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Abstract

This paper considers an environment which possesses nonconvexities and shows that under relatively mild additional assumptions the standard results of recursive competitive equilibrium theory for convex economies continue to hold. The nonconvexity arises from the fact that workers can only supply labour in one sector in any given period although they are free to move among sectors. In an example it is demonstrated that this environment may prove useful in examining cyclical and secular adjustments in the labour market.

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SECTION 1

INTRODUCTION

Two common features of many microeconomic models used to address aggregate prhenomena are convex environments and homogeneous agents. One of the advantages of this approach is the fact that equilibrium allocations can be characterized by studying properties of a concave programming problem rather than trying to establish properties of a solution to a general fixed point problem. One of the drawbacks to this approach is that the equilibrium allocations have the property that all agents receive identical bundles of commodities. In some cases this is problematic even at the aggregate level. The example which is focussed on here is that of aggregate movements in employment and hours/work. This is not the only instance where problems arise: the same problem exists when one considers aggregate data for investment and bankruptcies.

One way to overcome this difficulty is to introduce heterogeneity of agents' characteristics. An alternative approach is to introduce non-convexities into the environment. This is the approach taken here. The paper provides two main types of results. First, it demonstrates sufficient conditions for results in the homogeneous agent, convex environment theory of recursive competitive equilibrium (see Prescott and Mehra [5]) to hold in a certain class of non-convex economies. It is seen that some conditions must (apparently) be added in the dynamic case even though none are required in the static case. The second type of result concerns the implications of the model described for the dynamic behaviour of employment, hours/worker and real wages at the sectoral level. In the context of an example it is demonstrated that the model is consistent with the fact that changes in hours/worker lead changes in employment, employment displays some persistence, and hours/worker are more highly correlated with real wages than is employment (see Rogerson [6]).

As pointed out earlier, the model presented here is essentially an alternative to considering convex environments with heterogeneous agents. This raises an important question which is not dealt with in this paper. Another potential drawback of representative agent models appears to be their (possible) inability to produce fluctuations of the same magnitude as observed in time series for actual economies. Certainly this issue has not been resolved yet, however, recent work by Kydland [1], for example, demonstrates how heterogeneity can result in increased fluctuations in certain variables. This leaves open the question of whether or not non-convex economies with homogeneous agents can also produce such phenomena. The kind of model studied in this paper seems to suggest that the answer to this question is yes, however this is a question which needs to be posed in a careful manner and studied more explicitly. The model studied is both an infinite horizon version of Rogerson [6] and a version of Lucas and Prescott [3] which allows for sectoral changes and aggregate fluctuations to be analyzed simultaneously, a topic which has recently received some attention by Lilien [2].

The model presented here is set up in a way which emphasizes its application to studying labour markets, however, the methods used here will probably be useful in studying other problems of a similar nature, e.g. flows of capital between sectors or countries, and flows of resources between divisions of a corporation.

SECTION 2

THE ENVIRONMENT

The economy consists of a continuum of identical infinitely lived agents. There are three commodities: labour, capital and output. There are two sectors where production can take place and the production technology is subject to a single aggregate shock. Let $f_j(K,H,s($: $R_+ \times R_+ \times R_+ \to R_+$ be the production function specifying output in sector j when K units of capital and H man-hours of labour are used as inputs, and the state of nature is s. It will be assumed that $f_j(\cdot,\cdot,\cdot)$ is homogeneous of degree one and weakly concave in (K,H), strictly increasing and strictly concave in K,H separately, and $f_j(0,0,s) = 0$ for all s and j. It will be assumed that s follows a stationary first order Markov process taking values in $S \subset R_+$. The process for s is specified by F: $S \times S \to [0,1]$ where

$$F(s',s) = \text{Prob}[s_t \leq s' | s_{t-1} = s]$$

Capital is sector specific. There is an initial endowment of capital of each type and it cannot be accumulated, or transformed and it does not depreciate.

There is a continuum of workers,uniformly distributed along [0,1]. Each worker is endowed with one unit of sector one capital, one unit of sector two capital and one unit of time. Any fraction of the unit of time may be supplied as labour with the following restrictions:

(i) labour cannot be supplied in both sectors simultaneously.

(ii) if labour is supplied in different sectors in periods t and t+1 then a psychic cost is incurred in period t+1.

The nature of the above restrictions is such that it will be useful to distinguish formally between labour and capital supplied in sector one and labour and capital supplied to sector two. In any given period there will be five commodities and the following indexing system will be used:

commodity 1 = output

commodity 2 = labour in sector one

commodity 3 = labour in sector two

commodity 4 = capital in sector one

commodity 5 = capital in sector two

The one period consumption set of a worker is then given by:

$$X = \{(x_1,\ldots,x_5) \in R^5\colon x_1 \geq 0,\ x_2 \geq 0,\ -1 \leq x_2 \leq 0,\ -1 \leq x_3 \leq 0,$$
$$x_4 \geq -1,\ x_5 \geq -1,\ x_2 \cdot x_3 = 0\}$$

Define $U(x,\ell)\colon X \times \{1,2\} \to R$ by:

$$U(x,\ell) = \begin{cases} x + v(x_2+x_3) & \text{if either } \ell=1,\ x_3=0,\ \text{or } \ell=2,\ x_2=0 \\ x + v(x_2+x_3)-m & \text{if either } \ell=2,\ x_2\neq 0 \text{ or } \ell=1,\ x_3\neq 0 \end{cases}$$

If $\{x_t\}$ is a stochastic process where $x_t \in X$ with probability one for all t, and $\ell_0 \in \{1,2\}$ then the worker receives utility given by:

$$E\{\sum_{t=1}^{\infty} \beta^t U(x_t, \ell_{t-1})\}$$

where $0 < \beta < 1$ is a discount factor, ℓ_t is the locationof the worker in period t and is given by:

$$\ell_t = \begin{cases} 1 & \text{if } x_{t2} \neq 0 \\ 2 & \text{if } x_{t3} \neq 0 \\ \ell_{t-1} & \text{if } x_{t2} = 0 \text{ and } x_{t3} = 0 \end{cases}$$

E is the expectation operator and U is the one period utility function. Here m is the psychic cost associated with changing the sector of employment and $v: R \to R$ is the disutility associated with working. It is assumed that $v(\cdot)$ is strictly increasing, strictly concave and bounded on the interval $[-1,0]$.

