Rochester Center for Economic Research

Labor Market Uncertainty and Portfolio Choice Puzzles

Yongsung Chang Jay H. Hong Marios Karabarbounis

Working Paper No. 582
June 2014

University of Rochester

# Labor-Market Uncertainty and Portfolio Choice Puzzles*

**Yongsung Chang**
*University of Rochester*
*Yonsei University*

**Jay H. Hong**
*Seoul National University*

**Marios Karabarbounis**
*Federal Reserve Bank of Richmond*

June 16, 2014

**Abstract**

The standard theory of household-portfolio choice is hard to reconcile with the following facts: (i) Households hold a small amount of equity despite the higher average rate of return. (ii) The share of risky assets increases with the age of the household. (iii) The share of risky assets is disproportionately larger for richer households. We develop a life-cycle model with age-dependent unemployment risk and gradual learning about the income profile that can address all three puzzles. Young workers, on average asset poor, face larger labor-market uncertainty because of high unemployment risk and imperfect knowledge about their earnings ability. This labor-market uncertainty prevents them from taking too much risk in the financial market. As the labor-market uncertainty is gradually resolved over time, workers can take more financial risks.

**Keywords:** Portfolio Choice, Labor-Market Uncertainty, Learning.

*Emails: *yongsung.chang@rochester.edu*, *jayhong@snu.ac.kr* and *marios.karabarbounis@rich.frb.org*. For helpful suggestions we would like to thank seminar participants at ASU, the University of Virginia, Stony Brook, Queens, Rochester, the Federal Reserve Bank of Philadelphia, the Bank of Greece, the University of Piraeus, SED (Seoul), and the Federal Reserve Bank of Richmond. Any opinions expressed are those of the authors and do not necessarily reflect those of the Federal Reserve Bank of Richmond or the Federal Reserve System.

# 1 Introduction

Three stylized facts have been documented by Guiso, Haliassos, and Jappelli (2002), among others: (i) Households hold a small amount of equity despite the higher average rate of returns (equity premium puzzle). (ii) The share of risky assets increases with the age of the household. (iii) The share of risky assets is disproportionately larger for richer households. The standard life-cycle model of household portfolio choice is hard to reconcile with these facts. The standard theory predicts that the risky share is negatively correlated with the age and wealth of households, as a household with a constant relative-risk aversion should invest aggressively when young and gradually move toward a safer portfolio.

We show that the age-dependent labor-market uncertainty helps us to address all three portfolio choice puzzles. Young workers face much greater risk of being unemployed. According to the 2013 Current Population Survey, the average unemployment rate of male workers ages 20-24 is as high as 14%, whereas that of workers ages 50-54 is 5.8%. According to Topel and Ward (1992), the incidence of job turnover is highly concentrated among young workers. In the first 10 years after entering the labor market, a typical worker holds 7 jobs (about two-thirds of his career total). Moreover, young workers are uncertain about the shape of their income profiles and this uncertainty is resolved over time (Guvenen (2007) and Guvenen and Smith (2014)).

As a result, both *actual* and *perceived* labor-market uncertainty is much larger for young workers, who on average have little wealth or are in debt. Since the labor-market outcome is largely uninsurable, young investors, despite a longer investment horizon, would like to avoid exposing themselves to too much risk in the financial market. By contrast, older investors, who face less uncertainty in the labor market, can afford taking more risk in their financial investments.

We quantitatively evaluate this link between labor-market uncertainty and financial risk in an otherwise standard life-cycle model of portfolio choice (e.g., Cocco, Gomes, and Maenhout (2005)). Our model features (i) age-dependent unemployment risk, (ii) gradual learning about the income profile, (iii) noise in the Bayesian updating rule to achieve a plausible rate of learning, (iv) a portfolio choice between a risk-free bond and a risky stock, and (v) a limited ability for a household to insure against labor-market income risk.

Our model is calibrated to match the *age profiles* of unemployment risk, earnings volatility, and consumption dispersion in the data. The age-dependent unemployment risk is based on the estimates by Choi, Janiak, and Villena-Roldan (2014). The average productivity over the life-cycle is taken from Hansen (1993), which is estimated from the CPS. The stochastic process and cross-sectional dispersion of the life-cycle income profile are similar to those in

Guvenen and Smith (2014).

Our model helps us to address all three portfolio choice puzzles mentioned above. First, our model matches the average risky share of 0.45, which we estimated based on the Survey of Consumer Finances.[1] This low risky share is achieved under the relative risk aversion of 5, much lower than the typical value required in standard models. Second, the age profile of risky share closely tracks that in the data. According to the SCF, the average risky share increases by 0.92 percentage point annually over ages 21-65. In our model, it increases by 0.36 percentage point. Third, the risky share and wealth are mildly positively correlated across households in the SCF, 0.107. This correlation is mildly negative in our model: -0.147. While our model fails to generate a positive correlation, it partially reconciles the large gap between the data and the standard life-cycle model (our model without labor-market uncertainty), which predicts a strongly negative correlation, -0.638.

In our model, workers update their priors about their earnings ability in a Bayesian fashion. However, it is often found in various natural experiments that subjects are more conservative in changing their priors than the standard Bayesian rule implies (e.g., Edwards (1968)). In fact, under the standard Bayesian updating rule, while the model generates significant income risk over longer horizons, the uncertainty over shorter horizons (e.g., one to five years ahead) is resolved extremely fast. For example, within two years of entering the labor market, about two-thirds of next period's uncertainty (measured by the one-period forecast-error variance) is resolved. We view this rate of learning to be unrealistically fast, especially if one considers the intense career-search behavior of young workers. Typically, young workers shop around for jobs before settling into a long-term career. Topel and Ward (1992) document that the average number of jobs held by workers at the tenth year after entering the labor market is approximately 7. Kambourov and Manovskii (2008) report that the average probability that a young worker will switch occupations is approximately 25% on an annual basis. Frequent job switching may diminish the informational context from previous labor-market experience, especially if it takes place across completely different industries or occupations. We introduce an additional noise in the Bayesian updating formula as a shortcut to reflect the reset of priors due to moves between industries or occupations. Since it is difficult to obtain direct estimates of how fast priors converge to the true earnings ability, we rely on the evidence on occupational mobility such as Kambourov and Manovskii (2008) and Topel and Ward (1992). We also verify that our parameterization matches the increasing age profile of consumption dispersion in the data very well.

Our benchmark model introduces three new features into an otherwise standard life-cycle model of portfolio choice (e.g., Cocco, Gomes, and Maenhout (2005)): (i) age-dependent

[1] The detailed definition of risky share is explained in Section 2.

unemployment risk, (ii) imperfect information about the earnings profile, and (iii) noise in the updating rule. In matching the average risky share in the data, the imperfect information about the earnings profile contributes the most (64%), followed by the noise in the updating rule (24%) and the age-dependent unemployment risk (12%).

Our theory predicts that workers in an industry (or occupation) with highly volatile earnings should take less risk in their financial portfolios. We empirically test this prediction using the industry volatility of labor income estimated by Campbell, Cocco, Gomes, and Maenhout (2001). We find that a household whose head is working in an industry where the labor-income volatility is 10% larger than the mean makes safer financial investments, exhibiting a risky share 2.2% lower than the average. This is consistent with Angerer and Lam (2009), who find a negative correlation between labor-income risk and risky share of workers among the NLSY 1979 cohort.

Our work contributes to the existing literature on the life-cycle portfolio choice in several ways. The closest paper to ours is Cocco, Gomes, and Maenhout (2005). We extend their analysis in two important directions. First, we introduce age-dependent unemployment risk and gradual learning about the age-earning profile. We show that perceived uncertainty may be important for portfolio decisions, especially for young workers. Second, we test our model over a wider ranger of portfolio statistics, most notably the correlations between the risky share and financial wealth within and across age groups. Another closely related paper is Gomes and Michaelides (2005), who show, among others, that heterogeneity in risk aversion and Epstein-Zin preferences is not enough to account for the age profile of risky share. Wachter and Yogo (2010) analyze the life-cycle profile of portfolios as we do. They match the age profile in the data using non-homothetic utility and decreasing relative risk aversion, whereas we match the portfolio profile using age-dependent labor-market uncertainty and constant relative-risk-aversion preferences.

Our paper distinguishes itself from previous studies on the covariance between labor-market risk and stock returns. Benzoni, Collin-Dufresne, and Goldstein (2007) show how labor income and stock-market returns are likely to move together at a longer time horizon. As a result, stocks are riskier for young workers than for old. Storesletten, Telmer, and Yaron (2007) show that if labor income is perfectly correlated with stock returns, the age profile of risky share can exhibit a hump shape. Lynch and Tan (2011) show that the countercyclical volatility of labor-market income growth plays an important role in discouraging the stock-holdings motive for poor and young households. Huggett and Kaplan (2013) decompose human capital into the safe and risky components and find that the level of human capital and stock returns have a small positive correlation.

We closely follow Guvenen (2007) and Guvenen and Smith (2014) in modeling the uncertainty about earnings profile. Both papers examine the implications of imperfect information about income profile for consumption over the life cycle. Consistent with their results, we find that imperfect information coupled with heterogeneity in income profiles can match the linearly increasing dispersion of consumption along the life cycle. However, we take a step further and ask whether gradual learning about the income profile can also help to explain the portfolio allocation puzzle between risky and riskless financial assets. Wang (2009) studies portfolio choice with income heterogeneity and learning within an infinite horizon model. In contrast, we employ a life-cycle model with a particular focus on the relation between risky share and age. Finally, Campanale (2011) develops a life-cycle model in which investors learn about stock-market returns. While uninformed investors can purchase information about the stock market from informed investors, it is impossible to know a priori the unrealized path of lifetime earnings. Hence, our model makes a more realistic assumption about the investor's earnings profile.

The paper is organized as follows. In Section 2, based on the extensive data from the SCF, we document the stylized facts on household-portfolio profiles, including the three above-mentioned puzzles. In particular, we provide detailed statistics across various age/wealth groups. In Section 3 we present a simple 3-period model to illustrate the important interaction between labor-market uncertainty and portfolio choice. Section 4 develops a fully specified life-cycle model for our quantitative analysis. We then calibrate the model to match the age profiles of unemployment risk, earnings volatility and consumption dispersion in the data. In Section 5, we address three portfolio choice puzzles. We consider various specifications of the model to evaluate the marginal contribution of each component of labor-market uncertainty newly featured. Section 6 tests the prediction of our theory using the cross-industry variation of income risks. Section 7 concludes.

# 2 Life-Cycle Profile of Households' Portfolios

Using the 1998 Survey of Consumer Finances (SCF), we document some stylized facts on the life-cycle profile of households' portfolios. The SCF provides detailed information on the households' characteristics and their investment decisions. The survey is conducted every three years and the basic pattern we document here is similar across the surveys.[2] To be consistent with our model (where households face a choice between risk-free and risky investment),

[2]Benzoni and Chyruk (2009) show that the basic facts regarding the risky share of financial assets are the same in the 2001, 2004, and 2007 surveys. Moreover, Ameriks and Zeldes (2004) use the available SCF studies from 1983-1998. They find that both the unconditional and the conditional share weakly increase with age (or exhibit a hump shape) if time effects are controlled for but increase strongly with age if they control for cohort effects.

we classify assets in the SCF into two categories: "safe" and "risky" assets. (The detailed description of how to classify assets into these two categories is discussed below.) Several facts emerge:

1. *Participation*: Just a little over half (56.9%) of the population invests in risky assets. This participation rate shows a hump shape over the life cycle, with its peak around the average retirement age.

2. *Conditional Risky Share*: Households that invest in risky assets, on average, allocate about half (45.5%) of their financial wealth to risky assets. This conditional risky share increases monotonically over the life cycle.

3. *Unconditional Risky Share*: The participation rate and conditional risky share combined, the unconditional risky share exhibits a hump shape over the life cycle.

4. *Wealth Correlation*: Wealthier households, on average, allocate a larger fraction of their savings to risky assets.

In the SCF, some assets can be easily classified into one or the other. For example, checking, savings, and money market accounts are safe investments, while direct holdings of stocks is risky. However, other assets (e.g., mutual funds and retirement accounts) are invested in a bundle of safe and risky instruments. Fortunately, the SCF provides information on how these accounts are invested. The respondents are asked not only how much money they have in each account but also where they are invested. If respondents report that most of the money in those accounts is in bonds, money market, or other safe instruments, we classify them as safe investments. If they report that the money is invested in some forms of stocks, we categorize them as risky investment. If they report that the account involves investments in both safe and risky instruments, we assign half of the money into each category.[3]

More specifically, the assets considered safe are checking accounts, savings accounts, money market accounts, certificates of deposit, cash value of life insurance, U.S. government or state bonds, mutual funds invested in tax-free bonds or government-backed bonds, trusts and annuities invested in bonds, and money market accounts. The assets considered risky are stocks, brokerage accounts, mortgage-backed bonds, foreign and corporate bonds, mutual funds invested in stock funds, trusts and annuities invested in stocks or real estate and pension plans

[3]While the SCF provides information on how individual retirement accounts are invested, it is not the case with pension plans, e.g., 401(k), defined contribution plans and others. In this case, we classify half of the money invested in these accounts as safe assets and the rest as risky assets (because the average risky share is close to 50%). In Appendix B we recalculate the risky share with different split rules between safe and risky assets such as 80-20 or 20-80, for example. The average of the risky share is affected by the split rule, but the shape of the age profile is not.

Table 1: Household Savings by Account Types

| Account Type | Average (US $) | Participation |
|---|---|---|
| Total safe assets | 64,488 | 99.8% |
| Checking account | 3,483 | 86.9% |
| Savings account | 4,746 | 60.2% |
| Savings bond (safe) | 5,916 | 22.4% |
| Life insurance | 8,955 | 31.7% |
| Retirement accounts (safe) | 12,103 | 38.1% |
| Total risky assets | 89,403 | 56.7% |
| Stocks | 32,898 | 20.6% |
| Trust (risky) | 6,121 | 1.2% |
| Mutual funds (risky) | 12,574 | 16.3% |
| Retirement accounts (risky) | 26,376 | 45.2% |
| Total financial assets | 153,891 | 100.0% |

Note: Survey of Consumer Finances (1998).

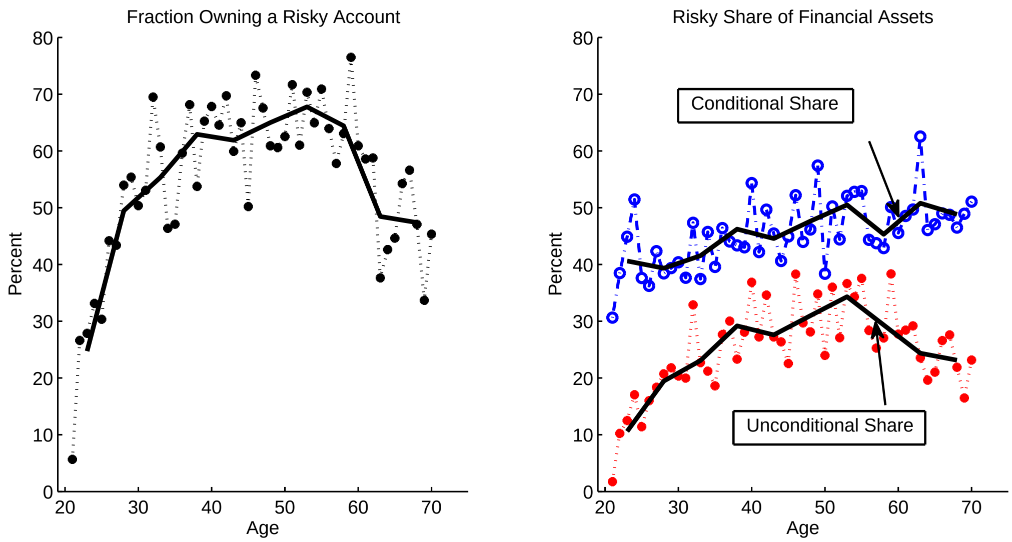
that are a thrift, profit sharing or stock purchase plan. Also considered a risky investment is the "share value of businesses owned but not actively managed excluding ownership of publicly traded stocks." We exclude the share value of *actively managed* business from our benchmark definition of risky investments. Appendix A provides more detailed information on how we construct our data from the SCF.

Table 1 summarizes the average amount (in 1998 dollars) held in specific accounts as well as the participation rate (the fraction of households that have a positive amount in that account). We restrict the sample to households that have a positive amount of assets. While 86.9% of households hold a checking account and 60.2% hold a savings account, only 20.6% directly own stocks. Nearly every household (99.8%) owns some form of safe assets, while only 56.7% invest in risky assets.

*Risky Share by Age*

We examine the risky share across different age and wealth groups. The risky share is defined as the total value of risky assets divided by the total amount of financial assets. Figure 1 shows participation rates, conditional (on participation) risky share, and unconditional risky share over the life cycle. The dotted line represents the average value by age and the solid line

Figure 1: Risky Share over the Life Cycle




Note: Survey of Consumer Finances (1998). The solid line represents the 5-year average. The left panel shows the participation rate (the fraction of households that invest in risky assets). The right panel shows the unconditional and conditional risky shares.

represents the 5-year average (e.g., 21-25, 26-30, etc.). In the left panel, the participation rate (the fraction of households who participate in risky investment) exhibits a hump shape over the life cycle with its peak just before the average retirement age. It increases from 24.7% in the age group of 21-25, to 55.3% in ages 31-35, reaches its peak of 67.7% in ages 51-55 and then decreases to 48.4% in ages 61-65.

