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ABSTRACT

This paper develops methods for solving and estimating rational expectations models with censored variables. The novel feature of the model is that private agents' decision rules depend on expectations of the censored variables. Appropriate econometric techniques are derived and are illustrated by application to Mexico's recent adjustable-peg exchange rate regime. A striking implication of the adjustable-peg model is that the forward premium will typically be serially correlelated even in the absence of a risk premium, reflecting serial correlation in expected devaluations due to serial correlation in the exogenous variables.

I. Introduction

The classic rational expectations model is comprised of a set of linear equations in variables that are continuous functions of underlying exogenous variables. This paper develops methods for solving and estimating rational expectations models with a particular type of non-linearity: some variables in the model are "censored", in the sense used in the econometric literature on limited variables. The novel feature of this paper is that individuals' decision rules depend on their expectations of future values of the limited variable.

There are many examples of economic phenomena where the variable of interest is usefully modelled as a censored variable. One example is a variant of the classic problem of Muth, the problem facing a farmer who must make planting decisions based on his forecast of next year's price. If there are farm price supports in place, the farmer will receive the support price unless the competitive equilibrium price is higher. Thus, the farmer (and the economist) may observe long periods of time where the price of the agricultural commodity is given by the support price and thus does not change, but there are conditions under which the equilibrium price will be above the support price, and the farmer must take this possibility into account when forecasting. In the terminology of the econometric literature on limited variables, the competitive equilibrium commodity price can be viewed as being "censored", since it is observed only occasionally, while the

¹Maddala [1983] develops estimation procedures for a model with price supports and/or ceilings, but in which agents do not form expectations of future prices.

underlying determinants of the equilibrium price (quantity planted, weather, demand conditions) are observed each period.

The decision rule describing the evolution of the censored variable is typically part of a larger model which the econometrician would like to estimate under the maintained hypothesis that expectations are formed rationally. This paper develops methods for solution and estimation of rational expectations models with limited variables, drawing on previous work from the literatures on limited dependent variables and on rational expectations econometrics. The novel feature of this paper is that economic agents must form expectations of (i.e., forecast) the limited variable. By contributing to the growing literature on nonlinear rational expectations models, this paper broadens significantly the class of rational expectations models that are econometrically tractable.

An application to adjustable-peg exchange rate regimes is included as a specific example. Under central bank management of an adjustable-peg exchange rate regime, the central bank wishes (for reasons unexplained in this paper) to peg the exchange rate for a period of time, changing the peg in response to movements in underlying economic conditions such as the money supply, reserves, or foreign inflation rates. When the peg is changed, the authorities are assumed to choose the new peg as a linear function of the rate that would obtain under a floating exchange rate rule. Thus, the floating rate can be viewed as a limited variable since this variable is only observed when the exchange rate peg is changed; the observed exchange rate is a censored transformation of the underlying floating rate. The exogenous determinants of the floating rate are, on the other hand, observed continuously.

Other examples of economic phenomena that can be modelled in this way include central bank policies for setting of discount rates, and a firm's choice of dividends as a function of earnings (as in Baxter (1987a)).

The paper is organized as follows. Section II uses a prototypical rational expectations model to illustrate model solution and estimation techniques for situations where some variables are treated as censored. As an example of an application of this technique, Section III presents a model of an adjustable-peg exchange rate regime, using the tools developed in Section II. Section IV implements empirically the model of Section III using data from Mexico's recent adjustable-peg period. In the process of solving the model, the techniques developed here also provide estimates of the probability of a change in the exchange rate in the next period, together with forecasts of next period's exchange rate, conditional or unconditional on a change in the level of the peg. Section V investigates the implication of this new approach to modeling exchange rates for the behavior of risk premia in the forward market for foreign exchange. Section VI contains concluding comments.

II. Censored Variables in Rational Expectations Models

In response to Lucas's (1976) critique of traditional macroeconometric modeling practices, techniques for specification and estimation of dynamic rational expectations models have been developed by Sargent, Hansen, and others. Subsequently, these techniques have been implemented in the investigation of a wide variety of problems, several examples are contained in Lucas and Sargent (1981).

This paper extends earlier work on estimation of dynamic rational expectations models to handle cases where some variables can be viewed as limited variables, i.e, variables that are not observable over their entire range. This section uses the following simple rational expectations model to illustrate these methods.

A simple linear rational expectations model without censoring

$$y_t^* = \beta X_t + u_{1t} \tag{1}$$

$$z_{t} = \alpha E_{t} y_{t+1}^{*} + u_{2t}$$
 (2)

$$X_{t} = \rho X_{t-1} + u_{3t}$$
 (3)

Equation (1) describes the evolution of a variable y_t^* which depends only on the exogenous variables X_t . Equation (2) describes the determination of a variable z_t which depends on agents' expectations of the <u>future</u> value of y^* . Equation (2) may be thought of as a reaction function; agents choose an action today, z_t , that depends on their expectation of tomorrow's y^* . For simplicity, the exogenous variables X_t are assumed to follow the first-order autoregressive process given in equation (3). The error terms, u_{1t} , u_{2t} , and u_{3t} are assumed i.i.d. normal random variables with zero means and variances σ_1^2 , σ_2^2 , and σ_3^2 . All covariances are assumed to be zero. This assumption is not needed to generate any of the results described below, but it substantially simplifies the exposition.

Application of standard solution techniques yields the following reduced form for this model:

$$y_t^* = \rho X_t + u_{1t}$$

$$z_t = \alpha \rho \rho X_t + u_{2t}$$

$$X_t = \rho X_{t-1} + u_{3t}$$

Note that the parameters of the model, α , β , and ρ , are just-identified.

The censored dependent variable problem

Now, suppose that instead of y_t^* , the variable of interest in equations (1) and (2) is a particular nonlinear transformation of y_t^* : call this variable y_t . The specific transformation studied in this paper is given by equation (4):

Thus, y_t may be viewed as a censored version of y_t^* : y_t is equal to y_t^* if y_t^* is greater than zero, and equals zero if y_t^* is less than zero. The model then becomes:

$$y_t = \beta X_t + u_{1t}$$
 (1')

$$z_{t} = \alpha E_{t} y_{t+1} + u_{2t}$$
 (2')

$$X_{t} = \rho X_{t-1} + u_{3t}$$
 (3)

The novel feature of this model is that agents' decisions (in equation (2')) depend on y_t , the censored version of y_t^* . This is different from standard linear rational expectations models (like the model consisting of equations (1)-(3) above) in which agents' decisions depend on y_t^* or a linear transformation of y_t^* . In cases where agents' decisions depend on a censored variable, the reduced-form decision rules are not linear in the exogenous variables, and standard estimation techniques will produce inconsistent parameter estimates. The purpose of this section is to develop appropriate model solution and estimation techniques for this problem.

An example of a case in which agents might care about the censored variable instead of the underlying variable include the farm price support example discussed in the introduction: the farmer cares about, and tries to forecast, the actual price he will receive--not the price that would clear the market in the absence of price supports. Another example is an adjustable-peg exchange rate regime; this example is developed in sections III-V below.

The econometrician is assumed to have data on X_t , z_t and y_t (the censored transformation of y_t^* for a sample of T observations indexed by $t=1,\ldots,T$. It is useful to denote by T_1 the set of all dates t for which y_t is positive.

In this model, standard estimation techniques using data for which y_t is positive will yield biased and inconsistent parameter estimates, since the expectation of the error term u_{1t} conditional on y_t 's being positive is not zero, i.e.,

$$E(u_{1t}|X_t;y_t>0) \neq 0$$
.

