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More on Marriage, Fertility, and the Distribution of Income

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Abstract

According to Pareto (1896), the distribution of income depends on "the nature of the people comprising a society, on the organization of the latter, and, also, in part, on chance." An overlapping generations model of marriage, fertility and income distribution is developed here. The "nature of the people" is captured by attitudes toward marriage, divorce, fertility, and children. Singles search for mates in a marriage market. They are free to accept or reject marriage proposals. Married agents make their decisions through bargaining about work, and the quantity and quality of children. They can divorce. Social policies, such as child tax credits or child support requirements, reflect the "organization of the (society)." Finally, "chance" is modelled by randomness in income, opportunities for marriage, and marital bliss.

Keywords: Fertility; Marriage and Divorce; Nash Bargaining; Income Distribution; Public Policy.

Subject Area: Macroeconomics.

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1. Introduction

At any point of time in the U.S. some adults are married while others are not, some women have large families and others have small ones, some families are rich, while others are poor, and some children can expect a bright future, others a dim one. Why do families differ so much and does it matter? This is the question addressed here. To answer it, an overlapping generations model of the family is built. The model has four key ingredients. First, marriage is modeled along the search-theoretic lines of Mortenson (1988). Each period males and females must make a decision on whether or not to stay with their mates. If an adult rejects his or her mate, then he or she is free to look for another one in the future. Second, in line with the work by Mansur and Brown (1980) and McElroy and Horney (1981), decisions within a marriage are arrived at via Nash Bargaining. Third, as in Barro and Becker (1988) and Razin and Ben-Zion (1975), adults decide how many children to have. Fourth, following the work of Becker and Tomes (1993) and Loury (1981), parents must decide how much time and goods to invest in their children. In addition to luck, these parental investments determine the productivity of a child when he or she grows up.

In the equilibrium modeled heterogeneity abounds. Some people are married, others are either divorced or single. There are large families and there are small ones. Households run the gamut from rich to poor. Some children can expect to lead fortunate lives, while others can't. As in the real world, family structure matters. In the model a significant number of children live with a single mother. Some of these mothers are unwed, others are divorced. These children grow up to earn much less than children raised in a two-parent family. The girls from single-parent families are also more likely to experience an out-of-wedlock birth or a divorce than the girls from two-parent families. And so the cycle perpetuates itself, implying a low degree of intergenerational mobility. There is also a negative relationship between income and fertility. That is, poor families tend to have more children. This exacerbates income inequality. To illustrate the model's mechanics two policy experiments are undertaken. Specifically, the effects of child tax credits and child support payments are investigated.

This not the only dynamic general equilibrium of marriage and divorce.[1] Aiyagari, Greenwood and Guner (2000) have combined the Mortenson (1988) paradigm with the Becker and Tomes (1993) framework to model the plight of single-parent families. In their analysis family size is held fixed. Husband and wife play a noncooperative Nash game. Regalia and Rios Rull (1999) also develop a model of marriage and divorce to analyze the rise in single motherhood since the 1970s. They attribute a significant fraction of this increase to the (relative) rise in female

[1] The need for dynamic general equilibrium models of the family has been noted by labor economists. For instance, according to McElroy (1997, p. 53) while there has been much work on partial equilibrium models of the household "little analysis has been based upon the appropriate general equilibrium framework, the marriage market." Weiss (1997, p.120) in his survey on the literature on marriage and divorce states that when "examining the economic contributions, the main obstacles is the scarcity of equilibrium models which carefully tie the individual behavior with the market constraints and outcomes. Consequently, we do yet have a convincing model which explains aggregate family formation and dissolution." The study for such models for policy analysis has been noted. "A model of marital search would be a more accurate descriptor of AFDC entry and exits ... " than a model of job search, says Moffitt (1992, 26). Hoynes (1997, p. 95) echoes this sentiment stating that relative to the classic, but static, Beckerian model of marriage "a dynamic model of marital search is a natural extension, but has yet to be developed in the literature."

wages. In their setup a single decision maker maximizes some common set of preferences for the family – the unitary preference model. Two questions arise. Why explore the utility of Nash bargaining as a solution concept for family decision making? And, is it important to factor fertility into general equilibrium analyses of the family? As will be seen, both of these ingredients have important implications for any analysis of family oriented public policies. Therefore, if society wants effective anti-poverty programs, they should be investigated. The case for including these features in dynamic general equilibrium models of marriage and divorce is now presented.

Nash Bargaining: So, why use Nash Bargaining to model decisions within the household? First, males and females may have differences in attitudes toward the desirable quantity and quality of children. In fact, this is inevitable if divorce is permitted. While it may reasonable to assume that a male and female share the same momentary utility in marriage, it is not reasonable to assume that they do upon divorce. For in life after divorce each party's income and expenditure will differ, they may remarry, etc. Forward-looking agents will take the possibility of divorce into account before and during marriage. This will lead to differences in attitudes toward kids, even if they share the same momentary utility in marriage. For instance, imagine that a couple would both like five children and believe that the woman should stay at home and raise them, at least provided that the marriage lasts. The woman realizes that if a divorce occurs she will be stuck raising five children and have no work experience. Consequently, when taking the possibility of divorce into account, she may prefer to have fewer children and go to work. Nash Bargaining allows for such differences in tastes to be easily reconciled. One party can effect transfers to the other until an agreement is attained.[2]

Second, there is evidence that allocations within the household are not decided in a manner consistent with a single decision maker who maximizes some common set of preferences for the family – the unitary decision model. For instance, when government child allowances were transferred from husbands to wives in Great Britain during the late 1970s intrahousehold resource allocations tilted toward wives – see Lundberg, Pollack and Wales (1997). Furthermore, the higher the ratio of eligible males to females in a population, the more resource allocations within a marriage favor the wife. According to Chiappori, Fortin, and Lacroix (1998) this finding is consistent with a Nash Bargaining model where each party takes into account the value of their options outside of the marriage. That is, the value of being single taking into account the probability of finding a future mate. This is exactly the type of framework that is modeled here. In a similar vein, Rubalcava and Thomas (2000) find that the presence of AFDC shifts resources allocations in low-income married households with children toward women, presumably because it raises the outside option of single life for women.[3]

Third, the assumed mode of household decision making matters. It has important implications for the public policy predictions that arise from models of marriage, divorce, and fertility. For example, take the case of child support payments studied here. These payment are designed to help the plight of children living with divorced mothers. Males will find marriages less attractive when they

[2] In a unitary decision model of marriage these differences in attitudes are difficult to resolve. Regalia and Rios Rull (1999) resolve this conflict by letting the woman in a match choose the number of children to have.

[3] In fact, Greenwood, Guner and Knowles (2000) use the model developed here to study AFDC. They find this, precisely.

have to make child support payments upon divorce. Suppose they do have to make these payments. In the parameterized version of the model presented, the equilibrium number of marriages plummets when a unitary decision model is assumed. There is only a moderate decline in the number of marriages, however, when Nash Bargaining is assumed. This occurs because young females make offsetting transfers to young males to make marriages viable. Hence, intrahousehold reallocations may have important implications for society's redistribution programs. This needs to be studied.

Fertility: Why is it important to include a fertility decision in models of marriage and divorce? First, to most, the decisions to get married and have children are inextricably linked. Therefore, it seems natural to model these two choices together. Furthermore, family structure and the well being of children are closely connected empirically. Other things equal, families with lower incomes tend to have more children [see Knowles (1999)]. Additionally, single mothers tend to have more children than married ones. Hence, resources per child are less in low-income families (often single-parent families), both because there is less income and because this income has to be spread over more members. This has implications for income inequality at a point in time and for the transmission of inequality across time. For example, it is well known that children from single-parent families are much more likely to drop out of school, to be unemployed, and to experience out-of-wedlock births [McLanahan and Sandefur (1994)]. It's an interesting question to ask why a woman should *choose* to have children out of wedlock. The answer to this can only come from a model where both the decisions to marry and have children are modelled explicitly.

Second, the reason for most anti-poverty programs is to improve the plight of children. To design effective public policy programs, the impact that anti-poverty schemes have on fertility must be taken into account. Take for example the child tax credit program studied here. With child tax credits, families will now have more income per child, other things equal. Thus, their children should be better off. But, other things may not be equal, if such a policy promotes larger family size. In the calculations undertaken here, a child tax credit fails to elevate the level of well being in society precisely due to an increase in family size.

2. Economic environment

Consider an economy populated by two groups of agents, females and males. Agents live for four periods: two periods as children, and two periods as adults. Let young and old refer to the first and second period of adulthood respectively. At any point in time, the female and male populations each consist of a continuum of children and a continuum of adults. Children become adults after they have been raised by their parents for two periods. Each adult is indexed by a productivity level. Let x denote the type (productivity) of an adult female, and z denote the type (productivity) of an adult male. Assume that x and z are contained in the sets $\mathcal{X} = \{x_1, x_2, \ldots, x_S\}$ and $\mathcal{Z} = \{z_1, z_2, \ldots, z_S\}$.

At the beginning of each period, there exists a marriage market for single agents. Any single agent can take a draw from this market. Agents are free to accept or reject a mate as they desire. If a single agent accepts a draw, s/he is married for the current period, provided of course, that the other person agrees too. Otherwise, the agent is single and can take a new draw at the beginning of the next period. Similarly, at the beginning of each period, married agents decide to remain married or get divorced. A divorced agent needs to remain single one period before having a new draw. Therefore, given the two-period overlapping generations structure remarriage is ruled out. Furthermore, assume that agents only match with people of the same generation.