The right panel shows the conditional and unconditional risky shares. The conditional share—the share among the households that invest in risky assets—monotonically increases over the life cycle. It increases from 40.6% in the 21-25 age group to 44.5% in the 41-45 age group, and then to 50.7% in the 61-65 age group. Since our model abstracts from the participation decision, when we compare the model and data, we will focus on the conditional risky share only. The average conditional risky share is 45.5%.[4] The unconditional risky share (participation rate times conditional risky share), also exhibits a hump shape. It rises from 10.5% at ages 21-25 to its peak of 34.3% at ages 51-55, and then decreases to 24.3% at ages 61-65. In sum, these life-cycle patterns of risky share clearly suggest that younger investors are reluctant to take financial risks, despite longer investment horizons and higher average

[4]The average risky share is defined as the average of risky shares across households, not the total amount of risky assets divided by total amount of financial assets. Under the alternative definition, the average risky share is 58.1% = $\frac{89{,}403}{153{,}891}$, much higher than ours. We prefer our definition because the latter definition is sensitive to an outlier, i.e., a few extremely rich households that make extensive risky investments.

rates of return to risky investments.

*Risky Share by Wealth*

We turn our attention to the relationship between the risky share and wealth (total financial assets). Table 2 reports the average risky share (both conditional and unconditional) across 5 quintile groups in the distribution of household wealth. Both measures of risky share show strong positive correlations with the amount of household wealth. Wealthier households take more risk in their financial investments. The unconditional risky share increases monotonically from 5.3% in the 1st quintile to 38.1% in the 3rd, and 64.9% in the 5th. The conditional risky share also increases from 35.9% in the 1st quintile to 44.4% in the 3rd, and 66.6% in the 5th. The participation rate (not reported in the table) also monotonically increases with wealth. In the 5th quintile of wealth, almost everyone (97%) makes risky investments.

Table 2: Risky Shares by Wealth

| | Risky Share | |
|---|---|---|
| Wealth Quintile | Unconditional | Conditional |
| $1^{st}$ | 5.3% | 35.9% |
| $2^{nd}$ | 25.9% | 40.5% |
| $3^{rd}$ | 38.1% | 44.4% |
| $4^{th}$ | 49.4% | 51.7% |
| $5^{th}$ | 64.9% | 66.6% |
| Average | 28.3% | 45.5% |

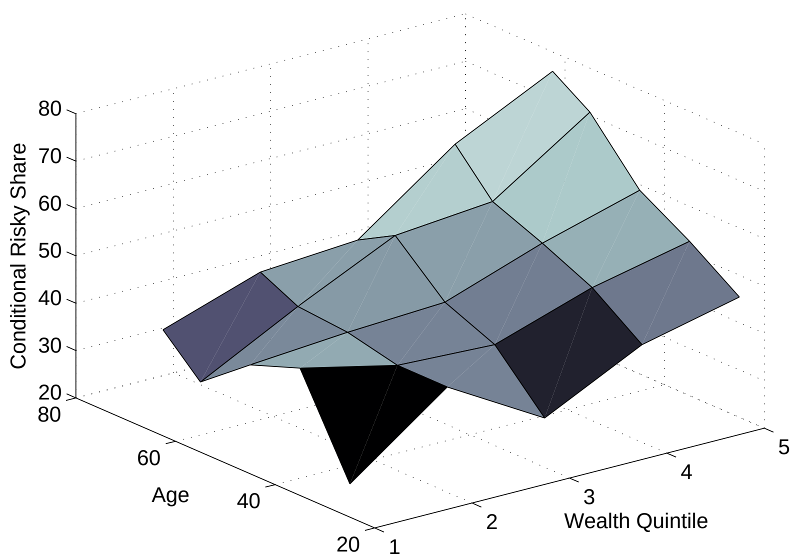
Note: Survey of Consumer Finances (1998)

Since older households are, on average, wealthier, one might suspect that the age effect drives this positive correlation between the risky share and wealth. To check whether this is the case in the data, Figure 2 plots the average conditional risky share across quintiles for the 5-year age group. While it is not always monotonic (perhaps due to a small cell size), it clearly shows a positive correlation between the risky share and wealth within each age group.

*Robustness (1): Actively managed businesses and homeownership*

So far, we have excluded two types of assets from household wealth: the amount of investment in their own business and the value of house(s). To see whether the age profile of risky share is robust to the inclusion of these assets, we re-calculate the conditional risky shares, including

Figure 2: Conditional Risky Share by Age and Wealth




each of these assets, in Table 3. The first column is our benchmark definition of risky shares when these assets are not included in the household's wealth.

In the second column ("Business included"), we include the net value of actively managed own businesses as a part of risky assets.[5] With the value of actively managed businesses included, the average risky share increases to 50.8% (from 45.5% according to our benchmark measure). However, the increasing pattern of the risky-share profile is unaffected. It increases from 44.9% at ages 21-30 to 54.4% at ages 61-65.

In the third column ("House included") we include the net worth of house(s) as a part of the household's risky assets. The net worth of house(s) is the sum of the house(s) value minus the amount borrowed as well as other lines of credit or loans the investor may have. We also add any investment in real estate such as vacation houses. With the net worth of houses included, the average risky share increases significantly, to 69.7%. However, again, the increasing pattern of the risky share profile remains unaffected. It increases from 61.4% at ages 21-30 to 76.8% at ages 61-65.

Next, we compare the risky shares (using our benchmark measure) of the households that own a business or a house. While the average risky share is slightly higher for business owners (47.9% compared to 45.0%), the increasing pattern of the age profile remains the same. They

[5]The net value of actively managed own business is the amount owed to the household by the business, subtracting the money owed by the household.

Table 3: Conditional Risky Shares under Alternative Definitions and by Household Type

| Age | Benchmark | Alternative | | Household Type | | | |
|---|---|---|---|---|---|---|---|
| | | Business included | House included | Own Business? No | Own Business? Yes | Own Home? No | Own Home? Yes |
| 21-30 | 40.2% | 44.9% | 61.4% | 40.1% | 41.5% | 40.8% | 42.3% |
| 31-40 | 44.3% | 50.2% | 66.9% | 44.7% | 41.2% | 37.0% | 47.0% |
| 41-50 | 45.9% | 51.9% | 70.7% | 45.1% | 49.7% | 40.8% | 47.1% |
| 51-60 | 48.0% | 52.7% | 74.1% | 47.6% | 50.4% | 38.0% | 50.1% |
| 61-65 | 50.1% | 54.4% | 76.8% | 48.4% | 56.4% | 46.9% | 50.7% |
| Average | 45.5% | 50.8% | 69.7% | 45.0% | 47.9% | 39.3% | 47.9% |

Note: Under "Business included" the net value of actively managed own businesses is included as a part of risky assets. Under "House included" the net worth of house(s) is included as a part of risky assets. The table also reports the conditional risky shares (using the benchmark definition) by samples. To increase our sample size, we used ages 61-70 instead of 61-65 to compute the average risky share for renters.

increase monotonically with age. There are many reasons for believing that homeownership should affect the portfolio choice over the life cycle (e.g., marriage, having children, etc.). For example, as noted by Cocco (2005), house price risk may crowd out stock holdings. The average risky share is higher for homeowners (47.9% compared to 39.3% for renters). However, the age profiles generally increase for both groups but not monotonically, especially for renters, perhaps due to the small sample size. Our simple analysis does not suggest that homeownership is the reason younger people choose to hold a smaller fraction of risky assets in their portfolios.

*Robustness (2): Net Savings and Retirement Accounts*

In our benchmark definition, safe and risky assets are defined as *gross* savings in safe and risky financial instruments, respectively. In our benchmark definition, consumer debt such as credit card debt and other consumer loans is ignored. The rationale for this is that it is not clear if liabilities should be subtracted from either the safe or the risky assets.[6] In Table 4 we calculate the risky share under the assumption that consumer debt is subtracted from the safe

[6] Imagine an individual with $1,000 in her checking account, $1,000 in stocks, and $500 debt to a relative. The individual may have placed the $500 in the checking account, in which case the net safe assets are $500 and the risky share is 66%. However, the individual may have invested the borrowed money in the stock market, in which case the net risky assets are $500 and the risky share is 33%. Since there is no information on how consumer debt is actually used, we choose to focus on gross savings.

Table 4: Conditional Risk Shares: Net Savings and Retirement Accounts

| Age | Benchmark | Net Savings | Retirement Accounts |
|---|---|---|---|
| 21-30 | 40.2% | 48.8% | 63.0% |
| 31-40 | 44.3% | 54.6% | 61.9% |
| 41-50 | 45.9% | 51.8% | 62.9% |
| 51-60 | 48.0% | 55.4% | 65.4% |
| 61-65 | 50.1% | 51.6% | 63.4% |
| Average | 45.5% | 53.0% | 63.2% |

Note: Under "Net Savings" the amount of net savings is used for the total amount of safe assets. "Retirement Account Only" reflects the risky share of assets in the retirement accounts.

assets. Given the significant fraction of people with debt (45.5% in our sample), the average risky share increases to 53.0% from 45.5% in Table 4. While the risky share still increases with age (from 48.8% at ages group 21-30 to 55.4% at ages 51-60), it is not monotonic. Finally, we examine how the savings in retirement accounts, which are considered long-term investments, are allocated between safe and risky assets. Table 4 shows that the risky share still mildly increases for ages 31-60 within the retirement accounts.

# 3 A Simple Portfolio-Choice Theory

Before developing a full-blown life-cycle economy, using a simple 3-period model, we illustrate how portfolio choice is affected by age, labor-market risk, and wealth.

A worker lives for three periods. Each period he receives income $y_t$ which is an i.i.d. random variable with a probability function $f(y_t)$. Preferences are given by

$$U = E \sum_{t=1}^{3} \beta^{t-1} \frac{{c_t}^{1-\gamma}}{1-\gamma}$$

where $\gamma$ is the coefficient of relative risk aversion and $c_t$ is consumption in period $t$. Two types of financial assets are available for savings. One is a risk-free bond, $b_t$, that pays a fixed gross return, $R$ and the other is a stock, $s_t$, that pays a stochastic gross return, $R_s = R+\mu+\eta$, where $\mu$ is the risk premium and $\eta$ is excess return drawn from a normal distribution of $N(0, \sigma_\eta^2)$. The probability density function associated with $\eta$ is denoted by $\pi(\eta)$. On average, the stock yields a higher rate of return than the bond to compensate for the risk associated with $\eta$:

$\mu > 0$.

Current income is divided between consumption, $c$, and savings, $b' + s'$. It is convenient to collapse total wealth into a single state variable $W = bR + sR_s$. Borrowing is not allowed for each investment ($b \geq 0$ and $s \geq 0$). The present value of utility in period $j$, $V_j$, can be written recursively when the next period's value is denoted with a prime ($'$):

$$V_j(W, y) = \max_{c,s',b'} \left\{ \frac{c^{1-\gamma}}{1-\gamma} + \beta \int_{\eta'} \int_{y'} V_{j+1}(W', y') df(y') d\pi(\eta') \right\} \qquad j = 1, 2, 3$$

$$\text{s.t.} \quad c + s' + b' = W + y$$

$$c \geq 0, \quad s' \geq 0, \quad b' \geq 0$$

where $V_4(\cdot, \cdot) = 0$.

*Case 1: No labor income (Samuleson Rule).*

Under the CRRA preferences, with no labor income in the future, a worker allocates savings according to the constant share between risky and safe assets, the so-called Samuelson (1969) Rule, so that the risky share is:[7]

$$\frac{s'}{s' + b'} \approx \frac{1}{\gamma} \frac{\mu}{\sigma_\eta^2}$$

This rule is intuitive. The risky share (i) increases in the risk premium, $\mu$, (ii) decreases in the risk aversion, $\gamma$, and (iii) decreases with the risk of stock returns, $\sigma_\eta$. According to this rule, the wealth, $W$, and investment horizon (age), $j$, are irrelevant for the portfolio decision, inconsistent with advice often provided by financial analysts.[8] The risky share is independent of wealth because of CRRA preferences. While the longer horizon provides an opportunity to weather the risk in stock returns, the variance of *total returns* also increases with the horizon. With CRRA preferences and i.i.d. stock returns the two effects cancel each other so that the risky share remains independent of the investment horizon.

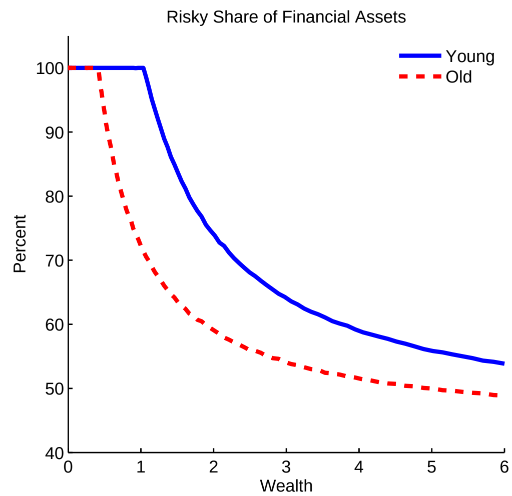
*Case 2: Deterministic labor income*

We now illustrate how labor-market uncertainty affects the risky share in a three-period example. In this example, the first period corresponds to "Young" worker, the second to the

[7] "No labor income" refers to the case where tomorrow's income $y' = 0$. Today's labor income $y$ is a part of "cash in hand," $W + y$.

[8] Many financial analysts advise investors with longer investment horizons (i.e., young investors) to take more risk. As Ameriks and Zeldes (2004) note, this advice is typically based on two observations: (i) Over time stocks provide better returns than bonds. (ii) With a longer time horizon, investors have more time to weather the ups and downs of the stock market.

Figure 3: Risky Share: Deterministic Labor Income



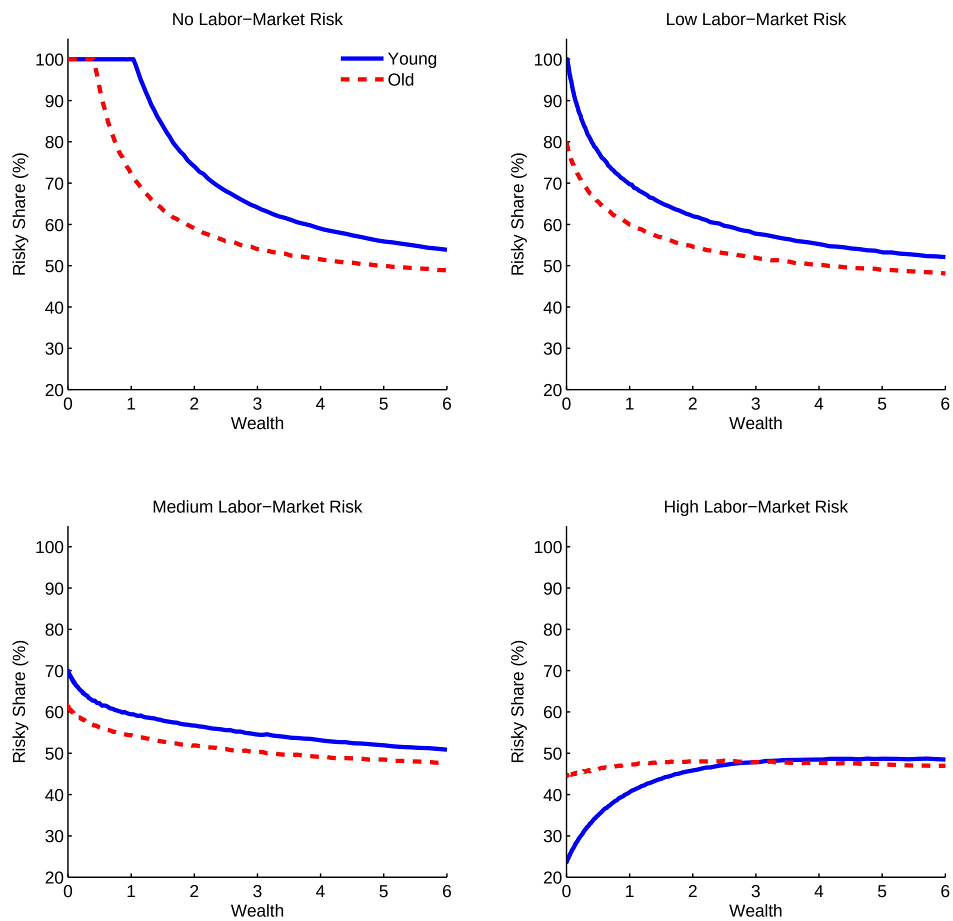

Note: Risky Share ($\frac{s'}{s'+b'}$) for "Young" and "Old".

"Old" worker, and the last to "Retired."

