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THE FIRST CONFERENCES ON THE THEORY OF ECONOMIC GROWTH

The first conferences on the theory of economic growth known to me were these:

I. Activity Analysis in the Theory of Growth and Planning held in Cambridge, England, at the University of Cambridge, July, 1963, sponsored by the International Economic Association, led by Edmond Malinvaud.

II. The Econometric Approach to Development Planning, held in Rome, Italy, at the Vatican, October, 1963.

III. Mathematical Models of Economic Growth, held in Rochester, New York, at the University of Rochester Department of Economics, July, 1964, sponsored by the Social Sciences Research Council, led by Lionel McKenzie.

IV. Optimal Growth, held in Stanford, California, at the Center for Advanced Study in the Behavioral Sciences, July, 1965, sponsored by the Mathematical Social Sciences Board, led by Kenneth Arrow.

Of these conferences the first two were not confined to growth theory but also included papers concerned with economic planning in the case of the Cambridge conference and with econometrics and planning in the case of the Vatican conference. I attended the conferences in Cambridge, Rochester, and Stanford, but I was not present at the Vatican. The papers presented to the Cambridge and Vatican conferences were published in volumes of proceedings which included reports of the discussions. These conferences were relatively short, lasting about one week, and papers prepared in advance were presented and discussed. The Rochester and Stanford conferences on the other hand were workshops. The Rochester conference lasted six weeks and the Stanford conference lasted at least four weeks. Thus much work was done at these conferences. Indeed some of the papers which were begun at the Rochester conference continued to be worked on at the Stanford conference and were published in a special issue of the Review of Economic Studies, January, 1967. However not all of the papers originating in these conferences were published in this issue and some papers were included there which did not originate in the conferences.

1. PRECURSORS OF THE CONFERENCES

To understand the proceedings of the conferences we should consider some of the immediately preceding events. A summer institute lasting eight weeks was held at Stanford in 1957 on Applications of Mathematics in Social Science Research. It was sponsored by the Committee on Mathematical Training of Social Scientists of the Social Science Research Council. The workshops devoted to economics were one on Linear Economic Models led by Robert Dorfman and one on International Trade and Taxation led by me. Neither of these paid much attention to growth theory, although Dorfman devoted a few days to the von Neumann model. The Rochester conference was also held under the auspices of the SSRC and the design of the conference was in imitation of the 1957 SSRC conference. Indeed, the Mathematical Social Science Board was in effect the successor of the SSRC committee and held four conferences subsequently devoted to economic growth, beginning with the Stanford conference in 1965.

Two events may have aroused the interest that led to these conferences. One was the publication in 1958 of the book by Dorfman, Samuelson, and Solow, or for short, DOSSO, entitled Linear Programming and Economic Analysis. This book contained studies in the context of Leontief type models, without utility functions, and gave a turnpike theorem for a two sector model without presenting a fully rigorous proof. Of course the book was known to Dorfman at the time of his 1957 workshop. It was the direct stimulus to my work and led me to prove a turnpike theorem for a simple Leontief model with variable coefficients. My paper was read to the Econometric Society in December,

1960. Unfortunately I submitted the paper to Metroeconomica which then stopped publication for an extended period following the death of the editor. After a wait of many months I resubmitted my paper to The Review of Economic Studies where it was published in 1963, well after the publication in 1961 of Morishima's paper. His paper gave a similar turnpike theorem for a Leontief model with a finite number of activities. My paper was entirely independent of Morishima's but in view of the priority of his publication I acknowledged his paper in mine. Metroeconomica also published an earlier version some time later, without showing me a galley proof, despite my attempt to withdraw it.

The other stimulus was a trip by John Hicks to the United States and Japan in 1960. He had been struggling with the DOSSO turnpike theorem in the context of a simple Leontief model. Among the places he visited were Berkeley and Osaka. In Berkeley he spoke with Radner among others and at Osaka with Nikaido and Morishima. These conversations stimulated Morishima and Radner to write the papers that were published, together with one by Hicks, in The Review of Economic Studies, February, 1961. I believe the papers by Morishima, Radner, and me gave the first fully proved turnpike theorems for multisector models. They all proved Samuelson turnpikes in von Neumann type models where there were no utility functions, but where the objective was to achieve the largest terminal stocks in certain ratios over a fixed time period. This is the way the problem had been put by Dorfman, Samuelson, and Solow. You will note that these papers did not consider infinite programs and thus did not consider the existence of optimal programs over the infinite horizon. The existence of optimal programs for finite periods was not questioned.