Females are only fecund for the first period of their adult life. Therefore, each period, young married couples and young single adult females decide how many children to have. A child has equal chances of being a female or a male. Let k denote the number of children a female has. Assume that k is contained in the set $\mathcal{K} = \{0, 1, ..., K\}$. Children stay with their mothers, if their parents get divorced. A divorced male has to pay child support payments to his former wife after divorce.

Agents are endowed with one unit of (nonsleeping) time in each period. Females must split this time between work, child-care, and leisure. Males divide their time between work and leisure. A married male has to spend a fixed amount of time per child on homework.

Married agents derive utility from the consumption of a public household good, from human capital investment in their children, from leisure, and from marital bliss. Consumption of this household good depends upon the number of adults and children in the family. Parents must decide how much time and goods to invest in their children. This determines the level of human capital possessed by their children. Parents treat their children equally. Single males care only about their own consumption of goods and leisure and they do not worry about human capital investment in their children. When a male marries a female with children, however, he derives utility from the human capital investment in his stepchildren. A single mother must make the decision on her own about how much time and money to invest in her kids.

After two periods with their mother, children are endowed with productivity levels that depend on the human capital investment received throughout their childhood. Each period the oldest adult males and females die and are replaced by the oldest children who enter into the marriage market.

2.1. Preferences

Females: Let the momentary utility function for a woman be

$$\begin{aligned} F\left(c,e,k,1-l-t\right) &\equiv U\left(c\right)+V\left(e,k\right)+R(1-l-t-\iota_f k) \\ &\equiv \frac{c^{\nu_f}}{\nu_f}+\omega_f\frac{k^{\xi_f}}{\xi_f}\frac{e^{\vartheta_f}}{\vartheta_f}+\delta_f\frac{(1-l-t-\iota_f k)^{\varsigma_f}}{\varsigma_f}. \end{aligned}$$

Here c is the consumption of household production, which is a public good for the family, k is the number of children, and e is human capital investment per child. Females allocate l units of their time for work, and t units of it for child care or nurture. They also incur a fixed time cost of ι_f per child.

Males: A male's attitude toward children depends upon his marital status. Males spend n units of their time working. The utility function for a married male is described by

$$\begin{aligned} M(c,e,k,1-n) &\equiv U\left(c\right)+P(e,k)+S(1-n-\iota_m k) \\ &\equiv \frac{c^{\nu_m}}{\nu_m}+\omega_m\frac{k^{\xi_m}}{\xi_m}\frac{e^{\vartheta_m}}{\vartheta_m}+\delta_m\frac{(1-n-\iota_m k)^{\varsigma_m}}{\varsigma_m}. \end{aligned}$$

Married males incur a fixed time cost of ι_m per child. The functions V and P imply that the married male's attitudes toward the welfare of children is allowed to differ from the female's. The utility function for a single male can be expressed simply as $M(c, e, 0, 1-n)$; a single male does not realize any utility from the children borne through previous relationships.

2.2. Household consumption

Let p denote the number of parents in a household. Then, the consumption for a household with p parents and k children is given by

$$c = \Psi(p,k)[Y(l,n;x,z) - d] - \gamma J(q), \text{ for } q = m, s,$$

where

$$\Psi(p,k) = \left(\frac{1}{p+bk}\right)^{\eta}, \ 0 < \eta < 1, \ 0 < b < 1,$$

and

$$Y(l,n;x,z) = \begin{cases} (xl + zn), & \text{for a married couple,} \\ xl, & \text{for a single woman,} \\ zn, & \text{for a single man,} \end{cases}$$

and where the indicator function J returns a value of one for a married household and zero otherwise so that $J(m) = 1$ and $J(s) = 0$.

The function Y has a clear interpretation under the above parameterization. The variables x and z can be thought of as the market wages for type-x females and type-z males. The function Ψ translates household production into the consumption realized by adult family members. There are scale effects in household consumption in the sense that each additional child costs less to feed and clothe than the one before. Still, it does cost more to maintain the extra child. Likewise, the second adult costs less than the first. The variable d represents the amount of household production that is used for investment in children. A single male will always set this to zero; because, either he has no children or he doesn't realize utility from them.

The parameter γ represents the quality of the match between a male and a female. Let $\gamma \in \mathcal{G} = \{\gamma_1, \gamma_2, ..., \gamma_m\}$ be a discrete random variable. For an unmarried couple this variable is drawn, after they are matched but before the marriage decision, according to distribution function $\Gamma(\gamma_h) = \Pr[\gamma = \gamma_h]$. For a married couple the variable γ then evolves over time according the process $\Delta(\gamma_n|\gamma_h) = \Pr[\gamma' = \gamma_n|\gamma = \gamma_h]$. Given the value drawn for γ', each party in a marriage decides whether to remain married.

2.3. Transmission of Human Capital

Human capital investment per child in a household with k children is given by

$$e = Q(t, d, k) \equiv \left(\frac{t}{k^{\kappa_1}}\right)^{\alpha} (\frac{d}{k^{\kappa_2}})^{1-\alpha}, \text{ for } 0 < \kappa_1, \kappa_2 < 1,$$

which transforms the child-care time of the mother, t, and the amount of the home produced good, d, into human capital investment, e. Recall that children are nurtured for two periods. At the end of every period the children of the oldest generation enter into the marriage market as single adults. The productivity levels for females are drawn from the distribution

$$\Xi(x_i|e_{-2} + e_{-1}) = \Pr[x = x_i|e_{-2} + e_{-1}],$$

and for males from

$$\Lambda(z_j|e_{-1} + e_{-1}) = \Pr[z = z_j|e_{-2} + e_{-1}],$$

where e_{-1} and e_{-2} indicate the human capital investment during the two periods of an agent's childhood. The distribution functions Ξ and Λ are stochastically increasing in $e_{-2}+$ e_{-1} in the sense of first-order stochastic dominance. Thus, higher human capital investment in children by parents increases the likelihood that children will be successful in life.

The conditional distribution Ξ is represented by a discrete approximation to a lognormal distribution with mean, $\mu_{x|e}$, and standard deviation, $\sigma_{x|e}$. Similarly, suppose that Λ is also given by a discrete approximation to a lognormal with mean, $\mu_{z|e}$, and standard deviation, $\sigma_{z|e}$. These conditional means are given by,

$$\mu_{x|e} = \mu_{z|e} = \ln[\varepsilon_1(e_{-2} + e_{-1})^{\varepsilon_2}], \text{ for } \varepsilon_2 \in (0, 1),$$

where the ε's are the parameters governing the technology that maps human capital investment by parents into productivity levels.

After the first period of adulthood the productivity levels for females and males evolve according to the following transition functions:

$$X(x_j|x_i) = \Pr[x' = x_j|x = x_i],$$

and

$$Z(z_j|z_i) = \Pr[z' = z_j|z = z_i],$$

where x' and z' denote the next-period values. These Markov chains are constructed to approximate an AR(1) in logarithms.[4]

3. Decision Making

3.1. Household Activity — Single Old Adults

A single old female of type x with k children will solve the following problem:

[4] The discrete approximations for Ξ, Λ, X, and Z follow the procedure outlined in Tauchen (1986).

$$G_2(x,k,z) = \max_{l,t,d} F\left(c,e,k,1-l-t\right) \qquad \text{P(1)}$$

subject to

$$c = \Psi(1,k)[Y(l,0;x,0) + A(z,k) - d]$$

and

$$e = Q(t,d,k),$$

where

$$A(z,k) = azN^s(z,k)k.$$

Here z denotes her former husband's productivity and the function $N^s(z,k)$ denotes his labor supply. The function A determines how much child support a former husband has to pay, which is assumed to be a fraction, a, of his current income, $zN^s(z,k)$, per child. Obviously, for a single old female who was never married $z = 0$.

Denote a single mother's level of human capital investment in her children by

$$e = E_2^s(x,k,z).$$

This implies that $E_2^s(x,k,z)$=$Q(T_2^s(x,k,z), D_2^s(x,k,z), k)$, where $T_2^s(x,k,z)$ and $D_2^s(x,k,z)$ are the decision rules for t and d that arise from P(1).

The maximized utility of a single old male is given by the following problem:

$$B_2(z,k) = \max_{n} M(c,0,0,1-n) \qquad \text{P(2)}$$

subject to

$$\begin{aligned} c &\equiv \Psi(1,0)[Y(0,n;0,z) - aznk] \\ &\equiv zn - aznk = zn(1-ak), \qquad 0 < a < 1, \end{aligned}$$

where k denotes the number of children for whom he has to pay child support. For a single old male who was never married $k = 0$.

3.2. Household Activity — Old Married Adults with k children

Nash Bargaining Problem: Consider a couple of type (x, z, γ, k) that is married in the second period. Assume that they make their decisions by applying the Nash solution to a fixed-threat bargaining game. Their problem is to solve

$$\max_{l,t,n,d} \; [F\,(c, e, k, 1 - l - t) - G_2(x, k, z)] \times [M(c, e, k, 1 - n) - B_2(z, k)] \qquad \text{P(3)}$$

subject to

$$c = \Psi(2, k)[Y(l, n; x, z) - d] - \gamma = \Psi(2, k)[xl + zn - d] - \gamma,$$

and

$$e = Q(t, d, k).$$

Here $B_2(z, k)$ and $G_2(x, k, z)$ and are the threat points for the husband and wife. They are the values of being single in the second period, and are given by the solutions for old single agent problems, P(1) and P(2).