First, consider the case where labor income is deterministic ($y > 0, \sigma_y^2 = 0$) so that there is no uncertainty in the labor-market outcome. Figure 3 plots the risky share ($\frac{s'}{s'+b'}$) of "Young" and "Old" for various levels of wealth. For both "Young" and "Old," the risky share decreases with wealth, completely opposite to what we saw in Section 2. When the labor income is deterministic, having a job is equivalent to holding a risk-free fixed-income asset. A worker with little wealth, because risk-free labor income makes up a large portion of total wealth, would like to allocate most of his savings to risky investments. In fact, according to the optimal policy function, whether young or old, a wealth-poor worker, whose $W$ is close to 0, allocates all his savings to stocks. As wealth increases, the risky share decreases. When wealth is large relative to labor income (where labor income becomes a negligible portion of total wealth), the risky share converges to the value implied by the Samuelson Rule.

The risky share decreases with age, again opposite to what we saw in the data. Since the young anticipate a longer stream of deterministic labor income (for the two remaining periods)–which is equivalent to holding a fixed-income asset, they are willing to take more risk in their financial investments. Figure 3 shows that this is true for any given level of wealth, unless the wealth is close to zero where the risky share is 100% for both young and old. In sum, with no uncertainty in the labor market, the risky share *decreases* with age and wealth, both of which are opposite to what we find in the SCF.

Figure 4: Risky Share: Stochastic Labor Income




Note: Risky shares ($\frac{s'}{s'+b'}$) of "Young" and "Old" for four values of labor-market variance ($\sigma_y$).

*Case 3: Stochastic labor income*

We now consider the case where labor income is stochastic. Figure 4 shows the optimal risky share for 4 different values of labor income risk, $\sigma_y$: zero (deterministic), low, medium and high. As the labor-income risk increases, a worker becomes less willing to make risky financial investments. The risky share declines for both young and old. Now, the large uncertainty in the labor market discourages workers making further risky financial investments. With a high enough labor-market risk (the last panel in Figure 4), (i) the young's risky share is lower than the old's, and (ii) it is increasing in financial wealth. Young investors, on average wealth poor, are highly exposed to risk, since their income consists mainly of highly volatile labor income. This example clearly illustrates that labor-market uncertainty is crucial for the relationship between the risky share and investors' age and financial wealth.

# 4 Life-Cycle Model

## 4.1 Economic Environment

*Demographics* The economy is populated by a continuum of workers with total measure of one. A worker enters the labor market at age $j = 1$, retires at age $j_R$, and lives until age $J$. There is no population growth.

*Preferences* Each worker maximizes the time-separable discounted life-time utility:

$$U = E \sum_{j=1}^{J} \delta^{j-1} \frac{{c_j}^{1-\gamma}}{1-\gamma} \tag{1}$$

where $\delta$ is the discount factor, $c_j$ is consumption in period $j$, and $\gamma$ is the relative risk aversion.[9] For simplicity, we abstract from the labor effort choice and assume that labor supply is exogenous when employed.

*Earnings Profile* We assume that the log earnings of a worker $i$ with age $j$, $Y_j^i$, is:

$$Y_j^i = \log z_j + y_j^i \quad \text{with} \quad y_j^i = a^i + \beta^i \times j + x_j^i + \varepsilon_j^i. \tag{2}$$

Earnings consist of common and individual-specific components. The component, $z_j$, represents the average age-earnings profile, which is assumed to be the same across workers and thus observable. The individual-specific component consists of deterministic $(a^i + \beta^i \times j)$ and stochastic $(x_j^i + \varepsilon_j^i)$ components. The deterministic component represents the individual-specific intercept $(a^i)$ and growth rate $(\beta^i)$ of the profile. The intercept of the earnings profile, $a^i$, is distributed across the population by $a^i \sim N(0, \sigma_a^2)$. The growth rate, $\beta^i$ is distributed by $\beta^i \sim N(0, \sigma_\beta^2)$. The stochastic component represents persistent $(x_j^i)$ and purely transitory $(\varepsilon_j^i)$ income shocks. The persistent component of the stochastic income shock, $x_j^i$, follows an AR(1) process:

$$x_j^i = \rho x_{j-1}^i + \nu_j^i, \qquad \text{with} \quad \nu_j^i \sim \text{ i.i.d. } N(0, \sigma_\nu^2) \tag{3}$$

where the transition probability is represented by a common finite-state Markov chain $\Gamma(x_j | x_{j-1})$. The transitory component follows $\varepsilon_j^i \sim$ i.i.d. $N(0, \sigma_\varepsilon^2)$, where the probability distribution of $\varepsilon$ is denoted by $f(\varepsilon)$. For simplicity, we will omit superscript $i$ from now on. Workers are

[9] Alternative preferences have also been proposed to address the portfolio choice puzzles. For example, Gomes and Michaelides (2005) use Epstein-Zin preferences with heterogeneity in both risk aversion and intertemporal elasticity of substitution and Wachter and Yogo (2010) use non-homothetic preferences. We adopt the standard preferences with constant relative risk aversion in order to highlight the role of labor-market uncertainty.

assumed to have imperfect knowledge about their own individual-specific components. A worker starts with a prior about $a$, $\beta$, and $x$, and updates these priors each period based on the realized value of total earnings. Thus, young workers on average face larger uncertainty about future earnings. This uncertainty is resolved over time as he gradually learns about his earnings ability based on successive realizations of labor income.

*Unemployment Risk* Each period workers face age-dependent unemployment risk. The probability of being unemployed for the worker of age $j$ is $p_j^u$. For simplicity, we assume that unemployment risk is not individual-history dependent.

*Savings* Financial markets are incomplete in two senses. First, workers cannot borrow. Second, there are only two types of assets for savings: a risk-free bond $b$ (paying gross return $R$ in consumption units) and a stock $s$ (paying $R_s = R + \mu + \eta$) where $\mu$ ($> 0$) represents the risk premium and $\eta$ is the stochastic rate of return.[10] Workers save to prepare for retirement (life-cycle savings) and to ensure themselves against labor-market uncertainty (precautionary savings).

*Social Security* The government runs a balanced-budget pay-as-you-go social security system. When a worker retires from the labor market at age $j_R$, he receives a social security benefit amount, $ss$, which is financed by taxing workers' labor incomes at rate $\tau_{ss}$.[11]

*Learning* In our benchmark model, workers do not have perfect knowledge about their lifetime profile of earnings. While the non-deterministic part of labor income, $y$, is observed, workers cannot perfectly distinguish between ability ($a$ and $\beta$) and luck ($x$ and $\varepsilon$). Instead, they update their priors in a Bayesian fashion. Given the normality assumption, a worker's prior belief is summarized by the mean and variance of intercept, $\{\mu_a, \sigma_a^2\}$, and those of slope, $\{\mu_\beta, \sigma_\beta^2\}$. Similarly, a worker's prior belief about the persistent component of the income shock is summarized by $\{\mu_x, \sigma_x^2\}$. When the prior beliefs over the covariances are denoted by $\sigma_{ax}$ ,$\sigma_{a\beta}$, and $\sigma_{\beta x}$, we can express the prior mean and variance as:

$$\mathbf{M}_{j|j-1} = \begin{bmatrix} \mu_a \\ \mu_\beta \\ \mu_x \end{bmatrix}_{j|j-1} \qquad \mathbf{V}_{j|j-1} = \begin{bmatrix} \sigma_a^2 & \sigma_{a\beta} & \sigma_{ax} \\ \sigma_{a\beta} & \sigma_\beta^2 & \sigma_{\beta x} \\ \sigma_{ax} & \sigma_{\beta x} & \sigma_x^2 \end{bmatrix}_{j|j-1} \tag{4}$$

[10] For simplicity, we abstract from the general equilibrium aspect by assuming exogenous average rates of returns to both stocks and bonds, and the labor supply decision.

[11] Ball (2008) analyzes financial investments for different levels of the social security benefit. He finds that the generosity of the social security system has little effect on portfolio choice.

where the subscript $j|j-1$ denotes the information at age $j$ before the actual earning $y_j$ is realized. The subscript $j|j$ denotes the information after earnings $y_j$ is realized, i.e., posterior.

When a worker exactly knows his ability ($a$ and $\beta$), the uncertainty about future earnings reflects the uncertainty in the stochastic component ($x$ and $\varepsilon$) only. However, when a worker does not have perfect knowledge about his ability, uncertainty about labor income includes the beliefs about his ability as well as the stochastic component. According to our calibration of the income profile, which is based on the estimates in the literature, income uncertainty over 1-5 periods ahead is resolved extremely fast, under the standard Bayesian updating formula. For example, within 3 years of a worker entering the labor market, almost 90% of the uncertainty about the next period's income is resolved. We argue that learning at this rate is not realistic, especially considering the frequent job changes of young workers in search of a good match. Typically, workers shop around for jobs before settling into a long-term career. According to Topel and Ward (1992), in the first 10 years entering the labor market, a typical worker holds 7 jobs (about two-thirds of his career total). Kambourov and Manovskii (2008) report that the average probability that a young worker will switch occupations is approximately 25% on an annual basis. Frequent job switching may diminish the informational context from previous labor-market experience, especially if it takes place across completely different industries or occupations. To achieve a more realistic speed of learning, we extend the standard Bayesian formula to include noise in the updating rule. One interpretation of such noise in the updating rule is a "reset" of priors, reflecting possible information loss due to occupational/industrial change.[12]

Specifically, when $q$ denotes this noise in the priors updating rule, the posterior means and variances at age $j$ are given by:

$$\mathbf{M}_{j|j} = \mathbf{M}_{j|j-1} + \begin{bmatrix} \frac{\sigma_a^2+\sigma_{a\beta}+\sigma_{ax}}{\sigma_a^2+\sigma_\beta^2 j^2+\sigma_x^2+\sigma_\varepsilon^2+\Gamma} \\ \frac{\sigma_{a\beta}+\sigma_\beta^2 j+\sigma_{\beta x}}{\sigma_a^2+\sigma_\beta^2 j^2+\sigma_x^2+\sigma_\varepsilon^2+\Gamma} \\ \frac{\sigma_{ax}+\sigma_{x\beta} j+\sigma_x^2}{\sigma_a^2+\sigma_\beta^2 j^2+\sigma_x^2+\sigma_\varepsilon^2+\Gamma} \end{bmatrix} (y_j - \mathbf{H}_j'\mathbf{M}_{j|j-1}) + \begin{bmatrix} q_j^a \\ q_j^\beta \\ q_j^x \end{bmatrix} \tag{5}$$

$$\mathbf{V}_{j|j} = \mathbf{V}_{j|j-1} - \begin{bmatrix} \frac{\sigma_a^2+\sigma_{a\beta}+\sigma_{ax}}{\sigma_a^2+\sigma_\beta^2 j^2+\sigma_x^2+\sigma_\varepsilon^2+\Gamma} \\ \frac{\sigma_{a\beta}+\sigma_\beta^2 j+\sigma_{\beta x}}{\sigma_a^2+\sigma_\beta^2 j^2+\sigma_x^2+\sigma_\varepsilon^2+\Gamma} \\ \frac{\sigma_{ax}+\sigma_{x\beta} j+\sigma_x^2}{\sigma_a^2+\sigma_\beta^2 j^2+\sigma_x^2+\sigma_\varepsilon^2+\Gamma} \end{bmatrix} \mathbf{H}_j'\mathbf{V}_{j|j-1} + \mathbf{Q}_j \tag{6}$$

where $\mathbf{H}_j = [\begin{array}{ccc} 1 & j & 1 \end{array}]'$ is a $(3\times 1)$ vector and $\Gamma = 2\sigma_{a\beta} j + 2\sigma_{ax} + 2\sigma_{\beta x} j$. The parameter

[12] It is often found in various natural experiments that subjects are more conservative in changing their priors than the standard Bayesian rule implies. For example, according to Edwards (1968) " ... it takes between 2 to 5 observations to make a subject change her prior beliefs to the same extent as what one observation would do for a Bayesian learner."

$q_j^\kappa$ for $\kappa \in \{a, \beta, x\}$ captures the additional noise in the updating rule. We assume $q_j^\kappa$'s are distributed as $\mathbf{q}_j \sim N(0, \mathbf{Q}_j)$ where $\mathbf{q}_j$ is the $(3 \times 1)$ vector on the right-hand side of (5), and the variance-covariance matrix of noise $\mathbf{Q}_j$ is a $(3 \times 3)$ matrix in which non-diagonal elements are zero (mutually independent from each other). We assume that the noise shock occurs after the update has taken place. Given that on expectation the shock will be zero, $q_j^\kappa$ will not affect the posterior of means *at the time of the update*, but only the posterior of the variances. Our formulation is computationally convenient because we do not need $q_j^\kappa$ as an additional state variable: it is included in the age $j$. In Appendix C, we formally derive these equations (5) and (6). Using the above formula, the next period's income follows the conditional distribution function:

$$F(y_{j+1}|y_j) = N(\mathbf{H}'_{j+1}\mathbf{M}_{j+1|j}\ ,\ \mathbf{H}'_{j+1}\mathbf{V}_{j+1|j}\mathbf{H}_{j+1} + \sigma^2_{\varepsilon_j}) \tag{7}$$

where

$$\begin{aligned}
\mathbf{M}_{j+1|j} &= \mathbf{R}\left[\mathbf{M}_{j|j-1} + \begin{bmatrix} \frac{\sigma_a^2+\sigma_{a\beta}+\sigma_{ax}}{\sigma_a^2+\sigma_\beta^2 j^2+\sigma_x^2+\sigma_\varepsilon^2+\Gamma} \\ \frac{\sigma_{a\beta}+\sigma_\beta^2 j+\sigma_{\beta x}}{\sigma_a^2+\sigma_\beta^2 j^2+\sigma_x^2+\sigma_\varepsilon^2+\Gamma} \\ \frac{\sigma_{ax}+\sigma_{x\beta} j+\sigma_x^2}{\sigma_a^2+\sigma_\beta^2 j^2+\sigma_x^2+\sigma_\varepsilon^2+\Gamma} \end{bmatrix} (y_j - \mathbf{H}'_j\mathbf{M}_{j|j-1})\right] \\
\mathbf{V}_{j+1|j} &= \mathbf{R}\mathbf{V}_{j|j}\mathbf{R}' + \mathbf{S}
\end{aligned}$$

with $\mathbf{R}$ denoting a $(3 \times 3)$ matrix whose diagonal elements are $(1, 1, \rho)$ and $\mathbf{S}$ denoting the covariance matrix of a shock vector, $[\ 0 \quad 0 \quad \nu^i_{j+1}\ ]$.

Let $k = \{e, u\}$ denote the employment status of a worker: employed or unemployed. As in our illustrative example in Section 3, we collapse the total financial wealth into one state variable $W = bR + sR_s$. The value function of a worker at age $j$ is:

$$\begin{aligned}
V_j^k(W, y, \mathbf{M}_{j|j-1}) = \max_{c^k, s', b'} \Bigg\{ & u(c^k) + \delta(1 - p_j^u) \int_{\eta'} \int_{y'} V^e_{j+1}(W', y', \mathbf{M}_{j+1|j}) dF_j(y'|y) d\pi(\eta') \\
& + \delta p_j^u \int_{\eta'} \int_{y'} V^u_{j+1}(W', y', \mathbf{M}_{j+1|j}) dF_j(y'|y) d\pi(\eta') \Bigg\}
\end{aligned} \tag{8}$$

$$\text{s.t.} \quad c^k + s' + b' = (1 - \tau_{ss}) \exp^{Y_j} \times \mathbf{1}\{k = e\} + ss \times \mathbf{1}\{j \geq j_R\} + W \tag{9}$$

where $\mathbf{1}\{\cdot\}$ is an indicator function, and $Y_j = \log z_j + y_j$. Note that every period, both employed and unemployed get a draw of $y$, which will affect the next period's $y'$ through $F_j(y'|y)$. However, a fraction $p_j^u$ of employed workers will lose their job $(k = u)$. The state variables include workers' wealth, $W$, the individual-specific component of labor income, $y$, and the prior mean ($\mathbf{M}_{j|j-1}$). Note that the prior about the second moment ($\mathbf{V}_{j|j-1}$) is not explicitly included in the value function, since the age ($j$) is a sufficient statistic.[13]

*Perfect Information Model* In order to evaluate the marginal contribution of each component of labor-market uncertainty on portfolio choice, we consider various specifications differing with respect to assumptions about (i) the knowledge about income profile, (ii) the age-dependent unemployment risk, and (iii) noise in the updating rule. The first alternative specification we consider is the standard life-cycle model without any of these three features. This specification is very similar to Cocco, Gomes, and Maenhout (2005). We will refer to this specification as the perfect information model (PIM). In this case, the value function of a $j$-year-old worker with an income profile of $\{a, \beta\}$ is:

$$V_j^{\{a,\beta\}}(W, x, \varepsilon) = \max_{c,s',b'} \left\{ u(c) + \delta \int_{\eta',x',\varepsilon'} V_{j+1}^{\{a,\beta\}}(W', x', \varepsilon') df(\varepsilon') d\Gamma(x'|x) d\pi(\eta') \right\} \tag{10}$$

$$\text{s.t.} \quad c + s' + b' = (1 - \tau_{ss}) \exp^{Y_j} + ss \times 1\{j \geq j_R\} + W.$$

The second alternative specification we consider is the perfect information model with age-dependent unemployment risk, which is referred to as the PIM with unemployment risk. Finally, we consider the benchmark model without noise in updating rule (i.e., benchmark with the standard Bayesian updating rule): Benchmark without noise.

