

Public Housing Units vs. Housing Vouchers:  
Accessibility, Local Public Goods, and Welfare

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# Public Housing Units vs. Housing Vouchers: Accessibility, Local Public Goods, and Welfare\*

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## Abstract

We develop a general equilibrium model of residential choice and study the effects of two housing aid policies, public housing units and housing vouchers. Land is differentiated by both residential accessibility and local public goods, and the provision levels of local public goods are determined by property tax revenues and neighborhood compositions. Households differ in their incomes and preferences for local public goods. Housing aid policies are financed by general income taxes. We discuss how the location of public housing units is a fundamental policy variable, in addition to the numbers and sizes of units, and argue that vouchers not only cause less distortion for social welfare compared to public housing, but may also improve overall welfare.

**Keywords:** Public housing, housing vouchers, housing policy, welfare, residential location choice, local public goods, endogenous sorting.

**JEL Classification:** H40, D60, H82, R13.

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# 1 Introduction

Governments intervene in housing markets all around the world. The forms of these interventions differ significantly across and within economies.<sup>1</sup> Although housing aid policies are among the most expensive welfare programs in many countries, our understanding of their effects is very limited. The existing research is mainly empirical and typically evaluates the effects of particular programs on certain outcomes (e.g., labor supply). Overlooked general equilibrium effects—which may introduce distortions that counterweigh the benefits measured in partial equilibrium analyses—and program-specific details can burden such analyses at times. Given the enormous costs of “experiments” that will provide clear answers to numerous important questions, there are many potential benefits to studying rich theoretical models that enable thorough comparisons of the effects of alternative policy proposals.

We develop a general equilibrium model of residential choice and study the effects of two housing aid policies, subsidized units and housing vouchers. A novel feature of our model is that the land is differentiated by both residential accessibility and local public goods.<sup>2</sup> Households differ in their incomes and preferences for one local public good, viz. education. The quality of education in a

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<sup>1</sup>See Smeeding et al. (1993) for cross country comparisons. Whereas the United States tends to provide cash subsidies (Olsen, 2003, comments on the large number of different programs in the U.S.), European countries are more inclined to directly provide physical structures, even when the spending is comparable (Priemus, 2000). In 2001, the United States spent slightly above 1.5% of its GDP on public aid on housing, and the counterpart in France is similar, comprising slightly above 1.7% of the GDP (Laferrère and Le Blanc, 2006). Yet the construction-subsidized rental sector (mainly the *habitation à loyer modéré*, or HLM in short) accommodates 17% of the households in France, with less than 2% for the U.S. counterpart. Even a simple policy like a cash subsidy can lead to very different outcomes when institutional details differ. For instance, Priemus (2001) states that whereas in the U.S., 100% of the additional rent is paid by tenants, tenants in the Netherlands only pay 25%, and the government is responsible for the rest. Moreover, in the Netherlands, there is no waiting list, and the rent subsidy is perceived as a “right.” As a result, the number of applicants rose from 348,000 in 1976 to 922,000 in 1996. In 1998, there were more than a million households receiving a housing allowance, out of more than 6 million households in the Netherlands. See Malpezzi (2012) for a post-2007-8 evaluation of U.S. housing policies.

<sup>2</sup>Two attributes that households consider when making residential choices, accessibility and local public goods and services, have traditionally been studied separately. Urban land use frameworks (based on pioneering works of Alonso, 1964; Muth, 1969; and Mills, 1972) focused on the implications of the trade-off between accessibility and space, and predicted households locating in rings around the center according to their types (see Straszheim, 1987, and Fujita, 1999, for a review of this literature), whereas Tiebout models focused on the implications of local public goods and services (e.g., Tiebout, 1956; Ellickson, 1971; Epple, Filimon, and Romer, 1984, 1993; Nechyba, 1999, 2000; Fernandez and Rogerson, 1996; among others), predicting strong stratification based on income and preferences for local public goods. The documented household sorting is stronger than that predicted by urban models and weaker than the sorting predicted by Tiebout models (Glaeser, Kahn, and Rappaport, 2000). There have been some recent attempts to merge the two lines of research and obtain a more realistic description of urban location patterns (de Bartolome and Ross, 2003; Hanushek and Yilmaz, 2007). Our approach, based on Hanushek and Yilmaz (2007), allows us to account for both key attributes in residential decision-making simultaneously, providing us with a rich model that captures several essential features of urban spatial structure.

neighborhood depends on both the composition of the population in the neighborhood and on the local educational expenditures. The expenditures are determined by property taxes, the rates of which are chosen by neighborhood residents through majority voting. The housing aid policies are financed by general income taxes. We study equilibrium outcomes such as rents, distribution of households within and across neighborhoods, and school qualities, and pay particular attention to welfare.

We first present the baseline equilibrium with no government intervention in the housing market. We observe a sorting of households according to their incomes and preferences in each neighborhood with respect to location. We also observe a (partial) sorting across neighborhoods. We then introduce public housing into the model. Public housing programs exhibit large variations from one market to another, and sometimes even within the same market. We combine some common elements of the widespread applications to construct the program that we adopt. Public housing units typically cause property-tax losses to the hosting neighborhood, and households respond by a stronger sorting across neighborhoods. In addition, the forced location and land consumption of public housing residents deviate their numeraire consumption and labor supply choices away from the efficient market allocation. Rents increase in general, and so does the education quality gap between neighborhoods. The result is a decrease in overall welfare. We then provide public housing units at alternative locations and find that, in fact, location matters. Our results suggest that household sorting further increases as public housing units move further from the city center because the fiscal burden problem created by public housing becomes more serious (and overall welfare decreases further) as the spatial distribution and choices of households further deviate from that of the “no-intervention” equilibrium.

As an alternative housing aid policy, we consider providing housing vouchers to the same participants and present comparisons. We find housing vouchers to be superior to public housing units in terms of welfare. The difference results mainly from two issues associated with public housing units: 1. the implications of stronger sorting between neighborhoods as a result of the fiscal burden problem that arises in the neighborhood that hosts the (property-tax exempt) public housing units; and 2. the deviations from the efficiency of market allocation caused by forced location and lot size choices under public housing.

The findings of this paper suggest that the intuition behind basic microeconomics textbook dis-

ussions for “in-cash versus in-kind” applies to a more general environment, one that includes endogenous local public goods, peer group externalities, and accessibility. The framework developed in this paper can be adapted to compare public housing and/or vouchers to other housing aid programs, or to compare the outcomes of any single program under different institutional details.

## 2 The Model

We incorporate a Tiebout economy into Alonso’s (1964) basic land use framework. Households in a monocentric city work at the Central Business District (CBD hereafter) and reside in the surrounding area. The distance of home to the workplace matters because of the pecuniary and time costs of commuting to work. A straight line that goes through the CBD (e.g., a river) divides the city into two jurisdictions, which we call *east* ( $e$ ) and *west* ( $w$ ). Each jurisdiction provides its residents a local public good, education. The provision level (quality) of education in a neighborhood is endogenous, and depends on both the composition of the households in the neighborhood and local spending (property tax revenues, the rates of which are determined by majority voting). Agents can move, and hence the tax rates and expenditure levels may differ between jurisdictions. We discuss this model in further detail below.

### 2.1 Households

Households choose a neighborhood (east or west), a location in that neighborhood ( $r$ , distance to the CBD), lot size  $s > 0$ , leisure  $l \in [0, 24]$ , and the consumption level of a composite consumer good  $z$ . The preferences of a household can be represented by the utility function  $U(q, s, z, l) = q^\alpha s^\eta z^\gamma l^\delta$ , with  $q > 0$  denoting neighborhood quality. We assume each household has one school-age child and measure neighborhood quality by the quality of education in the neighborhood school. One member of every household works for an (exogeneous) hourly wage  $W$ . Households differ in their wages and in their preferences for school quality. We name the higher income types *skilled* workers ( $S$ , earning  $W_S$ ) and the lower income types *unskilled* workers ( $U$ , earning  $W_U < W_S$ ), and assume that skilled workers value education more than unskilled workers do ( $\alpha_S > \alpha_U$ ).<sup>3</sup>

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<sup>3</sup>Tastes for education and incomes have been documented as strongly correlated. See Hastings et al. (2008) and the references therein.