Denote the level of human capital investment per child in a family with two old parents by

$$e = E_2^m(x, z, \gamma, k).$$

Let the resulting utility levels for an old husband and wife in a (x, z, γ, k)-marriage, or the values for M and F in P(3) evaluated at the optimal choices for l, t, n, d and the implied values for c and e, be represented by

$$H_2(x, z, \gamma, k),$$

and

$$W_2(x, z, \gamma, k). \tag{P'(3)}$$

3.3. Marriage — Old Adults

Consider an age-2 couple indexed by (x, z, γ, k). Each party faces a decision: should s/he choose married or single life for the period. Clearly, a married female will *want* to remain married if and only if $W_2(x, z, \gamma, k) \geq G_2(x, k, z)$; otherwise, it is in her best interest to get a divorce. Equally as clearly, a single female will *desire* to marry if and only if $W_2(x, z, \gamma, k) \geq G_2(x, k, 0)$; otherwise, she'll go it alone. Similarly, a married male would wish to remain so if and only if $H_2(x, z, \gamma, k) \geq B_2(z, k)$, while a single male will like to marry if and only if $H_2(x, z, \gamma, k) \geq B_2(z, 0)$.

The matching decision of the age-2 couple can summarized by the following indicator function:

$$I_2^q(x, z, \gamma, k) = \begin{cases} 1, \text{ if } W_2(x, z, \gamma, k) \geq G_2(x, k, J(q)z) \, and \; H_2(x, z, \gamma, k) \geq B_2(z, J(q)k) \\ 0, \text{ otherwise}, \end{cases} \tag{P(4)}$$

which is defined for $q = m, s$, and where $J(m) = 1$ and $J(s) = 0$. Note the indicator function depends upon the marital status, q, of the couple *at the time of the decision.*

3.4. Household Activity — Single Young Adults

Now, let the odds of drawing a single age-1 female of type x_i in the marriage market be represented by

$$\Phi_1(x_i), \text{ where } \Phi_1(x_i) \geq 0 \ \forall x_i \text{ and } \sum_{i=1}^{S} \Phi_1(x_i) = 1,$$

and the odds of meeting a single age-2 female of type x_i with k children in the marriage market be given by

$$\Phi_2(x_i, k), \text{ where } \Phi_2(x_i, k) \geq 0 \ \forall x_i \text{ and } \sum_{i=1}^{S} \sum_{k=0}^{K} \Phi_2(x_i, k) = 1.$$

Likewise, the odds of meeting a single age-i male of type z_i will be denoted by

$$\Omega_j(z_i), \text{ where } \Omega_j(z_i) \geq 0 \ \forall z_i \text{ and } \sum_{i=1}^{S} \Omega_j(z_i) = 1.$$

A key step in the analysis will be to compute these matching probabilities.

The programming problem for an one-period-old single type-x_i female is

$$\begin{aligned} G_1(x_i) &= \max_{k.l,t,d}\{F(c, e, k, 1-l-t) + \beta \sum_{k=1}^{S} \sum_{l=1}^{S} \sum_{n=1}^{m} \{W_2(x_k, z_l, \gamma_n, k) \ I_2^s(x_k, z_l, \gamma_n, k) \\ &\quad + G_2(x_k, k, 0) \ [1 - I_2^s(x_k, z_l, \gamma_n, k)]\} X(x_k|x_i) \Omega_2(z_l) \Gamma(\gamma_n)\}. \qquad \text{P}(5) \end{aligned}$$

subject to

$$c = \Psi(1, k)[Y(l, 0; x_i, 0) - d] = \Psi(1, k)[x_i l - d],$$

and

$$e = Q(t, d, k).$$

In the above problem β is the discount factor. Here $\Omega_2(z_l)\Gamma(\gamma_n)$ gives the probability that a single female of type x_i will meet a single male of type z_l and that their match will be of quality γ_n. Note that $W_2(x_k, z_l, \gamma_n, k)$ is given by the solution to the Nash Bargaining problem P(3) for a type-(x_k, z_l, γ_n, k) marriage.

Marriage is an option only if both parties agree; that is, when $I_2^s(x_k, z_l, \gamma_n, k) = 1$ — see P(4). The value $G_2(x_k, k, 0)$ of remaining single is given by the solution to the problem of an old single female, or by P(1). Last, note that in P(5) the indicator function $I_2^s(x_k, z_l, \gamma_n, k)$ chooses married or single life for the female when old depending upon what is her in best interest to do. Married life must also be feasible in the sense that her mate must agree.[5] This is incorporated into the indicator function's construction.

Let the utility-maximizing decision rules for the quantity and quality of children that solve this problem be represented by

$$k = K^s(x_i),$$

and

$$e = E_1^s(x_i, k) = E_1^s(x_i, K^s(x_i)).$$

The analogous recursion for a single male is

$$\begin{aligned} B_1(z_j) \quad = \quad & \max_n \{M(c, 0, 0, 1-n) + \beta \sum_{l=1}^{S} \sum_{i=1}^{S} \sum_{k=0}^{K} \sum_{n=1}^{m} \{H_2(x_i, z_l, \gamma_n, k) I_2^s(x_i, z_l, \gamma_n, k) \\ & + B_2(z_l, 0)[1 - I_2^s(x_i, z_l, \gamma_n, k)]\} \Phi_2(x_i, k) Z(z_l|z_j) \Gamma(\gamma_n)\}. \qquad \text{P(6)} \end{aligned}$$

subject to

$$c = \Psi(1, 0)Y(0, n; 0, z_j) = z_j n,$$

where $\Phi_2(x_i, k)\Gamma(\gamma_n)$ is the probability of meeting an old single female of type-x_i with k children and having a match quality of γ_n.

[5] That is, here there is a bilateral search problem, as opposed to the more typical unilateral job-search model, say as typified by the Andolfatto and Gomme (1996) and Hansen and Imrohoroglu's (1992) analyses of unemployment insurance.

3.5. Household Activity — Young Married Adults

Nash Bargaining Problem: Consider now the problem of a young married couple. Applying the Nash Bargaining solution to the fixed-threat bargaining game facing a young couple in a type-(x_i, z_j, γ_h) marriage gives

$$\max_{l,n,t,d,k} \{\{F(c, e, k, 1-l-t) + \beta \sum_{v=1}^{S} \sum_{l=1}^{S} \sum_{n=1}^{m} [W_2(x_v, z_l, \gamma_n, k)\ I_2^m(x_v, z_l, \gamma_n, k)$$
$$+G_2(x_v, k, z_l)[1 - I_2^m(x_v, z_l, \gamma_n, k)]]\Delta(\gamma_n|\gamma_h)X(x_v|x_i)\ Z(z_l|z_j) - G_1(x_i)\}$$
$$\times\{M(c, e, k, 1-n) + \beta \sum_{v=1}^{S} \sum_{l=1}^{S} \sum_{n=1}^{m} [H_2(x_v, z_l, \gamma_n, k) I_2^m(x_v, z_l, \gamma_n, k)$$
$$+ B_2(z_l, k)[1 - I_2^m(x_v, z_l, \gamma_n, k)]]\Delta(\gamma_n|\gamma_h)X(x_v|x_i)\ Z(z_l|z_j) - B_1(z_j)\}\} \quad \text{P(7)}$$

subject to

$$c = \Psi(2, k)[Y(l, n; x_i, z_j) - d] - \gamma_h = \Psi(2, k)[x_i l + z_j n - d] - \gamma_h, \quad (3.1)$$

and

$$e = Q(t, d, k). \quad (3.2)$$

The threat points $G_1(x_i)$ and $B_1(z_j)$ are given by the solutions to the problems for young single females and males.

Let the optimal decision rules for the quantity and quality of children in a type-(x_i, z_j, γ_h) young marriage be denoted by

$$k = K^m(x_i, z_j, \gamma_n),$$

and

$$e = E_1^m(x_i, z_j, \gamma_h, k) = E_1^m(x_i, z_j, \gamma_h, K^m(x_i, z_j, \gamma_h)).$$

Furthermore, let the expected lifetime utility for a young male and female arising out of a type-(x_i, z_j, γ_h)-marriage be represented by

$$H_1(x_i, z_j, \gamma_h), \quad \text{P'(7)}$$

and

$$W_1(x_i, z_j, \gamma_h).$$

3.6. Marriage — Young Adults

Then the marriage decisions for a randomly matched young couple, (x, z, γ), is given by

$$I_1^s(x,z,\gamma) = \begin{cases} 1, \text{ if } W_1(x,z,\gamma) \geq G_1(x) \textit{ and } H_1(x,z,\gamma) \geq B_1(z), \\ 0, \text{ otherwise.} \end{cases} \qquad \text{P(8)}$$

4. Equilibrium

4.1. Population Growth

The average number of children per female, k, is given by

$$\begin{aligned} \mathsf{k} &= \sum_{i=1}^{S}\sum_{j=1}^{S}\sum_{h=1}^{m} \Phi_1(x_i)\,\Omega_1(z_j)\Gamma(\gamma_h)I_1^s(x_i,z_j,\gamma_h)K^m(x_i,z_j,\gamma_h) \\ &\quad + \sum_{i=1}^{S}\Phi_1(x_i)\,[1 - \sum_{j=1}^{S}\sum_{h=1}^{m}\Omega_1(z_j)\Gamma(\gamma_h)I_1^s(x_i,z_j,\gamma_h)]K^s(x_i). \end{aligned}$$

