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Can a Representative-Agent Model Represent a Heterogeneous-Agent Economy? *

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Abstract

Accounting for observed fluctuations in aggregate employment, consumption, and real wage using the optimality conditions of a representative household often requires preferences that are incompatible with economic priors (e.g., Mankiw, Rotemberg, and Summers 1985). This discrepancy between the equilibrium model and the aggregate data is often viewed as evidence of the failure of labor-market clearing. We argue that such a conclusion is premature. We construct a model economy where all prices are flexible and all markets clear at all times but household decisions are not readily aggregated because of incomplete capital markets and the indivisible nature of the labor supply. We demonstrate that if we were to explain the model-generated aggregate time series using decisions of a "fictitious" stand-in household, such a household is likely to have a non-concave or unstable utility. Our analysis suggests that the representative-agent model often fails to represent an equilibrium outcome of a heterogeneous-agent economy.

Keywords: Representative-agent model, Aggregation, Heterogeneity, Incomplete Markets, Indivisible Labor, GMM Estimation
JEL Classifications: E24, E32, J21, J22

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1 Introduction

Modern business cycle theories posit that observed aggregate fluctuations in the U.S. economy correspond to optimal decisions of a stand-in household (e.g., Kydland and Prescott, 1982; King, Plosser, and Rebelo, 1988). In these models, the cyclical variation of aggregate consumption and employment is a result of the continuous optimum of a household that trades current and future goods and leisure in response to stochastic movements in prices. However, studies that use aggregate time-series data to test the hypothesis of intertemporal substitution often reach negative conclusions. For example, Mankiw, Rotemberg, and Summers (1985) (denoted MRS hereafter) found that the over-identifying restrictions implied by the theory are almost always rejected, the estimated parameters of preferences are highly unstable, and the utility function is often non-concave, leading to elasticities of wrong signs. This incompatibility between the representative-agent model and the aggregate data is often viewed as a failure of labor-market clearing, (e.g., Galí, Gertler, and Lopéz-Salido, 2007).

In this paper we argue that such a conclusion is premature. We demonstrate that an attempt to account for the aggregate behavior of a heterogeneous-agent economy by a "fictitious" representative household often fails. We construct a model economy where all prices are flexible and all markets clear at all times. In our model, individual households possess identical preferences but face a limit on the amount they can borrow and cannot perfectly insure against idiosyncratic productivity shocks (Aiyagari, 1994). Moreover, households supply their labor in an indivisible manner (Rogerson, 1988). Under this environment, the optimality condition for the choice of hours worked and consumption holds with inequality due to a discrete choice of labor supply. Those inequalities are carried over to the aggregate level, preventing a nice aggregation of individual optimality conditions.

The lack of systematic movement among consumption, hours worked, and productivity in the aggregate data has also resulted in the measurement of a considerable stochastic wedge between the representative-agent model and the aggregate data; see, e.g., Hall

(1997) and Chari, Kehoe, and McGrattan (2007). Time-varying factors in the marginal rate of substitution between commodity consumption and leisure (e.g., stochastic shifts in preferences, home production technology, or changes in labor-income tax rates) are proposed to account for this wedge. Our analysis also suggests that such a wedge may reflect imperfect aggregation rather than fundamental changes in preferences.[1]

The equilibrium path of our model economy under the exogenous aggregate productivity shocks reproduces the volatility and correlation structure of key aggregate variables (consumption, hours, and wages) from the U.S. economy. We then ask whether outcomes of our heterogeneous-agent model economy are readily characterized as realizations of an optimizing representative agent. We estimate three optimality conditions that a representative agent would face when choosing hours worked and commodity consumption. If we were to explain the model-generated aggregate time series using decisions of a stand-in household, such a household must have a highly unstable or non-concave utility: the estimated representative household often works longer hours and consumes more commodities when the real wage is low. Similar to the finding by MRS from the actual U.S. aggregate data, the generalized method of moments (GMM) estimates of preference parameters of a representative household are highly unstable or often have wrong signs.

To investigate the marginal contributions of each friction (capital market incompleteness and indivisibility of labor), we consider additional model economies that feature each friction only: the incomplete capital markets with divisible labor economy (referred to as "incomplete-markets" model) and the complete capital markets with indivisible labor economy (referred to as "indivisible-labor" model). According to the GMM estimation of model-generated aggregate time series, we find these economies can be well represented by optimal choices of a representative agent. In the "incomplete-markets" model (with divisible labor), the GMM estimates based on model-generated aggregate time series fairly accurately reveal the individual households' preference parameters. We

[1]Our result is consistent with that of Sheinkman and Weiss (1986), who showed that capital-market incompleteness can lead to a stochastic term in aggregate preferences.

show that the aggregation error in the optimality condition for the choice of consumption and hours worked reflects the ratio of the (CES) aggregate of the marginal utility of individual consumption to the marginal utility of aggregate consumption. While this ratio is, in principle, time-varying, its variation is quantitatively small because all households' consumptions tend to move together in response to aggregate productivity shocks (everyone is working). This type of approximate aggregation no longer works when the labor supply is indivisible. The individual optimality condition holds with inequality due to a discrete choice of labor supply and such inequality persists at the aggregate level. However, when capital markets are complete, despite an indivisible labor supply, the equilibrium of a heterogeneous-agent economy can be well described by an efficient allocation based on comparative advantage. In other words, the GMM estimates reveal the social planner's objective function, the equally weighted average of household utility functions.

Confronted with the inability of an equilibrium model to account for the joint behavior of aggregate consumption, hours worked, and wages, MRS proposed three hypotheses: (1) aggregation error, (2) economy-wide time-varying preferences, and (3) failure of market clearing.[2] While it is highly plausible that all of these have contributed to the discrepancy between the representative-agent model and the aggregate data, our analysis suggests that the incompatibility between the representative household's optimization and the aggregate data may reflect a poor aggregation — which results in a stochastic wedge in the aggregate condition — rather than a failure of market clearing or exogenous shifts in preferences. Nevertheless, our analysis also shows that when the model economy consists of heterogeneous agents and the individual optimality conditions are hard to aggregate, an attempt to account for the aggregate time series by an optimizing

[2]It is also well-known that low-wage and less-skilled workers enter the labor market during expansions and exit during recessions, making aggregate hours more volatile than the effective unit of hours (Hansen, 1993), and making aggregate wages less volatile than individual wages (Bils, 1985; Solon, Barsky, and Parker, 1994). However, this so-called compositional bias has an impact mostly on the volatilities, not on the correlations. Both in the model and in the data, the poor GMM estimates of preference parameters stem mostly from the lack of correlation between employment and productivity (wage), which is 0.03 (0.2).

behavior of the representative household fails. The relative risk aversion of consumption is significantly underestimated when the aggregate consumption Euler equation is used. The parameter that governs the behavior of the labor supply is estimated with great uncertainty, just like those from the actual aggregate data.

The paper is organized as follows. Section 2 briefly discusses the GMM estimate of three optimality conditions based on the aggregate U.S. time series. In Section 3 we compute the equilibrium fluctuations of the heterogeneous-agent economy with incomplete capital markets and indivisible labor using the bounded rationality method developed by Krusell and Smith (1998). In Section 4, based on the aggregate time series generated from the heterogeneous-agent model economy, we estimate three optimality conditions that a "fictitious" representative agent would satisfy. We also provide an example that illustrates the difficulty in aggregating individual optimality conditions when both frictions are present. Section 5 presents a summary.

2 GMM estimates based on aggregate data from the U.S. economy

Consider a representative household whose preferences are given by:

$$\max \ E_t \sum_{s=0}^{\infty} \beta^s \left\{ \frac{C_{t+s}^{1-\sigma} - 1}{1-\sigma} - \psi \frac{H_{t+s}^{1+\gamma}}{1+\gamma} \right\}$$

where C_t is consumption and H_t is hours worked in period t.[3] The preference parameters are β, the discount factor, σ, the inverse of the intertemporal substitution elasticity of consumption, γ, the inverse of the intertemporal substitution elasticity of hours, and a constant ψ. When the representative household follows the optimal path, three first-order conditions must hold:

$$\psi \frac{H_t^{\gamma}}{C_t^{-\sigma}} \frac{P_t}{W_t} - 1 = 0. \tag{S}$$

[3] We assume a utility function separable between consumption and hours worked, which is popular in both business cycle analysis and the empirical labor supply literature. However, non-separability does not change the main result of the article.

$$E_t\left[\beta\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}\frac{P_t(1+R_t)}{P_{t+1}}-1\right]=0. \tag{EC}$$

$$E_t\left[\beta\frac{H_{t+1}^{\gamma}}{H_t^{\gamma}}\frac{W_t(1+R_t)}{W_{t+1}}-1\right]=0. \tag{EL}$$

Here P_t is the nominal price of a unit of C_t, W_t is the wage rate, and R_t is the nominal return from holding a security between t and $t+1$. The static first-order condition (S) holds regardless of the household's decisions in the capital market. The Euler equation for consumption (EC) will hold even if labor supply cannot be freely chosen, and trading is not possible in many assets, as long as some asset exists that is either held in positive amounts or for which borrowing is possible. The Euler equation for leisure (EL) asserts that along an optimal path the representative household cannot improve its welfare by working one hour more at t and using its earnings W_t to purchase a security whose proceeds will be used to buy back $W_t(1+R_t)/W_{t+1}$ of leisure at $t+1$ in all states of nature.

