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Structural Spurious Regressions and A Hausman-type Cointegration Test*

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Abstract

This paper proposes two estimators based on asymptotic theory to estimate structural parameters with spurious regressions involving unit-root nonstationary variables. This approach motivates a Hausman-type test for the null hypothesis of cointegration for dynamic Ordinary Least Squares estimation using one of our estimators for spurious regressions. We apply our estimation and testing methods to four applications: (i) long-run money demand in the U.S.; (ii) long-run implications of the consumption-leisure choice; (iii) output convergence among industrial and developing countries; (iv) Purchasing Power Parity for traded and non-traded goods.

Keywords: Spurious regression, GLS correction method, Dynamic regression, Test for cointegration.

JEL Classification Numbers: C10, C15

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1 Introduction

Economic models often imply that certain variables are cointegrated. However, tests often fail to reject the null hypothesis of no cointegration for these variables. One possible explanation of these test results is that the error is unit-root nonstationary due to a nonstationary measurement error in one variable or nonstationary omittted variables. In the unit-root literature, when the stochastic error of a regression is unit-root nonstationary, the regression is technically called a spurious regression. This is because the standard t-test tends to be spuriously significant even when the regressor is statistically independent of the regressand in Ordinary Least Squares. Monte Carlo simulations have often been used to show that the spurious regression phenomenon occurs with regressions involving unit-root nonstationary variables (see, e.g., Granger and Newbold (1974), Nelson and Kang (1981, 1983)). Asymptotic properties of estimators and test statistics for regression coefficients of these spurious regressions have been studied by Phillips (1986, 1998) and Durlauf and Phillips (1988) among others.

The purpose of this paper is twofold. First, we propose a new approach to estimating structural parameters with spurious regressions. When structural parameters can be recovered from spurious regressions, we call these structural spurious regressions. Second, we propose a Hausman-type test for the null hypothesis of cointegration. This test is naturally motivated by the structural regression approach. We also show that this test can be used to test for cointegration even when the spurious regression is not structural under the alternative hypothesis.

As an example of a structural spurious regression, consider a regression to estimate the money demand function when money is measured with a nonstationary error. Currency held by domestic economic agents for legitimate transactions is very hard to measure since currency is held by foreign residents and is also used for black market transactions. Therefore, money may be measured with a nonstationary error. As shown by Stock and Watson (1993) among others, under the assumption that the money demand function is stable in the long-run, we have a cointegrating regression if all variables are measured without error. If the variables are measured with stationary measurement errors, we still have a cointegrating regression. However, if money is measured with a nonstationary measurement error, we have a spurious regression. We can still recover structural parameters under certain conditions. The crucial assumption is that the nonstationary measurement error is not cointegrated with the regressors.

Another example of a structural spurious regression is a regression of money demand with nonstationary omitted variables. Consider the case in which the money demand is stable in the long-run when a measure of shoe leather costs of holding money is included as an argument. If an econometrician omits the measure of shoe leather costs from the money demand regression, and if the measure is nonstationary, the regression error is nonstationary. Shoe leather costs of holding money are related to the value of time, and therefore to the real wage rate. Because the real wage rate is nonstationary in standard dynamic stochastic general equilibrium models with a nonstationary technological shock, the omitted measure of the shoe leather costs is likely to be nonstationary. In this case, the money demand regression that omits the measure is spurious, but we can still recover structural parameters under certain conditions. The crucial assumption is that the omitted variable is not cointegrated with the regressors.

Our structural spurious regression approach is based on the Generalized Least Squares (GLS) solution of the spurious regression problem analyzed by Ogaki and Choi (2001),[1] who use an exact small

[1] Another approach would be to take the first difference to induce stationarity and then use instrumental variables. This is the approach proposed by Lewbel and Ng (2005) for their Nonstationary Translog Demand System. Our approach exploits the particular form of endogeneity assumed by many authors in the cointegration literature and avoids the use of instrumental variables. Our approach yields more efficient estimators as long as the particular form of endogeneity is correctly specified. This is important especially when weak instruments cause problems.

sample analysis based on the conditional probability version of the Gauss-Markov Theorem. We develop asymptotic theory for two estimators motivated by the GLS correction: GLS corrected dynamic regression and feasible GLS (FGLS) corrected dynamic regression estimators. These estimators will be shown to be consistent and asymptotically normally distributed in spurious regressions. When the error term is in fact stationary, and hence the variables are cointegrated, the GLS corrected estimator is not efficient, but the FGLS corrected estimator, like the OLS estimator, is superconsistent. Hence the FGLS estimation is a robust procedure with respect to the error specification. The FGLS corrected estimator is asymptotically equivalent to the GLS corrected estimator in spurious regressions and it is asymptotically equivalent to the OLS estimator in cointegrating regressions.

In some applications, it is hard to determine whether or not the error in the regression is stationary or unit-root nonstationary because test results are inconclusive. In such applications, the FGLS corrected estimator is attractive because it is consistent in both situations as long as the method of the dynamic regression removes the endogeneity problem.

This approach naturally motivates a Hausman-type test[2] for the null hypothesis of cointegration against the alternative hypothesis of no cointegration (or a spurious regression) in the dynamic OLS framework. We construct this test by noting that while both the dynamic OLS and GLS corrected dynamic regression estimators are consistent in cointegration estimation, the dynamic OLS estimator is more efficient.[3] On the other hand, when the regression is spurious only the GLS corrected dynamic regression estimator is consistent. Hence, we could do a cointegration test based on the specifications on the error. We show that under the null hypothesis of cointegration the test statistics have a usual χ^2 limit distribution, while under the alternative hypothesis of a spurious regression, the test statistic diverges.

In some applications, the assumption that the spurious regression is structural under the alternative hypothesis is not very attractive. If the violation of cointegration arises from reasons other than a nonstationary measurement error, it is hard to believe that the resulting spurious regression is structural. For this reason, we relax the assumption that the spurious regression is structural and show that the Hausman-type cointegration test statistic still diverges under the alternative hypothesis.

Dynamic OLS is used in many applications of cointegration. However, few tests for cointegration have been developed for dynamic OLS with the exception of Shin's (1994) test. As in Phillips and Ouliaris (1990), the popular Augmented Dickey-Fuller (ADF) test for the null hypothesis of no cointegration was originally designed to be applied to the residual from static OLS rather than the residual from dynamic OLS. Because the OLS and dynamic OLS estimates are often substantially different, it is desirable to have a test for cointegration applied to dynamic OLS. Another aspect of our Hausman type test is that it is for the null hypothesis of cointegration. Ogaki and Park (1998) argue that it is desirable to test the null hypothesis of cointegration rather than that of no cointegration in many applications when economic models imply cointegration.

Using Monte Carlo experiments, we compare the finite sample performance of the Hausman-type test with the test proposed by Shin (1994), which is a locally best invariant test for the null of zero variance of a random walk component in the disturbances. The experiment results show that up to the sample size of 300 the Hausman-type test is dominant in both size and power. When the sample size increases, Shin's test is better in power, but it suffers from a serious oversize problem.

[2] This test can also be called Durbin-Wu-Hausman-type test as it is closely related to ideas and tests in Durbin (1954) and Wu (1973) as well as a family of tests proposed by Hausman (1978).

[3] After completing the first draft, it has come to our attention that the Hausman-type test was originally proposed by Fernández-Macho and Mariel (1994) for the static OLS cointegrating regression with strict exogeneity and without any serial correlation. The test probably has not been popular because these assumptions are hard to justify in applications and because the test was not developed for dynamic regressions.

In some applications, it is appropriate to consider the possibility that measurement error is I(1) and is not cointegrated with the regressors. For these applications, the ADF test is applicable under the null hypothesis of a structural spurious regression as shown in Hu and Phillips (2005). For such applications, we recommend that both the ADF test and Hausman-type test be applied because it is not clear which null hypothesis is more appropriate.

We apply our estimation and testing methods to four applications: (i) long-run money demand in the U.S.; (ii) long-run implications of the consumption-leisure choice; (iii) output convergence among industrial and developing countries; (iv) Purchasing Power Parity (PPP) for traded and non-traded goods. In the first two applications, we focus on estimating unknown structural parameters, while in the last two applications we purport to test for cointegration with the Hausman-type cointegration test, where we relax the assumption that the spurious regression under the alternative hypothesis is structural.

The rest of the paper is organized as follows. Section 2 gives econometric analysis of the model, including asymptotic theories and finite sample simulation studies. Section 3 presents models of nonstationary measurement error and nonstationary omitted variables. It then presents the empirical results of four applications. Section 4 contains concluding remarks.

2 The econometric model

Consider the regression model

$$y_t = \beta' x_t + \eta_t, \tag{1}$$

where $\{x_t\}$ is an m-vector integrated process generated by

$$\Delta x_{it} = v_{it}.$$

The error term in (1) is assumed to be

$$\eta_t = \sum_{i=1}^{m} \sum_{j=-k}^{k} \gamma_{i,j} v_{i,t-j} + e_t, \tag{2}$$

$$e_t = \rho e_{t-1} + u_t. \tag{3}$$

Assumption 1 *Assume that $v_t = (v_{1t}, \ldots, v_{mt})'$ and u_t are zero mean stationary processes with $E|v_{it}|^\alpha < \infty$, $E|u_t|^\alpha < \infty$ for some $\alpha > 2$ and strong mixing with size $-\alpha/(\alpha-2)$. We also assume that the method of dynamic regression removes the endogeneity problem, that is, $E(u_t v_s) = \mathbf{0}$ for all t, s. We call this the strict exogeneity assumption for the dynamic regression.*

The conditions on v_t and u_t ensure the invariance principles: for $r \in [0,1]$, $n^{-1/2}\sum_{t=1}^{[nr]} v_t \to_d V(r)$, $n^{-1/2}\sum_{t=1}^{[nr]} u_t \to_d U(r)$, where $V(r)$ is an m-vector Brownian motion with covariance $\sum_{j=-\infty}^{\infty} E(v_t v_{t-j}')$, and $U(r)$ is a Brownian motion with variance $\sum_{j=-\infty}^{\infty} E(u_t u_{t-j})$. The functional central limit theorem holds for weaker assumptions than assumed here (De Jong and Davidson (2000)), but the conditions assumed above are general enough to include many stationary Gaussian or non-Gaussian ARMA processes that are commonly assumed in empirical modeling.

Let $\mathbf{v}_t = (\Delta x_{1,t-k}, \ldots, \Delta x_{1,t}, \ldots, \Delta x_{1,t+k}, \ldots, \Delta x_{m,t-k}, \ldots, \Delta x_{m,t}, \ldots, \Delta x_{m,t+k})'$, and $\boldsymbol{\gamma} = (\gamma_{1,-k}, \ldots, \gamma_{1,0}, \ldots, \gamma_{1,k}, \ldots, \gamma_{m,-k}, \ldots, \gamma_{m,0}, \ldots, \gamma_{m,k})'$. We estimate the structural parameter β in the regression

$$y_t = \beta' x_t + \boldsymbol{\gamma}' \mathbf{v}_t + e_t. \tag{4}$$

The inference procedure about β differs according to different assumptions on the error term e_t in (3). When $|\rho| < 1$, e_t is stationary, and hence regression (4) is a cointegration regression with serially correlated error. When $\rho = 1$, e_t is a unit root nonstationary process and the OLS regression is spurious. Both models are important in empirical studies in macroeconomics and finance.

In the next two sections, we will study the asymptotic properties of different estimation procedures under these two assumptions. Under the assumption that $\rho = 1$, OLS is not consistent while both GLS correction and FGLS correction will give consistent and asymptotically equivalent estimators. Under the assumption that $|\rho| < 1$, the GLS corrected estimator is not efficient as it is $\sqrt{n}$ convergent, but the FGLS estimator is n convergent and asymptotically equivalent to the OLS estimator. Therefore, FGLS is robust with respect to the error specifications ($\rho = 1$ or $|\rho| < 1$).

2.1 Regressions with I(1) error

In this section we consider the situation when the error term is I(1), i.e., $\rho = 1$ in (3). The estimation methods we study are dynamic OLS, GLS correction, and FGLS correction.

2.1.1 The dynamic OLS spurious estimation

We start with dynamic OLS estimation of regression (4). Under the assumption of $\rho = 1$, this regression is spurious since for any value of β the error term is always I(1). In Appendix A, we show that the DOLS estimator $\hat{\beta}_{\text{dols}}$ has the following limit distribution:

$$(\hat{\beta}_{\text{dols}} - \beta_0) \to \left[\int_0^1 V(r)V(r)'dr\right]^{-1} \left[\int_0^1 V(r)U(r)dr\right]. \tag{5}$$

$\hat{\boldsymbol{\gamma}}$ in the estimation is also inconsistent with $\hat{\boldsymbol{\gamma}} - \boldsymbol{\gamma}_0 = O_p(1)$. As remarked in Phillips (1986, 1989), in spurious regressions the noise is as strong as the signal. Hence, uncertainty about β persists in the limiting distributions.

2.1.2 GLS corrected estimation

When $\rho = 1$, we can filter all variables in regression (4) by taking the full first difference, and use OLS to estimate

$$\Delta y_t = \beta' \Delta x_t + \boldsymbol{\gamma}' \Delta \mathbf{v}_t + u_t = \theta' \Delta z_t + u_t, \tag{6}$$

where $\theta = (\beta', \boldsymbol{\gamma}')'$ and $z_t = (x_t', \mathbf{v}_t')'$. This procedure can be viewed as GLS corrected estimation.[4]

If we let $\tilde{\theta}_{\text{dgls}}$ denote the GLS corrected estimator, then we can show that

$$\sqrt{n}(\tilde{\theta}_{\text{dgls}} - \theta_0) \to_d N(\mathbf{0}, \Omega), \tag{7}$$

where $\Omega = Q^{-1}\Lambda Q^{-1}$ with $Q = E(z_t z_t')$ and Λ is the long run variance matrix of $z_t u_t$. Thus β in a structural spurious regression can be consistently estimated (jointly with $\boldsymbol{\gamma}$) and the estimators are asymptotically normal. In the special case when $m = 1$, $\{v_{1t}\}$ and $\{u_t\}$ are *i.i.d.* sequences and $\eta_t = e_t$, (7) gives that

$$\sqrt{n}(\tilde{\theta}_{\text{dgls}} - \theta_0) \to_d N(0, \sigma_u^2/\sigma_{1v}^2),$$

where σ_u^2 and σ_{1v}^2 are variances of u_t and v_{1t}, respectively.

[4]This is a conventional GLS procedure when u_t is *i.i.d.*. When u_t is serially correlated as in our approach, we name this procedure GLS corrected dynamic estimation.

2.1.3 The FGLS estimation

To use GLS to estimate a regression with serial correlation in empirical work, a Cochrane-Orcutt FGLS procedure is usually adopted. This procedure also works for spurious regressions as shown by Phillips and Hodgson (1994). They show that the FGLS estimator is asymptotically equivalent to that in the differenced regression when the error is unit-root nonstationary. In the present paper, we will show that the FGLS correction to the dynamic regression provides a consistent and robust estimator to structural spurious regressions.

Let the residual from OLS regression (4) be denoted by $\hat{e}_t$,

$$\hat{e}_t = y_t - \hat{\beta}_n' x_t - \hat{\boldsymbol{\gamma}}_n' \mathbf{v}_t.$$

To conduct the Cochrane-Orcutt GLS estimation, we first run an AR(1) regression of $\hat{e}_t$,

$$\hat{e}_t = \hat{\rho}_n \hat{e}_{t-1} + \hat{u}_t. \tag{8}$$

It can be shown that $n(\hat{\rho}_n - 1) = O_p(1)$. Conduct the following Cochrane-Orcutt transformation of the data:

$$\tilde{y}_t = y_t - \hat{\rho}_n y_{t-1}, \quad \tilde{x}_t = x_t - \hat{\rho}_n x_{t-1}, \quad \tilde{\mathbf{v}}_t = \mathbf{v}_t - \hat{\rho}_n \mathbf{v}_{t-1}. \tag{9}$$

Then consider OLS estimation of the regression

$$\tilde{y}_t = \beta' \tilde{x}_t + \boldsymbol{\gamma}' \tilde{\mathbf{v}}_t + error = \theta' \tilde{z}_t + error, \tag{10}$$

where $\tilde{z}_t = (\tilde{x}_t', \tilde{\mathbf{v}}_t{}')'$. The OLS estimator of θ in (10) is computed as

$$\tilde{\theta}_{\text{fgls}} = \left[\sum_{t=1}^{n} \tilde{z}_t \tilde{z}_t' \right]^{-1} \left[\sum_{t=1}^{n} \tilde{z}_t \tilde{y}_t \right]. \tag{11}$$

The limiting distribution of $\tilde{\theta}_{\text{fgls}}$ can be shown to be the same as in (7). Intuitively, even though the dynamic OLS estimator is inconsistent, the residual is unit-root nonstationary because no linear combination of y_t and x_t is stationary. Therefore, ρ_n approaches unity in the limit, and $\tilde{z}_t$ behaves asymptotically equivalent to Δz_t. A detailed proof of results in this section is given in Appendix A.