To understand this formula, note that the probability of a type-(x_i, z_j, γ_h) marriage between young adults is $\Phi_1(x_i)\,\Omega_1(z_j)\Gamma(\gamma_h)I_1^s(x_i,z_j,\gamma_h)$. This match will generate $K^m(x_i, z_j, \gamma_h)$ kids. The odds that a woman will be type-x_i and remain single are $\Phi_1(x_i)\,[1-\sum_{j=1}^{S}\sum_{h=1}^{m}\Omega_1(z_j)\Gamma(\gamma_h)I_1^s(x_i,z_j,\gamma_h)]$. This woman will have $K^s(x_i)$ children. In a stationary equilibrium the growth rate of the population, g, will therefore be

$$g = \sqrt{\frac{\mathsf{k}}{2}}.$$

4.2. Matching Probabilities

Young Adults: The probabilities of meeting a young female and male of a given type in the marriage market are $\Phi_1(x)$ and $\Omega_1(z)$. To determine these probabilities, let $\Upsilon^{mm}(x_i, z_j, \gamma_h, x_k, z_l, \gamma_n)$ represent the fraction of females who were married in both periods and transited from state (x_i, z_j, γ_h) to (x_k, z_l, γ_n). Likewise, let $\Upsilon^{ss}(x_i, x_k)$ denote the fraction of females who were single in both periods, and transited from x_i to x_k, and $\Upsilon^{ms}(x_i, z_j, \gamma_h, x_k, z_l)$ denote the fraction of females who suffered a marriage breakup, etc. Hence,

$$\begin{aligned}
\Upsilon^{mm}(x_i, z_j, \gamma_h, x_k, z_l, \gamma_n) &\equiv \Phi_1(x_i)\,\Omega_1(z_j)\Gamma(\gamma_h)I_1^s(x_i, z_j, \gamma_h) \\
&\quad \times I_2^m(x_k, z_l, \gamma_n, k^m)\Delta(\gamma_n|\gamma_h)X(x_k|x_i)Z(z_l|z_j),
\end{aligned}$$

$$\begin{aligned}
\Upsilon^{ss}(x_i, x_k) &\equiv \Phi_1(x_i)\,[1 - \sum_{j=1}^{S}\sum_{h=1}^{m}\Gamma(\gamma_h)\Omega_1(z_j)I_1^s(x_i, z_j, \gamma_h)] \\
&\quad \times X(x_k|x_i)[1 - \sum_{l=1}^{S}\sum_{n=1}^{m}\Gamma(\gamma_n)I_2^s(x_k, z_l, \gamma_n, k^s)\Omega_2(z_l)],
\end{aligned}$$

$$\begin{aligned}
\Upsilon^{ms}(x_i, z_j, \gamma_h, x_k, z_l) &\equiv \Phi_1(x_i)\,\Omega_1(z_j)\Gamma(\gamma_h)I_1^s(x_i, z_j, \gamma_h)X(x_k|x_i)Z(z_l|z_j) \\
&\quad \times \{\sum_{n=1}^{m}\Delta(\gamma_n|\gamma_h)[1 - I_2^m(x_k, z_l, \gamma_n, k^m)]\},
\end{aligned}$$

$$\begin{aligned}
\Upsilon^{sm}(x_i, x_k, z_l, \gamma_n) &\equiv \Phi_1(x_i)\,[1 - \sum_{j=1}^{S}\sum_{h=1}^{m}\Gamma(\gamma_h)\Omega_1(z_j)I_1^s(x_i, z_j, \gamma_h)] \\
&\quad \times I_2^s(x_k, z_l, \gamma_n, k^s)\Gamma(\gamma_n)X(x_k|x_i)\Omega_2(z_l), \qquad (4.1)
\end{aligned}$$

where $k^m \equiv K^m(x_i, z_j, \gamma_h)$ and $k^s \equiv K^s(x_i)$.

Then, it is easy to see that the odds of meeting a young woman of type-x_r in the marriage market are given by

$$\begin{aligned}
\Phi_1(x_r) &= \{\sum_{i,j,k,l,h,n} \Xi(x_r|E_1^m(x_i,z_j,\gamma_h,K^m(x_i,z_j,\gamma_h)) + E_2^m(x_k,z_l,\gamma_n,K^m(x_i,z_j,\gamma_h))) \\
&\quad \times\Upsilon^{mm}(x_i,z_j,\gamma_h,x_k,z_l,\gamma_n)K^m(x_i,z_j,\gamma_h) \\
&\quad +\sum_{i,k}\Xi(x_r|E_1^s(x_i,K^s(x_i)) + E_2^s(x_k,K^s(x_i),0))\Upsilon^{ss}(x_i,x_k)K^s(x_i) \\
&\quad +\sum_{i,j,k,l,h}\Xi(x_r|E_1^m(x_i,z_j,\gamma_h,K^m(x_i,z_j,\gamma_h)) + E_2^s(x_k,K^m(x_i,z_j,\gamma_h),z_l)) \\
&\quad \times\Upsilon^{ms}(x_i,z_j,\gamma_h,x_k,z_l)K^m(x_i,z_j,\gamma_h) \\
&\quad +\sum_{i,k,l,n}\Xi(x_r|E_1^s(x_i,K^s(x_i)) + E_2^m(x_k,z_l,\gamma_n,K^s(x_i))) \\
&\quad \times\Upsilon^{sm}(x_i,x_k,z_l,\gamma_n)K^s(x_i)\}/\mathsf{k}. \qquad (4.2)
\end{aligned}$$

The probability of meeting a type-z_r young man is determined analogously:

$$\begin{aligned}
\Omega_1(z_r) &= \{\sum_{i,j,k,l,h,n} \Lambda(z_r|E_1^m(x_i,z_j,\gamma_h,K^m(x_i,z_j,\gamma_h)) + E_2^m(x_k,z_l,\gamma_n,K^m(x_i,z_j,\gamma_h))) \\
&\quad \times\Upsilon^{mm}(x_i,z_j,\gamma_h,x_k,z_l,\gamma_n)K^m(x_i,z_j,\gamma_h) \\
&\quad +\sum_{i,k}\Lambda(z_r|E_1^s(x_i,K^s(x_i)) + E_2^s(x_k,K^s(x_i),0))\Upsilon^{ss}(x_i,x_k)K^s(x_i) \\
&\quad +\sum_{i,j,k,l,h}\Lambda(z_r|E_1^m(x_i,z_j,\gamma_h,K^m(x_i,z_j,\gamma_h)) + E_2^s(x_k,K^m(x_i,z_j,\gamma_h),z_l)) \\
&\quad \times\Upsilon^{ms}(x_i,z_j,\gamma_h,x_k,z_l)K^m(x_i,z_j,\gamma_h) \\
&\quad +\sum_{i,k,l,n}\Lambda(z_r|E_1^s(x_i,K^s(x_i)) + E_2^m(x_k,z_l,\gamma_n,K^s(x_i))) \\
&\quad \times\Upsilon^{sm}(x_i,x_k,z_l,\gamma_n)K^s(x_i)\}/\mathsf{k}.
\end{aligned}$$

Old Adults: Next, how are the odds of meeting a single age-2 type-x female with k children, $\Phi_2(x,k)$, or of a single age-2 type-z male, $\Omega_2(z)$ determined in stationary equilibrium? This depends upon the number of single agents who remain unmarried from the previous period. So, how many are there? Again, the number of married and single one-period-old type-x_i females are given by $\Phi_1(x_i)\sum_{j=1}^{S}\sum_{h=1}^{m}\Omega_1(z_j)\Gamma(\gamma_h)I_1^s(x_i,z_j,\gamma_h)$ and $\Phi_1(x_i)\,[1-\sum_{j=1}^{S}\sum_{h=1}^{m}\Omega_1(z_j)\Gamma(\gamma_h)I_1^s(x_i,z_j,\gamma_h)]$. Given this supply of one-period-old single females, the quantity of two-period-old type-x_j single females will be $\sum_{i=1}^{S}X(x_j|x_i)\Phi_1(x_i)\,[1-\sum_{j=1}^{S}\sum_{h=1}^{m}\Omega_1(z_j)\Gamma(\gamma_h)I_1^s(x_i,z_j,\gamma_h)]$.

Let

$$\aleph(x_i,k)=\begin{cases}1, \text{ if } K^s(x_i)=k,\\ 0, \text{ otherwise,}\end{cases}$$

be an indicator function representing the number of children that a single one-year-old female of type-x_i has. Then, the odds of drawing a single two-period-old type-x_j female with k children in the marriage market will be

$$\begin{aligned}\Phi_2(x_j,k) &= \{\sum_{i=1}^{S}\aleph(x_i,k)X(x_j|x_i)\Phi_1(x_i)\,[1-\sum_{j=1}^{S}\sum_{h=1}^{m}\Gamma(\gamma_h)\Omega_1(z_j)I_1^s(x_i,z_j,\gamma_h)]\}\\ &\quad\div\{\sum_{j=1}^{S}\sum_{i=1}^{S}X(x_j|x_i)\Phi_1(x_i)\,[1-\sum_{j=1}^{S}\sum_{h=1}^{m}\Gamma(\gamma_h)\Omega_1(z_j)I_1^s(x_i,z_j,\gamma_h)]\}.\end{aligned}$$

The analogous formula for the odds of meeting a single two-period-old male of type-z_j, $\Omega_2(z_i)$, reads

$$\Omega_2(z_i)=\frac{\sum_{j=1}^{S}Z(z_i|z_j)\Omega_1(z_j)[1-\sum_{i=1}^{S}\sum_{h=1}^{m}\Gamma(\gamma_h)\Phi_1(x_i)\,I_1^s(x_i,z_j,\gamma_h)]}{\sum_{i=1}^{S}\sum_{j=1}^{S}Z(z_i|z_j)\Omega_1(z_j)[1-\sum_{i=1}^{S}\sum_{h=1}^{m}\Gamma(\gamma_h)\Phi_1(x_i)\,I_1^s(x_i,z_j,\gamma_h)]}. \tag{4.3}$$

It's now time to take stock of the situation so far.