If the static first-order condition (S) held exactly, one of (EC) and (EL) would be redundant. However, since (S) is unlikely to hold exactly in the data, we use the information in all three of these first-order conditions to estimate the parameters of the utility function. Following MRS, σ, γ, β and ψ are estimated by the GMM using the quarterly U.S. aggregate time series for the period 1964:I-2003:IV. Aggregate real per capita consumption is the sum of consumption expenditures on non-durable goods and services. The aggregate price is the price deflator that corresponds to our measure of consumption. Aggregate hours worked represent the total hours employed in the non-agricultural business sector. The nominal wage is the nominal hourly earnings of production and non-supervisory workers in the non-agricultural sector. The nominal interest rate is the 3-month Treasury bill rate. All quantities are divided by the working-age (ages between 16 and 65) population.

We use two sets of instruments in the GMM estimation. Instrument I consists of the following variables for periods $t-1$ and $t-2$: growth rates of consumption, real interest rates, hours worked, and real wages. Instrument II consists of the same variables as

Instrument I but for periods t and $t-1$. Hence, we can check through Instrument II if the estimates are severely affected when current variables are used as instrument variables. While it is common to include period t variables as instruments in the asset pricing literature (see Hansen, 2007; Cochrane, 2001, Chapter 10, for a detailed explanation of the GMM procedure), the existence of predetermined prices (such as sticky wages) may warrant excluding the period t-variable from the list of instruments (see MRS for this argument). We report the estimates using both instruments, and they are not very different from each other. The standard two-stage approach as in Hansen and Singleton (1982) is used in performing the GMM estimation. At the first stage, the identity weighting matrix is applied to get preliminary estimates of the coefficients. The inverse of Newey and West's (1987) heteroskedasticity and autocorrelation consistent (HAC) covarince matrix is used as the second-stage weighting matrix to derive asymptotically efficient estimates.[4]

Estimates in Table 1 basically replicate those in MRS. They also share the common shortcomings of preference parameter estimates in aggregate time series such as in the studies by Dunn and Singleton (1986), Hansen and Singleton (1982, 1984), and Ghysels and Hall (1990). According to these estimates, preferences are often found to be unreasonable. In the static first-order condition (S), the households are not risk averse enough. The estimate of σ is 0.210 (with standard error of 0.062) and 0.188 (0.067) with Instruments I and II, respectively. The marginal disutility from working is not increasing in hours worked, since the estimate of γ is negative: it is -0.569 (0.198) and -0.473 (0.210), with Instruments I and II, respectively. According to these estimates, households would often work longer hours when the real wage is low (i.e., consume less leisure despite the low real price of leisure).

In the Euler equation for consumption (EC), the intertemporal substitution elasticity of consumption turns to a negative value (-0.210 and -0.129, respectively, with Instruments I and II), although it is not statistically significant. In the Euler equation

[4] The HAC covariance matrix is calcuated with a Bartlett kernel and Newey and West's (1987) fixed bandwidth selection criterion.

for leisure (EL), the estimate for γ is 0.179 and 0.089, respectively, with Instruments I and II, implying a fairly elastic labor supply. One of the stylized facts in aggregate labor-market fluctuations is that hours worked vary greatly without a corresponding movement of wages. To account for these, the representative household must have had a very elastic labor-supply schedule. According to these point estimates, the implied value for the intertemporal substitution elasticity of hours worked ($1/\gamma$) is 5.6 and 11.2. These are clearly beyond the admissible values based on empirical micro studies such as those of MaCurdy (1981) and Altonji (1986). When all three optimality conditions are estimated together as a system of equations in the last column of the table, σ is -0.046 (with standard error of 0.027) and γ is 0.023 (0.044), according to which the representative household exhibits a non-concave utility in consumption and is willing to shift its work schedule even for a tiny movement in anticipated wage changes. Each optimality condition is rejected according to Hansen's (1982) J-test of over-identifying restrictions at the significance level of 5%. When the three optimality conditions are tested together, the intertemporal substitution hypothesis is not rejected at the significance level of 10%. When expenditures on non-durable goods (excluding services) are used for aggregate consumption, the estimation result moves slightly toward our economic priors. The estimate of σ in Table 2 is now between 0.136 (0.333) and 0.843 (0.049), depending on the optimality condition and instrument. However, the estimate of γ is still highly unstable (either negative or a small value), since it ranges between -0.450 (0.115) and 0.413 (0.138).

In sum, two features in the aggregate labor-market data led to the wrong sign or a small value of γ. A lack of systematic correlation between the cyclical components of hours worked and wages (which is 0.39 in the aggregate data after Hodrick-Prescott (HP) filtering) results in either non-concave or unstable utility. Accounting for the volatility of hours worked relative to wages (more precisely, relative to the real wage evaluated by the marginal utility of consumption) requires an elastic labor-supply schedule. (At business cycle frequencies, the ratio of the standard deviation of hours to that of wages is 1.52). The discrepancy between the optimality conditions and aggregate data is often interpreted as evidence of the failure of labor market clearing due to, say, sticky wages. In the next section, we show that a competitive equilibrium obtained from a reasonably calibrated heterogeneous-agent model can lead to estimates similar to those we see in the U.S data, which in turn implies that non-sensible estimates of preference parameters in the aggregate data do not necessarily reflect a failure of market clearing or stochastic components of preferences. Rather, they can reflect imperfect aggregation of individual optimality conditions.

3 The Benchmark Model

The model economy is based on Chang and Kim (2007) who extend Krusell and Smith's (1998) heterogeneous-agent model with incomplete capital markets (Aiyagari, 1994) to a model with an indivisible labor supply (Rogerson, 1988). Both frictions break the tight link between individual and aggregate labor-supply schedules. The indivisibility of labor implies that the optimality condition for hours worked holds as inequality at the individual level. The incompleteness of capital markets implies an imperfect aggregation of individual optimality conditions.

There is a continuum (measure one) of workers who have identical preferences but different productivity. Individual productivity varies exogenously according to a stochastic process with a transition probability distribution function $\pi_x(x'|x) = \Pr(x_{t+1} \leq x'|x_t = x)$. A worker maximizes his utility by choosing consumption c_t and hours worked h_t:

$$\max \ E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\sigma} - 1}{1-\sigma} - \psi \frac{h_t^{1+\gamma}}{1+\gamma} \right\}$$

subject to

$$a_{t+1} = w_t x_t h_t + (1 + r_t) a_t - c_t.$$

Workers trade claims for physical capital, a_t, which yields the rate of return r_t and depreciates at rate δ each period. They face a borrowing constraint: $a_t \geq \bar{a}$ for all t. Workers supply their labor in an indivisible manner; i.e., h_t takes either zero or $\bar{h}(< 1)$. If he works, a worker supplies $\bar{h}$ units of labor and earns $w_t x_t \bar{h}$, where w_t is the wage rate per effective unit of labor. The representative firm produces output according to a constant-returns-to-scale Cobb-Douglas technology in capital, K_t, and efficiency units of labor, L_t.

$$Y_t = F(L_t, K_t, \lambda_t) = \lambda_t L_t^{\alpha} K_t^{1-\alpha},$$

where λ_t is the aggregate productivity shock with a transition probability distribution function $\pi_\lambda(\lambda'|\lambda) = \Pr(\lambda_{t+1} \leq \lambda'|\lambda_t = \lambda)$. In this model economy, a technology shock is the only aggregate shock. This does not necessarily reflect our view on the source of business cycles. Since we would like to estimate aggregate preferences, we intentionally rule out shocks that may shift the labor-supply schedule itself and cause identification problems in estimating preferences (e.g., exogenous shifts in aggregate preferences, government spending, or the income tax rate).[5]

The value function for an employed worker, denoted by V^E, is:

$$V^E(a, x; \lambda, \mu) = \max_{a' \in \mathcal{A}} \left\{ \frac{c^{1-\sigma} - 1}{1-\sigma} - \psi \frac{\bar{h}^{1+\gamma}}{1+\gamma} \right.$$
$$\left. + \beta E\Big[\max \big\{ V^E(a', x'; \lambda', \mu'), V^N(a', x'; \lambda', \mu') \big\} \big| x, \lambda \Big] \right\}$$

subject to

$$c = wx\bar{h} + (1+r)a - a',$$

$$a' \geq \bar{a},$$

$$\mu' = \mathbf{T}(\lambda, \mu),$$

where $\mathbf{T}$ denotes a transition operator that defines the law of motion for the distribution of workers $\mu(a, x)$.[6] The value function for a non-employed worker, denoted by $V^N(a, x; \lambda, \mu)$, is defined similarly with $h = 0$. Then, the labor-supply decision is characterized by:

$$V(a, x; \lambda, \mu) = \max_{h \in \{0, \bar{h}\}} \big\{ V^E(a, x; \lambda, \mu), V^N(a, x; \lambda, \mu) \big\}.$$

[5] According to Krusell and Smith (1998), shocks to time preferences help to generate a more skewed wealth distribution, closer to the data. However, we also rule out this shock because we attempt to estimate preferences from the model-generated data.

[6] Let $\mathcal{A}$ and $\mathcal{X}$ denote sets of all possible realizations of a and x, respectively. The measure $\mu(a, x)$ is defined over a σ-algebra of $\mathcal{A} \times \mathcal{X}$.