Using the same indexing system it is possible to define a technology set Y(s) corresponding to the production functions specified earlier. For each $s \in S$ let

$$Y(s) = \{(y_1, y_2, \ldots, y_5) \in R^5: y_1 \geq 0,\ y_2 \leq 0,\ y_3 \leq 0,\ y_4 \leq 0,\ y_5 \leq 0,$$

$$y_1 \leq f_1(-y_2, -y_4, s) + f_2(-y_3, -y_5, s)\ .$$

The economy described above is completely specified by the following list:

$\mathcal{E} = (Y(s), v, B, m, F, X)$

SECTION 3

RECURSIVE COMPETITIVE EQUILIBRIUM

In this section we define the notion of a recursive competitive equilibrium for $\mathcal{E}$. In order to do this we need to define a state variable. The state variable for the economy $\mathcal{E}$ will be given by the pair $(\lambda,s) \in [0,1] \times S$, where λ is the fraction of workers who were in sector one at the end of last period and s is the current realization of the aggregate shock. A state variable for an individual will be the triple $(\lambda,s,\ell) \in [0,1] \times S \times \{1,2\}$ where λ,s are as above and ℓ is the location of the individual at the end of last period. We now have the following definition:

Definition: A recursive competitive equilibrium (RCE) for $\mathcal{E}$ is a list

$$(p(\lambda,s),\Lambda(\lambda,s),V(\lambda,s,\ell),\ x(\lambda,s,\ell,t),\ y(\lambda,s))$$

where

(i) p: $[0,1] \times S \to \mathcal{S}^5$ is continuous almost everywhere

(ii) Λ: $[0,1] \times S \to [0,1]$ is continuous almost everywhere

(iii) V: $[0,1] \times S \times \{1,2\} \to R$ is continuous, bounded and satisfies: $V(\lambda,s,\ell) = \max_{x,\ell'} \{U(x,\ell) + \beta EV(\Lambda(\lambda,s),s',\ell')\}$.

subject to $x \in X$

$$p(\lambda,s)\cdot x \leq 0$$

$$\ell' = \begin{cases} 1 & \text{if } x_2 \neq 0 \\ 2 & \text{if } x_3 \neq 0 \\ \ell & \text{if } x_2 = x_3 = 0 \end{cases}$$

(iv) For all $t \in [0,\lambda]$, all $(\lambda,s) \in [0,1] \times S$

$$x(\lambda,s,1,t) \in \underset{x,\ell'}{\text{argmax}}\{U(x,1) + BEV(\Lambda(\lambda,s),s',\ell')\}$$

subject to

$$x \in X$$

$$p(\lambda,s)\cdot x \leq 0$$

$$\ell' = \begin{cases} 1 & \text{if } x_2 \neq 0 \\ 2 & \text{if } x_2 \neq 0 \\ 1 & \text{if } x_2 = x_3 = 0 \end{cases}$$

For all $t \in (\lambda,1)$, all $(\lambda,s) \in [0,1] \times S$

$$x(\lambda,s,2,t) \in \text{argmax}\{U(x,2) + BEV(\Lambda(\lambda,s),s',\ell')\}$$

subject to $x \in X$

$$p(\lambda,s)\cdot x \leq 0$$

$$\ell' = \begin{cases} 1 & \text{if } x_2 \neq 0 \\ 2 & \text{if } x_3 \neq 0 \\ 2 & \text{if } x_2 = x_3 = 0 \end{cases}$$

(v) For all $(\lambda,s) \in [0,1] \times S$, $y(\lambda,s)$ is a solution to:

$$\max_{y}\ p(\lambda,s)\cdot y$$

subject to $y \in Y(s)$

(vi) $$\int_0^{\lambda} x(\lambda,s,1,t)dt + \int_0^{1} x(\lambda,s,2,t)dt = y(\lambda,s)$$

for all $\lambda,s \in [0,1] \times S$

(vii) For all $(\lambda,s) \in [0,1] \times S$

$$\Lambda(\lambda,s) = \text{meas}\{t \in [0,\lambda]: x_3(\lambda,s,1,t) = 0\} + \text{meas}\{t \in (\lambda,1]: x_2(\lambda,s,1,t) \neq 0\}$$

The interpretation is as follows. $p(\lambda,s)$ is a pricing function, $\Lambda(\lambda,s)$ is the law of motion for λ, $V(\lambda,s,\ell)$ is the value function for an individual, $x(\lambda,s,\ell,t)$ specifies the consumption bundles taken by workers, and $y(\lambda,s)$ is the production decision. Condition (iii) is the functional equation which V satisfies, condition (iv) states that current period consumption decisions are optimal, condition (v) states that firms maximize profits, condition (vi) says that markets clear, and condition (vii) states that expectations concerning the law of motion for λ are fulfilled.

SECTION 4

AN ALTERNATIVE FORMULATION OF THE ENVIRONMENT

In the economy $\mathcal{E}$ non-convexities were present in the consumption set X. This meant that in equilibrium, if there is any switching of workers between sectors that identical agents will be choosing different consumption bundles. As will be seen later, this feature makes it somewhat awkward to utilize the relation between optimal allocations and equilibrium allocations. In this section we present an alternative formulation of $\mathcal{E}$ which consists simply of adding certain kinds of lotteries to the consumption set. The advantage of doing this is that it will then be possible to generate equilibrium in which all agents choose the same allocation (ex ante) even though they end up consuming different bundles. We now present the changes to be made to the environment as specified in Section 1.

Define $\overline{X}_1 = \{x \in X: x_3 = 0\}$, $\overline{X}_2 = \{x \in X: x_2 = 0\}$

and

$$\overline{X} = \overline{X}_1 \times \overline{X}_2 \times [0,1] \times [0,1]$$

Define $\overline{U}: \overline{X} \times [0,1]$ by

$$\begin{aligned}\overline{U}(x^1,x^2,q^1,q^2,p) = {} & [pq^1 + (1-p)(1-q^2)]U(x^1,1) \\ & + [(1-p)(q^2) + p(1-q^1)]U(x^2,2) \\ & - [(1-q^1)p + (1-q^2)(1-p)]m\end{aligned}$$

The interpretation is as follows: p is the probability that the worker finds himself in sector one. He then chooses a contingency plan. q^1 is the probability that he remains in sector one contingent upon being in sector one and q^2 is the probability that he remains in sector two conditional upon being in sector two. Finally, x^j is the consumption bundle if the worker remains in or moves into sector j. Note that the worker will receive x^1 with probability $pq^1 + (1-p)(1-q^2)$ and x^2 with probability $(1-p)q^2 + p(1-q^1)$. Also expected psychic cost associated with moving is $m[p(1-q^1) + (1-p)(1-q^2)]$ since $p(1-q^1) + (1-p)(1-q^2)$ is the probability of moving.

Now suppose $\{\bar{x}_t\}$ is a stochastic process with $\bar{x}_t \in \bar{X}$ with probability one for all t, and $p_0 \in [0,1]$ is given. Then the consumer receives utility

$$E\{\sum_{t=1}^{\infty} B^t \bar{U}(\bar{x}_t, p_{t-1})\}$$

where p_t follows the path:

$$p_t = \bar{q}_t^1 p_{t-1} + (1-p_{t-1})(1-\bar{q}_t^2).$$

Note that if $\bar{x}_t \in \bar{X}$ then we will write $\bar{x}_t = (\bar{x}_t^1, \bar{x}_t^2, \bar{q}_t^1, \bar{q}_t^2)$.

