An Operational Theory of Monopoly Union-Competitive Firm Interaction

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I. INTRODUCTION

Union behavior has long been a focus of interest for labor economists. As a result there exists an extensive literature on many topics related to unionism. Recently a large literature has re-opened the debate on the size of the union-nonunion wage differentials by allowing in one way or another for endogeneity of union status. (See Lewis, 1986; and Parsley, 1980, for surveys of these studies). There have also been new debates on other union topics. One issue has been whether union-firm contracts are "inefficient" in the usual monopoly sense—i.e. is the wage-employment combination obtained by the union setting a wage and the firm choosing a point on its demand curve; or is the wage-employment combination on the contract curve? (See, for example, McDonald and Solow, 1981; MacCurdy and Pendavel, 1983; Oswald, 1982, and especially 1984, and Eberts and Stone (1986)). A related issue concerns the union objective function and whether anything can be learned from observed data (see Pencavel, 1984). Unions' membership in recent years has declined quite substantially. Attempts have been made to explain this pattern (Freeman, 1984; Neumann and Rissman, 1984). Strike activity has received some attention, in the form of both theoretical and empirical studies (Hayes, 1984; Reder and Neumann, 1980; Kennan, 1980). Finally, some authors have focussed more on the members that make up a union and the firms that make up the industry, rather than dealing directly with the aggregate entities "union" and "industry" (Lazear, 1983; Oswald, 1982, 1984).

While the literature has made substantial contributions to the understanding of the effects of unions, it suffers from two unfortunate limitations. First, as Pencavel (1984) notes, the standard union models have very few testable propositions. Second, though linked by a common focus on
unions, the literature is highly fragmentary; models in which a variety of issues may be addressed simultaneously are notable by their absence. Rather, the tendency has been towards utilization of distinct models to analyze each separate topic. Compare, for example, the models used to explain the pattern of intertemporal incidence of unions (Ashenfelter and Pencavel, 1969; Freeman, 1984), union-nonunion wage differentials by industry (Parsley, 1980), and strikes (Hayes, 1984). This disparate approach has ruled out a potentially fruitful source of predictions—namely restrictions on the covariation of several endogenous variables considered together—in addition to impeding the development of a coherent view of the whole set of union issues.

The recent literature dealing with union objective functions is in part a response to the lack of predictions from the standard monopoly union model. The strategy has been to search for restrictions on the objective function that would command the same general level of agreement as profit maximization for firms. Unfortunately, such agreement has not been forthcoming so that the predictions remain sensitive to the choice, among a wide class, of competing objective functions.

The essential departure of the model presented in this paper from earlier work is that the unions are assumed to use resources in their dealings with workers and firms. The strategy is thus to try and make progress by a more detailed specification of the environment in which unions operate rather than by developing different objective functions. The most important consequence of this assumption is that equilibrium will not generally imply unionization of all firms in an industry; i.e. incomplete union coverage. It transpires that this outcome allows the union to satisfy the constraint implied by the market demand for output in a somewhat less restrictive fashion than is typical. With this relaxation, clear predictions are straightforwardly obtained.
The analysis presented below makes use of a particular objective for the union. The exact nature of the predictions are of course related to this specification, and happily so, since discriminating among alternatives would be difficult otherwise. However, the central point is that the development of a model which yields incomplete coverage allows predictions to be made, and will do so for a variety of union objectives. Thus the problems of general agreement on an appropriate union objective function may be to a large extent side-stepped.

The possibility of incomplete union coverage that arises from union activities being costly also has the advantage of permitting the specification of a more general model than is usually the case. The standard monopoly models imply 100% unionization of an industry and thus cannot address the problem of union incidence, for example. The model presented in Section II below permits the simultaneous derivation of predictions concerning a whole collection of union issues from a simple general model. Section III displays the model's equilibrium, a wide variety of predictions being derived in Section IV. The model is first studied in a very elementary environment. That it can be embellished to accommodate and make predictions regarding a wider variety of issues, demonstrating the model's easy manipulability, is established in Section V. It is also shown that the theory is of assistance in interpreting some existing empirical "facts", and that at some cost, the model's detailed predictions can be stated in a fashion which is relatively undemanding in terms of data requirements. Section VI suggests ways of testing the model. Some conclusions and suggestions for further work are presented in Section VII.
II. **The Basic Model**

The primary departure of the analysis presented here from the standard union models in the literature is the incorporation of an explicit cost structure for the union activities. In this section the cost structure is introduced into a simple framework for the interaction of unions and firms. This makes it possible to highlight the essential differences produced by the introduction of union costs. In later sections elaborations of the basic model are presented. These retain the basic features of this section but permit a richer variety of union questions to be addressed.

The question of union objective functions has received a great deal of attention over many years with, as yet, little agreement as to an appropriate specification.\(^1\) For conventional union models the particular objective function is crucial to the basic predictions of the model. However, when union costs are introduced, many important predictions may be obtained that are not sensitive to the objective function. For the basic model the union's only activity is the collection of union dues. The natural objective function in this context is the maximization of these dues net of the costs of collecting them:

\[
D = dN\lambda - u(N,\lambda)
\]  

(1)

where \(d\) represents unit union dues; \(N\) is the number of union firms; \(\lambda\) is the number of workers hired by each union firm; and \(u(\cdot)\) is the cost of the union's activity—collecting dues from \(\lambda\) workers in \(N\) locations.

The importance of the cost structure in (1) is its potential for limiting union size to less than the entire industry. This is a major
departure from the union literature in which, as in the conventional monopoly
union model, for example, the level of unionization is always 100%. The
precise conditions for unionization to be less than 100% of the industry are
specified below. It is intuitively clear from inspection of (1), however,
that since revenue rises in the product $Nl$, for the union to be limited to
less than the entire industry, marginal costs must rise with $N$ and $l$. Thus,
the cost conditions needed are analogous to the cost conditions necessary for
limiting the size of multi-plant firms. The specification adopted here is as
follows: $u(\bullet)$ is assumed to be monotonically increasing in both arguments,
strictly convex, and twice continuously differentiable; $u(0,0) = u(0,l) = u(N,0) = 0$. Moreover, it is required that the cross effect $u_{Nl}$ not be "too
large". Using subscripts to denote partial derivatives, the restriction is:
$u_{Nl} - Nu_{Nl} > 0$.

In maximizing $D$, the union faces several constraints on its activities.
Conventional models normally assume that the union has some monopoly power
over selling labor. This assumption is retained here. It is also often
assumed that the union faces a single firm. This assumption is not retained.
Rather, the union is assumed to operate in an otherwise competitive
environment. The problem may then be cast as one of maximizing the returns
from having monopoly rights on one input in the production process. The value
of this monopoly will be affected by the nature of the environment in which
the union has to operate. In achieving the maximized value the union's
behavior will depend on parameters of the environment. This departure forms
the basis for the predictions about union behavior derived below.