The competitive equilibrium consists of a set of value functions, $\{V^E(a,x;\lambda,\mu), V^N(a,x;\lambda,\mu),\ V(a,x;\lambda,\mu)\}$, a set of decision rules for consumption, asset holdings, and labor supply, $\{c(a,x;\lambda,\mu),\ a'(a,x;\lambda,\mu),\ h(a,x;\lambda,\mu)\}$, aggregate capital and labor inputs, $\{K(\lambda,\mu),\ L(\lambda,\mu)\}$, factor prices, $\{w(\lambda,\mu),\ r(\lambda,\mu)\}$, and a law of motion for the distribution $\mu' = \mathbf{T}(\lambda,\mu)$ such that:

1. Individuals optimize:

   Given $w(\lambda,\mu)$ and $r(\lambda,\mu)$, the individual decision rules $c(a,x;\lambda,\mu)$, $a'(a,x;\lambda,\mu)$, and $h(a,x;\lambda,\mu)$ solve $V^E(a,x;\lambda,\mu)$, $V^N(a,x;\lambda,\mu)$, and $V(a,x;\lambda,\mu)$.

2. The representative firm maximizes profits:

$$w(\lambda,\mu) = F_1\big(L(\lambda,\mu), K(\lambda,\mu), \lambda\big)$$

$$r(\lambda,\mu) = F_2\big(L(\lambda,\mu), K(\lambda,\mu), \lambda\big) - \delta$$

   for all (λ,μ).

3. The goods market clears:

$$\int \big\{a'(a,x;\lambda,\mu) + c(a,x;\lambda,\mu)\big\} d\mu = F\big(L(\lambda,\mu), K(\lambda,\mu), \lambda\big) + (1-\delta)K$$

   for all (λ,μ).

4. Factor markets clear:

$$L(\lambda,\mu) = \int xh(a,x;\lambda,\mu)d\mu$$

$$K(\lambda,\mu) = \int a d\mu$$

   for all (λ,μ).

5. Individual and aggregate behaviors are consistent:

$$\mu'(A^0,X^0) = \int_{A^0,X^0} \left\{ \int_{\mathcal{A},\mathcal{X}} \mathbf{1}_{a'=a'(a,x;\lambda,\mu)}\ d\pi_x(x'|x)d\mu \right\} da'dx'$$

   for all $A^0 \subset \mathcal{A}$ and $X^0 \subset \mathcal{X}$.

3.1 Calibration

We briefly explain the choice of the model parameters. A detailed discussion of the calibration can be found in Chang and Kim (2006, 2007). The unit of time is a business quarter. We assume that the individual productivity shock (a source of the cross-sectional heterogeneity in our model economy) x_t follows an AR(1) process: $\ln x' = \rho_x \ln x + \varepsilon_x$, where $\varepsilon_x \sim N(0, \sigma_x^2)$. The values of $\rho_x = 0.939$ and $\sigma_x = 0.287$ reflect the persistence and standard deviation of innovation to individual wages in the Panel Study of Income Dynamics (PSID).[7] A working individual spends one-third of his discretionary time ($\bar{h} = 1/3$). The intertemporal substitution elasticity of consumption is one ($\sigma = 1$). The intertemporal substitution elasticity of hours worked is 0.4 ($\gamma = 2.5$). We set ψ such that the steady state employment rate is 60%. The discount factor β is chosen so that the quarterly rate of return to capital is 1% in the steady state. An aggregate productivity shock, λ_t, follows an AR(1) process: $\ln \lambda' = \rho_\lambda \ln \lambda + \varepsilon_\lambda$, where $\varepsilon_\lambda \sim N(0, \sigma_\lambda^2)$. We set $\rho_\lambda = 0.95$ and $\sigma_\lambda = 0.007$. Table 3 summarizes the parameter values of the benchmark economy.

3.2 Cross-sectional distribution and aggregate fluctuations of the model

As we investigate the aggregation and its implication to economic fluctuations, it is desirable for the model economy to possess a reasonable amount of cross-sectional heterogeneity and business-cycle volatility. We compare cross-sectional earnings and wealth—two important observable dimensions of heterogeneity in the labor market—found in the model and in the data. We also argue that the model-generated aggregate consumption, hours, and wages (variables used in the GMM estimation) exhibit business cycle properties similar to those in the data.

[7] These are maximum-likelihood estimates of Heckman (1979) correcting for a sample selection bias. See Chang and Kim (2006) for the details.

Table 4 summarizes both the PSID and the model's detailed information on wealth and earnings. The PSID denotes households' family wealth distribution in 1983 (1984 survey).[8] For each quintile group of wealth distribution, we calculate the wealth share, ratio of group average to economy-wide average, and the earnings share.

In both the data and the model, the poorest 20 percent of families in terms of wealth distribution were in debt. The PSID found that households in the 2nd, 3rd, 4th, and 5th quintiles own 0.50, 5.06, 18.74, 76.22 percent of total wealth, respectively, while, according to the model, they own 2.46, 10.22, 23.88, 65.49 percent, respectively. The average wealth of those in the 2nd, 3rd, 4th, and 5th quintiles is, respectively, 0.03, 0.25, 0.93, and 3.81 times larger than that of a typical household, according to the PSID. These ratios are 0.12, 0.51, 1.19, and 3.27, according to our model. Households in the 2nd, 3rd, 4th, and 5th quintiles of wealth distribution earn, respectively, 11.31, 18.72, 24.21, and 38.23 percent of total earnings, according to the PSID. The corresponding groups earn 15.06, 19.01, 23.59, and 32.63 percent, respectively, in the model. We argue that the model economy possesses a reasonable degree of heterogeneity, thus making it possible to study the effects of cross-sectional aggregation.

To obtain the aggregate fluctuations, we solve the equilibrium of the model using the "bounded rationality" method developed by Krusell and Smith (1998)—agents make use of a finite set of moments of μ in forecasting aggregate prices. As in Krusell and Smith (1998) we achieve a fairly precise forecast when we use the first moment of μ only (i.e., aggregate capital, K). The detailed description of our computation procedure is given in Chang and Kim (2007). Table 5 compares the cyclical property of key aggregate variables of the model economy to that in the U.S. aggregate data for 1964:I - 2003:IV. All variables are logged and de-trended by the HP filter. Our model with an aggregate productivity shock generates about 63% of business cycle volatility in the data: the standard deviation of output in the U.S. data is 2.04% and in our model it is 1.28%. This is not surprising because we allow only for aggregate productivity shocks. The relative

[8] Family wealth in the PSID reflects the net worth of houses, other real estate, vehicles, farms and businesses owned, stocks, bonds, cash accounts, and other assets.

(to output) volatilities of aggregate variables such as consumption, hours of work, and real wages are, however, pretty close to those in the data. They are, respectively, 0.43, 0.85, and 0.56, in the data, whereas they are 0.39, 0.76, and 0.50 in our model. The correlations with output are 0.83, 0.87, and 0.60, respectively, for consumption, hours, and real wages in the data. They are, respectively, 0.84, 0.87, and 0.68 in the model. One distinguishing aspect of the model is that hours worked is as volatile as that in the data but not highly correlated with wages (0.23 in our model and 0.39 in the data) – despite the fact that the only driving force in the simulation is the aggregate productivity shock. This is a striking result because the failure to generate a low correlation between hours and wages is known to be one of the most salient shortcomings of the RBC models. As we demonstrate below, the interaction between the indivisibility of labor and capital market incompleteness breaks a tight link between employment and wages at the aggregate level. In sum, our model reasonably well reproduces the business-cycle properties of aggregate variables used in the GMM estimation.

4 Estimation based on the model-generated aggregate data

4.1 Representative-agent model

In order to confirm that the GMM procedure recovers the true underlying preference parameters, we first estimate optimality conditions using the time series generated from the representative-agent model with productivity shocks (i.e., the standard real business cycle model). We assume that the preference parameters of the stand-in household are the same as those in the benchmark economy: $\sigma = 1$ and $\gamma = 2.5$. All parameters except for ψ are also identical to those in the benchmark model. We choose ψ so that the steady state hours worked is one-third. We estimate the optimality conditions based on the sample size of 160 observations, close to that in the U.S. quarterly time series data. We do not estimate the static first-order condition (S) because it holds exactly. The top panel of Table 6 reports the average and standard deviation of the estimates using the 2484 sets of estimations, each sample with 160 observations, three-fourths of which overlap with the next set. (We simulate 100,000 observations from the model and discard the first 500 observations.) We report the estimates based on Instrument I only because they are not greatly affected by the choice of instrument. According to the Euler equation for consumption (EC), the point estimate of σ is 0.670 (with standard error 0.197). According to the Euler equation for leisure (EL), the estimate of γ is 3.227 (0.252). When both equations are estimated jointly (System), σ is 0.754 (0.198) and γ is 3.019 (0.239). While the estimate for β is always 0.99 with high precision, the estimates of σ are smaller than the true value of 1 and those of γ are bigger than its true value of 2.5.

This small sample bias becomes negligible when we quadruple the sample size. The estimates in the bottom panel of Table 6 are based on 618 sets of estimation, each of which has a sample size of 640 observations, three-fourths of which overlap with the next set. According to these estimates, σ is 0.929 (with standard error of 0.093) and 1.008 (0.070), respectively, in (EC) and (System). The estimate of γ is 2.800 (0.197) and 2.651 (0.137) in (EL) and (System), respectively. Figure 1 exhibits the distribution (kernel density) of estimates for σ, γ, β, and J-statistic from both small-sample-size (solid line) and large-sample-size (dashed line) data sets generated from the model.[9] Both σ and γ are now highly concentrated around their true values, confirming that the GMM estimation accurately recovers true parameters with a large enough sample size.