We now consider the economy $\bar{E}$ completely specified by the following list:

$$\bar{E} = (Y(s), v, B, m, F, \bar{X})$$

where all objects are as defined above or in previous sections.

SECTION 5

SYMMETRIC RECURSIVE COMPETITIVE EQUILIBRIUM WITH LOTTERIES

In this section we define a notion of RCE for $\bar{E}$ in which all agents receive identical allocations. The state variable for the aggregate economy is still written as (λ,s) but now λ is interpreted as the expected number of agents in sector one at the end of last period. The state variable for an individual will be the triple (λ,s,p) where $p \in [0,1]$ is the probability that the individual was in section one at the end of last period Formally we now have:

<u>Definition</u>: A symmetric recursive competitive equilibrium with lotteries (SRCEL) for $\bar{E}$ is a list

$$(p(\lambda,s), \Lambda(\lambda,s), V(\lambda,s,p),\ x(\lambda,s,p),\ y(\lambda,s))$$

where:

(i) $p: [0,1] \times S \to \mathcal{S}^5$ is continuous almost everywhere

(ii) $\Lambda: [0,1] \times S \times [0,1]$ is continuous almost everywhere

(iii) $V: [0,1] \times S \times [0,1] \quad R$ is continuous, bounded and satisfies:

$$V(\lambda,s,p) = \max_{x,p'} \{\bar{U}(x,p) + BEV(\Lambda(\lambda,s),s',p')\}$$

subject to:

$$x \in \bar{X}$$
$$p(\lambda,s)\cdot x^1 \leq 0$$
$$p(\lambda,s)\cdot x^2 \leq 0$$
$$p' = pq^1 + (1-p)(1-q^2)$$

(iv) For all $(\lambda,s,p) \in [0,1] \times S \times [0,1]$

$$x(\lambda,s,p) \in \underset{x,p'}{\operatorname{argmax}}\{\overline{U}(x,p) + BEV(\Lambda(\lambda,s),s',p')\}$$

$$\text{subject to } x \in \overline{X}$$

$$p(\lambda,s)\cdot x^1 \leq 0$$

$$p(\lambda,s)\cdot x^2 \leq 0$$

$$p' = pq^1 + (1-p)(1-q^2)$$

(v) For all $(\lambda,s) \in [0,1] \times S$ $y(\lambda,s)$ solves:

$$\max_y p(\lambda,s)\cdot y$$

$$\text{subject to } y \in Y(s)$$

(vi) For all $(\lambda,s) \in [0,1] \times S$

$$x(\lambda,s,\lambda) = y(\lambda,s)$$

(vii) For all $(\lambda,s) \in [0,1] \times S$

$$\Lambda(\lambda,s) = \lambda q^1(\lambda,s,\lambda) + (1-\lambda)(1-q^2(\lambda,s,\lambda))$$

The interpretation is similar to that given before. Note that conditions (vi) and (vii) imply that all agents are receiving identical allocations. Also note that in conditions (iii) and (iv) nothing would change if the condition $p(\lambda,s)\cdot x^j \leq 0$ $j=1,2$ were replaced by

$$[pq^1 + (1-p)(1-q^2)]p(\lambda,s)\cdot x^1 + [(1-p)q^2 + p(1-q^1)]p(\lambda,s)\cdot x^2 \leq 0$$

This follows easily from the fact that utility is linear in consumption of output.

Formally, looking for a SRCEL for $\overline{\mathcal{E}}$ is equivalent to looking for an equilibrium in the following single agent economy: There is a single agent endowed with one unit of time. Each period the agent must decide what fraction of the period to spend in sector one and what fraction of the period to spend in sector two. In addition, the agent must decide what fraction of his or her time spent in sector one should be spent working and what fraction of his or her time spent in sector two should be spent working. What makes the problem interesting dynamically is that it is costly for the individual to alter from one period to the next the fraction of total time spent in each of the two sectors (although not relative time spent working as compared to resting in a given sector). Observe that the concavity of $v(\cdot)$ implies that the individual would prefer to spend the entire period in a given sector working a given number of total hours rather than divide the hours between two sectors.

This interpretation is essentially what will make it easier later on to connect optimal allocations with allocations generated by SRCEL rather than with those generated by RCE. In the next section we show that RCE for $\mathcal{E}$ and SRCEL for $\bar{\mathcal{E}}$ are equivalent in a certain sense.

SECTION 6

EQUIVALENCE OF RCE AND SRCEL

In this section we prove the following proposition:

Proposition 1: (p,Λ,V,x,y) *is a* RCE *for* $\mathcal{E}$ *iff there exists a* SRCEL *for* $\bar{\mathcal{E}}$ *of the form* $(p,\Lambda,\bar{V},\bar{x},y)$

Proof: Let (p,Λ,V,x,y) be a RCE for $\mathcal{E}$.

Define $\bar{V}(\lambda,s,p) = pV(\lambda,s,1) + (1-p)V(\lambda,s,2)$.

By definition of RCE

$$V(\lambda,s,1) = \max\{U(x,1) + BEV(\Lambda,s',\ell)\}$$

$$\text{subject to } x \in X$$

$$p(\lambda,s)\cdot x \leq 0$$

$$\ell = \begin{cases} 1 & \text{if } x_3 = 0 \\ 2 & \text{if } x_3 \neq 0 \end{cases}$$

Define $B(\lambda,s)$ to be the set of pairs (x,ℓ) satisfying the above constraints.

Hence $pV(\lambda,s,1) = \max_{(x,\ell)\in B(\lambda,s)} \{pU(x,1) + pBEV(\Lambda,s',\ell)\}$

$$= \max_{x^1,x^2,q^1} \{pq^1[U(x^1,1) + BEV(\Lambda,s',1)$$

$$+ p(1-q^1)[U(x^2,1) + BEV(\Lambda,s',2)]\}$$

$$\text{subject to } x^1 \in \overline{B}^1(\lambda,s)$$

$$x^2 \in \overline{B}^2(\lambda,s)$$

$$0 \le q^1 \le 1$$

where $\quad \overline{B}^j(\lambda,s) = \{x\in\overline{X}_j\colon p(\lambda,s)\cdot x\le 0\}$

