Can Everyone Benefit from Growth? Three Difficulties

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CAN EVERYONE BENEFIT FROM GROWTH? THREE DIFFICULTIES

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Can everyone benefit from growth?

Three difficulties

1. *Introduction*

When the resources available to a fixed population grow, is it possible to guarantee that all agents benefit? If final consumptions are determined by operating the price mechanism from equal division the answer is: not necessarily. The possibility that some agents be hurt is closely related to the "throw away" paradox, studied by Aumann and Peleg (1974), and the "immiserizing growth" paradox, discussed by Bhagwati (1958).

We ask whether this undesirable phenomenon can be avoided by any mechanism. A mechanism is ***Resource Monotonic*** (RM) if the utility of no agent ever decreases when the aggregate endowment increases. We show that any ***Pareto Optimal*** (PO) and RM mechanism may violate two basic properties frequently imposed in normative analyses of the problem of fair allocation (see Thomson and Varian, 1985, for a review of this literature). The first one is ***Individual Rationality from Equal Division*** (IR): every agent should be guaranteed the utility he would reach by consuming the n-th share of the resources, where n is the number of agents. The second property is that the mechanism select ***envy-free*** allocations (EF): no agent should prefer another agent's consumption to his own. Even a much weaker axiom than EF, requiring that ***no agent should receive more of every good than any other agent***, proves incompatible with PO plus RM.

The proofs of these results are by way of examples. The examples are far from pathological: they involve two goods, and two agents with convex and homothetic preferences.

2. *Preliminaries*

There are two goods and two agents, indexed by $i = 1,2$. For each i, agent i's preferences are represented by a continuous and monotone increasing utility function u_i defined on his consumption set $\mathfrak{R}^2_+$. An economy is a triple (u_1,u_2,ω), where $\omega \in \mathfrak{R}^2_+$ is the ***aggregate endowment***, to be allocated among the two agents. Since, for each of our results, the pair (u_1,u_2) will be kept fixed, we will simplify the notation by designating the economy by its aggregate endowment ω. A ***correspondence*** (at u) associates with every $\omega \in \mathfrak{R}^2_+$ a set $C(\omega)$ of pairs $(z_1,z_2) \in \mathfrak{R}^4_+$ such that $z_1 + z_2 = \omega$. We limit our attention to ***essentially single-valued correspondences***, namely correspondences such that for all $\omega \in \mathfrak{R}^2_+$, and for all (z_1,z_2) and $(z_1',z_2') \in C(\omega)$, we have $u_i(z_i) = u_i(z_i')$ for each i. Then, we can write $u_i(C(\omega))$ to mean $u_i(z_i)$ where $z = (z_1,z_2)$ is arbitrary in $C(\omega)$.

The correspondence C is ***Resource Monotonic*** (RM) if

for all $\omega,\omega' \in \mathfrak{R}^2_+$ with $\omega' \geqq \omega$, $u_i(C(\omega')) \geqq u_i(C(\omega))$ for $i = 1,2$.[1]

The correspondence C is ***individually rational from equal division of the aggregate resources*** (IR) if

for all $\omega \in \mathfrak{R}^2_+$, $u_i(C(\omega)) \geqq u_i(\omega/2)$ for $i = 1,2$.

The correspondence C is ***envy-free*** (EF) if

for all $\omega \in \mathfrak{R}^2_+$ and for all $(z_1,z_2) \in C(\omega)$, $u_i(z_i) \geqq u_i(z_j)$ for all i,j.

---

[1]Vector inequalities $x \gg y$, $x > y$, $x \geqq y$.

P is the Pareto correspondence. Λ is the 45° line. Given $x \in \Re^2$, $\pi(x,\Lambda)$ is the symmetric image of x with respect to Λ. Given x, $y \in \Re^2$, $\pi(x,y)$ is the symmetric image of x with respect to y.

3. *The results*

***Theorem* 1.** There is a profile $u = (u_1,u_2)$ of utility functions representing convex and homothetic preferences such that no correspondence (at u) satisfies PO, RM and IR together.

***Proof*:** A profile as desired is represented in Figure 1. Agent 2's preferences are symmetrical of agent 1's preferences with respect to Λ: $u_2(z_2) = u_1(\pi(z_2,\Lambda))$ for all $z_2 \in \Re^2_+$. Initially, $\omega \in \Lambda$.

For each i, let $\bar{z}_i$ be the maximizer of u_i on the line through $\omega/2$ of slope -1. I is agent 1's indifference curve through $\bar{z}_1$ and $J = \pi(I,\Lambda)$ is agent 2's indifference curve through $\bar{z}_2$. Note that $\bar{z} \equiv (\bar{z}_1,\bar{z}_2) \in P(\omega)$. I is made up of two segments with slopes inverse of each other.