The fact that y_t is positive (and thus y_t^* is positive) provides information about the error term. The correct expectation is given by:

$$E(u_{1t}|X_t,y_t>0) = E\{u_{1t}|u_{1t}>-X_t\beta\} = \sigma_1\lambda_t$$
 (5)

where

$$\lambda_{t} = f(\Psi_{t})/[F(\Psi_{t})]$$

$$\Psi_{t} = \{-X_{t}\beta\}/\sigma_{1}$$
(6)

where $F(\Psi_t)$ and $f(\Psi_t)$ denote the c.d.f. and p.d.f. of a standard normal random variable evaluated at Ψ_t , and where λ_t is the inverse of Mill's ratio. Thus, the correct expectation of y_t conditional on y_t being positive is:

$$E(y_t | X_t, y_t \ge 0) = X_t \beta + \sigma_1 \lambda_t.$$

If we knew the sequence λ_t , then standard estimation techniques could be applied, using only observations teT₁:

$$y_{t} = X_{t}\beta + \sigma_{1}\lambda_{t} + \xi_{1t}$$

$$\text{since } E(\xi_{1t}|y_{t} \ge 0) = 0.$$
(7)

Gronau (1973) and Heckman (1976, 1979) have developed a simple two-stage estimator for models with censored dependent variables. In this paper, the discussion of identification and estimation uses the Heckman technique. This procedure is particularly useful in models in which agents' expectations of a censored dependent variable enter elsewhere in the economic model, since construction of agents' expectations proceeds in a manner analogous to the procedure followed by an econometrician using the Heckman approach to estimation. In addition, it permits a direct test of whether the "selectivity bias" induced by censoring is econometrically important. The chief drawback to the Heckman estimator is that it is not a maximum likelihood estimator, although it is consistent and can be used as a starting point for maximum likelihood methods. This point is discussed more fully below.

The first step of the Heckman procedure is to estimate a probit model with a dependent variable that takes the value 1 if y_t is positive and takes the value 0 if it is not. The explanatory variables used are X_t . The output from this step is used to construct consistent estimates of the series λ_t ; call these estimates $\hat{\lambda}_+$.

For the second step, the estimated $\hat{\lambda}_t$ series is used on the right-hand-side of (7') in place of the λ_t .

$$y_{t} = X_{t}\beta + \sigma_{1}\hat{\lambda}_{t} + \xi_{1t} \tag{7'}$$

The error term \mathbf{f}_{1t} in the regression equation (7') has zero expectation and is orthogonal to the independent variables in the equation, thus, estimation of this equation may be undertaken using standard techniques.

Expectations of censored variables

The key step in solving the model--and the novel feature of this model compared to the earlier literatures on limited variables and on rational expectations econometrics--is determining how agents form expectations of the future value of the censored variable, $E(y_{t+1}|X_t,y_{t+1}>0)$ (see equation (2')) under the hypothesis that expectations are formed rationally. Since agents at time t condition their expectations on variables known at time t (i.e., variables dated t or earlier,) update equation (1') by one period and rewrite it as the sum of variables known at time t and variables unknown at time t (using equation (3)):

$$y_{t+1} = \beta X_{t+1} + u_{1,t+1}$$

$$= \beta \rho X_{t} + \{\beta u_{3,t+1} + u_{1,t+1}\}$$
(8)

This equation is now in a form where the results obtained above can be applied directly:

A recent paper by Eichengreen, Watson, and Grossman (1984) uses a probit model to investigate "Bank Rate Policy under the Interwar Gold Standard"; their paper involves estimation only of the government policy rule. This policy rule is not imbedded in a model which includes agents' reactions to--and forecasts of--government policy. As suggested in this Section, their procedure could lead to inconsistent paremeter estimates (because of simultaneous equation bias) if some of the explanatory variables in the government policy function are influenced by private agents, and these variables in turn depend on private agents' expectations of future government actions.

$$E(y_{t+1} | X_t, y_{t+1} \ge 0) = \beta \rho X_t + \sigma^* \lambda_t^*$$

where

$$\sigma^* = \left\{ var(\beta u_{3,t+1}^+ u_{1,t+1}^-) \right\}^{1/2} = \left\{ \beta^2 \sigma_3^2 + \sigma_1^2 \right\}^{1/2}$$

$$\lambda_t^* = f(\Psi_t^*) / [F(\Psi_t^*)]$$

$$\Psi_t^* = (\beta \rho / \sigma^*) X_t$$

Since agents are assumed to know the parameters of the model and to form expectations rationally, they calculate λ_t^* as above and arrive at a decision rule of the form:

$$z_{t} = \alpha E_{t}(y_{t+1}) + u_{2t}$$

$$= \alpha pr_{t}\{y_{t+1}>0 \mid X_{t}\} E_{t}\{y_{t+1} \mid X_{t}, y_{t+1}>0\} + \alpha pr_{t}\{y_{t+1}=0\}\{0\} + u_{2t}$$

$$= \alpha \beta \rho [F(Y_{t}^{*})X_{t}] + \alpha \sigma^{*}[F(Y_{t}^{*})\lambda_{t}^{*}] + u_{2t}$$
(9)

It is evident from equation (9) that the decision rule for \mathbf{z}_t is a nonlinear function of the exogenous variables, \mathbf{X}_t , since $\mathbf{F}(\mathbf{Y}_t^*)$ is a nonlinear function of \mathbf{X}_t .

In order to estimate an equation of the form of (9), the econometrician needs to calculate an estimate of $E_t \hat{y}_{t+1}$. Because of the rational expectations assumption, the econometrician and the agents in the economy calculate this expectation in the same way. To estimate λ_t^* the first step of the Heckman procedure is applied to equation (8). These estimates are then inserted into equations (5) and (6) to yield:

 $^{^3}$ Another approach to estimating (2') would be to replace $\mathbf{E}_t\mathbf{y}_{t+1}$ with the realized value of \mathbf{y}_{t+1} , putting the expectation error into the equation's error term, and then using an instrumental variables estimator. The resulting error term would be non-normal, but the estimate would be consistent. This approach is not used here since a key motivation of this paper is to study the properties of expectations of limited variables.

$$z_{t} = \alpha E_{t} \hat{y}_{t+1} + u_{2t}$$

$$= \alpha \beta \rho [F(\hat{Y}_{t}^{*})X_{t}] + \alpha \sigma^{*} [F(\hat{Y}_{t}^{*})\hat{\lambda}_{t}^{*}] + u_{2t} . \qquad (9')$$

Compare (9') to (9) above to see clearly the similarity between expectations formation under rational expectations and the procedure followed by the econometrician.

Collecting the equations of the model to be estimated, we have:

$$y_{t} = \beta X_{t} + \sigma_{1} \hat{\lambda}_{t} + \xi_{1t}$$
 (10)

$$z_{t} = \alpha \beta \rho [F(\hat{Y}_{t}^{*})X_{t}] + \alpha \sigma^{*}[F(\hat{Y}_{t}^{*})\hat{\lambda}_{t}^{*}] + u_{2t} \qquad \text{all t} \qquad (11)$$

$$X_{t} = \rho X_{t-1} + \nu_{t}$$
 all t (12)

It is straightforward to show that the parameters of interest, α , β , and ρ are identified in this system. As shown at the beginning of this section, if the model were a standard linear rational expectations model--one in which y_t was not censored--the parameter α would be just identified. It is interesting to note that the there is an overidentifying restriction arising from the presence of the $\hat{\lambda}_t$ and $\hat{\lambda}_t^*$ series in (10) and (11).

Hansen and Sargent (1980) have stressed that a hallmark of linear rational expectations models is the role of cross-equation restrictions in achieving identification, with these cross-equation restrictions deriving

The overidentifying restriction involves the covariance matrix as well as the parameters of the model. To see this, note that the parameters β and ρ are identified from equations (10) and (12). An estimate of α can be unraveled from the estimate of the coefficient on $[F(\Psi_t^*)X_t]$ in equation (11). The coefficient on $[F(\Psi_t^*)\lambda_t^*]$ is $\alpha\sigma^*$. Since $\sigma^* = [\beta^2\sigma_3^2 + \sigma_1^2]$, and an estimate of σ_3^2 and is available from the estimated covariance matrix, the presence of $\hat{\lambda}_t$ in (10) and $\hat{\lambda}_t^*$ in (11) provides an overidentifying, nonlinear restriction.

from the assumption that agents know the model's parameters and form expectations rationally. In this model, an overidentifying restriction is provided by the fact that agents form expectations about the censored \mathbf{y}_t in the same way that the econometrician corrects for sample selection bias.