Similarly

$$(1-p)V(\lambda,s,2) = \max_{x^3,x^4,q^2} \{(1-p)(1-q^2)[U(x^3,2) + BEV(\Lambda,s',1)$$

$$+ (1-p)q^2[U(x^4,2) + BEV(\Lambda,s',1)]\}$$

$$\text{subject to } x^3 \in \overline{B}^1(\lambda,s)$$

$$x^4 \in \overline{B}^2(\lambda,s)$$

$$0 \le q^2 \le 1$$

Clearly

$$pV(\lambda,s,1) + (1-p)V(\lambda,s,2) =$$

$$\max_{\substack{x^1,\ldots,x^4 \\ q^1,q^2}} \quad pq^1[U(x^1,1) + BEV(\Lambda,s',1)] + p(1-q^1)[U(x^2,1) + BEV(\Lambda,s',2)]$$

$$+ (1-p)(1-q^2)[U(x^3,2) + BEV(\Lambda,s',1)]$$

$$+ (1-p)q^2[U(x^4,2) + BEV(\Lambda,s',1)]$$

$$\text{subject to: } x^1 \in \overline{B}^1(\lambda,s)$$
$$x^2 \in \overline{B}^2(\lambda,s)$$
$$x^3 \in \overline{B}^1(\lambda,s)$$
$$x^4 \in \overline{B}^2(\lambda,s)$$
$$0 \leq q^1 \leq 1,\ 0 \leq q^2 \leq 2$$

The form of the objective function and constraints implies that the above can be written as

$$= \max_{x^1,x^2,q^1,q^2} \{[pq^1 + (1-p)(1-q^2)][U(x^1,1) + BEV(\Lambda,s',1)]$$
$$+ [(1-p)q^2 + p(1-q^2)][U(x^2,2) + BEV(\Lambda,s',2)]$$
$$- [p(1-q^1) + (1-p)(1-q^2)]m\}$$

$$\text{subject to: } x^1 \in \overline{B}^1(\lambda,s)$$
$$x^2 \in \overline{B}^2(\lambda,s)$$
$$0 \leq q^1 \leq 1,\ 0 \leq q^2 \leq 1$$

$$= \max_{x^1,x^2,q^1,q^2} \{\overline{U}(x,p) + BE\overline{V}(\Lambda,s',pq^1 + (1-p)(1-q^2))\}$$

$$\text{subject to: } x^1 \in \overline{B}^1(\lambda,s)$$
$$x^2 \in \overline{B}^2(\lambda,s)$$
$$0 \leq q^1 \leq 1,\ 0 \leq q^2 \leq 1$$

This shows that $\overline{V}(\lambda,s,p)$ satisfies condition (iii) of equilibrium.

Now assume that $\Lambda(\lambda,s) \leq \lambda$. Then it follows that

$$(1)\quad \max_x \{U(x,1) + BEV(\Lambda,s',1)\} = \max_x\{U(x,1) + BEV(\Lambda,s',2)\}$$

$$\text{subject to } x \in \bar{B}^1(\lambda,s) \qquad \text{subject to } x \in \bar{B}^2(\lambda,s)$$

and

$$(2)\quad \max_x \{U(x,2) + BEV(\Lambda,s',2)\} \geqq \max_x\{U(x,2) + BEV(\Lambda,s',1)\}$$

$$\text{s.t. } x \in \bar{B}^2(\lambda,s) \qquad \text{s.t. } x \in \bar{B}^1(\lambda,s)$$

Define

$$\bar{q}^1(\lambda,s,p) = \min\{1,\frac{\Lambda(\lambda,s)}{p}\}$$

$$\bar{q}^2(\lambda,s,p) = 1$$

$$\bar{x}^1(\lambda,s,p) = \underset{x}{\operatorname{argmax}}\ U(x,1) \quad \text{s.t.} \quad x \in \bar{B}^1(\lambda,s)$$

$$\bar{x}^2(\lambda,s,p) = \underset{x}{\operatorname{argmax}}\ U(x,2) \quad \text{s.t.} \quad x \in \bar{B}^2(\lambda,s)$$

If $\Lambda(\lambda,s) > \lambda$ if follows that

$$(3)\quad \max_x \{U(x,1) + BEV(\Lambda,s',1)\} \geq \max_x \{U(s,1) + BEV(\Lambda,s',2)\}$$

$$\text{s.t. } x \in \bar{B}^1(\lambda,s) \qquad \text{s.t. } x \in \bar{B}^2(\lambda,s)$$

and

$$(4)\quad \max_x \{U(x,2) + BEV(\Lambda,s',1)\} = \max_x \{U(x,2) + BEV(\Lambda,s',2)\}$$

$$\text{s.t. } x \in \bar{B}^1(\lambda,s) \qquad \text{s.t. } x \in \bar{B}^1(\lambda,s)$$

Define $\bar{q}^1(\lambda,s,p) = 1$

$$\bar{q}^2(\lambda,s,p) = \min\{1,\frac{1-\Lambda(\lambda,s)}{1-p}\}$$

$\bar{x}^1$, $\bar{x}^2$ as above

The definition of $\bar{V}$ and conditions (1) - (4) imply that $(p,\Lambda,\bar{V},\bar{x},y)$ is a SRCEL for $\bar{E}$.

This proves half of the proposition. We now prove the other half.

Let $(p,\bar{\Lambda},\bar{V},\bar{x},y)$ be a SRCEL for $\bar{E}$. By definition of equilibrium $\bar{V}$ satisfies

$$\bar{V}(\lambda,s,p) = \max_{x^1,x^2,q^1,q^2} \{[pq^1+(1-p)(1-q^2)]U(x^1,1)+[(1-p)q^2+p(1-q^2)]U(x^2,2)$$
$$- [p(1-q^1) + (1-p)(1-q^2)]m$$
$$+ BE\bar{V}(\Lambda,x',pq^1 + (1-p)(1-q^2))\}$$

$$\text{s.t.} \quad x^1 \in \bar{B}^1(\lambda,s)$$
$$x^2 \in \bar{B}^2(\lambda,s)$$
$$0 \leq q^1 \leq 1,\ 0 \leq q^2 \leq 1.$$

We first show that for λ,s fixed $\bar{V}$, is linear in p. Define an operator T mapping the set of bounded continuous functions on $[0,1] \times S \times [0,1]$ into itself by

$$(5) \quad Tf(\lambda,s,p) = \max_{x^1,x^2,q^1,q^2} \{[pq^1+(1-p)(1-q^2)]U(x^1,1)$$
$$+ [(1-p)q^2 + p(1-q^2)]U(x^2,2)$$
$$- [p(1-q^1) + (1-p)(1-q^2)]m$$
$$+ BEf(\Lambda,s',pq^1+(1-p)(1-q^2))\}$$

$$\text{s.t.} \quad x^1 \in \bar{B}^1(\lambda,s) \qquad x^2 \in \bar{B}^2(\lambda,s)$$
$$0 \leq q^1 \leq 1 \qquad 0 \leq q^2 \leq 1$$

Claim: If for any given pair (λ,s) f is linear in its third variable then so is Tf.