## 4.2 Calibration

We calibrate the model to be consistent with the income process and dispersion of consumption in the data. There are six sets of parameters: (i) life-cycle parameters, $\{j_R, J\}$, (ii) preferences $\{\gamma, \delta\}$, (iii) asset market structure $\{R, \mu, \sigma_\eta^2\}$, (iv) labor income process $\{z_j, \rho, \sigma_a^2, \sigma_\beta^2, \sigma_\varepsilon^2, \sigma_\nu^2\}$, (v) noise in the priors updating rule, $\{\mathbf{V}_{1|0}, Q_j\}$, and (vi) the social security system $\{\tau_{ss}, ss\}$. Unless stated otherwise, we keep the parameter values constant across the models. Table 5 reports all parameter values for the benchmark case.

*Life Cycle, Preferences, and Social Security* The model period is one year. Workers are born and enter the labor market at age 21 and live for 60 periods, $J = 60$, which corresponds to ages 21-80. Workers retire at age 65, $j_R = 45$, when they start receiving the social security benefit, $ss$. The social security tax rate $\tau_{ss} = 20\%$ targets a replacement ratio of 40% for an

[13] For simplicity, the unemployment spell does not interrupt the learning process in our model; thus, the speed of learning depends on age only, not individual-specific component. Making the learning individual-employment-history dependent will expand the state space enormously.

average productivity worker. The relative risk aversion, $\gamma$, is set to 5 to match the average risky share in our benchmark specification. This value is much lower than those typically adopted to match the average risky share in the literature. The discount factor, $\delta = 0.94$, is calibrated to match the wealth-to-income ratio of 3.5, the value commonly targeted in the literature.[14] Matching the wealth-to-income ratio is important for the quantitative analysis of portfolio choice. As documented in previous studies such as Storesletten, Telmer, and Yaron (2004), and Ball (2008), if the average asset holdings are too high relative to labor earnings, the labor-income risk becomes negligible. As shown in our illustrative examples in Section 3, the risky share converges to the value implied by the Samuelson Rule for an investor with a sufficiently large amount of wealth.

*Asset Returns* The rate of return to the risk-free bond $R = 1.02$ is based on the average real rate of returns to 3-month US Treasury bills for the post-war period. Following Gomes and Michaelides (2005), we set the equity premium $\mu$ to 4%. The standard deviation of the innovations to the rate of return to stock $\sigma_\eta$ is 18%, again, based on Gomes and Michaelides (2005).[15] We assume that the stock returns are orthogonal to labor-income risks.[16]

*Unemployment Risk* Based on the CPS for the period 1976-2013, Choi, Janiak, and Villena-Roldan (2014) estimate the transition rates from employment to unemployment over the life cycle. Figure 5, reproduced based on their estimates, clearly shows that the annual probability of moving to being employed this period to unemployment next period monotonically decreases with age. For example, a 21-year-old worker faces a 3.5% chance of moving from being employed to unemployed next year, whereas a 64-year-old worker faces a much smaller risk, less than 1%. We use these estimates for the age-dependent unemployment risk, $p_j^u$.

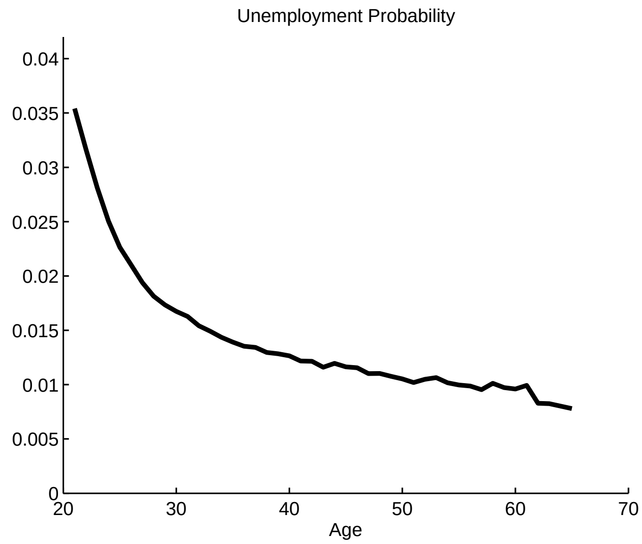
*Labor-Income Process* For the stochastic process of labor-income shocks, we use the estimates of Guvenen and Smith (2014). The dispersion of the individual-specific growth rate is $\sigma_\beta^2 = 0.03\%$. The AR(1) coefficient and the variance of innovation for the persistent income shocks are $\rho = 0.756$ and $\sigma_\nu^2 = 5.15\%$. The variance of the i.i.d. component is $\sigma_\varepsilon^2 = 1\%$. We choose the dispersion of the intercept, $\sigma_a^2 = 13\%$, to match the cross-sectional variance of

[14] In the perfect information model (PIM) we set $\delta = 1.101$. In this case, the model requires a large discount factor to match the wealth-to-income ratio observed in the data because (i) the precautionary savings motive against labor-market uncertainty is small and (ii) an increasing profile of earnings induces workers to borrow heavily early in life.

[15] Jagannathan and Kocherlakota (1996) report that for the period between 1926 and 1990, the standard deviation of annual real returns in the S&P stock price index was 21% as opposed to 4.4% in T-bills.

[16] The empirical evidence on the correlation between labor-income risk and stock market returns is mixed. While Davis and Willen (2000) find a positive correlation, Campbell, Cocco, Gomes, and Maenhout (2001) find a positive correlation only for specific population groups.

Figure 5: Unemployment Risk over the Life Cycle




Note: Estimates from Choi, Janiak, and Villena-Roldan (2014).

log consumption of young workers (age 25) in the data, as calculated by Guvenen (2007).[17] The deterministic earnings profile, which is common across workers, $z_j$, is taken from Hansen (1993). This parameterization leads to an increasing profile of variance in log-wages between ages 25 to 55, by approximately 0.35 point, fairly consistent with Heathcote, Storesletten, and Violante (2014) and Guvenen and Smith (2014). We assume that workers do not have any prior knowledge regarding their income profile upon entering the labor market. We view this assumption as a useful starting point to determine the effect of labor-market uncertainty on investment behavior. In Section 5, we consider the model specification where workers know to some degree their lifetime income profile and calibrate the amount of prior uncertainty based on Guvenen (2007) and Guvenen and Smith (2014). We show that our results remain largely intact, even with smaller amounts of prior uncertainty.

*Speed of Learning* We introduced additional noise in the priors updating rule to generate a more realistic gradual resolution of uncertainty. We assume that the noise, $q$, is drawn from the distribution $\mathbf{q}_j \sim N(0, Q_j)$. We also assume that the variance of noise $Q_j$, is a linearly decreasing function of age: $Q_j = \lambda_j \mathbf{V}_{1|0}$ where $\lambda_j = \overline{\lambda} + \frac{-\overline{\lambda}}{J_R - 1}(j-1)$ and $\mathbf{V}_{1|0}$ is the initial

[17] Studies like Deaton and Paxson (1994) and Storesletten, Telmer, and Yaron (2004) document a large increase in the variance of log-consumption around 30 points. Other studies like Heathcote, Storesletten, and Violante (2014) and Guvenen and Smith (2014) report a smaller rise, around 10 points, and a lower variance at age 25. As a result Guvenen and Smith (2014) find a smaller value for $\sigma_a^2$. We choose to follow the data estimates of Guvenen (2007) which basically fall between the aforementioned studies. We show that our results are fairly robust to values for $\sigma_a^2$ consistent with Guvenen and Smith (2014).

prior variance of ability parameters ($a$ and $\beta$). The variance of noise starts from $\overline{\lambda}\times$ $\mathbf{V}_{1|0}$, decreases over time, and converges to 0 by the time a worker retires from the labor market.

We interpret $q$ as a reset of priors about the earnings profile. Belief resetting may be particularly frequent among young workers, who hold a number of short-term and part-time jobs before settling into a long-term career. As a result, we link the frequency of belief resetting to occupational mobility patterns. According to Topel and Ward (1992), the average number of jobs held by workers within the first 10 years of entering the labor market is 7. Kambourov and Manovskii (2008) estimate that the average probability that workers between the ages of 23-28 will switch occupations is 0.26 for non-college-educated workers and 0.23 for those with some college education. Our parameterization suggests that $\lambda_j$ is on average 0.25 during the first 10 years in the labor market. This implies that for the first 10 years, workers reset their priors by 25% of their initial priors (or switch occupations with probability 25%). This is equivalent to workers having had 2.5 jobs in the first 10 years after entering the labor market.

One way to judge how fast income uncertainty is resolved over time is to check how fast the cross-sectional dispersion of consumption increases over the life cycle. As we will show in Section 5, our benchmark model closely tracks the age profile of the variance of log consumption in the data, as documented by Guvenen (2007).

# 5 Results

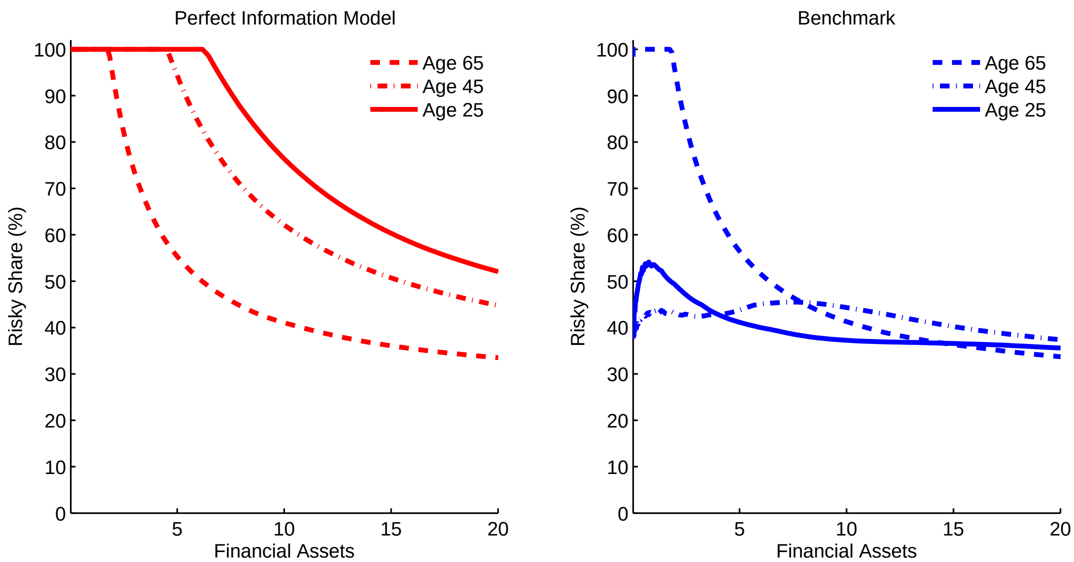
## 5.1 Policy Functions

In order to understand the basic economic mechanism of the model, we first compare the optimal portfolio choice (policy functions) of two specifications of the model: the perfect information model (PIM) and the benchmark. Figure 6 shows the optimal financial portfolio choice of a worker with median income at ages 25, 45, and 65, respectively. We show the risky share for a wide range of financial wealth (from 0 to 20). In our benchmark case, the average wealth is 7.02. In the PIM (left panel), consistent with our illustrative example in Section 3, the risky share decreases with both wealth and age. When the earnings ability is perfectly known, workers can predict future labor income (to the extent of the stochastic variation due to income shocks) fairly well. Poor workers at all three ages—25, 45, and 65—for whom (the relatively safe although stochastic) labor income makes up a large portion of total wealth, would like to invest their financial wealth in risky assets. This is also true for younger workers, who expect a long, relatively safe labor-income stream. For example, a 25-year-old worker with median labor income and average wealth level (about 7 in our model) would like to allocate almost all financial wealth to risky assets. In sum, a young and asset-poor worker exhibits a higher risky share when the earnings profile is perfectly known.

Table 5: Benchmark Parameters

| Parameter | Notation | Value | Target / Source |
|---|---|---|---|
| Life Cycle | $J$ | 60 | – |
| Retirement Age | $j_R$ | 45 | – |
| Risk Aversion | $\gamma$ | 5 | Average Risky Share =0.455 |
| Discount Factor | $\delta$ | 0.94 | Capital-Output Ratio=3.5 |
| Risk-free Rate | $R$ | 1.02 | Gomes and Michaelides (2005) |
| Equity-Risk Premium | $\mu$ | 0.04 | Gomes and Michaelides (2005) |
| Stock-Return Volatility | $\sigma_\eta$ | 0.18 | Gomes and Michaelides (2005) |
| Social Security Benefit | $ss$ | 0.40 | Replacement Ratio |
| Social Security Tax | $\tau_{ss}$ | 0.20 | Balanced Social Security Budget |
| Variance of Fixed Effect | $\sigma_a^2$ | 13% | Consumption Variance for Age 25 |
| Variance of Wage Growth | $\sigma_\beta^2$ | 0.030% | Guvenen and Smith (2014) |
| Variance of Transitory Component | $\sigma_\nu^2$ | 5.15% | Guvenen and Smith (2014) |
| Persistence Parameter | $\rho$ | 0.756 | Guvenen and Smith (2014) |
| Variance of i.i.d. component | $\sigma_\varepsilon^2$ | 1.0% | Guvenen and Smith (2014) |
| Common Age-Earnings Profile | $\{z_j\}_{j=21}^{65}$ | – | Hansen (1993) |
| Unemployment Risk | $\{p_j^u\}_{j=21}^{65}$ | Figure 5 | Choi, Janiak, and Villena-Roldan (2014) |
| Initial Variance of Noise | $\overline{\lambda}$ | 0.3 | Topel and Ward (1992), Kambourov and Manovskii (2008) |

Figure 6: Optimal Portfolio Choice for a Worker with Median Income




In our benchmark model (the right panel) where the income profile is uncertain, old workers are willing to take a lot of risk in making financial investments. The optimal portfolio choice of a 65-year-old worker, who retires next period, is similar to that of the PIM because the labor-income risk is pretty much irrelevant for him. A 45-year-old worker with an average amount of assets holds a risk share around 42%; hence, he is still cautious in making risky financial investments. A 25-year-old worker, who entered the labor market 4 years ago, faces two conflicting incentives for taking risk in financial investments. On the one hand, he would like to hedge against the large labor-market uncertainty. On the other hand, he would like to build up wealth quickly by taking advantage of the higher average rate of returns to stocks (life-cycle savings motive). According to our benchmark calibration, the wealth effect on risky share is not monotonic. The risky share increases with wealth when the wealth level is close to 0, indicating that the life-cycle savings motive dominates the desire to hedge against labor-market uncertainty for wealth-poor workers. The risky share declines with assets after the level of wealth exceeds 0.75 (one-tenth of the average wealth).

## 5.2 Comparison to Survey of Consumer Finances

We now generate statistics from a simulated panel of 10,000 workers from each model and compare them to those from the SCF. Table 6 presents the basic statistics regarding the three portfolio choice puzzles discussed in Section 2. The table reports the average risky share, the growth rate of average risky share by age, and the correlation between the risky share and wealth from two model specifications: the benchmark and the perfect information model (PIM). In both models we use the same values for all other parameters.

Table 6: Risky Shares: Data vs. Models

| Statistic | Data | PIM | Benchmark |
|---|---|---|---|
| Average risky share | 45.5% | 78.8% | 45.1% |
| Slope of profile (pp) | 0.92 | –1.24 | 0.36 |
| Correlation with wealth | 0.107 | –0.638 | –0.147 |

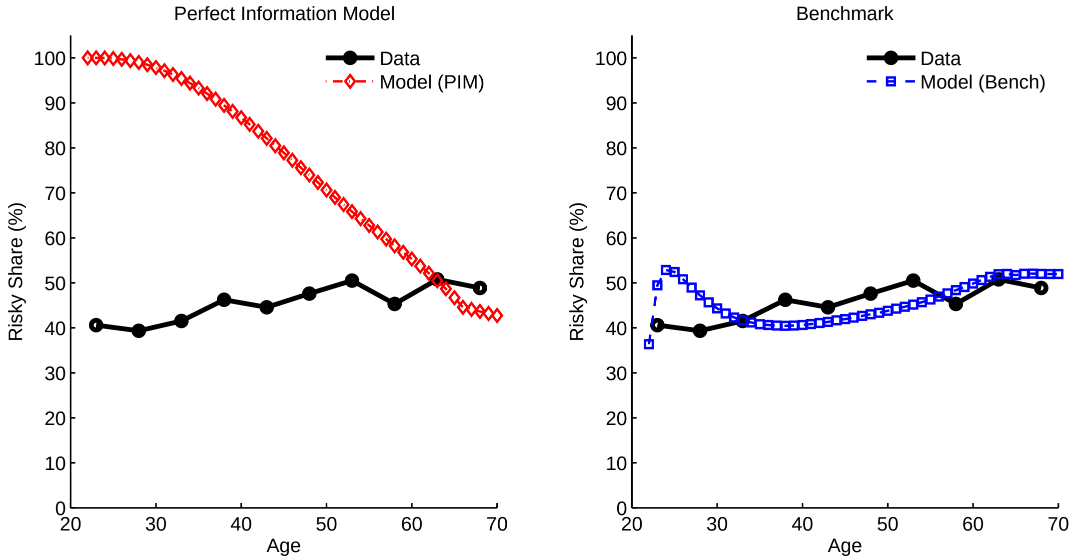
Note: The slope of the age profile refers to the average increase of the risky share (in percentage points) over the life cycle (ages 21 to 65). PIM refers to the perfect information model. The correlation with wealth is based on $Corr(W, \frac{s'}{s'+b'})$. The data statistics are based on the SCF.