In extracting the rents to its monopoly rights the union's activities
must ultimately raise the price of the final product above that which would
prevail under competitive free entry conditions; the increase being the source
of the surplus the union obtains. Thus the union must somehow limit entry into the industry. In this paper the union does so in precisely the fashion implicit in conventional models of unions--by threatening potential entrants with unionization. That this threat is credible in equilibrium is established below.

Of the firms producing in the industry, the union designates how many are unionized and how many are not. Unionized firms are required to pay their workers the union wage \( w \), while nonunion firms need not do so, and thus pay the competitive alternative wage, \( \hat{w} \), to workers in this industry. This specification rules out many types of union-firm interaction--lump-sum payments for example. The point of doing so is to force logical distance between a model of a unionized industry and what becomes (interpreting \( u(\cdot) \) as enforcement cost) a producer's cartel. That is, given enough freedom in its interaction with firms, the union effectively owns them.

Thus, there are three types of firms that the union has to deal with: union, nonunion, and potential entrants. Consider first the optimizing behavior of unionized firms.

Entrepreneurs are all equally good at operating firms, and have best alternatives valued at \( A \). The industry is assumed to be small relative to the economy, in which case \( A \) is the constant supply price of entrepreneurs.

The technology used to produce output \( Q \) has the number of workers \( L \) as its sole variable input:

\[
Q = f(L);
\]

(2)

\( f' > 0, f'' < -\epsilon \) for some \( \epsilon > 0 \), and \( \lim_{L \to 0} f' = \infty \).
At this point it is assumed that the union does not provide any services the firm finds either productive or harmful. Let \( h(Q) = f^{-1}(Q) \) be the labor input required to produce \( Q \); \( h' > 0, h'' > \zeta \) for some \( \zeta > 0 \), and \( \lim_{Q \to 0} h' = 0 \).

Then for any \( \bar{w} \), the firm's variable cost function is

\[
 c(\bar{w}, Q) = \bar{w}h(Q),
\]

and total costs are

\[
 F + c(\bar{w}, Q),
\]

where \( F = R + A \), and \( R \) is expenditure on fixed factors. Given the price of output, unionized firms earn profit

\[
 \pi = pQ - F - wh(Q)
\]

if they produce \( Q \). Unionized firms therefore produce either \( Q = 0 \) or \( Q = Q^* \), where

\[
 Q^* = \arg\max_{Q \in [0, \infty)} \{ pQ - F - c(w, Q) \},
\]

depending on whether \( pQ^* - F - c(w, Q^*) \geq 0 \).

In a similar fashion, nonunion firms face total costs \( F + c(\hat{w}, Q) \) and produce either \( Q = 0 \) or \( Q = \hat{Q} \), where
\[
\hat{Q} = \arg \max_{Q \in (0, \infty)} \{pQ - F - c(\hat{w}, Q)\},
\]

depending on whether \( \hat{\pi} \equiv p\hat{Q} - F - c(\hat{w}, \hat{Q}) \geq 0 \).

For given \( w \), union firms must earn zero profits. The argument is that non-negative profits are required to induce union firms to produce, while nonpositive profits are necessary if the union's threat to unionize entrants is not be regarded as an invitation. This result and the definition of \( Q^* \) imply unionized firms produce

\[
Q^* = q(w,F) \equiv \arg \min_{Q \in (0, \infty)} \frac{F + c(w,Q)}{Q}, \tag{6}
\]

and the equilibrium product price is

\[
p^*(w,F) = \frac{F + c[w,q(w,F)]}{q(w,F)} = wh'[q(w,F)]. \tag{7}
\]

Also, nonunion output \( \hat{Q} = \hat{q} (\hat{w},w,F) \) is the unique solution in \( Q \) to

\[
p^*(w,F) = \hat{wh}'(Q)
\]

Finally, assuming potential entrants regard the union's threat as credible, their behavior is simply abstention.

In summary, union firms produce \( q(w,F) \)--the output which minimizes their average cost given \( w \) and \( F \)--and must obtain zero profits in equilibrium. If there are any nonunion firms, they earn non-negative profit, and do so by producing \( \hat{q} (\hat{w},w,F) \). Potential entrants remain "potential", and the price of output is \( wh'[q(w,F)] \).

Consumers play a passive role in the analysis. Their behavior is summarized by a market demand function \( X = \varphi(p) \), where \( X \) is the total
quantity purchased and $p$ is the price of output. It is assumed that (i) $\varphi$ is twice continuously differentiable, with $\varphi' < 0$; (ii) for all $p \geq \bar{p}$, where $\bar{p} < \infty$, $\varphi(p) = 0$; (iii) for all $p > 0$, $\varphi(p) < \bar{\varphi}$ for some constant $\bar{\varphi}, 0 < \bar{\varphi} < \infty$; and

$$(iv) \quad p > \frac{F + c(\hat{w}, \hat{Q})}{\hat{Q}},$$

where $\hat{Q} = \arg\min Q \quad \frac{F + c(\hat{w}, Q)}{Q}.$

That is, if there were no union, demand is such that the good would indeed be produced in a free entry competitive equilibrium.

Workers are identical in all respects. They find all production activities in this industry equally distasteful and work an exogenously fixed number of hours in any firm. The employment options workers face are as follows. The best alternative to participation in the industry generates utility $V > 0$. Work in nonunion firms is also available, paying wage $\hat{w}$ and generating utility $v(\hat{w})$ where $v(\cdot)$ is an indirect utility; $v' > 0$. Union firms pay wage $w$ and the union levies dues of $d$. For the basic model, it is convenient to assume that union membership confers no consumption benefits or costs on workers. Under this assumption union workers obtain utility $v(w-d)$. Workers are assumed to be freely mobile so that

$$w - d = \hat{w} \quad \quad (8)$$

must hold.

The union's problem may now be written:

$$\max_{M,N,w} D(w,F) = (w-\hat{w})\pi(w,F) - u[N,\lambda(w,F)]$$
S.T. \( Nq(w,F) + Mq(\hat{w},w,F) = \varphi(p), \)

\[ p = wh'[q(w,F)], \]

\[ N \geq 0, \text{ and } M \geq 0. \]

where \( M \) is the number of nonunion firms. The first constraint is simply that the market for output clears. The second is that, as noted earlier, union firms must earn zero profits.

Since \( N > 0 \) is necessary for \( D > 0 \), \( N \geq 0 \) is satisfied provided the union chooses to operate, which is assumed. Further, non-negativity of \( M \) and \( p = wh'(\cdot) \) can be included in the first constraint. Proceeding this way, the programming problem becomes

\[
\max_{w,N} D(N,w) \quad \text{S.T.} \quad Nq(w,F) \leq \varphi[wh'[q(w,F)]]. \tag{9}
\]

The next section presents the solution to this problem.