Definition 4.1. *A stationary matching equilibrium can be represented by set of child quantity and quality allocation rules,* $K^m(x,z,\gamma)$, $K^s(x)$, $E_2^m(x,z,\gamma,k)$, $E_2^s(x,k,z)$, $E_1^m(x,z,\gamma,K^m(x,z,\gamma))$, and $E_1^s(x,K^s(x))$, *a set of marriage decision* *rules, $I_2^m(x,z,\gamma,k)$, $I_2^s(x,z,\gamma,k)$, and $I_1^s(x,z,\gamma)$, and a set of matching probabilities, $\Phi_1(x)$, $\Phi_2(x,k)$, $\Omega_1(z)$, and $\Omega_2(z)$, such that:*

1. *The child quality allocation rule $E_2^s(x,k,z)$ solves the old single female's household problem P(1).*

2. *The child quantity and quality allocation rules* $K^s(x)$ and $E_1^s(x,K^s(x))$ *solve the young single female's household problem P(5).*

3. *The child quality allocation rule $E_2^m(x,z,\gamma,k)$ solves the married old couple's Nash bargaining problem P(3).*

4. *The child quality and quantity allocation rules* $K^m(x,z,\gamma)$ and $E_1^m(x,z,\gamma,K^m(x,z,\gamma))$ *solve the young married couple's Nash bargaining problem P(7).*

5. *The marriage decision an old currently married couple and an old currently single one, $I_2^m(x,z,\gamma,k)$ and $I_2^s(x,z,\gamma,k)$, are described by P(4), in conjunction with P(1), P(2) and P'(3).*

6. *The marriage decision for a young couple, $I_1^s(x,z,\gamma)$, is described by P(8), in conjunction with P(5), P(6) and P'(7).*

7. *The matching probabilities, $\Phi_1(x)$, $\Phi_2(x,k)$, $\Omega_1(z)$, and $\Omega_2(z)$, are governed by the stationary distributions described by (4.2) to (4.3).*

At a general level, not much can be said about the properties of the above model since the solution involves a complicated fixed-point problem. On the one hand, in order to compute the solution to a young single agent's choice problem one needs to know the equilibrium matching probabilities. On the other hand, calculating the equilibrium matching probabilities requires knowledge about the solutions to each of the decision problems.

5. Some Computational Analysis

5.1. Benchmark Equilibrium

To gain some insight into the model's mechanics, its solution will be computed numerically.[6] To do this, values must be assigned to the model's parameters. These are listed in Table 1. The parameter values are not chosen to tune the model to be in perfect harmony with any features of the real world. Instead, they are picked to generate an equilibrium that displays several interesting characteristics

[6] Part of the numerical procedure used to compute the model's solution is outlined in the Appendix. The algorithm for finding the equilibrium type distributions, or the Φ's and Ω's, is similar to that employed in Aiyagari, Greenwood, and Guner (2000). For more detail, see that source.

that will now be discussed.

TABLE 1: Benchmark Parameter Values

Tastes	$\nu_f = 0.5$, $\omega_f = 1$, $\xi_f = 0.325$, $\vartheta_f = 0.2$,
	$\delta_f = 3, \iota_f = 0.05$, $\varsigma_f = 0.3$, $\beta = 0.67$,
	$\nu_m = 0.5, \omega_m = 1$, $\xi_m = 0.325$, $\vartheta_m = 0.35$,
	$\delta_m = 3$, $\iota_m = 0.0325$, $\varsigma_m = 0.3$.
Technology	$b = 0.30$, $\eta = 0.5$,
	$\alpha = 0.5$, $\kappa_1 = 0.4$, $\kappa_2 = 0.5$,
	$\varepsilon_1 = 15.15, \varepsilon_2 = 0.5$,
Stochastic Structure	$\mu_{x\|e} = \mu_{z\|e} = \ln[\varepsilon_1(e_{-2} + e_{-1})^{\varepsilon_2}]$, $\sigma_{x\|e} = \sigma_{x\|e} = 0.4$,
	$\rho_x = 0.7$, $\rho_z = 0.7$,
	$\Gamma(\gamma_1) = \Gamma(\gamma_2) = 0.5$, $\Delta(\gamma_1\|\gamma_1) = \Delta(\gamma_2\|\gamma_2) = 0.5$, $\gamma_1 = 2.5$, $\gamma_2 = 0$,
Simulation Control	$S = 15$, $K = 4$, $m = 2$,
Policy Variables	$a = 0.05$.

Properties of the Equilibrium: First, observe from Table 2 that at any point in time a significant proportion of the adult population is not married. In equilibrium some people are always single, others experience a divorce. At any time about 85% of the population is married.

Table 2: Marital Status

(Percentage Distribution)

	Young	Old
Married	86	85
Single	14	5
Divorced	–	10

Second, family income is related to marital status, as Table 3 illustrates. For example, family income for a household headed by a young single female is 17% of that for a married couple. This transpires for two reasons. To begin with, in a marriage there are two potential wage earners versus only one in a household with a single adult. Additionally, married males and females work more than unmarried ones — Table 4.[7]

Table 3: Family Income

	Young	Old
Married	1.00	1.00
Single — female	0.17	0.14
Single — male	0.36	0.41
Divorced — female		0.24
Divorced — male		0.33

Table 4: Time Allocations

	Male			Female		
	Married	Single	Divorced	Married	Single	Divorced
Work	0.60	0.44	0.41	0.37	0.27	0.27
Nurture	0	0	0	0.21	0.10	0.10
Leisure	0.34	0.56	0.59	0.33	0.46	0.52
Fixed	0.06	0	0	0.09	0.17	0.12

Third, fertility is also related to marital status. Single women have a much higher fertility rate than married women do. A young married woman has 1.8 kids

[7] Additionally, in the model married males tend to earn more than unmarried ones since they make better mates. This is true in the data, too – see Cornwall and Rupert (1997).

on average while a young single woman has 3.3. So, while 85% of the population is married, only 78.5% of children live in a household with two adults. On average a female has two children; therefore, the population is stationary.

Fourth, children from a single-female family tend to do much worse. This is because their mother doesn't have much time or money to invest in them. A single mother has less time for work, nurture, and leisure because she has more children on average; i.e., more of her time is absorbed on the fixed costs of child rearing. Since she earns less money than a married couple, she has less resources to invest in her offspring also. Additionally, single women tend to have more children than do married women. The result of these facts is a lower level of human capital investment per child in a single female family — Table 5.

Table 5: Investment in Human Capital

	Young	Old
Married	1.00	0.99
Single female	0.30	0.29
Divorced female		0.37

Table 6 shows the effect of family background on a female's income. A girl growing up in a household with a single mother can expect to enjoy only two-thirds of the family income of one growing up with both parents. She is much more likely (44% versus 20%) to experience an out of wedlock birth or a divorce than the girl from a two-parent home too — Table 7.

Table 6: Effects of Childhood History on Female Income

Childhood History	$m \to m$	$m \to s$	$s \to m$	$s \to s$
Expected Wage	1.00	0.79	0.75	0.54
Expected Family Income	1.00	0.87	0.85	0.68

Table 7: Effects of Childhood History on Female Marital Experience

Adult History	$m \to m$	$m \to s$	$s \to m$	$s \to s$
Childhood History				
$m \to m$	0.80	0.09	0.08	0.03
$m \to s$	0.73	0.11	0.11	0.06
$s \to m$	0.71	0.11	0.11	0.06
$s \to s$	0.56	0.17	0.16	0.12

The Income Distribution: Some income distribution statistics for both the US economy and the model are reported in Table 8. The figures for the US are based on a cross section of annual household income for 1992, as reported in the Panel Study on Income Dynamics (PSID). The table reports the cutoff levels of income corresponding to different percentiles of the income distribution. The number for the 1st percentile is normalized to one. Hence, in the US data a household who lies at the 5th percentile of the income distribution has an income 2.56 times greater than that of a household who is at the 1st percentile. The corresponding figure for the model is 2.00. While the model does a reasonable job matching the data, Table 8 shows that the poor in the model are relatively poorer than in the data, but the rich are not as rich. It is not surprising that the model does not generate enough skewness at the upper end of the income distribution. It does not allow for entrepreneurs, superstars, and other features of the labor market. The upshot is that the mean-to-median ratio in the data is 1.26, as compared with 1.12 in the model.

The Fertility-Income Relationship: Figure 1 shows the relationship between income and family size for both the model and the US. The data for the US comes from the Panel Study on Income Dynamics (PSID). The earnings variable is the present value of future lifetime household labor income at age 30, as calculated by Knowles (1999). In the data, fertility declines with labor income. The fertility variable is total number of children ever born to a woman, who is either head or spouse of the household head. The model replicates this relationship quite well.

It makes a difference whether family or per-capita income is used. When family income is adjusted for size, the situation portrayed in Table 3 changes. Single males do relatively better now, since they have no dependents. Perhaps, this is why they work the least. The situation for unmarried females is now even bleaker. Income per family member is only 16% of the level realized in a married household — Table 9. The distribution of income is more skewed when income per member is used. The mean-to-median ratio increases from 1.12 to 1.24 in the model. The rise is more modest in the data, from 1.26 to 1.33. The increase in the model is more significant than the one in data because the number of kids declines a little too sharply with family income — Figure 1.

Table 8: Income Distribution

Income Level at the Cutoff (normalized)

Percentile	Data	Model
1	1.00	1.00
5	2.56	2.00
10	3.85	2.84
25	6.79	6.79
50	14.37	13.97
75	23.44	19.58
90	34.07	25.96
95	43.21	30.56
99	77.5	40.92

Table 9: Family Income per Member

	Young	Old
Married	1.00	0.97
Single — female	0.16	0.12
Single — male	1.28	1.48
Divorced — female		0.26
Divorced — male		1.18

5.2. Some Comparative Statics Exercises

To gain some insight into the structure of the model, several comparative statics exercises will be undertaken now.