The city has a dense radial transportation system, and household commuting costs increase with distance to the CBD: if a household lives  $r$  miles away from the CBD, the cost of daily roundtrip commuting is  $ar$  dollars (pecuniary cost,  $a > 0$ ) and  $br$  hours (time cost,  $b > 0$ ), which converts to  $bWr$  dollars given the opportunity cost of time.<sup>4</sup> We normalize the price of the composite consumption good to one and denote the unit rent of land  $r$  miles away from the CBD by  $R(r)$ . Households pay a property tax on the value of land. Let  $\tau$  denote the property tax rate as a proportion of daily rent.<sup>5</sup> The budget constraint of a household can be written as

$$z + (1 + \tau)R(r)s + Wl = Y(r) \equiv 24W - (a + bW)r. \quad (1)$$

Given the market rent curves  $\{R_e(r), R_w(r)\}$  and quality-tax packages  $\{(q_e, \tau_e), (q_w, \tau_w)\}$  for each jurisdiction, type  $i \in \{S, U\}$  household solves the problem:

$$\begin{aligned} \max_{s, z, l, j \in \{e, w\}} \quad & U(q, s, z, l) = q_j^{\alpha_i} s^\eta z^\gamma l^\delta \\ \text{s.t.} \quad & z + (1 + \tau_j)R_j(r)s + W_i l = Y_i(r). \end{aligned} \quad (2)$$

## 2.2 Market Rent Curves and Allocation of Land

Land is owned by absentee landlords and auctioned off to the highest bidders. The reservation price of any given landlord,  $R_a$ , is determined by an alternative use of land such as agriculture and is independent of the location. For a given utility level  $\bar{u}$ , we can find the maximum rent a household is willing to pay per unit of land and optimal lot size  $r$  miles away from the CBD by solving the problem  $\Psi(r, \bar{u}, q, \tau) = \max_{s, z, l} \left\{ (Y(r) - z - Wl) / ((1 + \tau)s) \mid U(q, s, z, l) = \bar{u} \right\}$  to obtain the *bid rent function*

$$\Psi_i(r, \bar{u}, q_j, \tau_j) = \frac{k^{1/\eta}}{(1 + \tau_j)W_i^{\delta/\eta}} q_j^{\alpha_i/\eta} Y_i(r)^{(\eta + \gamma + \delta)/\eta} \bar{u}^{-1/\eta} \quad (3)$$

and the *bid-max lot size function*

$$s(r, \bar{u}, q_j, \tau_j) = \frac{\eta}{(\eta + \gamma + \delta)(1 + \tau_j)} \frac{Y_i(r)}{\Psi_i(r, \bar{u}, q_j, \tau_j)} \quad (4)$$

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<sup>4</sup>We ignore non-commute transportation costs and travel costs within the CBD.

<sup>5</sup>We provide details on this conversion in the calibration section below.

where  $k = \frac{\eta^\eta \gamma^\gamma \delta^\delta}{(\eta + \gamma + \delta)^{\eta + \gamma + \delta}}$ ,  $i \in \{S, U\}$ , and  $j \in \{e, w\}$ .<sup>6</sup> At an auction for a particular location  $r^*$ , the winner is the type with the highest bid rent curve at that location. Given the two types in the model, in each jurisdiction there are two bid rent curves. The *equilibrium rent curve*  $R_j(r)$  is the upper envelope of the bid rent curves of two types and the agricultural rent  $R_a$ . In equilibrium, there will be a distance in each jurisdiction  $r_{jf}^*$ , the *fringe distance*, beyond which no households reside. Because all bid rent curves are convex and decreasing, the equilibrium rent curve  $R_j(r)$  is also decreasing up to  $r_{jf}^*$  and is constant from that point on. Households with steeper bid rent curves locate closer to the CBD. Higher income increases the demand for land consumption and attracts households further away from the CBD, but it also increases the opportunity cost of commuting time.

Our city is a *closed city*, that is, population is given exogenously. Let  $L(r)$  represent the land density  $r$  miles away from the CBD and  $n_j(r)$  the equilibrium density function of the household population in jurisdiction  $j \in \{e, w\}$ . Suppose that the residents of the land at distance  $r$  in jurisdiction  $j$  are type  $i$  households in equilibrium. If the equilibrium level of utility of the type  $i$  agent,  $i \in \{S, U\}$ , is  $u_i^*$ , then  $n_j(r) = \frac{L(r)}{s(r, u_i^*)}$ . Let  $\bar{N}_S, \bar{N}_U$  denote the populations of the respective types. The *population constraint* for type  $i \in \{S, U\}$  can then be stated as

$$\int_0^\infty \frac{L(r)}{s_w(r)} I[t_w^*(r) = i] dr + \int_0^\infty \frac{L(r)}{s_e(r)} I[t_e^*(r) = i] dr = \bar{N}_i, \quad (5)$$

where  $t_j^*(r)$  is a function showing the type of the occupant at distance  $r$  in jurisdiction  $j$ , and  $I[\cdot]$  is an indicator function that takes the value 1 when the condition in brackets is satisfied and 0 otherwise. The population constraint implicitly assumes that the land market clears in each jurisdiction ( $\forall r \leq r_{jf}^*, s_j(r)n_j(r) = L(r)$ ).

## 2.3 Neighborhoods

The two neighborhoods may differ in the *quality of education* and *property tax rate*  $(q_j, \tau_j)$  packages they provide. Neighborhood schools are open to the residents of the community only. Admission is free. Schools are financed by property taxes on residential land. The quality of education  $q(\Pi, E)$  in a neighborhood is determined by (per-student) instructional expenditures  $E$  and peer quality

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<sup>6</sup>For derivations and a detailed discussion of the properties of these bid-rent functions, see Hanushek and Yilmaz (2007).

II. For a given group of students, an increase in instructional expenditures increases the quality of education ( $\partial q/\partial E \geq 0$ ). Given the equilibrium rent function  $R_j(r)$  and equilibrium tax rate  $\tau_j$ , we can calculate the tax base and total tax revenues to find the per-student expenditure in the public school system  $j \in \{e, w\}$ :

$$E_j = \frac{1}{N_j} \tau_j \int_0^{r_{jj}^*} R_j(r) L(r) dr, \quad (6)$$

where  $N_j$  denotes the number of students in neighborhood  $j$ .

Different groups of students may benefit differently from a given amount of instructional expenditures. This is captured by the peer quality component  $\Pi$  in our model ( $\partial q/\partial \Pi \geq 0$ ). Having high-achieving peers may matter in different ways: the students may learn directly from their peers, they may be motivated by the competition, or the teacher may teach at a higher level or at a more demanding pace. Because a student's characteristics are correlated with those of her parents, achievement at school would be correlated with the background variables of peers.<sup>7</sup> We assume that  $\Pi$  increases with the proportion of skilled households in the neighborhood and use the following tractable formulation:<sup>8</sup>

$$\Pi = c_0 + c_1 \exp\left(-c_2 \frac{N_U}{N_S}\right), \quad c_0, c_1, c_2 \geq 0. \quad (7)$$

We adopt the specification  $q_j = q(\Pi_j, E_j) = \Pi_j E_j$  for the *school quality*.

The *timing of events* is as follows: at the beginning of each period, households make residential choice decisions, expecting the previous period's quality-tax rate packages to prevail. They move in and vote for the tax rate. For a type  $i$  household, the most preferred tax rate  $\tau_i^* = \frac{\alpha_i}{\eta - \alpha_i}$  is the solution to the indirect utility maximization problem<sup>9</sup>

$$\tau_i^* = \underset{\tau}{\operatorname{argmax}} V(.) = \frac{k}{(1 + \tau_j)^\eta R(r)^\eta W_i^\delta} q_j^{\alpha_i} Y_i(r)^{\eta + \gamma + \delta} \quad (8)$$

for  $q_j = \Pi_j E_j$  and  $E_j = \tau_j \bar{R}_j$ . The households are *myopic* when voting; they do not consider the implications of their votes on migration patterns and the composition of neighborhoods. The

<sup>7</sup>See Sacerdote (2011) for a long list of studies that document significant effects of peer background on a student's own test scores in elementary and secondary schools.

<sup>8</sup>Alternative specifications give similar results. See, for example, Hanushek and Yilmaz (2007, 2012) and Hanushek, Sarpça, and Yilmaz (2011).

<sup>9</sup>The indirect utility function  $V(.)$  is decreasing in the property tax rate  $\tau$  while increasing in  $q$ , a function of  $\tau$ . When  $\eta > \alpha$  the preferences of a household are single-peaked. This ensures the existence of a voting equilibrium.



quality tax rate package may be different from what they expected; however, they are stuck until the beginning of the next period. Then they update their expectations, and the events start over again. We study the *stationary equilibrium*, which is attained when no one has an incentive to relocate in response to the voting results.

**Definition:** an *equilibrium* is a set of utility levels  $\{u_S^*, u_U^*\}$ , market rent curves  $\{R_e(r), R_w(r)\}$ , quality of education and property tax rate pairs  $\{(q_e, \tau_e), (q_w, \tau_w)\}$ , household population distribution functions  $\{n_e(r), n_w(r)\}$ , and location-type functions  $\{t_e^*(r), t_w^*(r)\}$  that show the equilibrium occupant of the location at distance  $r$  in community  $j$  such that:

- households' choices are determined by solving (2);
- the market rent function  $R_j(r)$  in each jurisdiction is determined through a bidding process among different types of households;
- same types of households obtain the same level of utility regardless of their choices;
- the tax rates in each jurisdiction are determined by majority voting by myopic voters;
- local governments' budgets balance in each jurisdiction, (6);
- labor and land markets clear;
- and the population constraints (5) hold.

## 2.4 Calibration

The equilibrium of our model can only be calculated numerically.<sup>10</sup> We specify parameter values to match certain statistics from mid-size U.S. cities in 2005. Normalizing the sum  $\eta + \gamma + \delta$  to 1, the solution to the household problem gives the optimal budget shares for leisure, consumption, and lot size as  $\delta$ ,  $\gamma$ , and  $\eta$ , respectively. In the U.S., the average number of hours of work per week in full-time jobs is 40, and the average annual earnings of workers are \$30,104 for high school graduates and \$58,114 for college graduates.<sup>11</sup> Accordingly, we set the hourly wages for unskilled and skilled households as  $W_U = 14$  and  $W_S = 27$ . In a 168 (= 24 \* 7) hour week, 40 hours of work implies a 0.762 budget share for leisure, so the budget share of non-leisure (actual expenditures) is (1-0.762).<sup>12</sup> The data on household expenditures suggest that expenditures on shelter constitute

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<sup>10</sup>The algorithm that is used for solving the models in this paper is available in the web appendix.