Our benchmark model matches the risky share in the data with the relative risk aversion of 5: 45.5% in the data and 45.1% in the benchmark model. In the PIM, which is similar to the standard life-cycle model without uncertainty about income profile and unemployment risk, this ratio is 78.8%. To match the average risky share of 45.5% in the data, the PIM requires a value of relative risk aversion above 15 under the same parameterization of the income process. But even in this case, the PIM fails to generate an increasing profile of risky share over the life cycle.

We next turn our attention to the age profile. Financial advisors often recommend that young investors, facing a longer investment horizon, take more risk in financial investments. However, our data based on the SCF show that the risky share on average increases by 0.92 percentage point each year between ages 21 and 65 (Table 6). Our benchmark model successfully reproduces an increasing—but to a lesser degree than that in the data—age profile of risky share. In our benchmark model, on average, the risky share increases by 0.36 percentage point. Young workers, faced with much larger uncertainty in the labor market, would like to avoid too much risk in making financial investments. As the labor-market uncertainty is gradually resolved over time, through updating priors about their ability and decreasing unemployment risk, they start taking more risks. By contrast, the PIM generates a risky-share profile that *decreases* on average by 1.24 percentage points each year between ages 21 and 65.

Figure 7 plots the age profile of the risky share from both models. In the PIM, the risky share (left panel) starts with 100% at age 21, monotonically declines to 77.2% at age 45, and to 44.6% at age 65. In our benchmark model, however, the age profile of the risky share is not monotonic. It starts with a low level of 36.3% at age 21, quickly increases to 52.4% at age 24, decreases to 40.6% at age 38, and gradually increases to 52.0% at age 65. This is

Figure 7: Risky Share over the Life Cycle: Model




Note: Left panel shows the total asset, bond, and stock holdings as well as risky shares for the perfect information model (PIM). Right panel show the benchmark model.

because a young worker faces two conflicting incentives to take risks in making investments. On the one hand, he would like to hedge against the large labor-market uncertainty. On the other hand, he would like to build up his savings (life-cycle savings motive) quickly by taking advantage of the risk premium. When the worker enters the labor market, the former effect dominates, suppressing the risky share, then quickly the latter (life-cycle savings) effect comes in, inducing him to take some risks for a while until he accumulates some assets. Overall, our model roughly tracks the age profile of the risky share in the SCF. We view this as a partial resolution in reconciling the tension between the data and theory about households' portfolio choice over the life cycle.

The third portfolio-choice puzzle is the correlation between the risky share and wealth. According to the SCF, the total amount of financial assets and the risky share are weakly positively correlated across households, with a correlation coefficient of 0.107: wealthy households tend to take more risk in making financial investments. In the PIM, this correlation is strongly negative, with a correlation coefficient of -0.638. A decreasing age profile has contributed to this large negative correlation between wealth and risky share. Our benchmark model partially corrects for this large discrepancy between the data and the standard model and the correlation between the risky share and wealth becomes mildly negative: -0.147. Young workers (on average, asset-poor) would like to hedge the labor-market uncertainty by investing more conservatively. Table 7 reports the average risky shares for 5 wealth groups. In the data, the average risky share gradually increases from 35.9% in the first quintile to 66.6% in the 5th quintile. By contrast, in the PIM, the risky share monotonically decreases from 99% in the first quintile to 50% in the 5th. According to our benchmark model, the risky share mildly decreases with wealth: 50% in the first to 41% in the 5th quintiles.

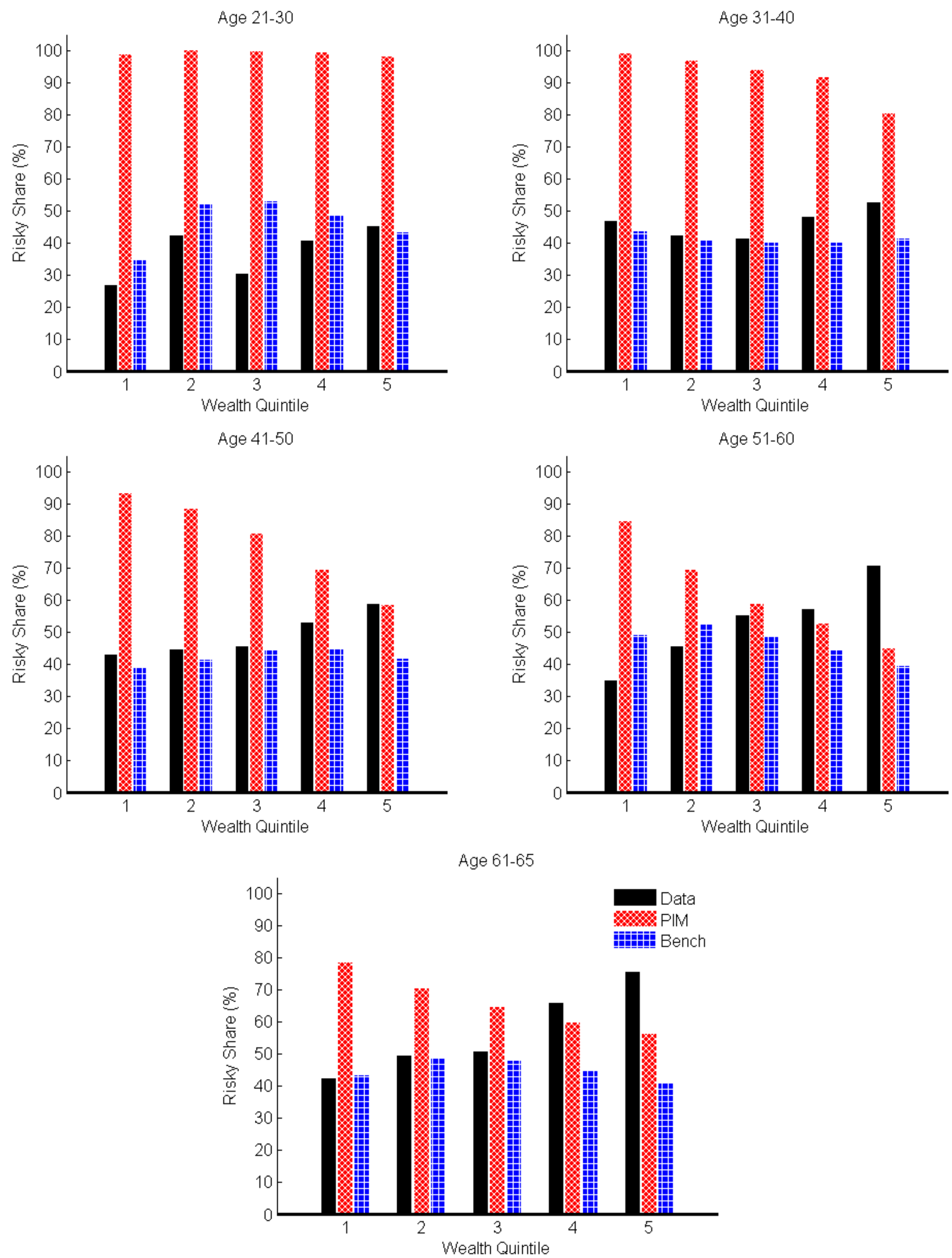
Table 7: Risky Share by Wealth

| Wealth Quintile | Data | PIM | Benchmark |
|---|---|---|---|
| $1^{st}$ | 35.9% | 98.9% | 50.0% |
| $2^{nd}$ | 40.5% | 95.1% | 45.4% |
| $3^{rd}$ | 44.4% | 85.6% | 44.6% |
| $4^{th}$ | 51.7% | 67.2% | 45.1% |
| $5^{th}$ | 66.6% | 49.2% | 41.0% |
| Average | 45.5% | 78.8% | 45.1% |

In the data, the positive correlation between the risky share and wealth still holds within all age groups. Figure 8 shows the average risky shares across wealth quintiles conditional on age (by age groups of 21-30, 31-40, 41-50, 51-60, and 61-65). In the data (solid), the risky share increases with the wealth within each age group. By contrast, the PIM (with $\Diamond$) predicts that the risky share monotonically decreases with wealth in all age groups. A poor worker—for whom labor income makes up a large portion of total wealth—would like to invest a larger portion of his financial wealth in risky assets. This is particularly true for young workers, ages 21-30, who allocate almost all of their financial wealth to risky assets. The benchmark model (with $\Box$), however, cannot match this pattern, as the risky share conditional on age is largely uncorrelated with wealth.

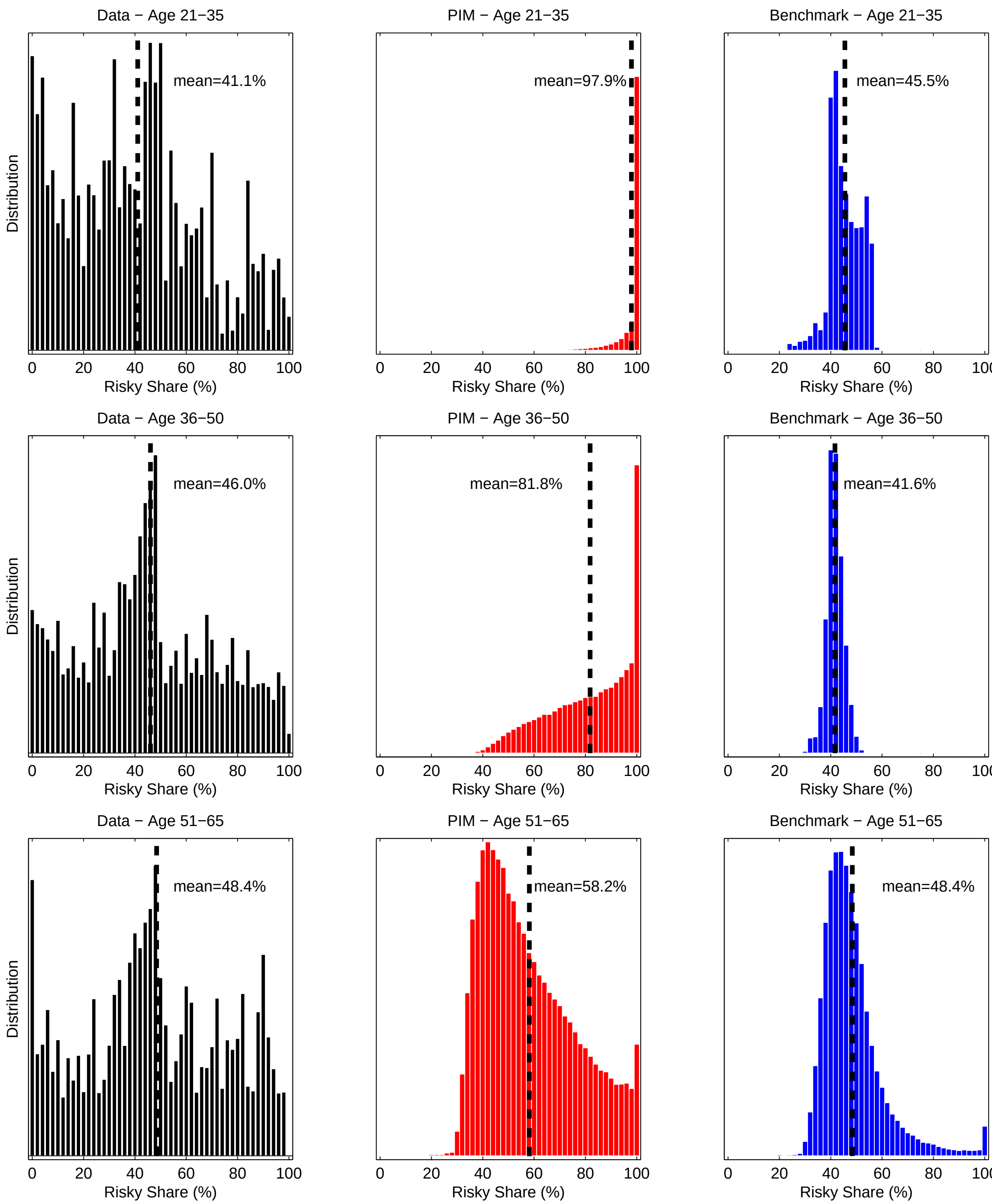
While we are mostly concerned about the first moment of the portfolio across households, that is, the average risky share over the life cycle, it is also informative to see how the entire distribution evolves with age. Figure 9 shows the cross-sectional distributions of the risky share for three 15-year groups, ages 21-35, 36-50, and 51-65. In the data (left panels), while the average risky share increases mildly, from 41.1% to 46.0%, and to 48.4%, respectively, for ages 21-35, 36-50 and 51-65, the portfolio choice is widely dispersed across all three age groups. The PIM (middle panels) generates a pattern of distribution by age that exhibits a sharp change in the skewness of distribution. On the other hand, our benchmark model (right panels) generates a much less skewed distribution along with a mild change in mean across all three age groups. Obviously, the portfolio choices are much more widely dispersed in the data: the risky share ranges from zero to one in all three age groups in the SCF. Even though

Figure 8: Risky Share and Wealth by Age Group




our model assumes heterogeneity in earnings profile, it fails to generate enough dispersions in portfolio distribution because it abstracts from many other important sources of heterogeneity such as inheritance, discount rate, and risk aversion, among others.

Figure 9: Distribution of Risky Share across Age Groups




## 5.3 Dispersion of Consumption by Age

It is a stylized fact that the cross-sectional dispersion of consumption increases over the life cycle. For example, Deaton and Paxson (1994) find that the variance of log consumption increases from 0.23 at age 25 to 0.5 at age 65. Storesletten, Telmer, and Yaron (2004) report a similar profile. Other studies find a lower, but still increasing shape. Guvenen (2007) reports that the variance of log consumption increases from 0.13 at age 25 to 0.4 at age 65. Guvenen and Smith (2014) and Heathcote, Storesletten, and Violante (2014) find the variance to be approximately 0.10 at age 25 to 0.20 at age 55. We follow the estimates of Guvenen (2007), which basically fall between the aforementioned studies. As part of our sensitivity analysis, we use a lower value of $\sigma_a^2$ that matches the lower estimates of Guvenen and Smith (2014) and Heathcote, Storesletten, and Violante (2014).

Figure 10 exhibits the age profile of the variance of log consumption in the data and those from the two models (PIM and benchmark). As Guvenen (2007) points out, a model without uncertainty about income profiles has difficulty generating the (linearly) increasing dispersion in consumption. For example, a standard life-cycle model with stochastic income shocks (with high persistence) produces a concave profile of consumption variance.[18] By contrast, a model where workers gradually learn their earnings abilities is able to generate an increasing age-profile of consumption variance: the cross-sectional dispersion of consumption, which depends on lifetime income, emerges as workers gradually learn about their earnings abilities. According to Figure 10 , the age profile of consumption dispersion from the benchmark model is in line with that in the data, whereas the PIM shows an increasing but concave profile.[19]

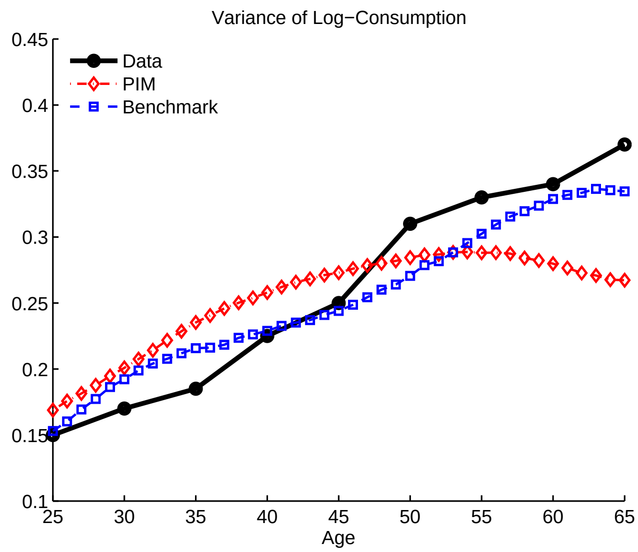
## 5.4 Decomposing the Contribution of Three Types of Uncertainty

We have introduced three types of labor-market uncertainty into the standard life-cycle model: (i) age-dependent unemployment risk, (ii) imperfect information about earnings ability, and (iii) noise in the Bayesian updating rule. In this section we evaluate the contribution of each element by considering two additional model specifications. The first additional model specification to consider is the PIM with age-dependent unemployment risk only, referred to as PIM with unemployment risk. The comparison of this model with the PIM will isolate the contribution of age-dependent unemployment risk. The second additional model to consider is the benchmark model without noise in the updating rule, i.e., $\overline{\lambda} = 0$. We call this version as Benchmark without noise.The comparison of this specification with the benchmark will provide a marginal contribution of noisy updating in matching the risky share in the data. Table 8 summarizes types of labor-market uncertainty in our various specifications. Table 9 reports the average risky share as well as those by wealth quintile. Figure 11 shows the age

[18] For example, the life-cycle model with an idiosyncratic income process similar to Storesletten, Telmer, and Yaron (2004).