2.2 Regressions with I(0) error

In this section we consider the asymptotic distributions of the three estimators (DOLS estimator, GLS corrected estimator and FGLS corrected estimator) under the assumption of cointegration, i.e., $|\rho| < 1$ (3).

2.2.1 The dynamic OLS estimation

Under the assumption of cointegration, the DGP of y_t is

$$y_t = \beta' x_t + \boldsymbol{\gamma}' \mathbf{v}_t + e_t, \quad e_t = \rho e_{t-1} + u_t, \quad |\rho| < 1. \tag{12}$$

Applying the invariance principle, for $r \in [0,1]$, $n^{-1/2} \sum_{t=1}^{[nr]} e_t \to E(r)$, where $E(r)$ is a Brownian motion with variance $\sum_{j=-\infty}^{\infty} E(e_t e_{t-j})$. The limiting distribution of the OLS estimator of β, which is asymptotically independent of $\hat{\boldsymbol{\gamma}}_n$, is known to be

$$n(\hat{\beta}_{\text{dols}} - \beta_0) \to_d \left(\int_0^1 V(r) V(r)' dr \right)^{-1} \left(\int_0^1 V(r) dE(r) \right). \tag{13}$$

2.2.2 GLS corrected estimation

Now, if we take the full first difference as we did in the spurious regressions, the regression becomes

$$\Delta y_t = \beta' \Delta x_t + \boldsymbol{\gamma}' \Delta \mathbf{v}_t + e_t - e_{t-1} = \theta' \Delta z_t + e_t - e_{t-1}. \tag{14}$$

Note that this transformation leads to loss in efficiency since the estimator $\tilde{\beta}_{\text{dgls}}$ is now $\sqrt{n}$ convergent rather than n convergent as the DOLS estimator. With some minor revisions to equation (7), the limiting distribution of the estimator in this case can be written as

$$\sqrt{n}(\tilde{\theta}_{\text{dgls}} - \theta_0) \rightarrow_d N(\mathbf{0}, \Omega^*), \tag{15}$$

where $\Omega^* = Q^{-1}\Lambda^* Q^{-1}$. Q is defined as following equation (7) and Λ^* is the long run variance matrix of vector $z_t \Delta e_t$. In the special case when $m = 1$, $\{v_{1t}\}$ and $\{u_t\}$ are *i.i.d.* sequences and $\eta_t = e_t$, $\Omega^* = 2\sigma_e^2(1 - \psi_e)/\sigma_{1v}^2$, where ψ_e is the first order autocorrelation coefficient of $\{e_t\}$.

2.2.3 The FGLS estimation

Instead of taking the full first difference, if we estimate the autoregression coefficient in the error and use this estimator to filter all sequences, we will obtain an estimator that is asymptotically equivalent to the DOLS estimator. Intuitively, in the case that the error $e_t = u_t$ is serially uncorrelated, then the AR(1) coefficient $\hat{\rho}_n$ will converge to zero, and hence the transformed regression will be asymptotically equivalent to the original regression. Or, if the error is stationary and serially correlated, then the AR(1) coefficient will be less than unity, and, as shown in Phillips and Park (1988), the GLS estimator and the OLS estimator in a cointegration regression are asymptotically equivalent.

If we conduct the Cochrane-Orcutt transformation (9) and estimate β in regression

$$\tilde{y}_t = \tilde{\beta}_n' \tilde{x}_t + \tilde{\boldsymbol{\gamma}}_n' \tilde{\mathbf{v}}_t + error, \tag{16}$$

then Appendix B shows that the limiting distribution of $\tilde{\beta}_n$ is the same as the limit of the OLS estimator given in (13). We summarize the FGLS corrected estimator in the following proposition:

Proposition 1 *Suppose Assumption 1 holds. In spurious regressions, the FGLS corrected estimator is asymptotically equivalent to the GLS corrected estimator, and its limit distribution can be written as*

$$\sqrt{n}(\tilde{\theta}_{\text{fgls}} - \theta_0) \rightarrow_d N(\mathbf{0}, \Omega).$$

In cointegration regressions, the FGLS corrected estimator is asymptotically equivalent to the DOLS estimator, and its limit distribution can be written as

$$n(\hat{\beta}_{\text{fgls}} - \beta_0) \rightarrow_d \left(\int_0^1 V(r)V(r)' dr \right)^{-1} \left(\int_0^1 V(r) dE(r) \right).$$

So FGLS is not only valid when the regression is spurious but also asymptotically efficient when the regression is cointegration.

Remarks: 1. If a constant is added to (4), we show in Appendix D that our methods still apply. In particular, the GLS or FGLS corrected estimators are asymptotically equivalent to that given in (7) under the assumption of spurious regressions.

2. If a trend term is added to (4) (this is the case in which the deterministic cointegration restriction is not satisfied in the terminology of Ogaki and Park (1988)), then the GLS corrected estimation leads to a singular covariance matrix for the estimator when ρ is less than one in absolute value. This is

because a trend term in (4) leads to a constant term in the first differenced regression (14) and because the long run variance of the first difference of e_t multiplied by a constant is zero. Therefore, our methods do not apply to regressions with time trends.

3. Under some conditions, the methods proposed in this paper also apply to other model configurations, such as regressions where the regressors have drifts. These extensions will be studied in future work.

2.3 Finite sample performance of the three estimators

From the above analysis, we show that the FGLS corrected estimation is a robust procedure with respect to error specifications. In this section, we use simulations to study its finite sample performances compared to the other two estimators. In the simulation we consider the case when x_t is a scalar variable and we generate v_t and u_t from two independent standard normal distributions and let $e_t = \rho e_{t-1} + u_t$. The structural parameter is set to $\beta = 2$, and $\boldsymbol{\gamma}'\mathbf{v}_t = 0.5v_t$. The number of iterations in each simulation is 5000 and in each replication, $100 + n$ observations are generated of which the first 100 observations are discarded.

Table 1 shows the bias and the mean square error (MSE) of all three estimators for $\rho = 0, 0.95$, and 1 . When $\rho = 0$, the regression is cointegration with *i.i.d.* error. It is clear that the DOLS estimator is the best one when $n = 50$. But when n reaches 100, the FGLS estimator becomes almost as good as the DOLS estimator. When $\rho = 0.95$, the regression is cointegration with serially correlated error. In this case, the GLS and FGLS estimators are much better than the DOLS estimator. When the sample size increases, the FGLS estimator becomes the best one. Finally when $\rho = 1$, the regression is spurious and the GLS corrected estimator performs the best as expected.

Figure 1 plots the three estimators when $n = 100$ as ρ approaches 1. The figures show that the DOLS estimator becomes flatter and flatter as $\rho \to 1$. The GLS estimator remain largely the same for ρ close to unity. And the FGLS estimator becomes a bit flatter when ρ reaches 1, but it still shows a clear peak around zero.

¿From the finite sample performance, it can be seen that the FGLS estimator is almost as good as the DOLS estimator in cointegrations and it significantly outperforms the DOLS estimator in spurious regressions. The GLS estimator is the best when ρ approaches 1, but it suffers from significant loss in efficiency when ρ is small. So we may want to take full difference only when we are very sure that the error is unit root nonstationary. Otherwise the FGLS estimator is a good choice.

2.4 Hausman specification test for cointegration

2.4.1 The test statistic and its asymptotic properties

In this section we construct a Hausman-type cointegration test based on the difference of two estimators: an OLS estimator ($\hat{\beta}_{\text{dols}}$) and a GLS corrected estimator ($\tilde{\beta}_{\text{dgls}}$). This is equivalent to comparing estimators in a level regression and in a differenced regression. The test is for the null of cointegrating relationships against the alternative of a spurious regression:

$$H_0 : |\rho| < 1; \quad \text{against} \quad H_A : \rho = 1.$$

Our discussions so far suggest that under the null of cointegration, both OLS and GLS are consistent but the OLS estimator is more efficient. However under the alternative of a spurious regression, only the GLS corrected estimator is consistent.

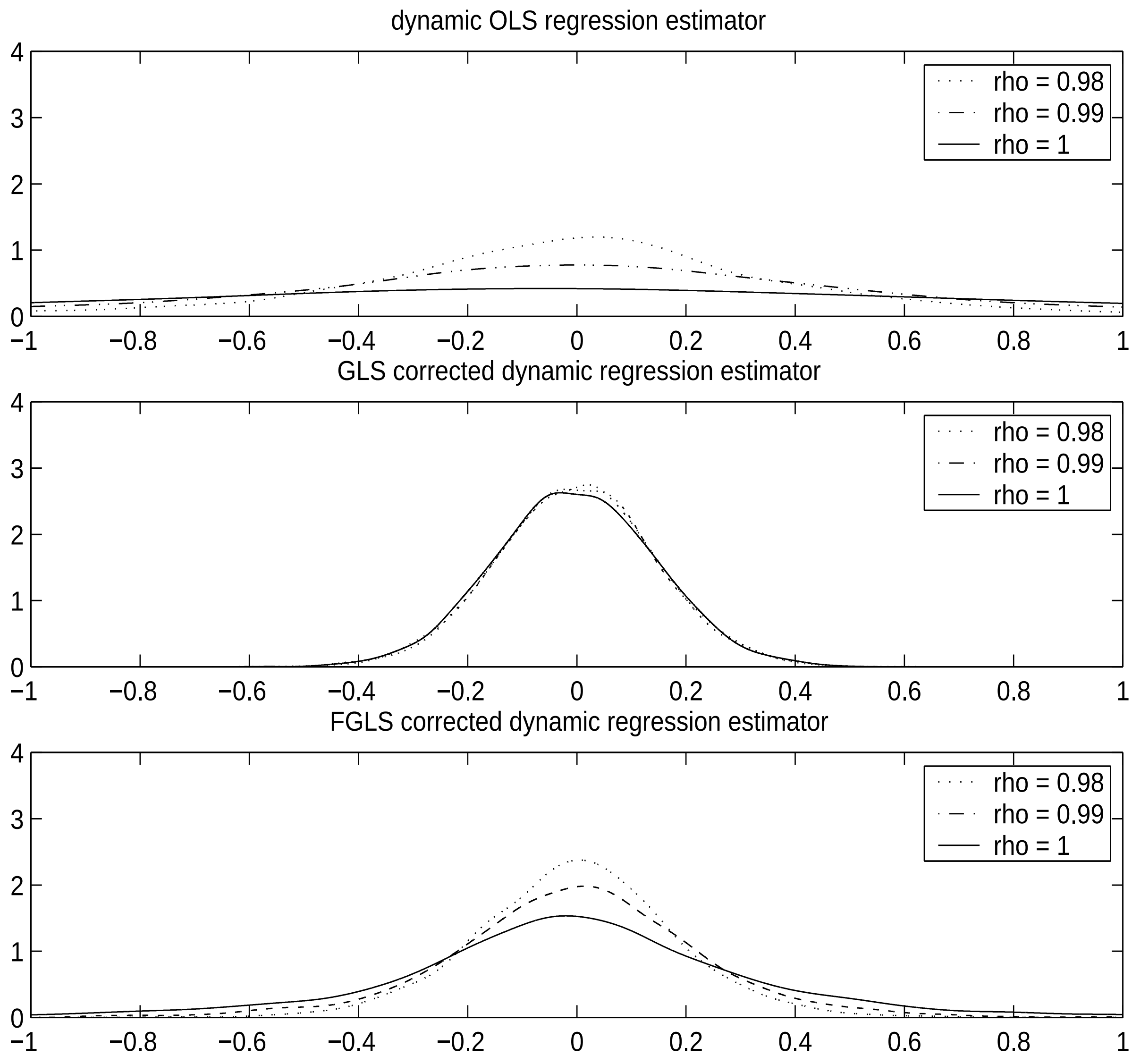

Figure 1: Comparison of three estimators when $n = 100$ and $\rho \to 1$

Let $\hat{V}_\beta$ denote a consistent estimator for the asymptotic variance of $\sqrt{n}(\tilde{\beta}_{\text{dgls}} - \beta)$. Under our assumptions, it converges to the corresponding submatrix of Ω^* under the null hypothesis and to the corresponding submatrix of Ω under the alternative. For example, when $m = 1$, $\{v_{1t}\}$ and $\{u_t\}$ are independent *i.i.d.* and $\eta_t = e_t$, take $\hat{V}_\beta = \left(\frac{1}{n}\sum_{t=1}^n \hat{w}_t^2\right) / \left(\frac{1}{n}\sum_{t=1}^n \Delta x_t^2\right)$, where $\hat{w}_t$ denotes the residuals from OLS estimation of differenced regression. Under the null of cointegration, $\hat{V}_\beta \to 2\sigma_e^2(1-\psi_e)/\sigma_{1v}^2$. Under the alternative of spurious regression, $\hat{V}_\beta \to \sigma_u^2/\sigma_{1v}^2$.

We define the Hausman-type test statistic as:

$$h_n = n(\tilde{\beta}_{\text{dgls}} - \hat{\beta}_{\text{dols}})'\hat{V}_\beta^{-1}(\tilde{\beta}_{\text{dgls}} - \hat{\beta}_{\text{dols}}). \tag{17}$$

Proposition 2 *Suppose Assumption 1 holds. Under the null hypothesis of cointegrations, $h_n \to \chi^2(m)$. Under the alternative of spurious regressions, $h_n = O_p(n)$.*

Proof: under the null of cointegration,

$$\begin{aligned}
& \sqrt{n}(\tilde{\beta}_{\text{dgls}} - \hat{\beta}_{\text{dols}}) \\
= \; & \sqrt{n}(\tilde{\beta}_{\text{dgls}} - \beta_0) - \sqrt{n}(\hat{\beta}_{\text{dols}} - \beta_0) \\
= \; & \sqrt{n}(\tilde{\beta}_{\text{dgls}} - \beta_0) + o_p(1) \\
\to \; & N(0, V_\beta),
\end{aligned}$$

where V_β is the asymptotic variance of $\tilde{\beta}_{\text{dgls}}$ under the assumption of cointegration. Therefore, if $\hat{V}_\beta$ is a consistent estimator for V_β,

$$h_n = n(\tilde{\beta}_{\text{dgls}} - \hat{\beta}_{\text{dols}})'(\hat{V}_\beta)^{-1}(\tilde{\beta}_{\text{dgls}} - \hat{\beta}_{\text{dols}}) \to \chi^2(m).$$

Under the alternative of spurious regressions,

$$\begin{aligned}
& \sqrt{n}(\tilde{\beta}_{\text{dgls}} - \hat{\beta}_{\text{dols}}) \\
= \; & \sqrt{n}(\tilde{\beta}_{\text{dgls}} - \beta_0) - \sqrt{n}(\hat{\beta}_{\text{dols}} - \beta_0) \\
= \; & O_p(1) + O_p(\sqrt{n}) \\
= \; & O_p(\sqrt{n}).
\end{aligned}$$

Hence, $h_n = O_p(n)$ under the alternative.

We can extend the test to allow endogeneity under the alternative. Consider the following DGP:

$$\begin{aligned}
y_t &= \beta' x_t + \boldsymbol{\gamma}'\mathbf{v}_t + \phi s_t + e_t, \\
e_t &= \rho e_{t-1} + u_t,
\end{aligned}$$

where $\{s_t\}$ satisfies the same conditions as u_t and v_t, but it is correlated with $\{v_t\}$. The statistic defined in (17) can be applied to test the hypotheses:

$$\begin{aligned}
& H_0' : |\rho| < 1 \quad \text{and} \quad \phi = 0 \\
\text{against} \qquad & H_A' : \rho = 1 \quad \text{and} \quad \phi \neq 0.
\end{aligned}$$

The asymptotics of h_n under the null H_0' are the same as that under H_0. Under the alternative H_A', we show in Appendix C that the DOLS estimator has the same asymptotic distribution as that under H_A and $h_n = O_p(n)$. Therefore, this Hausman-type test is consistent for the null hypothesis of cointegrations against the alternative of spurious regressions, regardless of whether the exogeneity assumption holds under the alternative.

2.4.2 Finite sample properties of the Hausman-type cointegration test

Before applying the Hausman-type cointegration test empirically, it will be instructive to examine its finite sample properties in comparison with other comparable tests under the same null hypothesis. To this end, we conduct a small simulation experiment based on the following dynamic regression model,

$$y_t = \gamma_1 \Delta x_{t+1} + \beta x_t + \gamma_2 \Delta x_{t-1} + e_t, \quad (18)$$

$$e_t = \rho e_{t-1} + u_t, \quad (19)$$

where $\gamma_1 = 0.3$, $\beta = 2$, $\gamma_2 = -0.5$, and setting $\rho = 0.9$ for the size performance and $\rho = 1$ for the power performance. We consider sample sizes of $n \in \{50, 100, 200, 300, 500\}$ that are commonly encountered in empirical analysis. In the simulations pseudo random numbers are generated using the GAUSS (version 6.0) RNDNS procedures. Each simulation run is carried out with 5,000 replications. At each replication $100 + n$ random numbers are generated of which the first 100 observations are discarded to avoid a start-up effect.