Alternative econometric techniques

The two-step Heckman procedure described above yields consistent parameter estimates, however, these estimates are inefficient relative to maximum likelihood. But because the Heckman estimator is consistent, it can be used as an initial estimator for maximum likelihood methods, for example, iterated estimation of the likelihood function using the method of Newton.

Amemiya (1973) constructs another consistent initial estimator that can be used either as an end in itself, or as a first round estimator to begin the iterative search process. Amemiya's estimator is an instrumental variables estimator, and, like the Heckman estimator, is not computationally difficult to produce. The Amemiya estimator can also be used in truncated samples (where, for some t, data on both y_t and X_t are missing) while the Heckman procedure cannot.

Finally, estimation of the model developed in this paper can be carried out using methods for estimation of Tobit models. The chief advantage of this approach is that the resulting estimates are maximum likelihood estimates. The disadvantage is that the role of expectations does not emerge as clearly, not is it possible to test directly the importance of the "selectivity bias." Finally, for more complicated sample selection problems (e.g., farm price supports will be in place next year only if Democrats are

elected, and whether Democrats are elected depends on a different set of variable than those determining the equilibrium price of the agricultural commodity), the Tobit model cannot be applied.

III. An Adjustable-Peg Exchange Rate Regime

As an application of the methods developed in Section II, this section develops a model of an adjustable-peg exchange regime. This model is concerned with the determination of exchange rates, prices, and money demand in a small country in which the government follows a time invariant policy rule for setting the exchange rate. While few countries--if any--are in fact on a pure floating rate regime or a pure fixed rate regime, until now it has not been possible to estimate a rational expectations model of a country on an intermediate regime. ⁵

The econometric methods developed in the last section permit the joint estimation of the parameters of the policy rule, of the money demand equation, and of the processes for the exogenous variables. In the interest of the conservation of space and the reader's patience, the model has been made as simple as possible while retaining key features of the problem.

Greater generality is allowed in the actual estimation of the model in Sections IV and V below. The structure of this model is the same as that of the model of Section II.

⁵Aspects of changes in regimes have been extensively studied by Flood and Garber (1980), (1983), (1984). A recent paper by Blanco and Garber (1986) examines speculative attacks in Mexico during the 1973-82 period. Related work by Grilli (1986) extends the Blanco and Garber model to allow both buying and selling attacks.

The Model

The model is described by the equations below; all variables are in natural logarithms.

The policy rule for setting the exchange rate is of the form:

$$e_{t}^{*} = E\{\tilde{e}_{t}^{|I|}_{t-1}, z_{t}^{|A|}, e_{t}^{*}\} + b + \phi_{t}$$

$$e_{t}^{*} = \begin{cases} e_{t}^{*} & \text{if } e_{t}^{*} \geq e_{t-1} \\ e_{t-1}^{*} & \text{if } e_{t}^{*} < e_{t-1} \end{cases}$$
(13)

where \mathbf{e}_{t} is the exchange rate peg set by the authorities in period t, \mathbf{e}_{t} is the floating rate that would obtain in period t if the exchange were allowed to float in that period, b is a constant, and \mathbf{e}_{t} is an i.i.d. $N(0,\sigma_{\mathbf{e}}^{2})$, error term which the authorities choose randomly each period. E(.) is the expectations operator, I_{t-1} is the information set available to the authorities as of time t-1 (it includes all variables dated t-1 and earlier), \mathbf{e}_{t} is output in period t, \mathbf{e}_{t} is domestic credit (the domestically-created component of the monetary base) and \mathbf{e}_{t} denotes foreign prices.

In this model, all exchange rate changes are devaluations. The authorities change the exchange rate if the value of e_t^* is greater than last period's exchange rate. The variable e_t^* is the "shadow peg"--the level of the peg that would obtain in the event of an exchange rate change. It is a linear function of the "shadow floating rate", \tilde{e}_t (the reduced form for \tilde{e}_t is derived below.) The "shadow peg" is a limited variable because it is observed only when the exchange rate actually changes, i.e., when the shadow peg exceeds the current peg. The specification (13) giving the "shadow peg" as a function of the "shadow exchange rate" is intended to capture the idea

that, in an adjustable-peg regime, the peg is changed when the prevailing peg is "too far" from the equilibrium floating rate. The parameters b and ϕ_t specify how far is "too far" from the point of view of the authorities. Note that the authorities do not observe the current money demand shock before choosing the current peg. The peg-setting rule (18) can easily be made more "realistic" by allowing the choice of the peg to depend on the level of reserves or other variables that seem to be relevant for policymakers' behavior. 6

Private agents' demand for base money is given by:

$$m_t - p_t = yz_t + \alpha(E_t p_{t+1} - p_t) + x_{1t}$$
 (14)

where p_t denotes the domestic price level in period t, x_{1t} is an i.i.d. $N(0,\sigma_1^2)$ error term, $E_t(\cdot)$ is the expectation of (·) conditional on the period t information set, and m_t is the (log of the) money supply. Domestic credit, d_t , is taken as exogenous both by private agents and the exchange-rate setting authority. The model's exogenous variables are assumed to follow random walks (possibly with drift):

$$d_t = \mu + d_{t-1} + x_{2t}$$
, $\mu > 0$ (15)

$$p_{t}^{*} = p_{t-1}^{*} + x_{3t}$$
 (16)

$$\mathbf{z}_{\mathsf{t}} = \mathbf{z}_{\mathsf{t}-1} + \mathbf{x}_{\mathsf{4}\mathsf{t}} \tag{17}$$

 x_{2t} , x_{3t} , and x_{4t} are each serially independent random variables with zero means and variances σ_2^2 , σ_3^2 , and σ_4^2 . For simplicity, $E(x_{1t}, x_{jt})$ is assumed to

⁶For this rule to be feasible, it must be the case that agents do not expect "buying attacks" to be profitable; a sufficiently expansionary domestic monetary policy will ensure this. In applications where it is necessary to allow both devaluations and revaluations, the exchange rate rule (13) can be generalized to allow this. The model would then be estimated using techniques for two-sided Tobit models.

be equal to zero for all t and j=2,3,4 (the innovations in the exogenous variables are contemporaneously uncorrelated with the shocks to money demand.) The exogenous variable have been assumed to follow random walks in order to simplify the solution and exposition. More complicated processes are allowed in the empirical implementation of the model in Sections IV and V.

Finally, purchasing power parity is assumed to hold:

$$p_{t} = p_{t}^{*} + e_{t}$$
 (18)

In the exchange rate rule (13), \tilde{e}_t is the log of the "shadow floating exchange rate", defined as the exchange rate that would obtain today if the country were on a floating-rate system. To find \tilde{e}_t the model is solved under the assumptions that: (i) the country is on a flexible rate regime, and (ii) under a flexible rate regime the monetary base is assumed to be completely backed by domestic credit. Under these assumptions, the solution for \tilde{e}_t is:

$$\tilde{e}_{t} = \mu + d_{t} - p_{t}^{*} - \gamma z_{t} - (1+\alpha)^{-1} x_{1t}$$
 (19)

 $^{^{7}}$ A more realistic approach would be to allow prices to depend on both the current peg, e_{t} , and the current shadow floating rate, \widetilde{e}_{t} . This modification is straightforward but is not pursued here.

This assumption is made in order to avoid having to keep track of the value of reserves in domestic currency units as the exchange rate changes. This assumption is reasonable if the reserve component of the monetary base is small under a floating rate regime. Since reserves are unnecessary to the operation of a floating rate regime, the government has an incentive to "eat up" reserves that may exist at the time of a regime switch.