[19] In our calibration we adopted the estimates of Guvenen and Smith (2014) for the slope of the profile, $\sigma_\beta^2$, and stochastic income shocks $\sigma_x^2$ and $\sigma_\varepsilon^2$ . We then chose $\sigma_a^2 = 0.13$ to match the consumption variance of young workers (initial dispersion in true ability).

Figure 10: Cross-Sectional Variance of Log Consumption by Age




Note: The age profile of the data is based on the estimate in Guvenen (2007).

profile of the risky share for all 4 model specifications.[20]

Table 8: Summary of Various Specifications

| | (1): PIM | (2): PIM w/ unemp risk | (3): Benchmark w/o noise | (4): Benchmark |
|---|---|---|---|---|
| Unemployment Risk | No | Yes | Yes | Yes |
| Imperfect Information | No | No | Yes | Yes |
| Noise in Updating | No | No | No | Yes |

Adding the age-dependent unemployment risk to the PIM slightly decreases the average risky share from 78.8% to 74.7% (the bottom row of Table 9). Figure 11 shows that the impact of unemployment risk on risky share is most important for young workers (line with "△"). For example, a 25-year-old worker who faces a 3% unemployment risk decreases the risky share from 99.6% to 87.5%. The impact of unemployment risk on the portfolio choice becomes negligible after age 40 when the annual unemployment risk becomes close to 1%.

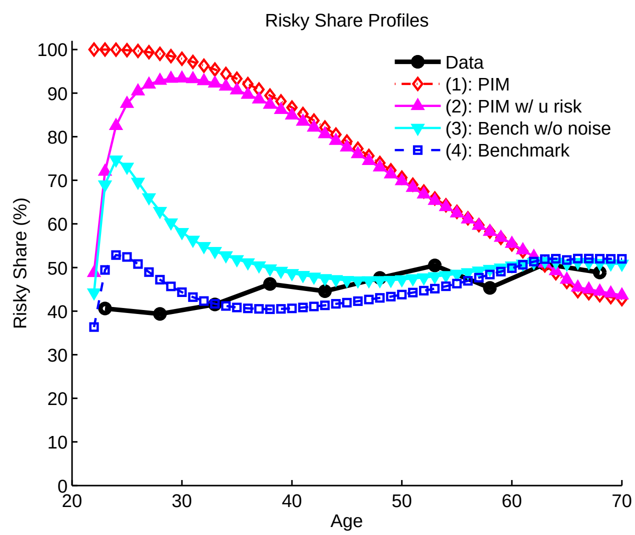
Introducing uncertainty about the income profile into this specification (thus moving from PIM with unemployment risk to Benchmark without noise) further decreases the average risky

[20] We recalibrate the discount factor so that all model specifications match the same wealth-income ratio.

Table 9: Risky Shares: Various Specifications

| Wealth Quintile | Data | (1): PIM | (2): PIM w/ unemp risk | (3): Benchmark w/o noise | (4): Benchmark |
|---|---|---|---|---|---|
| 1st | 35.9% | 98.9% | 80.1% | 65.7% | 50.0% |
| 2nd | 40.5% | 95.1% | 91.4% | 54.6% | 45.4% |
| 3rd | 44.4% | 85.6% | 84.7% | 51.6% | 44.6% |
| 4th | 51.7% | 67.2% | 67.6% | 49.9% | 45.1% |
| 5th | 66.6% | 49.2% | 50.3% | 42.4% | 41.0% |
| Average | 45.5% | 78.8% | 74.7% | 52.9% | 45.1% |

Figure 11: Risky Share Profiles from Models




Note: "Benchmark" features all three types of labor-market uncertainty: unemployment risk, imperfect information about the income profile, and noise in updating formula ($\overline{\lambda} = 0.3$). "PIM w/ u" refers to PIM with unemployment risk. "Benchmark w/o noise" refers to the benchmark without noise in updating formula ($\overline{\lambda} = 0$). In each model specification, all other parameters remain the same as those in Benchmark.

share to 52.9% in Table 9. The age profile of the risky share in this specification (with "$\bigtriangledown$") shows that the impact of imperfect information is significant across all age ranges, except the very young (ages 21-23). Large labor-market uncertainty affects the very young, borrowing-constrained investors in two conflicting ways. On the one hand, the uncertainty about labor-market outcomes discourages risky investment. On the other hand, it motivates workers to accumulate assets faster in these early years to insure against adverse labor-income shocks in the future. Relatively wealthier, very young investors can afford to invest in risky assets that generate higher returns on average. This explains why imperfect information has a small impact for ages younger than 25. Although sizable in its magnitude, the benchmark without noise model cannot fully account for the low risky share. With standard Bayesian learning, the long-run income risk perceived by workers can be significant, as Guvenen (2007) points out. However, for portfolio choice, labor-market uncertainty over short horizons is more important because workers can re-balance their portfolios every period. With standard Bayesian learning, short-run income risk is quickly resolved, which explains why the benchmark without noise model can only partially address the puzzles. Adding the noise in the updating rule reduces the average risky share to a level consistent with the data (45.5%). The most impact takes place for younger workers since noise is greater during the first years of one's career. We discuss more about the difference between short- and long-run income risk in the next subsection.

Looking across wealth quintiles, unemployment risk affects mostly wealth-poor workers. Combining age-dependent unemployment risk and imperfect information about ability brings down the risky share in all quintile groups significantly. Finally, the noise in the updating rule has the most impact on the 1st and 2nd quintile of wealth groups.

Overall, in accounting for the total decrease in average risky share from 78.8% (PIM) to 45.1% (benchmark), (i) age-dependent unemployment risk has contributed 12%, (ii) imperfect knowledge about the income profile has contributed the most, 65%, and (iii) noise in the priors-updating rule has contributed about 23%.

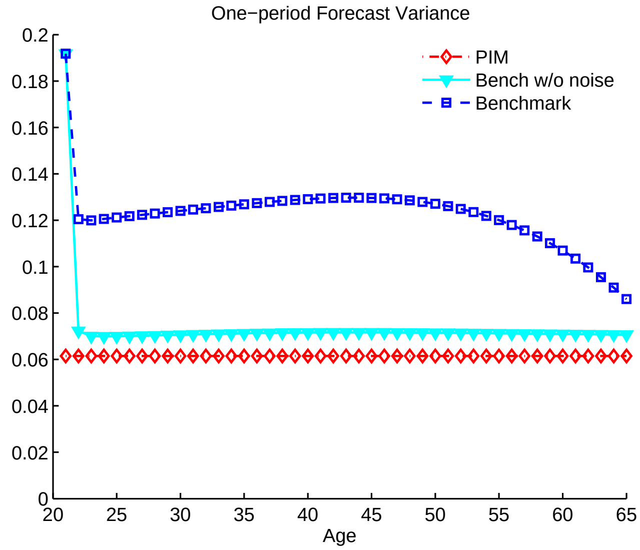
## 5.5 Speed of Learning: Short- vs. Long-run Uncertainty

In order to better match the risky share of young workers in the data, our model requires an additional noise in the Bayesian updating rule. Without this noise, the short-run uncertainty about the income profile is resolved extremely fast. To distinguish between short-run and long-run income risk, we use the forecast-error variance or minimum square error (MSE), as a measure of uncertainty proposed by Guvenen (2007) and Guvenen and Smith (2014). The forecast error variance is defined as:

$$\mathrm{MSE}_{j+s|j} = \mathbf{H}'_{j+s}\mathbf{V}_{j+s|j}\mathbf{H}_{j+s} + \sigma^2_{\varepsilon_j} \quad \text{with} \quad \mathbf{V}_{j+s|j} = \mathbf{R}^s\mathbf{V}_{j|j}\mathbf{R}'^s + \sum_{i=0}^{s-1}\mathbf{R}^i\mathbf{S}\mathbf{R}'^s.$$

The speed of learning about ability can be measured as the rate at which labor income risk (measured by the MSE) under imperfect information converges to the one under perfect information. We find that labor-market uncertainty at different horizons $s$ (e.g., short-run vs. long-run) resolves at different rates. In particular, we find that under the standard Bayesian

Figure 12: One-Period Forecast Error Variance of Income




Note: The one-period forecast-error variance about income by age groups for three model specifications: PIM, benchmark without noise, and benchmark.

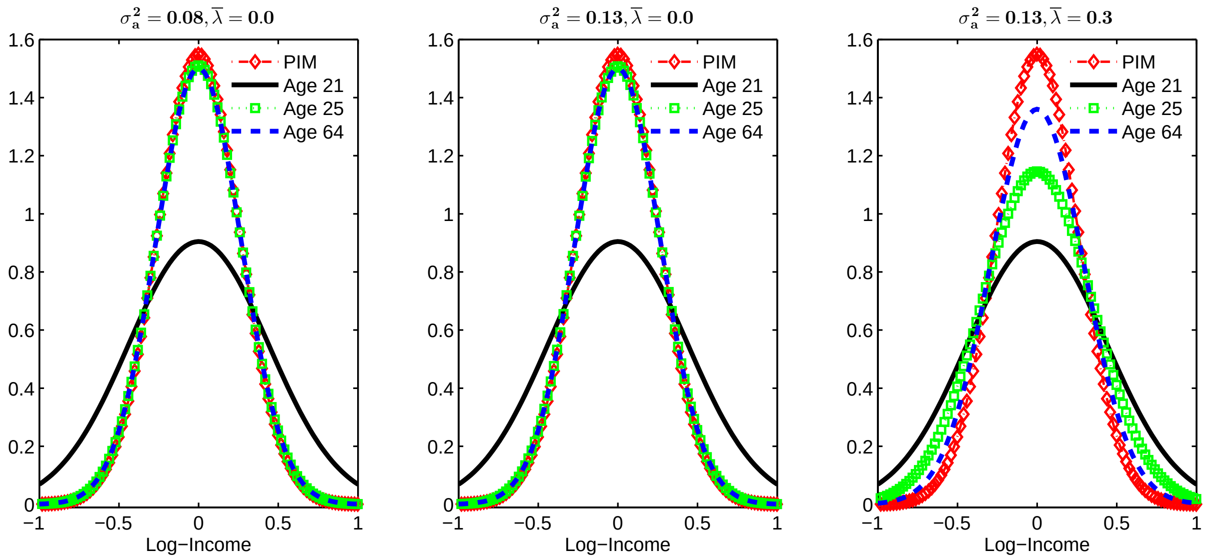
updating rule, the short-run uncertainty resolves extremely fast, even though the long-run uncertainty remains large.

Figure 12 shows the *one-period* forecast-error income variance, $\mathrm{MSE}_{j+1|j}$, by age, for three model specifications: PIM, benchmark without noise, and benchmark. The one-period forecast error income variance of the PIM (line with diamonds) reflects the uncertainty due to stochastic income shocks only ($\sigma_\nu^2$ and $\sigma_\varepsilon^2$). Thus, it is not age dependent, by construction. Without the noise in the updating rule (line with triangles), almost all uncertainty over a one-period horizon is resolved within a few years. We do not view this as realistic, considering the frequent job turnover at younger ages and the amount of time it takes for young workers to settle into a long-term career (e.g., Topel and Ward (1992)). We introduced noise into the Bayesian updating rule as a short cut to obtain a more realistic rate at which uncertainty is resolved. The effect can be seen in the line with squares. Uncertainty over a one-period horizon is significantly larger compared to the PIM and now is resolved at a much slower rate.[21]

To elaborate more on this issue, we explore how the noise in updating affects workers' belief (perceived probability) about the next period's income. Figure 13 plots the distributions of this belief for each of three specifications: (i) small dispersion in the intercept term of the

[21] As Guvenen (2007) points out, non-monotonicity occurs due to the growth component $\beta \times j$. The variance of this component increases exponentially with age, which can significantly slow down the resolution of uncertainty. This also takes place with $\bar{\lambda} = 0.0$ but the effect is much smaller.

Figure 13: Distribution of Beliefs for Next Period's Income



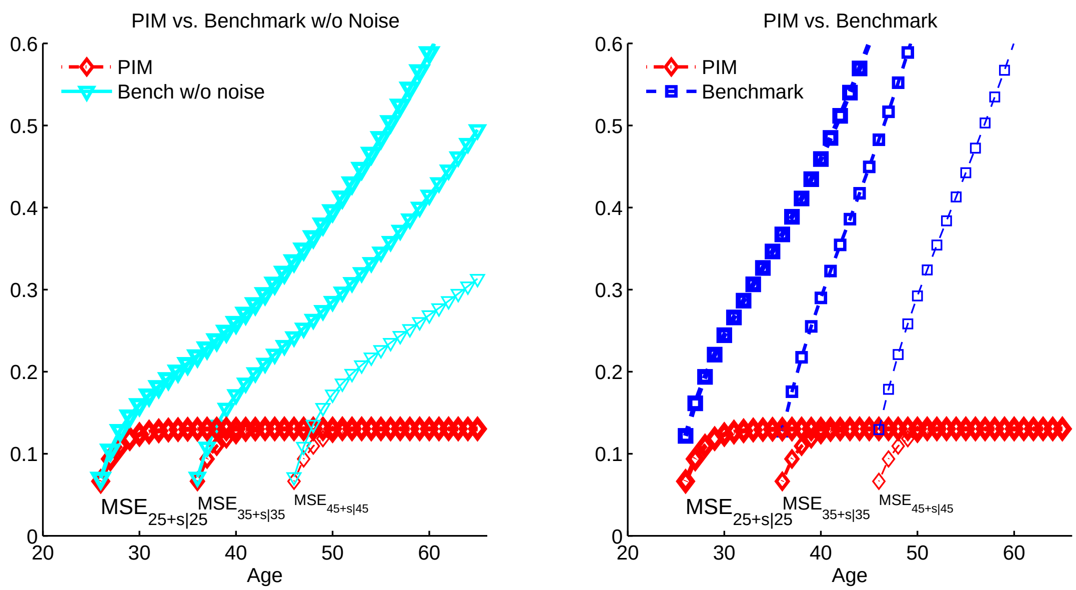

Note: Distribution of beliefs (perceived probability) for next period's log income. The variance of the intercept in the income profile is $\sigma_a^2$. The variance of noise relative to the initial prior variance of the income profile is $\bar{\lambda}$. PIM represents the probability distribution under perfect information about the profile.

ability profile: $\sigma_a^2 = 0.08$ and $\bar{\lambda} = 0$, (ii) benchmark without noisy updating: $\sigma_a^2 = 0.13$ and $\bar{\lambda} = 0$, and (iii) benchmark: $\sigma_a^2 = 0.13$ and $\bar{\lambda} = 0.3$. The reason we include the case with small dispersion in $a$, $\sigma_a^2 = 0.08$, is that this specification is identical to the income process used in Guvenen and Smith (2014). Note that our benchmark specification adopted a slightly larger value of $\sigma_a^2 = 0.13$ to match a slightly larger initial variance of consumption, i.e., cross-sectional variance of workers age 25. For comparison, we also plot the probability distribution for the next period's income in the PIM, which reflects the variance from the stochastic income shocks only, and thus does not depend on age and iscommon across three different specifications. We show the beliefs for three ages: 21 (new entrant), 25 (4 years of labor-market experience), and 64 (retire next year).

The left panel plots the distribution for $\sigma_a^2 = 0.08$ and $\bar{\lambda} = 0$, which corresponds to the parameterization of Guvenen and Smith (2014). A 21-year-old worker who just entered the labor market forecasts the next period's income with the probability distribution shown by the solid line. In 4 years, when he is 25 years old, his belief about the next period's income has shrunk significantly and is almost indistinguishable from that of the PIM. The middle panel plots the same distributions for $\sigma_a^2 = 0.13$ and $\bar{\lambda} = 0$ (benchmark without noise). Again, without noise in the updating rule, the belief quickly converges to that of the PIM. By age 25 (within 4 years after entering the labor market) the probability distribution becomes almost identical to that of the PIM where the income profile is perfectly known.[22] When we introduce noise into the updating rule (the right panel with $\sigma_a^2 = 0.13$ and $\bar{\lambda} = 0.3$), learning becomes much slower.

