CAN EVERYONE BENEFIT FROM GROWTH?
THREE DIFFICULTIES

Herve` Moulin *

and

William Thomson **

Working Paper No. 87

May 1987

*Virginia Polytechnic Institute and State University, Blacksburg, VA 24061.
**University of Rochester, Rochester, NY 14627.

Stimulating conversations with Youngsub Chun, Jacques Cremer, Ron Jones, John Roemer and Nick Tideman are gratefully acknowledged.

Moulin's research was supported by the NSF under grant SES 8419465. Thomson's research was supported by the NSF under grant SES 8511136.
Can everyone benefit from growth?

Three difficulties

1. Introduction

When the resources available to a fixed population grow, is it possible to guarantee that all agents benefit? If final consumptions are determined by operating the price mechanism from equal division the answer is: not necessarily. The possibility that some agents be hurt is closely related to the "throw away" paradox, studied by Aumann and Peleg (1974), and the "immiserizing growth" paradox, discussed by Bhagwati (1958).

We ask whether this undesirable phenomenon can be avoided by any mechanism. A mechanism is Resource Monotonic (RM) if the utility of no agent ever decreases when the aggregate endowment increases. We show that any Pareto Optimal (PO) and RM mechanism may violate two basic properties frequently imposed in normative analyses of the problem of fair allocation (see Thomson and Varian, 1985, for a review of this literature). The first one is Individual Rationality from Equal Division (IR): every agent should be guaranteed the utility he would reach by consuming the n-th share of the resources, where n is the number of agents. The second property is that the mechanism select envy-free allocations (EF): no agent should prefer another agent's consumption to his own. Even a much weaker axiom than EF, requiring that no agent should receive more of every good than any other agent, proves incompatible with PO plus RM.

The proofs of these results are by way of examples. The examples are far from pathological: they involve two goods, and two agents with convex and homothetic preferences.
2. Preliminaries

There are two goods and two agents, indexed by \( i = 1, 2 \). For each \( i \), agent \( i \)'s preferences are represented by a continuous and monotone increasing utility function \( u_i \) defined on his consumption set \( \mathbb{R}^2_+ \). An economy is a triple \((u_1, u_2, \omega)\), where \( \omega \in \mathbb{R}^2_+ \) is the aggregate endowment, to be allocated among the two agents. Since, for each of our results, the pair \((u_1, u_2)\) will be kept fixed, we will simplify the notation by designating the economy by its aggregate endowment \( \omega \). A correspondence (at \( u \)) associates with every \( \omega \in \mathbb{R}^2_+ \) a set \( C(\omega) \) of pairs \((z_1, z_2) \in \mathbb{R}^4_+\) such that \( z_1 + z_2 = \omega \). We limit our attention to essentially single-valued correspondences, namely correspondences such that for all \( \omega \in \mathbb{R}^2_+ \), and for all \((z_1, z_2)\) and \((z'_1, z'_2)\) \( \in C(\omega) \), we have \( u_i(z_i) = u_i(z'_i) \) for each \( i \). Then, we can write \( u_i(C(\omega)) \) to mean \( u_i(z_i) \) where \( z = (z_1, z_2) \) is arbitrary in \( C(\omega) \).

The correspondence \( C \) is Resource Monotonic (RM) if
\[
\text{for all } \omega, \omega' \in \mathbb{R}^2_+ \text{ with } \omega' \geq \omega, u_i(C(\omega')) \succeq u_i(C(\omega)) \text{ for } i = 1, 2. \]

The correspondence \( C \) is individually rational from equal division of the aggregate resources (IR) if
\[
\text{for all } \omega \in \mathbb{R}^2_+, u_i(C(\omega)) \succeq u_i(\omega/2) \text{ for } i = 1, 2.
\]

The correspondence \( C \) is envy-free (EF) if
\[
\text{for all } \omega \in \mathbb{R}^2_+ \text{ and for all } (z_1, z_2) \in C(\omega), u_i(z_i) \succeq u_j(z_j) \text{ for all } i, j.
\]

---

\(^1\)Vector inequalities \( x \gg y, x \succ y, x \succeq y \).
P is the Pareto correspondence. \( \Lambda \) is the 45\(^{\circ} \) line. Given \( x \in \mathbb{R}^2 \), \( \pi(x, \Lambda) \) is the symmetric image of \( x \) with respect to \( \Lambda \). Given \( x, y \in \mathbb{R}^2 \), \( \pi(x, y) \) is the symmetric image of \( x \) with respect to \( y \).