Solution of the model

As discussed above, the variable e* is usefully modeled as a limited variable. This variable is only observed under particular conditions--when the level of the exchange rate peg is changed by the government. The peg is changed when e* crosses a threshold: the threshold is the current value of the peg. Thus, the threshold level is known, but is not constant over time. It is useful to redefine the variable of interest as follows:

$$y_t = e_t^* - e_{t-1}$$
 (20)

i.e., the difference between the exchange rate given by the policy rule (13) and last period's pegged rate. Using (14) and (18) to substitute out for e*, we have:

$$y_{t} = (\mu + b) + d_{t} - p_{t}^{*} - e_{t-1} - \gamma z_{t} + \phi_{t}$$
 (21)

The econometrician only observes the variable y_t when it is positive; i.e., when there is a devaluation. The variables on the right-hand side of (21) are either exogenous or predetermined, and are observed in every period. Since y_t is a censored dependent variable, the basic Heckman technique can be applied to equation (21) to yield consistent parameter estimates.

As discussed in Section II, the new feature of this model is the fact that agents must form expectations of a limited variable. In this model, this feature arises in the money demand equation. Using (18) and definition (20), the money demand equation becomes:

$$\mathbf{m}_{t} = \mathbf{e}_{t} + \mathbf{p}_{t}^{*} + \mathbf{y}_{t} + \alpha(\mathbf{pr}_{t}\{\mathbf{y}_{t+1} \geq 0\} \mathbf{E}_{t}\{\mathbf{y}_{t+1} | \mathbf{y}_{t+1} \geq 0\}) + \mathbf{x}_{1t}$$
 (22)

Private agents' demand for money depends on $pr_t(y_{t+1} \ge 0)$: the probability, conditional on date t information, that there will be a devaluation in the

next period, and on $E_t\{y_{t+1}|y_{t+1}\geq 0\}$: the conditional expectation of the devaluation, given that one occurs. This conditional expectation is given by:

$$E_{t}\{y_{t+1}|y_{t+1}\geq 0\} = (\mu+b) + E_{t}d_{t+1} - E_{t}p_{t+1}^{*} - E_{t}e_{t} - \gamma E_{t}z_{t+1} + E_{t}(\phi_{t+1}|y_{t}\geq 0)$$

$$= (2\mu+b)+d_{t}-p_{t}^{*}-e_{t}-\gamma z_{t} + E_{t}\{\phi_{t+1}+x_{2,t+1}-x_{3,t+1}-\gamma x_{4,t+1}|y_{t+1}\geq 0\}$$
(23)

Following the same procedure as in Section II, agents in this model calculate:

$$E_{t} \{ y_{t+1} | y_{t+1} \ge 0 \} = (2\mu + b) + d_{t} - p_{t}^{*} - e_{t}^{-\gamma z} + \sigma^{*} \lambda_{t}^{*}$$
where

$$\sigma^* = \left\{ var(\phi_{t+1} + x_{2,t+1} - x_{3,t+1} - \gamma x_{4,t+1}) \right\}^{1/2} ,$$

$$\gamma^*_t = -\left\{ (2\mu + b) + d_t - p_t^* - e_t - \gamma z_t \right\} + \sigma^*$$

$$\lambda^*_t = f(\gamma^*_t) / [F(\gamma^*_t)] .$$

The unconditional one-period-ahead expected rate of devaluation is given by:

$$pr_{t} \{y_{t+1} \ge 0\} E_{t} [y_{t+1} | y_{t+1} \ge 0] = F(Y_{t}^{*}) [(2\mu + b) + d_{t} - p_{t}^{*} - e_{t}^{-\gamma z} + \sigma^{*} \lambda_{t}^{*}]$$
 (25)

Estimation of this model thus yields estimates of the "one-step-ahead" probabilities of devaluation, $\operatorname{pr}_t(y_{t+1}|y_{t+1}\geq 0)$, which are given by $F(Y_t^*)$, and estimates of next period's expected exchange rate, conditional on a devaluation taking place:

$$E_{t}\{y_{t+1}|y_{t+1}\geq 0\} = (2\mu+b)+d_{t}-p_{t}^{*}-e_{t}^{-\gamma z}+\sigma^{*}\lambda_{t}^{*}$$

Inserting (25) into the money demand equation (22) yields (nominal) money demand as a function of the exogenous and predetermined variables of the model:

$$m_{t} = \alpha(2\mu+b)F(\gamma_{t}^{*}) + e_{t} - \alpha[F(\gamma_{t}^{*})e_{t}] + p_{t}^{*} - \alpha[F(\gamma_{t}^{*})p_{t}^{*}] + \gamma z_{t}$$
$$- \alpha\gamma[F(\gamma_{t}^{*})z_{t}] + \alpha[F(\gamma_{t}^{*})d_{t}] + \alpha\sigma^{*}[F(\gamma_{t}^{*})\lambda_{t}^{*}] + x_{1t} . \qquad (26)$$

Estimation of the model

Since estimation of this model so closely mimics estimation of the model of Section II, discussion of the estimation procedure is kept quite brief. The first step in the estimation procedure is to implement the first stage of the Heckman procedure to remove the bias from the equation for the exchange rate, equation (21) (the dependent variable here is actually the rate of devaluation in period t.) From the output of this step, construct the $\hat{\lambda}_t$ series needed as an explanatory variable in (21) to remove the bias caused by the censoring of y_t . Thus, (21) becomes:

$$y_{t} = (\mu+b)+d_{t}-p_{t}^{*}-e_{t-1}^{-\gamma z}_{t} + \sigma_{\phi}^{\lambda}_{t} + \xi_{1t}$$
where
$$E(\xi_{1t}|y_{t}\geq 0) = 0,$$

$$\sigma_{\phi} = \left\{var(\phi_{t})\right\}^{1/2}$$
(27)

$$\Psi_{t} = -\left\{ (\mu+b) + d_{t} - p_{t}^{*} - e_{t-1}^{*} - \gamma z_{t} \right\} + \sigma_{\phi}$$

$$\lambda_{t} = f(\Psi_{t}) / F(\Psi_{t}) \qquad .$$

In estimating the money demand, the econometrician calculates $E_t(y_{t+1}|y_{t+1}\geq 0)$ just a private agent would, except that the econometrician must estimate λ_t^* , whereas private agents can calculate λ_t^* exactly. Thus, the money demand equation to be estimated becomes:

$$\begin{split} \mathbf{m}_{t} &= \alpha(2\mu + \mathbf{b}) \mathbf{F}(\hat{\mathbf{Y}}_{t}^{*}) + \mathbf{e}_{t} - \alpha[\mathbf{F}(\hat{\mathbf{Y}}_{t}^{*}) \mathbf{e}_{t}] + \mathbf{p}_{t}^{*} - \alpha[\mathbf{F}(\hat{\mathbf{Y}}_{t}^{*}) \mathbf{p}_{t}^{*}] + \mathbf{Y} \mathbf{z}_{t} \\ &- \alpha \mathbf{Y}[\mathbf{F}(\hat{\mathbf{Y}}_{t}^{*}) \mathbf{z}_{t}] + \alpha[\mathbf{F}(\hat{\mathbf{Y}}_{t}^{*}) \mathbf{d}_{t}] + \alpha \sigma^{*}[\mathbf{F}(\hat{\mathbf{Y}}_{t}^{*}) \hat{\lambda}_{t}^{*}] + \mathbf{X}_{1t} \end{split} .$$

It is now possible to estimate the reduced form of the model, imposing the cross-equation restrictions implied by the rational expectations hypothesis. As in the model of Section II, overidentifying restrictions are provided by the fact that private agents in the economy and the econometrician form expectations about limited variables in the same way.

The next section estimates a version of this model using data from Mexico's recent adjustable-peg exchange rate regime. The focus of that section is on estimating and interpreting parameter estimates, and investigating the properties of forecasts of devaluation and their associated probabilities.