We have shown that income uncertainty over short horizons is resolved extremely fast without the noise in updating. However, this is not true for uncertainty over a longer time horizon. Figure 14 shows the forecast error variance for all horizons by three age groups: 25, 35, and 45. The left panel compares the forecast error between the benchmark without noise model (plotted as triangles) and the PIM (plotted as diamonds). The right panel compares the forecast error between the benchmark (plotted as squares) and the PIM (plotted as diamonds). In the left panel the difference between the forecast variance is very small over a short time horizon (1-5 years) but increases significantly over a long time horizon. Uncertainty about the growth of the profile $\beta$ translates into substantial income risk, but only over longer horizons. In the right panel (benchmark) income risk is significantly larger under imperfect information over *both* a short and a longer time horizon.

Figure 14: Forecast Error Variance of Future Income: Different Horizon




Note: Left panel shows the $\mathrm{MSE}_{j+s|j}$ for three age groups $j = 25, 35, 45$ for all horizons $s$ for two model specifications: PIM and benchmark without noise. Right panel plots the same graphs for: PIM and benchmark.

This distinction between short- and long-run uncertainty is subtle but important for the

[22]The uncertainty is resolved at a faster rate in our model compared to the one using the parameterization by Guvenen and Smith (2014) because the signal-noise ratio is larger in our model (i.e., a larger $\sigma_a^2$ relative to $\sigma_\nu^2$ and $\sigma_\varepsilon^2$).

portfolio choice. The lifetime uncertainty about the earnings ability is important for the *total* savings, which is well illustrated by Guvenen (2007). However, for the portfolio choice, the labor-market uncertainty over the short horizon (e.g., one-period forecast-error variance) is more relevant because workers can adjust their portfolios frequently (e.g., every year in our model). In other words, for the decision on the total amount of savings, uncertainty at the longer horizon is important (because it takes a time to build up assets). For the decision on portfolio choice, uncertainty at the short-run horizon is more relevant because workers are able to adjust their portfolios frequently.

## 5.6 Sensitivity Analysis

We perform various sensitivity analyses to see whether our main results are robust with respect to different parameterizations. In particular, we are concerned with the robustness in four dimensions. First, we examine the case where workers have some private information about their ability upon entering the labor market. Second, we consider two alternative values of relative risk aversion: $\gamma = 3$ and $\gamma = 4$. Third, we see how the initial distribution of wealth (the wealth distribution of 21-year-old workers) affects the results. Finally, we consider the model with a smaller dispersion in the intercept of earnings profiles, $\sigma_a^2 = 0.08$, the value used in Guvenen and Smith (2014) for the direct comparison with Guvenen and Smith (2014). In each sensitivity analysis, we keep all other parameters of the model the same as those in our benchmark specification. Table 10 reports the results of these sensitivity analyses.

In our benchmark model we assumed that workers (or firms) are not informed about their earnings ability upon entering the labor market. This might be too extreme given that workers might have some private information that econometricians do not have access to. Indeed, Guvenen (2007) and Guvenen and Smith (2014) find that workers know a significant fraction of their lifetime income. In our model, the amount of prior knowledge is given by the matrix:

$$\mathbf{V}_{1|0} = \begin{bmatrix} (1-\psi_a)\sigma_a^2 & \sigma_{a\beta} & \sigma_{ax} \\ \sigma_{a\beta} & (1-\psi_\beta)\sigma_\beta^2 & \sigma_{\beta x} \\ \sigma_{ax} & \sigma_{\beta x} & \sigma_x^2 \end{bmatrix}$$

where all variances and covariances equal the population moments. In our benchmark we assume that $\psi_a = 0, \psi_\beta = 0$: workers have no more extra information than the econometricians do, upon entering the labor market. Following Guvenen (2007) and Guvenen and Smith (2014), we experiment with four sets of values: $\{\psi_a = 0, \psi_\beta = 0.60\}$, $\{\psi_a = 0, \psi_\beta = 0.80\}$, $\{\psi_a = 0.60, \psi_\beta = 0.60\}$, $\{\psi_a = 0.80, \psi_\beta = 0.80\}$.[23] The more the worker knows about his

[23] Guvenen (2007) and Guvenen and Smith (2014) examine prior uncertainty with respect to $\sigma_\beta^2$. Since in

Table 10: Risky Shares: Sensitivity Analysis

| Model | Average Risky Share | Slope of Profile (pp) | $Corr(W, \frac{s'}{s'+b'})$ |
|---|---|---|---|
| **Data** | **45.5%** | **0.96** | **0.106** |
| Benchmark | 45.1% | 0.36 | $-0.147$ |
| $\psi_a = 0, \psi_\beta = 0.60$ | 46.0% | 0.36 | $-0.183$ |
| $\psi_a = 0, \psi_\beta = 0.80$ | 46.9% | 0.38 | $-0.206$ |
| $\psi_a = 0.60, \psi_\beta = 0.60$ | 47.4% | 0.31 | $-0.231$ |
| $\psi_a = 0.80, \psi_\beta = 0.80$ | 49.4% | 0.26 | $-0.287$ |
| $\gamma = 4$ | 60.0% | 0.18 | $-0.228$ |
| $\gamma = 3$ | 84.0% | $-0.26$ | $-0.273$ |
| Initial Assets $= 0.1 \times \bar{W}$ | 45.2% | $-0.12$ | $-0.191$ |
| $\sigma_a^2 = 0.08, \overline{\lambda} = 0.3$ | 46.8% | 0.41 | $-0.178$ |
| $\sigma_a^2 = 0.08, \overline{\lambda} = 0.0$ | 54.3% | 0.40 | $-0.340$ |

Note: The benchmark features $\psi_a = 0$, $\psi_\beta = 0$, $\gamma = 5$, $\sigma_a^2 = 0.13$, $\overline{\lambda} = 0.3$, and zero initial assets.

income profile upon entering the labor market, the smaller the labor-market uncertainty he faces. As a result, the average risky share increases compared to the benchmark case. When a worker knows 80% of his profile ($\{\psi_a = 0.80, \psi_\beta = 0.80\}$), the average risky share increases to 49.6% compared to the benchmark 45.1%, while the risky share profile is still increasing (but at a smaller rate). The correlation with wealth becomes more negative: -0.28 compared to -0.14. In all other cases the results are close to those in our benchmark. As long as labor-income uncertainty takes a while to resolve, even small amounts of prior uncertainty can significantly deter young workers from taking financial risk.

The portfolio choice is obviously sensitive to the risk-aversion parameter. In our benchmark model, the relative risk aversion is 5. As we lower the relative risk aversion to $\gamma = 4$ and $\gamma = 3$, the risky share significantly increases to 60.0% and 84.0%, respectively. The increasing pattern of the age profile remains the same with $\gamma = 4$: the risky share increases by 0.18 percentage point on average. But the age profile shows a mild negative slope with $\gamma = 3$: the risky share decreases with age by -0.26 percentage point on average. The correlation with wealth also decreases to -0.228 and -0.273, respectively, with $\gamma = 4$ and $\gamma = 3$.

Young workers enter the labor market with zero assets in our benchmark model. While

our parameterization $\sigma_a^2$ is set to a larger value, we also experiment with the prior uncertainty regarding this parameter.

some young workers enter the labor market with significant amounts of debt in practice, many can borrow or rely on family financing. To reflect this possibility, we assume that young workers enter the labor market with a small amount of wealth, 10% of the economy-wide average wealth. While it has little impact on the average risky share (45.2%), it does affect the age profile of risk share. Now, the profile has a mild negative slope on average: it decreases by 0.12 percentage point each year.

In our benchmark parameterization, we choose the initial dispersion of ability, $\sigma_a^2 = 0.13$, to match the cross-sectional variance of log consumption of 25-year-old workers in the data (from Guvenen (2007)). We also experiment with $\sigma_a^2 = 0.08$ following Guvenen and Smith (2014), which targets a slightly smaller consumption dispersion. With the same amount of relative variance for the noise in the priors-updating rule (i.e., $\sigma_a^2 = 0.08$ and $\overline{\lambda} = 0.3$), the result is similar to that of the benchmark model. The average risky share slightly increases to 46.8% and the average risky share increases over the life cycle by 0.40 percentage point, similar to our benchmark. The correlation with wealth slightly decreases to -0.178. Without noise in the updating rule ($\sigma_a^2 = 0.08$ and $\overline{\lambda} = 0.0$), the results change noticeably but they are similar to our benchmark without noise ($\sigma_a^2 = 0.13$ and $\overline{\lambda} = 0.0$), reported in Table 9. Hence, our choice of a slightly larger $\sigma_a^2$ does not have a significant impact on the main results of our analysis on the portfolio choice.

# 6 Industry Income Volatility and Risky Share

According to our model, the portfolio choice depends on uncertainty in the labor market as well as that in the financial market. Our theory predicts that workers in jobs (e.g., industries or occupations) with highly volatile earnings should be conservative in making financial investments. Testing this implication is not simple because workers also self-select into different industries and occupations with known volatility. Despite this limitation, we examine the partial correlation between the risky share of household and the labor-income risk of industry.

The SCF provides information about households' main job including the industry. For labor-income risk measures, we use the estimates by Campbell, Cocco, Gomes, and Maenhout (2001) based on the PSID.[24] According to these estimates, workers in agriculture face the largest uncertainty in income with a average variance of income shock of 31.7%, whereas those in public administration face the smallest variance, 4.7%.[25]

[24] The income specification used by Campbell, Cocco, Gomes, and Maenhout (2001) is $\log(Y_{it}) = f(t, Z_{i,t}) + \nu_{i,t} + \varepsilon_{i,t}$ where $f(t, Z_{i,t})$ is a deterministic function of age and other characteristics, $\nu_{i,t}$ represents a permanent shock that evolves based on $\nu_{i,t} = \nu_{i,t-1} + u_{it}$, with $u_{i,t} \sim N(0, \sigma_u^2)$ while $\varepsilon_{i,t}$ is a temporary shock with $\varepsilon_{i,t} \sim N(0, \sigma_\varepsilon^2)$. The variances reported here are the sum of the estimated variances for $\sigma_u^2$ and $\sigma_\varepsilon^2$ for every industry.

[25] The variances of income shocks are 10.8% in construction, 8.9% in wholesale and retail trade, 9% in

Households' risky shares are regressed on the income-risk measure of the industry and other individual characteristics such as total income, age, education, number of children, and marital status. Table 11 reports the estimated coefficients and their standard errors of this regression. The statistically significant negative coefficient on labor-income risk confirms that larger labor-market risk crowds out financial risk. As the risk in the labor market (the variance of the labor-income shock) increases by 1%, the household's risky share decreases by 0.22 percentage point (with a standard error of 0.06). This is consistent with Angerer and Lam (2009), who find a negative correlation between labor-income risks and the share of risky assets from the NLSY 1979 cohort. The other coefficients are consistent with our economic priors. Workers with more education (a proxy for permanent income) and total income take more risks in making financial investments. So do older workers.

Table 11: Regression of Risky Share on Income Risk of Industry

| Dependent Variable = Household's Risky Share | | |
|---|---|---|
| Constant | $-0.8296^{***}$ | (0.0202) |
| **Industry's income risk** | $-0.2234^{***}$ | (0.0601) |
| Log Income | $0.0973^{***}$ | (0.0017) |
| Age | $0.0017^{***}$ | (0.0002) |
| Education | $0.1926^{***}$ | (0.0060) |
| Number of children | $-0.0121^{***}$ | (0.0016) |
| Marriage dummy | 0.0080 | (0.0052) |

Notes: The numbers in parentheses are standard errors. Measures of an industry's labor-income risk are based on Campbell, Cocco, Gomes, and Maenhout (2001).

# 7 Conclusion

We develop a quantitative life-cycle model to examine whether labor-market uncertainty helps us to understand the long-standing puzzles in household portfolio choice. (i) Households hold a small amount of equity despite the higher average rate of returns (equity premium puzzle). (ii) The share of risky assets increases with the age of the household. (iii) The share of risky assets is disproportionately larger for richer households.

The model features three types of labor-market uncertainty: a gradual learning about true

transportation and finance, 11.8% in business services, 6.7% in transportation and communications, and 5.2% in manufacturing.

earnings ability, age-dependent unemployment risk, and noise in the priors-updating rule. When the model is calibrated to match the age profiles of earnings, unemployment risk, and consumption dispersion in the data, the model provides a partial resolution to these puzzles.

The model is able to reproduce the average risk share (45 percent) in the data under the relative risk aversion of 5, which is a much smaller value than that required in the standard model. According to the SCF, the risky share increases on average by 0.92 percentage point each year between ages 21 and 65. Our model also generates an increasing—but to a lesser degree than that in the data—age profile of the risky share: it increases by 0.36 percentage point each year. The standard life-cycle model predicts a strongly negative correlation between the risky share and wealth, whereas it is mildly positive in the data. Our model partially corrects for this failure and generates a mildly negative correlation between the risky share and wealth.

In our model, young workers, on average asset poor, face larger labor-market uncertainty because of high unemployment risk and imperfect knowledge about their earnings ability. To hedge against this labor-market uncertainty, young investors are less willing to take risks in making financial investments. As labor-market uncertainty resolves over time, they take more risks in the financial market. Our theory predicts that workers in an industry with highly volatile earnings should take less risk in financial portfolio. Based on the regression of the household's risky share on the industry's labor-income risk, we find that a household working in an industry where higher labor-income volatility is 10% larger than the mean exhibits a risky share that is 2.2% lower than the average.

We argue that a complete theory of investment should consider the risk that investors face not only in the financial market but also elsewhere, especially in the labor market.

# References


Ameriks, John and P. Stephen Zeldes. 2004. "How do household portfolio shares vary with age?" *Working Paper* .

Angerer, Xiaonhong and Pok-Sang Lam. 2009. "Income Risk and Portfolio Choice: An Empirical Study." *The Journal of Finance* 64 (2):1037–1055.

Ball, Stephane. 2008. "Stock market participation, portfolio choice and pensions over the life cycle." *Working Paper* .

Benzoni, L. and O. Chyruk. 2009. "Investing over the life cycle with long-run labor income risk." *Economic Perspectives, Federal Reserve Bank of Chicago* (33, 3rd Quarter).

Benzoni, Luca, Pierre Collin-Dufresne, and S. Robert Goldstein. 2007. "Portfolio Choice over the Life-Cycle when the Stock and Labor Markets Are Cointegrated." *The Journal of Finance* 62 (5):2123–2167.

Campanale, Claudio. 2011. "Learning, ambiguity and life-cycle portfolio allocation." *Review of Economic Dynamics* 14 (2):339–367.

Campbell, J., J. Cocco, F. Gomes, and P. Maenhout. 2001. "Investing Retirement Wealth: A Life-Cycle Model." In *Risk Aspects of Investment-Based Social Security Reform*, edited by M. Feldstein and J. Campbell. NBER.

Choi, Sekyu, Alexandre Janiak, and Benjamin Villena-Roldan. 2014. "Unemployment, Participation and Worker Flows Over the Life Cycle." *Working Paper* .

Cocco, F. Joao. 2005. "Portfolio Choice in the Presence of Housing." *The Review of Financial Studies* 18 (2):491–533.

Cocco, F. Joao, J. Francisco Gomes, and J. Pascal Maenhout. 2005. "Consumption and Portfolio Choice over the Life-cycle." *The Review of Financial Studies* 18 (2):491–533.

Davis, Steven J. and Paul Willen. 2000. "Occupation-Level Income Shocks and Asset Returns: Their Covariance and Implications for Portfolio Choice." NBER Working Papers 7905, National Bureau of Economic Research, Inc.

Deaton, Angus and Christina Paxson. 1994. "Intertemporal Choice and Inequality." *Journal of Political Economy* 102 (3):437–67.

Edwards, Ward. 1968. *Conservatism in Human Information Processing.* Cambridge, MA: Cambridge University Press.

Gomes, J. F. and Alexander Michaelides. 2005. "Optimal life-cycle asset allocation: Understanding the empirical evidence." *The Journal of Finance* 60 (2):869–904.

Guiso, Luigi, Michael Haliassos, and Tullio Jappelli. 2002. *Household Portfolios*. Cambridge, MA: MIT Press.

Guvenen, Fatih. 2007. "Learning Your Earning: Are Labor Income Shocks Really Very Persistent?" *American Economic Review* 97 (3):687–712.

Guvenen, Fatih and Anthony A. Smith. 2014. "Inferring Labor Income risk and Partial Insurance from Economic choices." *Ecomometrica* :Forthcoming.

Hansen, D. Gary. 1993. "'The cyclical and secular behavior of labor input: Comparing efficiency units and hours worked." *Journal of Applied Econometrics* 97 (3):687–712.

Heathcote, Jonathan, Kjetil Storesletten, and Giovanni L. Violante. 2014. "Consumption and Labor Supply with Partial Insurance: An Analytical Framework." *American Economic Review* :Forthcoming.

Huggett, Mark and Greg Kaplan. 2013. "The money value of a man." *Working Paper* .

Jagannathan, Ravi and R. Narayana Kocherlakota. 1996. "Why Should Older People Invest Less in Stocks Than Younger People?" *Quarterly Review, Federal Reserve Bank of Minneapolis* 20 (3):11–23.

Kambourov, Gueorgui and Iourii Manovskii. 2008. "Rising occupational and industry mobility in the United States: 1968-1997." *International Economic Review* 49 (1):41–79.