Let C be a correspondence (at u) satisfying PO, RM and IR. By PO, either (i) $u_2(C(\omega)) \geqq u_2(\bar{z}_2)$ or (ii) $u_1(C(\omega)) \geqq u_1(\bar{z}_1)$. If (i) holds, let ω increase to ω' located below the ray passing through the origin and $\bar{z}_1$. By RM, $u_2(C(\omega')) \geqq u_2(C(\omega))$ and therefore if $z' \in C(\omega')$, then z'_2 lies on or above J. Hence, z'_1 lies on or below $K = \pi(J,\omega'/2)$, in contradiction with IR, since agent 1's indifference curve through $\omega'/2$ lies strictly above K.

If (ii) holds, we derive a similar contradiction by considering an increase of the aggregate resources from ω to $\pi(\omega',\Lambda)$.

(It is the fact that ω' is below the ray passing through the origin and $\bar{z}_1$ that permits us to choose preferences to be homothetic, as represented in Figure 1.)

Q.E.D.

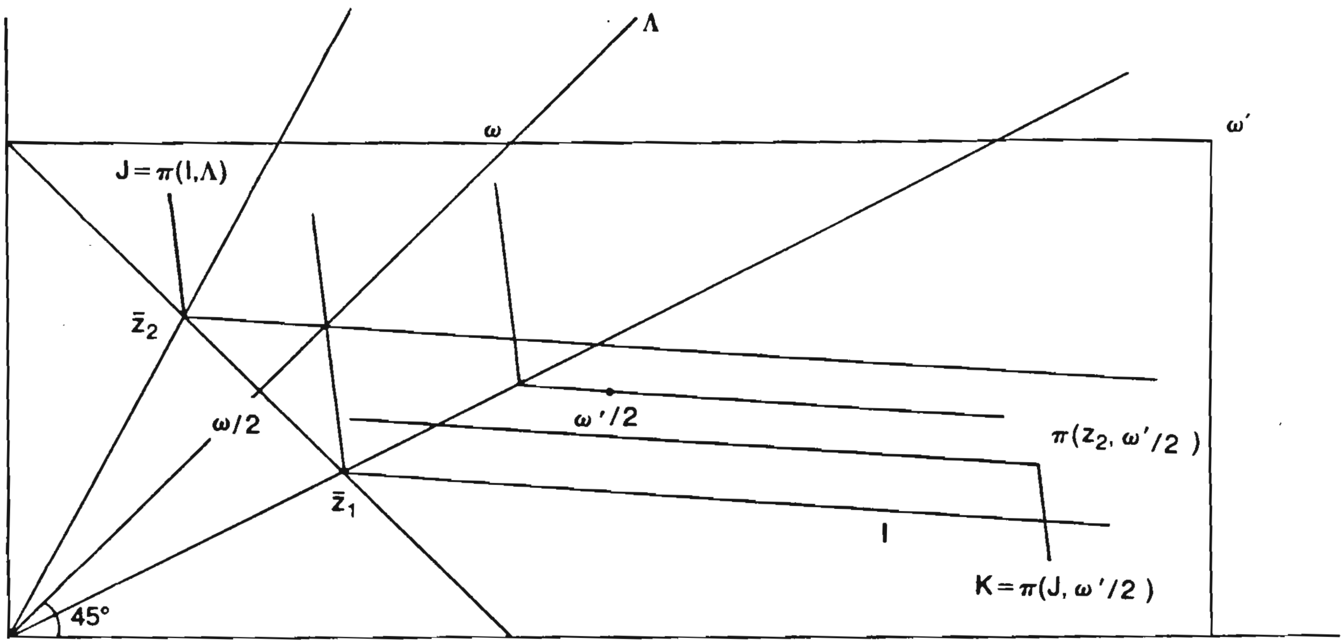

Figure 1

Note that the fact that z' is Pareto optimal after the change in aggregate resources is not used in this proof. This suggests that Theorem 1 can be strengthened. This is indeed the case. Let $\epsilon > 0$ be given.

The correspondence C is ***Individually rational above an ϵ-share of the aggregate resources*** (IR_ϵ) if

for all $\omega \in \mathfrak{R}^2_+$, $u_i(C(\omega)) \geqq u_i(\epsilon\omega)$ for $i = 1,2$.

Thus, IR is just $IR_{1/2}$. For $0 \leqq \epsilon < 1/2$, IR_ϵ entitles each agent to a smaller share of the resources than IR does. Our next result is that no matter how small this share is, provided it remains positive, IR_ϵ cannot be satisfied simultaneously with PO and RM.

Theorem 2. For all ϵ, $0 < \epsilon \leqq 1/2$, there is a profile $u = (u_1, u_2)$ of utility functions representing convex and homothetic preferences such that no correspondence (at u) can satisfy PO, RM and IR_ϵ together.