Proof of Claim: The maximization with respect to x^1 and x^2 is independent of p, q^1 and q^2. Let $\bar{x}^1(\lambda,s)$ and $\bar{x}^2(\lambda,s)$ correspond to optimal values of these variables. Note that the resulting expression is linear in q^1 and q^2 and moreover that the signs of the coefficients on q^1 and q^2 are independent of p. Hence optimal choices of q^1 and q^2 can be written as $\bar{q}^1(\lambda,s)$ and $\bar{q}^2(\lambda,s)$. Substituting these into equation (5) we see that Tf is linear in p for any given (λ,s). This completes the proof of the claim.

Since $\bar{V}$ can be obtained by repeatedly applying T to a given bounded continuous function, and the limit of a sequence of linear functions is also linear, this shows that $\bar{V}$ is linear in p for any given values of λ,s.

It follows that $\bar{V}(\lambda,s,p)$ is completely specified by $\bar{V}(\lambda,s,0)$ and $\bar{V}(\lambda,s,1)$ with

$$\bar{V}(\lambda,s,p) = p\bar{V}(\lambda,s,1) + (1-p)\bar{V}(\lambda,s,0).$$

Define $\quad V(\lambda,s,1) = \bar{V}(\lambda,s,1)$

$\qquad\qquad V(\lambda,s,2) = \bar{V}(\lambda,s,0).$

By definition of equilibrium

$$\overline{V}(\lambda,s,1) = \max_{q^1,x^1,x^2} \{q^1U(x^1,1) + (1-q^1)U(x^2,2) - (1-q^1)m + BEq^1\overline{V}(\Lambda,s',1) + BE(1-q^1)\overline{V}(\Lambda,s',0)\}$$

$$\text{s.t.} \quad x^1 \in \overline{B}^1(\lambda,s) \quad x^2 \in \overline{B}^2(\lambda,s)$$

$$0 \leq q^1 \leq 1$$

The expression in parentheses is linear in q^1, hence

$$\overline{V}(\lambda,s,1) =$$

$$\max\{\max_x\{U(x,1) + BE\overline{V}(\Lambda,s',1)\}, \max_x\{U(x,1)-m+BE\overline{V}(\Lambda,s',2)\}\}$$

$$\text{s.t.} \quad x \in \overline{B}^1(\lambda,s) \qquad\qquad \text{s.t.} \quad x \in \overline{B}^2(\lambda,s)$$

$$= \max_{(x,\ell)} \{U(x,1) + BEV(\Lambda,s',\ell)\}$$

$$\text{s.t.} \ (x,\ell) \in B(\lambda,s)$$

Hence $V(\lambda,s,1)$ satisfies condition (iii) of RCE. A similar argument works for $V(\lambda,s,2)$. From here the proof is essentially identical to that used to establish the first half the proposition, so it is not repeated. This completes the proof.

SECTION 7

PARETO OPTIMALITY

In this section we define an equal weight Pareto optimum for $\mathcal{E}$ and $\bar{\mathcal{E}}$. There will be no need to distinguish between $\mathcal{E}$ and $\bar{\mathcal{E}}$ formally since at the aggregate level there is no distinction between fractions and probabilities.

Definition: An (equal weight) recursive Pareto optimum (RPO) is a list $(W(\lambda,s),\ x^1(\lambda,s),\ x^2(\lambda,s),\ q^1(\lambda,s),\ q^2(\lambda,s))$ such that

(i) $W(\lambda,s)$: $[0,1] \times S \rightarrow R$ is continuous, bounded and satisfies

$$W(\lambda,s) = \max_{x^1,x^2,q^1,q^2} \{[\lambda q^1+(1-\lambda)(1-q^2)]U(x^1,1) + [\lambda(1-q^1)+(1-\lambda)q^2]U(x^2,2) - [\lambda(1-q^1)+(1-\lambda)(1-q^2)]m + BEW(\lambda q^1+(1-\lambda)(1-q^2),s')\}$$

subject to: $x^1 \in \bar{X}_1,\quad x^2 \in \bar{X}_2$

$$[\lambda q^1+(1-\lambda)(1-q^2)]x^1 + [\lambda(1-q^1)+(1-\lambda)q^2]x^2 \in Y(s)$$

$$0 \leq q^1 \leq 1$$

$$0 \leq q^2 \leq 1$$

$$\text{(ii) } W(\lambda,s) = [\lambda q^1(\lambda,s) + (1-\lambda)(1-q^2(\lambda,s)]U(x^1(\lambda,s),1)$$
$$+ [\lambda(1-q^1(\lambda,s))+(1-\lambda)q^2(\lambda,s)]U(x^2(\lambda,s),2)$$
$$- [\lambda(1-q^1(\lambda,s))+(1-\lambda)(1-q^2(\lambda,s))]m$$
$$+ BEW(\lambda q^1(\lambda,s)+(1-\lambda)(1-q^2(\lambda,s)),s')$$

and

$$x^1(\lambda,s) \in \overline{X}_1 \qquad x^2(\lambda,s) \in \overline{X}_2$$
$$0 \leq q^1(\lambda,s) \leq 1, \; 0 \leq q^2(\lambda,s) \leq 1$$

$$[\lambda q^1(\lambda,s)+(1-\lambda)(1-q^2(\lambda,s))]x^1(\lambda,s)$$
$$+ [\lambda(1-q^1)(\lambda,s)) + (1-\lambda)q^2(\lambda,s)]x^2(\lambda,s) \in Y(s)$$

It is relatively straightforward to show the existence of a RPO. However, this is not enough for our purposes as later we will want to connect RPO allocations with SRCEL allocation. To do this we will want $W(\lambda,s)$ to be concave in λ. This however is at first glance not very likely due to the fact that in the functional equation determining W the objective is not concave and the constraint set is not convex. However, it turns out that the nature of the problem is such that with some additional restrictions the result can still be obtained. To see this we first write the problem in the following equivalent form:

$$W(\lambda,s) = \max_{h^1,h^2,\Lambda} \{f_1(1,-\Lambda h^1,s) + f_2(1,-(1-\Lambda)h^2,s) + \Lambda v(h^1)$$
$$+ (1-\Lambda)v(h^2) - |\Lambda - \Lambda|m + BEW(\Lambda,s')\}$$

$$\text{subject to} \quad -1 \leq h^1 \leq 0$$
$$-1 \leq h^2 \leq 0$$
$$0 \leq \Lambda \leq 1$$

Note that if Λ is given the resulting problem is a strictly concave programming problem in h^1 and h^2. Let $h^1(\Lambda,s)$ and $h^2(\Lambda,s)$ be the unique solutions to this problem [Note: if $\Lambda = 1$ then take $h^2 = 0$ and if $\Lambda = 0$ take $h^1 = 0$]. Now consider the following equivalent problem:

$$(6) \quad W(\lambda,s) = \max_{\Lambda \in [0,1]} \{f_1(1,-\Lambda h^1(\Lambda,s),s) + f_2(1,-(1-\Lambda)\, h^2(\Lambda,s),s)$$
$$+ \Lambda v(h^1(\Lambda,s)) + (1-\Lambda)v(h^2(\Lambda,s))$$
$$- |\Lambda-\Lambda|m + BEW(\Lambda,s')\}$$