3. The results

**Theorem 1.** There is a profile \( u = (u_1, u_2) \) of utility functions representing convex and homothetic preferences such that no correspondence \((at u)\) satisfies PO, RM and IR together.

**Proof:** A profile as desired is represented in Figure 1. Agent 2's preferences are symmetrical of agent 1's preferences with respect to \( \Lambda \): \( u_2(z_2) = u_1(\pi(z_2, \Lambda)) \) for all \( z_2 \in \mathbb{R}^2_+ \). Initially, \( \omega \in \Lambda \).

For each \( i \), let \( z_1^i \) be the maximizer of \( u^i \) on the line through \( \omega/2 \) of slope -1. \( I \) is agent 1's indifference curve through \( z_1^i \) and \( J = \pi(I, \Lambda) \) is agent 2's indifference curve through \( z_2^i \). Note that \( \bar{z} \equiv (z_1^i, z_2^i) \in \pi(\omega) \). \( I \) is made up of two segments with slopes inverse of each other.

Let \( C \) be a correspondence \((at u)\) satisfying PO, RM and IR. By PO, either (i) \( u_2(C(\omega)) \geq u_2(\bar{z}_2^i) \) or (ii) \( u_1(C(\omega)) \geq u_1(\bar{z}_1^i) \). If (i) holds, let \( \omega \) increase to \( \omega' \) located below the ray passing through the origin and \( \bar{z}_1^i \). By RM, \( u_2(C(\omega')) \geq u_2(C(\omega)) \) and therefore if \( z' \in C(\omega') \), then \( z'_2 \) lies on or above \( J \). Hence, \( z_1^i \) lies on or below \( K = \pi(J, \omega'/2) \), in contradiction with IR, since agent 1's indifference curve through \( \omega'/2 \) lies strictly above \( K \).

If (ii) holds, we derive a similar contradiction by considering an increase of the aggregate resources from \( \omega \) to \( \pi(\omega', \Lambda) \).
(It is the fact that $\omega'$ is below the ray passing through the origin and $\bar{z}_1$ that permits us to choose preferences to be homothetic, as represented in Figure 1.)

Q.E.D.

Note that the fact that $z'$ is Pareto optimal after the change in aggregate resources is not used in this proof. This suggests that Theorem 1 can be strengthened. This is indeed the case. Let $\varepsilon > 0$ be given.

The correspondence $C$ is **individually rational** above an $\varepsilon$-share of the aggregate resources (IR$_{\varepsilon}$) if

for all $\omega \in \mathbb{R}^2_+$, $u_i(C(\omega)) \geq u_i(\varepsilon \omega)$ for $i = 1, 2$.

Thus, IR is just IR$_{1/2}$. For $0 \leq \varepsilon < 1/2$, IR$_{\varepsilon}$ entitles each agent to a smaller share of the resources than IR does. Our next result is that no
matter how small this share \( \varepsilon \) is, provided it remains positive, \( \text{IR}_\varepsilon \) cannot be satisfied simultaneously with PO and RM.

**Theorem 2.** For all \( \varepsilon, 0 < \varepsilon \leq 1/2 \), there is a profile \( u = (u_1, u_2) \) of utility functions representing convex and homothetic preferences such that no correspondence (at \( u \)) can satisfy PO, RM and \( \text{IR}_\varepsilon \) together.

**Proof:** Given \( \varepsilon, 0 < \varepsilon \leq 1/2 \), let \( a \in \mathbb{R}_+ \) be such that \( a > (1-\varepsilon)/\varepsilon \) and \( u = (u_1, u_2) \) be defined by \( u_1(x, y) = \min\{x/a, y\} \) for all \((x, y) \in \mathbb{R}_+^2\) and \( u_2(z_2) = u_1(\pi(z_2, A)) \) for all \( z_2 \in \mathbb{R}_+^2 \).