It should also be mentioned that a symposium on production functions and economic growth appeared in the Review of Economic Studies, June, 1962, before the conferences occurred. This group of papers contained Arrow's paper on learning by doing and Joan Robinson's paper on the Golden Age that maximizes consumption per capita. Of course the Golden Age was also described by others in this same period. Arrow's paper has had a significant influence on the development of the New Growth Theory. However the important idea for the conferences we are concerned with was the Golden Age which turned out to provide a turnpike for the growth model which combined the Ramsey objective of maximizing a utility sum over an infinite horizon with the multisector production model of von Neumann. Finally Uzawa published two papers on deterministic economic growth in the two sector neoclassical model in 1962 and 1963 in the Review of Economic Studies and a paper on optimal growth in this model in the same journal in 1964. These papers had an influence on both the Rochester and the Stanford conferences. Notice how many of the early papers were published in the Review of Economic Studies. The editorial advisors in those years were Nicholas Kaldor and Ursula Hicks and the chairman of the Economic Studies Board was Ian Little. John Hicks was in the background.

2. THE CAMBRIDGE CONFERENCE

Altogether five papers on economic growth were presented to the Cambridge conference held in 1963. The papers labeled as studies on growth theory were given by Koopmans, Allais, and me. They were in the style of the von Neumann model. The paper of Koopmans was entitled Economic Growth at a Maximal Rate. It was largely concerned with presenting in diagrammatic form the turnpike theory for von Neumann models stemming from the papers of Radner, Morishima, and me . The von Neumann model is not easy to illustrate with diagrams since it involves at least two capital goods located at two different times, so the model is at least four dimensional. However by a clever device of representing inputs summing to one by their ratios and outputs by their quantities, Koopmans was able to reduce the dimensionality to three. Then the diagram could be projected on the plane of the paper. With this diagram he was able to illustrate both the Radner turnpike theorem and the complications that I had found when the assumption of strict convexity of the production set near the von Neumann ray was not met. I am not sure how much of new material on von Neumann facets was available at the conference but it was mentioned in the published paper. Koopmans pointed out the lack of economic

realism in the von Neumann model, even after the objective of maximal accumulation from given initial stocks had been introduced by DOSSO. He expected this deficiency to be overcome eventually.

My paper in the Cambridge conference was devoted to establishing a turnpike theory in the maximal accumulation problem where all the possible complications were present, not only the lack of strict convexity of the production set but also the presence of subeconomies with different maximal growth rates depending on the set of goods to be accumulated. I also showed that the possibility existed that the supremum of the rates of growth for a certain subset of goods might not be attainable, but that this did not invalidate the turnpike theorem. Perhaps I made the mistake of burying my most interesting result in a lemma. My Lemma 11 asserts that the supremum of the rates at which the *ith* good appears in a feasible path of balanced growth is the growth rate of a von Neumann equilibrium. It is critical that the definition of the equilibrium contains the condition introduced by Kemeny, Morgenstern, and Thompson in 1956 that the output have a positive value at the equilibrium prices. My theorems were stated in terms of the von Neumann facet.

The paper of Allais gave a detailed exposition of his version of the Golden Rule or as he termed it the capitalistic optimum. In his statement the capitalistic optimum is realized at an interest rate equal to the rate of growth of primary income. Then the maximal sustainable output of consumption goods will be realized per unit of primary income, that is,
income derived from labor and natural resources.

Radner's paper was classified as planning theory rather than growth theory but it is actually concerned with planning for optimal growth. Thus his discussion introduces a social utility function. He used Bellman's optimality principle to characterize the value function when certain boundedness conditions are satisfied. He is able to give methods for approximating optimal policies in models with infinite horizons by means of stationary policies. In the case of a linear logarithmic economy the procedure was made explicit. In the discussion Radner seems to have conjectured the result recently achieved by Kaganovich that in a closed model with a homothetic utility function and no joint production the von Neumann ray would be a turnpike. It is not clear that the printed paper included everything he presented orally in the conference.