4.2 Heterogeneous-agent model

We now apply the same GMM procedure to the aggregate time series generated from our benchmark heterogeneous-agent model. According to Table 7, the estimate for σ based on the small sample size (160 observations in each estimation) is 1.116 (0.079) and 1.107 (0.064) in (S) and (System), respectively. The estimation based on the large sample size (640 observations in each estimation) delivers similar values. While the estimate for the

[9] In estimating the kernel density, we used a Gaussian kernel with Silverman's (1986) automatic bandwidth selection criterion is used.

intertemporal substitution elasticity of consumption is close to the assumed value for individual households, when (EC) is estimated alone, the estimate of σ is below that of households. It is only 0.422 (0.220) and 0.639 (0.127), respectively, for small and large sample sizes, which resemble the low value of risk aversion often reported in the literature based on the aggregate consumption Euler equation (e.g., Hansen and Singleton, 1984; Ghysels and Hall, 1990).

The estimation result of the intertemporal substitution elasticity of hours worked is striking. As we found from the actual U.S. data (e.g., Table 1, Table 2, or MRS), γ is estimated to be either negative or close to zero (although statistically insignificant). According to the small-sample-size estimation, the estimate of γ is $-0.065(0.160)$, -0.158 (0.143) and 0.002 (0.101), respectively, in (S), (EL), and (System). This pattern persists in the large-sample-size estimation. They are all negative values and occasionally statistically significant: -0.139 (0.075), -0.235 (0.064) and -0.013 (0.051), respectively, in (S), (EL) and (System). We noted earlier that U.S. aggregate data led to the wrong sign or small value of γ for two reasons: (i) a lack of systematic correlation between hours worked and wages results in either non-concave or unstable utility, and (ii) accounting for the volatility of hours worked relative to wages requires an elastic labor-supply schedule. Our heterogeneous-agent model also shows similar patterns of relative volatility and correlation in aggregate employment and wages and has led to similar GMM estimates of γ.

Figure 2 exhibits the kernel density of estimates based on both small-sample-size (solid line) and large-sample-size (dashed line) data sets. The estimates for σ are clustered between 1 and 1.5 in both (S) and (System) among small-sample-size estimates and a similar pattern persists in the estimates based on the large sample size, while the estimates are more clustered as the sample size increases. Interestingly, estimates based on (EC) exhibit a somewhat bimodal distribution at 0 and 0.5 among small-sample estimates. With a large sample size the estimates are clustered around 0.64. Estimates of γ exhibit either a wrong sign or a small positive number regardless of the sample size. They are distributed between -0.5 and 0.5, with more concentration among the large-sample-size estimates. In terms of the J-statistic (bottom row), the hypothesis that a "representative" household optimally chooses hours worked and consumption is often rejected, similar to the pattern we observe from the GMM estimation based on actual U.S. data. In particular, with a large sample size, the intertemporal substitution hypothesis is rejected at the frequency of 98 out of 100.

When the model economy consists of heterogeneous agents and the individual optimality conditions are hard to aggregate, an attempt to account for the aggregate time series by an optimizing behavior of the representative household fails. The relative risk aversion of consumption is significantly underestimated when the aggregate consumption Euler equation is used. The parameter that governs the behavior of the labor supply is estimated with great uncertainty regardless of the equation and instrument, just like those from the actual aggregate data.

4.3 Auxiliary model economies

In our benchmark model economy with heterogeneous agents the difficulty of aggregation stems from two frictions: incomplete capital markets and indivisible labor. To distinguish the contribution of each, we consider two additional model economies that feature each friction only: the "incomplete-markets" model with divisible labor and the "indivisible-labor" model with complete capital markets.

"incomplete-markets" model Households can choose any length of working hours but still face the borrowing constraint and (uninsurable) idiosyncratic productivity shocks. This is essentially the same specification as in Krusell and Smith (1998) with endogenous choice of leisure. The equilibrium of this economy can be defined similar to that of the benchmark model with the worker's value function with divisible labor, $V^D(a, x; \lambda, \mu)$:

$$V^D(a, x; \lambda, \mu) = \max_{a' \in \mathcal{A}, h \in (0,1)} \left\{ \ln c - B\frac{h^{1+\gamma}}{1+\gamma} + \beta E\Big[V^D(a', x'; \lambda', \mu')\big|x, \lambda\Big] \right\}$$

subject to

$$c = w(\lambda, \mu)xh + (1 + r(\lambda, \mu))a - a',$$

$$a' \geq \bar{a},$$

$$\mu' = \mathbf{T}(\lambda, \mu).$$

"Indivisible-labor" model The next model economy we consider allows for complete capital markets but maintains indivisible labor and heterogeneity through idiosyncratic productivity shocks. The equilibrium of this economy can be replicated by an allocation made by a social planner who maximizes the equally weighted utility of the population. For an efficient allocation, the planner assigns workers with higher productivity to work. If a worker's productivity is above x_t^*, he supplies $\bar{h}$ hours of labor. The planner's value function in the complete market, denoted by $V^C(K, \lambda)$, and the decision rules for aggregate consumption, $C(K, \lambda)$, and cut-off productivity, $x^*(K, \lambda)$, satisfy the following Bellman equation:

$$V^C(K, \lambda) = \max_{C, x^*} \left\{ \ln C - B \frac{\bar{h}^{1+\gamma}}{1+\gamma} \int_{x^*}^{\infty} \phi(x)dx + \beta E\Big[V^C(K', \lambda') \big| \lambda \Big] \right\}$$

subject to

$$K' = F(K, L, \lambda) + (1 - \delta)K - C,$$

where $L = \bar{h} \int_{x^*}^{\infty} x\phi(x)dx$ is the aggregate effective unit of labor, and $\phi(x)$ is the cross-sectional productivity distribution of workers (unconditional distribution of $\pi_x(x'|x)$). The cut-off productivity x^* satisfies:

$$\frac{1}{C} F_L(K, L, \lambda)\bar{h}x^* = B \frac{\bar{h}^{1+\gamma}}{1+\gamma}. \tag{1}$$

The left-hand side is the utility gain from assigning the marginal worker to production. The marginal worker supplies $\bar{h}x^*$ units of effective labor, and the marginal product of labor is F_L. The right-hand side represents the disutility incurred by this worker. The upshot is that, under complete capital markets, there is a well-defined efficiency condition for labor supply and consumption at the aggregate level.

GMM estimates from the model-generated aggregate data Except for β and ψ, the same parameter values are used across all models. For the "indivisible-labor" model, β is set to 0.99 and ψ is chosen to be consistent with 60% employment along with $\bar{h} = 1/3$. For the "incomplete-markets" model, β and ψ are jointly searched to be consistent with average hours of 0.2 ($= 60\% \times 1/3$) and an interest rate of 1% in a steady state. These economies are simulated by the same aggregate productivity shocks.

Table 8 shows the parameter estimates from the aggregate time series of the "incomplete-markets" (with divisible labor) model. Despite incomplete capital markets, the aggregate data fairly accurately reveal the individual preference parameters with a high statistical precision. With a large sample size, σ is 1.058 (0.024), 0.828 (0.072), and 0.855 (0.893), according to (S), (EC), and (System), respectively. The labor supply parameter also reveals the value assumed at the individual household level. With a large sample, γ is 2.588 (0.095), 2.828 (0.258), and 2.625 (0.148), according to (S), (EL), and (System), respectively. Figure 3 shows that the estimates are also highly concentrated around their means. The capital-market incompleteness alone does not generate a large aggregation error because, with a divisible labor supply, in response to aggregate productivity shocks, hours and consumption are highly correlated across households, allowing for a fairly precise aggregation. To illustrate this, we provide a simple example in Section 4.4 below.

According to Table 9, aggregate consumption from the "indivisible-labor" (with complete capital markets) economy reveals the relative risk aversion of individual households. The estimate of σ is 0.963 (0.101) and 1.011 (0.064), according to (EC) and (System) with a large sample size.[10] The labor supply elasticity at the aggregate level is, however, very different from that of households. The estimates of γ are 0.840 (0.096) and 0.793 (0.065), respectively, for (EL) and (System) with a large sample size, implying a labor supply elasticity of 1.19 and 1.26, higher than the individual elasticity of 0.4. While

[10]Note that we do not estimate the static first order condition (S) for this model economy because it holds exactly in (1). Potentially, the estimation of (S) is still possible because aggregate hours and wages are subject to the so-called compositional bias. However, the composition bias does not a have enough time-varying component as the economy moves near the deterministic steady state.

aggregate preferences are not necessarily identical to individual preferences, the GMM estimates based on the model-generated aggregate time series reveal the social planner's objective function, the (equally) weighted average of household utility functions—there is a well-defined efficiency condition under complete capital markets. Figure 4 confirms that the distributions of parameter estimates are concentrated around their means.

We have shown that when individual optimality conditions are hard to aggregate (due to incomplete capital markets *and* indivisible labor), an attempt to account for the aggregate time series by an optimizing behavior of the representative household often ends up with non-sensible estimates for preferences. MRS interpreted the non-sensible preference parameters estimated from the aggregate time series as evidence of the failure of market clearing. Our analysis suggests that the incompatibility between the equilibrium outcome of a representative household's optimization and the aggregate data may actually reflect poor aggregation rather than the failure of the market. Nevertheless, our analysis shows that equilibrium outcomes of a heterogeneous-agent economy cannot be easily represented by a stand-in household.