Proof: Given ϵ, $0 < \epsilon \leqq 1/2$, let $a \in \Re_+$ be such that $a > (1-\epsilon)/\epsilon$ and $u = (u_1, u_2)$ be defined by $u_1(x,y) \equiv \min\{x/a, y\}$ for all $(x,y) \in \Re^2_+$ and $u_2(z_2) \equiv u_1(\pi(z_2, \Lambda))$ for all $z_2 \in \Re^2_+$.

Let C be a correspondence (at u) satisfying PO, RM and IR_ϵ. Then, let $\omega \equiv (1,1)$ and $\omega' \equiv (a,1)$. By IR_ϵ, $u_1(C(\omega')) \geqq u_1(\epsilon\omega')$, and therefore $u_2(C(\omega')) \leqq (1-\epsilon)/a$. Since $\omega \leqq \omega'$, RM implies that $u_2(C(\omega)) \leqq (1-\epsilon)/a$. By a symmetrical argument involving the bundles ω and $\omega'' \equiv (1,a)$, we obtain $u_1(C(\omega)) \leqq (1-\epsilon)/a$. However, by the choice of a, the allocation $z \equiv (z_1, z_2)$ where $z_1 \equiv (a/(a+1), 1/(a+1))$ and $z_2 \equiv \pi(z_1, \Lambda)$ is such that $z_1 + z_2 = \omega$ and $u_i(C(\omega)) \leqq (1-\epsilon)/a < u_i(z_i) = 1/(a+1)$ for $i = 1,2$. This is in violation of PO.

Q.E.D.

Next, we establish the incompatibility of PO, RM and EF. In fact, we prove a stronger result involving the following requirement, discussed in Thomson (1983).

The correspondence C satisfies ***No-Domination*** (ND) if

for all $\omega \in \Re^2_+$, and for all $z = (z_1, z_2) \in C(\omega)$, neither $z_1 \gg z_2$ nor $z_2 \gg z_1$.

A correspondence violating ND obviously generates envy.

Theorem 3. There is a profile $u = (u_1, u_2)$ of utility functions representing convex and homothetic preferences such that there is no correspondence (at u) satisfying PO, RM and ND together.

Proof: An economy as desired is represented in Figure 2. Let $u = (u_1, u_2)$ be defined by $u_1(x,y) \equiv \min\{x + 100y, 41x + 3y\}$ for all $(x,y) \in \Re^2_+$ and $u_2(z_2) \equiv u_1(\pi(z_2, \Lambda))$ for all $z_2 \in \Re^2_+$. $\bar{z}$ is defined as in the proof of Theorem 1.