III. **EQUILIBRIUM IN THE BASIC MODEL**

The Lagrangian for problem (9) is

\[
\Lambda(N,w,\lambda) = (w-\hat{w})Nl(w,F) - u[Nl(w,F)] + \lambda[\varphi - Nq(w,F)],
\]

where \( \lambda \geq 0 \) is an undetermined multiplier. First-order conditions for a maximum are (asterisks denoting optimal values)

\[
\frac{\partial \Lambda}{\partial w} = N^*l + (w^*-\hat{w})N^* \frac{\partial l}{\partial w} - u \frac{\partial l}{\partial w} + \lambda^*[\varphi' \cdot w^*h' \cdot \frac{\partial q}{\partial w}] \\
- N^* \frac{\partial q}{\partial w} = 0, \tag{10}
\]

\[
\frac{\partial \Lambda}{\partial N} = (w^*-\hat{w})l - u - \lambda^*q = 0, \tag{11}
\]
and \[ \frac{\partial \lambda}{\partial \lambda} = \psi - N^*q \geq 0, \] \hspace{1cm} (12)

where \( \lambda^* > 0 \) only if (12) is an equality. Second-order necessary conditions are long expressions with the usual interpretations. If \( \lambda^* = 0 \), \( \pi(N, w) \) must be strictly concave in \((N, w)\) for \((N, w)\) in a neighborhood of \((N^*, w^*)\). For \( \lambda^* > 0 \), the locus of \((N, w)\) pairs for which \( \pi(N, w) = \pi(N^*, w^*) \) must have "more curvature" than the locus of \((N, w)\) pairs satisfying \( \psi - Nq = 0 \), in the neighborhood of \((N^*, w^*)\). 4

Consider the interpretation of (10). First, the derivatives \( \partial \lambda / \partial w \) and \( \partial q / \partial w \) are required. Differentiating and rearranging:

\[ \frac{1}{q} \frac{\partial c}{\partial Q} - \frac{1}{2} \frac{[F + c(w, q)]}{q} = 0 \] \hspace{1cm} (13)

and

\[ \frac{2}{\partial Q} c > 0. \]

It follows that (recall \( c(w, Q) \equiv wh(Q) \))

\[ \frac{\partial q}{\partial w} = \frac{\psi/q - h'}{\psi} < 0 \] \hspace{1cm} (14)

and since \( l(w, F) \equiv h[q, (w, F)], \)

\[ \frac{\partial l}{\partial w} = h' \frac{\partial q}{\partial w} < 0. \] \hspace{1cm} (15)

A rise in the union wage rate reduces output and therefore labor input in unionized firms.

Returning to (10), an increment to the wage generates greater dues from \( N \) workers. However, the number of workers hired by each union firm falls, reducing both dues collected and union costs. If there are no nonunion firms,
in which case \( \lambda^* \geq 0 \), an increment to the wage also affects the constraint—hence the last term in (10).

To examine the last effect further, consider the constraint \( Nq - \varphi = 0 \). For given \( w \), reductions in \( N \) always cause \( Nq < \varphi \), and conversely for increases in \( N \). \( Nq - \varphi = 0 \) therefore defines a unique \( N \) for each \( w \): \( N(w) \equiv \varphi/q \). Under the assumptions made above, \( N(w) \) is twice differentiable. As \( w \to 0 \), (11) implies \( q \to \infty \), so \( N \to 0 \) is required to satisfy \( Nq - \varphi = 0 \) provided \( \varphi \) is bounded, as is assumed; \( N(0) = 0 \). On the other hand, since \( [F + c(w,q)]/q \) is an increasing function of \( w \) (with derivative bounded away from 0), raising \( w \) eventually generates \( [F + c(\tilde{w},q)]/q \neq \tilde{p} \) for some \( \tilde{w} < w \), in which case \( \varphi(\tilde{p}) = 0 \) implies \( N(\tilde{w}) = 0 \). \( N(w) = \varphi/q \) can be thought of as the locus of \((N,w)\) pairs for which aggregate nonunion output is 0. More generally, \( N(w,\alpha) \equiv (\varphi - \alpha)/q \) is the locus of \((N,w)\) pairs for which nonunion output equals some \( \alpha \geq 0 \). The slope of this locus is

\[
\frac{\partial N(w,\alpha)}{\partial w} = \frac{1}{2} \left[ \frac{\varphi'h - Nq}{q} \right]
\]

which may take on either sign. That is, an increase in \( w \) lowers each union firm's output, and thus \( Nq \) for given \( N \). But the price of the product must rise too, and so quantity demanded is reduced. Whether the number of union firms required to produce \( \varphi - \alpha \) rises or falls depends on whether the output effect, operating through the output of each union firm, exceeds or falls short of the product demand effect. If \( \partial N(w,0)/\partial w > 0 \) an increase in \( w \) slackens the constraint, hence the last term in (10).

In (11), an increment to \( N \) yields dues from \( I \) workers and raises union costs. If there are no nonunion firms, an addition to \( N \) also tightens the constraint.
One issue which immediately emerges is whether the union will choose to unionize all the firms in the industry: Will there be complete union coverage? Perhaps the most obvious way to answer this question is to point out that coverage will be incomplete if and only if the global maximum of \( D(N,w) \) with respect to \((N,w)\) lies inside the constraint. For this condition to hold, it is necessary that \( D(N,w) \) indeed have a global maximum for finite \((N,w)\). The possibility of infinite \( N \) for given \( w \) is ruled out by \( u_{NN} > 0 \). The possibility of perpetual wage increases, for given \( N \), is best analyzed by rewriting \( D(N,w) \) as
\[
D(N,w) = wNl - [u(N,l) + wN]<.\]

The usual monopoly-type necessary condition that \( l(w,F) \) be elastic in \( w \) emerges. Thus the convexity of \( u(*) \) and, sufficient convexity in \( h(*) \), gives \( D(*) \) a global maximum for finite \((N,w)\), in which case the existence of nonunion firms depends on the location of the constraint. Consequently, there are two cases---complete and incomplete coverage which has important consequences for the predictive content of the model.

Finally, is the union's threat to unionize any potential entrant credible in equilibrium? The assumptions required for the existence of a zero profit competitive equilibrium with a determinate firm size are also sufficient to guarantee the answer to be in the affirmative. Loosely, to achieve zero profit competitive equilibrium when firms possess U-shaped average cost curves and demand is arbitrary, the minimum point on the union firms' average cost curve must occur at a level of output which is "small" relative to demand (see the account in Sonnenschein, 1982). The output
for which price equals marginal cost for nonunion firms must also be small. Given these assumptions, increments to demand can be accommodated by entry of nonunion firms producing at the output for which price equals marginal cost, or union firms producing at minimum average cost. Under these assumptions, in the union's problem the number of firms designated union and nonunion are appropriately (as was implicit above) treated as continuous variables. It follows that at the union's optimum \((N^*, w^*)\) either

(i) the union is indifferent about whether to unionize a small number of potential entrants given \(w^* - \lambda^* = 0\) and \(\pi_N = 0\);

or

(ii) the union is indifferent about whether to unionize a small number of potential entrants and adjust \(w^*\) slightly--\(\lambda^* > 0\), \(\pi_N > 0\) and \(\pi \geq 0\).