The final paper on growth theory was given by Chakravarty. It is devoted to the question of a proper preference function over investment programs which are essentially of infinite or at least indeterminate length. There is a suggestion of maximizing over finite periods where the terminal objective is to reach a capital stock that will allow the maintenance of some chosen rate of growth for all future time. There was only one paper given in Cambridge which achieved fame. This was not on economic growth. It was the paper of Malinvaud on a planning procedure that became the inspiration for a literature on that subject.

3. THE VATICAN CONFERENCE

The Vatican conference, which involved some of the same people as the Cambridge conference, was held only three months later. Just as in Cambridge many of the papers were not directly concerned with economic growth and *a fortiori* not with optimal economic growth. The papers of most interest to us were given by Koopmans, Malinvaud, Morishima, and Allais. Since I did not attend this conference, I will not discuss it in any detail. However, just as in the case of the Cambridge conference one paper, by Koopmans, achieved fame and, together with the 1966 paper of David Cass, inspired a literature on what came to be called the neoclassical model of optimal growth. In this model there is only one good which serves both as capital good and consumption good, the horizon is infinite, and utility may be either discounted or undiscounted. The novelty as compared with Ramsey is that population is growing, and the criterion that is adopted for optimality is the maximization of the sum of the utility of per capita consumption over the infinite horizon. In the one sector model they prove convergence of the capital stock to the stock of the Golden Age.

A paper to much the same effect was given by Malinvaud, but it was quickly overshadowed by the Koopmans paper. Malinvaud considered the case of utility which might be either discounted or not and proposed a criterion which is very close to the overtaking criterion proposed later by Atsumi and Weizsäcker. He defines a program as optimal if there is no alternative program which achieves a larger utility sum (perhaps discounted) in some finite initial period and gives no less utility in any subsequent period. In the 1964 paper of Uzawa the model was extended to two goods, one a capital good and one a consumption good. He made use of the condition that the consumption sector is more capital intensive than the capital goods sector.

Morishima gave a paper describing a log linear model which bears some resemblance to Radner's paper in Cambridge but suffers from the weakness that the amount of consumption is set equal to the wage bill. Morishima says "The purpose of this study is to extend the recent results of growth economics (especially the turnpike theorem) to a model with endogenous population growth and flexible consumption demands." The most interesting feature is to relate the rate of growth of population to levels of consumption and therefore to the interest rate, since real wages are inversely related to the interest rate. The rate of growth of population must equal the interest rate in equilibrium. The consequence is that the turnpike is a balanced path whose growth rate is endogenous. The highest equilibrium growth rate also has the highest equilibrium consumption level. Under rather artificial assumptions on the determination of the wage rate Morishima shows that the highest equilibrium growth path is locally stable. He also considers the effect on the equilibrium balanced growth path of technical progress. For modern interests such a theory suffers from the fact that the optimal program is not likely to be relevant to a competitive equilibrium, since consumer tastes only affect current consumption, not the choice between consumption today and consumption later.

Pasinetti in his contribution is very critical of the students of balanced growth for failing to allow for technical progress and a biased change in consumer demand as income increases. However, so far as I can see, he does not model these factors, though he talks about linear models of Leontief or Sraffa type, and goes on at great length about the things that such models should allow for, and do not. Allais presents a paper of immense length, nearly three hundred pages, with about the same message as his Cambridge talk. I was impressed by his attempt to provide an empirical basis for the contention that physical capital saturation is present in all countries and the differences in income are explained by differences in social institutions and culture.

4. THE ROCHESTER CONFERENCE

The Rochester conference in 1964 was organized by me. I was given this opportunity by Tjalling Koopmans through the Social Science Research Council. Tjalling was scheduled to run a conference, but some difficulty arose which Tjalling felt compromised his position and left him unable to continue. This conference was not a short meeting such as we are now having, where only papers already prepared are presented, but a six week workshop where people collaborated and held discussions following presentations of preliminary research. It is my impression that the most successful interaction took place between Nikaido, Gale, and McFadden. Nikaido presented a working note at the very beginning on a Samuelson turnpike, that is, a turnpike in the simple Leontief model with a terminal objective. He introduces a feasible program of exogenous demand and proves a strong turnpike theorem. This is in the spirit of DOSSO. Very quickly Gale and then McFadden added short comments using a one sector model. McFadden showed that the necessary and sufficient condition for feasibility of an infinite consumption path c_t is that $\sum c_t/\rho^t$ be less or equal to the initial stock where ρ is the growth factor. Two weeks later he introduced a utility function and (he says on the suggestion of Mirrlees and Gale) defines a maximal path with an overtaking criterion and proves that in the one sector model the necessary and sufficient condition for the path to be maximal is that it satisfy a feasibility condition with equality and the condition that $\rho^t u'(c_t) = u'(c_0)$ for all t. Later, giving credit