Table 2 reports selected finite sample properties of the Hausman-type cointegration test together with a residual based test under the null of cointegration due to Shin (1994, Shin's test) who extended the KPSS test in the parametrically corrected cointegrating regression. In the simulations the lengths of lead and lag terms for DOLS and DGLS are chosen by the BIC rule.[5] A nonparametric estimation method for long run variance estimation is employed using the QS kernel with the bandwidth of 'integer$[8(n/100)^{1/4}]$'. The results in Table 2 illustrate two points. First, the empirical size of the Hausman-type test is close to the nominal size, in particular when sample size is relatively large, whereas Shin's test suffers from a serious oversize problem. Second, in terms of power, the Hausman-type test dominates Shin's test for moderate sample sizes that are very likely to be encountered empirically. Shin's test seems more powerful when n is relatively large, but only at the cost of severe size distortions. Overall, our simulation results provide evidence in favor of the Hausman-type test.

3 Empirical applications

In this section we apply the GLS-type correction methods and the Hausman-type cointegration test to analyze four macroeconomic issues: (i) long-run money demand in the U.S.; (ii) long-run implications of the consumption-leisure choice; (iii) output convergence among industrial and developing countries; (iv) Purchasing Power Parity (PPP) for traded and non-traded goods. Tthe main purpose of the first two applications is to illustrate the spurious regression approach to estimating unknown structural parameters. Identification of the structural parameters in these two applications are based on nonstationary measurement error or nonstationary omitted variables that are explained in the following two subsections. The main purpose of the last two applications is to apply the Hausman-type cointegration test. The alternative hypothesis is not taken as structural spurious regressions in the last two applications.

3.1 A model of nonstationary measurement error

Consider a set of variables that are cointegrated. One model of a structural spurious regression is based on the case in which one of the variables is measured with nonstationary measurement error. Let y_t^0 be the true value of y_t, and assume that

[5]It is an interesting research topic to investigate the performance of various lag length selection rules, but would be beyond the scope of this paper.

$$y_t^0 = \beta' x_t + \boldsymbol{\gamma}^{0\prime}\mathbf{v}_t + e_t^0 \tag{20}$$

is a dynamic cointegrating regression that satisfies the strict exogeneity assumption.[6] Let y_t be the measured value of y_t^0, and assume that the measurement error satisfies

$$y_t - y_t^0 = \boldsymbol{\gamma}^{m\prime}\mathbf{v}_t + e_t^m, \tag{21}$$

where e_t^m is I(1) and its expectation conditional on x_s for all s is zero. Here, the crucial assumption for identification is that the measurement error is not cointegrated with x_t. Then

$$y_t = \beta' x_t + \boldsymbol{\gamma}'\mathbf{v}_t + e_t \tag{22}$$

where $\boldsymbol{\gamma} = \boldsymbol{\gamma}^0 + \boldsymbol{\gamma}^m$, and e_t is I(1) and satisfies the strict exogeneity assumption.[7]

3.2 A model of nonstationary omitted variables

Another case that leads to a structural spurious regression is a model of nonstationary omitted variables.

$$y_t = \beta' x_t + \theta' x_t^0 + \boldsymbol{\gamma}^{1\prime}\mathbf{v}_t + \boldsymbol{\gamma}^{0\prime}\mathbf{v}_t^0 + e_t^0 \tag{23}$$

where x_t^0 is a vector of I(1) variables and $\mathbf{v}_t^0$ is a vector of leads and lags of the first differences of x_t^0. We imagine that the econometrician omits x_t^0 from his regression. We assume that

$$\theta' x_t^0 + \boldsymbol{\gamma}^{0\prime}\mathbf{v}_t^0 = \boldsymbol{\gamma}^{m\prime}\mathbf{v}_t + e_t^m, \tag{24}$$

where e_t^m is I(1) and its expectation conditional on x_s for all s is zero. Here, the crucial assumption for identification is that the x_t^0 is not cointegrated with x_t. Then

$$y_t = \beta' x_t + \boldsymbol{\gamma}'\mathbf{v}_t + e_t \tag{25}$$

where $\boldsymbol{\gamma} = \boldsymbol{\gamma}^1 + \boldsymbol{\gamma}^m$, and e_t is I(1) that satisfies the strict exogeneity assumption.

3.3 U.S. money demand

The long-run money demand function has often been estimated under a cointegrating restriction among real balances, real income, and the interest rate. The restriction is legitimate if the money demand function is stable in the long-run and if all variables are measured without nonstationary error. Indeed, Stock and Watson (1993) found supportive evidence of stable long-run M1 demand by estimating cointegrating vectors. However, either if money is measured with a nonstationary measurement error or if nonstationary omitted variables exist, then we have a spurious regression and the estimation results based on a cointegration regression are questionable.

First, consider the model of a nonstationary measurement error described above. To be specific, we follow Stock and Watson (1993) and assume that the dynamic regression error is stationary and the strict exogeneity assumption holds for the dynamic regression error when money is correctly measured. We then assume that money is measured with a multiplicative measurement error. We assume that the log measurement error is unit-root nonstationary, and that the residuals of the projection of the log

[6] Note that any variable can be chosen as the regressand in a cointegrating regression. Therefore, we choose the variable with nonstationary measurement error as the regressand.

[7] Here, we assume that the dimensions of $\boldsymbol{\gamma}^0$ and $\boldsymbol{\gamma}^m$ are the same without loss of generality, because we can add zeros as elements of $\boldsymbol{\gamma}^0$ and $\boldsymbol{\gamma}^m$ as needed.

measurement error on the leads and lags of the regressors in the dynamic regression satisfy the strict exogeneity assumption. Given that a large component of the measurement error is arguably currency held by foreign residents and black market participants, the log measurement error is likely to be very persistent. Therefore, the assumption that the log measurement error is unit root nonstationary may be at least a good approximation. The assumption that the measurement error is not cointegrated with the regressors is plausible if the error is mainly due to currency held by foreign residents.

Second, consider the model of nonstationary omitted variables. A possible omitted variable is a measure of the "shoe leather cost" that represents transaction costs. For example, in the literature of money demand estimation, the real wage rate has sometimes been used as a regressor for this reason. Because the real wage rate is I(1) in standard dynamic stochastic general equilibrium models with an I(1) technological shock, the omitted measure of the "shoe leather cost" is to be nonstationary. If the real wage rate is the omitted variable, the assumption that it is not cointegrated with the regressors that include log income is not very plausible. However, it is possible that the true omitted variable that represents the "shoe leather cost" is not the real wage rate and that it is not cointegrated with log income.

We apply our GLS correction methods to estimate long-run income and interest elasticities of M1 demand.[8] To this end, the regression equations are set up with the real money balance ($\frac{M}{P}$) as regressand and income (y) and interest (i) as regressors. Following Stock and Watson (1993), the annual time series for M1 deflated by the net national product price deflator is used for $\frac{M}{P}$, the real net national product for y and the six month commercial paper rate in percentage for i. $\frac{M}{P}$ and y are in logarithms while three different regression equations are considered depending on the measures of interest. We have tried the following three functional forms. Equation 1 has been studied by Stock and Watson (1993).

$$
\begin{aligned}
\ln\left(\frac{M}{P}\right)_t &= \alpha + \beta \ln(y_t) + \gamma i_t + e_t, \quad \text{(equation 1)}\\
\ln\left(\frac{M}{P}\right)_t &= \alpha + \beta \ln(y_t) + \gamma \ln(i_t) + e_t, \quad \text{(equation 2)}\\
\ln\left(\frac{M}{P}\right)_t &= \alpha + \beta \ln(y_t) + \gamma \ln\left[\frac{1+i_t}{i_t}\right] + e_t. \quad \text{(equation 3)}
\end{aligned}
$$

It is worth noting that the liquidity trap is possible for the latter two functional forms. When the data contain periods with very low nominal interest rates, the latter two functional forms may be more appropriate.

Table 3 presents the point estimates for β (income elasticity of money demand) and γ based on the three estimators under scrutiny: dynamic OLS, GLS corrected dynamic regression estimator, and FGLS corrected dynamic regression estimator.[9] Several features emerge from the table. First, all the estimated coefficients have theoretically 'correct' signs: positive signs for income elasticities and negative signs for γ for the first two functional forms and positive signs for γ for the third functional form. Second, GLS corrected estimates of the income elasticity are implausibly low for all three functional forms for low values of k and increase to more plausible values near one as k increases. The fact that the results become more plausible as k increases suggests that the endogeneity correction of dynamic regressions works in this application for moderately large values of k such as 3 and 4. The results for lower values of k are consistent with those of low income elasticity estimates of first differenced regressions that

[8] Readers are referred to Appendix F for the empirical guidelines on the use of estimation and testing techniques developed in this paper.

[9] For the FGLS corrected dynamic regression estimator, the serial correlation coefficient of the error term is estimated before being applied to the Cochrane-Orcutt transformation. This coefficient is assumed to be unity in the GLS corrected dynamic regression estimator which is equivalent to regressing the first difference of variables without a constant term.

had been used in the literature before 1980. Therefore, the estimators in the old literature of first differenced regressions before cointegration became popular are likely to be downward biased because of the endogeneity problem. Third, all point estimates of the three estimators are very similar, and the Hausman-type test fails to reject the null hypothesis of cointegration for large enough values of k. Hence, there is little evidence against cointegration. However, it should be noted that a small random walk component is very hard to detect by any test for cointegration. Therefore, it is assuring to know that all three estimators are similar for large enough values of k, and the estimates are robust with respect to whether the regression error is I(0) or I(1).

We report the value of k chosen by the Bayesian Information Criterion (BIC) rule throughout our empirical applications in order to give some guidance in interpreting results. It is beyond the scope of this paper to study detailed analysis of how k should be chosen because this issue has not been settled in the literature of dynamic cointegrating regressions.

Table 3 also reports the results when the ADF test is applied to the residual of OLS. The results show evidence against the null hypothesis of structural spurious regressions and hence corroborate the results from the Hausman-type test under the opposite null hypothesis.

3.4 Long-run implications of the consumption-leisure choice

Consider a simplified version of Cooley and Ogaki's (1996) model of consumption and leisure in which the representative household maximizes

$$U = E_0[\sum_{t=0}^{\infty} \delta^t e(t)]$$

where E_t denotes the expectation conditioned on the information available at t. We adopt a simple intraperiod utility function that is assumed to be time and state-separable and separable in nondurable consumption, durable consumption, and leisure

$$e(t) = \frac{C(t)^{1-\beta} - 1}{1-\beta} + v(l(t))$$

where $v(\cdot)$ represents a continuously differentiable concave function, $C(t)$ is nondurable consumption, and $l(t)$ is leisure.

The usual first order condition for a household that equates the real wage rate with the marginal rate of substitution between leisure and consumption is given as:

$$W(t) = \frac{v'(l(t))}{C(t)^{-\beta}}$$

where $W(t)$ is the real wage rate. We assume that the stochastic process of leisure is (strictly) stationary in the equilibrium as in Eichenbaum, Hansen, and Singleton (1988). An implication of the first order condition is that $\ln(W(t)) - \beta \ln(C(t)) = \ln(v'(l(t)))$ is stationary. When we assume that the log of consumption is difference stationary, this implies that the log of the real wage rate and the log of consumption are cointegrated with a cointegrating vector $(1, -\beta)'$.

We first consider the model of the measurement error. We assume that $(W(t))$ is measured with a multiplicative measurement error, so that $\ln(W(t))$ is measured with a measurement error $\epsilon(t)$. We assume that this measurement error is unit-root nonstationary. One component of the measurement error arises because it is difficult to measure fringe benefits. Therefore, we expect the measurement error to be very persistent, and the assumption of unit-root nonstationarity may be a good approximation.

We also assume that the measurement error and log consumption are not cointegrated for identificaiton. This assumption does not rule out nonlinear long-run relationships between these variables.

Consider a regression

$$\ln(W^m(t)) = a + \beta \ln(C(t)) + e(t), \tag{26}$$

where $W^m(t)$ is the measured real wage rate, and $e(t) = -\epsilon(t) + \ln(v'(l(t))) - a$. If $\epsilon(t)$ is stationary, then $e(t)$ is stationary, and Regression (26) is a cointegrating regression as in Cooley and Ogaki. In this simple version, the preference parameter β is the Relative Risk Aversion (RRA) coefficient, which is equal to the reciprocal of the intertemporal elasticity of substitution (IES). Cooley and Ogaki show that the same regression can be used to estimate the reciprocal of the long-run IES when preferences for consumption are subjected to time nonseparability such as habit formation. For simplicity, we interpret β as the RRA coefficient in this paper.

If $\epsilon(t)$ is unit-root nonstationary, then Regression (26) is a spurious regression because $e(t)$ is nonstationary in this case. Hence, the standard methods for cointegrating regressions cannot be used. However, the preference parameter β can still be estimated by the spurious regression method. The key assumption for identification is that the measurement error is not cointegrated with the regressor as discussed above. As long as the log multiplicative measurement error and log consumption are not linearly related in the long-run by a linear cointegration relationship, the assumption is satisfied. It should be noted that either consumption or the wage rate can be used as the regressand in the case of cointegration. However, if the wage rate is measured with nonstationary error and consumption is not, then the wage rate should be chosen as the regressand.

Another interpretation of this application is based on the nonstationary omitted variables for this application. A possible omitted variable represents demographic changes. In our analysis, per capita real consumption series was constructed by dividing constant 1982 dollar consumption series by civilian noninstitutional adult (age 16 and over) population. If this population series does not capture the trend of all the demographic change that affects the relationship between the real wage rate and consumption, we have a nonstationary omitted variable. As long as the omitted demographic change and log consumption are not linearly related in the long-run by a linear cointegration relationship, the assumption that the omitted variable is not cointegrated with the regressor is satisfied. Another possible omitted variable is human capital if human capital affects the marginal rate of substitution between leisure and consumption.

Table 4 presents the estimation results for the RRA coefficient (β) based on various estimators. We used the same data set that Cooley and Ogaki used.[10] The results in Table 4 illustrate several points. First, all point estimates for β have the theoretically correct positive sign. Second, for nondurables (ND), GLS-corrected dynamic regression estimates of β are much lower than Dynamic OLS estimates for all values of k. As a result, the Hausman-type cointegration test rejects the null hypothesis of cointegration for all values of k at the 1 percent level. Therefore, the evidence supports the view that Regression (26) is a spurious regression, and the true value of the RRA coefficient is likely to be much lower than the dynamic OLS estimates. Both the GLS corrected and the robust FGLS corrected dynamic regression estimation results are consistent with the view that the RRA coefficient is about one for the value of k chosen by BIC. For nondurables plus services (NDS), GLS corrected dynamic regression estimates of β are much lower than Dynamic OLS estimates for small values of k. As a result, the Hausman-type cointegration test rejects the null hypothesis of cointegration at the 5 percent level when k is 0, 1, and 2. It still rejects the null hypothesis of cointegration at the 5 percent level when k is 3. It does not reject the null hypothesis when k is 4 or 5. According to the BIC rule, k is chosen to be 3, and there is some evidence against cointegration. However, because the GLS corrected

[10]See Cooley and Ogaki (1996, page 127) for a detailed description of the data.

dynamic regression estimates get closer to dynamic OLS estimates as k increases, the evidence is not very strong. It is likely that a small random walk component exists for the error term of the regression for NDS making it a spurious regression. The robust FGLS corrected dynamic regression estimates are close to both GLS corrected dynamic regression estimates and dynamic OLS estimates as long as k is 3 or greater.

Table 4 also reports ADF test results when the test is applied to the residual of static OLS. The lag length for the ADF test has been chosen by the sequential t-test method advocated by Campbell and Perron (1991). We do not have any evidence against the null hypothesis of structural spurious regressions for ND. This hypothesis cannot be rejected at the 5 percent level, but can be rejected at the 10 percent level for NDS.

Thus, we have fairly strong evidence that we have a spurious regression for ND and some evidence that we have spurious regression for NDS. The true value of RRA is likely to be about one for both ND and NDS.

3.5 Output convergence across national economies

In this section, we apply the techniques to reexamine a long standing issue in macroeconomics, the hypothesis of output convergence. For this application and the next, our main purpose is not to estimate unknown structural parameters but to test the null hypothesis of cointegration with the Hausman-type test. For this purpose, we do not need the strict exogeneity assumption under the alternative hypothesis of no cointegration (or a spurious regression).