In either case the threat to unionize any individual firm, or a small coalition for that matter, is entirely credible. Consistent treatment of a large union and competitive firms implies the former has the power to threaten credibly.

IV. **Predictions from the Basic Model**

The assumptions that the union uses costly resources in its activities results in the possibility of less than 100% unionization of an industry—the incomplete coverage case. In this case the constraint in (9) does not bind; the presence of nonunion firms frees the union from having to adjust the number of unionized firms in a particular way to clear the market for output when \(w\) is perturbed. Instead demand can be accommodated by adjustments to the number of nonunion firms.
This additional freedom yields a large class of new predictions that
could not be obtained from the standard model. Two examples: First, it
permits predictions on the relation between union coverage and wage
differentials in competitive industries. By contrast, in the standard
monopoly model coverage does not even vary. Second, compared with the
complete coverage case, it predicts radically different responses in union
behavior to changes in industry demand when unionization is less than 100%.
Since the incomplete coverage case provides the bulk of the new predictions,
this case receives emphasis below.

**Incomplete Coverage Equilibrium**

In the incomplete coverage equilibrium, (12) may be ignored and
(10)-(11) simplified to:

\[
D = w^\times \lambda + (w^\times - \hat{w})\lambda \frac{\partial L}{\partial w} - u \frac{\partial L}{\partial w} = 0
\]

(17)

and

\[
D = (w^\times - \hat{w})\lambda - u = 0
\]

N

(18)

Second-order conditions require \(D_{\lambda\lambda} \leq 0, D_{\lambda\lambda} \leq 0\) and \(D_{w^\times w^\times} - D_{\lambda\lambda} \geq 0\)
when all are evaluated at \((N^\times, w^\times)\). Assume all three inequalities hold
strictly.

First consider changes in the demand for the product. Since \(\varphi(p)\) does
not appear in (17) or (18), neither \(w^\times\) nor \(N^\times\) depends on it, from which it
follows immediately that \(p^\times\) does not vary either. Accordingly, all changes in
\(\varphi(p)\) are fully captured by the increment to \(\varphi(p^\times)\), and the sole response is
in terms of entry or exit of nonunion firms:
\[ dM^* = \frac{1}{q} \Phi(p^*). \]

This result stands very much in contrast with most of the traditional literature. The characteristics of product demand have figured prominently in discussions of the determination of union wages and employment since Marshall (1896). The literature dealing with union attitudes towards tariffs, government programs to stimulate various industries, etc, all assumes that union employment and wages will be affected by such measures. The above result suggests that such will only be true for unions with complete coverage. Only those unions, therefore, would have an incentive to engage in lobbying activities to affect product demand.

Next, consider variations in the value of workers' alternative \( V \).

Since \( dV = \nu^d \hat{w} \), such a change translates into \( d\hat{w} \). Application of the usual calculus to (17) and (18) yields: \(^5\)

\[ \frac{d\hat{w}^*}{dw} > 0 \quad \text{and} \quad \frac{dN^*}{dw} < 0. \]

Thus a rise in the value of alternative opportunities generates an increase in the union wage and a decline in the number of union firms. The intuition is just that the initial rise in \( \hat{w} \) operates as a factor price increase for the union, which therefore responds by scaling back operations directly, via reducing \( N^* \), and indirectly through the reduction in \( l \) induced by raising \( w^* \).

The basic results \( dw^*/d\hat{w} > 0 \) and \( dN^*/d\hat{w} < 0 \) immediately imply a variety of results about other attributes of the union sector of the industry.
First, consider the union–nonunion wage differential \( \delta \equiv w - \hat{w} \). A tedious derivation indicates that \( \delta \) will rise with an increment to \( \hat{w} \) unless the increase in \( w \) generates a sharp absolute increase in the elasticity of \( l(w,F) \) with respect to \( w \). The leading case is clearly \( d\delta/d\hat{w} > 0 \). This result offers sharp contrast with the usual monopoly union model, which has inherently ambiguous predictions regarding the union differential.

Surprisingly, the existing literature almost entirely ignores the determinants of the wage differential.

Union output and employment fall at both the firm and aggregate level. Since unionized firms produce \( q(w^*,F) \),

\[
\frac{dq}{dw} = \frac{\partial q}{\partial w^*} \frac{dw^*}{dw} < 0,
\]

and

\[
\frac{dl}{dw} = h^* \frac{\partial q}{\partial w} < 0.
\]

Total output of, and employment in, unionized firms are \( Nq \) and \( N^*l \), respectively, in which case

\[
\frac{d}{dw} N^*q = q \frac{dN^*}{dw} + N^* \frac{dq}{dw} < 0
\]

and

\[
\frac{d}{dw} N^*l = l \frac{dN^*}{dw} + N^* \frac{dl}{dw} < 0.
\]

Furthermore, since \( l(w,F) \) is elastic with respect to \( w \), payments to union workers \( w^*N^*l \) fall in total:
\[
\frac{d}{dw} w^N*^L = N^* \left( \frac{d}{dw} w^L \right) \frac{dN^*}{dw} + w^L \frac{dN^*}{dw} < 0,
\]
as well as when measured as a fraction of total factor payments in unionized firms,
\[
\frac{d}{dw} \left( \frac{w^L}{F + w^L} \right) = \left[ \frac{1}{N^* w^L} - \frac{1}{F + w^L} \right] \frac{d}{dw} w^L < 0.
\]

Two final predictions for changes in \( \hat{w} \) concern union profits and the equilibrium product price. First, the envelope theorem implies \( dD/d\hat{w} < 0 \) - the amount of resources agents might be willing to expend to acquire the union monopoly position or organize workers declines as \( \hat{w} \) rises. Second, \( dw/d\hat{w} > 0 \) implies an increase in the level of minimum average cost, and hence \( \frac{dp^*}{dw} > 0 \),

where \( p^* \) is the equilibrium price of the product. It is nevertheless true that although \( p^* \) rises, both the revenue of each unionized firm \( (p^*q) \) and the revenue of the union sector \( (N^*p^*q) \) fall when \( \hat{w} \) rises. Such must occur simply because factor payments \( (F+w^L) \) and \( N^*[F+w^L] \) fall and union firms must earn zero profits.