<sup>11</sup>U.S. Statistical Abstract.

<sup>12</sup>We assume that people sell all of their hours and buy back leisure at the wage rate.

about 20% of the budget of an average household.<sup>13</sup> This implies budget shares of 4.76% ( $= (1 - 0.762) * 20%$ ) for housing and 19% for consumption. We chose the most desired property tax rates for the two household types as 1.97% (for high valuation) and 1.04% (for low valuation)—slightly higher/lower than the U.S. average of 1.40%—and calibrate the  $\alpha$ 's accordingly.<sup>14</sup> The average population density in a city with a population of 1 to 2.5 million is about 4.53 people per acre.<sup>15</sup> This defines an endogenous calibration target that is achieved by setting agricultural rent  $R_a = \$731$  per acre per month. The utility function parameters consistent with all of these are  $\alpha_L = 0.014$ ,  $\alpha_H = 0.021$ ,  $\delta = 0.762$ , and  $\gamma = 0.19$ .<sup>16</sup>

(Table 1 about here)

We calculate the commuting costs assuming that households drive to work. The pecuniary cost can be calculated based on the cost of owning and operating an automobile. In 2004, the pecuniary cost per mile was \$0.56, and we set  $a = 1.1$ . Assuming the commuting speed in the city is 20 miles per hour, we set  $b = 0.13$ . We assume 1.5 million households populate the city. When computing the equilibrium, we target for a (endogenous) fringe distance (city radius) approximately 15 miles in each jurisdiction. The proportion of college graduates in the U.S. is about 30%. We expect this proportion to be slightly higher in a city. Hence, we set the proportion of skilled households to 40%. We choose the parameters of the school quality function to  $c_0 = 0.1$ ,  $c_1 = 1.3677$  and  $c_2 = 0.05743$  to match certain endogenous calibration targets to their estimates in the empirical literature.

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<sup>13</sup>Using data from U.S. Statistical Abstract, the share of shelter in total expenditures can be calculated as 21% for households with incomes less than \$70K, 19.6% for those with incomes between \$70K–80K, 19% for those with incomes between \$80K–100K, and 19.5% for those with incomes \$100K and over. With just two income levels in the model, the calibration of budget shares is consistent with these figures.

<sup>14</sup>We conducted an additional analysis to check the robustness of our results to changes in the levels of  $\alpha_i$ 's and  $\tau_i$ 's. The levels we select for  $\alpha_i$ 's affect the most desired tax rates of skilled and unskilled households by about  $\pm 20%$  or more. This analysis shows that our welfare comparisons are robust to such changes in property tax rates and utility parameters. We report the results from this additional analysis in the web appendix.

<sup>15</sup>U.S. Census Bureau, Population Estimates Files.

<sup>16</sup>Property taxes are paid over property value (the present value of the perpetual rent stream), whereas the model is written for a day. The conversion can be done as follows: the total annual rent is  $365 * R * s$ , and the property taxes paid in a year amount to  $\tau * 365 * R * s$ . With an annual interest rate  $i$ , the property value (the present value of the perpetual rent stream) is  $(365 * R * s) / i$ . The annual property tax rates given above are calculated as the ratio of annual tax paid  $\tau * 365 * R * S$  to the property value, i.e.,  $\tau * i$  for  $i = 0.0253$ .

## 2.5 Baseline Equilibrium

Figure 1 displays a map of the city and Table 2 summarizes some properties of equilibrium. Several observations are immediate: (1) neighborhoods are heterogeneous, both household types exist in each neighborhood; (2) there is (partial) Tiebout sorting across neighborhoods;<sup>17</sup> and (3) there is income-sorting with respect to distance within neighborhoods: Unskilled households choose smaller lots closer to the CBD, whereas skilled households live on larger lots away from the CBD.

(Figure 1, Table 2 about here)

The results are intuitive. Costly commuting causes the market rents in each jurisdiction to decrease in distance to the CBD. Higher income increases the land demand and attracts households further away from the CBD, where the land is cheaper. High income also increases the opportunity cost of commuting time, but our calibration suggests that this effect is dominated by the former, consistent with residential patterns in the U.S. As a result, in each community unskilled households occupy a semicircle around the CBD. The skilled households reside in a semi-ring that surrounds the semicircle of unskilled households. The outer end of the skilled households' semi-ring is the fringe distance, the land beyond which is left for agricultural use.<sup>18</sup> The intuition behind heterogeneous communities is also straightforward: households choose the lot size, distance to the CBD, and the community (with a given quality of education and a property tax rate) simultaneously. Hence, lower taxes or lower rents in a community (hence larger lots) can compensate for lower quality of education and vice versa.

Notice that about 70% of skilled households live in the same neighborhood, constituting a 55% majority of the population there. As a result, the voting outcome is the higher tax rate. Without loss of generality, we refer to the higher tax neighborhood as the *west* school district throughout the paper. Because the two neighborhoods have comparable populations, higher taxes and rents mean higher educational expenditures per student in the west (\$3,653 vs \$2,027). Also, the peer

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<sup>17</sup>The first two observations highlight an advantage of our approach over Tiebout frameworks that do not consider accessibility: the partial sorting across heterogeneous communities is consistent with empirical findings on household sorting (e.g., Davidoff, 2005).

<sup>18</sup>The “ring” structure, dating back to von Thunen’s model of land use (1826), is replaced by semi-rings here because of the jurisdiction boundaries that identify the differing bundles of the local public good. In general, the widths of rings may differ between neighborhoods. The ordering of households around the CBD, however, is the same in each district.

quality is higher in the west neighborhood; thus the quality of education exceeds that in the east.<sup>19</sup> This difference in the quality of education is capitalized into rents; rents in the west are about 25% higher on average than those in the east.

### 3 Public Housing

We study two housing aid policies in this and the following sections: government-provided subsidized units and housing vouchers. These programs are financed by uniform income taxes on earnings of *all households regardless of their location* (and participant contributions in the case of public housing units). Only unskilled households are eligible. The government sets the maximum number of participants  $N_P$  exogenously. If the number of program applicants exceeds  $N_P$ , participants are selected by a lottery.<sup>20</sup> Table 3 summarizes the main differences between the two policies we consider.

(Table 3 about here)

#### 3.1 Public Housing Model

The government announces that public housing units will be located between  $r_P$  and  $r_P + w_P$  miles from the CBD in one of the neighborhoods.<sup>21</sup> All unskilled households that consider this to be in their interest apply, and a lottery selects  $N_P$  participants in the case of excess demand. The rents

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<sup>19</sup>The higher tax/expenditure community providing a higher quality of education with higher per-student-expenditure may mislead one to overemphasize the role of expenditures on school quality. Hanushek, Sarpça, and Yilmaz (2011) show that the existence of a private sector for education breaks the link between expenditures and school quality.

<sup>20</sup>Rationing is an important feature of housing aid policies in the U.S. Downs (1991) states “*Congress has never provided enough funds for vouchers to make them entitlements, so they must be rationed to a fraction of all eligible households.*” Using a panel data set collected by the Housing Authority of the City of Pittsburgh, Epple et al. (2009) estimate that for each family that leaves public housing there are about four families that would like to move into the vacated unit.

<sup>21</sup>It is possible to build public housing in both neighborhoods, such as a full-ring on the map, and guarantee certain outcomes such as desegregation exogenously. In the equilibrium of such a model, we would obtain results similar to the baseline equilibrium, with weaker sorting and a smaller gap in the quality of education across jurisdictions. Typically, however, public housing is not built in the best neighborhoods, because of both costs and political reasons. Moreover, building public housing in a neighborhood is likely to cause some residents to relocate, altering the composition and the desirability of the neighborhood. Therefore, keeping one of the neighborhoods free from public housing gives us a chance to observe the interesting effects of such a program on its hosting neighborhood, resulting inbound-outbound migration, and the effects of this migration on neighborhood composition/characteristics as well as on local public good provision in both hosting and neighboring localities.

are determined by the same auction process as described above, with bidders excluding the  $N_P$  program participants and including the government. For any location in the public housing band, the government pays the maximum rent that households would have been willing to pay, had this land been available to them. This land is then divided into lots with equal size  $s_P$  and allocated to  $N_P$  participants with subsidized rents. A program participant household pays a fixed price  $\overline{R_P}$  as its program contribution. Denote the income tax rate by  $\theta$ . This policy reduces a public housing resident's problem to leisure-consumption choice only:

$$\begin{aligned} \max_{z,l} U_P(q_e, s_P, z, l) &= q_e^{\alpha_U} s_P^\eta z^\gamma l^\delta \quad s.t. & (9) \\ z + \overline{R_P} + (1 - \theta)W_U l &= Y_P(r) = 24(1 - \theta)W_U - (a + b(1 - \theta)W_U)r, \end{aligned}$$

whereas other households of type  $i \in \{S, U\}$  solve the original problem (2) in the presence of income taxes:

$$\begin{aligned} \max_{s,z,l,j \in \{e,w\}} U(q, s, z, l) &= q_j^{\alpha_i} s^\eta z^\gamma l^\delta \quad s.t. & (10) \\ z + (1 + \tau_j)R_j(r)s + (1 - \theta)W_i l &= Y_i(r) = 24(1 - \theta)W_i - (a + b(1 - \theta)W_i)r, \end{aligned}$$

taking rents and neighborhood quality-tax pairs as given.<sup>22</sup>

The cost of public housing is financed by income tax revenues and participant contributions, and equals the sum of rents and any payments to the local government. Public housing units are exempt from property taxes; a payment—known as *Payment in Lieu of Tax*, or *PILOT*—is made to compensate the local government in the east for some of the property tax revenue that it loses, in recognition of the services it provides to public housing residents.<sup>23</sup> The time constraint of a household gives the labor supply as all of the time except leisure and commute time:  $n = 24 - l - br$ . The solution to (10) gives the optimal leisure of a household residing at distance  $r$  as

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<sup>22</sup>An incentive problem arises when public housing residents do not pay property taxes, as their most preferred tax rate would be 100% when voting. Even though the equilibrium described in the next section is not affected by this, there may be some alternative specifications that may result in different types of equilibria depending on whether public housing residents are allowed to vote or not. For consistency, we assume that public housing residents vote for the same tax rate as other unskilled households.