IV. Empirical Implementation of the Adjustable-Peg Model

The adjustable-peg model of Section III was estimated for Mexico's recent adj stable-peg period, 1973-1982, using quarterly data. 9,10 The model of that section was modified slightly for the purpose of estimation. These modifications allow more general processes for the exogenous variables, and include in the exchange-rate setting equation additional variables which seem to be empirically important determinants of devaluations. Also, a constant term has been added to the money demand equation.

Herminio Blanco and Peter Garber generously provided their Mexican data for the 1973-1982 period. Estimation was carried out using quarterly data for the period 1973:1-1982:2.

The model was also estimated as a Tobit model in order to see whether maximum likelihood methods produced substantially different results. The resulting parameter estimates and behavior of variables such as expected devaluations, probabilities of devaluation, etc., were similar to those presented here.

The model's endogenous variables are the exchange rate and money demand, which are described by the equations below:

Policy rule for the exchange rate:

 $y_t = e_{t-1}^* = b + E_t(\tilde{e}_t | I_{t-1}, d_t, p_t^*, z_t, \Delta R_t) - e_{t-1} + \Delta R_t + \phi_t$ (28) where ΔR_t is the change in the log of foreign exchange reserves, denominated in SDR's. The exchange rate e_t is set by the following rule:

$$e_{t} = \begin{cases} e_{t}^{*} & \text{if } e_{t}^{*} \geq e_{t-1} \\ e_{t-1} & \text{otherwise} \end{cases}$$

or, equivalently, devaluations are set according to the following:

$$e_t - e_{t-1} = \begin{cases} y_t & \text{if } y_t \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

Money demand equation:

$$m_{t} = k + e_{t} + p_{t}^{*} + \gamma z_{t} + \alpha [pr_{t} \{y_{t+1} \ge 0\}] E_{t} [y_{t+1} | y_{t+1} \ge 0] + x_{1t}$$
 (29)

The model's exogenous variables are domestic credit, the foreign price level, and domestic output. Low-order integrated processes were found to fit the Mexican data reasonably well. This finding corroborates Nelson and Plosser's (1982) finding that many economic time series are well-modelled by low-order integrated processes. The following processes were fit to the data; coefficient estimates are presented in Table 1 (all variables are in logarithms): 11

Domestic credit:

$$d_{t} - d_{t-1} = \mu_{d} + x_{2t}$$
 (30)

 $^{^{11}}$ Estimation of a VAR system showed a lack of Granger causality between any of the variables.

Foreign price level:

$$p_{t}^{*} - p_{t-1}^{*} = \mu_{p} + \theta(p_{t-1}^{*} - p_{t-2}^{*}) + x_{3t}$$
 (31)

Output:

$$z_{t} - z_{t-1} = \mu_{z} + x_{4t}$$
 (32)

Estimation of the exchange rate rule

Substituting out for $E_t(\widetilde{e}|\)$ in equation (31), the exchange rate rule to be estimated becomes:

$$y_t = (\mu_d - \mu_p - \nu_z + b + k) + d_t - p_t^* - e_{t-1} - \nu_z + \delta(\Delta R_t) + \phi_t$$
 (33)

Estimation of the devaluation rule (33) requires a two stage procedure, since y_t is a limited dependent variable. ¹² The results of the estimation are shown in Table 2. The coefficient estimates are generally of the predicted sign and magnitude, with devaluations depending positively on the level of domestically-created money, and negatively on output and the level of foreign exchange reserves. The estimate of δ , the coefficient on ΔR_t , the percentage change in reserves, is -.62: a 1% drop in reserves causes the authorities to set the peg .62% higher in the event of a devaluation. This action is presumably taken to stop the reserve drain, and remove the incentive for speculative attacks. The coefficient on foreign prices is of the wrong sign, but has a standard error so large that the hypothesis that the coefficient is actually equal to its theoretically correct magnitude cannot be rejected at usual significance levels. The coefficient of λ_t is an estimate of σ_ϕ : the standard error of the exchange-rate setting rule. The estimate of σ_ϕ is .31.

For the purposes of estimation, a "devaluation event" is a devaluation of more than .5%.

Testing the importance of selectivity bias involves testing whether the coefficient on λ_t is significantly different from zero; for this hypothesis, the "uncorrected" standard error of .06 is appropriate, ¹³ leading to a failure to reject the hypothesis of no sample selection bias. Thus there is an econometrically significant bias induced by the exchange rate pegging of the the authorities. The effect of this bias is that, if one were to estimate a floating exchange rate model for Mexico during this period, one would obtain inconsistent parameter estimates.

This equation fails to identify the parameter b, althought it will be identified given estimates of the money demand equation and the processes for the exogenous variables.

Estimation of money demand parameters

The reduced-form version of the money demand equation is (substituting into (29) using equations (30)-(33)):

$$\begin{split} \mathbf{m}_{t} &= \mathbf{k} + \mathbf{e}_{t} + \mathbf{p}_{t}^{*} + \mathbf{v}z_{t} + \alpha(2\mu_{d}^{-2}\mu_{p}^{-2}\mathbf{v}\mu_{z}^{+b+k})F(\mathbf{v}_{t}^{*}) - \alpha[F(\mathbf{v}_{t}^{*})d_{t}] - \alpha[F(\mathbf{v}_{t}^{*})\mathbf{p}_{t}^{*}] \\ &- \alpha\theta[F(\mathbf{v}_{t}^{*})(\mathbf{p}_{t-1}^{*} - \mathbf{p}_{t-2}^{*})] - \alpha[F(\mathbf{v}_{t}^{*})\mathbf{e}_{t}] - \alpha\mathbf{v}[F(\mathbf{v}_{t}^{*})z_{t}] - \alpha\delta[F(\mathbf{v}_{t}^{*})\Delta R_{t}] \\ &+ \alpha\sigma^{*}[F(\mathbf{v}_{t}^{*})\lambda_{t}^{*}] + \mathbf{x}_{1t} \end{split} \tag{34}$$

Estimation of the money demand equation requires a two-stage procedure: the first stage generates the λ_t^* series needed as an independent variable in equation (34). Output from this first stage is used to construct values for expected devaluations and probabilities of devaluation, which are discussed below.

¹³Because the Heckman procedure has the "generated regressors" problem discussed by Pagan (1984a,b), standard errors from the OLS equation run in the second stage are not the correct standard errors for testing most hypotheses. However, they are correct for testing for selectivity bias.

Estimation of equation (34) produces estimates of γ, αγ, αδ, and several estimates of α . The results of estimating this equation are given in Tables 3a and 3b. 14 The dependent variable is the monetary base (MO) in Table 3a, and is M1 in Table 3b. While the estimates of y (the income elasticity of money demand) -- 2.01 for MO and 1.35 for M1--are not unreasonable, given findings for other countries and other time periods (e.g., Goldfeld (1973)), the estimates of $\alpha \gamma$, $\alpha \delta$, and the several estimates of α (the semi-elasticity of demand for money with respect to the inflation rate) vary widely and are often of implausible sign and magnitude, with the median estimate of α being about zero. Hence, the money demand equation is also estimated in the constrained form given by equation (29), imposing the appropriate estimate of y obtained from estimation of (34). (An estimate of y is needed to construct the term $[pr_{+}\{y_{++1}\geq 0\}]E_{+}[y_{++1}|y_{++1}\geq 0]$). The results of estimating equation (29) are given in Tables 4a and 4b. The estimate of α obtained from estimation of (29) MO as dependent variable was -.53, and the estimate of α using M1 as dependent variable was -.21. Finally, another constrained version of the money demand equation was estimated, imposing the previously-estimated income elasticity estimates and, in addition, imposing the theoretical coefficients of 1.0 on the variables $\mathbf{e_t}$ and $\mathbf{p_t^*}$. The results are given in Tables 5a and 5b: with MO as dependent variable, the resulting estimate of α is -.96, and with M1 as dependent variable, the estimate of α

Because some of the variables in the money demand equation are generated from earlier portions of the estimation, (for example, all variables involving $F(\hat{\Psi}_{\mathbf{t}}^*)$), the reported standard errors are too low. In the exchange rate equation, the standard errors corrected for the "generated regressors" problem are three to four times larger than the uncorrected standard errors (unadjusted errors are not reported). This provides a rough guide for adjusting the standard errors presented in Table 3 onward.

is -.45. Thus, the semi-elaticities of demand for MO and M1 appear, in the constrained regressions, to be in the range found by by other researchers for other countries.