As a key proposition of the neoclassical growth model, the convergence hypothesis has been popular in macroeconomics and has attracted considerable attention in the empirical field, particularly during the last decade. Besides its important policy implications, the convergence hypothesis has been used as a criterion to discern the two main growth theories, exogenous growth theory and endogenous growth theory. However, it remains the subject of continuing debate mainly because the empirical evidence supporting the hypothesis is mixed. Nevertheless, the established literature based on popular international datasets such as the Summers-Heston (1991) suggests as a stylized fact output convergence among various national economies: convergence among industrialized countries but not among developing countries and not between industrialized and developing countries.

Given that a mean stationary stochastic process of output disparities between two economies is interpreted as supportive evidence of stochastic convergence, unit-root or cointegration testing procedures are often used by empirical researchers to evaluate the convergence hypothesis. In this vein, our techniques proposed here fit in the study of output convergence. We consider four developing countries (Columbia, Ecuador, Egypt, and Pakistan) along with four industrial countries (Germany, Luxemburg, New Zealand, Switzerland). The raw data are extracted from the *Penn World Tables* of Summers-Heston (1991) and consist of annual real GDP per capita (RGDPCH) over the period of 1950-1992. The following two regression equations are considered with regard to the cointegration relation.

$$y_t^D = \alpha + \beta y_t^I + e_t, \tag{27}$$
$$y_t^I = \alpha + \beta y_t^I + e_t, \tag{28}$$

where y_t^D and y_t^I denote log real GDP per capita for developing and industrial countries, respectively.

Tables 5-1 and 5-2 report the results which exhibit a large variation in estimated coefficients. Recall that our interest in this application lies in the cointegration test based on the Hausman-type test. As can be seen from Table 5-1, irrespective of country combinations, the null hypothesis of cointegration can be rejected when developing countries are regressed onto industrial countries, indicating that there is little evidence of output convergence between developing countries and industrial countries. The picture changes dramatically when industrial countries are regressed onto industrial countries as in (28). Table 5-2 displays that the Hausman-type test fails to reject the null of cointegration in all cases considered. Our finding is therefore consistent with the notion of convergence clubs.

3.6 PPP for traded and non-traded goods

As a major building block for many models of exchange rate determination, PPP has been one of the most heavily studied subjects in international macroeconomics. Despite extensive research, the empirical evidence on PPP remains inconclusive, largely due to econometric challenges involved in determining its validity. As is generally agreed, most real exchange rates show very slow convergence which makes estimating long-run relationships difficult with existing statistical tools. The literature suggests a number of potential explanations for the very slow adjustment of relative price: volatility of the nominal exchange-rate, market frictions such as trade barriers and transportation costs, imperfect competition in product markets, and the presence of non-traded goods in the price basket. According to the commodity-arbitrage view of PPP, the law of one price holds only for traded goods and the departures from PPP are primarily attributed to the large weight placed on nontraded goods in the CPI. This view has obtained support from many empirical studies based on disaggregated price indices. They tend to provide ample evidence that prices for non-traded goods are much more dispersed than for their traded counterpart and consequently non-traded goods exhibit far larger deviations from PPP than traded goods. Given that general price indices involve a mix of both traded and non-traded goods, highly persistent deviations of non-traded goods from PPP can lead to the lack of conclusive evidence on the long run PPP relationship. As in the previous application, our main purpose for this application is not to estimate unknown structural parameters but to test the null hypothesis of cointegration with the Hausman-type test.

Let p_t and p_t^* denote the logarithms of the consumer price indices in the base country and foreign country, respectively, and s_t be the logarithm of the price of the foreign country's currency in terms of the base country's currency. Long-run PPP requires that a linear combination of these three variables be stationary. To be more specific, long-run PPP is said to hold if $f_t = s_t + p_t^*$ is cointegrated with p_t such that $e_t \sim I(0)$ in

$$f_t^T = \alpha + \beta p_t^T + e_t,$$
$$f_t^N = \alpha + \beta p_t^N + e_t,$$

where the superscripts T and N denote the price levels of traded goods and non-traded goods, respectively.

Following the method of Stockman and Tesar (1995), Kim (2004) recently analyzed the real exchange rate for total consumption using the general price deflator and the real exchange rate for traded and non-traded goods using implicit deflators for non-service consumption and service consumption, respectively.[11] We use Kim's dataset to apply our techniques to the linear combination of sectorally decomposed variables. Table 6 presents the results using quarterly price and exchange rate data for six countries: Canada, France, Italy, Japan, U.K., and U.S. for the period of 1974 Q1 through 1998 Q4. With the Canadian dollar used as numeraire, Table 6 presents the estimates for β which should be close to unity according to long-run PPP. For traded goods, estimates are above unity in most cases, but the variation across estimates does not seem substantial, resulting in non-rejection of the null of

[11]For details, see the Appendix for the description of the data. We thank JB Kim for sharing the dataset.

cointegration in all cases considered. By sharp contrast, the Hausman-type cointegration test rejects the null hypothesis in every country when the price for non-traded goods is used. It is noteworthy that there exists considerable difference between GLS-corrected estimates for β and their DOLS and FGLS counterparts which are far greater than unity. That is, supportive evidence of PPP is found for traded goods but not for non-traded goods, congruent with the general intuition as well as the findings by other studies in the literature such as Kakkar and Ogaki (1999) and Kim (2004).[12]

4 Concluding remarks and future work

In this paper, we developed two estimators to estimate structural parameters in spurious regressions: GLS corrected dynamic regression and FGLS corrected dynamic regression estimators. A GLS corrected dynamic regression estimator is a first differenced version of a dynamic OLS regression estimator. Asymptotic theory shows that, under some regularity conditions, the endogeneity correction of the dynamic regression works for the first differenced regressions for both cointegrating and spurious regressions. This result is useful because it is not intuitively clear that the endogeneity correction works even in regressions with stationary first differenced variables.

For the purpose of the estimation of structural parameters when the possibility of nonstationary measurement error or nonstationary omitted variables cannot be ruled out, we recommend the FGLS corrected dynamic regression estimators. They are consistent both when the error is I(0) and I(1). They are asymptotically as efficient as dynamic OLS when the error is I(0) and as GLS corrected dynamic regression when the error is I(1). This feature may be especially attractive when the FGLS corrected dynamic estimator is extended to a panel data setting when some regression errors are I(0) and the others are I(1). This extension is studied by Hu (2005).

We also developed the Hausman-type cointegration test by comparing the dynamic OLS regression and GLS corrected dynamic regression estimators. As noted in the Introduction, this task is important not merely because few tests for cointegration have been developed for dynamic OLS but because tests for the null hypothesis of cointegration are useful in many applications. For this test, the spurious regression obtained under the alternative hypothesis does not have to be structural.

We applied our estimation and testing methods to four applications: (i) long-run money demand in the U.S.; (ii) long-run implications of the consumption-leisure choice; (iii) output convergence among industrial and developing countries; (iv) Purchasing Power Parity (PPP) for traded and non-traded goods.

In the first application of estimating the money demand function, the results suggest that the endogeneity correction of the dynamic regression works with a moderately large number of leads and lags for the GLS corrected dynamic regression estimator. The GLS corrected dynamic regression estimates of the income elasticity of money demand are very low with low orders of leads and lags, and then increase to more plausible values as the order of leads and lags increases. Dynamic OLS estimates are close to the GLS corrected dynamic regression estimates for a large enough order of leads and lags, and we find little evidence against cointegration with the Hausman-type cointegration test. The FGLS corrected dynamic regression estimates are very close to the GLS corrected dynamic regression and the dynamic OLS estimates for sufficiently large order of leads and lags.

In the second application of the long-run implications of the consumption-leisure choice, we found strong evidence against cointegration when nondurables are used as the measure of consumption with the

[12]Engel (1999) finds little evidence for long-run PPP for traded goods with his variance decomposition method. However, it should be noted that his method is designed to study variations of real exchange rates over relatively shorter periods compared with cointegration-type methods that are designed to study long-run relationships.

Hausman-type cointegration test. We also found some evidence against cointegration when nondurables plus services are used as the measure of consumption. We estimate the RRA coefficient to be about one with both GLS corrected and FGLS corrected dynamic regression estimators.

Hence, in these first two applications, the FGLS corrected dynamic regression estimator works well in the sense that it yield estimates which are close to those of the estimator that seems to be correctly specified. This is confirmed by our simulation results in Section 2 that the small sample efficiency loss from using the FGLS corrected dynamic regression estimator is negligible for reasonable sample sizes. Therefore, we recommend the robust FGLS corrected dynamic regression estimator when the researcher is unsure about whether or not the regression error is I(0) or I(1). This is important because it is difficult to detect a small random walk component in the error term when the error is actually I(1) and to detect a small deviation from a unit-root when the dominant autoregressive root is very close to one when the error is actually I(0).

In the third application, we applied the Hausman-type cointegration test to log real output of pairs of countries to study output convergence across national economies. Our test results are consistent with the stylized fact of convergence clubs in that we reject the null hypothesis of cointegration between developing and developed countries while failing to reject the null hypothesis of cointegration between two developed countries. Finally, we apply the Hausman-type cointegration test to study long-run PPP. Our test results support the commodity-arbitrage view that long-run PPP holds for traded goods but not for non-traded goods.

In future work, it will be important to study the choice of k, the number of leads and lags in the endogeneity correction. Another aspect that will be useful in empirical work is the study of possible deterministic time trend and seasonal effects in the model.

Appendix

Appendix A: Proof of results in section 2.1

To show the distribution of the OLS estimator in regression (4) with $\rho = 1$, define $\mathbf{X} = [x_1, \ldots, x_n]'$, $\mathbf{V} = [\mathbf{v}_1, \ldots, \mathbf{v}_t]'$, and $\mathbf{e} = [e_1, \ldots, e_n]'$. Also define $M_v = I_n - \mathbf{V}(\mathbf{V}'\mathbf{V})^{-1}\mathbf{V}$ and $M_x = I_n - \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}$. Then the OLS estimator for β and $\boldsymbol{\gamma}$ can be written as

$$\begin{aligned}\begin{bmatrix} \hat{\beta}_n - \beta_0 \\ \hat{\boldsymbol{\gamma}}_n - \boldsymbol{\gamma}_0 \end{bmatrix} &= \begin{bmatrix} \mathbf{X}'\mathbf{X} & \mathbf{X}'\mathbf{V} \\ \mathbf{V}'\mathbf{X} & \mathbf{V}'\mathbf{V} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{X}'\mathbf{e} \\ \mathbf{V}'\mathbf{e} \end{bmatrix} \\ &= \begin{bmatrix} (\mathbf{X}'M_v\mathbf{X})^{-1} & -(\mathbf{X}'M_v\mathbf{X})^{-1}\mathbf{X}'\mathbf{V}(\mathbf{V}'\mathbf{V})^{-1} \\ -(\mathbf{V}'\mathbf{V})^{-1}\mathbf{V}'\mathbf{X}(\mathbf{X}'M_v\mathbf{X})^{-1} & (\mathbf{V}'M_x\mathbf{V})^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{X}'\mathbf{e} \\ \mathbf{V}'\mathbf{e} \end{bmatrix}.\end{aligned}$$

We are mostly interested in the structural parameter $\hat{\beta}$. Write its limit distribution as

$$\begin{aligned}\hat{\beta}_n - \beta_0 &= (\mathbf{X}'M_v\mathbf{X})^{-1}\mathbf{X}'\mathbf{e} - (\mathbf{X}'M_v\mathbf{X})^{-1}\mathbf{X}'\mathbf{V}(\mathbf{V}'\mathbf{V})^{-1}\mathbf{V}'\mathbf{e} \\ &= \left(\frac{\mathbf{X}'M_v\mathbf{X}}{n^2}\right)^{-1}\left(\frac{\mathbf{X}'\mathbf{e}}{n^2}\right) - \frac{1}{n}\left(\frac{\mathbf{X}'M_v\mathbf{X}}{n^2}\right)^{-1}\left(\frac{\mathbf{X}'\mathbf{V}}{n}\right)\left(\frac{\mathbf{V}'\mathbf{V}}{n}\right)^{-1}\left(\frac{\mathbf{V}'\mathbf{e}}{n}\right) \\ &= \left(\frac{\mathbf{X}'M_v\mathbf{X}}{n^2}\right)^{-1}\left(\frac{\mathbf{X}'\mathbf{e}}{n^2}\right) + o_p(1) \\ &\to \left[\int_0^1 V(r)V(r)'dr\right]^{-1}\left[\int_0^1 V(r)U(r)dr\right] \equiv h_1,\end{aligned}$$

which gives equation (5). For $\hat{\boldsymbol{\gamma}}$, we can write

$$\hat{\boldsymbol{\gamma}}_n - \boldsymbol{\gamma}_0 = -\left(\frac{\mathbf{V}'\mathbf{V}}{n}\right)^{-1}\left(\frac{\mathbf{V}'\mathbf{X}}{n}\right)\left(\frac{\mathbf{X}'M_v\mathbf{X}}{n^2}\right)^{-1}\left(\frac{\mathbf{X}'\mathbf{e}}{n^2}\right) + \left(\frac{\mathbf{V}'M_x\mathbf{V}}{n}\right)^{-1}\left(\frac{\mathbf{V}'\mathbf{e}}{n}\right) = O_p(1).$$

Alternatively, we can define that $y_t^* = y_t - \left(\sum_{t=1}^n y_t\mathbf{v}_t'\right)\left(\sum_{t=1}^n \mathbf{v}_t\mathbf{v}_t'\right)^{-1}\mathbf{v}_t$, $x_t^* = x_t - \left(\sum_{t=1}^n x_t\mathbf{v}_t'\right)\left(\sum_{t=1}^n \mathbf{v}_t\mathbf{v}_t'\right)^{-1}\mathbf{v}_t$, and $e_t^* = e_t - \left(\sum_{t=1}^n e_t\mathbf{v}_t'\right)\left(\sum_{t=1}^n \mathbf{v}_t\mathbf{v}_t'\right)^{-1}\mathbf{v}_t$. Note that $n^{-1/2}x_t^* = n^{-1/2}x_t + o_p(1)$ and $n^{-1/2}e_t^* = n^{-1/2}e_t + o_p(1)$. Then regression (4) can be written as

$$y_t^* = \beta' x_t^* + e_t^*. \tag{29}$$

The OLS estimator for β in regression (4) and (29) are identical. Write

$$\begin{aligned}\hat{\beta}_n - \beta_0 &= \left[n^{-2}\sum_{t=1}^n x_t^* x_t^{*'}\right]^{-1}\left[n^{-2}\sum_{t=1}^n x_t^* e_t^*\right] \\ &= \left[n^{-2}\sum_{t=1}^n x_t x_t'\right]^{-1}\left[n^{-2}\sum_{t=1}^n x_t e_t\right] + o_p(1) \\ &\to \left[\int_0^1 V(r)V(r)'dr\right]^{-1}\left[\int_0^1 V(r)U(r)dr\right].\end{aligned}$$

To show the limit distribution of the GLS corrected estimator in regression (6), write

$$\sqrt{n}(\tilde{\theta}_{\text{dgls}} - \theta_0) = \left[n^{-1}\sum_{t=1}^n \Delta z_t \Delta z_t'\right]^{-1}\left[n^{-1/2}\sum_{t=1}^n \Delta z_t u_t\right]. \tag{30}$$

For the denominator,

$$n^{-1}\sum_{t=1}^{n}\Delta z_t\Delta z_t' = \begin{bmatrix} n^{-1}\sum_{t=1}^{n} v_t v_t' & n^{-1}\sum_{t=1}^{n} v_t\Delta\mathbf{v}_t' \\ n^{-1}\sum_{t=1}^{n}\Delta\mathbf{v}_t v_t' & n^{-1}\sum_{t=1}^{n}\Delta\mathbf{v}_t\Delta\mathbf{v}_t' \end{bmatrix} \to \begin{bmatrix} \Sigma_v & \Gamma_{v,\Delta\mathbf{v}}' \\ \Gamma_{v,\Delta\mathbf{v}} & \Gamma_{\Delta\mathbf{v},\Delta\mathbf{v}} \end{bmatrix} \equiv Q, \tag{31}$$

where Σ_v is the variance matrix of $\{v_t\}$ and Γ is a matrix with elements computed from the autocovariances of $\{v_t\}$. In the special case when $m = 1$, $\eta_t = \gamma_{1,0}v_{1t} + e_t$,

$$n^{-1}\sum_{t=1}^{n}\Delta z_t\Delta z_t' = \begin{bmatrix} n^{-1}\sum_{t=1}^{n} v_{1t}^2 & n^{-1}\sum_{t=1}^{n} v_{1t}(v_{1t}-v_{1,t-1}) \\ n^{-1}\sum_{t=1}^{n} v_{1t}(v_{1t}-v_{1,t-1}) & n^{-1}\sum_{t=1}^{n}(v_{1t}-v_{1,t-1})^2 \end{bmatrix} \to \sigma_{1v}^2 \begin{bmatrix} 1 & 1-\psi_{1v} \\ 1-\psi_{1v} & 2(1-\psi_{1v}) \end{bmatrix},$$

where σ_{1v}^2 is the variance of v_{1t} and ψ_{1v} is its first order autocorrelation coefficient. If we further assume that v_{1t} is *i.i.d.*, the limit matrix becomes:

$$Q = \sigma_{1v}^2 \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}.$$

For the numerator, the assumptions on the innovation processes ensure that CLT holds:

$$n^{-1/2}\sum_{t=1}^{n}\Delta z_t u_t = \begin{bmatrix} n^{-1/2}\sum_{t=1}^{n} v_t u_t \\ n^{-1/2}\sum_{t=1}^{n}\Delta\mathbf{v}_t u_t \end{bmatrix} \to N(0,\Lambda), \tag{32}$$

where Λ is the long run covariance matrix of the vector $\Delta z_t u_t$:

$$\Lambda = \begin{bmatrix} \sum_{j=-\infty}^{\infty} E(v_t v_{t-j}' u_t u_{t-j}) & \sum_{j=-\infty}^{\infty} E(v_t\Delta\mathbf{v}_{t-j}' u_t u_{t-j}) \\ \sum_{j=-\infty}^{\infty} E(\Delta\mathbf{v}_t v_{t-j}' u_t u_{t-j}) & \sum_{j=-\infty}^{\infty} E(\Delta\mathbf{v}_t\Delta\mathbf{v}_{t-j}' u_t u_{t-j}) \end{bmatrix}.$$

When $m = 1$, v_{1t}, u_t are both *i.i.d.* and $\eta_t = \gamma_{1,0}v_{1t} + e_t$,

$$\Lambda = \sigma_{1v}^2\sigma_u^2 \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}.$$

Hence, for the quantity defined in (30), we have the limit distribution given in (7):

$$\sqrt{n}(\tilde{\theta}_{\text{dgls}} - \theta_0) \to N(\mathbf{0},\Omega)$$

where $\Omega = Q^{-1}\Lambda Q^{-1}$.