It is relatively straightforward to show that if $h^1(\Lambda,s)$ is decreasing and concave in Λ and $h^2(\Lambda,s)$ is increasing and concave in Λ, and W is concave in λ that the above problem is a (strictly) concave programming problem. For the remainder of this section we will assume that these conditions are met. In a later section we will consider sufficient conditions to guarantee these conditions. Under these conditions it is possible to show that (6) has a unique bounded continuous solution and that $W(\lambda,s)$ is strictly concave in λ. It follows that there is a unique RPO. It remains true that $W(\lambda,s)$ satisfies

$$W(\lambda,s) = \max_{\substack{c,h^1,h^2,H^1,H^2 \\ \Lambda,K^1,K^2}} \{c+\Lambda v(-h^1)+(1-\Lambda)v(-h^2)-|\Lambda-\Lambda|m+BEW(\Lambda,s')\}$$

$$\text{subject to} \quad c \leq f_1(K^1,H^1,s) + f_2(K^2,H^2,s)$$

$$0 \leq K^1 \leq 1 \qquad 0 \leq K^2 \leq 1$$

$$0 \leq h^1 \leq 1 \qquad 0 \leq h^2 \leq 1$$

$$0 \leq H^1 \leq \Lambda h^1 \qquad 0 \leq H^2 \leq (1-\Lambda)h^2$$

$$0 \leq \Lambda \leq 1$$

The programming problem on the right hand side of the above expression has a unique solution. Let $(c^*,\Lambda^*,h^{1*},h^{2*},H^{1*},H^{2*},K^{1*},K^{2*})$ be the unique solution for a given (λ,s). It follows that there exist constants $\mu_1,\ldots,\mu_8$ (depending upon (λ,s)) such that $(c^*,\Lambda^*,h^{1*},h^{*2},H^{1*},H^{2*},K^{1*},K^{2*})$ solves:

(7)
$$\max_{\substack{c,h^1,h^2,H^1,H^2 \\ K^1,K^2,}} \{c+\Lambda v(-h^1)+(1-\Lambda)v(-h^2)-|\Lambda-\Lambda|m+BEW(\Lambda,s')$$
$$+ \mu_1[f_1(K^1,H^1,s) + f_2(K^2,H^2,s)-c] + \mu_2[1-K^1]$$
$$+ \mu_3[1-K^2] + \mu_4[1-h^1] + \mu_5[1-h^2] + \mu_6[\Lambda h^1-H^1]$$
$$+ \mu_7[(1-\Lambda)h^2-H^2] + \mu_8[1-\Lambda]$$

Now assume that $f_1(K,H,s)$ and $f_2(K,H,s)$ are both continuously differentiable in K and H, $K,H > 0$. By the assumptions in section one we have that f_{1K}, f_{1H}, f_{2K}, and f_{2H} are all strictly positive. Also assume that $\lim_{H\to 0} f_j(K,H,s) = +\infty$ for all K,s. It is easy to see that these conditions imply that $c^* > 0$, $K^{1*} = K^{2*} = 1$, $H^{1*} > 0$ and $H^{2*} > 0$ for all (λ,s). It then follows that the following conditions hold:

$$\mu_1 = 1$$

(8)
$$\mu_1 f_{1K}(1,H^{1*},s) = \mu_2$$
$$\mu_2 f_{2K}(1,H^{2*},s) = \mu_3$$
$$\mu_1 f_{1H}(1,H^{1*},s) = \mu_6$$
$$\mu_1 f_{2H}(1,H^{2*},s) = \mu_7$$

It follows that $\mu_1,\mu_2,\mu_3,\mu_6,\mu_7$ are all strictly positive and unique. Moreover, since the choices of $H^{1*},H^{2*},K^{1*},K^{2*}$ vary continuously with (λ,s) $\mu_1,\mu_2,\mu_3,\mu_6,\mu_7$ also vary continuously with (λ,s). These functions will be used to generate an equilibrium pricing function.

Note that problem (7) implies:

$(K^{1*},K^{2*},H^{1*},H^{2*})$ satisfies

$$\underset{K^1,K^2,H^1,H^2}{\text{maximize}} \quad \mu_1[f_1(K^1,H^1,s) + f_2(K^2,H^2,s)] - \mu_2K^1-\mu_3K^2-\mu_6H^1-\mu_7H^2$$

$$\text{s.t.} \quad K^1 \geq 0 \quad K^2 \geq 0 \quad H^1 \geq 0 \quad H^2 \geq 0$$

and

$(c^*,h^{1*},h^{2*},\Lambda^*)$ satisfies

$$\text{maximize } \{c+\Lambda v(-h^1)+(1-\Lambda)v(-h^2)-|\Lambda-\Lambda|m+BEW(\Lambda,s')$$

$$\text{subject to: } \mu_1 c \leq \mu_2+\mu_3+\mu_4\Lambda h^1+\mu_5(1-\Lambda)h^2$$

$$0 \leq h_1 \leq 1 \quad 0 \leq h_2 \leq 1 \quad 0 \leq \Lambda \leq 1$$

SECTION 8

THE FUNCTIONS $h^1(\Lambda,s)$ AND $h^2(\Lambda,s)$

In this section we consider the following problem:

$$\max_{h\in[0,1]} \quad g(\lambda h) - \lambda v(h)$$

where $\lambda \in [0,1]$ is given. We want to establish properties of the solution $h(\lambda)$. We prove the following proposition.

Proposition 2: *If* $h(\lambda) \in (0,1)$, $g \in C^3$, $v \in C^3$, $g' > 0$, $g'' < 0$, $g''' \geq 0$, $v' > 0$, $v'' > 0$, $v''' \leq 0$ *then* $h''(\lambda) > 0$.

Proof: The first order necessary condition is

(9) $$g'(\lambda h) = v'(h)$$

This defines a function $h(\lambda)$. By the assumptions on g and v, h will be twice differentiable. Differentiating both sides of (9) gives:

(10) $$g''(\lambda h)[h+\lambda h'] = v''(h)h'$$

or

$$h' = \frac{g''(\lambda h)h}{[v''(h)-\lambda g''(\lambda h)]}$$

Note that $h' < 0$, $h + \lambda h' > 0$. Differentiating both sides of (10) with respect to λ gives:

$$g'''(\lambda h)h[h+\lambda h'] + g''(\lambda h)[h'+\lambda h''+h'] = v'''(h)h'h' + v''(h)h''$$

Hence

$$h'' = \frac{g'''(\lambda h)h[h+\lambda h']+g''(\lambda h)[2\lambda h']-v'''(h)(h')^2}{v''(h) - g''(\lambda h)}$$

Note that $g''' > 0$, $v''' < 0$ imply $h'' > 0$, which completes the proof.