Let \( C \) be a correspondence (at \( u \)) satisfying PO, RM and \( \text{IR}_\varepsilon \). Then, let \( \omega \equiv (1, 1) \) and \( \omega' \equiv (a, 1) \). By \( \text{IR}_\varepsilon \), \( u_1(C(\omega')) \geq u_1(\varepsilon \omega') \), and therefore \( u_2(C(\omega')) \leq (1-\varepsilon)/a \). Since \( \omega \leq \omega' \), RM implies that \( u_2(C(\omega)) \leq (1-\varepsilon)/a \). By a symmetrical argument involving the bundles \( \omega \) and \( \omega'' \equiv (1, a) \), we obtain \( u_1(C(\omega)) \leq (1-\varepsilon)/a \). However, by the choice of \( a \), the allocation \( z \equiv (z_1, z_2) \) where \( z_1 \equiv (a/(a+1), 1/(a+1)) \) and \( z_2 \equiv \pi(z_1, A) \) is such that \( z_1 + z_2 = \omega \) and \( u_1(C(\omega)) \leq (1-\varepsilon)/a < u_1(z_i) = 1/(a+1) \) for \( i = 1, 2 \). This is in violation of PO.

Q.E.D.

Next, we establish the incompatibility of PO, RM and EF. In fact, we prove a stronger result involving the following requirement, discussed in Thomson (1983).

The correspondence \( C \) satisfies \textbf{No-Domination (ND)} if

for all \( \omega \in \mathbb{R}_+^2 \), and for all \( z = (z_1, z_2) \in C(\omega) \), neither \( z_1 \gg z_2 \) nor \( z_2 \gg z_1 \).
A correspondence violating ND obviously generates envy.

**Theorem 3.** There is a profile \( u = (u_1, u_2) \) of utility functions representing convex and homothetic preferences such that there is no correspondence (at \( u \)) satisfying PO, RM and ND together.

**Proof:** An economy as desired is represented in Figure 2. Let \( u = (u_1, u_2) \) be defined by \( u_1(x, y) \equiv \min\{x + 100y, 41x + 3y\} \) for all \((x, y) \in \mathbb{R}_+^2\) and \( u_2(z_2) \equiv u_1(\pi(z_2, \Lambda)) \) for all \( z_2 \in \mathbb{R}_+^2 \). \( \bar{z} \) is defined as in the proof of Theorem 1.

![Figure 2](image-url)

Aggregate resources increase from \( \omega \equiv (20, 20) \) to \( \omega' \equiv (50, 20) \).

Let \( z \in C(\omega) \) and \( z' \in C(\omega') \). By PO, either (i) \( u_2(z_2) \geq u_2(\bar{z}_2) \) or (ii) \( u_1(z_1) \geq u_1(\bar{z}_1) \). Assume first that (i) holds. By RM, \( u_2(z_2') \geq u_2(\bar{z}_2) \). By
ND. \( z'_2 \) belongs to the \( 2^{nd} \) and \( 4^{th} \) quadrants from \( \omega'/2 \). The set of all \( z'_2 \) satisfying both conditions is the four-sided figure with vertices \( \bar{z}_2, a, b \) and \( c \). No such \( z'_2 \) is the second component of an allocation \((z'_1, z'_2) \in P(\omega')\). Indeed, in the Edgeworth box of the economy \( \omega' \) obtained by placing the origin of agent 1's consumption space at \( \omega' \), \( P(\omega') \) is the union of the segments \([0,d]\) and \([d,\omega']\). (In Figure 2, we have represented the agents' indifference curves through a generic point \( y \) of the Pareto set).

Assuming now that (ii) holds, we obtain the desired conclusion by considering an increase of aggregate resources from \( \omega \) to \( \pi(\omega',\Lambda) \).

Q.E.D.

4. Concluding comments

A. The axiom of Resource Monotonicity was first studied by Chun and Thomson (1984) and extensively investigated by Roemer (see, in particular, 1986).