<sup>23</sup>Given that public housing complexes are built/subsidized with public money (and that their rent incomes are lower and their expenses are higher than similar for-profit buildings) it may be counterproductive to tax them the same as other rental properties. A survey on tax exemption and tax assessment policies related to affordable housing complexes and PILOTs in the U.S. found that over 60% of states authorize localities to exempt affordable housing complexes from local property taxes (Lunney, 2009).

$l_i^*(r) = \delta \frac{Y_i(r)}{(1-\theta)W_i}$  for  $i \in \{S, U\}$ . For a household that lives in public housing, optimal leisure is  $l_P^*(r) = \frac{\delta}{\delta+\gamma} \frac{Y_P(r) - \overline{R}_P}{(1-\theta)W_U}$ . We can define a  $t_j^*(r)$  function that shows the type of occupant at distance  $r$  in jurisdiction  $j$  (whether skilled, unskilled, or in public housing). Let  $I[.]$  be an indicator function that takes the value 1 when the condition in brackets is satisfied and 0 otherwise. The public housing program budget constraint is:

$$\int_{r_P}^{r_P+w_P} R_e(r)L(r)dr + PILOT = \theta \sum_{j \in \{e, w\}} \left[ \int_0^{r_{fj}^*} \frac{L(r)}{s_j(r)} \left( \sum_{i \in \{S, U, P\}} I[t_j^*(r) = i] (24 - l_i^*(r) - br) W_i \right) dr \right] + N_P \overline{R}_P. \quad (11)$$

The LHS adds up the total rent cost of the public housing band and the payment to the local government, the choice of which we discuss below. Consider the first term on the RHS: the inside summation identifies which household type resides at a particular location and calculates their labor income in equilibrium. The integral calculates the total labor income using the household density at that location. The outside summation adds the labor income of two communities. A fraction  $\theta$  of this gives us the income tax revenues. The second term on the RHS is the sum of participant contributions. Equilibrium now also requires program budget constraint (11) holds.

### 3.2 Equilibrium

We first study a model in which the government provides public housing units on the land that is between 4 and 6 miles away from the CBD in the *east* neighborhood. This causes most skilled households to reside in the west, which then becomes the higher rent/tax/school quality neighborhood.<sup>24</sup> In the baseline equilibrium unskilled households lived in this band in the east, and the lot sizes ranged from 0.109 to 0.154 acres (at 4 and 6 miles away from the CBD) with an average lot size of 0.128 acres. The average monthly rent was \$3,520 per acre. A public housing unit measures 0.160 acres now. This is about 25% larger than the average unit within that band in the baseline equilibrium. This band accommodates about 15% of all unskilled households. We exogenously set the income tax rate at 0.57% to match the endogenous calibration target for the participant contribution for public housing of \$339 so that the utility increase from public housing is equivalent

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<sup>24</sup>The choice of neighborhood does not matter. Knowing where public housing units will locate, households adjust their bids and sort accordingly, and the neighborhood with public housing becomes the lower-tax/quality/rent neighborhood.

to that from a 10% income subsidy in equilibrium and (11) holds.

In practice, the basis for deciding upon an appropriate PILOT amount varies across municipalities. One of the recommendations in Lunney (2009) is to “*lower the property value on which the tax rate is assessed by basing the assessment on actual income and expenses rather than on potential market rate figures.*” A typical public housing complex would have a lower rent income and higher operating costs than a comparable private rental complex. Kenyon and Langley (2010) report that “*Some (municipalities) ask tax-exempt institutions to pay a specific proportion of the property taxes the institution would owe if taxable. (...) in many cases PILOTs are completely ad hoc and negotiated without any apparent basis.*” These and other resources suggest that property taxes collected over total rent *income* of the public housing program ( $PILOT = \tau_e N_P \overline{R_P}$ ) would be an appropriate choice. With our calibration, this corresponds to 42% of the property taxes that would be owed in equilibrium if the public housing band was fully taxable (for example, the city of Boston seeks 25%, Kenyon and Langley, 2010). Bowman, et al. (2009) estimate that the state-level foregone property tax revenues because of PILOTs range between 1.5% to 10% of property tax revenues, and the national average is around 5%. Our specification implies a 3.42% loss. Choosing a smaller PILOT would make public housing residents even less desirable neighbors, strengthening our arguments against public housing that will follow. (We comment on the implications of choice of a larger PILOT in the conclusion.)

We present the welfare effects of this policy in the first row of Table 4. Our welfare measure is the change in rents necessary to provide households with their utility level in baseline equilibrium. A negative number indicates that the household type is worse off, because rents need to be decreased to make those households indifferent to the baseline allocation. As a measure of the change in overall welfare, we calculate the change in rents necessary to keep aggregate utility at the baseline equilibrium level and present it in column 4 (AU). Public housing residents are better off, whereas all other households are worse off compared to baseline equilibrium; overall welfare (as measured by AU) is lower.

(Table 4 about here)

Table 5 presents some properties of the new equilibrium, and Figure 2 displays a map of the city. Sorting is stronger compared to that under baseline equilibrium: 75% of skilled households live in

the west (up from 69% in baseline) and constitute the majority there.

(Figure 2, Table 5 about here)

Rents are higher than those in the baseline equilibrium in both neighborhoods. The intuition is straightforward: the public housing policy removes a substantial amount of *would-be-inhabited* land from the private market. The non-recipients, who are either the skilled or the less fortunate unskilled workers, compete for the remaining land. However: 1. the average income of these households—which affects their willingness to pay for land—that compete in the private market is higher than before (because the households who left the private market are exclusively unskilled); and 2. the amount of accessible land is less than before (because public housing took out more land from the market than its residents would obtain on their own, if they had a choice). As a result, other things being equal, one would expect to see higher rents in equilibrium. However, other things are not equal, as there is also a fiscal burden issue: the property tax revenues from the public housing band are less than what would be collected from the same area without tax exemptions. This makes public housing residents undesirable neighbors, and the magnitude of this problem increases in the gap between the property tax revenues at market rents and the PILOT. For skilled households (with stronger preferences for education), the west is now a better option than before: the marginal effects of a tax dollar on per-student expenditures are higher. As a result, sorting is stronger, and the difference in quality of education is higher between two neighborhoods compared with the baseline equilibrium, as both the spending and the peer quality in the west (east) are higher (lower) than before. The computed equilibrium quantifies the described effects.

We also observe that the public housing residents decrease their labor supply by about 6.5% on average to 38.6 hours per week, down from 41.3 hours in baseline equilibrium. This is a result of two effects: 1. income taxes decrease the relative price of leisure; and 2. rent subsidy expands the budget set for leisure and consumption, but not for housing, resulting in a disproportionate increase in the consumption of the first two. Additional analysis confirms the presence of such a strong effect under several alternative formulations.<sup>25</sup> These findings are consistent with the empirical findings on housing aid policies and labor supply.<sup>26</sup>

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<sup>25</sup>Results from the additional analysis are available in the web appendix.

<sup>26</sup>Clearly, it is beyond the scope of this paper to review this literature. Among others, see Bingley and Walker (2001), Hulse and Randolph (2005), Olsen et al (2005), and the references therein.



## **The Size of a Public Housing Unit**

In the analysis presented above, a public housing unit is 25% larger compared with the average unit that was on the band now occupied by public housing units in the baseline equilibrium. Our results regarding welfare, however, are not driven by this particular choice of size. Note that if they were given a choice, public housing residents would now choose a lot that is slightly larger than their choices in the baseline equilibrium because of the income effect of the subsidy. A further increase in the size of public housing units, however, may hurt the public housing residents, as the net cost (subsidized rent) has to increase with the size for the program budget to hold: first, the lots are larger, and second, the overall level of rents is higher because the removal of accessible land from the private market is more excessive than before. Note that although the population density decreases in the east with larger public housing units, the property tax loss becomes a larger concern, as it applies to a larger area in the neighborhood. (We would see the opposite effects with smaller units.)