for assistance to Gale, Mirrlees, and Nikaido, he provided a note in which he proved for a continuous time version of this model that the necessary and sufficient condition for the existence of a maximal path is that the utility function be bounded above. This is as far as he got in the workshop, but this research was continued and eventuated in his contribution to the Symposium on Optimal Infinite Programmes in the Review of Economic Studies, January, 1967, where the results were extended to general linear models. He showed *inter alia* that good programs in Gale's sense exist if and only if the welfare function is bounded above.

Gale presented a discussion which clearly prefigured his classic paper in the 1967 Symposium, but I do not have a paper for him. Jim Mirrlees presented two papers. He offered an extension of the Ramsey argument to a two sector model like that used by Uzawa in his 1964 optimal growth paper. Mirrlees' innovation is to use a concave utility function in place of the quantity consumed of the consumption good. This allows him to avoid bang-bang solutions. He also attempted to extend the Ramsey model to the case of uncertainty but apparently met a snag he was unable to overcome. I tried to discover from him by *E*-mail what the difficulty was, but he could not recall, and had lost the paper. He was interested in determining the conditions under which the introduction of uncertainty would cause the level of consumption in the present period to decrease. In a two period model he derives the condition that the product of consumption and the marginal utility of consumption $u'(c)$ should be a convex function. This follows if $2u''(c) + cu'''(c)$ is non-negative. His difficulty seemed to arise in attempting to extend this criterion to the infinite optimal path. The role of the third derivative has since been widely recognized in the literature on saving by individuals.

Inada, Tsukui, and Kurz also presented papers. Of course, everyone gave talks, including Hiroshi Atsumi, who was a Rochester graduate student, and Emmanuel Drandakis. They both published papers which must have benefited from their attendance in the conference. Atsumi on optimal growth in a two sector model with the overtaking criterion and an infinite horizon, with the first turnpike theorem where a method was used that could be generalized to *n*-sectors. His paper appeared in the Review of Economic Studies, April, 1965. Drandakis on a generalization of my turnpike theorem for a generalized Leontief model to an infinite horizon. His paper appeared in Econometrica, April, 1966.

To continue with the account of papers presented in the conference. Inada gave papers concerned with the stability problem in the two-sector model which Uzawa had described in his articles in the Review of Economic Studies, 1961 and 1963, mentioned above. Inada abandons the neoclassical assumption of a constant savings ratio by supposing that the savings ratio depends on social preferences given the production possibility frontier for consumption goods and capital goods. He requires this choice to satisfy the weak axiom of revealed preference. However the method of explicitly maximizing a utility sum is not used. In a second paper he deals with two sector models with irreversible investment where it is necessary to explain the distribution of investment between sectors. He assumes that only rental income is saved and gives a rule for the distribution of rental income in one sector over the two sectors.

Tsukui proved a Samuelson turnpike theorem in a generalized Leontief model similar to my model in Econometrica, 1963. He assumes a finite number of processes and proves that the optimal path will actually enter the von Neumann facet after some time and will converge to the unique von Neumann equilibrium under his assumptions. Tsukui's paper appeared in the Symposium issue. Tsukui also applied this argument to a planning model of Japanese growth in Econometrica, April, 1966.

The papers offered by Kurz referred back to the article by Uzawa on optimal growth in two-sector models. He replaced the objective of maximizing the discounted sum of per capita consumption by that of maximizing the discounted sum of the utility of per capita consumption over the infinite future. He proves stability of the Golden Age under the assumption that the consumption sector is relatively more capital intensive. It is worth noting the role of the papers of Uzawa on the two sector model in preparing the groundwork for several of these papers.

Gale and I did not present papers. However it is clear from the summary of the presentations that

we began the research on the papers which eventually appeared in the Symposium issue in Gale's case and in the Hicks festschrift, Value, Capital, and Growth, 1968, in my case.