Following Roemer, we interpret it as a natural implication of public ownership of resources.

B. A consequence of our results: the Walrasian correspondence from equal division does not satisfy RM. This is because (i) when preferences are homothetic and initial endowments are proportional, and in particular equal, this correspondence is essentially single-valued, and (ii) it satisfies PO, IR, EF and ND. (A direct proof of this negative feature of the Walrasian correspondence is straightforward.)

C. Consider now the more familiar set up in which each agent starts out with a vector of resources \( \omega_i \) which he owns. There, several resource monotonicity
properties can be formulated. First, we may require that an agent's utility
does not decrease when his own initial endowment increases (ceteris paribus).
Call this property $RM_1$. (Its violation by the price mechanism is precisely
the throw-away paradox). If preferences are strongly monotone, \( u_i(z_i) > \)
u_i(z_i') if \( z_i > z_i' \) it is not difficult to construct correspondences satisfying
PO. Individual Rationality \( (u_i(z_i) \geq u_i(\omega_i) \text{ for each } i) \) and $RM_1$.
Postlewaite (1979) provides examples. They are obtained by selecting utility
representations for the preferences and equating utility gains from the image
of the initial endowment. (These are the egalitarian solutions of bargaining
theory.) Similarly, by equating utility gains from the image of the zero
consumption, we obtain correspondences satisfying PO and RM, as noted by
Hurwicz (1978), but not Individual Rationality.

A second resource monotonicity requirement is that an agent's utility
should not decrease when the resources of some other agent increase, ceteris
paribus. Call this property $RM_2$: it is more difficult to justify than $RM_1$, as
well as harder to meet. The fact that the Walrasian mechanism does not
satisfy $RM_2$ was noted by Thomson (1978).

The proof of Theorem 1 can easily be adapted to show (still with convex
and homothetic preferences,) that the three requirements PO, $RM_2$ and
Individual Rationality are mutually incompatible.

D. Beyond the impossibility results uncovered in this note, one would like to
understand what features of preferences make the Resource Monotonicity axiom
so hard to meet. All our examples, as always in the literature on the
transfer paradox, exhibit strong complementarity of the two goods. Whether or
not this is the deep explanation of our impossibilities will be the subject of future research (see Moulin, 1987).
References


WP#33 OIL PRICE SHOCKS AND THE DISPERSION HYPOTHESIS, 1900 ~ 1980
by Prakash Loungani, January 1986

WP#34 RISK SHARING, INDIVISIBLE LABOR AND AGGREGATE FLUCTUATIONS
by Richard Rogerson, (Revised) February 1986

WP#35 PRICE CONTRACTS, OUTPUT, AND MONETARY DISTURBANCES
by Alan C. Stockman, October 1985

WP#36 FISCAL POLICIES AND INTERNATIONAL FINANCIAL MARKETS
by Alan C. Stockman, March 1986

WP#37 LARGE-SCALE TAX REFORM: THE EXAMPLE OF EMPLOYER-PAID HEALTH
INSURANCE PREMIUMS
by Charles E. Phelps, March 1986

WP#38 INVESTMENT, CAPACITY UTILIZATION AND THE REAL BUSINESS CYCLE
by Jeremy Greenwood and Zvi Hercowitz, April 1986

WP#39 THE ECONOMICS OF SCHOOLING: PRODUCTION AND EFFICIENCY IN PUBLIC
SCHOOLS
by Eric A. Hanushek, April 1986

WP#40 EMPLOYMENT RELATIONS IN DUAL LABOR MARKETS (IT'S NICE WORK IF YOU
CAN GET IT!)
by Walter Y. Oi, April 1986

WP#41 SECTORAL DISTURBANCES, GOVERNMENT POLICIES, AND INDUSTRIAL OUTPUT IN
SEVEN EUROPEAN COUNTRIES
by Alan C. Stockman, April 1986

WP#42 SMOOTH VALUATIONS FUNCTIONS AND DETERMINANCY WITH INFINITELY LIVED
CONSUMERS
by Timothy J. Kehoe, David K. Levine and Paul R. Romer, April 1986