Elasticity on Quality, ϑ_f: Suppose that the elasticity on the quality of children in the female's utility function is lowered from 0.2 to 0.19. What happens? The return at the margin from investing time and resources in children declines more rapidly now. Hence, parents will tend to invest less in their offspring. Instead, they will choose to have more children. That is, they now prefer quantity relative to quality. Married females now have 2.0 children on average (versus 1.8 earlier) while single ones have 3.7 (as compared with 3.3). The population's annualized growth rate increases to 0.73% [$= (1.075^{1/10} - 1) \times 100\%$]. Since there is less investment per child, the average quality of the mating pool drops. The fraction of married agents falls by about 3 percentage points.

The Fixed Time Costs of Childrearing, ι_f and ι_m: Let the fixed time cost of raising a child for a female drop. Specifically, let ι_f fall from 0.05 to 0.04. Since the cost of raising a child has fallen, there are more children in equilibrium. Married females now have 2.1 children on average while single ones have 3.9. Since single females have the most children, the attractiveness of being a single mother increases. This, too, raises the average number of children per female. These factors lead the population's annualized growth rate to increase to 0.98%. The long-run quality of the mating pool drops. The increase in the quantity of children comes at the expense of their quality. All parents invest less per child. There are also more single mothers and they invest less in their children than do married ones. These tendencies operate to lower the long-run quality of the matching pool. As a result of these factors, in the new equilibrium the number of marriages falls by about 4 percentage points.

Leisure Elasticity, ς_f: What will happen if the utility function for women is made more elastic with respect to leisure? In particular, let $\varsigma_f = 0.35$ as opposed to 0.30. Women are now willing to work more — both at home and in the market — since the disutility from working is not rising as fast in terms of effort. There is now more investment of both goods and time in children. Since married women work the most this increases the benefit of marriage. The quality of the matching pool also rises. The upshot of this is that the number of young single mothers falls by about 0.7 percentage points. Married women have more children, since at the margin the disutility from raising more of them has dropped. The population's growth rate decreases slightly (because the number of young single women drops).

Consumption Elasticities, ν_f and ν_m: Consider the impact of making the utility function more curved in consumption. Reset $\nu_f = \nu_m = 0.4$, as opposed to the value of 0.5 adopted earlier. The number of marriages now rises by 8.5 percentage points. The population's growth rate increases to 0.6% per period. The question is, why? When the marginal utility from consumption declines faster, parents divert more of their income into children. They choose to increase both the quantity and quality of their offspring. Additionally, the extra consumption that males realize from single life is valued less. There will be less children living with a single parent. These considerations increase the long-run quality of the mating pool. The number of marriages rises, therefore, on these accounts.

Shock Structure: How does the structure of the shocks affect the equilibrium? To explore this, the degree of persistence in the matching shock is increased. Now, $\Delta(\gamma_1|\gamma_1) = \Delta(\gamma_2|\gamma_2) = 0.9$. This leads to drop in the rate of marriage among the young (from 86 to 74%). When there is a bad match quality shock it will now persist into the future making marriage less attractive. Since there are more single mothers, the population's growth rate increases to about 0.5% per period. Likewise, increasing persistence in either or both of the type shocks has a similar effect.

5.3. Nash Bargaining

How does Nash Bargaining work in the model? The Nash Bargaining solution solves a Pareto problem between husband and wife — the details are in the Appendix. Therefore, there exists some set of weights ρ and $(1-\rho)$ such that solving a type-(x_i, z_j, γ_h) young couple's Nash Bargaining problem, P(8), is equivalent to solving the Pareto problem

$$\begin{aligned}
&\max_{l,n,t,d,k} \{(1-\rho)\{F(c,e,k,1-l-t) + \beta \sum_{v=1}^{S}\sum_{l=1}^{S}\sum_{n=1}^{m}[W_2(x_v, z_l, \gamma_n, k)\ I_2^m(x_v, z_l, \gamma_n, k) \\
&+G_2(x_v, k, z_l)[1 - I_2^m(x_v, z_l, \gamma_n, k)]]\Delta(\gamma_n|\gamma_h)X(x_v|x_i)\, Z(z_l|z_j)\} \\
&+\rho\{M(c,e,k,1-n) + \beta \sum_{v=1}^{S}\sum_{l=1}^{S}\sum_{n=1}^{m}[H_2(x_v, z_l, \gamma_n, k) I_2^m(x_v, z_l, \gamma_n, k) \\
&\ + B_2(z_l, k)[1 - I_2^m(x_v, z_l, \gamma_n, k)]]\Delta(\gamma_n|\gamma_h)X(x_v|x_i)\, Z(z_l|z_j)\}\},
\end{aligned}$$

subject to (3.1) and (3.2).[8] The Pareto weight ρ reflects the husband's bargaining power and is endogenously determined as a function of the state (x_i, z_j, γ_h).

Figure 2 shows how this weight behaves as a function of the state (x, z, γ). Take the case where the match quality variable has the high value. Observe that the male's bargaining strength increases with the level of his productivity, z, and decreases with his wife's, x. The same is true when the match quality variable takes on the low value. As can be seen, most matches end with a marriage. When match quality is low, nobody want to be stuck with a low type. Thus, the degree of assortative mating is fairly low. This may be an artifact of two-period nature of the model. If you reject your mate today, then you only have one more chance

[8] This does *not* say that the model's general equilibrium is Pareto optimal. In general, it's not.

in the future. Hence, this may be rectified by adding more periods. Alternatively, this could be fixed up by having an individual's draw on the marriage market being influenced by his or her type.[9]

Now, suppose that the model is solved holding the weight ρ fixed across states. For example let $\rho = 0.5$, which gives husband and wife an equal say in family decision making, so to speak. The number of marriages plummets in equilibrium from about 85 to 49%. Why? When the weights are fixed, utility can't be transferred from one party to the other in order to prevent a breakup and therefore not nearly as many marriages are sustainable. The degree of positive assortative mating is much higher than under the Nash Bargaining solution. Figure 3 shows the set of sustainable marriages in the economy with Nash Bargaining — i.e., the set of (x, z, γ) for which $I_1^s(x, z, \gamma) = 1$. With a good match quality shock virtually all matches are sustainable. Even when the quality of the match is low most matches are sustainable. No female, however, wants a male from the low end of the distribution. Males aren't quite as choosy. When each party's bargaining power is held fixed, there is a high degree of assortative mating as Figure 4 illustrates. Now, when the quality of match is poor most marriages aren't sustainable.

6. Two Public Policy Experiments

Child tax credits are designed to elevate the welfare of all children in the economy. They transfer income away from families without children to families with them. Child support payments are targeted at those children who experience a family

[9] Fernandez and Rogerson (1999) study the relationship between marital sorting and inequality. In their work, agents are exogenously married according to some probability structure that depends on their type.

breakup because their parents get divorced. Here, to ease the devastating impact that a divorce can have on family income, governments require fathers to pay child support to their former wives. To illustrate how a model such as this can be used, consider the effects of these two public policies.

6.1. Child Tax Credits

Suppose that all families with children, both single and two-parent families, are eligible to collect a child subsidy. This subsidy provides a tax credit per child equal to 0.5% of the average level of income in the benchmark economy. It is financed by a lump-sum tax equal to 1.0% of income in the benchmark economy. What are the effects of this policy?

On the upside, the beneficial effects of the policy are twofold. First, poor families will get extra income that should allow them to invest more time and resources in their children. Second, it should make marriage a more attractive option for males, since single males are taxed without receiving any subsidy. On the downside, the attractiveness of marriage for females, however, might decline. Second, the beneficial aspects of this policy for children may be dissipated by larger family size.

The long-run health of the economy is not helped by this policy. First, the percentage of single mothers increases by about 4.5 percentage points. The percentage of children living with a young single mother rises by about 7 percentage points. This transpires because young single mothers tend to have more children than married ones, and because the policy promotes fertility. The (annualized) population growth rate rises from 0.13 to 1.07%. Single mothers now have 3.9 children as compared with 3.3 for the benchmark economy. Married women now average 2.1 children (versus 1.8 previously).

To understand the model's mechanics, it pays to artificially decompose the experiment into short- and long-run effects. For the short-run effects consider the impact of the child tax credit holding fixed the type distributions for young agents, or Φ_1 and Ω_1. This shuts down the effects on the economy from any induced changes in parental human capital investments. The percentage of single mothers rises by 2 percentage points. Both single and married women have more children (3.8 and 2.0). Married couples also substitute quality for quantity of children. The rise in female headship also reduces the average level of human capital investment in children. These effects operate to reduce the long-run quality of the mating pool, leading to a further 3 point rise in the percentage of single mothers.

Average income in the economy falls by about 11%. This occurs because there is now much less human capital investment in children. First, the increase in female headship is associated with a reduction in investment in children. Single mothers have less wherewithal — in terms of both time and goods — than married couples. Second, with an increase in the quantity of children there is a fall in their quality. As the price of having an extra child drops parents — married or otherwise — substitute quantity for quality. Figure 5 shows the impact of a child tax credit on the steady-state utility distributions for males and females. The policy makes males worse off in the sense that the utility distribution for the benchmark economy stochastically dominates the one for the economy with the child tax credit. This isn't the case for females. Women in the lower strata of the economy are better off with a child tax credit. The rest are slightly worse off. The poorest women have the largest number of children so a tax credit helps them the most. Since women value children more than men (single men don't value them at all), the overall effect of the tax credit on women's expected utility is less detrimental than it is for men.