## **Within-district Location of Public Housing Units**

In the analysis presented above, the public housing units are provided 4 miles away from the CBD. At that location, a public housing unit is 25% larger than a typical lot at the same location in the baseline model (and is close in size to what the public housing residents would obtain on their own with the income effect of the subsidy). We utilize the spatial features of our model by comparing the welfare outcomes with public housing units provided at different locations. To isolate the effects of location, we now keep the sizes and the number of subsidized units—hence the total area occupied by public housing—the same while varying the distance of the public housing band to the CBD. If moved *closer to the CBD*, the (same-size) public housing unit becomes larger compared with a typical lot around that location. Although the commuting costs decrease for the public housing residents, their rents are higher closer to the CBD, and the subsidy covers a smaller share of this higher rent. Public housing also replaces a larger number of households that would occupy the same area without public housing. This has two implications: first, property-tax loss in the east increases (because the market rents are higher when closer to the CBD, but PILOT is proportional to subsidized rents); and second, the number of households competing for land in the private market increases, as public housing replaces smaller lots (more households) when closer to the CBD. (We

would observe the opposite effects as the public housing units are moved *away from the CBD*.)

## **Welfare Comparisons**

How welfare is affected by changes in the size and location of public housing units depends on the relative magnitudes of the effects described above, the associated implications on the sorting of households in response, and the resulting levels and differentials of rents, tax rates, and school qualities in equilibrium.

Rows 2 to 6 of Table 4 summarize the welfare changes associated with the size of a public housing unit ranging from 25% smaller (-25%) to 75% larger (+75%) compared with the average unit on the area occupied by public housing band in the baseline equilibrium. The last column shows that overall welfare decreases as we move away from our initial choice of 25% larger in either direction. Varying the lot size between -25% and +25% seems to have only a small effect on public housing residents' welfare, but increases beyond +25% make the public housing residents significantly worse off as: 1. they would be forced to obtain lots that are too large compared to their optimal choice under this income effect; 2. holding total government subsidies and the number of public housing residents constant, the share of subsidies in this forced lot-size consumption would be too small.

Rows 7 to 10 of Table 4 summarize the welfare effects when public housing units are provided at 0, 2, and 6 miles away from the CBD. We find that the welfare of public housing residents increases as public housing units are provided further away from the CBD, suggesting that the decrease in the net cost of the land outweighs the increase in commuting costs.

A deviation from our initial choice of +25% or from our initial choice of four miles in either direction hurts households outside of public housing, because of the changes in the magnitude of the fiscal burden problem (arising from changes in the amount of property-tax loss and population density) and the implied general equilibrium effects described above.

In each of the cases we have studied, the total utility gain of public housing residents is smaller than the total utility loss of the rest of the population, so the change in overall welfare (as measured by AU) is negative.

## 4 Housing Vouchers

### 4.1 Voucher Model

Instead of providing physical units with subsidized rents, the government can simply redistribute the income tax revenues to low-income households in the form of housing vouchers. In this scenario, each of the  $N_P$  program participants obtains a voucher towards rent with the amount  $\nu_P$ . For the government budget to balance, income tax revenues must equal the total amount of these vouchers:

$$N_P \nu_P = \theta \sum_{j \in \{e, w\}} \left[ \int_0^{r_{fj}^*} \frac{L(r)}{s_j(r)} \left( \sum_{i \in \{S, U, P\}} I[t_j^*(r) = i] (24 - l_i^*(r) - br) W_i \right) dr \right] \quad (12)$$

A household's problem is the same as the one in (10), but a voucher recipient's housing expenditures are those exceeding  $\nu_P$  only. This is equivalent to creating a third household type with the same preferences as unskilled households, but with the kinked budget constraint

$$z + \max\{0, (1 + \tau_j)R_j(r)s - \nu_P\} + W_U l = Y_P(r) = 24(1 - \theta)W_U - (a + b(1 - \theta)W_U)r. \quad (13)$$

Because household utility increases in lot size, no household will spend less on housing than the voucher amount. Whether the household will spend more depends on the voucher amount and some model parameters. The leisure choice of a voucher recipient is  $l_P^*(r) = \delta \frac{Y_P(r) + \nu_P}{(1 - \theta)W_U}$  if the household spends greater than the voucher amount, and  $l_P^*(r) = \frac{\delta}{\gamma + \delta} \frac{Y_P(r)}{(1 - \theta)W_U}$  otherwise. The land is allocated according to the competitive auction mechanism described in Section 2. An additional equilibrium condition is that the program budget (12) holds.

### 4.2 Equilibrium

For comparison, we keep the (number of) recipients and the tax rate the same as the public housing policy; this implies a housing voucher worth \$227 per month.<sup>27</sup> Figure 3 provides a map of the city

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<sup>27</sup>Rationing of housing vouchers to a fraction of eligible households is consistent with practices in the U.S. (Downs, 1991). The lottery specification has the additional benefit of keeping the program size the same as under public housing. An alternative implementation is to provide vouchers to every household below a certain income level. With just two income levels in the model, this means providing vouchers to *all* unskilled households, thus increasing the number of program participants from 9% to 60% of the population. Holding the income tax rate constant decreases the voucher amount to \$34 per household. The findings of this exercise (available in the web appendix) are quite similar to those in this section, so we refrain from repeating those details here.

and Table 6 summarizes some properties of equilibrium. Vouchers shift land demand, increasing rents in both neighborhoods. The equilibrium rents, however, are lower than those under public housing policy, because there is no excessive removal of accessible land from the market as there is under public housing.

Sorting of households is stronger than that under the baseline equilibrium, close to the public housing equilibrium levels: 74% of skilled households live in the west (as opposed to 69% in baseline) and constitute a 56% majority there. The major cause of this is the increase in land demand in the east: all voucher recipients reside in the east (where unskilled households are a majority) because of their weaker preferences for education. The quality of education in the west is slightly higher than both the baseline and public housing models, because of both sorting and the higher expenditures. A policy maker concerned with education of the poor may prefer vouchers over public housing: the quality in the east, the poorer neighborhood, is higher than that under the public housing model.

The fiscal burden problem we described in the public housing units section is of no concern here, as voucher recipients still pay property taxes at the market rent level. However, households without vouchers are hurt by both the higher rents and income taxes. The equilibrium utility levels of non-recipients (both skilled and unskilled households) are higher than those under public housing, but lower than baseline equilibrium levels. The (average) utility level of voucher recipients falls below that of public housing recipients. We present the change in household welfare according to their types in the last row of Table 4. The welfare loss to non-recipients is less than that under public housing policy, and the change in total welfare (as measured by AU) is positive.

(Figure 3, Table 6 about here)

Income taxes hurt work incentives by decreasing the relative price of leisure. The income effect of vouchers also allows recipients to increase their leisure consumption. But voucher recipients work more than public housing recipients (42 vs. 38.6 hours/week). This should not be puzzling: the public housing program also creates an income effect, but by restricting households' location-lot size choices and their housing expenditures, it also causes a disproportionate increase in consumption of the other two commodities available for purchase, the composite good and leisure. Housing voucher recipients are not restricted in terms of their residential choices, they reside further away from unskilled households, obtaining larger lots and increasing their leisure proportionally. Even

though they work less than the typical unskilled resident living at the same distance in baseline equilibrium—a result of the income effect caused by the voucher—they work longer hours than they would if they received public housing instead.

### 4.3 A Note on Desegregation

We have discussed several reasons a housing voucher program might be preferred over a public housing program with the same budget. Another advantage of housing vouchers over public housing units is that the vouchers do not impose restrictions on households' location choices. Hence, a policy maker with concern over the patterns of household sorting across neighborhoods may be particularly interested in how housing vouchers can influence these patterns.

The equilibrium neighborhood compositions under the two programs are, however, almost identical in the above analysis. The west community provides a higher quality of education at the cost of higher taxes on land consumption. The skilled households have stronger preferences (and willingness to pay) for education compared with voucher recipients, so they outbid the voucher recipients on the west land away from the CBD. However, unskilled households without vouchers value proximity to the CBD and outbid voucher recipients on the west land close to the CBD. The unskilled households demand smaller lots than voucher recipients and therefore are not as affected by larger taxes as much as voucher recipients who demand relatively larger lots. As a result, voucher recipients are not observed residing in the west. We have also mentioned that increasing the number of voucher recipients without changing the income tax rates resulted in a similar equilibrium.