To derive the limit distribution for the FGLS estimator, we first derive the limit distribution for $\hat{\rho}_n$ in regression (8). Write the process of $\hat{e}_t$ as

$$\begin{aligned} \hat{e}_t &= y_t - \hat{\theta}_{\text{dols}}' z_t \\ &= e_t + (\theta_0 - \hat{\theta}_{\text{dols}})' z_t \\ &= e_{t-1} + (\theta_0 - \hat{\theta}_{\text{dols}})' z_t + u_t \\ &= \hat{e}_{t-1} + (\theta_0 - \hat{\theta}_{\text{dols}})'\Delta z_t + u_t \\ &= \hat{e}_{t-1} + g_t, \quad \textit{say}. \end{aligned}$$

¿From this expression, we can see that $\hat{e}_t$ is a unit-root process with serially correlated error g_t. The OLS estimator $\hat{\rho}_n$ can be written as

$$\hat{\rho}_n - 1 = \frac{\sum_{t=1}^{n}\hat{e}_{t-1}g_t}{\sum_{t=1}^{n}\hat{e}_{t-1}^2}. \tag{33}$$

If we let $b = (1, (\beta_0 - \hat{\beta}_n)')'$, $a_t = (e_t, x_t')'$, we can write

$$\begin{aligned} \hat{e}_t &= e_t + (\beta_0 - \hat{\beta}_n)' x_t + (\boldsymbol{\gamma}_0 - \hat{\boldsymbol{\gamma}}_n)' \mathbf{v}_t \\ &= b' a_t + (\boldsymbol{\gamma}_0 - \hat{\boldsymbol{\gamma}}_n)' \mathbf{v}_t. \end{aligned} \tag{34}$$

Then the denominator in (33) can be written as

$$n^{-2} \sum_{t=1}^{n} \hat{e}_t^2 = b' n^{-2} \sum_{t=1}^{n} a_t a_t' b + o_p(1) \to \alpha' \int_0^1 A(r) A(r)' dr \ \alpha,$$

where $\alpha = (1, -h_1')'$ and $A(r) = (U(r), V(r)')'$.

The numerator in (33) can be written as

$$\hat{e}_{t-1} g_t = [b' a_t + (\boldsymbol{\gamma}_0 - \hat{\boldsymbol{\gamma}}_n)' \mathbf{v}_t][(\beta_0 - \hat{\beta}_n)' v_t + (\boldsymbol{\gamma}_0 - \hat{\boldsymbol{\gamma}}_n)' (\mathbf{v}_t - \mathbf{v}_{t-1}) + u_t].$$

The sum of this quantity is $O_p(n)$. We omit the details here since we will not make use of the distribution form of $\hat{\rho}_n$. We let c denote the limit, i.e., $n^{-1} \sum_{t=1}^{n} \hat{e}_{t-1} g_t \to c$.

Then,

$$n(\hat{\rho}_n - 1) = \frac{n^{-1} \sum_{t=1}^{n} \hat{e}_{t-1} g_t}{n^{-2} \sum_{t=1}^{n} \hat{e}_{t-1}^2} \to_d c \left(\alpha' \int_0^1 A(r) A(r)' dr \ \alpha \right)^{-1}. \tag{35}$$

In fact, in our following computations, we only make use of the fact that

$$n(\hat{\rho}_n - 1) = O_p(1).$$

Below we show how to derive the limit distribution for $\tilde{\theta}_{\text{fgls}}$. For the sequence of $\tilde{y}_t$, we can write it as

$$\begin{aligned} \tilde{y}_t &= y_t - \hat{\rho}_n y_{t-1} \\ &= \theta_0' z_t + e_t - \hat{\rho}_n (\theta_0' z_{t-1} + e_{t-1}) \\ &= \theta_0' (z_t - \hat{\rho}_n z_{t-1}) + (e_t - e_{t-1}) + (1 - \hat{\rho}_n) e_{t-1} \\ &= \theta_0' \tilde{z}_t + u_t + (1 - \hat{\rho}_n) e_{t-1}. \end{aligned}$$

Now, we can write

$$\hat{\theta}_{\text{fgls}} - \theta_0 = \left[\sum_{t=1}^{n} \tilde{z}_t \tilde{z}_t' \right]^{-1} \left[\sum_{t=1}^{n} \tilde{z}_t [u_t + (1 - \hat{\rho}_n) e_{t-1}] \right]. \tag{36}$$

Write the denominator as

$$\sum_{t=1}^{n} \tilde{z}_t \tilde{z}_t' = \begin{bmatrix} \sum_{t=1}^{n} \tilde{x}_t \tilde{x}_t' & \sum_{t=1}^{n} \tilde{x}_t \tilde{\mathbf{v}}_t' \\ \sum_{t=1}^{n} \tilde{\mathbf{v}}_t \tilde{x}_t' & \sum_{t=1}^{n} \tilde{\mathbf{v}}_t \tilde{\mathbf{v}}_t' \end{bmatrix}.$$

First,

$$\begin{aligned} \sum_{t=1}^{n} \tilde{x}_t \tilde{x}_t' &= \sum_{t=1}^{n} (x_t - \hat{\rho}_n x_{t-1})(x_t - \hat{\rho}_n x_{t-1})' \\ &= \sum_{t=1}^{n} [(1 - \hat{\rho}_n) x_{t-1} + v_t][(1 - \hat{\rho}_n) x_{t-1} + v_t]' \\ &= (1 - \hat{\rho}_n)^2 \sum_{t=1}^{n} x_{t-1} x_{t-1}' + (1 - \hat{\rho}_n) \sum_{t=1}^{n} [x_{t-1} v_t' + v_t x_{t-1}'] + \sum_{t=1}^{n} v_t v_t'. \end{aligned}$$

Hence,

$$
\begin{aligned}
& n^{-1}\sum_{t=1}^{n}\tilde{x}_t^2 \\
= \;& n(1-\hat{\rho}_n)^2\left(n^{-2}\sum_{t=1}^{n}x_{t-1}x_{t-1}'\right)+(1-\hat{\rho}_n)\left(n^{-1}\sum_{t=1}^{n}x_{t-1}v_t'+n^{-1}\sum_{t=1}^{n}v_t x_{t-1}'\right)+n^{-1}\sum_{t=1}^{n}v_t v_t' \\
= \;& n^{-1}\sum_{t=1}^{n}v_t v_t' + o_p(1) \to \Sigma_v.
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
n^{-1}\sum \tilde{x}_t\tilde{\mathbf{v}}_t' &= n^{-1}\sum_{t=1}^{n}v_t(\mathbf{v}_t-\mathbf{v}_{t-1})'+(1-\hat{\rho}_n)n^{-1}\sum_{t=1}^{n}v_t\mathbf{v}_{t-1}' \\
&\quad +(1-\hat{\rho}_n)n^{-1}\sum_{t=1}^{n}x_{t-1}(\mathbf{v}_t-\mathbf{v}_{t-1})'+(1-\hat{\rho}_n)^2 n^{-1}\sum_{t=1}^{n}x_{t-1}\mathbf{v}_{t-1}' \\
&= n^{-1}\sum_{t=1}^{n}v_t(\mathbf{v}_t-\mathbf{v}_{t-1})'+o_p(1)\to\Gamma_{v,\Delta\mathbf{v}}.
\end{aligned}
$$

Finally, $n^{-1}\sum_{t=1}^{n}\tilde{\mathbf{v}}_t\tilde{\mathbf{v}}_t' = \frac{1}{n}\sum_{t=1}^{n}\Delta\mathbf{v}_t\Delta\mathbf{v}_t' + o_p(1) \to \Gamma_{\Delta\mathbf{v},\Delta\mathbf{v}}$. Hence,

$$
n^{-1}\sum_{t=1}^{n}\tilde{z}_t\tilde{z}_t' = n^{-1}\sum_{t=1}^{n}\Delta z_t\Delta z_t' + o_p(1) \to_p Q. \tag{37}
$$

Next, consider the numerator in (36)

$$
\sum_{t=1}^{n}\tilde{z}_t[u_t+(1-\hat{\rho}_n)e_{t-1}] = \left[\begin{array}{c}\sum_{t=1}^{n}\tilde{x}_t[u_t+(1-\hat{\rho}_n)e_{t-1}] \\ \sum_{t=1}^{n}\tilde{\mathbf{v}}_t[u_t+(1-\hat{\rho}_n)e_{t-1}]\end{array}\right].
$$

It is not hard to see that $n^{-1}\sum_{t=1}^{n}\tilde{z}_t[u_t+(1-\hat{\rho}_n)e_{t-1}] \to_p 0$. Intuitively, $\tilde{z}_t$ behaves asymptotically like the differenced regressors $(v_t', \Delta\mathbf{v}_t')'$, and u and v are uncorrelated by assumption. Our remaining task is to show that

$$
n^{-1/2}\sum_{t=1}^{n}\tilde{z}_t[u_t+(1-\hat{\rho}_n)e_{t-1}] = n^{-1/2}\sum_{t=1}^{n}\Delta z_t u_t + o_p(1) \to N(\mathbf{0},\Lambda). \tag{38}
$$

This can be shown using similar arguments in proving (37). Combining (38) with (37), we obtain the limit distribution for $\tilde{\theta}_{\text{dgls}}$ as given in (11).

Appendix B: Proof of results in section 2.2

To show the limit distribution of the dynamic OLS estimator in the cointegration, define

$$
H_n = \left[\begin{array}{cc} I_m n & 0 \\ 0 & I_{m(2k+1)}n^{1/2}\end{array}\right]. \tag{39}
$$

We can write

$$
H_n(\hat{\theta}_{\text{dols}}-\theta_0) = \left[\begin{array}{c} n(\hat{\beta}_n-\beta_0) \\ n^{1/2}(\hat{\boldsymbol{\gamma}}_n-\boldsymbol{\gamma}_0)\end{array}\right] = \left[\begin{array}{cc} n^{-2}\sum_{t=1}^{n}x_t x_t' & n^{-3/2}\sum_{t=1}^{n}x_t\mathbf{v}_t' \\ n^{-3/2}\sum_{t=1}^{n}\mathbf{v}_t x_t' & n^{-1}\sum_{t=1}^{n}\mathbf{v}_t\mathbf{v}_t'\end{array}\right]^{-1}\left[\begin{array}{c} n^{-1}\sum_{t=1}^{n}x_t e_t \\ n^{-1/2}\sum_{t=1}^{n}\mathbf{v}_t e_t\end{array}\right].
$$

For the denominator,

$$\begin{bmatrix} n^{-2}\sum_{t=1}^{n} x_t x_t' & n^{-3/2}\sum_{t=1}^{n} x_t \mathbf{v}_t' \\ n^{-3/2}\sum_{t=1}^{n} \mathbf{v}_t x_t' & n^{-1}\sum_{t=1}^{n} \mathbf{v}_t \mathbf{v}_t' \end{bmatrix} \to \begin{bmatrix} \int_0^1 V(r)V(r)'dr & 0 \\ 0 & \Gamma_{\mathbf{v},\mathbf{v}} \end{bmatrix}. \quad (40)$$

Thus, the estimator of the I(1) and I(0) components are asymptotically independent. For the numerator,

$$\begin{bmatrix} n^{-1}\sum_{t=1}^{n} x_t e_t \\ n^{-1/2}\sum_{t=1}^{n} \mathbf{v}_t e_t \end{bmatrix} \to_d \begin{bmatrix} \int_0^1 V(r)dE(r) \\ N(0, \Lambda_{\mathbf{v},e}) \end{bmatrix}, \quad (41)$$

where $\Lambda_{\mathbf{v},e}$ is the long run variance of $\mathbf{v}_t e_t$. Equation (13) then follows.

To show the limit distribution for FGLS estimator in regression (16), write

$$\begin{aligned} & n^{-1}\sum_{t=1}^{n} \hat{e}_t^2 \\ = \; & n^{-1}\sum_{t=1}^{n} e_t^2 + 2n^{-1}\left(\sum_{t=1}^{n} e_t z_t' H_n^{-1}\right) H_n(\theta_0 - \hat{\theta}_{\text{dols}}) + n^{-1}(\theta_0 - \hat{\theta}_{\text{dols}})' H_n \left(H_n^{-1}\sum_{t=1}^{n} z_t z_t' H_n^{-1}\right) H_n(\theta_0 - \hat{\theta}_{\text{dols}}) \\ = \; & n^{-1}\sum_{t=1}^{n} e_t^2 + o_p(1) \to \sigma_e^2. \end{aligned}$$

Similarly, we can show that

$$n^{-1}\sum_{t=1}^{n} \hat{e}_t \hat{e}_{t-1} = n^{-1}\sum_{t=1}^{n} e_t e_{t-1} + o_p(1) \to \psi_e \sigma_e^2,$$

where ψ_e is the first order autocorrelation coefficient of $\{e_t\}$. Then the OLS estimator is

$$\hat{\rho}_n = \frac{n^{-1}\sum_{t=1}^{n} \hat{e}_t e_{t-1}}{n^{-1}\sum_{t=1}^{n} \hat{e}_t^2} \to_p \psi_e.$$

Conduct the Cochrane-Orcutt transformation (9) and estimate

$$\tilde{y}_t = \beta' \tilde{x}_t + \boldsymbol{\gamma}' \tilde{\mathbf{v}}_t + error.$$

For the sequence of $\tilde{y}_t$, we can write it as

$$\tilde{y}_t = \beta_0' \tilde{x}_t + \boldsymbol{\gamma}_0' \tilde{\mathbf{v}}_t + \tilde{e}_t,$$

where $\tilde{e}_t = e_t - \hat{\rho}_n e_{t-1}$. Using the same weight matrix H_n, write

$$\begin{bmatrix} n(\tilde{\beta}_n - \beta_0) \\ n^{1/2}(\tilde{\boldsymbol{\gamma}}_n - \boldsymbol{\gamma}_0) \end{bmatrix} = \left[H_n^{-1} \begin{bmatrix} \sum_{t=1}^{n} \tilde{x}_t \tilde{x}_t' & \sum_{t=1}^{n} \tilde{x}_t \tilde{\mathbf{v}}_t{}' \\ \sum_{t=1}^{n} \tilde{\mathbf{v}}_t \tilde{x}_t' & \sum_{t=1}^{n} \tilde{\mathbf{v}}_t \tilde{\mathbf{v}}_t{}' \end{bmatrix} H_n^{-1} \right]^{-1} \begin{bmatrix} n^{-1}\sum_{t=1}^{n} \tilde{x}_t \tilde{e}_t \\ n^{-1/2}\sum_{t=1}^{n} \tilde{\mathbf{v}}_t \tilde{e}_t \end{bmatrix}. \quad (42)$$

Define that $\tilde{E}(r) = (1-\psi_e)E(r)$ and $\tilde{V}(r) = (1-\psi_e)V(r)$. By Lemma 2.1 in Phillips and Ouliaris (1990), $n^{-1/2}\tilde{x}_{[nr]} \to \tilde{V}(r)$ and $n^{-1/2}\sum_{t=1}^{[nr]} \tilde{e}_t \to \tilde{E}(r)$. Therefore we can show that

$$\begin{aligned} n^{-2}\sum_{t=1}^{n} \tilde{x}_t \tilde{x}_t' &\to \int_0^1 \tilde{V}(r)\tilde{V}(r)'dr \\ n^{-3/2}\sum_{t=1}^{n} \tilde{x}_t \tilde{\mathbf{v}}_t' &\to 0 \\ n^{-1}\sum_{t=1}^{n} \tilde{\mathbf{v}}_t \tilde{\mathbf{v}}_t{}' &\to P, \quad say. \end{aligned}$$

Hence, the limit of the denominator in (42) is

$$H_n^{-1}\left[\sum_{t=1}^{n} z_t z_t'\right] H_n^{-1} \to \left[\begin{array}{cc} (1-\psi_e)^2 \int_0^1 V(r)V(r)'dr & 0 \\ 0 & P \end{array}\right].$$

Next, consider the numerator in (42). In fact, we are only interested in the first element,

$$n^{-1}\sum_{t=1}^{n} \tilde{x}_t\tilde{e}_t \to \int_0^1 \tilde{V}(r)d\tilde{E}(r) = (1-\psi_e)^2 \int_0^1 V(r)dE(r).$$

Therefore, we obtain the limit distribution for $\tilde{\beta}_{\text{fgls}}$,

$$\begin{aligned} n(\tilde{\beta}_{\text{fgls}} - \beta_0) &\to \left(\int_0^1 \tilde{V}(r)\tilde{V}(r)'dr\right)^{-1}\left(\int_0^1 \tilde{V}(r)d\tilde{E}(r)\right) \\ &= \left(\int_0^1 V(r)V(r)'dr\right)^{-1}\left(\int_0^1 V(r)dE(r)\right), \end{aligned}$$

which is the same as the limit of $\hat{\beta}_{\text{dols}}$.

