Money and Market Incompleteness in Overlapping-Generations Models

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MONEY AND MARKET INCOMPLETENESS IN

OVERLAPPING-GENERATIONS MODELS

by

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I. Introduction

The overlapping-generations model of Samuelson (1956) is an increasingly popular environment for studying a wide range of macroeconomic phenomena. Its popularity seems due, at least in part, to the fact that the model allows heterogeneity of agents while remaining mathematically tractable. In addition, it has been proposed by Wallace (1980) and others that this model be taken seriously as a model of a monetary economy. This view stems from the fact that there exist equilibria in these models in which fiat money has positive value.

The equilibria of overlapping-generations (OLG) models can exhibit patterns of consumption, investment, and asset prices that can differ markedly from those obtained in other widely-used models, notably, the model of an infinitely-lived, representative agent (ILA).[1] The OLG model differs in two important ways from the ILA model: (i) in the OLG model agents have finite lifetimes, and (ii) the OLG model is characterized by market incompleteness, since the sequential structure of the model typically prohibits individuals from engaging in trade before they are born and after they are dead. Thus, a question currently receiving a great deal of attention in the macroeconomics literature is the extent to which the dynamics in OLG models derive from the model's generic market incompleteness. A related question is whether, in the OLG framework, the function of money is to completes markets.

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[1] Lucas (1981) is an an example of a paper that makes use of this model.

In a recent paper, Marshall, Sonstelie, and Gilles (1987) (henceforth, MSG) study a non-stochastic sequential OLG economy without production or storage opportunities. In that setting, the market incompleteness of OLG models is unimportant in the sense that equilibria in the sequential, incomplete markets setting are also equilibria in an environment with complete markets. In order to prove the proposition that the market "incompleteness" of the sequential market view is not important to equilibrium allocations, MSG establish an equivalence between equilibria in the "timeless" or "complete" market structure--with appropriate wealth transfers--and monetary equilibria in the "sequential" or "incomplete" market structure. The proposition that money does not "complete markets", then, follows immediately.

But money obviously does <u>something</u> in OLG models, because monetary equilibria in deterministic, sequential OLG economies can be quite different from equilibria where no money or other durable asset is permitted. The answer provided by MSG is that it is appropriate to view the function of money in these models as that of redistributing wealth.

Since the predictions of stochastic OLG models are increasingly being compared to actual time series, the purpose of the present paper is to study within a stochastic setting the effects on equilibrium allocations and prices of the OLG model's built-in market incompleteness, and to examine the role of money. The specific model employed here is of an economy with stochastic endowments, and without production opportunities. Section II describes the model and examines the equilibria that are obtained under timeless and sequential market structures; several examples are presented. Section III discusses the extent to which money completes markets in the sequential market structure. Section IV presents the paper's conclusions.

II. A Stochastic Model without Production

This section examines market incompleteness and the role of money in a stochastic OLG framework with two-period-lived agents and no storage or production opportunities. Agents' endowments are assumed to follow a stationary stochastic process. In this model it is generally not the case that equilibria in timeless Arrow-Debreu markets can be replicated in sequential markets. The reason is that in the Arrow-Debreu market structure, agents can diversify away the idiosyncratic risk associated with stochastic first-period-of-life endowments; in the sequential market structure, they cannot. Thus the fact that markets are incomplete in the sequential set-up has important consequences for equilibrium consumption paths. The simple model discussed below illustrates this point; it will be easy to see that it must hold true for more complicated models--for example, ones in which agents live more than two periods, or with more states of nature, or with more complicated stochastic processes on endowments.

The Model

The stochastic process for endowments is assumed to be such that an agent's endowment at date t depends only on the state that obtains at date t; let $w_t(t,s_t)$, $w_t(t+1,s_{t+1})$ denote the lifetime endowment profile for an agent born at time t in state s_t and if s_{t+1} is the state that obtains at time t+1 (i.e., when he is old). For simplicity of exposition the state of nature s_t is assumed to be an i.i.d. random variable; the probability of state s occurring at time t is denoted $\pi(s)$.

To take a specific example, suppose that there are three possible states of nature, s=1, 2, 3, and let the endowment profile for this economy be given by:

	endowment	
	young	**old**
state = 1	$w_y+\theta$	$w_o-\theta$
state = 2	w_y	w_o
state = 3	$w_y-\theta$	$w_o+\theta$

where $\theta \le \min(w_y, w_o)$ so that endowments are always nonnegative.

This example has been constructed so that the economy-wide endowment is constant at the level (w_y+w_o); there is no aggregate risk even though there is idiosyncratic (individual) risk. If individuals in this economy are risk averse they will want to diversify away this idiosyncratic risk. As is demonstrated below, the extent of risk-reduction that agents can achieve depends on the market structure assumed for the economy. Two alternative market structures are examined.

Timeless (Arrow-Debreu) Markets

Suppose that, in the economy described above, agents are allowed to meet "outside of time" in Arrow-Debreu markets, and are allowed to trade their endowments for consumption goods indexed by (t,s) where the index s denotes the state of nature in which consumption takes place, and the index t takes on the values t=1,2,3,... In the sequential economy studied below, this index will be interpreted as the date of consumption.

Agents in this economy are distinguished by the index j=1,2,3,... In the sequential economy studied next, the index j will denote the agent's birthdate. Since in that economy agents are alive only in two periods, agent j values consumption only in periods j and j+1. He is assumed to have a utility function which exhibits a constant rate of relative risk aversion. Thus, in the context of timeless markets, agent j's expected utility is given by the following :

$$EU_j = \sum_{t=j}^{j+1} \sum_{s=1}^{3} \pi(s) \left\{\frac{1}{1-\sigma}\right\}\left\{c_j(t,s)^{1-\sigma} - 1\right\} \tag{1}$$

The problem facing agent j is to maximize (1) subject to the budget constraint:

$$\sum_{t=-\infty}^{\infty} \sum_{s=1}^{3} r(t,s)c_j(t,s) \leq \sum_{t=-\infty}^{\infty} \sum_{s=1}^{3} r(t,s)w_j(t,s) \tag{2}$$

where r(t,s) is the price in the Arrow-Debreu markets of a unit of the consumption good (t,s).[2]

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[2]As is usual in OLG economies that begin at date t=1, there are difficulties engendered by the fact that the old agents at t=1 are different from all other generations, having had no "young" period of life. The purpose of the present paper is better served by having all agents in the economy look alike. Thus, the economy has no first period, and the index t has range (-∞, +∞).

Given the built-in stationarity of this economy, it is natural to search for a stationary equilibrium where the consumption levels depend on the state of nature, s, but not on the "date" of consumption, t. In particular, since the aggregate endowment is constant in this economy, and because there is no discounting, it is natural to conjecture (and easy to verify) that the following is an equilibrium for the economy facing the timeless market structure:

$c_j(t,s) = (w_y+w_o)/2$	for $t = j,j+1$ and for all j,s
$c_j(t,s) = 0$	for all $t \neq j,j+1$ and for all j,s
$r(t,s) = \lambda$	for all t,s and for any constant λ

It is worth noting at this point that if the economy were assumed to start at a specific point in time, say t=1, the equilibrium in the timeless Arrow-Debreu market structure would look different. This is the type of economy studied by MSG (in a deterministic version), so it is worth investigating briefly the differences in equilibria that arise from the existence of an initial period. Because of the existence of the initial old generation which has endowment only on period t=1, no intertemporal trade is possible. However, individuals will trade claims to goods at a point in time, in order to diversify away all idiosyncratic risk. The resulting equilibrium will then be:

$c_j(t,s) = w_y$	for $t = j$ and for all j,s
$c_j(t,s) = w_o$	for $t = j+1$ and for all j,s
$c_j(t,s) = 0$	for all $t \neq j,j+1$ and for all j,s
$r(t,s_t)/r(t+1,s_{t+1}) = [c_j(t+1,s_{t+1})/c_j(t,s_t)]^{\sigma}$	
$\quad = [w_o/w_y]^{\sigma}$	for all t,j,s.

Thus, the timeless-markets equilibrium in economy with an initial period looks very different from that obtained in the economy with an infinite past. With an initial period, agents can smooth their consumption path relative to their endowment, but they cannot smooth it as much as they would like to since no intertemporal trade is possible. In the economy with an infinite past, this perfect smoothing is possible.

Sequential markets

This section studies the equilibrium obtaind in a sequential market structure--one in which agents are allowed to trade only after they are born. By the time agent j is born in time period t=j, his endowment for his first period of life ($w_t(t,s)$) has already been realized. He is consequently unable to diversify away the risk associated with first-period-of-life endowments, in contrast to the situation in the timeless, Arrow-Debreu market structure. And since no intertemporal trade can take place in a sequential OLG model with two-period-lived homogeneous agents, the young agent cannot trade claims on his stochastic second-period-of-life endowments. Thus, in a sequential market structure, (and leaving aside for the moment the possiblity of monetary equilibria), every agent is stuck with consuming his stochastic endowment and cannot achieve any reduction in his idiosyncratic risk. This is a familiar result from deterministic OLG models with two-period-lived homogeneous agents, and holds whether the economy has an initial period or whether it has been running forever.

It is clear from this simple example that the incompleteness of markets associated with the sequential OLG model has important consequences for equilibrium consumption and interest rates. In expected utility terms, risk-averse agents are worse off with the sequential structure since they bear idiosyncratic risk that is diversifiable in the Arrow-Debreu structure. Thus, comparison of the real equilibria obtained in the timeless and the sequential market structures suggests that the market incompleteness inherent in the sequential structure is important for the difference in the real allocations achieved by these two structures.

Monetary equilibrium in the sequential economy

This section characterizes the properties of monetary equilibria in stochastic OLG economies and investigates the role of money in these models. Specifically, is the role of money in stochastic OLG models the role suggested by MSG: solely that of redistributing wealth? And, if not, does money overcome any problems associated with market incompleteness?

To ensure that money is valued in equilibrium, we assume that the endowment profile of the economy satisfies the necessary conditions for an equilibrium with valued fiat money that are found in Peled (1982). In the simple economy above, this necessary condition says that the young agent's endowment is greater than the old agent's endowment in every state of nature. There is a constant amount, M, of fiat money initially held by the old.

A monetary equilibrium for this economy is found by solving the problem described by equations (3)-(8) below. Only the young agents have a non-trivial decision problem; the old agents supply their currency inelastically. The young agents in the economy will find themselves in one of three states of nature, thus, it is natural to seek a stationary equilibrium where there is one price for each state of nature. If this guess is correct, it follows immediately that consumption of every agent depends only on the current state. The notation in equations (3)-(8) has been simplified as follows to reflect this guess:

$c_y(s)$ denotes the young agent's consumption in state s: $s=1,2,3$

$c_o(s,s')$ denotes the consumption of an agent who was young in state s and old in state s': $s,s'=1,2,3$

$w_y(s)$, $w_o(s)$ denotes young and old endowments in state s: $s=1,2,3$

$p(s)$ is the price of money (the inverse of the price level): $s=1,2,3$.

Thus the decision problem of an agent born in state s is to choose $c_y(s)$, $c_o(s,s')$, $s,s'=1,2,3$ and state-dependent consumption loans, $\ell(s,s')$ to maximize their lifetime utility function:

$$U(c_y(s),c_o(s,s')) = \left\{\frac{1}{1-\sigma}\right\}\left\{c_y(s)^{1-\sigma} - 1\right\} + \sum_{s'}\left\{\frac{\pi(s)}{1-\sigma}\right\}\left\{c_o(s,s')^{1-\sigma} - 1\right\} \tag{3}$$

subject to the budget constraints:

$$c_y(s) \le w_y(s) - \sum_{s'} q(s,s')\ell(s,s') - p(s)M \tag{4a}$$

$$c_o(s,s') \le w_o(s') + \ell(s,s') + p(s')M \quad . \tag{4b}$$

where $q(s,s')$ is the price in state s of a unit of consumption delivered tomorrow if tomorrow's state is s'. Equation (4a) is the budget constraint for a young agent born in state s. It says that his consumption when young must be less than or equal to his young-period endowment, minus the present value of consumption loans he makes and his accumulation of cash balances. Equation (4b) is the budget constraint for this agent when he is old, given that state s' obtains in his old age. His consumption is constrained by his old-age endowment, plus the value of consumption loans made while young which pay off in state s', plus the current value of his cash balances. The agent's consolidated or "present value" budget constraint is found by multiplying (4b) by q(s,s'), summing over s', and substituting into (4a), to obtain:

$$c_y(s) + \sum_{s'} q(s,s')c_o(s,s') \leq w_y(s) + \sum_{s'} q(s,s')w_o(s,s') + \left\{ \sum_{s'} q(s,s')p(s') - p(s) \right\} M \qquad (5)$$

Equation (5) shows clearly the necessary condition for no arbitrage opportunities in money; the condition is:

$$\sum_{s'} q(s,s')p(s') - p(s) \leq 0 \qquad (6)$$

with this condition holding with equality if money is valued in equilibrium. If this condition did not hold, an agent could make an infinite amount of money by buying money today, and selling claims to money tomorrow.[3]

The consumption demands are found by solving the first-order conditions for the maximization problem above; these demands are:

[3] Suppose $\sum_{s'} q(s,s')p(s') > p(s)$. Then an agent would wish to sell claims to one unit of money (say, $1) tommorow in each state of nature, i.e., claims to $1 for sure. This would yield him $\Sigma q(s,s')p(s')$ goods units in receipts. This amount is enough to buy the $1 needed to back the claims, p(s) goods units, and still have some goods left over.

$$c_y(s) = \left[\frac{\Omega(s)}{1 + \sum_{s'} q(s,s') \left[\frac{q(s,s')}{\pi(s)}\right]^{-1/\sigma}} \right] \tag{7a}$$

$$c_o(s,s') = \frac{q(s,s')}{\beta\, \pi(s)} \left[\frac{\Omega(s)}{1 + \sum_{s'} q(s,s') \left[\frac{q(s,s')}{\pi(s)}\right]^{-1/\sigma}} \right] \tag{7b}$$

where $\Omega(s) = \{w_y(s) + \sum_{s'} q(s,s')w_o(s,s')\}$ is the wealth of an agent born in state s.

Finally, equilibrium consumption must satisfy the economy's resource constraint in each period:

$$c_y(s) + c_o(s) \leq w_y(s) + w_o(s) \qquad \text{for all } s, t. \tag{8}$$

Some examples

There do not exist analytic functions giving equilibrium values for prices $p(s)$, $q(s,s')$ and consumptions $c_y(s)$, $c_o(s,s')$ as functions of σ, $w_y(s)$, $w_o(s)$, $\pi(s)$, and M, even for the case of logarithmic utility (σ=1). An iterative scheme (outlined in the Appendix) was used to compute steady-state values for these prices and quantities; several examples are exhibited in Tables 1-4. While this paper does not provide a proof of uniqueness of the steady state equilibrium or a proof of convergence, in each case considered the algorithm quickly converged to the same steady state regardless of the starting point.

Several characterics of the monetary equilibria exhibited in Tables 1-4 are worth discussing. First, compared to the autarky equilibria obtained in the sequential economy without money, the sequential monetary equilibria exhibit smoother consumption patterns and higher utility. The equilibrium variability of consumption decreases as the parameter of relative risk aversion, σ, rises. Equilibrium asset prices exhibit correspondingly decreased variability as σ rises. The amount of consumption individuals would be willing to surrender to escape the randomness of the sequential nonmonetary economy and return to the Arrow-Debreu market structure ranges in the examples above from 3% (in Table 2, where agents have a low level of risk aversion) to 32% (in Table 4, where agents have a high level of risk aversion, and where a mean-preserving-spread has increased the probability weights on the extreme outcomes). The amount of consumption individuals would be willing to surrender to escape the randomness of the sequential monetary economy and return to the Arrow-Debreu market structure ranges in the examples above from .3% (in Table 2) to .9% (in Table 4). These results suggest that the monetary equilibrium is much "closer"--in an expected utility sense and in a "distance" sense--to the Arrow-Debreu equilibrium than to the autarky sequential markets equilibrium.

In none of the examples presented does the monetary equilibrium replicate the allocations of the Arrow-Debreu market structure. The only case in which the monetary equilibrium produces identical allocations to the Arrow-Debreu equilibrium is the case (not exhibited in the tables) in which old agents have zero endowment in every state of nature.

III. Does Money Complete Markets?

The analysis of the previous section demonstrates that the economy's market structure has important implications for equilibrium consumption and asset prices. Further, whether money (or some other durable asset) is allowed to have value in the sequential OLG economy affects equilibrium allocations and prices of other assets. But it does not follow immediately that money "completes markets" in OLG models, since Marshall, Sonstelie, and Gilles (1986) have convincingly argued that it does not, at least in deterministic environments. They established that, for any monetary equilibrium in the sequential economy, it is possible to redistribute wealth in such a way that the timeless Arrow-Debreu market structure replicates the consumption patterns of the original sequential equilibrium.

The question therefore remains of whether this correspondence holds in stochastic models. Specifically: can we effect transfers of state-dependent endowments in such a way that the equilibrium in the timeless Arrow-Debreu market replicates the equilibrium of the monetary economy? The previous examples of sequential monetary equilibria demonstrate clearly that this is not possible. The reason is that in the sequential monetary equilibria, the consumption patterns of agents alive contemporaneously are not perfectly correlated. But we know that in timeless Arrow-Debreu markets agents will diversify away all idiosyncratic risk in a way that will leave their consumption paths perfectly correlated; this is true whether or not the economy has an initial period. Thus, in timeless Arrow-Debreu markets, transfers of state-dependent wealth may alter the <u>scale</u> of one agent's consumption relative to another's, but it cannot reproduce consumption patterns in which agents bear diversifiable risk.

Thus, there is an important sense in which markets are incomplete in stochastic OLG frameworks. The incompleteness stems from the inability of agents to insure the endowment risk of their first period of life. Even if the realization of an agent's endowment on his birthdate is the mean of the distribution, his consumption path is altered relative to the timeless markets path because in future periods of his life he knows he will be trading with (then-young) agents facing birthdate risk (see the examples in Tables 1-4).

Given that markets are incomplete in stochastic OLG models with sequential trading, is it the case that money "completes markets"? Since introducing money into the sequential market structure alters consumption paths in the direction of the paths obtained in the timeless market structure (i.e., it smooths consumption relative to the nonmonetary sequential equilibrium), it can reasonably be argued that money at least partially acts as a replacement for the missing market for insurance of birthdate risk. In this case, the monetary equilibrium is identical to the timeless-markets equilibrium.

IV. Conclusions

This paper has investigated the extent to which the dynamics of stochastic OLG models arise from the model's generic incompleteness of markets, and has examined the extent to which money serves to complete markets. In contrast to results obtained by MSG within a deterministic framework, the market incompleteness inherent in the OLG framework has real effects on consumption and asset prices. In stochastic models, it is not the case that money may be viewed as simply redistributing wealth among agents.

In the stochastic model studied here, the addition of money in the sequential market structure alters the equilibrium in a way that suggests that money partially "completes markets", since it brings consumption paths closer to those obtained in timeless, Arrow-Debreu markets and, perhaps more importantly, increases utility.

Although this paper is not intended to address the question of whether the OLG model is an attractive model of money, it is useful to ask whether this paper's results shed any light on the question. Wallace (1980) states that: "Since getting fiat money to have value is necessary for any non-trivial theory of it,...[one approach is to] model explicitly the notion that money facilitates exchange."[4] In the monetary equilibria presented in Section II, money seems to facilitate at least some portion the exchanges that would have taken place in the timeless market structure, but which are impossible in the sequential market structure. In this sense, then, the present paper supports the view that OLG models may be useful models of money. What is disturbing about this view, however, is the well-known result that <u>any</u> durable asset can fill this role, and will be preferred to fiat money if it has a positive rate of return. Also, tax/subsidy schemes implemented by the government can reproduce the monetary equilibrium. Thus, the model does not explain one key puzzle of monetary economics: why is fiat money valued in economies with durable, productive assets?

The analysis of this paper suggests that there may be a non-equivalence between equilibria in timeless, Arrow-Debreu market structures and equilibria in sequential markets whenever agents are constrained against making mutually

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[4] Wallace (1980), p. 50.

beneficial exchanges by trading constraints imposed by the sequential structure of the economy. This could potentially occur in deterministic models if, for example, there is a productive externality or if human capital accumulation affects output available to the economy, and where the human capital is transferrable (at least in part). A situation could arise where the young would have like to have paid those born before them to invest more in human capital accumulation, but are prohibited from doing so in the sequential structure (but not in the timeless, Arrow-Debreu market structure).

The analysis of this paper suggests that the market incompleteness of OLG models has important effects, at least within stochastic versions of this model. The extent to which this market incompleteness is important in more general deterministic versions of the model remains as an open question.

Table 1

A simple example

Logarithmic utility: $\sigma=1$
Money stock: $M=1$
Three i.i.d. states, equal state probabilities: $\pi(s) = 1/3$, $s=1,2,3$.

Endowments:

state	young	old
1	6	4
2	7	3
3	8	2

Equilibrium consumption levels:

state	young	old
1	4.2293	5.7707
2	4.9342	5.0658
3	5.6391	4.3609

Equilibrium contingent claim prices, $q(s,s')$:

s \ s'	1	2	3
1	.24430	.27829	.32327
2	.28501	.32467	.37715
3	.32573	.32105	.43103

Equilibrium money prices, $p(s)$:

$p(1) = 1.7707$
$p(2) = 2.0658$
$p(3) = 2.3609$

Constant consumption level yielding the same expected utility as the monetary equilibrium above: 4.9662

Constant consumption level yielding the same expected utility as the autarky equilibrium: 4.4781

These utility-equivalent constant consumption levels should be compared to the constant level of 5.00 which is feasible, and which is achieved in the Arrow-Debreu market structure.

Table 2

A decrease in relative risk aversion

Endowments and parameters as in example 1, except σ=.33 instead of σ=1.

Endowments:

state	young	old
1	6	4
2	7	3
3	8	2

Equilibrium consumption levels:

state	young	old
1	4.0889	5.9111
2	4.9616	5.0384
3	5.8469	4.1531

Equilibrium contingent-claim prices, q(s,s'):

s \ s'	1	2	3
1	.29480	.31092	.33161
2	.31443	.33163	.35369
3	.33212	.35029	.37359

Equilibrium money prices, p(s):

p(1) = 1.9111
p(2) = 2.0384
p(3) = 2.1531

Constant consumption level yielding the same expected utility as the monetary equilibrium above: 4.9827

Constant consumption level yielding the same expected utility as the autarky equilibrium: 4.8319

These utility-equivalent constant consumption levels should be compared to the constant level of 5.00 which is feasible, and which is achieved in the Arrow-Debreu market structure.

Table 3

An increase in relative risk aversion

Endowments and parameters as in example 1, except σ=3.0 instead of σ=1.0.

Endowments:

state	young	old
1	6	4
2	7	3
3	8	2

Equilibrium consumption levels:

state	young	old
1	4.4726	5.5274
2	4.9382	5.0618
3	5.3587	4.6413

Equilibrium contingent-claim prices, q(s,s'):

s \ s'	1	2	3
1	.17548	.22909	.29796
2	.23688	.30926	.40223
3	.30345	.39617	.51527

Equilibrium money prices, p(s):

p(1) = 1.5274
p(2) = 2.0618
p(3) = 2.6413

Constant consumption level yielding the same expected utility as the monetary equilibrium above: 4.9584

Constant consumption level yielding the same expected utility as the autarky equilibrium: 3.4968

These utility-equivalent constant consumption levels should be compared to the constant level of 5.00 which is feasible, and which is achieved in the Arrow-Debreu market structure.

Table 4

A Mean-Preserving Spread relative to Table 3

All parameters as in Table 3, except:

$\pi(s)=.43333$ for $s=1,3$
$\pi(s)=.13333$ for $s=2$

Endowments:

state	young	old
1	6	4
2	7	3
3	8	2

Equilibrium consumption levels:

state	young	old
1	4.4590	5.5410
2	4.9217	5.0783
3	5.3395	4.6605

Equilibrium contingent-claim prices, q(s,s'):

s \ s'	1	2	3
1	.22435	.08991	.37903
2	.30258	.12136	.51120
3	.38734	.15522	.65439

Equilibrium money prices, p(s):

p(1) = 1.5410
p(2) = 2.0783
p(3) = 2.6605

Constant consumption level yielding the same expected utility as the monetary equilibrium above: 4.9548

Constant consumption level yielding the same expected utility as the autarky equilibrium: 3.4026

These utility-equivalent constant consumption levels should be compared to the constant level of 5.00 which is feasible, and which is achieved in the Arrow-Debreu market structure.

Appendix

Algorithm to compute the monetary equilibria of the examples of Section II.

Read in values of σ, M, $\pi(s)$, $w_y(s)$, $w_o(s)$.

NOTE: the algorithm also works for transition probabilities $\pi(s,s') \neq \pi(s')$

Read in initial values for $q(s,s')$ -- any positive numbers.

Compute $\Omega(s)$ (wealth of an agent born in state s) for s=1,2,3, using equation (7c).

Compute $c_y(s)$, $c_o(s)$ for s=1,2,3, using equations (7a) and (7b)

Compute $p(s)$ for s=1,2,3 using the individual's budget constraint, equation (4a), and the equilibrium condition that $\ell(s,s')=0$

compute new values for $q(s,s')$, s,s'=1,2,3 from the following equation , which is equation (7b) divided by equation (7a):

$$q(s,s') = \pi(s') * \left\{ \frac{c_y(s)}{c_o(s')} \right\}^{\sigma}$$

Check the no-arbitrage condition, equation (6).
If money is valued (which is the case in all the examples) and if the right-hand side of (6) is negative and within a specified tolerance band around zero, cease iteration.
If not, iterate until this condition is satisfied.

no-arbitrage condition not satisfied

no-arbitrage condition satisfied

Print out equilibrium consumption plans and asset prices.

References

Lucas, Robert E., Jr. (1978) "Asset Prices in an Exchange Economy," Econometrica, 46:6, Nov. 1978, 1426-45.

Marshall, John, J. Sonstelie, and C. Gilles (1987) "Money and Redistribution: Revisionist Notes on a Problem of Samuelson", forthcoming, Journal of Monetary Economics.

Peled, Dan (1982) "Informational Diversity over Time and the Optimality of Monetary Equilibria," Journal of Economic Theory 28, 255-74.

Samuelson, Paul (1958) "An Exact Consumption Loan Model of Interest with or without the Social Contrivance of Money," Journal of Political Economy 66:6, Dec. 1958, 467-82.

Wallace, Neil (1980) "The Overlapping Generations Model of Fiat Money", in Models of Monetary Economies, edited by John H. Kareken and Neil Wallace, Federal Reserve Bank of Minneapolis, pp. 49-82.

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