Lynch, W. Anthony and Sinan Tan. 2011. "Labor income dynamics at business-cycle frequencies: implications for portfolio choice." *Journal of Financial Economics* 101 (2):333–359.

Samuelson, Paul A. 1969. "Lifetime Portfolio Selection by Dynamic Stochastic Programming." *The Review of Economics and Statistics* 51 (3):239–46.

Storesletten, Kjetil, Christopher I. Telmer, and Amir Yaron. 2004. "Consumption and risk sharing over the life cycle." *Journal of Monetary Economics* 51 (3):609–633.

Storesletten, Kjetil, I. Christopher Telmer, and Amir Yaron. 2007. "Asset Pricing with Idiosyncratic Risk and Overlapping Generations." *Review of Economic Dynamics* 10 (4):519–548.

Topel, H. Robert and P. Michael Ward. 1992. "Job mobility and the careers of young men." *Quarterly Journal of Economics* 107 (2):439–479.

Wachter, A. Jessica and Motohiro Yogo. 2010. "Why Do Household Portfolio Shares Rise in Wealth?" *The Review of Financial Studies* 23 (11):3929–3965.

Wang, Neng. 2009. "Optimal consumption and asset allocation with unknown income growth." *Journal of Monetary Economics* 56 (4):524–534.

# Appendix

# A Data: Survey of Consumer Finances

**General Description** The Survey of Consumer Finances is a cross-sectional survey conducted every 3 years. It provides detailed information on the finances of families in the U.S.A. Respondents are selected randomly with a strong attempt to select families from all economic strata. The "primary economic unit" consists of an economically dominant single individual or couple (married or living as partners) in a household and all other individuals who are financially dependent on that individual or couple. In a household with a mixed-sex couple the "head" is taken to be the male. One set of the survey cases was selected from a multistage area-probability design and provides good coverage of characteristics broadly distributed in the population. The other set of survey cases was selected based on tax data. This second sample was designed to disproportionately select families that were likely to be relatively wealthy. Weights compensate for the unequal probabilities of selection. To deal with respondents who were unable to provide a precise answer, the survey gives the option of providing a range. While the number of respondents is 4,309, the number of observations is 21,545. This is because variables that contained missing values have been imputed five times, drawing repeatedly from an estimate of the conditional distribution of the data. Multiple imputation offers a couple of advantages over singly imputed data.

**Example of Survey** We provide an example of the questionnaire related to checking accounts. The following questions are asked, among others. 1) Do you have any checking accounts at any institution? 2) How many checking accounts do you have? 3) How much is in this account? (What was the average over the last month?). In some other accounts like individual retirement accounts the respondent is asked specifically how the money is invested. The questions are: 1) Do you have any individual retirement accounts? 2) How much in total is in your IRA(s)? 3) How is the money in this IRA invested? Is most of it in certificates of deposit or other bank accounts, most of it in stocks, most of it in bonds or similar assets or what? The possible answers are: 1) CDs/ Bank accounts; money market; 2) Stock; Mutual funds; 3) Bonds/ Similar assets; T-Bills; Treasury notes; 4) Combinations of 1, 2, 3; 5) Combinations of 2, 3; 6) Combinations of 1, 2; 7) Universal life policy or other similar insurance products, 8) Annuity, 9) Commodities, 10) Real estate/mortgages, 11) Limited partnership/ Other similar investments, 12) Brokerage accounts, 13) Split/ Other.

**Construction of Variables** In this section we explain the types of assets we categorize as safe and risky. In all our definitions, we make use of weights, variable $X42001$.

- *Checking accounts, Money Market Accounts*: The variables X3506, X3510, X3514, X3518, X3522, X3526 report the amount of money the respondent has in six different accounts. The respondent is asked whether each of these accounts is a checking account or a money market account. Responses can be found in variables X3507, X3511, X3515, X3519, X3523, X3527. We define **Checking Accounts** (and respectively **Money Market Accounts**) as the sum of these accounts.
- *Savings accounts*: We define the sum of variables X3804, X3807, X3810, X3813, X3816 as **Savings Accounts**.
- *Certificates of Deposit*: The variable X3721 gives the amount of money in certificates of deposits. We define **Certificates of Deposit** to be equal to this variable as long as the account does not belong to

someone unrelated to the household (variable $X7620 < 4$).

- *Saving bonds*: We define as **Savings Bonds(safe)** the sum of variables X3902 (money saved in U.S. government savings bonds), variable X3908 (face value of government bonds) and variable X3910 (money in state and municipal bonds). We define as **Savings Bonds (risky)** the sum of variables X3906 (face value of mortgage-backed bonds), variable X7934 (face value of corporate bonds) and variable X7633 (face value of foreign bonds).

- *Life Insurance*: Variable X4006 gives the cash value of life insurance policies, while variable X4010 gives the amount currently borrowed using these policies. We define as **Life Insurance** the amount given by X4006-X4010.

- *Miscellaneous assets and debts*: This category gives the amount of money the respondent is owed by friends, relatives or others, money in gold or jewelry and others. Variable X4018 gives the total amount owed and X4022, X4026, X4030 the dollar value in these types of assets. Variable X4032 is the amount owed by the respondent. We define **Miscellaneous Assets** as X4018+ X4022 + X4026 + X4030-X4032.

- *Other Consumer Loans*: Variables X2723, X2740, X2823, X2840, X2923, X2940 give the amount still owed in loans like medical bills, furniture, recreational equipment or business loans. Using variables X6842-X6847 we make sure these loans are not part of business loans and we define the variable **Other Consumer Loans** to be equal to X2723 + X2740 + X2823 + X2840 + X2923 + X2940.

- *Brokerage Accounts*: Variable X3930 gives the amount the total dollar value of all the cash or call money accounts, and the variable X3932 gives the current balance of margin loans at a stock brokerage. We define **Brokerage Accounts** to be equal to X3039-X3932.

- *Mutual Funds*: Variable X3822 gives the total market value of all the stock funds, variable X3824 the total market value of all of the tax-free bond funds, variable X3826 the total market value of all government-backed bonds, variable X3828 the total market value of other bond funds, and variable $X3830$ the total market value of all of the combination funds or any other mutual funds of the respondent. We define as **Mutual Funds (safe)** the sum of variables X3824+X3826+X3828+0.5×X3830 and as **Mutual Funds(risky)** the sum of variables X3822+0.5×X3830.

- *Publicly Traded Stocks*: Variable X3915 gives the total market value of stocks owned by the respondent, and variable X7641 the market value of stocks of companies outside the U.S. We define **Stocks** to be equal to X3915+X7641.

- *Annuities*: Variable X6820 gives the total dollar value of annuities. Variable X6826 reports how the money is invested. We define **Annuities(safe)** to be equal to X6820 if X6826=2 (Bonds/interest; CDS/Money Market) and equal to 0.5×X6820 if X6826=5 (Split between Stocks/Interest; Combination of Stocks, Mutual Fund, CDS). We define **Annuities(risky)** to be equal to X6820 if X6826=1 or =3 (Stocks; Mutual Funds or Real Estate) and equal to 0.5× X6820 if X6826=5.

- *Trust*: Variable X6835 gives the total dollar value of assets in a trust. Variable X6841 reports how the money is invested. We define **Trust(safe)** to be equal to $X6835$ if $X6841 = 2$(Bonds/interest; CDS/Money Market) and equal to 0.5×X6835 if X6841=5 (Split between Stocks/Interest; Combination of Stocks, Mutual Fund, CDS). We define **Trust(risky)** to be equal to X6835 if X6841=1 or =3 (Stocks; Mutual Funds or Real Estate) and equal to 0.5× X6835 if X6841=5.

- *Individual Retirement Accounts*: Variables $X3610, X3620, X3630$ report how much money in total is in individual retirement accounts. Variable $X3631$ reports how the money is invested. We define the variable **IRA(safe)** to be equal to $X3610 + X3620 + X3630$ if $X3631 = 1$ (money market) or $X3631 = 3$ (Bonds/ Similar Assets; T-Bills) or X3631=11 (Universal life policy). IRA(safe) equals $\frac{2}{3}$(X3610 + X3620 + X3630) if X3631=4 (combination of money market-stock mutual funds-bonds and T-bills), equal to $\frac{1}{2}$ (X3610 + X3620 + X3630) if X3631=5 (combination of stock mutual funds-bonds and T-bills), and equal to $\frac{1}{2}$ (X3610 + X3620 + X3630) if X3631=6 (combination of money market-stock mutual funds) or X3631=-7 (split). Similarly we define the variable **IRA(risky)** to be equal to $X3610 + X3620 + X3630$ if $X3631 = 2$ (stocks) or $X3631 = 14$ (Real Estate/Mortgages) or $X3631 = 15$ (Limited Partnership) or $X3631 = 16$ (Brokerage account). IRA(risky) equals $\frac{1}{3}(X3610 + X3620 + X3630)$ if $X3631 = 4$ (combination of money market-stock mutual funds-bonds and T-bills), equal to $\frac{1}{2}(X3610 + X3620 + X3630)$ if $X3631 = 5$ (combination of stock mutual funds-bonds and T-bills), and equal to $\frac{1}{2}(X3610 + X3620 + X3630)$ if $X3631 = 6$ (combination of money market-stock mutual funds) or $X3631 = -7$ (split).

- *Pensions*: The variables $X4226, X4326, X4426, X4826, X4926, X5026$ give the total amount of money in present pension accounts. We subtract any possible loans against these accounts by using the variables $X4229, X4328, X4428, X4828, X4928, X5028$. Variables $X4216, X4316, X4416, X4816, X4916, X5016$ provide information on how the money is invested. We define **Pensions(risky)** if any of the latter variables equals 3 (Profit Sharing Plan) or 4 (Stock purchase plan). Other than these two options the SCF does not provide many details regarding pension plans. For example, respondents can report that the money is invested in a 401K without further information on how the money are invested. In this case, we split the money half into **Pensions(safe)** and the other half into **Pensions(risky)**. As mentioned in the text, we experiment with other split rules and show our findings in Table B-1 of Appendix B.

- *Business*: Variables $X3129, X3229, X3329$ report the net worth of businesses, variables $X3124, X3224, X3324$ and $X3126, X3226, X3326$ the amount owed to the business and the amount owed by the business, respectively. Finally, variable $X3335$ gives the share value of any remaining businesses. We define **Actively Managed Business** to be equal to $X3129 + X3229 + X3329 + X3124 + X3224 + X3324 - X3126 - X3226 - X3326 + X3335$. Similarly we define **Non-Actively Managed Business** as the sum of $X3408 + X3412 + X3416 + X3420 + X3424 + X3428$.

- *Housing*: Variable $X513, X526$ gives the value of the land the respondent (partially) owns, variable $X604$ the value of the site, and variable $X614$ the value of the mobile home the respondent owns. Variable $X623$ gives the total value of home and site if he owns both. Variable $X716$ is the value of home/apartment/property that the respondent owns (partially). We define **Value of the Home** as the sum of the above variables. Variables $X1706, X1806, X1906$ give the total value of property such as vacation houses or investment in real estate. Variables $X1715, X1815, X1915$ give the amount of money owed in loans associated with this property. We define **Net house worth** to be equal to Value of the Home $+X1706 + X1806 + X1906 - X1715 - X1815 - X1915$.

*Definitions*

**Safe Assets** = Checking Accounts + Money Market Accounts + Savings Accounts + Certificates of Deposit + Savings Bonds(safe) + Life Insurance + Miscellaneous Assets + Mutual Funds(safe) + Annuities(safe) + Trust(safe) + IRA(safe)+ Pensions(safe)

**Risky Assets**= Savings Bonds(risky) + Brokerage Accounts + Stocks + Mutual Funds(risky) + Annuities(risky) + Trust(risky) + IRA(risky)+ Pensions(risky) + Non-Actively Managed Business

**Risky Share**= $\frac{\text{Risky Assets}}{\text{Safe Assets + Risky Assets}}$

# B Division of Pension Plans Between Riskless and Risky Assets

As mentioned in the main text, the SCF does not provide exact information on how pension plans, such as 401(k)s, are invested. For our benchmark definition of the risky share, we categorized half of the money invested in these accounts as safe asset holdings and half as risky assets. Our choice of an equal split related to the average risky share is close to 50%. In this section we re-calculate the risky share of financial assets using alternative split rules. In particular, we experiment with two extreme cases: a rule that allocates 80% of the money in these accounts to safe assets (and 20% in risky), and a rule that allocates 20% of these money in safe assets (and 80% in risky). We report our findings in Table B-1. The average risky share is sensitive to our choice. Naturally, if we allocate most of the money to safe assets, the risky share will decrease to 38.2%. If we allocate most of the money to risky assets, the risky share will increase to 52.6%. However, the increasing age-pattern documented under our benchmark definition remains intact.

Table B-1: **Portfolio Choice for Different Split Rules**

| Age group | Benchmark 50-50 | 80-20 | 20-80 |
|---|---|---|---|
| 21-30 | 39.9% | 32.9% | 47.0% |
| 31-40 | 43.8% | 34.6% | 53.0% |
| 41-50 | 46.0% | 37.9% | 54.2% |
| 51-60 | 47.9% | 40.9% | 54.8% |
| 61-65 | 50.7% | 46.4% | 55.0% |
| Average | 45.5% | 38.2% | 52.6% |

# C Updating Rule

We provide a brief description of our updating rule of priors about the earnings profile. Consider a simpler case where income $y_t = z_t + \varepsilon_t$ depends on a persistent component $z_t = \rho z_{t-1} + \eta$, with $\eta \sim N(0, \sigma_\eta^2)$ and an i.i.d. component $\varepsilon \sim N(0, \sigma_\varepsilon^2)$. Every period a worker observes income $y_t$ and updates his prior about the true value of $z_t$. After the update has taken place, the worker receives noise $q_t \sim N(0, \sigma_q^2)$. The prior belief is summarized by two parameters: the mean of the distribution $\mu_{z_{t|t-1}}$ and the variance $\sigma^2_{z_{t|t-1}}$. After observing $y_t$ the worker will form a posterior belief, which in turn is summarized by $\mu_{z_{t|t}}$ and $\sigma^2_{z_{t|t}}$. The posterior mean will depend on the prior mean, the realized income and the realized shock. In particular

$$\mu_{z_{t|t}} = \omega\mu_{z_{t|t-1}} + (1-\omega)y_t + q_t$$

Our basic assumption is that only $y_t$ is known at the time of updating. The noise shock $q_t$ will occur at the end of the period. Since the shock will be zero in expectation, the worker will just assign weights to the prior mean and the income. To pick $\omega$ the worker minimizes the variance of $\mu_{z_{t|t}}$, so:

$$\min_{\omega} \mathbf{E}(\mu_{z_{t|t}} - z_t)^2 = \min_{\omega} \mathbf{E}(\omega\mu_{z_{t|t-1}} + (1-\omega)y_t + q_t - z_t)^2 =$$

$$\min_{\omega} \mathbf{E}(\omega\mu_{z_{t|t-1}} - \omega z_t + (1-\omega)\varepsilon + q_t)^2 = \min_{\omega} \mathbf{E}(\omega\mu_{z_{t|t-1}} - \omega(\rho z_{t-1} + \eta) + (1-\omega)\varepsilon + q_t)^2$$

$$= \min_{\omega} \mathbf{E}(-\omega\eta + (1-\omega)\varepsilon + q_t)^2 == \min_{\omega} \mathbf{E}(-\omega\eta + (1-\omega)\varepsilon)^2 + \sigma_q^2$$

The last equation made use of $\mu_{z_{t|t-1}} = \rho z_{t-1}$. We can see that parameter $\sigma_q^2$ will not affect the optimal weight $\omega$. Minimizing this expression we have $\omega = \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_\eta^2}$. Plugging $\omega$ back we get

$$\mu_{z_{t|t}} = \omega\mu_{z_{t|t-1}} + (1-\omega)y_t + q_t \rightarrow \mu_{z_{t|t}} = \mu_{z_{t|t-1}} + \frac{\sigma_\eta^2}{\sigma_\varepsilon^2 + \sigma_\eta^2} y_t + q_t$$

This is equivalent to equation (5) in the text. Doing the same with the variance:

$$\mathbf{E}(-\omega\eta + (1-\omega)\varepsilon)^2 + \sigma_q^2 = \mathbf{E}(-\frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_\eta^2}\eta + \frac{\sigma_\eta^2}{\sigma_\varepsilon^2 + \sigma_\eta^2}\varepsilon)^2 + \sigma_q^2 = \frac{\sigma_\varepsilon^4\sigma_\eta^2 + \sigma_\varepsilon^2\sigma_\eta^4}{(\sigma_\varepsilon^2 + \sigma_\eta^2)^2} + \sigma_q^2 =$$

$$= \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_\eta^2}\sigma_\eta^2 + \sigma_q^2 = \sigma_\eta^2 - \frac{\sigma_\eta^2}{\sigma_\varepsilon^2 + \sigma_\eta^2}\sigma_\eta^2 + \sigma_q^2$$

This is equivalent to equation (6) in the text. Note that if $\sigma_q^2 = 0$, we go back to the standard Bayesian learning.