4.4 An illustrative example

In this subsection, we illustrate why it is so difficult to aggregate individual optimality conditions when both frictions—incomplete capital markets and indivisible labor—are present. To make the analysis simple, we construct an example in a static environment. We also abstract from the idiosyncratic productivity (x). Suppose a worker maximizes the utility, $\frac{c^{1-\sigma}-1}{1-\sigma} - \psi\frac{h^{1+\gamma}}{1+\gamma}$, given the budget constraint, $c = wh + ra$. When the labor supply is divisible, the optimality condition for the choice of hours worked and consumption is

$$h(a) = \left(\frac{w}{\psi \cdot c(a)^{\sigma}}\right)^{\frac{1}{\gamma}}. \tag{2}$$

Suppose the cross-sectional distribution of individuals' asset holdings is denoted by $\xi(a)$. Aggregating (2) for *all* workers yields:

$$H = \left(\frac{w}{\psi \cdot C^{\sigma}}\right)^{\frac{1}{\gamma}} \cdot \int \left(\frac{c(a)}{C}\right)^{-\frac{\sigma}{\gamma}} d\xi(a) \tag{3}$$

where H and C are aggregate hours and consumption. Equation (3) is almost identical to the individual optimality condition (2) except for the last term. Rearranging (3) we obtain an aggregate relation similar to the static first-order condition—Equation (S)—in a representative-agent model:

$$\frac{\psi H^r}{C^{-\sigma}} = w \frac{\left(\int c(a)^{\frac{-\sigma}{\gamma}} d\xi(a)\right)^{\gamma}}{C^{-\sigma}} \tag{4}$$

The last term $\chi = \frac{\left(\int c(a)^{\frac{-\sigma}{\gamma}} d\xi(a)\right)^{\gamma}}{C^{-\sigma}}$ reflects the ratio of the CES aggregate of the marginal utility of individual consumption to the marginal utility of aggregate consumption. For a representative-agent model, $\chi = 1$ as the distribution of asset holdings, $\xi(a)$, is degenerate. When aggregate disturbances are introduced in a dynamic model, this ratio χ is time-varying because aggregate consumption as well as the distribution, $\xi(a)$, changes over time. According to the business-cycle accounting adopted by Hall (1997) and Chari, Kehoe, and McGrattan (2007), the variation of χ will show up as a time-varying wedge between the marginal rate of substitution and the real wage. According to our simulation of the "incomplete-markets" (with divisible-labor) model, the variation of this wedge is quantitatively small (the standard deviation of the wedge relative to that of output is only 0.09). With a divisible labor supply, *all* households' consumption tends to move together in response to aggregate productivity shocks, leaving the ratio χ (the ratio of CES aggregate of the marginal utility of individual consumption to the marginal utility of aggregation consumption) virtually unaffected. Thus, the estimation of (3) fairly accurately reveals the true preference parameters in (2).

When the labor supply is indivisible, however, this *approximate* aggregation no longer works. With a discrete choice of labor supply (either 0 or $\bar{h}$), the individual worker's optimality condition for the choice of consumption and hours worked holds with inequality. Specifically, in the static environment we just described, an individual worker decides to work ($h = \bar{h}$) if

$$\Delta U(a) = \frac{c_e(a)^{1-\sigma}}{1-\sigma} - \frac{c_u(a)^{1-\sigma}}{1-\sigma} \geq \psi \frac{\bar{h}^{1+\gamma}}{1+\gamma} \tag{5}$$

where $c_e(a) = w\bar{h} + ra$ and $c_u(a) = ra$ denote consumption when the worker is working (employed) and not working (unemployed), respectively. The left-hand side, $\Delta U(a)$, reflects the additional utility of consumption from earnings and the right-hand side the disutility from working. Given the strict concavity and continuity of the utility function, there exists a unique reservation asset holdings, a^R, below which workers supply $\bar{h}$ hours. In addition, thanks to the strictly decreasing marginal utility of consumption, $\Delta U(a) = w \cdot \bar{h} \cdot \tilde{c}(a)^{-\sigma}$ where $c_u(a) < \tilde{c}(a) < c_e(a)$ (mean-value theorem). Then, we can express Equation (5) as:

$$\bar{h} \leq \left(\frac{w}{\tilde{\psi} \cdot \tilde{c}(a)^{\sigma}}\right)^{\frac{1}{\gamma}} \quad \text{for } a \leq a^R. \tag{6}$$

where $\tilde{\psi} = \frac{\psi}{1+\gamma}$. Aggregating (6) for labor-market participants ($a < a^R$) yields:

$$H \leq \left(\frac{w}{\tilde{\psi} \cdot C^{\sigma}}\right)^{\frac{1}{\gamma}} \int^{a \leq a^R} [\frac{\tilde{c}(a)}{C}]^{\frac{-\sigma}{\gamma}} d\xi(a) \tag{7}$$

Equation (7) shows the difficulties in deriving a meaningful aggregate relation when the labor supply is indivisible. First of all, the inequality at the individual level carries over to the aggregate level. Second, when aggregate disturbances are introduced in a dynamic model, the reservation asset level (a^R) for labor-market participation itself is time varying. Third, the ratio of the CES aggregate of the marginal utility of consumption of *participants* to the marginal utility of aggregate consumption ($\int^{a \leq a^R} [\frac{\tilde{c}(a)}{C}]^{\frac{-\sigma}{\gamma}} d\xi(a)$) is more likely to move because the aggregate productivity shock has a bigger impact on those who participate the labor market. Moreover, it is not obvious how to measure $\tilde{c}(a)$ in practice. To summarize, when the labor supply is indivisible, individual optimality condition holds with inequality and the aggregation of those inequalities does not necessarily yield a meaningful relation in terms of observable aggregate variables. When the capital markets are complete, however, despite the indivisible labor supply, there is a well-defined aggregate efficiency condition (recall Equation (1) in Section 4.3).

Finally, we note that the difficulty in aggregating individual optimality conditions is distinctly different from the "bounded rationality" (often refereed to as approximate aggregation) in Krusell and Smith (1998). The bounded rationality refers to the class of equilibrium where agents' *information* set is limited. In a heterogeneous-agent model with incomplete capital markets, the equilibrium depends on the entire distribution of assets. In practice, it is impossible to include the entire distribution, an infinite-dimensional object, as a state variable. Krusell and Smith show that, in a certain class of models, using limited information about the distribution (i.e., the first moment only) is almost as good as using all the information when predicting prices.[11] For example, agents make their labor-supply decisions (i.e., Equation (5)), using the forecasted wages based on aggregate productivity and capital only. Yet, they seldom regret their decisions because the realized wages are almost identical to their forecasts.[12] Two aspects of the model allows for a successful forecasting of prices by the first moment only. First, given the Cobb-Douglas aggregate production technology, the equilibrium prices (marginal products) depends on the first moment only (e.g., aggregate capital). Second, agents try to stay away from the borrowing constraint around which their decision rules are highly non-linear. On the other hand, the real difficulty of deriving a meaningful aggregating relations lies in the fact that the labor-market participation condition holds with inequality in (6) which will be true even under the perfect foresight about the aggregate state.

5 Summary

The cyclical behavior of aggregate hours worked, wages, and consumption is hard to reconcile with the equilibrium outcome of the representative-agent model with standard preferences. Attempts to estimate preferences based on optimality conditions of a stand-in household often fail to deliver economically meaningful estimates. Either a commodity or leisure has to be an inferior good for the observed allocation to be an optimum. Unreasonable estimates of preference parameters are interpreted as evidence that the economy operates outside the labor-supply schedule in the short run due to, say, sticky wages. We

[11]See Krusell and Smith (2006) for various applications of this method.

[12]In our benchmark-model simulation, the forecasting function of equilibrium prices (w and r) yields R^2 of 0.997 and 0.988, respectively. See Appendix C in Chang and Kim (2007) for the accuracy of forecasting functions of prices for the benchmark model.

demonstrate that this incompatibility between the equilibrium of a representative-agent model and the aggregate data can reflect a failure of aggregation rather than that of the market. Nevertheless, our analysis suggests that outcomes of a heterogeneous-agent economy are not readily represented by an optimum of a representative-agent model with stable preferences.

References

Aiyagari, S. Rao (1994): "Uninsured Idiosyncratic Risk and Aggregate Saving," *Quarterly Journal of Economics*, 109(3), 659–684.

Altonji, Joseph G. (1986): "Intertemporal Substitution in Labor Supply: Evidence from Micro Data," *Journal of Political Economy*, 94(3:pt.2), S176–S215.

Bils, Mark (1985): "Real Wages over the Business Cycle: Evidence from Panel Data," *Journal of Political Economy*, 93(4), 666–689.

Chang, Yongsung, and Sun-Bin Kim (2006): "From Individual to Aggregate Labor Supply: A Quantitative Analysis Based on a Heterogeneous Agent Macroeconomy," *International Economic Review*, 47(1), 1–27.

Chang, Yongsung, and Sun-Bin Kim (2007): "Heterogeneity and Aggregation: Implications for Labor-Market Fluctuations," *American Economic Review*, 97(5), 1939–1956.