Appendix C: Proof of results in section 2.4

In the extended test, where we allow endogeneity under the alternative, the regression can be written as:

$$y_t = \beta' x_t + \boldsymbol{\gamma}' v_t + (\phi s_t + e_t).$$

Define $\mathbf{s} = [\phi s_1 + e_1, \ldots, \phi s_n + e_n]'$. Note that $n^{-1/2}\mathbf{s} = n^{-1/2}\mathbf{e} + o_p(1)$. Similar as in Appendix A, the OLS estimators for β under the alternative of a spurious regression can be written as

$$\begin{aligned} \hat{\beta}_n - \beta_0 &= (\mathbf{X}'M_v\mathbf{X})^{-1}\mathbf{X}'\mathbf{s} - (\mathbf{X}'M_v\mathbf{X})^{-1}\mathbf{X}'\mathbf{V}(\mathbf{V}'\mathbf{V})^{-1}\mathbf{V}'\mathbf{s} \\ &= \left(\frac{\mathbf{X}'M_v\mathbf{X}}{n^2}\right)^{-1}\left(\frac{\mathbf{X}'\mathbf{e}}{n^2}\right) - \frac{1}{n}\left(\frac{\mathbf{X}'M_v\mathbf{X}}{n^2}\right)^{-1}\left(\frac{\mathbf{X}'\mathbf{V}}{n}\right)\left(\frac{\mathbf{V}'\mathbf{V}}{n}\right)^{-1}\left(\frac{\mathbf{V}'\mathbf{e}}{n}\right) + o_p(1) \\ &= \left(\frac{\mathbf{X}'M_v\mathbf{X}}{n^2}\right)^{-1}\left(\frac{\mathbf{X}'\mathbf{e}}{n^2}\right) + o_p(1) \\ &\to \left[\int_0^1 V(r)V(r)'dr\right]^{-1}\left[\int_0^1 V(r)U(r)dr\right]. \end{aligned}$$

Due to endogeneity, the estimator in the differenced regression is not consistent either. The estimators $(\tilde{\beta}_{\text{dgls}}' - \beta_0', \tilde{\boldsymbol{\gamma}}_{\text{dgls}}' - \boldsymbol{\gamma}_0')' \to Q^{-1}\phi(E(v_t\Delta s_t)', E(\Delta\mathbf{v}_t\Delta s_t)')'$. Let $\bar{\beta}$ denote the limit of $\tilde{\beta}_{\text{dgls}}$, then

$$\begin{aligned} &\sqrt{n}(\tilde{\beta}_{\text{dgls}} - \hat{\beta}_{\text{dols}}) \\ =\ &\sqrt{n}(\tilde{\beta}_{\text{dgls}} - \bar{\beta}) - \sqrt{n}(\hat{\beta}_{\text{dols}} - \bar{\beta}) \\ =\ &O_p(1) + O_p(\sqrt{n}) \\ =\ &O_p(\sqrt{n}). \end{aligned}$$

Finally, in the differenced regression, the variance estimate still converges. Therefore, the Hausman-type test statistic is of order n under the alternative of spurious regressions no matter whether exogeneity holds or not.

Appendix D: Regression with a constant

In this section, we consider the model with a constant term, e.g.

$$y_t = \delta + \beta' x_t + \boldsymbol{\gamma}' \mathbf{v}_t + e_t. \tag{43}$$

As in Appendix A, we can also estimate the parameter δ and β in regression

$$y_t^* = \delta + \beta' x_t^* + e_t^*. \tag{44}$$

The OLS estimator in regression (44) can be written as

$$\begin{aligned}
\begin{bmatrix} n^{-1/2}(\hat{\delta}_n - \delta_0) \\ \hat{\beta}_n - \beta_0 \end{bmatrix} &= \begin{bmatrix} 1 & n^{-3/2}\sum_{t=1}^n x_t^{*\prime} \\ n^{-3/2}\sum_{t=1}^n x_t^* & n^{-2}\sum_{t=1}^n x_t^* x_t^{*\prime} \end{bmatrix}^{-1} \begin{bmatrix} n^{-3/2}\sum_{t=1}^n e_t^* \\ n^{-2}\sum_{t=1}^n x_t^* e_t^* \end{bmatrix} \\
&= \begin{bmatrix} 1 & n^{-3/2}\sum_{t=1}^n x_t' \\ n^{-3/2}\sum_{t=1}^n x_t & n^{-2}\sum_{t=1}^n x_t x_t' \end{bmatrix}^{-1} \begin{bmatrix} n^{-3/2}\sum_{t=1}^n e_t \\ n^{-2}\sum_{t=1}^n x_t e_t \end{bmatrix} + o_p(1) \\
&\to \begin{bmatrix} 1 & \int V(r)dr \\ \int V(r)dr & \int V(r)V(r)'dr \end{bmatrix}^{-1} \begin{bmatrix} \int U(r)dr \\ \int V(r)U(r)dr \end{bmatrix}.
\end{aligned}$$

Therefore, the OLS estimator of the intercept diverges in the spurious regression. Similarly, we can show that $\hat{\boldsymbol{\gamma}} - \boldsymbol{\gamma}_0$ in regression (43) is also $O_p(1)$. Note that the estimators $\hat{\delta}$, $\hat{\beta}$ and $\hat{\boldsymbol{\gamma}}$ estimated in this way are numerically identical to that estimated in regression (43).

If we do GLS correction or the differenced regression, the constant is canceled so the limit results are the same as given by (7). Finally, consider the Cochrane-Orcutt FGLS estimation. Let $\hat{e}_t$ denote the OLS residual from (43),

$$\hat{e}_t = y_t - \hat{\delta}_n - \hat{\beta}_n' x_t - \hat{\boldsymbol{\gamma}}_n' \mathbf{v}_t.$$

Then do another OLS estimation of

$$\hat{e}_t = \hat{\rho}_n \hat{e}_{t-1} + \hat{u}_t.$$

Write

$$\begin{aligned}
\hat{e}_t &= y_t - \hat{\delta}_n - \hat{\beta}_n' x_t - \hat{\boldsymbol{\gamma}}_n' \mathbf{v}_t \\
&= e_{t-1} + (\delta_0 - \hat{\delta}_n) + (\beta_0 - \hat{\beta}_n)' x_t + (\boldsymbol{\gamma}_0 - \hat{\boldsymbol{\gamma}})' \mathbf{v}_t + u_t \\
&= \hat{e}_{t-1} + [(\beta_0 - \hat{\beta}_n)' v_t + (\boldsymbol{\gamma}_0 - \hat{\boldsymbol{\gamma}}_n)'(\mathbf{v}_t - \mathbf{v}_{t-1}) + u_t] \\
&= \hat{e}_{t-1} + g_t,
\end{aligned}$$

which takes the same form as in the previous section where no constant is included. Hence we still have

$$\hat{\rho}_n - 1 = \frac{\sum_{t=1}^n \hat{e}_{t-1} g_t}{\sum_{t=1}^n \hat{e}_{t-1}^2}.$$

Write the process of $\hat{e}_t$ as:

$$\hat{e}_t = y_t - \hat{\delta}_n - \hat{\beta}_n' x_t - \hat{\boldsymbol{\gamma}}_n' \mathbf{v}_t = (\beta_0 - \hat{\beta}_n)'(x_t - \bar{x}) + (\boldsymbol{\gamma}_0 - \hat{\boldsymbol{\gamma}}_n)'(\mathbf{v}_t - \bar{\mathbf{v}}) + (e_t - \bar{e}). \tag{45}$$

Comparing equation (45) with (34), the only difference is that all terms in (45) are subtracted by their sample means. This leads to demeaned Brownian motions instead of standard Brownian motions in the limit of the distribution of $\hat{\rho}_n$. Using similar methods as in Appendix A, we can show that

$$n(\hat{\rho}_n - 1) = O_p(1).$$

Next, conduct the Cochrane-Orcutt transformation as in (9), and consider the OLS estimator in the regression

$$\tilde{y}_t = \tilde{\beta}_n' \tilde{x}_t + \tilde{\boldsymbol{\gamma}}_n' \tilde{\mathbf{v}}_t + error = \tilde{\theta}' \tilde{z}_t + error.$$

Write the estimator

$$\tilde{\theta}_{\text{fgls}} = \left[\sum_{t=1}^{n} \tilde{z}_t \tilde{z}_t' \right]^{-1} \left[\sum_{t=1}^{n} \tilde{z}_t \tilde{y}_t \right].$$

For $\tilde{y}_t$, write

$$\begin{aligned} \tilde{y}_t &= y_t - \hat{\rho}_n y_{t-1} \\ &= (1-\hat{\rho}_n)\delta_0 + \beta_0' \tilde{x}_t + \boldsymbol{\gamma}_0' \tilde{\mathbf{v}}_t + e_t + (1-\hat{\rho}_n) e_{t-1} \\ &= \theta_0' \tilde{z}_t + (1-\hat{\rho}_n)\delta_0 + e_t + (1-\hat{\rho}_n) e_{t-1}. \end{aligned}$$

Thus we can write

$$\hat{\theta}_{\text{fgls}} - \theta_0 = \left[\sum_{t=1}^{n} \tilde{z}_t \tilde{z}_t' \right]^{-1} \left[\sum_{t=1}^{n} \tilde{z}_t [(1-\hat{\rho}_n)\delta_0 + e_t + (1-\hat{\rho}_n) e_{t-1}] \right]. \qquad (46)$$

The only difference between (46) and (36) is that (46) has one additional term $(1-\hat{\rho}_n)\delta_0$. However, since δ_0 is a constant and $1-\hat{\rho}_n = O_p(n^{-1})$, this term vanishes in the limit. Therefore, using the Cochrane-Orcutt transformation, the limit distribution of the estimators is the same regardless of whether we have a constant in the data generating process of the data. In either case, we have the same result as given by (7).

Above are discussions under the assumption of spurious regressions. Under the assumption of cointegration, the OLS estimator $\hat{\beta}$ is still n convergent. By similar arguments, we can also show that the FGLS estimators are still asymptotically equivalent to the OLS estimators in the cointegration. Thus Proposition 1 holds when there is a constant in the DGP of y_t. Also, the GLS corrected estimator has the same property as when there is no constant since this constant is removed in differenced regression. Therefore, the Hausman-type cointegration test statistic has the same asymptotics as given in Proposition 2.

Appendix E: Data descriptions

In the first two empirical analysis we use the same data set as in Stock and Watson (1993, page 817) for the U.S. money demand, and the data set of Cooley and Ogaki (1996, page 127) for the long-run intertemporal elasticity of substitution. Readers are referred to the original work for further details on data.

Per capita output series are extracted from the *Penn World Tables* of Robert Summers and Alan Heston (1991). They are annual data on real GDP per capita (RGDPCH) for four developing countries (Columbia, Ecuador, Egypt, and Pakistan) along with four industrial countries (Germany, Luxemburg, New Zealand, and Switzerland) over the period of 1950-1992.

In the PPP application we borrow the dataset from Kim (2004) who constructed the real exchange rate for total consumption using the general price deflator and the real exchange rate for traded and non-traded goods using implicit deflators for non-service consumption and service consumption, respectively. Data are quarterly observations spanning from 1974 Q1 to 1998 Q4. The exchange rates for Canada, France, Italy, Japan, the United Kingdom, and the United States are taken from the International Financial Statistics (IFS) CD-ROM, and bilateral real exchange rates of traded and non-traded goods classified by type and total consumption deflators from the Quarterly National Accounts and Data Stream are studied.

Appendix F: Guidelines on empirical application

Procedures for GLS- and FGLS- corrected estimations

Step 1 Choose a length (p) of lead and lag terms using popular lag selection rules such as AIC, BIC, or their modified versions due to Ng and Perron (2001). Given that the lead and lag length selection issue has not been settled in the dynamic OLS literature, we recommend to report results from different order together with BIC as a rough guideline. To correct for the endogeneity problem Instrumental Variable (IV) approach can be also applied. The IV approach is appealing as it does not involve choosing proper length of leads and lags, but the downside must be that it is not easy to find good instruments in practice.

$$y_t = \sum_{k=1}^{p} \gamma_k \Delta x_{t+k} + \beta x_t + \sum_{k=1}^{p} \phi_k \Delta x_{t-k} + e_t,$$

Step 2 (*GLS-corrected estimation*) Filter all variables in the above equation by taking full difference

$$\begin{aligned} \Delta y_t &= \sum_{k=1}^{p-1} \gamma_k \Delta^2 x_{t+k} + \beta \Delta x_t + \sum_{k=1}^{p-1} \phi_k \Delta^2 x_{t-k} + \Delta e_t, \\ \Delta y_t &= \theta' \Delta z_t + \Delta e_t \end{aligned}$$

Step 2' (*FGLS-corrected estimation*) Retrieve the OLS residuals such that

$$\hat{e}_t = y_t - \sum_{k=1}^{p} \hat{\gamma}_k \Delta x_{t+k} + \hat{\beta} x_t + \sum_{k=1}^{p} \hat{\phi}_k \Delta x_{t-k},$$

and obtain $\hat{\rho}$ from regressing $\hat{e}_t$ onto $\hat{e}_{t-1}$. After 'n' iterations $\hat{\rho}_n$ can be obtained. The variables are transformed such that $\tilde{y}_t = y_t - \hat{\rho}_n y_{t-1}$, $\tilde{x}_t = x_t - \hat{\rho}_n x_{t-1}$, and $\Delta\tilde{x}_{t+k} = \Delta x_{t+k} - \hat{\rho}_n \Delta x_{t+k-1}$.

Step 3 Apply OLS to estimate $\theta = \{\gamma_1, \cdots, \gamma_{p-1}, \beta, \phi_1, \cdots, \phi_{p-1}\}$. The obtained estimates are the (F)GLS corrected estimates of θ.

The Hausman-type cointegration test

Step 1 Obtain the DOLS and GLS-corrected estimates for the parameters. We recommend to report the results from different order of lead and lag terms together with the one chosen by the BIC rule as a guideline. When selecting lead and lag lengths through the BIC rule, it is recommended to choose the lengths for DOLS and DGLS separately. That is, $\hat{\theta}_{dols}$ is obtained using the BIC lag length from DOLS regression equation while $\tilde{\theta}_{dgls}$ using the BIC lag length selected from DGLS regression equation as described above.

Step 2 Compute $\hat{V}_\beta$, a consistent estimate for the long run variance matrix of $\sqrt{n}(\tilde{\theta}_{dgls} - \theta)$ using Heteroskedasticity and autocorrelation consistent (HAC) estimator. In the empirical part of this paper we adopted the long run variance estimator from Andrews and Monahan (1992) with a quadratic spectral (QS) kernel using prewhitening. Readers are also referred to the recent study by Sul, Phillips, and Choi (2005) who propose a recursive demeaning and recursive Cauchy estimation to reduce the small sample bias in prewhitening coefficient estimates as well as a sample-size-dependent boundary condition rule that substantially enhances power without compromising size.