WP#43 AN OPERATIONAL THEORY OF MONOPOLY UNION-COMPETITIVE FIRM INTERACTION
by Glenn M. MacDonald and Chris Robinson, June 1986

WP#44 JOB MOBILITY AND THE INFORMATION CONTENT OF EQUILIBRIUM WAGES:
PART 1, by Glenn M. MacDonald, June 1986

WP#45 SKI-LIFT PRICING, WITH APPLICATIONS TO LABOR AND OTHER MARKETS
by Robert J. Barro and Paul M. Romer, May 1986, revised April 1987
WP#46  FORMULA BUDGETING: THE ECONOMICS AND ANALYTICS OF FISCAL POLICY UNDER RULES, by Eric A. Hanushek, June 1986

WP#48  EXCHANGE RATE POLICY, WAGE FORMATION, AND CREDIBILITY by Henrik Horn and Torsten Persson, June 1986

WP#49  MONEY AND BUSINESS CYCLES: COMMENTS ON BERNANKE AND RELATED LITERATURE, by Robert G. King, July 1986

WP#50  NOMINAL SURPRISES, REAL FACTORS AND PROPAGATION MECHANISMS by Robert G. King and Charles I. Plosser, Final Draft: July 1986

WP#51  JOB MOBILITY IN MARKET EQUILIBRIUM by Glenn M. MacDonald, August 1986

WP#52  SECRECY, SPECULATION AND POLICY by Robert G. King, (revised) August 1986

WP#53  THE TULIPMANIA LEGEND by Peter M. Garber, July 1986

WP#54  THE WELFARE THEOREMS AND ECONOMIES WITH LAND AND A FINITE NUMBER OF TRADERS. by Marcus Berliant and Karl Dunz, July 1986

WP#55  NONLABOR SUPPLY RESPONSES TO THE INCOME MAINTENANCE EXPERIMENTS by Eric A. Hanushek, August 1986

WP#56  INDIVISIBLE LABOR, EXPERIENCE AND INTERTEMPORAL ALLOCATIONS by Vittorio U. Grilli and Richard Rogerson, September 1986

WP#57  TIME CONSISTENCY OF FISCAL AND MONETARY POLICY by Mats Persson, Torsten Persson and Lars E. O. Svensson, September 1986

WP#58  ON THE NATURE OF UNEMPLOYMENT IN ECONOMIES WITH EFFICIENT RISK SHARING, by Richard Rogerson and Randall Wright, September 1986

WP#59  INFORMATION PRODUCTION, EVALUATION RISK, AND OPTIMAL CONTRACTS by Monica Hargraves and Paul M. Romer, September 1986

WP#60  RECURSIVE UTILITY AND THE RAMSEY PROBLEM by John H. Boyd III, October 1986

WP#61  WHO LEAVES WHOM IN DURABLE TRADING MATCHES by Kenneth J. McLaughlin, October 1986

WP#62  SYMMETRIES, EQUILIBRIA AND THE VALUE FUNCTION by John H. Boyd III, December 1986

WP#63  A NOTE ON INCOME TAXATION AND THE CORE by Marcus Berliant, December 1986
WP#64 INCREASING RETURNS, SPECIALIZATION, AND EXTERNAL ECONOMIES: GROWTH AS DESCRIBED BY ALLYN YOUNG. By Paul M. Romer, December 1986

WP#65 THE QUIT-LAYOFF DISTINCTION: EMPIRICAL REGULARITIES. by Kenneth J. McLaughlin, December 1986

WP#66 FURTHER EVIDENCE ON THE RELATION BETWEEN FISCAL POLICY AND THE TERM STRUCTURE. by Charles I. Plosser, December 1986

WP#67 INVENTORIES AND THE VOLATILITY OF PRODUCTION by James A. Kahn, December 1986

WP#68 RECURSIVE UTILITY AND OPTIMAL CAPITAL ACCUMULATION, I: EXISTENCE, by Robert A. Becker, John H. Boyd III, and Bom Yong Sung, January 1987