Chari, V.V., Patrick Kehoe, and Ellen McGrattan (2007): "Business Cycle Accounting," *Econometrica*, 75(3), 781–836.

Cochrane, John H. (2001): *Asset Pricing*. NJ: Princeton: Princeton University Press.

Dunn, Kenneth B., and Kenneth J Singleton (1986): "Modeling the Term Structure of Interest Rates under Non-separable Utility and Durability of Goods," *Journal of Financial Economics*, 17(1), 27–55.

Galí, Jordi, Mark Gertler, and J. David Lopéz-Salido (2007): "Markups, Gaps, and the Welfare Costs of Business Fluctuations," *Review of Economics and Statistics*, 89(1), 44–59.

Ghysels, Eric, and Alastair Hall (1990): "Are Consumption-based Intertemporal Capital Asset Pricing Models Structural?," *Journal of Econometrics*, 45(1-2), 121–139.

Hall, Robert E. (1997): "Macroeconomic Fluctuations and the Allocation of Time," *Journal of Labor Economics*, 15(1:pt.2), S223–S250.

Hansen, Gary D. (1993): "The Cyclical and Secular Behaviour of the Labour Input: Comparing Efficiency Units and Hours Worked," *Journal of Applied Econometrics*, 8(1), 71–80.

Hansen, Lars P. (1982): "Large Sample Properties of Generalized Method of Moments Estimators," *Econometrica*, 50(4), 1029–1054.

Hansen, Lars P. (2007): "Generalized Method of Moments Estimation," in *The New Palgrave's Dictionary of Economics*, ed. by Stephen Durlauf, and Lawrence Blume. Macmillan, forthcoming.

Hansen, Lars P., and Kenneth J. Singleton (1982): "Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models," *Econometrica*, 50(5), 1269–1286.

Hansen, Lars P., and Kenneth J. Singleton (1984): "Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models: Errata," *Econometrica*, 52(1), 267–268.

Heckman, James J. (1979): "Sample Selection Bias as a Specification Error," *Econometrica*, 47(1), 153–162.

King, Robert G., Charles I. Plosser, and Sergio T. Rebelo (1988): "Production, Growth and Business Cycles: I. The Basic Neoclassical Model," *Journal of Monetary Economics*, 21(2-3), 195–232.

Krusell, Per, and Anthony A. Smith, Jr. (1998): "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*, 106(5), 867–896.

Krusell, Per, and Anthony A. Smith, Jr. (2006): "Quantitative Macroeconomic Models with Heterogeneous Agents," in *Advances in Economics and Econometrics: Theory and Applications, Ninth World Congress, Volume 1*, ed. by Richard Blundell, Whitney K. Newey, and Torsten Persson. London: Cambridge University Press, 298–340.

Kydland, Finn E., and Edward C. Prescott (1982): "Time to Build and Aggregate Fluctuations," *Econometrica*, 50(6), 1345–1370.

MaCurdy, Thomas E. (1981): "An Empirical Model of Labor Supply in a Life-Cycle Setting," *Journal of Political Economy*, 89(6), 1059–1085.

Mankiw, N. Gregory, Julio J. Rotemberg, and Lawrence H. Summers (1985): "Intertemporal Substitution in Macroeconomics," *Quarterly Journal of Economics*, 100(1), 225–251.

Newey, Whitney K., and Kenneth D. West (1987): "A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix," *Econometrica*, 55(3), 703–708.

Rogerson, Richard (1988): "Indivisible Labor, Lotteries and Equilibrium," *Journal of Monetary Economics*, 21(1), 3–16.

Sheinkman, Jose A., and Laurence Weiss (1986): "Borrowing Constraints and Aggregate Economic Activity," *Econometrica*, 54(1), 23–45.

Silverman, Bernard W. (1986): *Density Estimation for Statistics and Data Analysis.* London: Chapman and Hall.

Solon, Gary, Robert Barsky, and Jonathan A. Parker (1994): "Measuring the Cyclicality of Real Wages: How Important Is Composition Bias?," *Quarterly Journal of Economics*, 109(1), 1–25.

Table 1: PARAMETER ESTIMATES BASED ON U.S. DATA (NON-DURABLES & SERVICES)

Equations	(S)	(EC)	(EL)	System
Instrument I				
σ	0.210	−0.210		−0.046
	(0.062)	(0.330)		(0.027)
γ	−0.569		0.179	0.023
	(0.198)		(0.145)	(0.044)
β		0.994	0.997	0.996
		(0.002)	(0.0008)	(0.0004)
ψ	0.156			0.113
	(0.017)			(0.011)
J-statistic	14.368	14.985	14.400	14.493
p-value	0.026	0.036	0.045	0.186
Instrument II				
σ	0.188	−0.129		−0.040
	(0.067)	(0.308)		(0.024)
γ	−0.473		0.089	0.0006
	(0.210)		(0.122)	(0.035)
β		0.995	0.997	0.996
		(0.002)	(0.0008)	(0.0004)
ψ	0.164			0.112
	(0.017)			(0.011)
J-statistic	15.710	14.793	14.987	15.083
p-value	0.015	0.039	0.036	0.208

Table 2: Parameter estimates based on U.S. data (Non-durables)

Equations	(S)	(EC)	(EL)	System
Instrument I				
σ	0.834	0.243		0.589
	(0.045)	(0.381)		(0.034)
γ	−0.444		0.379	0.074
	(0.108)		(0.152)	(0.058)
β		0.996	0.997	0.996
		(0.002)	(0.0008)	(0.0006)
ψ	9.790			5.110
	(1.336)			(0.716)
J-statistic	6.048	11.664	12.229	12.308
p-value	0.418	0.112	0.093	0.422
Instrument II				
σ	0.843	0.136		0.624
	(0.049)	(0.333)		(0.029)
γ	−0.450		0.413	0.018
	(0.115)		(0.138)	(0.050)
β		0.996	0.996	0.996
		(0.001)	(0.0008)	(0.0005)
ψ	10.189			5.765
	(1.470)			(0.710)
J-statistic	5.203	12.957	13.570	13.657
p-value	0.518	0.073	0.059	0.418

Table 3: Parameters of the benchmark model economy

Parameter	Description
$\alpha = 0.64$	Labor share in production function
$\beta = 0.9785504$	Discount factor
$\sigma = 1$	Inverse of intertemporal substitution elasticity of consumption
$\gamma = 2.5$	Inverse of intertemporal substitution elasticity of leisure
$\psi = 151.28$	Utility parameter
$\overline{h} = 1/3$	Labor supply if working
$\bar{a} = -2.0$	Borrowing constraint
$\rho_x = 0.939$	Persistence of idiosyncratic productivity shock
$\sigma_x = 0.287$	Standard deviation of innovation to idiosyncratic productivity
$\rho_\lambda = 0.95$	Persistence of aggregate productivity shock
$\sigma_\lambda = 0.007$	Standard deviation of innovation to aggregate productivity

Table 4: Characteristics of Wealth Distribution

	Quintile					
	1st	2nd	3rd	4th	5th	Total
PSID						
Share of wealth	-.52	.50	5.06	18.74	76.22	100
Group average/population average	-.02	.03	.25	.93	3.81	1
Share of earnings	7.51	11.31	18.72	24.21	38.23	100
Benchmark Model						
Share of wealth	-2.05	2.46	10.22	23.88	65.49	100
Group average/population average	-.10	.12	.51	1.19	3.27	1
Share of earnings	9.70	15.06	19.01	23.59	32.63	100

Table 5: Cyclical property of aggregate variables: benchmark model

Variable	U.S. Data	Model
σ_Y	2.04%	1.28%
σ_C/σ_Y	0.43	0.39
σ_H/σ_Y	0.85	0.76
σ_W/σ_Y	0.56	0.50
$cor(Y,C)$	0.83	0.84
$cor(Y,H)$	0.87	0.87
$cor(Y,W)$	0.60	0.68
$cor(H,W)$	0.39	0.23

Note: All variables are logged and de-trended by the HP filter. The volatility of output is measured by its standard deviation and that of all other variables is measured by the standard deviations relative to output.

Table 6: Parameter estimates : representative-agent model

Equations	(EC)	(EL)	System
Small Sample Size:			
σ	0.670		0.754
	(0.197)		(0.198)
γ		3.227	3.019
		(0.252)	(0.239)
β	0.990	0.990	0.990
	(0.0002)	(0.0002)	(0.0001)
Size	0.170	0.084	
Large Sample Size:			
σ	0.929		1.008
	(0.093)		(0.070)
γ		2.800	2.651
		(0.197)	(0.137)
β	0.990	0.990	0.990
	(0.0001)	(0.0001)	(0.0001)
Size	0.129	0.049	

Note: For upper (lower) panel, means and standard errors are calculated from 2484 (618) estimations. Each estimation has a sample size of 160 (640) observations. "Size" represents the empirical size (fraction of estimates rejected) of J-test with nominal size of 5%.

Table 7: Parameter estimates: heterogeneous-agent model

Equations	(S)	(EC)	(EL)	System
Small Sample Size:				
σ	1.116	0.422		1.107
	(0.079)	(0.220)		(0.064)
γ	−0.065		−0.158	0.002
	(0.160)		(0.143)	(0.101)
β		0.990	0.990	0.990
		(0.0002)	(0.0003)	(0.0002)
Large Sample Size:				
σ	1.116	0.639		1.095
	(0.034)	(0.127)		(0.023)
γ	−0.139		−0.235	−0.013
	(0.075)		(0.064)	(0.051)
β		0.990	0.990	0.990
		(0.0001)	(0.0001)	(0.0001)

Note: For upper (lower) panel, means and standard errors are calculated from 2484 (618) estimations. Each estimation has a sample size of 160 (640) observations.