The importance of this result is that it establishes sufficient conditions under which $h^1(\lambda,s)$ and $h^2(\lambda,s)$ have the properties assumed in thelast section. Essentially the result says that the marginal product of labour must decrease at an increasing rate and that the marginal disutility of work should increase at a decreasing rate. Note that any Cobb-Douglas type function or quadratic function used as a production function or utility function satisfies these conditions.

SECTION 9

EQUILIBRIUM AND OPTIMALITY

In this section we present results connecting SRCEL and RPO. Connections between RCE and RPO follow from the equivalence result of section 6. The proofs are not included in this section as they are identical to those of Prescott and Mehra (5) in section 7 of their paper. (See also Prescott and Lucas (4)).)

<u>Proposition 3</u>: *If* $(\bar{p}(\lambda,s),\bar{\Lambda}(\lambda,s),\bar{V}(\lambda,s,p),\bar{x}(\lambda,s,p),\bar{y}(\lambda,s))$ *is a* SRCEL *then* $(W(\lambda,s),x^1(\lambda,s),x^2(\lambda,s),q^1(\lambda,s),q^2(\lambda,s))$ *is a* RPO *where*

$$W(\lambda,s) = \bar{V}(\lambda,s,\lambda),$$
$$x^1(\lambda,s) = \bar{x}^1(\lambda,s,\lambda)$$
$$x^2(\lambda,s) = \bar{x}^2(\lambda,s,\lambda)$$
$$q^1(\lambda,s) = \bar{q}^1(\lambda,s,\lambda)$$
$$q^2(\lambda,s) = \bar{q}^2(\lambda,s,\lambda)$$

<u>Proposition 4</u>: *If* $(W(\lambda,s),x^1(\lambda,s),x^2(\lambda,s),q^1(\lambda,s),q^2(\lambda,s))$ *is a* RPO *then there is a* SRCEL $(\bar{p}(\lambda,s),\bar{\Lambda}(\lambda,s)\bar{V}(\lambda,s,p),\bar{x}(\lambda,s,p),\bar{y}(\lambda,s))$ *where:*

$$\bar{p}(\lambda,s) = (\mu_1(\lambda,s),\mu_2(\lambda,s),\mu_3(\lambda,s),\mu_6(\lambda,s),\mu_7(\lambda,s))$$
$$\Lambda(\lambda,s) = \Lambda^*(\lambda,s)$$
$$V(\lambda,s,\lambda) = W(\lambda,s)$$
$$\bar{x}(\lambda,s,\lambda) = (x^1(\lambda,s),x^2(\lambda,s),q^1(\lambda,s),q^2(\lambda,s)).$$

The importance of these propositions are that they establish the result that if there is a unique RPO then there is also a unique SRCEL. In section 7 we established conditions which are sufficient for there to be a unique RPO.

SECTION 10

EXTENSIONS TO INCLUDE UNEMPLOYMENT

The model which has been used to this point allows the ratio of employment in the two sectors to fluctuate but it does not allow aggregate employment to fluctuate. Allowing for this feature is quite simple. As was done in Rogerson [6], we need only specify a search technology which requires that individuals transferring from one sector to another spend some time unemployed. This technology may be deterministic (e.g. it takes one period to transfer), or random (e.g. it takes one period with probability one half and two periods also with probability one half). It could also be specified so that search intensity is determined endogenously. It is clear that the existence and optimality results obtained earlier will continue to hold.

Some additional interpretations will be discussed in the section where an example is presented.

SECTION 11

SOME EXAMPLES

In this section we consider an example specified by the following:

$S = \{1,2,3\}$

$$[P_{ij}] = [\text{Prob}[s_t=j|s_{t-1}=i]] = \begin{vmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{vmatrix}$$

$$f_1(1,H,s) = \begin{cases} 2(H-\frac{1}{2}H^2) & s=1 \\ (H-\frac{1}{2}H^2) & s=2 \\ .5(H-\frac{1}{2}H^2) & s=3 \end{cases}$$

$$f_2(1,H,s) = \begin{cases} .583\ H & s=1 \\ .514\ H & s=2 \\ .400\ H & s=3 \end{cases}$$

$$v(h) = \frac{-h^2}{2}$$

$B = .950$

$m = .077$

Assume that the initial state is $(\lambda,s) = (1,1)$.

Given the linear technology in sector two it is harmless to assume a group of workers who are located in sector two who by assumption cannot move into sector one. Assume that in all other respects these workers are identical to the other workers and that there is an equal number of them.

We are primarily interested in certain implications of the model and hence do not concentrate here on how an equlibrium is constructed. This is covered in the appendix.

It turns out that the SRCEL for this economy given a starting condition of $(\lambda,s) = (1,1)$ is such that the aggregate state will only take on six values:

$$(\lambda,s) \in \{(1,1),(.9,1),(1,2),(.9,2),(.9,3),(1,3)\}.$$

The variables of interest for us are the following:

$\Lambda(\lambda,s)$: $\Lambda(1,1) = 1$

$\Lambda(.9,1) = 1$

$\Lambda(1,2) = 1$

$\Lambda(.9,2) = .9$

$\Lambda(.9,3) = .9$

$\Lambda(1,3) = .9$

$w(\lambda,s) = (w^1(\lambda,s),\ w^2(\ ,s))$: $w(1.1) = (.667,.583)$

$w(.9,1) = (.667,.583)$

$w(1,2) = (.500,.514)$

$w(.9,2) = (.526,.514)$

$w(.9,3) = (.345,.400)$

$w(1.3) = (.345,.400)$

$h(\lambda,s) = h^1(\lambda,s),h^2(\lambda,s))$: $h(\lambda,s) = w(\lambda,s)$

$H(\lambda,s) = (H^1(\lambda,s),H^2(\lambda,s)$: $H(\lambda,s) = (\Lambda(\lambda,s)h^1(\lambda,s),(2-\Lambda(\lambda,s))h^2(\lambda,s))$

If we concentrate on the dynamics of the adjustment process in sector one we observe the following:

(1) Hours/worker always decreases (increases) before employment decreases (increases).

(2) Employment displays some "persistence".

The stationary distribution of (employment, hours/worker, real wage) in sector one is given by:

(1,.667,.667) with probability 1/4

(1,.500,.500) with probability 1/4

(.9,.526,.526) with probability 1/4

(.9,.345,.345) with probability 1/4

Note that the correlation between wages and hours/worker is 1 but that the correlation between employment and wages is only .65.

The environment of this example displays two prominent features:

(i) Both sectors are affected in the same direction by the aggregate shock, but sector one is relatively better off when s=1 and sector two is relatively better off when s=3 even though sector one is better "on average".

(ii) When the economy starts to go down (or up) there is some uncertainty as to whether this is going to continue or only last for one period.