Expectations of devaluation

The output from the first stage of the estimation of the money demand equation above can be used to calculate one-step-ahead estimates of the probability of a devaluation, together with predictions of the size of next period's devaluation, conditional or unconditional on the event of a devaluation taking place (refer to Section III for discussion of how these quantities are calculated.) Figures 1-4 exhibit these probabilities and predictions.

Figure 1 graphs devaluation "events" against the model's estimate of the one-step-ahead probability of a devaluation. The devaluation "event" is a binary variable which takes the value 1 if a devaluation occurred, and 0 otherwise. Examination of these two figures shows that the probability of devaluation rises throughout the period under study, and does not rise particularly sharply with any devaluation events except the ones in 1981-82. Figure 2 graphs the one-step-ahead probability of devaluation against the actual size of the devaluation. The probability of devaluation is highest for the largest devaluation: the one that occurred in 1982:1.

Figure 3 plots the unconditional expected devaluation against the actual devaluation, and Figure 4 plots the conditional expected devaluation against the actual devaluation. In Figure 3, the unconditional expected devaluation series exhibits peaks at dates which coincide with actual devaluations. For

the periods where no devaluations or small devaluations took place-- 1973:3 to 1976:2, and 1978:1 to 1981:4--the fact that the unconditional expectation of devaluation persistently exceeded the expected rate is evidence of growing pressure for a devaluation. In Figure 4, for the two largest devaluations (1976:3 and 1982:1) the conditional expected devaluation is of roughly the same magnitude as the actual devaluation. For the 1973:3 to 1976:2 period, the conditional expected devaluation is very high, but the corresponding probability of devaluation is very low (less than 20% throughout this period.)

Testing the model

Testing the adequacy of this model's fit involves testing whether the parameter restrictions implied by the model are rejected by the data. Many of these parameter restrictions are nonlinear, and many involve elements of the covariance matrix. An important step for future research is to devise and implement appropriate statistical tests of this type of model.

V. Adjustable-Peg Regimes and the Behavior of Forward Exchange Rates

A feature of foreign exchange markets that is the subject of many theoretical and empirical inquiries is the relationship between forward and spot exchange rates. This section examines the behavior of forward rates, forward premia, and risk premia in the context of the adjustable-peg model of Sections III and IV. Simple tests of market efficiency are also discussed.

The forward rate, f_t , for delivery in period t+1, is the sum of the expected future spot rate, $E_t e_{t+1}$, and a risk premium, RP_t :

$$f_{+} = E_{+}e_{++1} + RP_{+} . (35)$$

The model of Sections III and IV generates an estimate of $E_{t}e_{t+1}$. Figure 5 plots the one-period-ahead prediction of the spot rate, $E_{t}e_{t+1}$, against f_{t} , the market forward rate for delivery at time t+1. The two series look very much alike--apparently, the model predicts devaluations about as well as the forward market. It is perhaps surprising that the two series look so much alike, since the forward rate was not included in the estimation of the model of Section IV, either as a dependent or independent variable. The advantage of not having included a forward rate equation in that model is that the model can be (informally) tested by investigating the extent to which the model's implied forward rate matches up with the actual forward rate--in this case, they are very close. We turn next to the behavior of the forward premium. The forward premium is defined as the forward rate minus the current spot rate:

$$PP_t = f_t - e_t$$

In the adjustable-peg model, the forward premium is given by:

$$FP_{t} = E_{t}e_{t+1} + RP_{t} - e_{t}$$

$$= pr_{t}(\hat{e}_{t+1} \ge e_{t}) \left\{ E_{t}\hat{e}_{t+1} | \hat{e}_{t+1} \ge e_{t} \right\} + [1 - pr_{t}(\hat{e}_{t+1} \ge e_{t})]e_{t} + RP_{t} - e_{t}$$

$$= e_{t} + pr_{t}(y_{t+1} \ge 0)E_{t}(y_{t+1} | y_{t+1} \ge 0) + RP_{t} - e_{t}$$

$$= pr_{t}(y_{t+1} \ge 0)E_{t}(y_{t+1} | y_{t+1} \ge 0) + RP_{t}. \tag{36}$$

The forward premium for Mexico is graphed in Figure 6. Equation (36) shows that the forward premium will typically be serially correlated even in the absence of a risk premium; this reflects serial correlation in the (unconditional) expected devaluation caused by serial correlation in the model's forcing variables (e.g., money supplies and output.) It is

well-known that forward rates typically carry a large premium over spot rates when a devaluation is thought to be imminent—this phenomenon has earned the name "the peso problem." This effect is captured in equation (36). But equation (36) also shows that there will likely be a small, serially correlated forward premium whenever a government is following an adjustable—peg rule, either explicitly, or implicitly (as in the EMS, where exchange rates are fixed against other EMS countries, but are subject to revision from time to time.)

An autoregression for the forward premium is presented in Table 6: the forward premium is, as predicted, serially correlated. As discussed above, this does not constitute evidence of a time-varying risk premium. Note that the estimate of the constant term in the autoregression, which is interpretable as the average level of the risk premium, is insignificantly different from zero.

In order to investigate the existence and properties of a risk premium, the researcher would like to have a measure of the <u>ex ante</u> risk premium, defined as the current forward rate minus the current expectation of the spot rate that will prevail in the delivery period. Denote this ex ante risk premium by:

$$EARP_{t} = f_{t} - E_{t}e_{t+1} (37)$$

The expected spot rate is unobservable, however, so researchers often use the ex post risk premium in empirical work; this ex post risk premium is given by:

$$EPRP_{t} = f_{t} - e_{t+1} \tag{38}$$

The difference between the ex ante risk premium and the ex post risk premium

is the expectation error made by agents in period t in forecasting the spot rate in period t+1:

$$EPRP_{t} - EARP_{t} = E_{t}e_{t+1} - e_{t+1}$$
(39)

The model of Section III provides estimates of the expected future spot rate, E_{t^2t+1} . Thus, an estimate of the ex ante risk premium can be computed, and its properties studied. The ex ante risk premium and the ex post risk premium are graphed in Figure 7. The expectation error $E_{t^2t+1} - e_{t^2t}$ is graphed in Figure 8.

An autoregression for the ex ante risk premium, presented in Table 7, shows that the risk premium is serially uncorrelated and has a mean insignificantly different from zero. Thus, the serial correlation in the forward premium is due entirely to serial correlation in the expected rate of devaluation, and is not due to a time-varying risk premium. Without having developed an explicit rational expectations econometric model of the exchange rate, it would not have been possible to investigate separately the two components of the forward premium: the expected devaluation and the risk premium.

Testing Market Efficiency

The following regression has often been used to test efficiency of forward markets:

$$e_{t+1} = \alpha_0 + \alpha_1 f_t + u_t$$
where

$$u_{t} = E_{t}e_{t+1} - e_{t+1} - (RP_{t} - \overline{RP})$$
,

and where \overline{RP} is the mean level of the risk premium. Testing efficiency involves testing the hypothesis that $\alpha_1=1$ (α_0 provides an estimate of \overline{RP} .)

Since this is a rational expectations model, the expectation error is a zero mean random variable:

$$E(E_{t}e_{t+1}-e_{t+1}) = 0.$$

But the expectation error is not normally distributed, even if the forcing variables are all normal. This would invalidate any statistical tests which relied on normality, such as those described below.

Equation (40) was nevertheless run on the Mexican data; the results are given in Table 8. The point estimate of α_1 is .99, with a standard error of .06, thus, we fail to reject the hypothesis of efficiency: α_1 =1. The R² for this equation is .90.