Table 8: PARAMETER ESTIMATES: "INCOMPLETE-MARKETS" MODEL

Equations	(S)	(EC)	(EL)	System
Small Sample Size:				
σ	1.057	0.578		0.936
	(0.050)	(0.181)		(0.885)
γ	2.587		3.310	2.657
	(0.162)		(0.370)	(0.271)
β		0.990	0.990	0.990
		(0.0001)	(0.0002)	(0.0002)
Large Sample Size:				
σ	1.058	0.828		0.855
	(0.024)	(0.072)		(0.893)
γ	2.588		2.828	2.625
	(0.095)		(0.258)	(0.148)
β		0.990	0.990	0.990
		(0.0001)	(0.0001)	(0.0001)

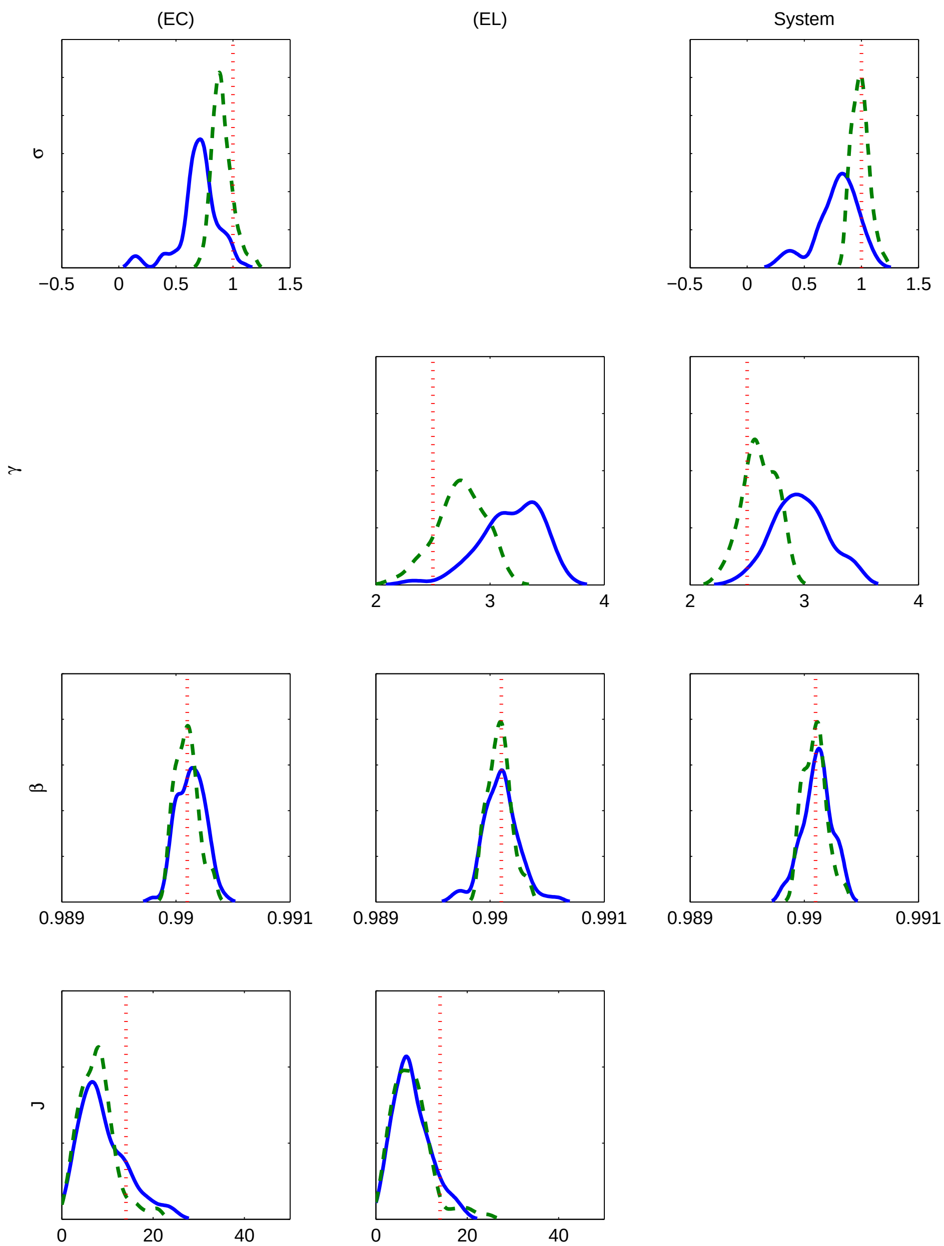
Note: For upper (lower) panel, means and standard errors are calculated from 2484 (618) estimations. Each estimation has a sample size of 160 (640) observations.

Table 9: Parameter estimates: "Indivisible-labor" model

Equations	(EC)	(EL)	System
Small Sample Size:			
σ	0.672		0.742
	(0.209)		(0.201)
γ		1.057	0.964
		(0.121)	(0.114)
β	0.990	0.990	0.990
	(0.0002)	(0.0002)	(0.0002)
Large Sample Size:			
σ	0.963		1.011
	(0.101)		(0.064)
γ		0.840	0.793
		(0.096)	(0.065)
β	0.990	0.990	0.990
	(0.0001)	(0.0001)	(0.0001)

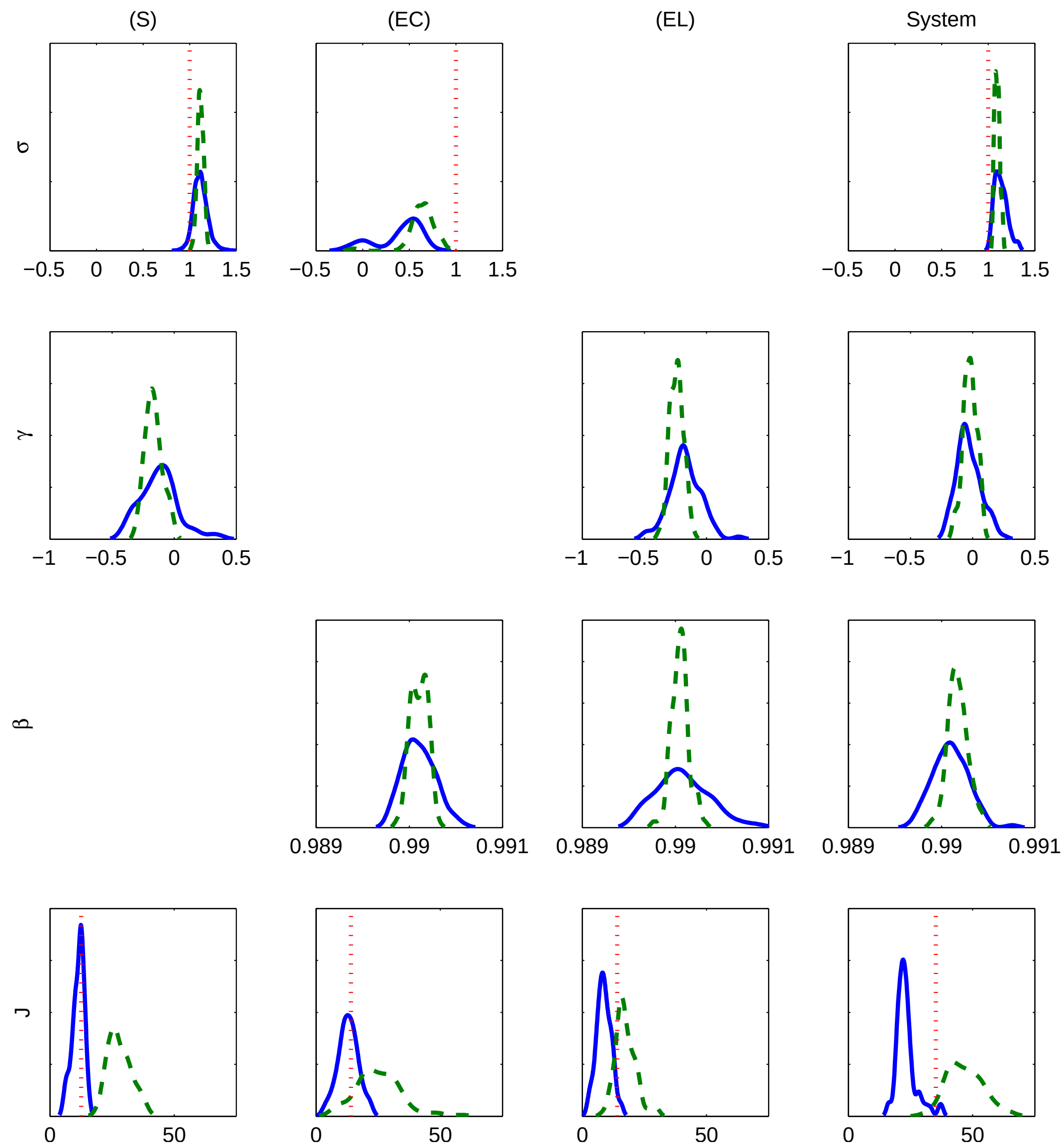
Note: For upper (lower) panel, means and standard errors are calculated from 2484 (618) estimations. Each estimation has a sample size of 160 (640) observations.