We conducted an additional analysis with alternative parameterizations that may allow us to observe voucher recipients in the west. Increasing tax rates while keeping the number of recipients the same did not cause enough increase in the bids of voucher recipients in the west. We have observed, however, that it is possible to induce voucher recipients residing in the west with both a larger program size and a larger income tax rate, increasing the voucher recipients' land bids in both neighborhoods and allowing them to outbid some households in the west. We present the spatial distribution of households in the equilibrium of one such model in Figure 4, in which 25% of unskilled households receive housing vouchers for \$537 per month. This program is financed by an income tax rate of 1.5%. In the equilibrium of this model, about one fifth of the voucher recipients live in the west, occupying a narrow semi-ring between the unskilled households' semicircle around

the CBD and the skilled households' semi-ring. Other aspects of the equilibria remain qualitatively the same for our purposes, so we refrain from a detailed discussion here.<sup>28</sup>

(Figure 4 about here)

## 5 Concluding Remarks

Green and Malpezzi (2003) survey a vast literature on the housing market and housing policies in the U.S., and argue that *“Most economists like vouchers because they are generally more efficient than other programs. (...) But in the United States, political support is generally stronger for programs tied more closely to the consumption of specific goods (housing, food, and medical care) than for income support.”* This paper attempts to contribute to this related debate. In particular, this paper explicitly highlights the importance of the location of public housing to equilibrium outcomes such as rents, neighborhood compositions, educational opportunities, labor supply decisions, and social welfare. We explain the channels through which such location effects work. Using a rich general equilibrium model that combines land use theory with a Tiebout framework, we provide a comparison of public housing and housing voucher policies, and conclude that vouchers not only cause less distortion for social welfare compared with public housing, but they also improve overall welfare. The difference between the two policies is mainly driven by the implications of: 1. property-tax losses of the neighborhood under public housing; and 2. the forced location and lot size choices under public housing. Therefore, even if the fiscal burden problem of public housing is solved (e.g., with higher PILOTs), vouchers may still be preferred because of the freedom of choice in location and lot sizes. The results of our analysis are consistent with the findings from previous studies that compare in-kind versus in-cash welfare programs, verifying their validity in a richer framework with accessibility, local public goods, and peer group externalities.

We also find that public housing policies tends to discourage the labor supply, especially for the unskilled workers who reside in public housing, as some empirical literature has suggested. Voucher recipients work the same hours as they did before the vouchers. This seems to strengthen the in-cash rather than in-kind arguments even further. Additional analysis (the results of which are available in the web appendix) reveals that our findings are robust to changes in the levels for utility

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<sup>28</sup>Detailed results are available from the authors.

parameters (which determine most desired property tax rates) as well as to the incorporation of a housing industry.<sup>29</sup>

In many countries, central governments contribute to the financing of local public goods and services in differing extents. If the share of local financing decreases in our model, the sorting would be weaker, as it becomes less effective on the local provision level. An implication is that unskilled households constitute the majority in both neighborhoods above a certain share of central financing. In that case, the voting outcome would be the lower tax rate in both neighborhoods, and the quality difference between the neighborhoods would be less. Another implication is that desegregation could be obtained more easily with vouchers than as presented in this paper. Future work will investigate these issues in detail.

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<sup>29</sup>Our framework is based on Alonso's model (1964), which assumes that each household manages the construction of its house by itself. An alternative approach is Muth's model (1969) in which households derive utility from consuming housing space  $H$ , produced by competitive firms using  $\Lambda$  units of land and  $K$  units of capital with the production function  $H = AK^a\Lambda^{1-a}$  for  $a \in (0, 1)$  and  $A > 0$ . Then, as land gets more expensive closer to the CBD, the share of capital to land in the construction of housing space increases, i.e., taller/multi-unit buildings are observed. We incorporate this housing industry into our framework and repeat the analysis presented in this paper. This change in the formulation does not alter our qualitative findings regarding welfare comparisons, while adding considerable complexity to the analysis. Detailed results are available from the authors.

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Table 1: Calibration Parameters

$\delta$	0.762
$\gamma$	0.19
$\eta$	0.048
$W_U$	14
$W_S$	27
$\alpha_L$	0.014
$\alpha_H$	0.021
$a$	1.1
$b$	0.13
$c_0$	0.1
$c_1$	1.3677
$c_2$	0.0574
$N_S/(N_S + N_U)$	0.40

Table 2: Baseline Equilibrium

	West	East
Average Monthly Rent (per Acre)	\$2729	\$2194
Average Rent in S area	\$2204	\$1433
Average Rent in U area	\$5416	\$3368
Average Lot Size S (Acres)	0.791	1.209
Average Lot Size U	0.173	0.275
Property Tax Rate	1.97%	1.04%
School Quality	14	7
<i>Distribution of Households Across Neighborhoods</i>		
S	69%	31%
U	42%	58%
<i>Neighborhood Population Breakdown</i>		
S	53%	26%
U	47%	74%

Table 3: Housing Aid Policies

	Public Housing Units	Housing Vouchers
Location & Lot size Choice	By policy-maker	By voucher recipients
Rent Paid	Fixed, below market rent	Reduced by voucher amount
Property Taxes	Households do not pay property taxes, the program makes a <i>payment in lieu of taxes (PILOT)</i> to the local government	Same as without voucher

Table 4: Welfare

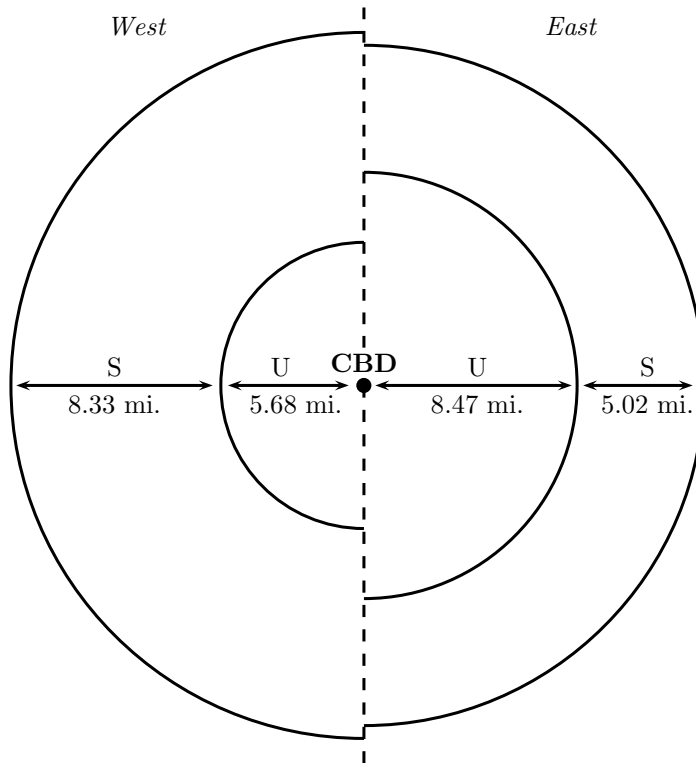
		S	U	P	AU
Public Housing		<b>-5.64</b>	<b>-5.39</b>	<b>85.67</b>	<b>-0.35</b>
	-25%	-9.24	-5.71	81.16	<b>-2.41</b>
	0	-9.28	-6.45	86.63	<b>-2.57</b>
Size of Public Housing	<b>+25%</b>	<b>-5.64</b>	<b>-5.39</b>	<b>85.67</b>	<b>-0.35</b>
	+50%	-9.55	-8.04	63.51	<b>-4.49</b>
	+75%	-9.72	-8.81	44.80	<b>-5.87</b>
	0-4.47 miles	-9.83	-9.20	33.53	<b>-6.71</b>
	2-4.90 miles	-9.61	-8.41	52.72	<b>-5.23</b>
Location of Public Housing	<b>4-6.00 miles</b>	<b>-5.64</b>	<b>-5.39</b>	<b>85.67</b>	<b>-0.35</b>
	6-7.48 miles	-9.29	-6.20	88.37	<b>-2.37</b>
Vouchers		-3.29	-3.31	59.01	<b>0.54</b>

Table 5: Public Housing

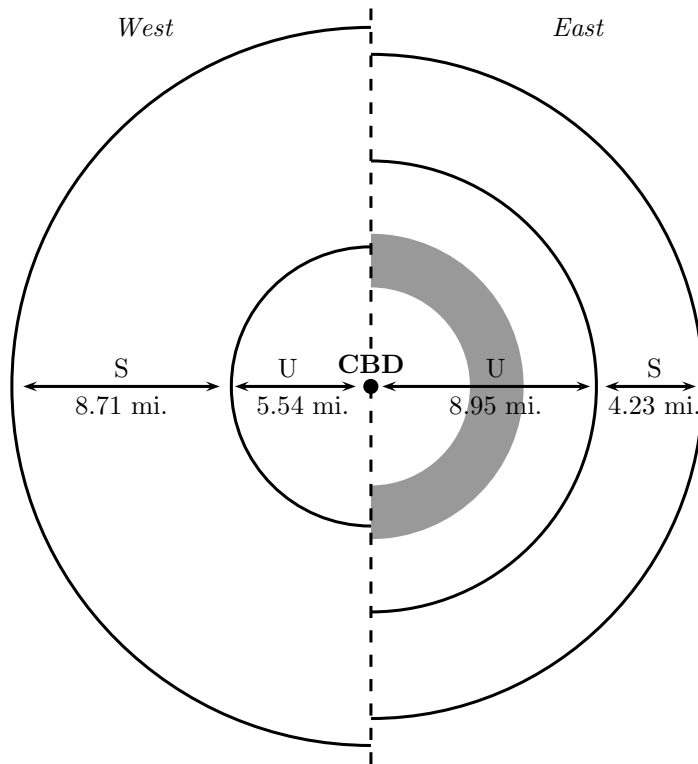
Income Tax Rate: 0.57%		
	West	East
Average Monthly Rent (per Acre)	\$2764	\$1955
Average Lot Size S (Acres)	0.770	1.261
Average Lot Size U	0.165	0.301
Average Lot Size P	-	0.320
Property Tax Rate	1.97%	1.04%
School Quality	14.15	6.02
<i>Distribution of Households Across Neighborhoods</i>		
S	75%	25%
U	41%	59%
		(14% P)
<i>Neighborhood Population Breakdown</i>		
S	55%	22%
U	45%	78%
		(19% P)