To investigate whether the model's prediction of the future spot rate was an unbiased predictor of the realized future spot rate, the following regression was run:

$$e_{t+1} = \alpha_0 + \alpha_1 E_t e_{t+1} + u_t$$
 (41)

The results are presented in Table 9: the resulting estimate of α_1 was 1.01, with a standard error of .04. Thus, the model's predictions appear unbiased. The R^2 for this regression is .95.

Efficiency tests are sometimes performed in differenced form, to avoid "taking credit for getting the level right":

$$e_{t+1} - e_t = \alpha_0 + \alpha_1 (f_t - e_t) + u_t$$
 (42)

Equation (42) was estimated, and the results are presented in Table 10. The point estimate for α_1 is 3.27, with a standard error of .78. Since the true standard error is higher, it is possible that the data marginally fails to reject the hypothesis that α_1 =1. The R² for this equation is .34.

Finally, the following regression was run, to examine whether the model's predictions of exchange rate changes is unbiased:

$$e_{t+1}^{-}e_{t}^{-}=\alpha_{0}^{+}+\alpha_{1}^{(E_{t}}e_{t+1}^{-}e_{t}^{-})+u_{t}^{-}$$
 (43)

Table 11 contains the results of this regression: the estimate of α_1 is .85, with a standard error of .12. Thus, the model's predictions of devaluations appear to be unbiased.

One prediction of the model developed in this paper is that the extent of serial correlation in forward premia depends on the degree of exchange rate "management", as well as on the serial correlation properties of such variables as the money stock, output, and foreign prices. Thus, the model has strong, testable implications for cross-country differences in serial correlation of forward premia, under the assumption that risk premia behave similarly across countries, or are absent. A related paper (Baxter (1987b)) investigates these implications using data from a sample of several countries which differ in their degree of exchange rate management.

VI. Conclusions

In this paper, methods for estimating rational expectations models have been extended to include models in which some of the variables are censored. The novel feature of the model is that private agents' decision rules depend on expectations of future values of the censored variables. Decision rules that are linear functions of the expected future value of the censored variables turn out to be nonlinear functions of the model's exogenous variables. Using Heckman's perspective that censoring amounts to sample selection bias, it is shown that standard econometric techniques applied to this model will yield inconsistent parameter estimates. Appropriate econometric techniques are derived for this problem, and are illustrated by application to Mexico's recent adjustable-peg exchange rate regime. The

Heckman estimation procedure has the advantage of highlighting the symmetry of agents' expectations on the one hand, and the procedure followed by the econometrician on the other. The model's reduced form exhibits the cross-equation parameter restrictions typical of rational expectations econometric models. In addition, extra restrictions arise from the similarity between private agents' and the econometricians' forecasts of the censored variable.

In the application studied here, it was found that the selectivity bias induced by the exchange rate pegging behavior of the authorities was econometrically important. Thus, application of standard exchange rate models for fixed or floating rate regime would yield inconsistent parameter estimates, leading to incorrect inference concerning effects of other economic variables on exchange rates.

A striking implication of the adjustable-peg model is that the forward premium will typically be serially correlated even in the absence of a risk premium, reflecting serial correlation in expected devaluations caused by serial correlation in the model's forcing variables. Having developed methods for solving and estimating models with censored variables, it is now possible to undertake a rigorous econometric analysis of the so-called "peso problem." In addition, it is possible to construct estimates of the ex ante risk premium, and to study directly its statistical properties. In the Mexican case, although the forward premium is highly serially correlated, there is no evidence of a time-varying risk premium.

The adjustable-peg model and its estimation are intended as a first illustration of the methods developed in this paper. The theoretical model

presented in Section III is highly simplified, and in futher applications it would be desirable to extend the model in the direction of greater realism.

In order to develop our understanding of the workings of adjustable-peg exchange rate regimes, it is necessary to study adjustable-peg regimes implemented in other countries and in other time periods.

But the methods developed here are applicable to a far greater range of problems. Several potential applications were mentioned in the introduction: farm price support programs, discount rate setting by the Fed, and dividend setting by firms. Another application in the area of international trade is to world markets in a commodity where import (or export) restrictions bind only part of the time. ¹⁵

There is clearly no shortage of problems where censoring or selectivity is thought to be important, and many of these problems (especially in the area of labor economics) have been extensively studied. The chief contribution of the current paper is that it represents a first step toward integration of the literature on limited variables and the literature on rational expectations. The methods developed here extends the class of econometrically tractable rational expectations models to include models in which individuals' decisions depend on their expectations of the future value of a limited variable.

 $^{^{15}{}m This}$ application was suggested by Adrian Pagan.

Table 1
Stochastic processes for exogenous variables

Domestic credit:

Foreign price level:

$$p_{t}^{*} - p_{t-1}^{*} = \mu_{p}^{+} + \theta(p_{t-1}^{*} - p_{t-2}^{*}) + x_{3t}$$

$$\frac{\hat{\mu}_{p}}{-} \qquad \hat{\theta} \qquad \hat{\sigma}_{3}^{-}$$
estimate | .0117 | .3867 | .0054 | .3867 | .0054 | .1603)

Output:

$$z_t - z_{t-1} = \mu_z + x_{4t}$$
 (32)

Table 2
Estimation of exchange rate rule

$$y_t = a_0 + a_1 e_{t-1} + a_2 d_t + a_3 z_t + a_4 p_t^* + a_5 (\Delta R_t) + a_6 \hat{\lambda}_t + \xi_{1t}$$

The series $\boldsymbol{\hat{\lambda}}_t$ was obtained from the first step of the Heckman procedure.

VARIABLE	THEORETICAL COEFFICIENT	ESTIMATED COEFFICIENT	STANDARD ERROR
constant	$(\mu_{\rm d} - \mu_{\rm p} - \gamma \mu_{\rm z} + b + k)$	46.98	22.21
e _{t-1}	-1.0	-1.28	.40
a _t	1.0	1.43	1.31
zt	- y	-5.90	3.54
p*	-1.0	2.00	6.87
ΔRt	6	62	.48
λ̂t	σ	.31	. 20

Note: theory predicts $\gamma>0$, $\delta<0$, and $\sigma>0$.

Number of observations: 13 Degrees of freedom: 6

 $R^2 = .98 \qquad \overline{R}^2 = .96$

Durbin-Watson statistic: 2.21

Table 3a

Estimation of unrestricted money demand equation
MO as dependent variable

$$\begin{split} \text{MO}_{\texttt{t}} &= \text{a}_0 + \text{a}_1 \text{e}_{\texttt{t}} + \text{a}_2 \text{p}_{\texttt{t}}^* + \text{a}_3 \text{z}_{\texttt{t}} + \text{a}_4 \text{F}(\hat{\gamma}_{\texttt{t}}^*) + \text{a}_5 [\text{F}(\hat{\gamma}_{\texttt{t}}^*) \text{d}_{\texttt{t}}] + \text{a}_6 [\text{F}(\hat{\gamma}_{\texttt{t}}^*) \text{p}_{\texttt{t}}^*] \\ &+ \text{a}_7 [\text{F}(\hat{\gamma}_{\texttt{t}}^*) (\text{p}_{\texttt{t}-1}^* - \text{p}_{\texttt{t}-2}^*)] + \text{a}_8 [\text{F}(\hat{\gamma}_{\texttt{t}}^*) \text{e}_{\texttt{t}}] + \text{a}_9 [\text{F}(\hat{\gamma}_{\texttt{t}}^*) \text{z}_{\texttt{t}}] + \text{a}_{10} [\text{F}(\hat{\gamma}_{\texttt{t}}^*) \Delta \text{R}_{\texttt{t}}] \\ &+ \text{a}_{11} [\text{F}(\hat{\gamma}_{\texttt{t}}^*) \lambda_{\texttt{t}}^*] + \text{x}_{1t} \end{split}$$

THEORETICAL VARIABLE COEFFICIENT		ESTIMATED COEFFICIENT	STANDARD ERROR
constant	k	-13.48	23.31
e _t	1.0	. 34	.31
p*	1.0	06	1.61
z t	Y	2.01	2.32
F(Ψ̂*)	$\alpha(2\mu_{\mathrm{d}}^{-2}\mu_{\mathrm{p}}^{-2}\nu_{\mathrm{z}}^{+\mathrm{b+k}})$	20.26	59.7 5
F(Υ̂*)d _t	-α	63	1.48
F(Ψ̂*)p*	α	11.70	8.55
F(Ŷ*)(p* _{t-1} - ₁	p* _{t-2}) αθ	. 96	12.92
F(Ŷ*)e _t	α	03	1.29
F(Ŷ*)z _t	αγ	-5.91	6.52
F(Ŷ*)ΔR _t	αδ	12	.74
F(Ŷ*)λ̂*	-ασ*	1.45	1.85

Note: Theory predicts $\alpha<0$, $\gamma>0$, $\delta<0$.