Figure 1: Kernel density of parameter estimates: representative-agent model



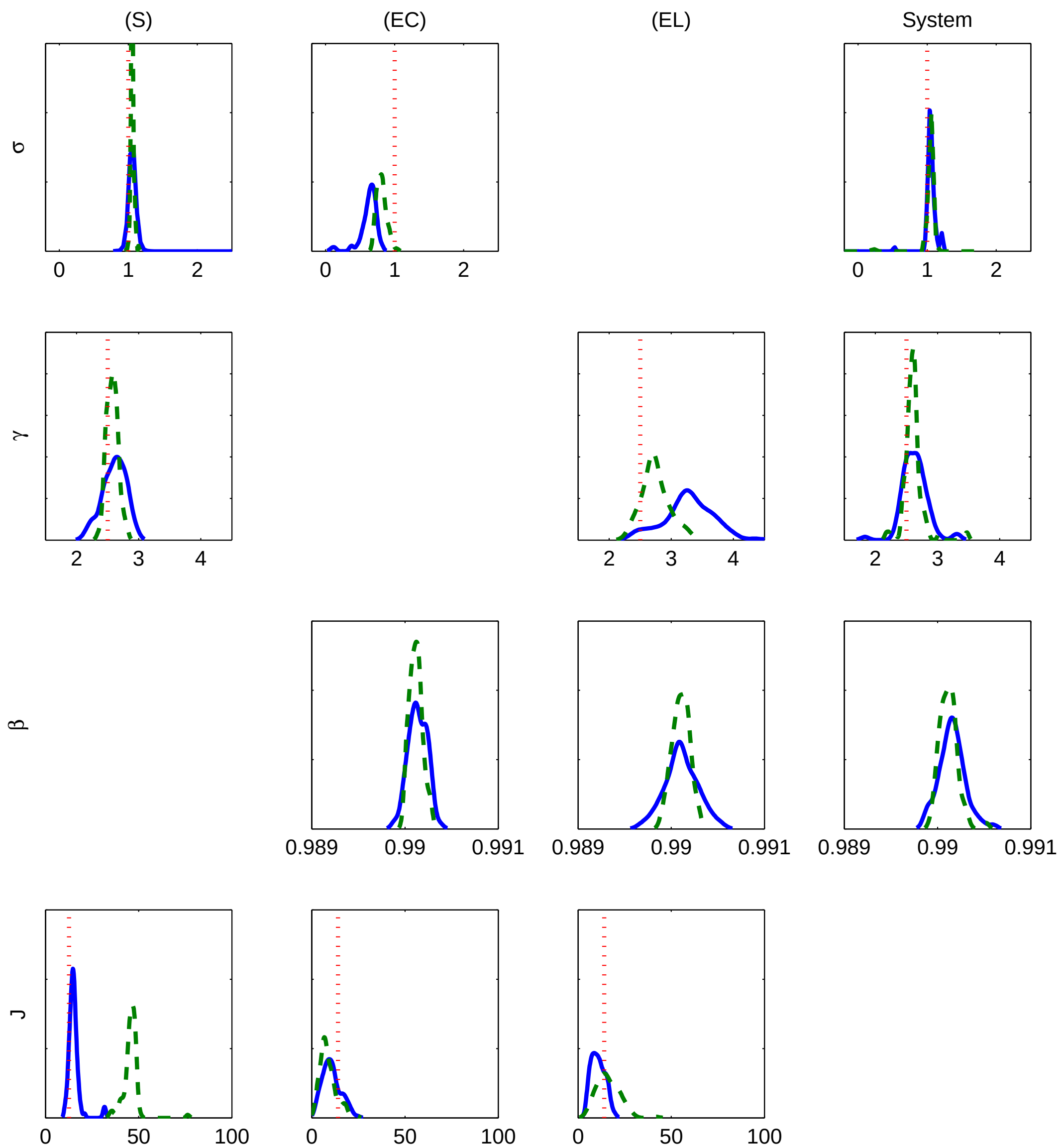
Note: The density of parameter estimates are calculated by the kernel method (Gaussian kernel with automatic bandwidth). The solid line represents the small-sample-size estimates (160 observations for each estimation), while the dashed line describes the large-sample-size estimates (640 observations for each estimation). The vertical dotted lines in the bottom panels represent 5% critical values of the J-statistic.

Figure 2: Kernel density of parameter estimates: heterogeneous-agent model



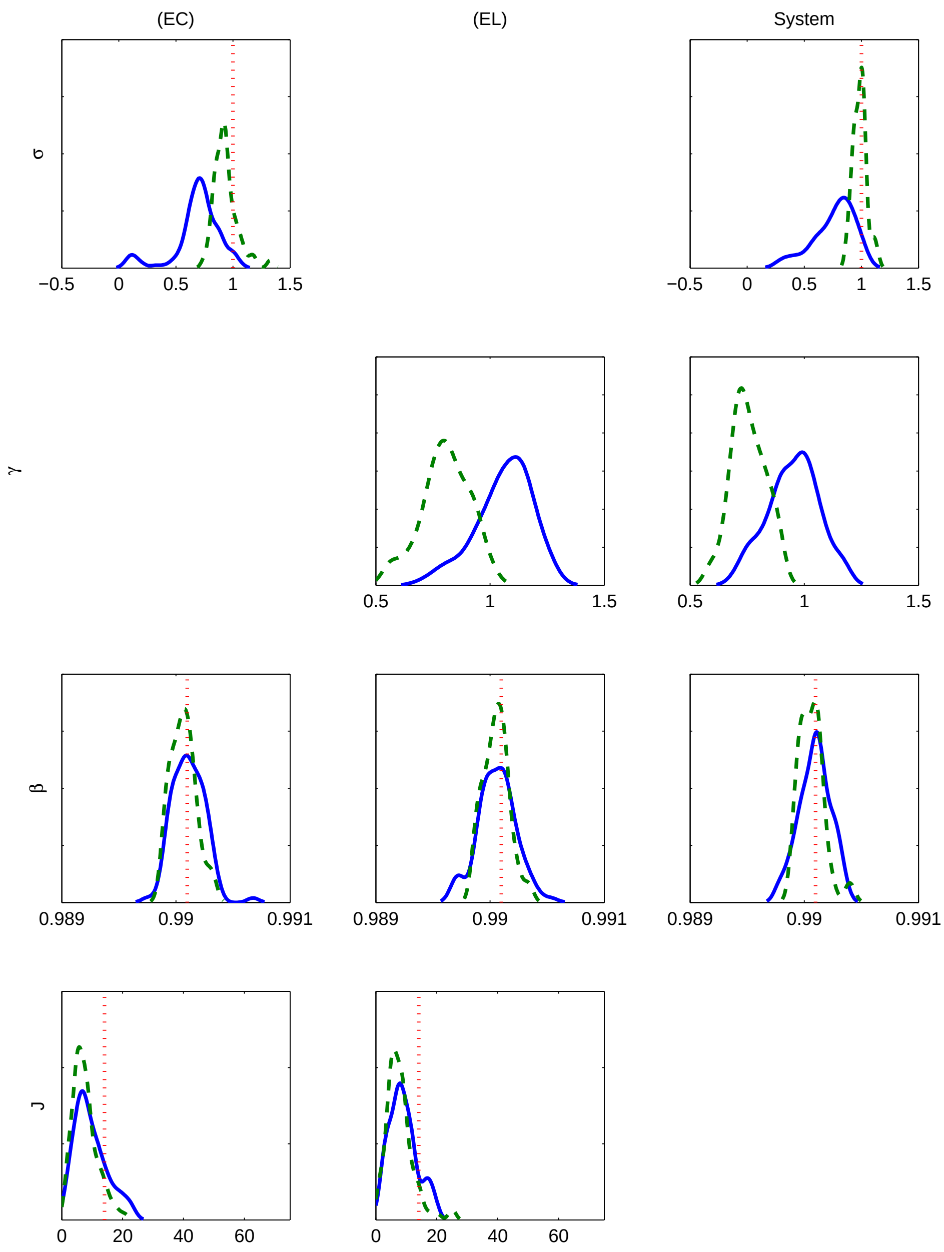
Note: The density of parameter estimates are calculated by the kernel method (Gaussian kernel with automatic bandwidth). The solid line represents the small-sample-size estimates (160 observations for each estimation), while the dashed line describes the large-sample-size estimates (640 observations for each estimation). The vertical dotted lines in the bottom panels represent 5% critical values of the J-statistic.

Figure 3: Kernel density of parameter estimates: "incomplete-markets" model



Note: The density of parameter estimates are calculated by the kernel method (Gaussian kernel with automatic bandwidth). The solid line represents the small-sample-size estimates (160 observations for each estimation), while the dashed line describes the large-sample-size estimates (640 observations for each estimation). The vertical dotted lines in the bottom panels represent 5% critical values of the J-statistic.

Figure 4: KERNEL DENSITY OF PARAMETER ESTIMATES: "INDIVISIBLE-LABOR" MODEL



Note: The density of parameter estimates are calculated by the kernel method (Gaussian kernel with automatic bandwidth). The solid line represents the small-sample-size estimates (160 observations for each estimation), while the dashed line describes the large-sample-size estimates (640 observations for each estimation). The vertical dotted lines in the bottom panels represent 5% critical values of the J-statistic.