Table 6: Housing Vouchers

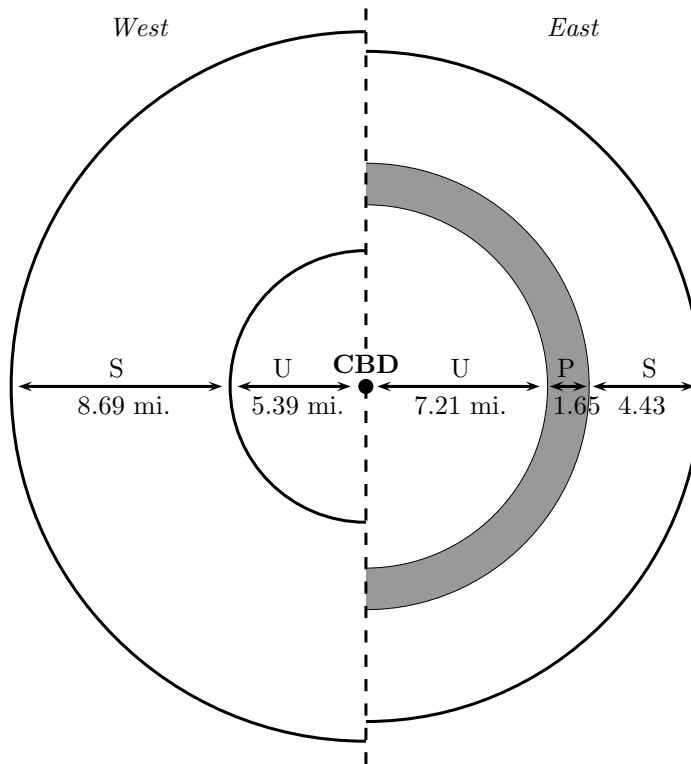
Income Tax Rate: 0.57%		
	West	East
Average Monthly Rent (per Acre)	\$2741	\$2186
Average Lot Size S (Acres)	0.771	1.246
Average Lot Size U	0.166	0.247
Average Lot Size V	-	0.423
Property Tax Rate	1.97%	1.04%
School Quality	14.25	6.56
<i>Distribution of Households Across Neighborhoods</i>		
S	74%	26%
U	39%	61%
		(14% V)
<i>Neighborhood Population Breakdown</i>		
S	56%	22%
U	44%	78%
		(18% V)



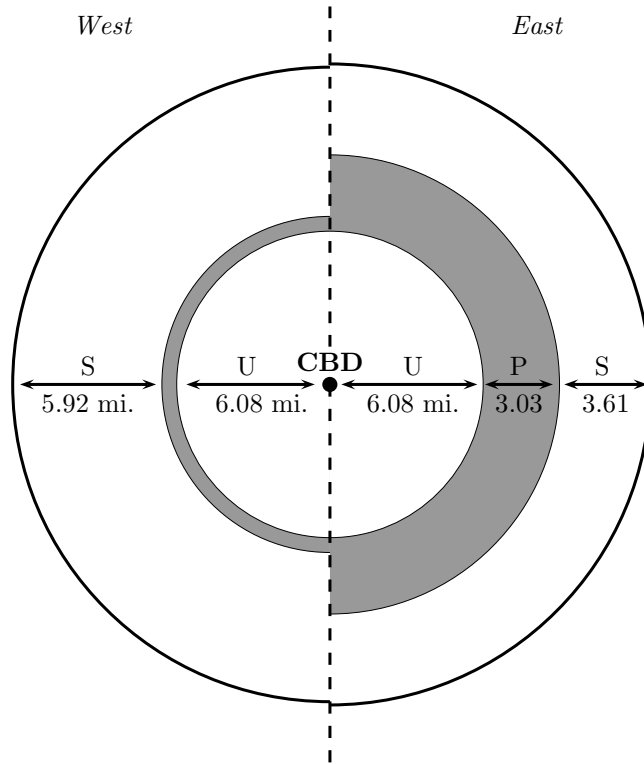
**Figure 1:** Spatial Distribution of Households in the Baseline Equilibrium. The outer semi-rings (marked by S) are the areas in which *skilled* households reside. The inner semicircles (marked by U) are the areas in which *unskilled* households reside.



**Figure 2:** Spatial Distribution of Households in the Public Housing Equilibrium. The outer semi-rings (marked by S) are the areas in which *skilled* households reside. The inner semicircles (marked by U) are the areas in which *unskilled* households reside. The shaded semi-ring is the public housing band between 4 and 6 miles from CBD.



**Figure 3:** Spatial Distribution of Households in the Vouchers Equilibrium. The outer semi-rings (marked by S) are the areas in which *skilled* households reside. The inner semicircles (marked by U) are the areas in which *unskilled* households reside. The shaded semi-ring is the area in which voucher recipients reside.



**Figure 4:** Map of an equilibrium with voucher recipients residing in both neighborhoods.



# Appendix to “Public Housing Units vs. Housing Vouchers: Accessibility, Local Public Goods, and Welfare”

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29.06.2012

NOT FOR PUBLICATION,  
ACCESSIBLE AT:

[http://a.permanent.url.will.be.assigned/web\\_appendix.pdf](http://a.permanent.url.will.be.assigned/web_appendix.pdf)

## A.1 Robustness and Extensions

### A.1.1 Utility Parameters and Tax Rates

The most desired property tax rates for the two household types are chosen to be slightly higher/lower than the U.S. average of 1.40%, and the utility parameters  $\alpha_H$  and  $\alpha_L$  are calibrated according to equation (8) in the paper. We conduct an additional analysis with alternative specifications for the levels of  $\alpha_i$ 's, which affect desired tax rates by about  $\pm 20\%$  or more. Table A.1 reports representative results from these alternative specifications along with the original specification given in the middle row of each panel. An inspection of the columns reporting the changes in overall welfare (AU) under two policies shows our findings are robust to such changes in property tax rates.

Table A.1: Welfare II - Property Tax Rates

$(\alpha_H, \alpha_L; \tau_H^*, \tau_L^*)$	Public Housing				Vouchers			
	S	U	P	AU	S	U	P	AU
(.018, .014; 1.52, 1.04)	-4.85	-5.18	81.74	<b>-0.34</b>	-2.93	-2.72	59.42	<b>1.03</b>
(.021, .014; 1.97, 1.04)	-5.64	-5.39	85.67	<b>-0.35</b>	-3.29	-3.31	59.01	<b>0.54</b>
(.024, .014; 2.54, 1.04)	-5.26	-5.20	82.04	<b>-0.25</b>	-3.01	-2.72	59.42	<b>0.97</b>
(.021, .012; 1.97, 0.84)	-4.89	-4.90	76.36	<b>-0.17</b>	-2.99	-2.73	59.50	<b>0.99</b>
(.021, .014; 1.97, 1.04)	-5.64	-5.39	85.67	<b>-0.35</b>	-3.29	-3.31	59.01	<b>0.54</b>
(.021, .016; 1.97, 1.26)	-5.20	-5.49	88.16	<b>-0.08</b>	-3.00	-2.78	59.23	<b>0.96</b>

### A.1.2 Labor Supply

One of our findings is that households decrease their labor supply when provided with public housing units. In this section we provide representative results from additional analysis in which we vary: 1. The sizes of public housing units; 2. The location of the public housing units; 3. Utility (and tax rate) parameters. Table A.2 summarizes average working hours per week, comparing the labor supply of public housing residents and housing voucher recipients to labor supply of the households that live on the public housing location in benchmark equilibrium.<sup>1</sup> One row in each panel (+25%, 2-4.90, and (.014, .021), in bold) represents the original model for reference. Public housing recipients work less than voucher recipients (The only exception arises when public housing is built at the CBD and a public housing resident’s travel cost becomes zero).

Table A.2: Average Weekly Labor Supply of Housing Aid Participants (hours/week)

	Benchmark	Public Housing	Housing Vouchers
<b>Size</b>			
-25%	41.2	35.6	42.0
0	41.2	37.3	42.0
<b>+25%</b>	<b>41.3</b>	<b>38.6</b>	<b>42.0</b>
+50%	41.3	40.6	42.0
+75%	41.3	42.3	42.0
<b>Location</b>			
0-4.47	40.7	42.5	42.0
2-4.90	40.9	41.1	42.0
<b>4-6.00</b>	<b>41.3</b>	<b>38.6</b>	<b>42.0</b>
6-7.48	41.7	37.2	42.0
<b>(<math>\alpha_L, \alpha_H</math>)</b>			
(.012,.021)	41.3	39.1	41.7
(.014,.018)	41.3	39.0	41.7
<b>(.014,.021)</b>	<b>41.3</b>	<b>38.6</b>	<b>42.0</b>
(.014,.024)	41.3	38.9	41.7
(.016,.021)	41.3	38.8	41.7

<sup>1</sup>The band occupied by public housing hosted unskilled households in benchmark equilibrium; see Figure 1.