WP#69 MONEY AND MARKET INCOMPLETENESS IN OVERLAPPING-GENERATIONS MODELS. by Marianne Baxter, January 1987

WP#70 GROWTH BASED ON INCREASING RETURNS DUE TO SPECIALIZATION by Paul M. Romer, January 1987

WP#71 WHY A STUBBORN CONSERVATIVE WOULD RUN A DEFICIT: POLICY WITH TIME-INCONSISTENT PREFERENCES by Torsten Persson and Lars E.O. Svensson, January 1987

WP#72 ON THE CONTINUUM APPROACH OF SPATIAL AND SOME LOCAL PUBLIC GOODS OR PRODUCT DIFFERENTIATION MODELS by Marcus Berliant and Thijs ten Raa, January 1987

WP#73 THE QUIT-LAYOFF DISTINCTION: GROWTH EFFECTS by Kenneth J. McLaughlin, February 1987

WP#74 SOCIAL SECURITY, LIQUIDITY, AND EARLY RETIREMENT by James A. Kahn, March 1987

WP#75 THE PRODUCT CYCLE HYPOTHESIS AND THE HECKSCHER-OHLIN-SAMUELSON THEORY OF INTERNATIONAL TRADE by Sugata Marjit, April 1987

WP#76 NOTIONS OF EQUAL OPPORTUNITIES by William Thomson, April 1987

WP#77 BARGAINING PROBLEMS WITH UNCERTAIN DISAGREEMENT POINTS by Youngsub Chun and William Thomson, April 1987

WP#78 THE ECONOMICS OF RISING STARS by Glenn M. MacDonald, April 1987

WP#79 STOCHASTIC TRENDS AND ECONOMIC FLUCTUATIONS by Robert King, Charles Plosser, James Stock, and Mark Watson, April 1987
WP#80  INTEREST RATE SMOOTHING AND PRICE LEVEL TREND-STATIONARITY
by Marvin Goodfriend, April 1987

WP#81  THE EQUILIBRIUM APPROACH TO EXCHANGE RATES
by Alan C. Stockman, revised, April 1987

WP#82  INTEREST-RATE SMOOTHING
by Robert J. Barro, May 1987

WP#83  CYCLICAL PRICING OF DURABLE LUXURIES
by Mark Bils, May 1987

WP#84  EQUILIBRIUM IN COOPERATIVE GAMES OF POLICY FORMULATION
by Thomas P. Cooley and Bruce D. Smith, May 1987

WP#85  RENT SHARING AND TURNOVER IN A MODEL WITH EFFICIENCY UNITS OF HUMAN CAPITAL
by Kenneth J. McLaughlin, revised, May 1987

WP#86  THE CYCLICALITY OF LABOR TURNOVER: A JOINT WEALTH MAXIMIZING HYPOTHESIS
by Kenneth J. McLaughlin, revised, May 1987

WP#87  CAN EVERYONE BENEFIT FROM GROWTH? THREE DIFFICULTIES
by Herve Moulin and William Thomson, May 1987
To order copies of the above papers complete the attached invoice and return to Christine Massaro, W. Allen Wallis Institute of Political Economy, RCER, 109B Harkness Hall, University of Rochester, Rochester, NY 14627. Three (3) papers per year will be provided free of charge as requested below. Each additional paper will require a $5.00 service fee which must be enclosed with your order. For your convenience an invoice is provided below in order that you may request payment from your institution as necessary. Please make your check payable to the Rochester Center for Economic Research. Checks must be drawn from a U.S. bank and in U.S. dollars.

W. Allen Wallis Institute for Political Economy

Rochester Center for Economic Research, Working Paper Series

OFFICIAL INVOICE

Requestor’s Name

Requestor’s Address

Please send me the following papers free of charge (Limit: 3 free per year).

WP# _____ WP# _____ WP# _____

I understand there is a $5.00 fee for each additional paper. Enclosed is my check or money order in the amount of $___________. Please send me the following papers.

WP# _____ WP# _____ WP# _____
WP# _____ WP# _____ WP# _____
WP# _____ WP# _____ WP# _____
WP# _____ WP# _____ WP# _____