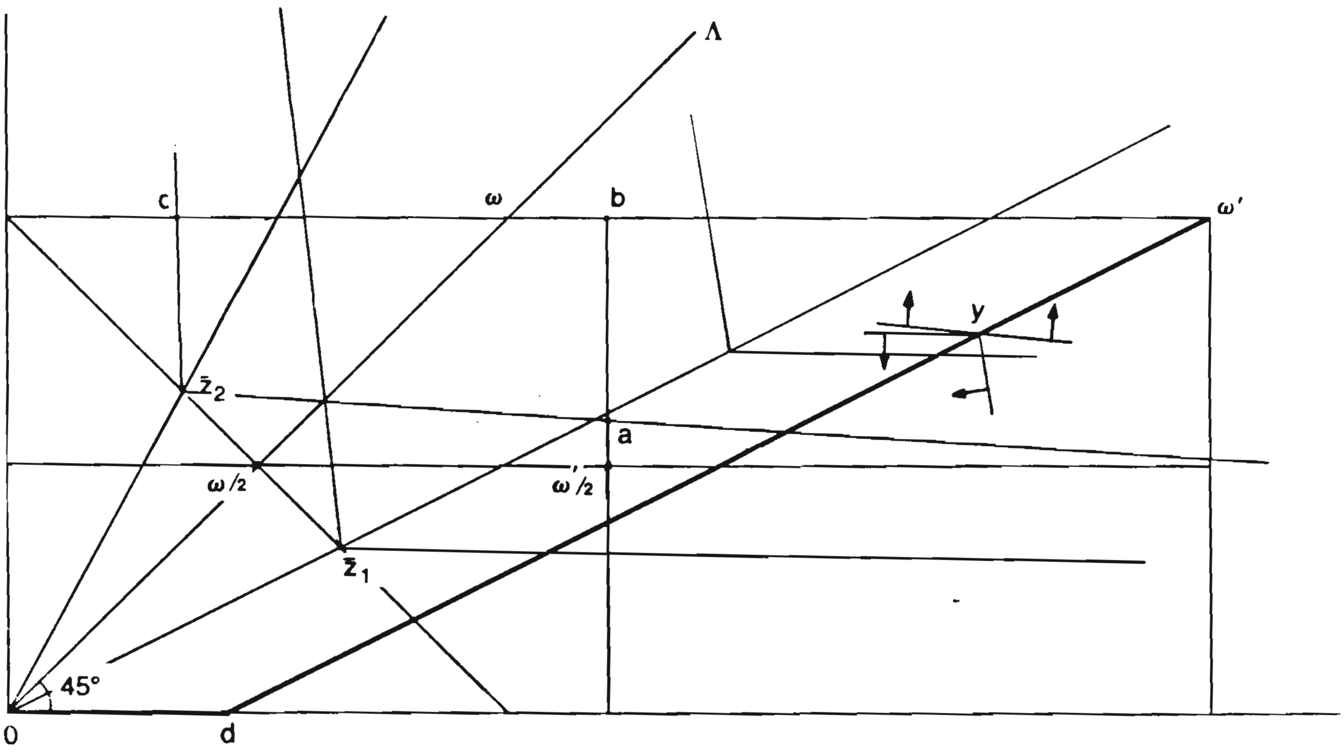

Figure 2

Aggregate resources increase from $\omega \equiv (20,20)$ to $\omega' \equiv (50,20)$.

Let $z \in C(\omega)$ and $z' \in C(\omega')$. By PO, either (i) $u_2(z_2) \geqq u_2(\bar{z}_2)$ or (ii) $u_1(z_1) \geqq u_1(\bar{z}_1)$. Assume first that (i) holds. By RM, $u_2(z_2') \geqq u_2(\bar{z}_2)$. By

ND, z_2' belongs to the II^{nd} and IV^{th} quadrants from $\omega'/2$. The set of all z_2' satisfying both conditions is the four-sided figure with vertices $\bar{z}_2$, a, b and c. No such z_2' is the second component of an allocation $(z_1', z_2') \in P(\omega')$. Indeed, in the Edgeworth box of the economy ω' obtained by placing the origin of agent 1's consumption space at ω', $P(\omega')$ is the union of the segments $[0,d]$ and $[d,\omega']$. (In Figure 2, we have represented the agents' indifference curves through a generic point y of the Pareto set).

Assuming now that (ii) holds, we obtain the desired conclusion by considering an increase of aggregate resources from ω to $\pi(\omega',\Lambda)$.

Q.E.D.

4. *Concluding comments*

A. The axiom of Resource Monotonicity was first studied by Chun and Thomson (1984) and extensively investigated by Roemer (see, in particular, 1986). Following Roemer, we interpret it as a natural implication of public ownership of resources.

B. A consequence of our results: the Walrasian correspondence from equal division does not satisfy RM. This is because (i) when preferences are homothetic and initial endowments are proportional, and in particular equal, this correspondence is essentially single-valued, and (ii) it satisfies PO, IR, EF and ND. (A direct proof of this negative feature of the Walrasian correspondence is straightforward.)

C. Consider now the more familiar set up in which each agent starts out with a vector of resources ω_i which he owns. There, several resource monotonicity properties can be formulated. First, we may require that an agent's utility does not decrease when his ***own*** initial endowment increases (ceteris paribus). Call this property RM_1. (Its violation by the price mechanism is precisely the throw-away paradox). If preferences are strongly monotone, ($u_i(z_i) > u_i(z_i')$ if $z_i > z_i'$) it is not difficult to construct correspondences satisfying PO, Individual Rationality ($u_i(z_i) \geqq u_i(\omega_i)$ for each i) and RM_1. Postlewaite (1979) provides examples. They are obtained by selecting utility representations for the preferences and equating utility gains from the image of the initial endowment. (These are the egalitarian solutions of bargaining theory.) Similarly, by equating utility gains from the image of the zero consumption, we obtain correspondences satisfying PO and RM, as noted by Hurwicz (1978), but not Individual Rationality.

A second resource monotonicity requirement is that an agent's utility should not decrease when the resources of some *other* agent increase, ceteris paribus. Call this property RM_2: it is more difficult to justify than RM_1, as well as harder to meet. The fact that the Walrasian mechanism does not satisfy RM_2 was noted by Thomson (1978).

The proof of Theorem 1 can easily be adapted to show (still with convex and homothetic preferences,) that the three requirements PO, RM_2 and Individual Rationality are mutually incompatible.

D. Beyond the impossibility results uncovered in this note, one would like to understand what features of preferences make the Resource Monotonicity axiom so hard to meet. All our examples, as always in the literature on the transfer paradox, exhibit strong complementarity of the two goods. Whether or not this is the deep explanation of our impossibilities will be the subject of future research (see Moulin, 1987).

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