The above results constitute unambiguous predictions in response to changes in \( \hat{w} \) for a variety of aspects of union behavior, at both the individual union firm and aggregate union sector levels. An additional feature of interest in the union literature, however, is the extent of union organization in the industry as measured by the fraction of total employment.
which is unionized. In order to consider the changes in this entity, predictions on the nonunion component of the industry are required. Consider first, the aggregate behavior of nonunion firms. Recall that these firms simply fill in the difference between total union output and quantity demanded at the equilibrium price. When \( \hat{w} \) rises, total union output falls; however the equilibrium price rises so that total quantity demanded also falls. Consequently, for a given reduction in union output, the change in nonunion output depends on the price elasticity of demand for the project. When product demand is not too elastic, for example, the impact of changes in \( \hat{w} \) on union employment, \( N_u \), translates into a change in the fraction unionized \( N_u/(N_u + M_u) \) [where \( l \equiv h(q) \)] of the same sign.

A final set of experiments considered in this section involves the union cost function \( u(N, l) \). Implicitly \( u(N, l) \) is the solution to a cost minimization problem wherein the union uses factors to collect dues. If the price of the \( j^{th} \) factor used by the union is denoted \( r_j \), and both \( u_{jr}^N > 0 \) and \( u_{jr} > 0 \), it is trivial to obtain

\[
\frac{dw^x}{dr_j} > 0
\]

and

\[
\frac{dn^x}{dr_j} < 0.
\]
That is, an increment to \( r_j \) causes a scaling back of union operations directly, by reducing \( N \), and indirectly through \( \partial \lambda / \partial w < 0 \).

Since the change in \( r_j \) has no direct impact on union or nonunion firms (i.e. given \( w^* \)), the predictions obtained for this experiment are qualitatively identical to those following from an increase in \( \hat{w} \).

Discussions of the effects of union costs on union incidence across industries may readily be incorporated into the theoretical structure outlined here. A particularly simple specification has been adopted for the cost function—a specification sufficient to produce the possibility of incomplete coverage. In many of the discussions of union costs it is argued that costs are higher per member when there are a large number of individual plants, when there is higher turnover etc. These arguments may easily be cast in the form of appropriate restrictions on \( u(\cdot) \). Fixed costs, both with respect to \( N \) and \( l \) may be added. Further, since differences in the sizes of firms will affect costs across different industries, variations in firms' fixed costs will induce variation in the observed union costs and hence be relevant for predictions concerning industrial union incidence. Predictions for these costs are readily obtained.\(^6\)

**Complete Coverage Equilibrium**

When the constraint \( M \geq 0 \) is binding, \( \lambda^* > 0 \) in (10)–(12) provided \( M = 0 \) strictly dominates \( M = \epsilon > 0 \). In this case (12) can be solved for \( N(w) = \phi/q \), and

\[
D[N(w), w] \equiv (w - \hat{w}) \frac{\phi \lambda(w, F)}{q(w, F)} - u[\phi(q(w, F), \lambda(w, F))].
\]
Necessary conditions for a maximum are

$$D = 0 \quad \frac{\partial}{\partial w} \quad \quad (19)$$

and

$$D_{ww} \leq 0$$

for $$w = w^*$$ solving (19). $$D_{ww} < 0$$ is imposed.

The complete coverage case is vastly more difficult to analyze when compared to the incomplete coverage setting. Most of the predictive content of the latter model obtained because $$N$$ and $$w$$ could be manipulated independently, which fails in the former. Referring back to (16), and imposing the complete coverage restriction $$a = 0$$, the sign of the required relationship between $$N$$ and $$w$$ is simply not available. Consequently, the major building block underlying the predictions provided above—a firm handle on movements in $$w^*$$ and $$N^*$$—is no longer in place. This accounts for the lack of predictions from the monopoly union model, where the entire industry is unionized, as recently emphasized by Pencavel (1984). Some headway can be made for changes in $$\hat{w}$$. Like the standard monopoly model, alterations in $$\hat{w}$$ move the union’s objective function and do not affect the constraint. In the usual way,

$$\frac{\partial w^*}{\partial w} \quad \frac{\partial}{\partial w} \quad \quad (19)$$

After some manipulation,

$$D = \frac{\phi l}{q} + (w^*-w) \frac{\partial l}{\partial w} - u \frac{\partial k}{\partial w} + [(w-w)l - u] \frac{dN}{d w}$$

in which case
\[
\begin{align*}
D &= - \left\{ \frac{\varphi}{q} \frac{\partial l}{\partial w} + \frac{\partial N}{\partial w} \right\} \\
&= - \left\{ \frac{\varphi}{q} \frac{\partial l}{\partial w} + \frac{\partial q}{q} \right\} \left[ \varphi' \frac{\partial l}{\partial w} - \frac{\partial q}{\partial w} \right].
\end{align*}
\]

Now \(-\varphi' > 0\), and

\[
- \left\{ \frac{\varphi}{q} \frac{\partial l}{\partial w} - \frac{\partial q}{q} \right\}
\]

\[
= - \frac{\partial q}{q} \left[ \frac{h'}{q} - \frac{h}{q} \right] > 0
\]

by convexity of \(hn\), and (14). Consequently,

\[
\frac{dw_x}{dw} > 0.
\]

Given this result,

\[
\frac{dq}{dw} < 0
\]

and

\[
\frac{d\hat{l}}{d\hat{w}} < 0.
\]

follow as before, as does \(dD/d\hat{w} < 0\) and \(dp*/d\hat{w} > 0\).

The prediction \(d\hat{l}/d\hat{w} > 0\), to be obtained under mild restrictions in the incomplete coverage case, does not appear to follow here even as a "leading case". Effects on the rest of the endogenous entities require knowledge of \(dN/dw\). If \(dN/dw < 0\), which could in principle be checked, the predictions for \(Nq, N\hat{l}\), and \(w*N\hat{l}\) continue to hold.
In summary, for increases in \( \hat{w} \), the complete coverage case provides either the same qualitative predictions as does the incomplete coverage case (though they differ quantitatively), or no predictions apart from \( d\hat{w}^x/d\hat{w} > 0 \), \( dq/d\hat{w} < 0 \) and \( dl/d\hat{w} < 0 \).

At this point it appears that the above are all the operational restrictions (which do not depend on specific parameter values) that the complete coverage model places on the data. This outcome stands in sharp contrast with the incomplete coverage case. For changes in demand, in particular, the independence results for most of the endogenous variables in the model may be contrasted with the general non-independence in the complete coverage case.

V. ELABORATIONS AND DISCUSSION

The basic model presented above is easy to manipulate and generates a variety of predictions. However, the setting assumed was deliberately simple to highlight the main result of incomplete coverage. Given a richer setting there are numerous other issues on which the model can shed light. In this section some of these elaborations are explored. Further, discussion of the effect of perturbing some of the model's assumptions is called for, as is information on whether the model assists in understanding existing empirical work. Finally, it is shown that at some cost, the model's predictions can be stated in other ways, and that doing so may yield a return in terms of reduced informational requirements for testing the theory.