### A.1.3 Vouchers for All Unskilled Households

Rationing of housing vouchers to a fraction of eligible households is consistent with the practices in the U.S. (Downs, 1991). The lottery specification in the paper has the additional benefit of keeping the program size the same as under public housing. An alternative implementation is to provide vouchers to every household below a certain income level. With just two income levels in the model, this means providing vouchers to all unskilled households, increasing the number of program participants from 9% to 60% of the population. Holding income tax rate constant decreases the voucher amount to \$34 per household. The findings of this exercise given in Table A.3 are quite similar to those of the lottery specification in the paper.

Table A.3: Housing Vouchers II  
(Every Unskilled Household Gets Vouchers)

Income Tax Rate: 0.57%		
	West	East
Average Monthly Rent (per Acre)	\$2727	\$2192
Property Tax Rate	1.97%	1.04%
School Quality	13.96	6.93
<i>Distribution of Households Across Neighborhoods</i>		
S	70%	30%
U	41%	59%
<i>Neighborhood Population Breakdown</i>		
S	53%	26%
U	47%	74%

### A.1.4 Housing Industry

Our framework is based on Alonso’s model (1964) which assumes that each household manages the construction of its house by itself. An alternative approach is Muth’s model (1969) in which households derive utility from consuming housing space  $H$ , produced by competitive firms using  $\Lambda$  units of land and  $K$  units of capital with the production function:

$$H = AK^a \Lambda^{1-a} \tag{1}$$

for  $a \in (0, 1)$  and  $A > 0$ . Then as land gets more expensive closer to the CBD, the share of capital to land in the construction of housing space increases, i.e., taller/multi-unit buildings are observed.

We incorporate this housing industry to our framework, and repeat the analysis presented so far in this paper. This change in the formulation does not alter our qualitative findings regarding welfare comparisons, while adding considerable complexity to the analysis. We solve the equilibrium without government intervention (benchmark), with public housing, and with housing vouchers. We calibrate the parameters of the housing production function so that *the ratio of housing space to land* is about 16 near the CBD and 1 near fringe in the benchmark model.<sup>2</sup>

<sup>2</sup>These can be interpreted as the number of floors of buildings, but one should also keep in mind that it is a

We initially locate the public housing units at 4 miles away from the CBD, keeping the ratio of housing space to land at the benchmark level at this distance (about 4). We set the public housing unit size same as the average unit in the same area in benchmark. We have also solved alternative models with: 1. The public housing unit size 25% smaller and 25% larger than the average unit in the same area in benchmark; 2. The ratio of housing space to land is 3 and 5; 3. The location of public housing units at 3 and 5 miles away from the CBD. We summarize results from this analysis in Table 7 below. The middle rows in top three panels (titled as “same,” “4,” “4 miles”) represent the main public housing model, and the very bottom row in the table presents the welfare implications of the voucher model with housing industry.

An inspection of Table A.4 reveals that housing vouchers are preferable to public housing units in this extended model too. We also solve this extended model with alternative utility-tax parameters. Table A.5 presents the counterpart to Table A.1 with housing industry.

In the first and last rows of the table, where the utility parameters of the two types get close to each other, the overall welfare under public housing is slightly greater than that under vouchers. However, a closer inspection of the Table reveals that households that do not participate in the program still prefer housing vouchers to public housing units. The higher overall welfare results from the large difference from program participants’ utility levels.

Table A.4: Models with Housing Industry I - Size, Share of Capital, and Location

	S	U	P	AU
<b>Size</b>				
-25%	-5.23	-4.67	70.39	-0.47
same	-5.13	-4.64	81.96	0.11
+25%	-5.06	-4.63	81.93	0.15
<b>Housing Space/Land</b>				
3	-6.01	-5.47	83.09	-0.67
4	-5.13	-4.64	81.96	0.11
5	-4.61	-4.13	77.99	0.44
<b>Location</b>				
3 miles	-5.08	-4.78	86.45	0.25
4 miles	-5.13	-4.64	81.96	0.11
5 miles	-5.04	-4.46	68.04	-0.39
<b>Vouchers</b>				
	-2.84	-3.34	56.22	0.60

continuous variable. The implied parameters for the production function are  $a=0.70$  and  $A=0.00149$ . Given the change in tax base, we also recalibrate the parameters of the education production function to facilitate comparisons with the models in Sections 3 and 4 of the paper. Further details are available from the authors.

Table A.5: Models with Housing Industry II - Utility Parameters

$(\alpha_H, \alpha_L; \tau_H, \tau_L)$	Public Housing				Vouchers			
	S	U	P	AU	S	U	P	AU
(.018, .014; 1.52, 1.04)	-4.60	-3.29	79.97	<b>0.97</b>	-3.03	-2.94	56.16	<b>0.73</b>
(.021, .014; 1.97, 1.04)	-5.13	-4.64	81.96	<b>0.11</b>	-2.84	-3.34	56.22	<b>0.60</b>
(.024, .014; 2.54, 1.04)	-8.79	-17.81	46.27	<b>-9.94</b>	-3.12	-2.95	56.01	<b>0.65</b>
(.021, .012; 1.97, 0.84)	-3.79	-3.66	67.26	<b>0.54</b>	-2.49	-3.16	56.69	<b>0.87</b>
(.021, .014; 1.97, 1.04)	-5.13	-4.64	81.96	<b>0.11</b>	-2.84	-3.34	56.22	<b>0.60</b>
(.021, .016; 1.97, 1.26)	-5.17	-4.02	89.61	<b>0.75</b>	-2.96	-3.00	56.16	<b>0.72</b>

## A.2 The Algorithm used for Solving the Benchmark Equilibrium

The following algorithm is based on the sequence of events described in the paper. Also see Figure A1 below. The school district  $j$  could be East or West. If we know the bid-rent curve of a household type in one district, we can draw the bid-rent curve for the same type in the other district as well, since identical households obtain the same level of utility regardless of where they live.

1. Define model parameters and discretize the space.
2. Equation 3 (and some algebra) suggests that bid-rent curve  $\Psi_1$  of a household is steeper than that  $\Psi_2$  of another if and only if for all  $r$  at which  $\Psi_1(r, \cdot) = \Psi_2(r, \cdot)$ ,  $\frac{\eta_1 + \gamma + \delta}{\eta_1} \frac{\eta_2}{\eta_2 + \gamma + \delta} \frac{(a+bW_1)Y_2(r)}{(a+bW_2)Y_1(r)} > 1$ . Moreover, the household with a steeper bid-rent curve locates closer to the employment center. Check this ratio and determine the spatial order of households. In our paper, unskilled households have a steeper bid-rent curve and locate closer to the employment center.
3. Randomize the initial tax rate/quality of education package in each district.
4. Initialize fringe distance,  $r_{fj}^*$ . Find  $u_S^*$  by using equation 3 in the paper and the fact that at  $r_{fj}^*$ , the rent is the agricultural rent  $R_a$ . Use this information to calculate bid-rent and lot sizes for skilled households in both districts.
5. Initialize  $r_{SUj}^*$ . Calculate the rent at  $r_{SUj}^*$  by using the bid-rent function of skilled households. Then find  $u_U^*$  by using equation 3. Calculate bid-rent and lot sizes for unskilled households in both districts.
6. Determine the land area that skilled households outbid unskilled households. By using lot sizes, calculate the population of skilled households.
7. If it is larger (smaller) than the target value, go back to 5 and increase (decrease)  $r_{SUj}^*$ . If it equal to the target value, move on to the next step.
8. By using lot sizes, calculate the population of unskilled households. If it is larger (smaller) than the target value, go back to 4 and decrease (increase)  $r_{fj}^*$ . If it equal to the target value, move on to the next step.
9. Find majority winner property tax rates, tax bases, and quality of education in each school district.
10. Go back to step 3 and update tax rate/education package. Repeat until the current period tax rate/education packages are equal to those in the last period.

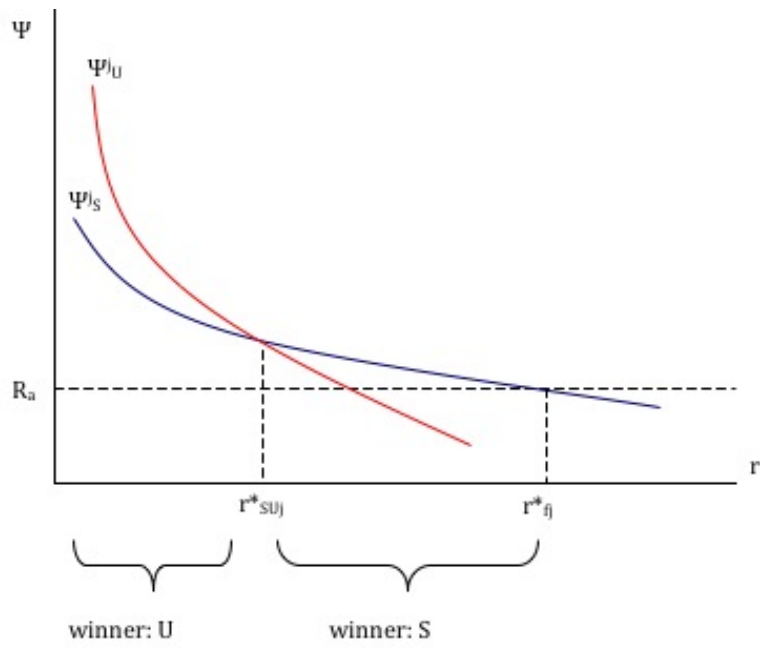


Figure A1