Intuitively, it seems clear that the characteristics of the results generated by this example are directly caused by these features in conjunction with the two other important features of the environment: the nonconvexity and the fixed cost. At this point no formal result is presented. However, note that if the fixed cost of moving is zero then the equilibrium for the above example does not display the properties highlighted above: Fluctuations in hours/worker would not lead fluctuations in employment, employment would not display persistence and wages and employment would be perfectly correlated.

The type of model which has been presented in this paper also has implications for the impact of a secular trend in relative productivities across sectors on the nature of cyclical fluctuations. For example, if in the previous example we increase the marginal product of labour in sector two for all states s, by a small amount the change on the SRCEL will be that employment in sector 1 is unchanged when s=1 or s=2 but when s=3 employment will be less. The reason for this is that when $s_t=1$ or $s_t=2$ (and $s_{t-1}=1$) the opportunity of working in sector two is irrelevant and hence increasing the value of working there has no effect in these situations. However, when s=3 this value is important and hence increasing it changes the number of workers who switch. The relevance of this intuition is that it suggests that a larger trend will conceivably produce larger cyclical fluctuations. It may also produce a permanent switch of some workers into sector two. The kind of model studied in this paper suggests that this process will be most intense in bad times.

SECTION 12

DISCUSSION

The example presented in the last section is a special case of the model presented in earlier sections and is very similar to the structure studied in the context of a two period model in some earlier work. Its defects are thus similar and are not discussed here.

A few extensions or alternative interpretations are worth noting. The programming problem which determines a Pareto optimum is entirely consistent with m being interpreted as an adjustment cost, i.e. it is costly for firms to adjust their number of employees and costless to adjust the hours worked per worker. This is clearly on extreme version of a model where it is costly to adjust both quantitites but it is more costly to adjust the number of employees.

If one interprets m as a search cost and thinks of workers leaving sector one during bad times and then being recalled during good times it might be natural to consider the case where m is encountered only when moving from sector one to sector two but not vice versa. It seems clear that this assumption would not affect the nature of the results.

The assumptions of the utility function being linear in consumption is relatively strong. The results concerning Pareto optimal allocations is valid for the case where utility is concave consumption. Optimal allocations will have the property that all agents have equal consumption in any given period. The problem arises when trying to define an appropriate state variable for individuals in the definition of equilibrium. Insuring that all workers consume the same amount of output requires a fairly extensive set of risk sharing opportunities in the presence of shocks which are not independent and the nature of the non-convexity present in the environment. Also, with utility concave in consumption it will turn out that the introduction of lotteries into the consumption space is essential in producing optimal allocations.

SECTION 13

COMPARISON WITH AN ALTERNATIVE MODEL

In an earlier paper the method of generating variations in both employment and hours/worker used in this paper was contrasted with that where a fixed time cost of going to work was used. Essentially what was found was that in generating aggregate movements in these two variables the approaches were similar in results. The main difference was that the approach used here relies on heterogeneity of shocks across firms whereas the other relies on heterogeneity across consumers. The timing of changes in hours/worker and employment produced in the example in this paper is consistent with that observed in the data. However, any version of the alternative model which is consistent with joint movements in hours/worker and employment can explain that reductions in hours/worker lead reductions in employment but not that increases in hours/worker lead increases in employment. The reason for this is essentially that in the fixed time cost model there is no penalty for switching between working and not working.

SECTION 14

CONCLUSION

The main results of this paper have been at an abstract level. It has been shown that with a few additional assumptions the results of recursive competitive theory continue to hold in a certain class of non-convex economies. Furthermore, it was demonstrated through an example that the class of economies under consideration apparently has a rich set of predictions concerning aggregate behaviour of the labour market. A clear priority for future research is a detailed investigation of the properties of the time series the model can produce, similar to those discussed in section 11.

$$(w_{\lambda s}, r_s) = \left(\frac{\theta_{1s}}{1+\lambda\theta_{1s}}, \theta_{2s}\right)$$

For convenience we will assume that $I_{\lambda s} = 0$ for all λ,s. Using the approach in section 2,it follows that if we have an equilibrium then the following conditions must be met:

(1) $V(1,1,1) = u(w_{11}) + \beta V(1,2,1)$

(2) $V(\phi,1,1) = u(w_{11}) + \beta V(1,2,1)$

(3) $V(\phi,1,2) = u(w_{11}) - m + \beta V(1,2,1)$

(4) $V(\phi,2,1) = u(w_{12}) + \frac{1}{2}\beta[V(\phi,1,1) + V(\phi,3,1)]$

(5) $V(\phi,2,2) = u(r_2) + \frac{1}{2}\beta[V(\phi,1,2) + V(\phi,3,2)]$

(6) $V(1,2,1) = u(w_{12}) + \frac{1}{2}\beta[V(1,1,1) + V(1,3,1)$

(7) $V(1,3,1) = u(w_{13}) + \beta V(\phi,2,1) = r_3 - m + \beta V(\phi,2,2)$

(8) $V(\phi,3,1) = u(w_{13}) + \beta V(\phi,2,1)$

(9) $V(\phi,3,2) = u(r_3) + \beta V(\phi,2,2)$

where $u(\cdot)$ is the one period indirect utility function. It is straightforward to show that $u(w) = w^2/2$.

It is also true that some inequalities must also hold, but we will come back to this later. One usually thinks of the economic environment being taken as given and then solving for the equilibrium. Here however, we reverse this slightly. We will take ϕ and the θ_{is} as given and will treat m as endogenous. The reason for this is that with this approach the wages are determined and the equations are linear in the values of V and m. The system of equations listed above can easily be solved, as it is triangular, and hence requires only repeated substitution. In particular equations 4,6,7,8 imply that

APPENDIX

The objective of this appendix is to demonstrate how to construct equilibria of the type displayed in the paper. An important feature of the example was that the economy could only find itself in one of the small number (six) of aggregate states:

$$(\lambda,s) \in \{(1,1),(\phi,1),(\phi,2),(\phi,3),(1,2),(1,3)\},$$

where $\phi \in [0,1]$. Now assuming that this is an equilibrium, we can easily determine what the wage and rental price of capital must be in each sector. This comes from simply fixing λ and looking at the static equilibrium for each sector. Since the technology in sector two is linear, the wage in that sector will not depend upon the number of workers in that sector. Let $(w_{\lambda s}, r_s)$ and $I_{\lambda s}$ correspond to the wage vector and income from capital when a fraction λ of the workforce is in sector one and the state of nature is s. (Note the difference between this and the state variable (λ,s) where λ is the fraction of agents in sector one at the end of last period). Now, if technologies are more generally specified as:

$$f_1(1,H,s) = \theta_{1s}(H - \tfrac{1}{2}H^2) \qquad s \in \{1,2,3\}$$

$$f_2(1,H,s) = \theta_{2s}H \qquad s \in \{1,2,3\}$$

then it follows that

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