Koopmans and Chakravarty were visitors to the conference. Koopmans repeated his paper on the Concept of Economic Growth. I do not have enough information to report on the two talks of Chakravarty. John Chipman and Hugh Rose were also visitors.

5. THE STANFORD CONFERENCE

The third conference in which I participated in this early period was held in Stanford at the Center for Advanced Research in Behavioral Sciences under the leadership of Kenneth Arrow. This conference was also a workshop which lasted about four weeks. I was the initiator of the workshop as the economics member on the Mathematical Social Sciences Board, or MSSB. The MSSB was administered by Preston Cutler of the Center, but the project was initiated by Duncan Luce and Patrick Suppes. NSF provided the money over a period of years. I expected that the research begun at Rochester would be continued in the Stanford workshop and many of the participants in the Rochester workshop did in fact attend the Stanford workshop, in particular, Gale, McFadden, and me, who worked at completing the papers begun in Rochester and later published. Kurz and Mirrlees also attended both workshops, and Koopmans, who was a visitor in Rochester, came full time to Stanford. Newcomers were Arrow, Hahn, Radner, Srinavasan, Uzawa, and Weizsäcker. Visitors were Bruno Horvat, Allen Manne, and Hans Vosgerau.

Gale and I presented models which had benefited from a year's further study. The summaries by the conference reporter were as follows:

"Gale presented a multisectoral activity model, including labor as an input. He defined utility as a function of activity levels and assumed strict concavity at a point. He then proved that any good program (not infinitely worse than the optimal stationary program) approaches the optimal stationary programs."

"McKenzie presented a model for considering asymptotic properties of optimal paths under general assumptions about technology. He proved convergence in two stages: convergence of the optimum path to a path in the von Neumann facet is proved, and convergence of paths lying in the facet."

Arrow discussed the problem of choosing a discount factor for use in the public sector. He supposes that if the capital market were perfect it would be appropriate simply to adopt the discount factor prevailing in the private sector. However he believes this to be unrealistic. He defines a natural rate of interest equal to the sum of a utility rate of interest plus the elasticity of marginal utility of consumption times the rate of increase of income. By use of both fiscal policy and monetary policy it may be possible to arrange to have the appropriate rate of investment in both public and private sectors.

Uzawa presented a long paper on optimum fiscal and monetary policy in a two sector model where one good is a private good available for either consumption or investment and the other good is a public consumption good. I do not have a copy of his paper but I recall that Arrow was impressed by the presence of 125 numbered equations. This corresponds exactly to his paper in a book of Adelman and Thorbecke, published in 1969.

McFadden presented a paper on the existence of efficiency prices. I only have the summary but it is clear that this was material included in his article for the Symposium issue of the Review of Economic Studies.

von Weizsäcker gave a paper which was intended to address the problem of uncertainty about the future. He suggested setting an accumulation target, for a long enough horizon, lying between 0 and the Golden Rule, and argues that this is sufficient to insure that the path chosen will be close to the optimal path. Of course this is suggestive of the type of planning suggested by Tsukui for Japan.

Koopmans spoke on future directions of research. The summary of his talk follows:

"Koopmans discussed some troublesome aspects of optimal growth models: (1) crucial assumptions are not verifiable, (2) problems with infinite horizons, (3) distortions of the distant future due to uncertainty, (4) models cannot incorporate all available information, and (5) the need for clarification of the axiomatic basis for preference orderings."

Manne and Srinivasan gave a paper on the formation of the Fourth Indian Five Year Plan. It really amounted to showing that no effective planning was occurring partly because adequate information was not available, but also because the goal of improving the living standards of the poorest people was not taken seriously. Also there were no provisions for revising the plan when aspects of the plan did not materialize. Moreover the material balances are described only for the final year of the plan and not on an annual basis. One critical paragraph reads as follows: "All who have read the various documents have been shocked by the vagueness, ambiguity and unwonted eloquence of these documents. One constantly feels that there is an attempt at soft-pedalling all the failures of the past, a shirking from a frank discussion of the fundamental issues of policy, and an attempt to promise everything to everybody, as in an election manifesto of a political party. The publication of a plan coincides with the tension-charged campaign for the general election in the country."

Mirrlees gave a paper discussing planning in mixed economies with "surplus labor". The summary reads "He analyzed the significance for investment planning of sub-optimum levels of aggregate investment and of non-optimum distribution of consumption. The criterion function to be maximized is a function of consumption, with different accounting prices for the consumption of different groups, and of the value of savings, valued at an accounting price. Estimation of the accounting prices is then discussed." This research foreshadows his joint brochure with Ian Little on criteria for choosing investment projects in developing countries.

Sheshinski and Burmeister gave a paper on the non-switching controversy treating the case of fixed capital. Also several technical papers were presented with, I assume, an educative purpose. Arrow spoke on duality in Hamilton-Pontryagin theory deriving the differential equations for p and k. Kalman talked about control theory, deriving the conditions for controllability, reachability, constructability, and observability. Finally at the start of the conference there were two round tables, one on population and growth led by Mirrlees and one on uncertainty and growth led by Radner.

In the round table on population and growth Mirrlees chose two criteria. If population is determined exogenously Mirrlees favored the criterion to maximize the integral over the planning period of population size times the utility of per capita consumption. However, if population is a policy variable he favored an objective equal to this integral divided by the integral of population size over the period. His periods were finite. The discussion was extensive. I was surprised to see the part I played in the discussion. I first suggested that the criterion should yield decisions on the rate of growth of population which would be made by free choice if individuals bore the full social cost of having children. I also gave a discussion of the consequence of purely selfish behavior by the present population. This might approximate maximizing the sum of per capita utility discounted by the rate of population growth, which we were tending to use in our models. I also suggested that continuity of generations might be enough to keep the optimal path under selfish optimization close to that under unselfish optimization. Koopmans noted that if population was a policy variable it would be necessary for a planner to choose a zero for the utility function.

In the round table on uncertainty and growth one might expect to find some signs of Radner's classic paper on competitive equilibrium with uncertainty in Econometrica, 1972, but they are hard to find. However, he did present a recursive formulation of a competitive equilibrium. The summary reads "Suppose we deal with a finite horizon T, and look at period $T-1$. Then all decision-makers have (different amounts of) information on the past actions of others, and the only remaining uncertainty is about the environment. The extended Arrow-Debreu model applies, yielding a definition of efficiency. By considering periods $T-2$, $T-3, ...,$ 1, we get recursive definitions of equilibrium and

efficiency." There is no sign of the definition of asymptotic properties of optimal paths in terms of distribution functions in the manner of Brock and Mirman. The only turnpike discussed is for a Samuelson type theorem where the analog to the von Neumann coefficient of expansion is the maximum over goods of the expectation of the log of the expansion factor over the period.

6. LATER MSSB CONFERENCES AND FURTHER DEVELOPMENTS

Further conferences on growth theory were held on the initiative of the MSSB in subsequent years. In 1967 a Symposium on Economic Growth led by Uzawa was held at the University of Chicago. This was a short conference in the style of our conference here. Papers delivered included Uzawa's paper on the Penrose effect, a paper by Foley and Sidrauski on Government Debt, Stabilization Policies, and Economic Growth, and a paper by Goldman on Successive Planning and Continual Planning Revision. In 1968 another short conference was held at Brown University on Money and Growth led, I believe, by Jerome Stein. Papers were read by S. C. Tsiang, Stein, Hahn, Levhari and Patinkin, Foley and Sidrauski, and others. In the summer of 1971 a conference on Uncertainty in Markets was held in Berkeley led by Roy Radner, which included some papers on optimal growth, in particular, the two papers of Brock and Mirman on growth with uncertainty, the undiscounted and the discounted cases. This is also where Bernt Stigum's papers on competitive equilibrium with infinitely many commodities appeared, papers later published in Econometrica, which were missed by Mas-Colell in his survey of this subject in the Handbook of Mathematical Economics, Volume 4. Two further conferences were held in the period before 1976 sponsored by MSSB on the Hamiltonian approach to economic dynamics, at Squam Lake in New Hampshire, in facilities owned by Dartmouth College, and at the University of Pennsylvania. These were led by David Cass and Karl Shell. Their accomplishment will be described shortly.

Let us consider how the research program initiated by Ramsey and von Neumann and further defined by Samuelson and Solow, and Uzawa, was promoted by these conferences. The first conference, in Cambridge, is the only one that devoted much of its attention to the Samuelson turnpike theorem in the von Neumann model, although the inadequacy of the objective function was recognized. However there were two papers where the Ramsey objective of maximizing a sum of utilities over the infinite horizon was used. Radner uses the optimality principle of Bellman to approximate the State Valuation function, as he calls it, or simply the value function in the modern terminology. He proves that a stationary policy exists that comes arbitrarily close to generating an optimal path. Under his assumptions the finite initial utility sums are uniformly bounded. He considers both finite and infinite paths, and both open and closed, that is, von Neumann style, models. He gives an algorithm for finding the optimal policy in his linear logarithmic economy, which is a multisector model with a discount factor. Finally Chakravarty discussed the problem of preference functions for investment programs. But the turnpike problem for optimal paths was not addressed in a model with an infinite horizon and a utility function.

In the conference at the Vatican a stability problem was addressed by Morishima in a multisector model of the linear logarithmic type developed by Radner. However the decision to consume or to save was not solved by maximizing utility but by the arbitrary rule that workers only consume and capitalists only save. So his results do not refer to the modern turnpike notion. Of course, the papers of Koopmans and Malinvaud essentially solved the problem of the optimality turnpike for one sector deterministic models with exogenous population growth, strictly concave utility, and strictly convex technology, with or without a discount factor.

Finally in the Rochester conference the modern view of the existence of optimal paths and their asymptotic properties in multisector models was clearly recognized. Of course, the turnpike problem was only addressed in models without a discount factor. However between this conference and its successor at Stanford the problems which had been solved by Koopmans and Malinvaud at the Vatican

were solved for the case of many goods and no discounting using the overtaking criterion. I suspect the overtaking criterion was brought to the Rochester conference by Mirrlees who had seen the paper of Weizsäcker that was published in the Review of Economic Studies, April, 1965. Weizsäcker used the one sector model. Thus in these four conferences we had passed from problems defined by the von Neumann closed model with a terminal objective to open Ramsey models with infinite horizons and the objective of maximizing the sum of utility over time. Several further major developments were to occur before reaching the current period in this research program, the proof of stability theorems in multisector models when discount factors are present and utility sums are maximized, the proof of these existence and stability theorems when uncertainty is present, the establishment of the relation of the theorems in optimality models to the properties of competitive equilibria over time, and determining conditions that lead to complex optimal paths. Most recently the so-called New Economic Growth has appeared where there is unbounded growth in the manner of von Neumann models and turnpike theorems like the Samuelson turnpike are proved, but with infinite paths and Ramsey objectives rather than terminal objectives.

Turnpike theorems in multisector models with more than two sectors and discount factors were first reported in the conference held at Squam Lake. The critical papers were given by José Scheinkman, Cass and Shell, and William Brock and Scheinkman. They established the turnpike theorem for models satisfying some strict convexity conditions, that is, no nontrivial von Neumann facets were allowed. These papers were published in a symposium issue of the Journal of Economic Theory, February, 1976.

The problem of proving a turnpike theorem with uncertainty was addressed by Leonard Mirman and William Brock and reported to the Berkeley conference. Their papers were published in the Journal of Economic Theory, 1972, and the International Economic Review, October, 1973. The key innovation was to describe the turnpike in the one sector case as a distribution function of capital stocks rather than as a particular capital stock. They first dealt with the discounted case and then in the later paper with the undiscounted case. The discounted case was extended to the multisector case in 1978 by Brock and Majumdar.

These were the researches in which the conference series begun by the SSRC and continued by the MSSB were important. However, one further development in this research program was made in which these conferences were not active. This is to establish the relevance of the turnpike in optimal growth models for asymptotic competitive equilibrium. The problem was addressed by Robert Becker and by Truman Bewley. Important further results were reached by Makoto Yano. This completes the story for the model that marries the Ramsey objective with the von Neumann model.

However, in the tradition of the two sector model an endogenous or New Growth Theory was initiated by Paul Romer and Robert Lucas, following in the footsteps of Uzawa. This has served to re-ignite the interest of the wider public in the subject of growth economics. Finally the study of chaotic optimal paths in one sector models was begun by Michele Boldrin and others, carried further by Mitra, Yano, Nishimura, and Majumdar and others. This literature is surveyed by Boldrin and Woodford in the Journal of Monetary Economics in 1990. Perhaps this brings us up to the papers that are being presented at this conference.

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