Endogenous Fertility: Is it important to include a fertility decision in models of marriage and divorce? The answer is yes. To see this, redo the above experiment holding fixed the distribution of fertility across young woman. The effects of the child tax credit on a woman's fertility are therefore shut down. The presence of a child tax credit now has little impact on family structure. The percentage of single mothers living in the economy now rises slightly, an increase of only 0.3 percentage points (compared with 4.5 earlier). The percentage of children living a young single mother moves up by 0.5 percentage points (versus 7.0 before). Average income in the economy now rises by 2.1% (as opposed to -11.4% previously)! When fertility is held fixed, families invest more in each of their children. This has an uplifting effect on society. The welfare gains from a child tax credit may be completely wiped away (and even reversed), however, by an increase in family size (especially for young single mothers).

6.2. Child-Support Payments

The per-child rate of support is set in the benchmark equilibrium at 5.0% of the male's income. What is the effect of this policy? The answer obtained by comparing the benchmark equilibrium to one without child support.

The removal of child support leads to a 0.65 point drop in the percentage of marriages. This is caused by both a rise in the number of young single females (0.8 percentage points) and an increase in divorces among the old (0.3 percentage points). Average income falls by about 1%. The rate of growth in the population rises ever so slightly from 0.13 to 0.19%. These effects seem moderate. The question is why.

One would expect that child support would make marriage and divorce less attractive for males and more attractive for females. The net impact will depend on which party is more likely to walk from a marriage. When child support is eliminated, marriages between high-type males and low-type females turn out to be more likely to break up. Without child support, a high-type male demands more than his low-type wife is willing to bear. Marriages between low-type males and high-type females, however, are less likely to dissolve. With child support in place, high-type females ask for more than a low-type male is willing to contribute to a marriage. The net effect on the equilibrium number of divorces is very small. Some of the drop in the equilibrium number of marriages derives from the fact that divorced mothers now invest less in their children (about 7% drop in e) and this drives down the long-run quality of the mating pool. This can be seen by examining the impact of removing child support, which is done by holding the type distributions for young agents, or Φ_1 and Ω_1, fixed. Again, this turns off the effects on the economy from any induced changes in parental human capital investments. When this is done the number of marriages drops by 0.45 percentage points. Hence, about 0.20 percentage points of the fall in the number of marriages is due to the drop in the long-run quality of the mating pool.

Nash Bargaining, again: The elimination of child support leads to some interesting reallocations within the family. When child support is eliminated an older female has a lower threat point. So her husband has relatively more bargaining power. Let $\mathcal{B}_2$ and $\mathcal{C}_2$ denote the combinations of (x, z, γ, k) that generate viable marriages among the old in the benchmark and no-child support equilibriums. The old male's weight increases for each and every $(x, z, \gamma, k) \in \mathcal{B}_2 \cap \mathcal{C}_2$. The average weight for males rises from 0.57 to 0.60. Older females do indeed work more.[10] Their leisure falls by almost 4 percentage points. Almost all of this is due to increased work in the market. (These changes are also due in part to the fact that high-type women constitute a larger fraction of marriages now.) Now, consider the impact on a young male's weight. Denote by $\mathcal{B}_1$ and $\mathcal{C}_1$ the combinations of (x, z, γ) that generate viable marriages among in the benchmark and no-child support steady states. Surprisingly, a young male's weight decreases for each and every $(x, z, \gamma) \in \mathcal{B}_1 \cap \mathcal{C}_1$! Why? A young female realizes that the gains from being married when she is old are lower when there is no child support in place. Hence, she will be more reluctant to marry when she is young. She demands more from her young suitor. Figure 6 shows the decline in the young male's weight, ρ, that occurs when child-support is withdrawn — the figure shows the average weight for each type of married male. On average, the young male's weight falls from 0.61 to 0.60. Therefore, some of the gains that males realize when child support is removed are redistributed back to females. A young married female's leisure rises by 1.8 percentage points, on average.

Last, the manner in which households undertake their decision-making appears to be important for analyzing the consequences of economic policy. To see this, suppose that the Nash bargaining weights are held at their benchmark values when child support payments are eliminated. Now, the equilibrium number of marriages plummets by 10 percentage points. Average income drops by 18%. A

[10] To calculate the average one needs to know how many type-(x, z, γ, k) marriages there are. The distribution of marriages will be different for the benchmark and no-child support economies. The average was computed using the distribution from the benchmark economy — so as to not contaminate the changes in the male's weights with the shift in the distribution.

marriage is no longer as flexible as before. One party is less able to transfer utility to the other in order to keep the marriage viable.[11]

7. Conclusion

An overlapping generations model of marriage, divorce, and the quantity and quality of children is developed here to study the distribution of income. Singles meet in a marriage market and are free to accept or reject marriage proposals from the opposite sex. Likewise, married agents must decide whether or not to remain with their current spouses. Within a marriage, decisions about how much to work, the number of children, and the amount of time and money to invest per child are decided by Nash Bargaining. In the model's general equilibrium, some adults are married while others aren't. Some females have children in wedlock, others out of it. Marital status and income are related. Families headed by a single mother are the poorest. Likewise, fertility and income are also related. Fertility declines with income. Single mothers have the most children. Children raised by a single mother have a greater tendency (relative to other children) to grow up poor due to a lack of human capital investment. The distribution of income is more skewed when family size is taken into account.

Can social policies be designed to improve the society's welfare? Future generations of the prototype model may shed insight on such questions. To illustrate

[11] An efficient marriage contract would specify, at the time of marriage, childsupport and alimony payments as a function of each parties type. Such contracts will not be time consistent, so enforceability is an issue. Modelling such contracts would greatly complicate the current analysis. Flinn (forth.) analyzes the determination of child support payments between divorced parents.

how the model could be used in such a context the impact of child tax credit and child support payments are considered. When the number of children is held fixed, child tax credits increase the amount of income per child. But, the number of children cannot be held fixed since the policy promotes an increase in family size. It also reduces the attractiveness of marriage for females. On net, child tax credits fail to elevate the well being of society.

Child support payments are aimed to insulate children from the drop in family income that occurs when their parents divorce. Child support payments should make divorce more attractive for females and less attractive for males. The effect on the equilibrium number of marriages is small. This is because child support payments reduce marital breakups between high-type males and low-type females, but promote breakups between low-type males and high-type females. This experiment highlights the fact that the form of household decision making may be important for designing public policy. Child support payments transfer resources away from husbands toward wives, other things equal. This strengthens the hand of married women vis à vis their husbands. With Nash bargaining utility can be transferred away from a husband to a wife to keep a marriage sustainable, so long as it is in the husband's interest to do so. But, to the extent that single males have the option to remain unmarried, part of this transfer will be undone by renegotiating the terms of marriage. Last, the model is still too crude to place confidence in the results for these two policy experiments. Future generations of the model, however, may be able to enlist in public service.

8. Appendix A: Algorithm for Nash Bargaining

Representing the Nash Bargaining Problem as a Pareto Problem: Consider the Nash Bargaining problem when the number of children, k, is held fixed. The Nash Bargaining problem can be reformulated as a Pareto problem, a fact demonstrated later. Therefore, for some Pareto weight $\rho(k) \in (0,1)$ it solves

$$\max_{0 \le n,l,t \le 1, d} \{(1-\rho(k))[F(c,e,k,1-l-t) - G(x,k,z) + \rho(k)[M(c,e,k,1-n) - B(z,k)]\}, \qquad \text{P}(11)$$

subject to the constraints for household production and human capital investment. Given the presence of the inequality constraints this is a nontrivial Kuhn-Tucker problem. For instance, in some marriages the woman will work in the market, while in others she won't.

Consider the case where an interior solution obtains. The first-order conditions for an interior solution are:

$$(1-\rho(k))F_c + \rho(k)M_c = -\rho(k)\frac{M_n}{\Psi z}, \tag{8.1}$$

$$(1-\rho(k))F_c + \rho(k)M_c = -(1-\rho(k))\frac{F_l}{\Psi x}, \tag{8.2}$$

$$[(1-\rho(k))F_e + \rho(k)M_e]Q_t = -(1-\rho(k))F_t, \tag{8.3}$$

and

$$\Psi[(1-\rho(k))F_c + \rho(k)M_c] = [(1-\rho(k))F_e + \rho(k)M_e]Q_d. \tag{8.4}$$

Observe that when $\rho(k) = [F + \mathsf{W} - G]/\{[M + \mathsf{H} - B] + [F + \mathsf{W} - G]\}$ the solution to the Pareto problem P(11) will correspond with the solution to

$$\max_{0 \le n,l,t \le 1, d} \{\ [F(c,e,k,1-l-t) + \mathsf{W} - G(x,k,z)]$$

$$\times[M(c, e, k, 1-n) + \mathsf{H} - B(z, k)]\}, \qquad \text{P(12)}$$

subject to the constraints for household production and human capital investment, and where W and H are the continuation values associated with the married state. This fact is readily verifiable by comparing the first-order conditions associated with the two problems while imposing the condition $\rho(k) = [F + \mathsf{W} - G]/\{[M + \mathsf{H} - B] + [F + \mathsf{W} - G]\}$. This shows that the solution to the Nash Bargaining problem solves a Pareto problem.

