at all Pareto efficient and egalitarian equivalent allocations (cfr Thomson [4], Theorem 5). This criticism however is sharply weakened by the result presented in this paper since no-domination appears too demanding as soon as we impose Pareto efficiency anyway.

Appendix

In this appendix, we prove the following theorem:

Theorem 2 *In every 2-agent division economy, there exists at least one Pareto efficient allocation at which there is no weak domination.*

We begin by introducing some notation. Let $\Omega \in I\!R^L_{++}$ be now any vector of any number L of goods. Let R_1, R_2 be any continuous and *weakly* monotonic preferences over $I\!R^L_+$. The definition of N, Z, and $C(R_i, z_0)$ are straightforwardly adapted from the terminology used in the proof of Theorem 1. A division $z = (z_1, z_2) \in Z$ is *weakly Pareto efficient* if for all

$z' = (z'_1, z'_2) \in Z, [z'_i \, R_i \, z_i \quad \forall i \in N] \Rightarrow [z'_k \, I_k \, z_k$ for al least one $k \in N]$. Let $WP \subseteq Z$ denote the set of weakly Pareto efficient divisions, and P the set of (strongly) Pareto efficient divisions. There is *weak domination* at $z = (z_1, z_2)$ if $z_i \geq z_j$ for some $i, j \in N$. Let $D_{i/j} \subset Z$ denote the set of feasible allocations at which agent i's bundle weakly dominates agent j's bundle.

Proof of Theorem 2

Let us first suppose that $(\frac{\Omega}{2}, \frac{\Omega}{2}) \in WP \setminus P$. Then, w.l.o.g. there exists $z = (z_1, z_2) \in P$ such that $(z_1 \, I_1 \, \frac{\Omega}{2}$ and $z_2 \in m(R_2, C_2(R_1, z_1))$. By assumption, $z_2 \, P_2 \, \frac{\Omega}{2}$, so that $z \notin D_{1/2}$. But let us suppose that $z \in D_{2/1}$. Therefore, by monotonicity of preferences R_1,

$$[x \in \mathbb{R}^L_+ \text{ and } \exists \lambda \in [0,1] \text{ s.t. } x = \lambda \frac{\Omega}{2} + (1-\lambda) z_1] \Rightarrow [x \, I_1 \, z_1]$$

By continuity of preferences R_2, and since $z_2 \, P_2 \, \frac{\Omega}{2}$, there exist $\delta \in \mathbb{R}^L_{++}$ and $x \in \mathbb{R}^L_+$ such that $x = \lambda \frac{\Omega}{2} + (1-\lambda) z_1$ for some $\lambda \in [0,1]$ and $\frac{\Omega}{2} \, I_2 \, (\Omega - x - \delta)$. Therefore, since $(x + \delta) \, P_1 \, x$, we have $(\frac{\Omega}{2}, \frac{\Omega}{2}) \notin WP \setminus P$, a contradiction. Let us suppose instead that $(\frac{\Omega}{2}, \frac{\Omega}{2}) \notin WP$. Then, by continuity of the preferences, there exists $z = (z_1, z_2) \in P$ such that $z_1 \, P_1 \, \frac{\Omega}{2}$ and $z_2 \, P_2 \, \frac{\Omega}{2}$, which respectively imply that $C_2(R_1, z_1) \cap D_{2/1} = \emptyset$ and $C_1(R_2, z_2) \cap D_{1/2} = \emptyset$, so that there is no weak domination at y.

End of the proof of Theorem 2

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