Step 3 Construct the test statistic

$$h_n = n(\tilde{\beta}_{\text{dgls}} - \hat{\beta}_{\text{dols}})'\hat{V}_{\beta}^{-1}(\tilde{\beta}_{\text{dgls}} - \hat{\beta}_{\text{dols}}) \to \chi^2(m)$$

Table 1: The bias and square root of the mean square error of three estimators

ρ	n	DOLS estimator		GLS corrected estimator		FGLS corrected estimator	
		Bias	Square root of MSE	Bias	Square root of MSE	Bias	Square root of MSE
$\rho = 0$	$n = 50$	-0.0000	0.0290	0.0031	0.2156	-0.0000	0.0296
	$n = 100$	-0.0002	0.0169	0.0004	0.1448	-0.0002	0.0170
	$n = 500$	-0.0000	0.0040	0.0010	0.0634	-0.0000	0.0040
$\rho = 0.95$	$n = 50$	-0.0007	0.3555	0.0019	0.2104	0.0000	0.2162
	$n = 100$	0.0070	0.2415	-0.0010	0.1472	0.0036	0.1375
	$n = 500$	0.0006	0.0773	-0.0007	0.0637	-0.0000	0.0475
$\rho = 1$	$n = 50$	-0.0347	1.4769	0.0024	0.2103	-0.0095	0.5968
	$n = 100$	-0.0113	1.2809	0.0028	0.1439	0.0031	0.3635
	$n = 500$	-0.0086	0.9895	0.0003	0.0643	-0.0011	0.1167

Table 2: Finite sample performance of the Hausman-type cointegration test

	Hausman-type Test		Shin's Test	
T	power	size (5%)	power	size (5%)
50	0.621	0.114	0.249	0.141
100	0.688	0.072	0.402	0.171
200	0.754	0.050	0.652	0.199
300	0.783	0.039	0.775	0.184
500	0.816	0.040	0.882	0.181

Note: The Hausman-type cointegration test is stipulated in section 2.4. Nonparametric estimator of long run variance is used based on the QS kernel with the bandwidth of 'integer$[8(T/100)^{1/4}]$'.

Table 3: Application to Long Run U.S. Money Demand

Estimator	k	Equation 1		Equation 2		Equation 3	
		$\hat{\beta}$	$\hat{\gamma}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\beta}$	$\hat{\gamma}$
DOLS	0	0.944 (0.054)	-0.090 (0.015)	0.889 (0.057)	-0.308 (0.058)	0.850 (0.085)	0.906 (0.280)
	1	0.958 (0.048)	-0.096 (0.014)	0.884 (0.046)	-0.313 (0.045)	0.843 (0.066)	0.915 (0.216)
	2	0.970 (0.051)	-0.101 (0.014)	0.879 (0.044)	-0.320 (0.043)	0.837 (0.072)	0.941 (0.233)
	3	0.975 (0.055)	-0.104 (0.015)	0.871 (0.036)	-0.328 (0.035)	0.832 (0.062)	0.975 (0.205)
	4	0.967 (0.054)	-0.108 (0.015)	0.855 (0.029)	-0.334 (0.028)	0.824 (0.065)	0.995 (0.215)
	BIC [lag]	[3]		[5]		[5]	
GLS-corrected	0	0.407 (0.081)	-0.014 (0.004)	0.419 (0.079)	-0.086 (0.022)	0.388 (0.078)	0.300 (0.082)
	1	0.654 (0.119)	-0.025 (0.010)	0.685 (0.115)	-0.177 (0.046)	0.643 (0.115)	0.506 (0.148)
	2	0.837 (0.134)	-0.050 (0.013)	0.848 (0.130)	-0.248 (0.053)	0.787 (0.133)	0.620 (0.161)
	3	0.856 (0.145)	-0.067 (0.017)	0.884 (0.140)	-0.289 (0.061)	0.816 (0.146)	0.725 (0.185)
	4	0.962 (0.161)	-0.086 (0.022)	0.898 (0.151)	-0.283 (0.067)	0.811 (0.153)	0.654 (0.195)
	BIC [lag]	[2]		[2]		[5]	
FGLS-corrected AR(1)	0	0.942 (0.052)	-0.083 (0.023)	0.893 (0.049)	-0.290 (0.079)	0.858 (0.071)	0.850 (0.435)
	1	0.888 (0.040)	-0.065 (0.009)	0.872 (0.035)	-0.278 (0.030)	0.815 (0.045)	0.744 (0.115)
	2	0.940 (0.045)	-0.081 (0.010)	0.901 (0.036)	-0.309 (0.031)	0.840 (0.054)	0.797 (0.128)
	3	0.980 (0.050)	-0.096 (0.011)	0.905 (0.029)	-0.330 (0.026)	0.851 (0.046)	0.912 (0.124)
	4	1.010 (0.045)	-0.108 (0.011)	0.886 (0.025)	-0.333 (0.023)	0.833 (0.051)	0.895 (0.133)
	BIC [lag]	[4]		[5]		[5]	
FGLS-corrected AR(2)	0	0.942 (0.052)	-0.083 (0.023)	0.893 (0.049)	-0.290 (0.079)	0.858 (0.071)	0.850 (0.435)
	1	0.900 (0.039)	-0.069 (0.009)	0.872 (0.038)	-0.276 (0.031)	0.809 (0.049)	0.722 (0.118)
	2	0.948 (0.042)	-0.086 (0.010)	0.894 (0.033)	-0.312 (0.029)	0.839 (0.049)	0.830 (0.131)
	3	0.991 (0.044)	-0.100 (0.011)	0.907 (0.029)	-0.332 (0.026)	0.853 (0.050)	0.903 (0.128)
	4	1.012 (0.042)	-0.109 (0.010)	0.889 (0.026)	-0.335 (0.023)	0.827 (0.061)	0.856 (0.142)
	BIC [lag]	[3]		[5]		[5]	
Hausman-Test	0	4.147		2.643		2.270	
	1	0.691		0.041		0.113	
	2	0.549		0.007		0.012	
	3	0.001		0.092		0.010	
	4	0.485		0.246		0.188	
ADF-based Test	0	-3.768‡		-3.128‡		-2.726†	

Note:

$$\ln\left(\frac{M}{P}\right)_t = \alpha + \beta \ln(y_t) + \gamma i_t + e_t, \quad \text{(equation 1)}$$

$$\ln\left(\frac{M}{P}\right)_t = \alpha + \beta \ln(y_t) + \gamma \ln(i_t) + e_t, \quad \text{(equation 2)}$$

$$\ln\left(\frac{M}{P}\right)_t = \alpha + \beta \ln(y_t) + \gamma \ln\left[\frac{1+i_t}{i_t}\right] + e_t. \quad \text{(equation 3)}$$

'GLS-corrected (FGLS-corrected)' denotes the GLS (FGLS) corrected dynamic regression estimator. Figures in the parenthesis represent standard errors. 'k' denotes the maximum length of leads and lags. In FGLS corrected estimation, the serial correlation coefficient in error term is estimated before being applied to the Cochrane-Orcutt transformation, whereas it is assumed to be unity in GLS corrected estimation which is analogous to regressing the first difference of variables without a constant term. Hausman test represents the Hausman-type cointegration test as stipulated in section 2.4. The test statistic is constructed as $(\hat{\Gamma}_{dgls} - \tilde{\Gamma}_{dols})\Sigma(\hat{\Gamma}_{dgls} - \tilde{\Gamma}_{dols})' \to \chi^2(2)$ where $\Gamma = [\beta, \gamma]$ and $\Sigma = \begin{bmatrix} var(\tilde{\beta}_{dgls}) & cov(\tilde{\beta}_{dgls}, \tilde{\gamma}_{dgls}) \\ cov(\tilde{\beta}_{dgls}, \tilde{\gamma}_{dgls}) & var(\tilde{\gamma}_{dgls}) \end{bmatrix}$. The critical values of $\chi^2(2)$ are 4.61, 5.99 and 9.21 for 10%, 5%, and 1% significance levels. The critical values of the ADF-based tests are -2.88 and -2.57 for 5% and 10% significance levels. ‡(†) represents that the null hypothesis can be rejected at 5% (10%).

Table 4: Application to Preference Parameter (β) Estimation

Estimator	k	DOLS	GLS-corrected	FGLS (AR(1))	FGLS (AR(2))	Hausman test	ADF-based test
ND	0	1.865 (0.218)	0.222 (0.061)	1.874 (0.151)	1.874 (0.151)	140.35‡	-2.302
	1	1.865 (0.192)	0.628 (0.091)	0.983 (0.087)	0.983 (0.087)	102.46‡	
	2	1.870 (0.181)	0.720 (0.102)	1.160 (0.089)	1.160 (0.089)	60.38‡	
	3	1.873 (0.193)	0.850 (0.110)	1.199 (0.097)	1.199 (0.097)	41.53‡	
	4	1.877 (0.204)	0.963 (0.117)	1.207 (0.107)	1.207 (0.107)	37.01‡	
	5	1.880 (0.196)	1.041 (0.123)	1.391 (0.104)	1.391 (0.104)	34.68‡	
	6	1.888 (0.202)	1.129 (0.125)	1.437 (0.109)	1.488 (0.109)	32.63‡	
	7	1.891 (0.200)	1.185 (0.129)	1.533 (0.111)	1.558 (0.114)	35.05‡	
	BIC [lag]	[0]	[3]	[4]	[4]		
NDS	0	1.102 (0.052)	0.480 (0.066)	1.106 (0.095)	1.106 (0.095)	15.93‡	-2.867†
	1	1.103 (0.052)	0.796 (0.080)	0.912 (0.041)	0.932 (0.035)	8.22‡	
	2	1.102 (0.042)	0.855 (0.084)	0.952 (0.034)	0.911 (0.038)	3.88†	
	3	1.100 (0.041)	0.924 (0.085)	0.978 (0.032)	0.975 (0.031)	1.60	
	4	1.099 (0.036)	0.967 (0.087)	0.998 (0.030)	1.000 (0.029)	0.98	
	5	1.095 (0.033)	0.995 (0.088)	1.018 (0.028)	1.009 (0.029)	0.58	
	6	1.091 (0.030)	1.019 (0.089)	1.031 (0.025)	1.025 (0.025)	0.44	
	7	1.088 (0.029)	1.031 (0.091)	1.040 (0.023)	1.042 (0.024)	0.77	
	BIC [lag]	[0]	[3]	[3]	[3]		

Note: Results for $W(t) = \frac{v'(l(t))}{C(t)^{-\beta}}$. 'GLS-corrected (FGLS-corrected)' denotes the GLS (FGLS) corrected dynamic regression estimator. Figures in parenthesis represent standard errors. 'k' denotes the maximum length of leads and lags. In FGLS corrected estimation, the serial correlation coefficient in the error term is estimated before being applied to the Cochrane-Orcutt transformation, whereas it is assumed to be unity in GLS corrected estimation which is analogous to regressing the first difference of variables without constant term. Hausman test represents the Hausman-type cointegration test as stipulated in section 2.4. The test statistic is constructed as $\frac{(\hat{\beta}_{dgls}-\tilde{\beta}_{dols})^2)}{Var(\tilde{\beta}_{dgls})} \rightarrow \chi^2(1)$. The critical values of $\chi^2(1)$ are 2.71, 3.84 and 6.63 for ten, five, and one percent significance levels. The critical values of the ADF-based tests are -2.88 and -2.57 for 5% and 10% significance levels. ‡(†) represents that the null hypothesis can be rejected at 5% (10%) significance level.

Table 5-1: Application to Output Convergence (Regressand: developing countries; Regressor: industrial countries)

Regressand	Regressor	k	DOLS	GLS-corrected	FGLS-corrected	Hausman Test
COL	GER	0	0.680 (0.156)	0.464 (0.091)	0.635 (0.030)	3.668†
		1	0.755 (0.179)	0.501 (0.111)	0.963 (0.109)	9.261‡
		2	0.876 (0.245)	0.490 (0.130)	1.087 (0.121)	11.294‡
		3	0.963 (0.235)	0.409 (0.154)	0.979 (0.058)	13.996‡
		4	1.105 (0.267)	0.607 (0.169)	1.029 (0.067)	11.654‡
		BIC	[0]	[0]	[3]	
	LUX	0	0.923 (0.144)	0.322 (0.097)	0.915 (0.009)	4.715‡
		1	0.953 (0.208)	0.642 (0.151)	0.669 (0.143)	6.544‡
		2	0.993 (0.136)	0.633 (0.167)	0.892 (0.103)	4.494‡
		3	1.035 (0.118)	0.657 (0.193)	1.030 (0.095)	3.094†
		4	1.087 (0.093)	0.739 (0.223)	1.119 (0.075)	3.012†
		BIC	[1]	[1]	[1]	
	NZL	0	1.218 (0.454)	0.363 (0.117)	1.219 (0.053)	13.819‡
		1	1.213 (0.368)	0.650 (0.199)	0.931 (0.270)	9.067‡
		2	1.203 (0.309)	0.600 (0.241)	1.425 (0.221)	3.104†
		3	1.218 (0.330)	0.748 (0.261)	1.608 (0.235)	2.205
		4	1.178 (0.328)	0.788 (0.294)	1.801 (0.191)	0.109
		BIC	[0]	[0]	[1]	
	SWI	0	0.972 (0.349)	0.493 (0.121)	0.969 (0.021)	4.219‡
		1	0.967 (0.343)	0.663 (0.162)	0.875 (0.284)	3.127†
		2	0.948 (0.327)	0.662 (0.189)	1.350 (0.248)	4.359‡
		3	0.901 (0.291)	0.567 (0.212)	1.435 (0.175)	6.130‡
		4	0.926 (0.437)	0.576 (0.210)	1.491 (0.212)	5.398‡
		BIC	[0]	[0]	[1]	
ECU	GER	0	0.784 (0.311)	0.344 (0.159)	0.779 (0.009)	5.410‡
		1	0.816 (0.400)	0.472 (0.191)	0.742 (0.413)	3.464†
		2	0.873 (0.583)	0.472 (0.231)	0.956 (0.503)	4.720‡
		3	0.913 (0.596)	0.357 (0.273)	1.259 (0.289)	5.272‡
		4	1.041 (0.743)	0.501 (0.322)	1.245 (0.252)	3.305†
		BIC	[0]	[0]	[1]	
	LUX	0	1.078 (0.697)	0.129 (0.157)	1.067 (0.034)	8.272‡
		1	1.158 (0.661)	0.393 (0.263)	0.085 (0.293)	11.803‡
		2	1.261 (0.410)	0.397 (0.291)	0.631 (0.246)	4.465‡
		3	1.347 (0.696)	0.734 (0.301)	0.573 (0.407)	3.543†
		4	1.438 (0.518)	0.848 (0.343)	1.076 (0.362)	1.487
		BIC	[0]	[0]	[1]	
	NZL	0	1.496 (0.418)	0.299 (0.182)	1.505 (0.091)	5.338‡
		1	1.499 (0.446)	0.608 (0.331)	0.988 (0.366)	5.694‡
		2	1.487 (0.536)	0.662 (0.387)	1.087 (0.436)	3.645†
		3	1.520 (0.573)	0.795 (0.409)	1.367 (0.465)	3.419†
		4	1.535 (0.631)	0.928 (0.462)	1.934 (0.486)	0.065
		BIC	[0]	[0]	[1]	
	SWI	0	1.155 (0.439)	0.287 (0.202)	1.176 (0.052)	10.777‡
		1	1.139 (0.405)	0.368 (0.294)	0.819 (0.321)	8.719‡
		2	1.086 (0.400)	0.418 (0.326)	1.139 (0.294)	5.337‡
		3	1.022 (0.340)	0.470 (0.352)	1.354 (0.235)	4.840‡
		4	1.061 (0.378)	0.563 (0.352)	1.332 (0.246)	0.492
		BIC	[0]	[0]	[1]	

Note: See the notes in Table 2. Annual data covering 1950-1992 are used for four developing countries (COL: Columbia; ECU: Ecuador; EGT: Egypt; PAK: Pakistan) and four industrial countries (GER: Germany; LUX: Luxemburg; NZL: New Zealand; SWI: Switzerland). The regression equation is $ln(y_{DEV}) = \alpha + \beta ln(y_{IND}) + e$.

Table 5-1: Continued–

Regressand	Regressor	k	DOLS	GLS-corrected	FGLS-corrected	Hausman Test
		0	0.916 (0.108)	0.372 (0.133)	0.851 (0.009)	5.032‡
		1	1.011 (0.145)	0.550 (0.143)	1.224 (0.094)	6.349‡
	GER	2	1.125 (0.256)	0.663 (0.165)	1.278 (0.140)	6.886‡
		3	1.160 (0.342)	0.691 (0.192)	1.263 (0.143)	7.081‡
		4	1.233 (0.449)	0.767 (0.223)	1.227 (0.151)	3.663†
		BIC	[1]	[1]	[1]	
		0	1.228 (0.270)	0.059 (0.136)	1.203 (0.056)	16.382‡
		1	1.313 (0.362)	0.568 (0.198)	0.596 (0.199)	15.242‡
	LUX	2	1.390 (0.141)	0.611 (0.229)	1.239 (0.107)	5.224‡
		3	1.436 (0.144)	0.831 (0.264)	1.280 (0.122)	3.104†
		4	1.476 (0.100)	1.025 (0.287)	1.396 (0.091)	0.951
EGT		BIC	[5]	[1]	[1]	
		0	1.674 (0.317)	0.299 (0.155)	1.658 (0.115)	11.892‡
		1	1.737 (0.301)	0.829 (0.257)	1.666 (0.260)	6.955‡
	NZL	2	1.794 (0.388)	1.090 (0.298)	1.653 (0.331)	2.523
		3	1.837 (0.456)	1.355 (0.324)	1.940 (0.412)	2.061
		4	1.852 (0.594)	1.473 (0.359)	2.719 (0.497)	0.203
		BIC	[3]	[1]	[4]	
		0	1.344 (0.243)	0.388 (0.169)	1.311 (0.075)	13.502‡
		1	1.422 (0.266)	0.666 (0.229)	1.464 (0.246)	5.930‡
	SWI	2	1.498 (0.387)	0.991 (0.235)	1.571 (0.372)	3.723†
		3	1.537 (0.483)	1.080 (0.268)	1.636 (0.462)	5.455‡
		4	1.756 (0.453)	1.176 (0.297)	2.005 (0.254)	4.184‡
		BIC	[2]	[2]	[2]	
		0	0.746 (0.112)	0.328 (0.155)	0.696 (0.007)	4.188‡
		1	0.858 (0.144)	0.454 (0.177)	0.930 (0.096)	6.657†
	GER	2	0.981 (0.193)	0.526 (0.213)	0.994 (0.103)	2.638
		3	1.020 (0.269)	0.727 (0.241)	1.066 (0.131)	1.212
		4	0.999 (0.339)	0.771 (0.283)	0.985 (0.157)	0.127
		BIC	[0]	[0]	[1]	
		0	0.980 (0.233)	0.267 (0.148)	0.972 (0.031)	7.201‡
		1	1.014 (0.257)	0.315 (0.245)	0.649 (0.188)	7.329‡
	LUX	2	1.055 (0.167)	0.330 (0.282)	0.935 (0.127)	4.463‡
		3	1.059 (0.196)	0.617 (0.321)	0.932 (0.164)	0.864
		4	1.038 (0.169)	0.809 (0.355)	0.937 (0.140)	0.001
		BIC	[0]	[0]	[1]	
PAK		0	1.354 (0.318)	0.294 (0.177)	1.347 (0.079)	2.775†
		1	1.377 (0.268)	0.675 (0.314)	1.392 (0.258)	2.642
	NZL	2	1.382 (0.273)	0.598 (0.381)	1.453 (0.251)	2.090
		3	1.447 (0.311)	0.936 (0.409)	1.454 (0.303)	0.161
		4	1.538 (0.582)	1.409 (0.384)	1.701 (0.633)	1.559
		BIC	[4]	[0]	[4]	
		0	1.121 (0.185)	0.557 (0.182)	1.088 (0.048)	3.709†
		1	1.177 (0.176)	0.655 (0.260)	1.320 (0.172)	3.034†
	SWI	2	1.254 (0.169)	0.722 (0.304)	1.413 (0.143)	0.767
		3	1.333 (0.202)	1.036 (0.312)	1.579 (0.170)	1.055
		4	1.542 (0.170)	1.208 (0.331)	1.692 (0.133)	0.982
		BIC	[0]	[0]	[3]	

Note: Refer to the notes in Table 2. Annual data covering 1950-1992 are used for four developing countries (COL: Columbia; ECU: Ecuador; EGT: Egypt; PAK: Pakistan) and four industrial countries (GER: Germany; LUX: Luxemburg; NZL: New Zealand; SWI: Switzerland). The regression equation is $ln(y_{DEV}) = \alpha + \beta ln(y_{IND}) + e$.

Table 5-2: Application to Output Convergence (Regressand: industrial countries; Regressor: industrial countries)

Regressand	Regressor	k	DOLS	GLS-corrected	FGLS-corrected	Hausman Test
		0	1.291 (0.246)	0.623 (0.112)	1.330 (0.044)	2.970†
		1	1.302 (0.207)	1.018 (0.162)	0.719 (0.079)	1.219
	LUX	2	1.312 (0.174)	1.094 (0.174)	0.849 (0.077)	2.314
		3	1.294 (0.222)	1.040 (0.184)	0.706 (0.101)	2.046
		4	1.293 (0.262)	1.074 (0.178)	0.694 (0.119)	0.224
		BIC	[4]	[4]	[1]	
		0	1.829 (0.125)	0.494 (0.159)	1.889 (0.078)	8.602‡
		1	1.802 (0.165)	1.089 (0.240)	1.465 (0.165)	4.908‡
GER	NZL	2	1.775 (0.105)	1.268 (0.285)	1.658 (0.096)	3.161†
		3	1.760 (0.060)	1.480 (0.301)	1.643 (0.056)	0.029
		4	1.692 (0.076)	1.671 (0.174)	1.746 (0.079)	0.318
		BIC	[5]	[5]	[5]	
		0	1.494 (0.087)	0.811 (0.147)	1.531 (0.064)	2.006
		1	1.482 (0.127)	1.260 (0.150)	1.305 (0.121)	0.631
	SWI	2	1.452 (0.127)	1.326 (0.169)	1.379 (0.107)	0.223
		3	1.413 (0.118)	1.339 (0.171)	1.331 (0.090)	0.023
		4	1.369 (0.184)	1.347 (0.148)	1.421 (0.134)	0.329
		BIC	[4]	[4]	[4]	
		0	0.726 (0.239)	0.702 (0.126)	0.678 (0.045)	1.914
		1	0.800 (0.168)	0.581 (0.136)	0.909 (0.092)	4.105‡
	GER	2	0.925 (0.191)	0.536 (0.163)	0.938 (0.088)	2.793†
		3	1.030 (0.265)	0.723 (0.185)	1.039 (0.111)	1.971
		4	1.104 (0.328)	0.838 (0.209)	1.000 (0.133)	3.667†
		BIC	[0]	[1]	[1]	
		0	1.289 (0.633)	0.412 (0.176)	1.290 (0.040)	3.015†
		1	1.256 (0.852)	0.796 (0.294)	0.445 (0.447)	1.272
LUX	NZL	2	1.227 (0.555)	0.797 (0.354)	1.231 (0.503)	0.201
		3	1.255 (0.886)	1.098 (0.362)	0.464 (0.814)	0.067
		4	1.188 (0.676)	1.277 (0.383)	2.311 (0.608)	0.094
		BIC	[3]	[1]	[1]	
		0	1.040 (0.499)	0.603 (0.184)	1.035 (0.007)	0.219
		1	1.016 (0.642)	0.898 (0.240)	0.932 (0.480)	0.617
	SWI	2	1.006 (0.461)	0.771 (0.273)	1.443 (0.469)	0.126
		3	0.984 (0.616)	0.875 (0.309)	1.578 (0.626)	0.009
		4	0.932 (0.900)	0.967 (0.336)	1.766 (0.845)	0.157
		BIC	[1]	[1]	[1]	

Note: Refer to the notes in Table 2. Annual data covering 1950-1992 are used for four developing countries (COL: Columbia; ECU: Ecuador; EGT: Egypt; PAK: Pakistan) and four industrial countries (GER: Germany; LUX: Luxemburg; NZL: New Zealand; SWI: Switzerland). The regression equation is $ln(y_{IND}) = \alpha + \beta ln(y_{IND}) + e$.

Table 5-2: Continued–

Regressand	Regressor	k	DOLS	GLS-corrected	FGLS-corrected	Hausman Test
		0	0.509 (0.044)	0.395 (0.127)	0.501 (0.046)	0.130
		1	0.498 (0.060)	0.449 (0.149)	0.507 (0.047)	0.053
	GER	2	0.498 (0.075)	0.459 (0.175)	0.488 (0.055)	2.174
		3	0.520 (0.096)	0.285 (0.177)	0.511 (0.060)	2.115
		4	0.517 (0.142)	0.315 (0.170)	0.452 (0.081)	0.106
		BIC	[5]	[5]	[5]	
		0	0.676 (0.232)	0.292 (0.125)	0.671 (0.044)	0.605
		1	0.703 (0.233)	0.552 (0.202)	0.314 (0.161)	0.948
NZL	LUX	2	0.731 (0.133)	0.576 (0.224)	0.485 (0.105)	2.393
		3	0.734 (0.209)	0.483 (0.206)	0.496 (0.164)	0.389
		4	0.745 (0.184)	0.628 (0.228)	0.597 (0.144)	0.014
		BIC	[5]	[5]	[5]	
		0	0.797 (0.043)	0.613 (0.145)	0.783 (0.013)	0.155
		1	0.801 (0.050)	0.742 (0.201)	0.791 (0.050)	0.060
	SWI	2	0.778 (0.047)	0.817 (0.224)	0.769 (0.046)	0.203
		3	0.763 (0.057)	0.681 (0.227)	0.789 (0.055)	0.039
		4	0.844 (0.080)	0.812 (0.238)	0.832 (0.080)	0.068
		BIC	[0]	[0]	[3]	
		0	0.634 (0.045)	0.531 (0.097)	0.639 (0.045)	0.077
		1	0.649 (0.079)	0.623 (0.100)	0.653 (0.063)	0.042
	GER	2	0.671 (0.119)	0.650 (0.114)	0.665 (0.078)	0.406
		3	0.676 (0.179)	0.572 (0.130)	0.672 (0.091)	0.078
		4	0.709 (0.278)	0.672 (0.149)	0.696 (0.122)	4.580
		BIC	[1]	[1]	[1]	
		0	0.831 (0.340)	0.350 (0.107)	0.848 (0.022)	0.934
		1	0.850 (0.244)	0.724 (0.162)	0.421 (0.142)	0.626
SWI	LUX	2	0.857 (0.214)	0.746 (0.183)	0.469 (0.137)	0.606
		3	0.845 (0.250)	0.702 (0.214)	0.423 (0.169)	1.101
		4	0.824 (0.268)	0.611 (0.238)	0.442 (0.182)	0.133
		BIC	[0]	[1]	[1]	
		0	1.206 (0.073)	0.503 (0.119)	1.233 (0.014)	1.700
		1	1.219 (0.098)	0.991 (0.187)	1.148 (0.106)	1.219
	NZL	2	1.224 (0.095)	1.005 (0.232)	1.171 (0.095)	0.634
		3	1.233 (0.088)	1.056 (0.260)	1.180 (0.091)	0.036
		4	1.234 (0.116)	1.194 (0.272)	1.263 (0.124)	0.051
		BIC	[1]	[1]	[1]	

Note: Refer to the notes in Table 2. Annual data covering 1950-1992 are used for four developing countries (COL: Columbia; ECU: Ecuador; EGT: Egypt; PAK: Pakistan) and four industrial countries (GER: Germany; LUX: Luxemburg; NZL: New Zealand; SWI: Switzerland). The regression equation is $ln(y_{IND}) = \alpha + \beta ln(y_{IND}) + e$.

Table 6: Application to PPP for traded and non-traded goods

Estimator	k	Traded Goods					Non-traded Goods				
		FRA	ITA	JPN	U.K.	U.S.	FRA	ITA	JPN	U.K.	U.S.
DOLS	0	1.149	1.379	1.558	1.306	1.053	1.872	2.142	2.357	2.059	1.711
		(0.312)	(0.165)	(0.326)	(0.201)	(0.198)	(0.165)	(0.241)	(0.299)	(0.278)	(0.443)
	1	1.179	1.456	1.485	1.439	1.078	1.887	2.175	2.376	2.066	1.728
		(0.424)	(0.217)	(0.459)	(0.262)	(0.276)	(0.151)	(0.222)	(0.299)	(0.290)	(0.441)
	2	1.195	1.511	1.442	1.533	1.092	1.898	2.216	2.399	2.067	1.748
		(0.515)	(0.277)	(0.646)	(0.342)	(0.328)	(0.161)	(0.237)	(0.331)	(0.272)	(0.473)
	3	1.186	1.531	1.390	1.571	1.102	1.888	2.250	2.397	2.054	1.762
		(0.524)	(0.308)	(0.561)	(0.392)	(0.381)	(0.165)	(0.247)	(0.346)	(0.261)	(0.511)
	4	1.195	1.553	1.388	1.613	1.109	1.871	2.287	2.402	2.042	1.763
		(0.502)	(0.353)	(0.471)	(0.412)	(0.401)	(0.159)	(0.252)	(0.343)	(0.233)	(0.484)
	BIC	[0]	[0]	[0]	[2]	[0]	[0]	[0]	[5]	[1]	[0]
GLS-corrected	0	0.833	1.114	1.086	1.030	0.919	0.375	0.448	0.372	0.351	0.159
		(0.393)	(0.381)	(0.411)	(0.365)	(0.140)	(0.178)	(0.176)	(0.198)	(0.171)	(0.080)
	1	0.984	1.086	1.221	1.324	1.027	0.864	0.900	0.888	0.988	0.397
		(0.454)	(0.436)	(0.477)	(0.419)	(0.161)	(0.315)	(0.309)	(0.352)	(0.295)	(0.142)
	2	1.259	1.333	1.415	1.516	1.103	1.127	1.104	1.046	1.001	0.569
		(0.469)	(0.454)	(0.477)	(0.430)	(0.167)	(0.369)	(0.368)	(0.408)	(0.355)	(0.166)
	3	1.374	1.391	1.246	1.560	1.158	1.255	1.171	1.192	1.088	0.751
		(0.501)	(0.485)	(0.504)	(0.460)	(0.176)	(0.422)	(0.422)	(0.456)	(0.407)	(0.185)
	4	1.549	1.661	1.248	1.600	1.184	1.577	1.411	1.379	1.166	0.785
		(0.520)	(0.511)	(0.535)	(0.492)	(0.188)	(0.456)	(0.468)	(0.503)	(0.457)	(0.208)
	BIC	[0]	[0]	[0]	[0]	[0]	[0]	[0]	[0]	[1]	[0]
FGLS-corrected	0	1.156	1.358	1.605	1.269	1.049	1.868	2.129	2.354	2.063	1.712
		(0.259)	(1.163)	(0.783)	(0.125)	(0.054)	(0.232)	(1.223)	(0.760)	(0.156)	(0.073)
	1	1.229	1.456	1.607	1.248	0.766	1.909	2.005	1.947	1.671	0.291
		(0.329)	(0.149)	(0.339)	(0.162)	(0.242)	(0.141)	(0.201)	(0.257)	(0.226)	(0.159)
	2	1.178	1.487	1.717	1.323	0.802	1.932	2.102	2.095	1.864	0.357
		(0.355)	(0.181)	(0.396)	(0.217)	(0.266)	(0.152)	(0.218)	(0.286)	(0.217)	(0.191)
	3	1.152	1.463	1.625	1.376	0.822	1.920	2.158	2.173	1.983	0.392
		(0.339)	(0.204)	(0.318)	(0.250)	(0.301)	(0.157)	(0.233)	(0.311)	(0.214)	(0.216)
	4	1.252	1.603	1.506	1.423	0.826	1.922	2.285	2.202	2.052	0.631
		(0.333)	(0.245)	(0.277)	(0.272)	(0.284)	(0.153)	(0.240)	(0.310)	(0.189)	(0.234)
	BIC	[1]	[1]	[2]	[1]	[1]	[2]	[1]	[5]	[1]	[3]
Hausman Test	0	0.182	0.883	0.256	0.080	0.132	12.667‡	20.200‡	16.524‡	13.972‡	18.459‡
	1	0.022	0.209	0.002	0.002	0.006	5.097‡	10.603‡	9.597‡	8.118‡	17.424‡
	2	0.171	0.109	0.051	0.001	0.134	2.317	6.604‡	6.494‡	4.912‡	16.549‡
	3	0.553	0.058	0.039	0.001	0.222	0.424	3.363†	3.738†	3.019†	14.485‡
	4	0.173	0.008	0.016	0.011	0.007	0.096	1.934	0.874	1.813	12.988‡

Note: Results are for $f_t^T = \alpha + \beta p_t^T + e_t$ and $f_t^N = \alpha + \beta p_t^N + e_t$ using Canada as a base country. Figures in parenthesis represent standard errors. 'k' denotes the maximum length of leads and lags. Hausman test represents the Hausman-type cointegration test as stipulated in section 2.4. The test statistic is constructed as $\frac{(\hat{\beta}_{dgls} - \tilde{\beta}_{dols})^2)}{Var(\tilde{\beta}_{dgls})} \rightarrow \chi^2(1)$. The critical values of $\chi^2(1)$ are 2.71, 3.84 and 6.63 for ten, five, and one percent significance level. ‡(†) represents that the null hypothesis of $\hat{\beta}_{dgls} = \tilde{\beta}_{dols}$ can be rejected at 5% (10%) significance level.

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