Solving the Nash Bargaining problem: It is easier to solve numerically the Pareto problem P(11) than the Nash Bargaining problem P(12). The Nash bargaining problem can only be easily solved on the set of viable marriages. In advance it is hard to know what this set is. To compute the solution to the Pareto problem requires finding the weight $\rho(k)$ that maximizes the product of the net gains from marriage, again holding fixed the number of children, k. So, the algorithm proceeds by making a guess for $\rho(k)$. The problem P(11) is then solved using this guess. This involves numerically solving the set of equations (8.1) to (8.4), or their analogues that incorporate the appropriate Kuhn-Tucker conditions — a married woman may not work in the market, for instance. This gives values for F and M. The weight is then updated using the formula

$$\rho(k) = \min\{\max\{\frac{[F + \mathsf{W} - G]}{[M + \mathsf{H} - B] + [F + \mathsf{W} - G]}, \delta\}, 1 - \delta\},$$

for some small $\delta > 0$. Therefore, $0 < \rho(k) < 1$. The Pareto problem is then recomputed using the new weight. The algorithm proceeds until a fixed point is found. This gives the values of $M + \mathsf{H} - B$ and $F + \mathsf{W} - G$ for a fixed number of kids, k. Sometimes a fixed point cannot be found, because the marriage is not viable. For a marriage to be viable, $M + \mathsf{H} - B$ and $F + \mathsf{W} - G$ must both exceed zero. Observe that if $M + \mathsf{H} - B < 0$ then $\rho(k) > 1$, while if $F + \mathsf{W} - G < 0$ then $\rho(k) < 0$. Therefore, it is easy deduce which marriages are viable or not. For example, set $\rho(k) = 1 - \delta$ and solve the Pareto problem P(11). If $M + \mathsf{H} - B < 0$ then there is no viable marriage from the male's perspective.

Last, when the number of kids is also a choice variable the algorithm then picks $k \in \mathcal{K}$ over the set of viable marriages to maximize the Nash Product:

$$\max_{k \in \mathcal{K}} [F + \mathsf{W} - G][M + \mathsf{H} - B].$$

Now, let k^* denote the solution to the above problem and define ρ by $\rho = \rho(k^*)$. This is the weight used in the couple's Pareto problem outlined in Section 5.3.[12]

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[12] The number of kids is discrete. It is still true, however, that the Nash Bargaining problem solves a Pareto problem. Suppose that $k^* = \arg\max_{\mathsf{k}} [X(k)Y(k)]$, for some functions X and Y. Then k^* must also solve $\max_{\mathsf{k}} [\rho X(k) + (1-\rho)Y(k)]$, when $\rho = Y(k^*)/[X(k^*) + Y(k^*)]$. Suppose not. Then the optimal value of the objective function would be proportional to $Y(k^*)X(k) + X(k^*)Y(k)$, a fact that is readily seen by substituting in for ρ. But, this can't be an optimum. By choosing $k = k^*$ the maximand could be increased since the Nash product maximizes $X(k)Y(k)$.

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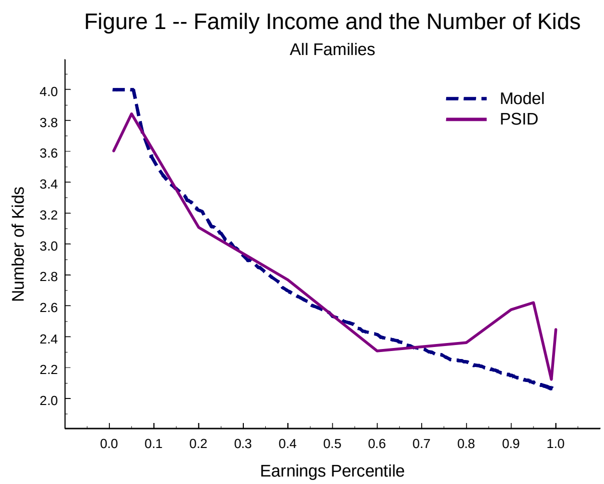
Figure 1 -- Family Income and the Number of Kids
All Families
Model
PSID
Number of Kids
2.0
2.2
2.4
2.6
2.8
3.0
3.2
3.4
3.6
3.8
4.0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Earnings Percentile

Figure 8.1:

FIGURE 2 -- Male's Weight, Nash Bargaining

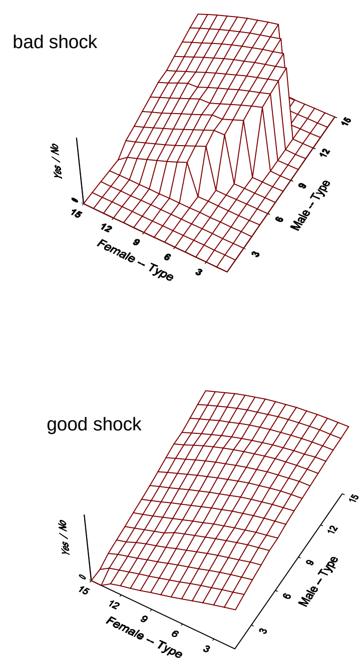

Figure 8.2:

FIGURE 3 -- Viable Marriages, Nash Bargaining

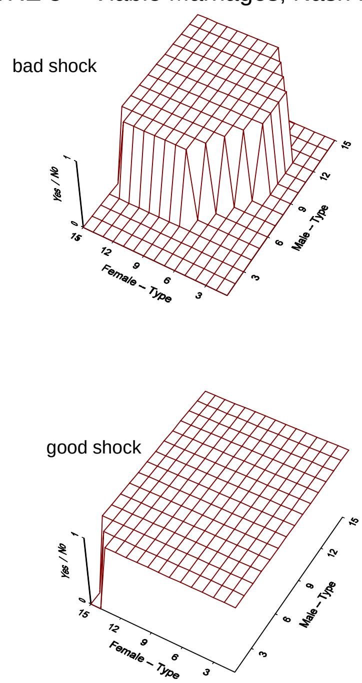

Figure 8.3:

FIGURE 4 --- Viable Marriages, Fixed Weight

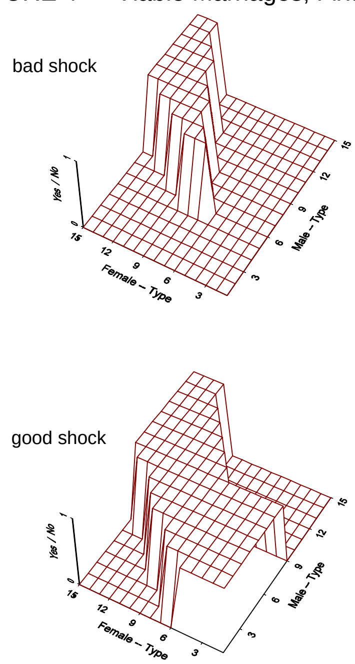

Figure 8.4:

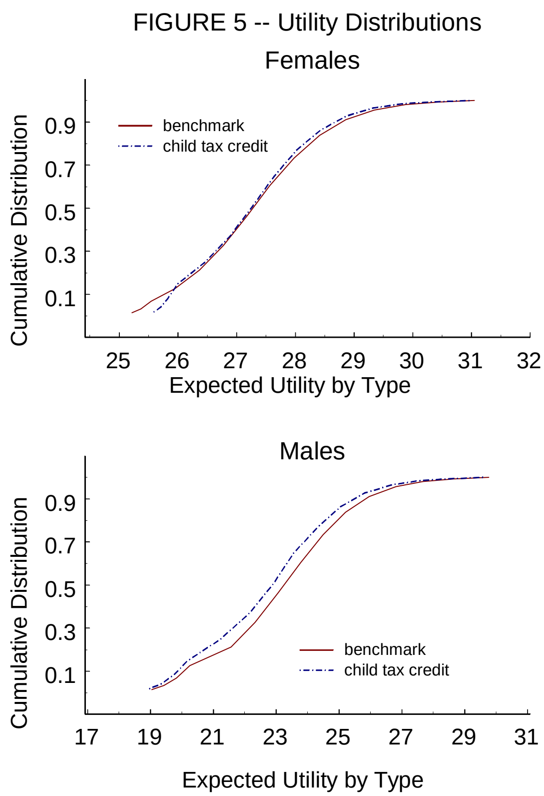
FIGURE 5 -- Utility Distributions
Females
benchmark
child tax credit
Cumulative Distribution
0.9
0.7
0.5
0.3
0.1
25
26
27
28
29
30
31
32
Expected Utility by Type
Males
benchmark
child tax credit
Cumulative Distribution
0.9
0.7
0.5
0.3
0.1
17
19
21
23
25
27
29
31
Expected Utility by Type

Figure 8.5:

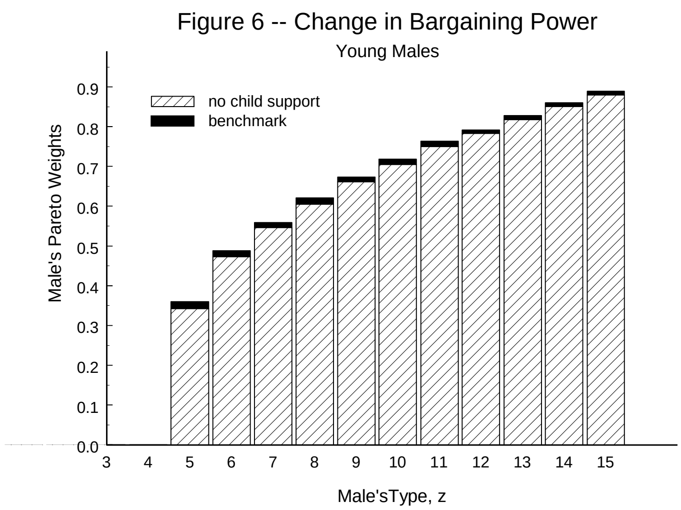
Figure 6 -- Change in Bargaining Power
Young Males
no child support
benchmark
Male's Pareto Weights
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
3
4
5
6
7
8
9
10
11
12
13
14
15
Male'sType, z

Figure 8.6: