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Abstract

We consider the problem of fair division in exchange economies. We formulate and study three concepts of equity designed to capture informal notions of "equal opportunities". The central concept is that of a "family of choice sets". Given such a family $\mathcal{B}$, an allocation is alternatively required to be such that (i) for some B in $\mathcal{B}$ each agent i maximizes his satisfaction in B at z_i, (ii) there is B in $\mathcal{B}$ such that each agent i is indifferent between z_i and the maximizer of his satisfaction in B, (iii) for each i there is B_i in $\mathcal{B}$ such that z_i maximizes agent i's satisfaction in the union of the B_j and z_i is in B_i. Most of the standard concepts of equity can be obtained as particular cases of these general definitions by appropriately choosing $\mathcal{B}$. We identify conditions on $\mathcal{B}$ guaranteeing that the resulting allocations satisfy additional desirable requirements. These conditions imply that in fact equal income Walrasian allocations are necessarily among the equitable allocations. Finally, we apply the definitions to economies with public goods.

Notions of Equal Opportunities

1. *Introduction*

In this paper, three notions of "equal opportunities" are formulated and applied to exchange economies and to public good economies. Their relation to other notions of equity is also examined.

The concept that has met with the greatest success in the analysis of distributional questions in exchange economies is probably that of an envy-free allocation, that is, an allocation such that no agent would prefer switching bundles with anyone else. However, an outcome at which one agent envies another agent may well be equitable if it is the result of a *process* in which all agents have had ***equal opportunities***.

For instance, consider the problem of allocating some indivisible object and suppose that there exists no means of effecting monetary compensations across agents. Then, the random mechanism assigning all agents equal chances of winning the object is ex ante equitable, although it will generate allocations with envy. "Equal opportunities" may also mean that the "transition" mechanism that takes agents from the initial position to the final position is fair; disparities of incomes may be found acceptable in societies where it is nevertheless thought to be a fundamental principle of fairness that the educational process (the transition mechanism) give all children equal opportunities to realize their potential. Finally, if agreement exists on the transition mechanism, equal opportunities may mean equal or "equivalent" initial resources. This idea was pursued in Thomson (1983), where certain invariance properties of final allocations with respect to exchanges of initial endowments were formulated and studied.

Another notion that has been the object of some discussion is that of equal opportunities to trade at efficiency prices. Variants of this idea, based as well on the Walrasian mechanism, have also been suggested. There, "equal opportunities" is taken to mean having the same set of consumptions from which to choose. However, there is no reason why a notion of "equal opportunities as equal choice sets" should have to involve Walrasian concepts.

In fact, our purpose here is to show that concepts of "equal opportunities as equal, ***or equivalent***, choice sets" can be developed independently of, although in a manner compatible with, Walrasian notions. We will propose three definitions. For convenience, the allocations satisfying these definitions will simply be called "equitable."

Our point of departure is a ***class of choice sets***, assumed to be given. At first, no restrictions are imposed on the class. This will imply a great generality in our definitions. The emphasis of our study however is on the second step, where for each of the three concepts we identify the restrictions placed on the family of choice sets by requiring that the resulting equitable allocations satisfy additional desirable properties. It turns out that several of these restrictions, in turn, imply that ***the set of equitable allocations necessarily contains***, and in some cases is equal to, ***the set of Walrasian allocations from equal division.***

We conclude this introduction by briefly and informally stating the three concepts. Given a class of choice sets, first we say that ***an allocation is an equal opportunity allocation relative to that class*** if there is a member of the class such that, for each agent, his component of the allocation maximizes his satisfaction in that choice set. Next, we declare an allocation ***equal opportunity equivalent relative to the class*** if there is one member of the class such that each agent finds his component of the allocation indifferent to the maximizer(s) of his satisfaction in the choice set. Finally, we say that an allocation ***exhibits no envy of opportunities relative to the class*** if, for each agent, there is a member of the family in which he maximizes his satisfaction at his component of the allocation, and such that each other agent prefers his component of the allocation to any point of that choice set.

Although the usefulness of these notions will ultimately have to be judged by an examination of how well they perform in general situations, we limit ourselves to applying them to standard classes of economies, economies with private goods only, and economies with private goods and public goods. We establish several existence and non-existence results and we clarify the relationship of these concepts to other concepts that have been discussed in the literature.

2. *Preliminaries.*

There are ℓ commodities and n agents. Each agent i has a preference relation defined over $\mathfrak{R}_+^\ell$, which is representable by a continuous utility function u_i. $T_i(u_i,z_i)$, with $z_i \in \mathfrak{R}_+^\ell$, is the set of consumptions that agent i, with utility function u_i, weakly prefers to z_i. We assume that the aggregate endowment Ω, and the technology according to which the public goods, if there are any, are produced, are known and fixed. Therefore, an economy can simply be denoted by a list of utility functions $u = (u_1,\ldots,u_n)$. U denotes a domain of economies. We only consider domains of strictly monotonic preferences. U^C is the domain of ***classical economies***, that is, economies where in addition preferences are convex. $D \equiv \{z_0 \in \mathfrak{R}_+^{\ell} | z_0 \leqq \Omega\}$[1] is the set of consumptions dominated by Ω. A is the set of ***feasible allocations***. For exchange economies, $A \equiv \{z \in \mathfrak{R}_+^{\ell n} | \Sigma z_i = \Omega\}$. P(u) is the set of ***Pareto-efficient allocations of u***. For exchange economies, $\bar{\omega} = (\Omega/n,\ldots,\Omega/n)$ is ***equal division***, $I(u) \equiv \{z \in A | u_i(z_i) \geqq u_i(\Omega/n) \quad \forall i\}$ is the set of allocations that are ***individually rational from equal division for u*** and IP(u) is the intersection of I(u) and P(u). More generally, the intersection of two correspondences F and G is denoted FG. $\Delta^{\ell-1}$ is the $(\ell-1)$-dimensional simplex.

Finally, we introduce the fundamental concept of an envy-free allocation (Foley, 1967) and we list some of its properties. A more detailed review of these properties, together with a complete bibliography, is given in Thomson and Varian (1985).

Definition. An allocation $z \in A$ is ***envy-free for u*** if for all i and j, $u_i(z_i) \geqq u_i(z_j)$. Let ***EF(u)*** be the set of these allocations.

In exchange economies, envy-free and efficient allocations exist under weak assumptions on preferences.[2] The allocations obtained by operating the Walrasian mechanism from equal division enjoy both properties. In economies with production however, there may be no such allocations. Finally, an allocation may be envy-free and efficient without Pareto-dominating equal division, although in classical economies all allocations Pareto-dominating equal division are envy-free.

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[1]Vector inequalities $x \geqq y$, $x \geq y$, $x > y$.

[2]The most general existence results are due to Varian (1974) and Svensson (1982). Neither author requires convexity of preferences.

The situation with public goods is quite different. We will illustrate some of the differences in the case of one private good and one public good. Using the Kolm triangle representation, we find that the set of envy-free allocations is the vertical segment through the top vertex. In general, this segment intersects the efficient set at a finite number of points. In Figure 1, this intersection is a singleton denoted z*. Any one of the points of the

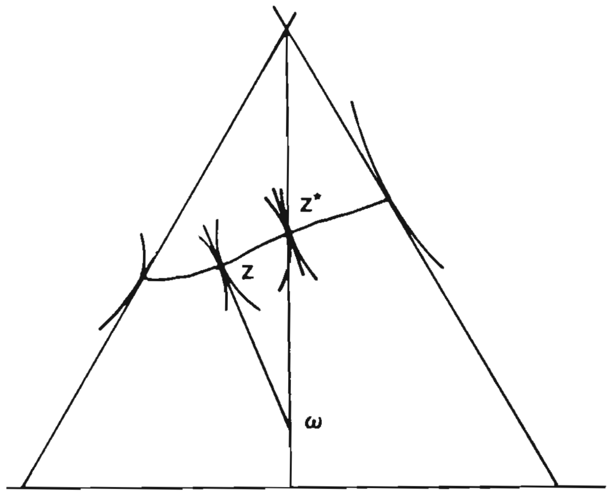

Figure 1

segment is a point of equal division. By operating the Lindahl mechanism, (which from a number of viewpoints, is the natural counterpart for public good economies of the Walrasian mechanism), from an arbitrary point of equal division, we do not in general reach an envy-free allocation: in Figure 1, ω is a point of equal division, but z, obtained by operating the Lindahl mechanism from ω, is not in EF(u). It is also worth noting that the Lindahl mechanism does not necessarily treat identical agents identically, in contrast with the Walrasian mechanism. Finally, the set of envy-free and efficient allocations contains the set of allocations Pareto-dominating a point of equal division ω for almost no choice of ω.

Recently, Sato[3] (1984) has proposed an alternative definition of equity for economies for public goods. According to this concept, the results given above for exchange economies do have counterparts in which the Lindahl correspondence plays the role of the Walrasian correspondence. However, it is fair to say that the concept is not as natural as Foley's original concept.

3. *Equal Opportunities.*

The following notion is briefly discussed in Thomson and Varian (1985). It simply and directly says that all agents should face the same choice set, as suggested by Kolm (1973), so that whatever differences exist between the final bundles can be entirely attributed to differences in tastes. It is noted in Thomson and Varian however that this choice set cannot be specified once and for all since the various choices made from it by the agents typically will not be compatible. Let us imagine instead that we have access to a whole family $\mathfrak{B}$ of choice sets. If $\mathfrak{B}$ is rich enough, then for each economy, compatibility of choices will hold for some $B \in \mathfrak{B}$.

Definition. An allocation $z \in A$ is an ***equal opportunity allocation relative to*** $\mathfrak{B}$ ***for*** u if there exists $B \in \mathfrak{B}$ such that for each i, z_i maximizes u_i in B. Let $\mathbf{EO}(\mathfrak{B},u)$ be the set of these allocations.

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[3] Sato (1985) also proposed a definition of "fairness in terms of consumer surplus" for economies with a single private good. This definition coincides with egalitarian-equivalence (see below) of net trades when the reference bundle is required to be proportional to the unit vector relative to the private good.

Two natural requirements on $\mathcal{B}$ are the following:

(α) for each $u \in U$, $EO(\mathcal{B},u) \cap P(u) \neq \phi$, and

(β) for each $u \in U$, $\phi \neq EO(\mathcal{B},u) \subset P(u)$.

Note that (β) implies (α).

We now apply the definition to exchange economies. Lemma 1 follows directly from it.

Lemma 1. $EO(\mathcal{B},.) \subset EF$. Also, if $\Omega/n \in B$ for all $B \in \mathcal{B}$, then, $EO(\mathcal{B},.) \subset I$.

Let $\mathcal{W}$ be the family of "budget sets" through the average endowment, hereafter called the **Walrasian family**: given $p \in \Delta^{\ell-1}$, $W_p \equiv \{z_0 \in \mathbb{R}^{\ell}_{+} | pz_0 = p\Omega/n\}$ and $\mathcal{W} \equiv \{W_p | p \in \Delta^{\ell-1}\}$. Under standard assumptions on preferences, $\mathcal{W}$ satisfies (β). Let **W(u)** be the set of Walrasian allocations from equal division for u.

Lemma 2. $EO(\mathcal{W},.) = W$.

Are there other examples of families satisfying (β)? Yes. Consider the following one.

Example: Assume n = 2. Let $\mathcal{B}' \equiv \mathcal{W} \cup \{K\}$, where K is the choice set depicted in Figure 2. K is piece-wise linear and $K \cap D$ is symmetric with respect to $\Omega/2$.

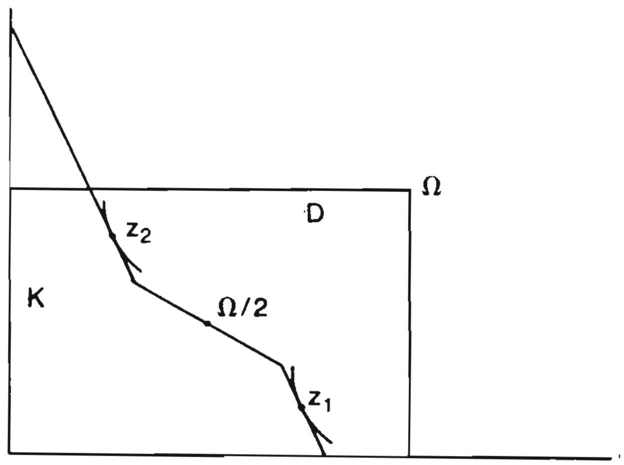


Figure 2

If two points z_1 and z_2 of K add up to Ω, and are maximizers over K of agent 1 and agent 2's utilities respectively, then the allocation $z \equiv (z_1, z_2)$ is Pareto-efficient (whether or not preferences are convex). For all u, $EO(\mathscr{B}', u) \supset W(u)$, and for some u, such as the u depicted in Figure 2, the containment is strict. (A possible interpretation for the shape of K: by having the price of the good that is measured on the vertical axis be relatively higher for small and for large quantities, one encourages intermediate consumptions. This may be a desirable social objective for some goods). Finally, note that the same kind of example could be constructed for an arbitrary number of commodities.

Are there other, perhaps less artificial, examples of families $\mathscr{B}$ satisfying (β)? Is it possible to characterize all families satisfying (β) or at least all the allocations obtainable in this way? Although we do not have a complete answer to these questions, we show next that *under a fairly weak additional assumption on the richness of the family of choice sets, the Walrasian allocations from equal division cannot be avoided*. First, however, we note that the concept proposed in this section (this will also be true of the one proposed in the last section), satisfies an interesting property recently introduced in the implementation literature. Since the property is far from being always satisfied, this first piece of information will be quite useful in helping us understand the implications of our definitions.

Definitions. A *correspondence on U* associates with every $u \in U$ a non-empty subset of A. A correspondence on U, φ, is ***monotonic*** if for all u, $u' \in U$, for all $z \in \varphi(u)$, if $T_i(u_i', z_i) \subset T_i(u_i, z_i)$ for all i, then $z \in \varphi(u')$.

The importance of the concept of monotonicity for the theory of implementation was discovered by Maskin (1977).

There is a close relation between monotonic correspondences and the Walrasian correspondence, (which, as easily checked, is monotonic in the interior of A). This relationship is described in the following Lemma, which follows from Hurwicz (1979). It appears in this form in Thomson (1982b). See also Gevers (1985). We omit the proof, which is straightforward.

***Proposition* 1.** Let φ, defined on a domain U, be such that

(i) $U \supset U^L$, the class of linear economies (economies in which each u_i is a linear function),

(ii) φ is monotonic,

(iii) if $u \in U^L$ and $\bar{\omega} \in P(u)$, then $\varphi(u) \supset IP(u)$.

Then $\varphi \supset W$.

We apply this result to $\varphi \equiv EO(\mathcal{B},.)$. (i) is a very mild coverage assumption. The fact that $EO(\mathcal{B},.)$ satisfies (ii) is easy to check. Finally, since it is very natural to require that if equal division $\bar{\omega}$ is efficient, then $\bar{\omega}$, as well as any allocation that is Pareto-indifferent to $\bar{\omega}$, is equitable, it is appealing to impose on $\mathcal{B}$ the even weaker requirement that $EO(\mathcal{B},.)$ satisfies (iii). Then, Proposition 1 says that $EO(\mathcal{B},.) \supset W$.

The same conclusion can be derived on the basis of other considerations, as explained below. First, we introduce a definition.

Definition. Let B be a choice set. The consumption $z_i \in B$ is ***useful on the domain U*** if it is the ith component of an allocation $z \in A$ such that for some economy $u \in U$, $z \in EO(\{B\},u)$. Also, $B \in \mathcal{B}$ is ***useful on U*** if every point of B is useful on U.

A consumption $z_i \in B$ may be a maximizer on B of some $u_i \in U$ and yet may never appear as the i^{th} component of a list of maximizing consumptions defining a feasible allocation. Therefore, it is natural to delete it from the set of possible choices. Such deletion would certainly simplify the specification of this set. To illustrate the notion somewhat more concretely, note that if W_p is the Walrasian choice set through Ω/n normal to p, no point of W_p that is not dominated by Ω is useful. On the other hand, every point of W_p dominated by Ω (the set they constitute is $W_p \cap D$) is useful on the domain of classical economies.

The next result is that usefulness of all $B \in \mathcal{B}$, together with a few minor conditions, implies that the equity correspondence $EO(\mathcal{B},.)$ contains the interior Walrasian correspondence from equal division. These assumptions, which are somewhat technical, are discussed after the statement of the theorem.

Theorem 1. Let $\mathcal{B}$ be a class of choice sets and U be a domain such that

(i) $U \supset U^C$, the domain of classical economies,

(ii) $\phi \neq EO(\mathcal{B},.)$ and $EO(\mathcal{B},.) \cap \text{int } A \subset P$,

(iii) every $B \in \mathcal{B}$ is useful on U.

(iv) every $B \in \mathcal{B}$ is closed and has a boundary which is a C^1 manifold.

Then $EO(\mathcal{B},.) \supset \text{int } W$, the interior Walrasian correspondence.

The proof of Theorem 1 is relegated to an appendix. We limit ourselves here to a few comments on the assumptions. Assumption (i) is a natural coverage assumption and the first part of (ii) is assumed to avoid trivialities. The interiority condition in the second part of (ii) cannot be escaped. Indeed, under the other assumptions of the theorem, non-efficient allocations on the boundary of A are possible. The first part of (iv) is for mathematical convenience. The second part of (iv) essentially says that the choice sets should be "reasonable looking". It would in particular be satisfied if the choice sets were defined by systems of inequalities (perhaps expressing various quantity constraints). The assumption of usefulness (iii) is also motivated by considerations of simplicity. Intuitively, in order to guarantee (β), a smaller family $\mathcal{B}$ will be necessary if all of its elements are useful. In fact, it follows directly from Theorem 1 that the smallest useful family satisfying (β) is the Walrasian family.

Although the above results identify circumstances in which the Walrasian allocations cannot be avoided, they do not tell us all that can be accomplished. To show that the notion of equal opportunities studied here can lead to other allocations, even under the assumptions of Theorem 1, we exhibit a family $\mathcal{B}$ such that, for n=2, EO($\mathcal{B}$,.) is in fact equal to FP. This family generalizes the example K seen earlier.

***Lemma* 3.** Let n=2. Then, there exists $\mathcal{B}$ such that EO($\mathcal{B}$,.) = FP.

***Proof*.** For each p, p' $\in \Delta^{\ell-1}$, and for all d $\in \mathfrak{R}_+$, let $B(p,p',d) \equiv \{z \in \mathfrak{R}_+^{\ell} | z \leqq \Omega,\ pz=p\Omega/2,\ p'\Omega/2-d < pz < p'\Omega/2 + d\} \cup \{z \in \mathfrak{R}_+^{\ell} | z \leqq \Omega,\ p'z = p'\Omega/2 + d,\ pz \leqq p\Omega/2\} \cup \{z \in \mathfrak{R}_+^{\ell} | z \leqq \Omega,\ p'z = p'\Omega/2 - d,\ pz \geqq p\Omega/2\}$.

Q.E.D.

We now turn to the case of public good economies. Things are much less satisfactory here, as indicated by the following negative result.

***Theorem* 2.** Suppose $n=2$ and $\ell=2$. There is no family $\mathcal{B}$ such that for all $u \in U^C$, $\phi \neq EO(\mathcal{B},u) \subset P(u)$.

Proof. Let $u \in U^C$ be an economy with differentiable u_i's such that (i) there is no point of $P(u)$ where all agents have the same marginal rates of substitution, and let $z \in EO(\mathcal{B},u) \cap P(u)$ be given. Since $z \in A$, (ii) all z_i have the same public good component. Since $z \in EO(\mathcal{B},u)$, (iii) there is $B \in \mathcal{B}$ such that for each i, agent i maximizes u_i at z_i. Since $u \in U^C$ and $\ell = 2$, (ii) and (iii) imply that $z_1 = z_2 \equiv z_0$. By (i), B has a kink at z_0. Since $z \in P(u)$, the agents' marginal rates of substitution at z_0 add up to the marginal rate of transformation at the corresponding production point (the Samuelson condition). Then, let $u' \equiv (u_1', \ldots, u_n)$ be obtained from u by replacing agent 1's utility function by one for which his marginal rate of substitution at z_0 is different from what it was initially and yet z_0 is still a maximizer over B. Then $z \in EO(\mathcal{B},u')$ but $z \notin P(u')$, since the Samuelson condition does not hold anymore.

Q.E.D.

4. *Equal Opportunity Equivalence.*

The next concept generalizes various ideas due to Pazner and Schmeidler (1978), Pazner (1977), Mas-Colell (1980) and Moulin (1986).

First, we recall a definition, due to Pazner and Schmeidler.

Definition. An allocation $z \in A$ is ***egalitarian-equivalent for u*** if there is a "reference bundle" $z_0 \in \mathfrak{R}_+^\ell$ such that for each i, $u_i(z_i) = u_i(z_0)$. Let *EE(u)* be the set of these allocations.

An appealing feature of this concept is that the existence of egalitarian-equivalent and efficient allocations is guaranteed under very general circumstances, even in production economies. However, an egalitarian-equivalent and efficient allocation may violate the following very desirable property

Definition. An allocation $z \in A$ satisfies the ***no-domination*** condition if there is no pair $\{i,j\}$ such that $z_i \geq z_j$.

Moreover, it is possible for an egalitarian-equivalent allocation to *strongly* violate the no-domination condition, in the sense that one agent receives everything and the others nothing. Finally, there are economies where the no-domination condition is violated by all egalitarian-equivalent and efficient allocations (see appendix). Proofs of these assertions as well as a more comprehensive evaluation of the concept can be found in Thomson (1987).

Our point of departure here however, is the idea of evaluating the equity of an allocation by comparing it to non necessarily feasible allocations (indeed the list $(z_0,\ldots,z_0)$, where z_0 is the reference consumption in the definition of egalitarian-equivalence, is not in general a feasible allocation). This fundamental idea can be applied in other ways. For instance, Pazner (1977) proposed the following definition.

Definition. An allocation $z \in A$ is ***envy-free equivalent for u*** if there is z' $\in EF(u)$ such that $u_i(z_i) = u_i(z'_i)$ for all i. Let ***EFE(u)*** be the set of these allocations.

Our next definition is obtained by combining the idea of equal opportunities with that of egalitarian-equivalence.

Definition. An allocation $z \in A$ is *Equal Opportunity Equivalent relative to $\mathcal{B}$ for u* if there exists $B \in \mathcal{B}$ such that for each i, z_i is indifferent for agent i to the maximizer of u_i on B. Let ***EOE($\mathcal{B}$,u)*** be the set of these allocations.

Note that for all $\mathcal{B}$, EFE $\supset$ EOE($\mathcal{B}$,.) $\supset$ EO($\mathcal{B}$,.). The correspondence EOE($\mathcal{B}$,.) is not in general monotonic, but it has that property for some $\mathcal{B}$ (for instance, if $\mathcal{B} = \mathcal{W}$; see Lemma 4).

The usefulness of this definition is made clear by the fact that several of the standard concepts of equity can be derived from it by appropriately choosing $\mathcal{B}$. Indeed, we have

***Lemma* 4.** If $\mathcal{B} = \mathcal{W}$, then EOE($\mathcal{B}$,.) = EO($\mathcal{W}$,.) = W.

***Lemma* 5.** If $\mathcal{B} = \{\{z_0\} | z_0 \in \mathfrak{R}^{\ell}\}$, then EOE($\mathcal{B}$,.) = EE.

***Lemma* 6.** If $\mathcal{B} = \{\{z_1,\ldots,z_n\} | (z_1,\ldots,z_n) \in A\}$, then EOE($\mathcal{B}$,.) = EFE.

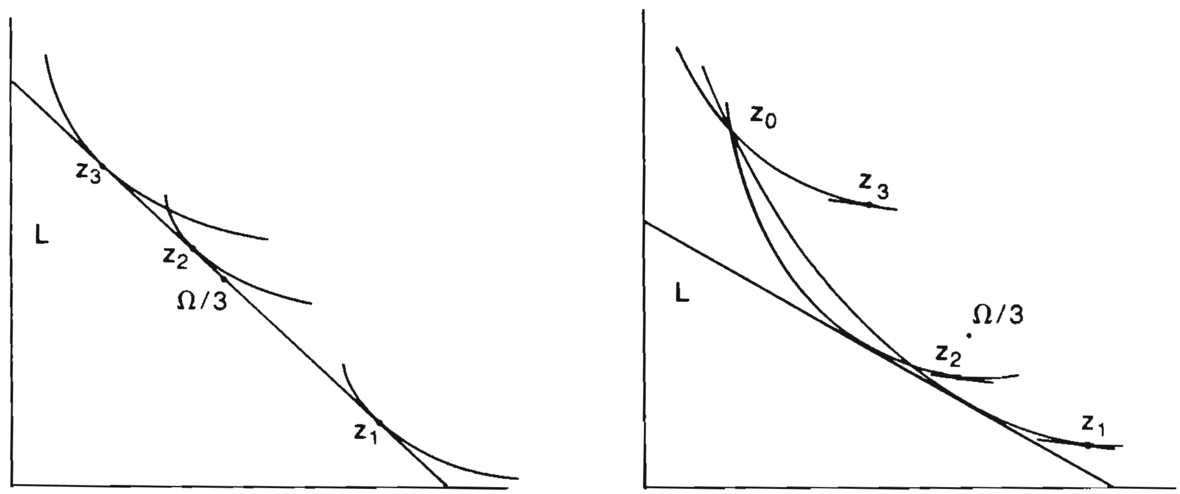
Let $\mathcal{L}$ be the family of linear choice sets. If we are interested only in efficient allocations, we can limit ourselves to the subfamily $\mathcal{L}'$ of choice sets not containing Ω/n in their interior, nor being dominated by Ω/n. Indeed, no list of n bundles indifferent (agent by agent) to n bundles taken from a linear choice set violating one or the other of these restrictions could ever define an efficient allocation, provided preferences are monotonic.

***Lemma* 7.** If n=2, EOE($\mathcal{L}$,.) = EE. If n>2, there is no necessary containment between EOE($\mathcal{L}$,u) $\cap$ P(u) and EE(u) $\cap$ P(u), even if $u \in U^C$.

Proof: The first statement follows from the fact that if $z \in$ EE(u), then the two indifference surfaces through z_1 and z_2 intersect, and this is equivalent to saying that they have a common hyperplane of support which can serve as choice set $L \in \mathcal{L}$ to show that $z \in$ EOE($\mathcal{L}$,u). The second statement is established by the examples of Figure 3. In Figure 3a, $z \in$ EO($\mathcal{L}$,u) $\cap$ P(u) but

$z \notin EE(u)$ since the three indifference curves through the three consumptions z_1, z_2 and z_3 have no point in common. In Figure 3b, $z \in EE(u) \cap P(u)$ but $z \notin EO(\mathcal{L},u)$ since the only common line of support to agents 1 and 2's indifference curves through z_1 and z_2 is not a line of support to agent 3's indifference curve through z_3.

Q.E.D.



(a) (b)

Figure 3

The no-domination condition can be violated by allocations in $EOE(\mathfrak{B},u)$ even if $\mathfrak{B} = \mathcal{L}'$. The next Theorem gives a condition on $\mathfrak{B}$ for this not to happen. Then, the only allocations that remain admissible are Walrasian allocations from equal division!

***Theorem* 3.** A necessary and sufficient condition for a subfamily $\mathfrak{B}$ of $\mathcal{L}'$ to be such that for all u and for all $z \in EOE(\mathfrak{B},u)$, z satisfies the no-domination condition, is that $\mathfrak{B} \subset \mathcal{W}$. (Then, in fact, $z \in W(u)$).

Proof: The proof is illustrated in Figure 4. Let $L \in \mathcal{L}' \setminus \mathcal{W}$ be given.

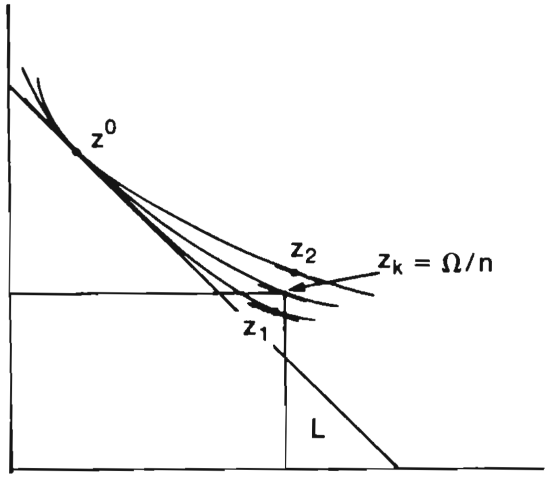


Figure 4

Since Ω/n does not dominate L and $\Omega/n \notin$ int L, there are (i) z_1 and z_2 symmetric of each other with respect to Ω/n with $z_1 < z_2$, (ii) z_0 on the boundary of L, and (iii) an economy u such that each agent has an indifference curve tangent to L at z_0; agents 1 and 2's indifference curves through z_0 pass through z_1 and z_2 respectively; for all $k > 2$, agent k's indifference curve through z_0 passes through Ω/n; finally, these indifference curves have parallel lines of support at z_1, z_2 and Ω/n respectively. Note that $z \in$ EOE($\{L\}$,u) and yet by (i), z violates the no domination condition.

Q.E.D.

Given $p \in \Delta^{\ell-1}$, let $\mathcal{L}(p)$ be the subfamily of $\mathcal{L}$ of all linear choice sets normal to p.

Theorem 4. $\text{EOE}(\mathcal{L}(p),.) \cap P \neq \phi$ for all $p \in \Delta^{\ell-1}$.

Proof: (The proof is essentially the same as the proof that egalitarian-equivalent and efficient allocations exist for a reference allocation proportional to a given allocation.) Index the family $\mathcal{L}(p)$ by incomes: $\mathcal{L}(p) = \{L(p,I) | I \in \Re_+\}$. For each $I \in \Re_+$, let $v_i(I) \equiv \max\{u_i(z_i) | z_i \in L(p,I)\}$. As I varies from 0 to infinity, the vector $v(p) \equiv (v_1(I),\ldots,v_n(I))$ traces out a monotone path in utility space. Let $\bar{v}$ be the intersection of the path with the boundary of u(A). Finally, let $\varphi(u) \equiv \{z \in A | u(z) = \bar{v}\}$. The correspondence φ so defined satisfies the desired requirements.

Q.E.D.

The notion of equal opportunity equivalence is perfectly well defined in public good economies, and for natural families $\mathcal{B}$, $EOE(\mathcal{B},.) \neq \phi$. Before stating a general existence result, however, we first explain how to identify

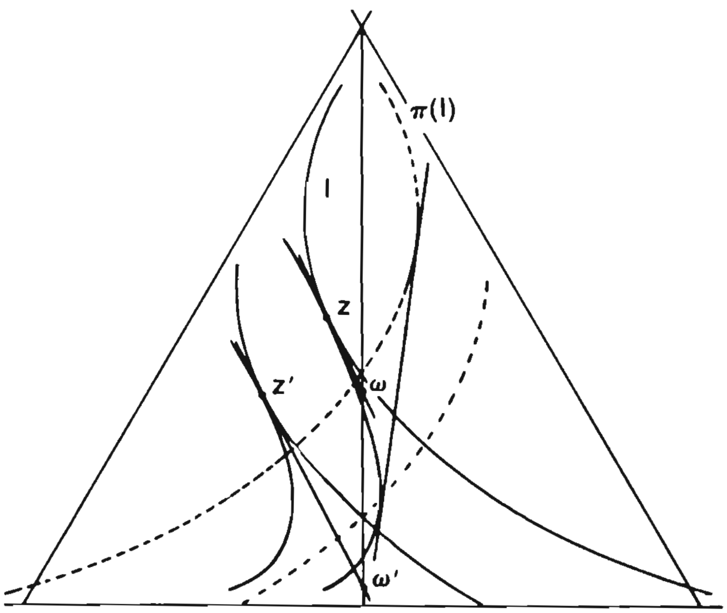


Figure 5

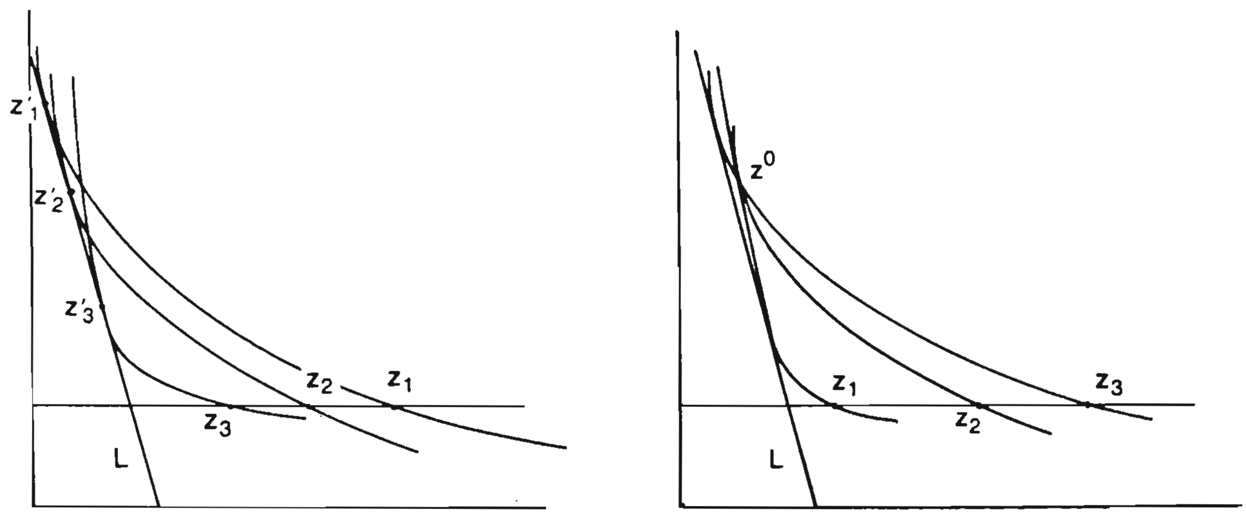
the set $EOE(\mathcal{L},u) \cap P(u)$ in the Kolm triangle, (that is, in the case of two agents, one private good, one public good, produced according to a linear technology, one unit of the private good yielding one unit of the public good). Let $z \in P(u)$ be given. To see whether $z \in EOE(\mathcal{L},u)$, take the symmetric image of I, agent 1's indifference curve through z_1, with respect to the equal division line. This is the dashed curve labeled $\pi(I)$ in Figure 5. If $\pi(I)$ and agent 2's indifference curve through z_2 have a common tangency line, as is the case in Figure 5, then $z \in EOE(\mathcal{L},u)$.

The set $EOE(\mathcal{L},u)$ is connected. Its end-points are obtained by finding two indifference curves such that the symmetric image of one is contained in, and tangent to, the other.

The counterparts of Lemmas 5 and 6 hold for public good economies: EE and EFE can be obtained by appropriately choosing $\mathfrak{B}$.

We also have the following counterpart of Lemma 7.

Lemma 8. If n=2, $EOE(\mathcal{L},.) = EE$. If $n > 2$, there is no necessary containment between $EOE(\mathcal{L},u) \cap P(u)$ and $EE(u) \cap P(u)$, even if $u \in U^C$.



(a) (b)

Figure 6

Proof: The proof of the first statement is similar to that of the first statement of Lemma 5. The proof of the second statement is given by the examples of Figure 6. In that Figure, units of measurement of the goods and Ω should be understood to be chosen as that $z \in P(u)$.

Q.E.D.

On the other hand, the counterpart of Lemma 4 does not hold. Although a Lindahl allocation from some point of equal division may be in EOE($\mathcal{L}$,u), as illustrated by point z of Figure 5,(indeed, z can be obtained by operating the Lindahl mechanism from ω), not all Lindahl allocations from some point of equal division need be in the set. This is illustrated by point z' which results from operating the Lindahl mechanism from ω'. Indeed, the symmetric image with respect to the equal division line of agent 1's indifference curve through z_1' does not intersect agent 2's indifference curve through z_2'.

We do have, however, the counterpart of the existence theorem. Given the family $\mathcal{L}(p)$ of linear choice sets normal to the price $p \in \Delta^{\ell-1}$, the same argument as in Theorem 4 shows that $EOE(\mathcal{L}(p),.) \cap P(.) \neq \phi$. Of course, the allocations in the set will accidentally be envy-free.

Theorem 5. $EOE(\mathcal{L}(p),.) \cap P(.) \neq \phi$ for all $p \in \Delta^{\ell-1}$.

Theorem 4, (this also holds for Theorem 5), can be generalized by observing that the only relevant properties of the family $\mathcal{L}(p)$ that matter in its proof are shared by all of the following families $\mathcal{B} = \{B(\lambda) | \lambda \in \mathfrak{R}_+\}$.

(i) $B(0) = \{0\}$,

(ii) for all $\lambda, \lambda' \in \mathfrak{R}_+$ if $\lambda \leqq \lambda'$, then $B(\lambda) \subset B(\lambda')$,

(iii) for all $r > 0$, there exists λ such that $B(\lambda) \supset \{z \in \mathfrak{R}^{\ell}_{+} \mid \|z\| \leqq r\}$,

(iv) B is a continuous function of λ.

It is easy to verify that for all family $\mathfrak{B}$ satisfying (i) - (iv) $EOE(\mathfrak{B},.) \cap P(.) \neq \phi$.

We conclude this section by illustrating with the help of one more example the compatibility of our approach with other approaches. Mas-Colell (1980) and Moulin (1986) consider public good economies and propose to select allocations z satisfying the following property: there is a constant return to scale technology such that each agent i is indifferent between z_i and what he would be able to achieve on his own if he had acess to the technology, given his initial endowment. Here, we assume that agents have collective ownership of the economy's resources, instead of each being initially endowed with some vector of goods, but otherwise, this notion coincides with the notion of equal opportunity equivalence relative to the family of linear choice sets passing through a single point. It is of particular interest that Moulin arrived at this allocation mechanism via the axiomatic route, while our derivation here is based on intuitive considerations of equity.

5. *No Envy of Opportunities.*

We close with the formulation of another concept, which generalizes a definition proposed by Varian (1976)[4] and further studied by Archibald and Donaldson (1979).

Definition. An allocation $z \in A$ exhibits ***No Envy of Opportunities relative to*** $\mathcal{B}$ ***for u*** if for each i, there is $B_i \in \mathcal{B}$ such that z_i maximizes u_i on B_i and for no pair $\{i,j\}$, agent i prefers any point of B_j to z_i. Let **NEO($\mathcal{B}$,u)** be the set of these allocations.

For all $\mathcal{B}$, EO($\mathcal{B}$,.) is a monotonic correspondence. Suppose that $z \in A$ is such that for each i, agent i maximizes u_i in some $B_i \in \mathcal{B}$, but for a pair $\{i,j\}$, $B_i \subset B_j$. This situation, which is considered by Archibald and Donaldson when the choice sets are the budget sets defined by the supporting hyperplanes to the agents' indifference curves at their respective consumptions, is called by them one of "strong inequality". Then, (if in fact, $B_i \subset \text{int } B_j$,) the existence of envy of opportunities can be inferred simply on the basis of monotonicity of preferences.

It is clear that for all $\mathcal{B}$, EO($\mathcal{B}$,.) $\subset$ NEO($\mathcal{B}$,.) $\subset$ EFE.

Also, we have the following elementary facts.

Lemma 9. If $\mathcal{B} = \mathcal{L}$, and all agents have smooth preferences in int $\mathcal{R}^{\ell}_{+}$, then NEO($\mathcal{B}$,u) $\cap$ int A $\subset$ W(u).

Lemma 10. Let $\mathcal{B} = \{\{z_0\} | z_0 \in \mathcal{R}^{\ell}_{+}\}$. Then NEO($\mathcal{B}$,.) = EF.

---

[4]Varian's definition applies only to allocations in P(u) and implicitly assumes the Walrasian mechanism to be operated. Given an allocation $z \in P(u)$, imagine that each agent i is able to trade from z_i at the prices supporting z. The set of allocations so obtained is defined to be his opportunity set. Say that an allocation $z \in P(u)$ is ***opportunity fair*** if no agent i prefers any point of the opportunity set of any other agent to his own consumption z_i.

There are allocations in NEO($\mathcal{L}$,u) that are not in P(u), as illustrated in Figure 7a for u ϵ U^C. Also, there are allocations in NEO($\mathcal{L}$,u) $\cap$ P(u) that are not in W(u), even if agents have smooth preferences in int $\mathfrak{R}^{\ell}_{+}$. This is illustrated in Figure 7b, also for u ϵ U^C. There, z is a boundary allocation.

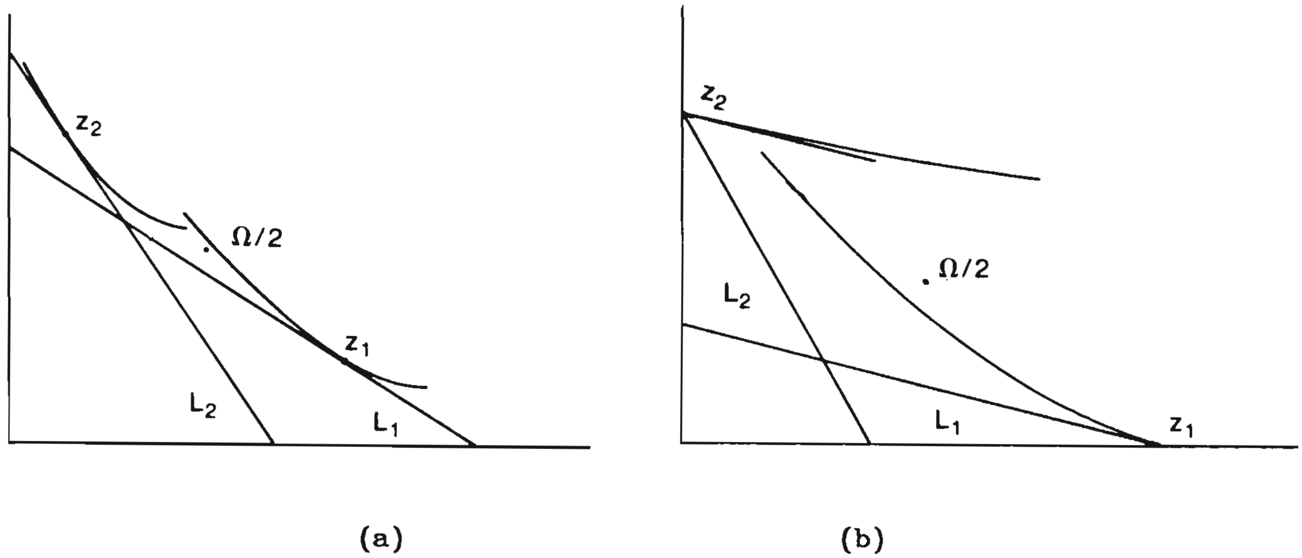


Figure 7

In the Edgeworth box, NEO($\mathcal{L}$,u) is a subset of the set of points that are "beyond" both offer curves drawn from equal division. Indeed, if z ϵ A is such that the line of support to agent i's indifference curve at z_i goes above $\Omega/2$, then agent j, if given access to this choice set, will necessarily be able to do better than z_j.

In the public good case, NEO($\mathcal{L}$,u) is empty for some u ϵ U^C. In the Kolm triangle, for z to be in this set, z first of all should be a point of the vertical segment through the top vertex since NEO($\mathcal{L}$,.) $\subset$ EF. In addition, the lines of support to the agents' indifference curves at z should be symmetric of each other with respect to the segment. There is no guarantee that such a z exists and even if it does, it typically will not be efficient.

There may be weakenings of the above definition that would allow existence in general situations. Here is one attempt, which is motivated by the following variant, which appears in Thomson (1982), of the notion of an envy-free allocation. It says that no agent should prefer the ***average*** of what the other agents have received to what he has received.

Definition. An allocation $z \in A$ is ***average-envy-free for u*** if for no agent i, $u_i(z_i) \geqq u_i(\sum_{j \neq i} z_j/(n-1))$.

Then we have:

Definition. An allocation $z \in A$ exhibits ***No Envy of Opportunities on Average relative to $\mathcal{B}$ for u*** if for each i, there is $B_i \in \mathcal{B}$ such that z_i maximizes u_i in B_i and for no i, agent i prefers any point of $\sum_{j \neq i} B_j/(n-1) \equiv \{z_i \in \mathcal{R}_+^{\ell} | z_i = (\sum_{j \neq i} z_j)/(n-1)$ and for each $j \neq i$, $z_j \in B_j\}\}$.

If agent i does not envy the opportunities of any of the other agents, he may nevertheless envy their average opportunities, and conversely. This can be seen by means of simple examples.

Although this definition may be useful in some contexts, it will not help us solve our existence problem in the public good case since when n=2, it coincides with the definition opening this section.

Here is another possible weakening of the main definition of this section, which may be promising.

Definition. An allocation $z \in A$ is ***No-Envy of Opportunities Equivalent relative to $\mathcal{B}$ for u*** if for each i, there is $B_i \in \mathcal{B}$ such that $u_i(z_i) = u_i(z_i^*)$, where z_i^* is the maximizer of u_i on B_i and for no pair $\{i,j\}$, agent i prefers any point of B_j to z_i^*. Let ***NEOE***$(\mathscr{B},u)$ be the set of these allocations.

It is clear that for all $\mathscr{B}$, NEOE$(\mathscr{B},.)$ contains both NEO$(\mathscr{B},.)$ and EOE$(\mathscr{B},.)$. However, since the latter correspondence is non-empty for natural choices of $\mathscr{B}$, even in public good economies, (Theorems 4 and 5), one could argue that our last definition is *too much* of a generalization. So we will not pursue its analysis.

6. *Concluding comment*

The concepts presented here can be adapted in a straightforward way to the problem of evaluating the equity of a trade vector instead of that of an allocation. Instead of considering sets of possible consumptions, we would consider sets of possible *trades*. Given a family $\mathscr{T}$ of sets of possible trades, the three main definitions of this paper can be rewritten by replacing $\mathscr{B}$ by $\mathscr{T}$.

Appendix A

In this appendix, we give the proof of Theorem 1.

Let $p \in \Delta^{\ell-1}$ be given and u be an economy in which (v) all agents have linear preferences with indifference curves normal to p. By (i), $u \in U$ and by (ii), there is $z \in EO(\mathscr{B},u)$. Let then $B \in \mathscr{B}$ be such that each agent i maximizes u_i over B. By (v), $pz_i = pz_j$ for all i,j and $pz_0 \leqq pz_1$ for all $z_0 \in$ B. Since $z \in A$, then $pz_i = p\Omega/n$ for all i, and by (iii), $pz_0 = p\Omega/n$ for all $z_0 \in B$. Therefore, B is a subset of W_p, the Walrasian choice set through equal division relative to the prices p.

In the rest of the proof, all topological notions should be understood to be relative to the plane of equation $pz_0 = p\Omega/n$.

We claim that (vi) int B = int($W_p \cap D$). Supposing otherwise, let z_0 on the boundary of int($W_p \cap D$)\int B be given. By (iv), $z_0 \in B$ and by (iii), there is $u \in U$, $z \in A$ and i such that $z_i = z_0$ and z_j maximizes u_j on B for all j. In fact, we can take u to be as in the first part of the proof. Now, let C be a convex subset of int($W_p \cap D$)\int B containing z_0 on its boundary. By (iv), C can be taken to have a non-empty interior. Let u_i' be such that its indifference surface through z_0 is not supported at z_0 by the price p, and $T_i(u_i',z_0) \cap W_p = C$. Such a u_i' can be found in U, by (i). Let $u' \equiv (u_1,\ldots,u_i',\ldots,u_n)$. Then, for each i, z_i maximizes u_i' in B so that $z \in EO(\mathscr{B},u')$. However, $z \notin P(u')$, in contradiction with (ii). Therefore, (vi) holds, and since $p \in \Delta^{\ell-1}$ was arbitrary, we conclude that $EO(\mathscr{B},.) \supset$ int W.

Q.E.D.

Appendix B

In Figure 10, we summarize the main existence and inclusion results of the paper. They concern private good economies and pertain to the families of choice sets that we took most frequently as examples. An entry such as "$\supset W$" at the intersection of the row labelled NEO and of the column labelled $\mathcal{L}$ means that "NEO($\mathcal{L}$,.) $\supset$ W".

In Figure 11, we indicate the containment relations between the equity notions proposed here and the notions of an envy free allocation s and that of an envy free allocation. These inclusions hold for an arbitrary family $\mathcal{B}$.

Families Equity of Concepts Choice Sets	$\{z_0\}$	$\{z_1,\ldots,z_n\}$ $(z_1,\ldots,z_n)$ $\in A$	$\mathcal{W}$, the Walrasian family through equal division	$\mathcal{L}$, the family of linear choice sets	comprehensive hull of td, for t>0, d fixed in $\Re^{\ell}_{+}$	budget set of slope p, p fixed in $\Delta^{\ell-1}$ and income $I > 0$
EO EO ∩ P	ϕ	EF EF ∩ P	W W	⊃ W W	ϕ	ϕ
NEO NEO ∩ P	EF EF ∩ P	EF EF ∩ P	W W	⊃ W ⊃ W	ϕ	ϕ
EOE EOE ∩ P	EE EE ∩ P	EFE EFE ∩ P	W W	n = 2 EE n > 2 ⊃ EE EE ⊃	EE_d $EE_d \cap P$	EE_p EE_p
NEOE NEOE ∩ P	EFE EFE ∩ P	EFE EFE ∩ P	W ⊂ W ⊂	⊂ W ⊂ W	EE_d $EE_d \cap P$	EE_p EE_p

Figure 10

$$\begin{array}{ccccc} NEO(\mathcal{B},.) & \subset & NEO(\mathcal{B},.) & \subset & EF \\ \cap & & \cap & & \cap \\ EOE(\mathcal{B},.) & \subset & NEO(\mathcal{B},.) & \subset & EFE \end{array}$$

Figure 11

Appendix C

Here, we present a fourth notion of equity. This notion has been relegated to an appendix because it is not based on evaluating choice sets. However, it does capture part of what can be understood by the phrase "equal opportunities", and in that sense it does have its place in this paper. In spirit, it is perhaps closest to notions developed in Thomson (1983).

Assume that some choice has been made of a mechanism designed to allocate gains from trade. One might argue that by operating this mechanism from a point of equal division, an equitable allocation will necessarily be reached. Let us generalize this idea by declaring $z \in A$ equitable if it is Pareto-indifferent to an allocation that could be obtained in this way for ***some*** choice of a point of an equal division, ***perhaps not a feasible one***.

Formally, let φ: $U \times A \to A$ be a correspondence.

Definition. An allocation $z \in A$ is an ***equal opportunity allocation relative to the mechanism*** $\boldsymbol{\varphi}$ ***for u*** if there exist an egalitarian allocation $\omega^0 = (\omega_0, \ldots, \omega_0)$ and $z^0 \in \varphi(u, \omega^0)$ such that for all i, $u_i(z_i) = u_i(z_i^0)$. Let $\boldsymbol{\varphi}$**-EO(u)** be the set of these allocations.

We emphasize that the allocations ω^0 and z^0 appearing in this definition are not in general feasible.

Figure 12 illustrates this definition when φ is the Walrasian correspondence.

Note that W-EO(.) $\supset$ W. However, for most u, there are allocations in W-EO(u) that are not in W(u).

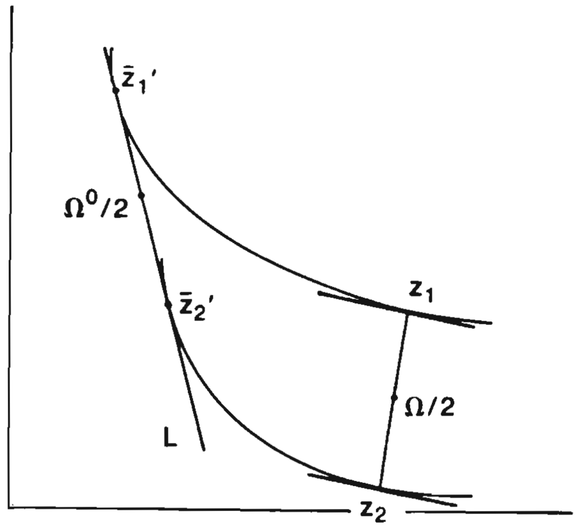


Figure 9

For n = 2, W-EO(.) = EE. Indeed, given u, the two agents' indifference curves cross ($z \in EE(u)$) if and only if they admit a common line of support L. Given $\bar{z}_1$ and $\bar{z}_2$, two points of contact of these indifference curves with L, let $\omega_0 \equiv (\bar{z}_1+\bar{z}_2)/2$. Then $z \in$ W-EO(u) with ω_0 as reference point of equal division.

More generally, we have W-EO(.) = EOE($\mathcal{L}$,.).

The implications of this definition for other choices of φ will be left to future research.

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