1. A More Active Role for the Union

In the simple model presented above, the union was viewed purely as a dues collection agent. This simple treatment provided very clean analysis,
but is less than adequate for several reasons. One is that the intraindustry union-nonunion wage differential is predicted to equal the dues \( w^* = d \) and comparison of standard estimates of the differential \( w/w^* \approx 0.15 \) with typical levels of dues \( d/w^* \approx 0.01 \) would reject that hypothesis readily. Second, unionized firms seem to organize work differently, and in a fashion which is not readily explicable as a simple response to higher wages (see the data in Duncan and Stafford (1980) for example).

The point of this subsection is to show that the analysis is easily augmented to allow unions a role in the structure of production. The extension can be made more complicated, and presumably a study focusing on this issue would do so, but the simple route taken here suffices to make the point.

Suppose the union provides firms with services converting \( L \) units of labor input into \( \gamma L \) units, \( \gamma \neq 1 \)—possibly at a cost of raising \( F \) by the factor \( \phi > 1 \)—and in so doing generates services to union workers which they value at \( s \). (Here \( \gamma \), \( \phi \), and \( s \) are taken as exogenous, but they need not be). The monitoring type activities, making large assembly lines efficient, that are frequently discussed (again see Duncan and Stafford) can be treated by specifying \( \gamma > 1 \) and \( \phi > 0 \) and \( s < 0 \). On the other hand, \( \gamma < 1 \) and \( s > 0 \) may represent the on-the-job social aspect of union membership. In either case, the union is thought of as the agent who internalizes external economies which prevent individual firms from offering these services on their own.

Proceeding in this fashion, the modifications required in the above analysis are simply the replacement of (8) by \( \gamma \).

\[
\hat{w} + s - d = w,
\]
and then union firm's cost function by

$$F + (w/\gamma)h(Q).$$

The union's problem is then analogous to (9):

$$\max_{N,w} (w + s - \hat{w})N\ell(w,F,\gamma) - \bar{U}[N,\ell(w,F,\gamma); \gamma,s],$$

where

$$\ell(w,F,\gamma) \equiv h[q(w,F,\gamma)],$$

$$q(w,F,\gamma) = \arg\min_{Q} \frac{F + (w/\gamma)h(Q)}{Q},$$

and \(\bar{U}[N,\ell(\cdot); \gamma,s]\) is the cost of providing the \((\gamma,s)\) package to \(\ell\) workers at \(N\) firms.

This richer structure allows considerations of a new set of issues; for example, the effects of the union on labor productivity, and relative profitability and size of union and nonunion firms. There is little theoretical work that provides a framework within which to generate predictions on unions and firm (or plant) sizes. Parsley's (1980) survey of the empirical work links discussion of firm size primarily to its relation with the degree of concentration in product markets. Lazear's (1983) model does not have an explicit prediction for the size of union versus nonunion firms. In that setting this comparison would depend on the sign of the correlation between entrepreneurial abilities that lead to a large firm sizes with the abilities to resist unionization. If these were positively correlated, for example, nonunion firms would also be relatively large. In the present model, the relative size of union and nonunion firms could be
parameterized in terms of union-nonunion differences in $\gamma$ and $\phi$. If the productivity effects of unions do operate by making large scale production line processes efficient through solving the monitoring problems, this would be captured by $\gamma > 1$ and $\phi > 1$. The size difference across firms will then depend on the relative magnitude of $\gamma$ and $\phi$, and hence the capital intensity of union versus nonunion firms. The larger is $\phi$, the more likely it is that union firms will be larger than nonunion firms. The implications for firm size differences and the wage differential would then follow from an analysis of the effects of changes in $\gamma$ and $\phi$, analogous to that presented above.

Turning to the productivity and profitability issue, perhaps the most influential area of recent empirical analyses of unionism has been the "Harvard School" productivity studies (Freeman and Medoff, 1979, 1984 (Ch. 11); Clark, 1980). The implications of the findings in some of these studies for firm profits in the unionized sector have also generated discussion of the paradoxical result that lower costs under unionism are associated with lower firm profits. (Freeman and Medoff, 1984; Ruback and Zimmerman, 1984). The hypothesis that unions contribute positively to the production process implies $\gamma > 1$. Imposing this restriction, the relationship between profits and productivity across union and nonunion sectors is readily examined in the present model. Ruback and Zimmerman (1984) provides some evidence—that unionization lowers equity value. Freeman and Medoff (1984, Table 12.1) also conclude that unions reduce profitability of the unionized firms. This outcome is always implied by the model since a hitherto nonunion firm in the industry is making positive profits. Any change that causes the union to unionize a larger fraction of the existing firms in the industry (say, a reduction in union marginal costs) will imply lower profits for these
firms. The traditional monopoly union model in which a union takes over a competitive industry would predict no change in the profit levels of on-going firms before and after unionization since all would be making zero profits in both situations. Both Freeman and Medoff (1984) and Ruback and Zimmerman (1984) refer to casual evidence that operating firms always resist unions. This would not be a prediction of the traditional monopoly model since the firm knows it will be unionized, and thus it would not pay to use resources in a vain attempt to prevent this. On the other hand, in the present model, firms operating in an industry in which unionization is increasing will have an incentive to resist unionization since there will, in equilibrium, be nonunion firms making positive profits. It will in general pay to improve the firms' chance of being one of these nonunion firms. A related prediction is that the profitability of a firm entering the industry and being unionized (\(N^x\) increasing) should not be greatly affected by this change.

Lazear (1983) presents an explicit analysis of firms spending resources to prevent their becoming unionized; see also Kuhn (1985). Because of the complexity of Lazear's model, the amount spent to combat unionism was considered exogenous. In particular, no attempt was made to relate this to the size of the wage differential. In the simpler model of the present paper, the amount of resources spent—and more particularly its relationship to other variables in the model—can readily be derived. All that is required is a mechanism for the allocation of firms to the nonunion sector based on resources spent. The basic analysis would not change in any important way provided that these resources could not be captured by the union.
2. **Strikes**

The model analyzed above does not admit the possibility of strikes. The setting is one of complete information, and incomplete information appears to be a prerequisite for strikes to emerge as equilibrium outcomes.

An incomplete information environment in which strikes may be analyzed is as follows. Suppose that having decided to enter an industry, each firm learns the value of a firm-specific cost parameter—a location advantage, for example—the knowledge of which is private information. Assume further that the union chooses a collection of firms to unionize, as above. Having done so it is to the union's advantage to attempt to learn the firm's private information. A strike can be used to pursue this end. Specifically, equilibrium can involve the union offering two wage rates, \( w_1 \) and \( w_2 \): \( w_1 > w_2 \). Union firms can choose to pay \( w_2 \), but only by inducing a strike of length \( \tau \). The triple \((w_1, w_2, \tau)\) can be chosen along with \( N \) to maximize union profits subject to the strike length-wage combination yielding truthful revelation of cost parameters by firms, in Revelation Principle style.

Proceeding in this fashion, union wages, number of union firms and strike length are all endogenous and jointly determined. Predictions regarding the response of each to changes in the economic environment may be obtained.

3. **Union Effects on Skill Accumulation**

That the presence of unions might alter the worker's training choice in various ways has been pointed out in a series of recent papers; see, for example, Weiss (1985) and the references therein. Though the model used here is not a dynamic one, it can still prove to be a useful framework for addressing the type of questions raised in those papers.
One example will suffice. The model set out above treats the union as in some way interacting with L workers at each of N locations. It is not hard to construct a situation in which, unless dealing with skilled workers renders this process much less costly, the union finds the possibility that union firms will seek to substitute more skilled (though still unionized) workers a binding constraint on its behavior. Much like a product market monopolist, the union would prefer not to see a decrease in demand for its product. Here the product is "bodies", and the decline in demand comes via substitution of skill for bodies. In such a situation, the possibility that workers' skill might be augmented generally causes the union to lower union wages and raise the number of union firms.

4. Unconditional Predictions, and the Interpretation of Some Existing Empirical Work

The set of predictions derived above can be stated in several different ways. Pursuing the alternative representations is usually not costless, but may reduce the information required to test the model.

To proceed, note that the conventional route of defining

\[ w^* = \omega(w, F, \ldots), \]

\[ N^* = \eta(w, F, \ldots), \]

\[ \quad \ldots \]

\[ p^* = \rho(w, F, \ldots), \]

has been followed. These equations represent the reduced form of the model, and its predictions are in terms of the partial derivatives of \( \omega(\cdot) \), \( \eta(\cdot) \), etc. These predictions can be stated in an alternative form because all
exogenous variables in the model influence $w^*$ and $N^*$ in opposite directions. From this result it might be tempting to assert that the unconditional covariance of $w^*$ and $N^*$ (where the variation in $w^*$ and $N^*$ is induced by variations in underlying exogenous variables) is negative. In general this statement is correct if (but not only if) the exogenous variables are drawn from a distribution independently of one another. Given this restriction, which represents the cost of reducing informational demands, $\text{Cov}(w^*, N^*) < 0$ is implied. Moreover, since the results on $d$, $q$, $l$, $N^*l$, $N^*q$, $w^*N^*l$, $\hat{q}$, $\hat{l}$, $p^*$, $\hat{w}$, and $\pi$ are obtained from the changes in $w^*$ and $N^*$, the theory has implications for most unconditional correlations between arbitrary pairs of these variables.

Viewing the predictions this way is useful for interpretation of existing empirical results. For example, that $\text{Cov}[w^*, w^*l/(w^*l+F)] < 0$ is predicted provides an explanation for the result (see, for example, Rosen (1970)) that union labor and other factors appear to be good substitutes; that is, $\text{Cov}[-] < 0$ is usually interpreted as evidence of a substitution elasticity in excess of unity.

A second example is that the standard Lewis (1963) approach to estimation of average industry union-nonunion wage differentials can be given an interpretation in terms of underlying parameters. Letting $\bar{w}$ equal the industry average wage rate and $\mu$ the fraction unionized.

$$\ln \bar{w} \approx \ln \hat{w} + \mu \ln \frac{w}{\hat{w}},$$

assuming the geometric average approximates the arithmetic average.

Then using
\[ \ln \frac{\bar{w}}{w} = \ln 1 + \left( \frac{\bar{w} - \hat{w}}{w} \right) \equiv \nu, \]

where \( \nu \) is the percentage union-nomunion differential,

\[ \ln \bar{w} \equiv \ln \hat{w} + \mu \nu. \]

The theory implies \( \nu \) should vary with exogenous features of the industry (which it seems to; see for example MacDonald and Evans (1981)), and that given \( \nu \) (i.e. holding all exogenous factors relevant to \( \nu \) fixed), the coefficient of \( \nu \) in an estimated version of the above equation should yield an estimate of \( \nu \). This property holds because for constant \( \nu \), variation in \( \mu \) and \( \ln \bar{w} \) is induced by variation in \( \phi(p) \).

A final example concerns observed wage and price rigidities. The theory presented in this paper is a partial equilibrium model. Thus it is not in its present form appropriate for discussion of macroeconomic issues. However, individual industry evidence of wage rigidity is often alluded to in macro discussions of unemployment (e.g., Taylor, 1983). There are also micro studies of wage rigidities stemming from union behavior (e.g., Grossman, 1984). Apparent wage and price rigidities follow from the product market independence results of the present model in cases where unions have less than 100% coverage. Perhaps ironically, it is the "weaker" unions, in the sense of generating less than full coverage, that yield the wage rigidities. Since the model is partial equilibrium in nature, there are no implications for aggregate unemployment.
5. Model Assumptions and Robustness

Of the assumptions made above, there are two which particularly deserve some comment. The first is that the union organizes firms rather than workers. It is straightforward to demonstrate that if the union instead designates some workers "union", labels others "nonunion", and threatens all other workers with union status if they work in the industry, then the equilibrium outcomes are precisely those analyzed above for all union variables. The difference is that firms hiring nonunion workers bid their price up from $\hat{w}$ to $w$, thus transferring the profits earned under the previous scheme by nonunion firms to nonunion workers. The union variables are not sensitive to this part of the specification because all that changes is the manner in which entry is restricted and all methods which succeed are equally useful. To put the point differently, in the equilibrium of the model presented above, if the union simply labelled the $N^\star L$ workers unionized, it would not seek to alter $w^\star$ because the formal problem simply involves replacing $N$ with $L_u^u/L$, where $L_u^u$ is the total number of workers labelled unionized.

Second, the "equilibrium concept" used here is leader-follower, with the union leading. Moreover, given the total rents extracted by the union, there is no redistribution of the rents between workers and the union which would make both workers and the union better off. It has become fashionable (see McDonald and Solow (1981), Macurcy and Pencavel (1983) and Oswald (1982, 1984)) to include firms in the coalition. That is, wages as well as the allocation of labor are taken to be "efficient" from the standpoint of workers, firms and the union, with rents being extracted from consumers (their role again being followers excluded from the coalition). The point to note
here is simply that the results presented above depend only in detail on the exclusion of firms from the coalition. If unions are still modelled as using resources, conclusions akin to those presented above arise in an "efficient contracts" setting, and for the same reason—the number of union firms (or workers) and the union wage are not so constrained by the precise structure of demand. In particular, the very strong separation of union variables from \( \varphi(p) \) continues to hold.

VI. TESTING THE MODEL

As demonstrated in the previous section, the proposed model has predictions for a wide variety of empirical phenomena connected with unions. It may also be used to interpret much of the existing empirical literature. In this section some tests are proposed that concentrate on the major predictions of the model. More especially, the tests concern predictions that most easily differentiate the present model from others in the existing literature. These predictions therefore focus on the basic result concerning the independence of union behavior and industry demand conditions, in cases where union coverage is less than 100%.

The standard monopoly models predict some non-zero response of union employment levels and wage rates in response to changes in product market demand conditions. In this setting, empirical measures that divide "industries" between union and nonunion members or firms would have to be interpreted either as first aggregating over distinct industries, or as union and nonunion types of work in the same industry. Either way, the union–nonunion differential should be somehow sensitive to product market conditions. Similarly, union employment levels, and hence coverage, would
also react to product changes.

In the present model neither the union wage nor union employment are predicted to be sensitive to product market conditions where unionization is less than 100%. Union coverage, which is well defined in this model, is predicted to be inversely related to product market conditions. Thus a test which is both crucial for the present model and which readily differentiates it from other models in the literature would involve a comparison of the reaction of union wage rates and employment (or coverage) when product market conditions change for industries which are 100% unionized with those that are not. In particular, finding that changes in product demand had significant effects on union wages and employment under incomplete coverage would cast serious doubt on the model. This prediction is the strongest of all because it is essentially nonparametric, in the sense that the model predicts it under all empirical parameterizations.

The model is a long-term model. The product market changes that are used for a test should therefore be permanent. In addition they should be separated from changes in other exogenous variables--especially the alternative wage, or union costs--that would affect union behavior. Thus, an appropriate time period for the test would be one where there was general wage rate stability and where there was no recent trade union legislation. Under these conditions, the absence of a different reaction of union wages and employment across the two coverage "regimes", and in particular, a finding of sensitivity of union variables to product market conditions in the less than full coverage case, would be major evidence against the model proposed in this paper.
VII. CONCLUSIONS AND SUGGESTIONS FOR FUTURE RESEARCH

The preceding sections have presented and discussed a model of unions which incorporates the assumption that the union uses resources in its activities--i.e. that it faces operating costs. This assumption has led to the possibility of industries being less than 100% unionized. The result of this possibility is that many union issues which either could not be discussed or for which there were no clear predictions in standard monopoly models of unionism may now be addressed and predictions derived. The strongest predictions concern the major differences of unions in the complete and incomplete coverage cases and the independence of union wage rates, employment, etc. from conditions of product demand in the incomplete coverage case.

This approach offers an alternative to that of many recent studies that have attempted to expand the generality and predictive content of union models by exploring alternative objective functions.

This model is set in a competitive industry environment and employs a simple objective function. As a result, it is very easy to manipulate and may readily be extended to consider a wide variety of union issues. Some of these extensions were sketched in Section V; many more are possible. The precise details of the predictions are, of course, dependent on the particular objective function that has been used. However, many of the strong product demand independence results are insensitive to changes in the objective function. An interesting future task would be to relax the competitive environment assumption--most particularly contrasting it with a regulated environment. The issue of the extent to which unions lose when industries are deregulated could then be addressed.
FOOTNOTES

1 See for example, Dunlop (1950); and more recently Oswald (1982, 1984), Macurdy and Pencavel (1983) and Pencavel (1984).

2 This assumption can be relaxed at the cost of including more algebra and minor restrictions on the expansion paths of the multiple input technology.

3 Note that if the union permits any nonunion firms to operate in the industry, those firms will earn positive profits if \( w > \hat{w} \), as is shown to be the case below. That these profits are not eroded by competition from outside is ensured by the union's threat. That they do not accrue to the union is implied by the restriction that the union only obtains revenue through collection of dues from union members. It is also shown below that the union will not generally try to obtain these rents by choosing full union coverage (100% unionization). An implication is that nonunion firms will be willing to devote positive effort to retaining that status.

While initially disturbing, the positive profits earned by nonunion firms are simply a manifestation of the presence of the monopoly "inefficiency" assumed in the model, and are that part of the total "monopoly profit" which the union does not obtain owing to the limited means through which the union is permitted to earn revenue.

In an earlier version of the paper, the union made "quantity dependent" threats--any firm producing a level of output in a specified range would be unionized. Zero profits for all firms was the outcome. The union's problem so obtained is identical to that presented below. The point here is that due to the manner in which the union collects surplus, some surplus will inevitably escape its grasp. Where this lost surplus ends up matters little for the predictions.
4 Since \( I(w,F) \) is a continuous function of \( w \), \( \pi(N,w) \) is continuous too. It is shown below that the set of \((N,w)\) pairs from which the union might pick \{(N,w)|Nq - \varphi \leq 0, N \geq 0, w \geq 0\} is compact and non-empty. Thus the problem in (9) has a solution.

\[
\frac{d\pi}{dw} = D_{wN} D_{w} - D_{NN} D_{w}.
\]

It was assumed that \( \pi_{NN} < 0 \). Also,

\[
D_{ww} = -\pi_{NN} \frac{\partial I}{\partial w} > 0 \text{ from (15)},
\]

and

\[
D_{Nw} = -I < 0.
\]

Further

\[
D_{wN} = I + (w^* - w) \frac{\partial I}{\partial w} - u \frac{\partial I}{\partial N} = \frac{u}{N} - u \frac{\partial I}{\partial N} \text{ from (17)} < 0 \text{ from (15)}.
\]

Accordingly,

\[
\frac{d\pi}{dw} > 0.
\]

Similarly,

\[
\frac{d\pi}{dw} = D_{NN} D_{w} - D_{wN} D_{w} < 0
\]

since \( D_{NN} < 0 \).
See the earlier version of this paper, MacDonald and Robinson (1985), for details on the predictions.

The data in Duncan and Stafford imply that \( \frac{s}{\hat{w}} \) is in the neighborhood of .12, in which case \( w + s - d = \hat{w} \) is not implausible. That \( d \) is not large is consistent with the notion (which can be included in the model) that there is some degree of competition for the union role.

Implicit here is the restriction that though \( \gamma > 1 \) could obtain, in equilibrium union firms remain at a cost disadvantage. If this relation fails, the model can still be analyzed, but the nonunion firms earn zero profits, union firms earn non-negative profits, and the union threatens potential entrants with nonunion status. If the union-nonunion production differences was purely in terms of \( \gamma (\delta=1) \), the restriction implies \( w/\gamma > \hat{w} \) at the optimum. More generally, if \( \gamma \neq 1 \) also implies \( \delta \neq 1 \), \( w/\gamma > \hat{w} \) is not required.

This material is from a U.W.O. thesis which seeks to embed the basic insights of Hayes in a suitably modified version of the present setting; see Stirling (1985).

The model used here is much like that presented above. In equilibrium, workers are indifferent about their level of skill accumulation, firms optimally choose skilled or unskilled workers given the configuration of wages and dues, and the union leads all other agents.
REFERENCES


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