Number of observations: 35 Degrees of freedom: 23

 $R^2 = .97 \qquad R^2 = .95$

Durbin-Watson statistic: 1.90

Table 3b

Estimation of unrestricted money demand equation
M1 as independent variable

The equation estimated was:

$$\begin{split} \text{M1}_{\text{t}} &= \text{a}_{0} + \text{a}_{1} \text{e}_{\text{t}} + \text{a}_{2} \text{p}_{\text{t}}^{*} + \text{a}_{3} \text{z}_{\text{t}} + \text{a}_{4} \text{F}(\hat{\gamma}_{\text{t}}^{*}) + \text{a}_{5} [\text{F}(\hat{\gamma}_{\text{t}}^{*}) \text{d}_{\text{t}}] + \text{a}_{6} [\text{F}(\hat{\gamma}_{\text{t}}^{*}) \text{p}_{\text{t}}^{*}] \\ &+ \text{a}_{7} [\text{F}(\hat{\gamma}_{\text{t}}^{*}) (\text{p}_{\text{t-1}}^{*} - \text{p}_{\text{t-2}}^{*})] + \text{a}_{8} [\text{F}(\hat{\gamma}_{\text{t}}^{*}) \text{e}_{\text{t}}] + \text{a}_{9} [\text{F}(\hat{\gamma}_{\text{t}}^{*}) \text{z}_{\text{t}}] + \text{a}_{10} [\text{F}(\hat{\gamma}_{\text{t}}^{*}) \Delta R_{\text{t}}] \\ &+ \text{a}_{11} [\text{F}(\hat{\gamma}_{\text{t}}^{*}) \hat{\lambda}_{\text{t}}^{*}] + \text{x}_{1t} \end{split}$$

VARIABLE	THEORETICAL COEFFICIENT	ESTIMATED COEFFICIENT	STANDARD ERROR	
constant	k	-10.94	13.74	
e _t	1.0	.21	.19	
z t	Y	1.35	1.39	
p* t	1.0	1.18	.98	
F(Υ̂*)	$\alpha(2\mu_{\mathbf{d}}^{-2}\mu_{\mathbf{p}}^{-2})\mu_{\mathbf{z}}^{+b+k}$	18.68	35.41	
F(Ŷ*)d _t	-α	.02	. 90	
F(γ̂*)p*	α	7.08	5.19	
F(γ̂*)(p*	p* _{t-2}) αθ	-2.56	7.53	
F(Ŷ*)e	α	16	.78	
F(ψ̂*)z _t	αγ	-4.48	3.95	
F(γ̂*)ΔR _t	αδ	.06	.44	
F(Ŷ*)λ*	-ασ*	1.00	1.06	

Note: Theory predicts $\alpha < 0$, $\gamma > 0$, $\delta < 0$.

Number of observations: 36 Degrees of freedom: 24

 $R^2 = .99$ $R^2 = .99$

Durbin-Watson statistic: 1.91

Table 4a

Estimation of a restricted money demand equation
MO as dependent variable

$$MO_{t} = a_{0} + a_{1}e_{t} + a_{2}p_{t}^{*} + 2.01z_{t} + a_{3}[pr_{t}\{y_{t+1} \ge 0\}]E_{t}[y_{t+1}|y_{t+1} \ge 0] + x_{1t}$$

The value y=2.01 obtained in estimation of the unrestricted money demand equation (see Table 3a) was used here as the coefficient of z_t and in the construction of the variable $[pr_t\{y_{t+1}\geq 0\}]E_t[y_{t+1}|y_{t+1}\geq 0]$; this variable is called EXPDEV in the table. Theory predicts $a_3=\alpha<0$.

VARIABLE	THEORETICAL COEFFICIENT	ESTIMATED COEFFICIENT	STANDARD ERROR
constant	k	-19.16	. 83
e _t	1.0	.31	. 13
p*	1.0	1.15	. 23
EXPDEV	α	54	. 20

Number of observations: 35 Degrees of freedom: 31

 R^2 : .96 \overline{R}^2 : .96

Durbin-Watson statistic: 2.06

Table 4b

Estimation of restricted money demand equation
M1 as dependent variable

$$\texttt{M1}_{\texttt{t}} = \texttt{a}_0 + \texttt{a}_1 \texttt{e}_{\texttt{t}} + \texttt{a}_2 \texttt{p}_{\texttt{t}}^* + \texttt{1.35z}_{\texttt{t}} + \texttt{a}_3 [\texttt{pr}_{\texttt{t}} \{ \texttt{y}_{\texttt{t+1}} \geq \texttt{0} \}] \texttt{E}_{\texttt{t}} [\texttt{y}_{\texttt{t+1}} | \texttt{y}_{\texttt{t+1}} \geq \texttt{0}] + \texttt{x}_{\texttt{1t}}$$

The value y=1.35 obtained in estimation of the unrestricted money demand equation (see Table 3b) was used here as the coefficient of z_t and in the construction of the variable $[pr_t\{y_{t+1}\geq 0\}]E_t[y_{t+1}|y_{t+1}\geq 0]$; this variable is called EXPDEV in the table. Theory predicts $a_2 = \alpha < 0$.

VARIABLE	THEORETICAL COEFFICIENT	ESTIMATED COEFFICIENT	STANDARD ERROR
constant	k	-15.54	.58
e _t	1.0	. 15	.08
p*	1.0	2.19	.15
EXPDEV	α	21	.14

Number of observations: 36 Degrees of freedom: 32

 R^2 : .98, \overline{R}^2 : .98

Durbin-Watson statistic: 1.99

Table 5a

Estimation of restricted money demand equation MO as independent variable

The equation estimated was:

$$MO_{t} = a_{0} + e_{t} + p_{t}^{*} + 2.01z_{t} + a_{1}[pr_{t}(y_{t+1} \ge 0)]E_{t}[y_{t+1}|y_{t+1} \ge 0] + x_{1t}$$

The difference between this equation and that of Table 4a is that the variables e_t and p_t^* are constrained to have their theoretically correct values. The value y=2.01 obtained in estimation of the unrestricted money demand equation was used here as the coefficient of z_t and in the construction of the variable $[pr_t\{y_{t+1}\geq 0\}]E_t[y_{t+1}|y_{t+1}\geq 0]$; this variable is called EXPDEV in the table. Theory predicts $a_1=\alpha<0$.

VARIABLE	THEORETICAL COEFFICIENT	ESTIMATED COEFFICIENT	STANDARD ERROR
constant	k	-20.40	.04
EXPDEV	α	96	.37

Number of observations: 35 Degrees of freedom: 33

 R^2 : .87, \overline{R}^2 : .86

Durbin-Watson statistic: .65

Table 5b

Estimation of restricted money demand equation M1 as independent variable

The equation estimated was:

$$m_t = a_0 + e_t + p_t^* + 1.35z_t + a_1[pr_t\{y_{t+1} \ge 0\}]E_t[y_{t+1}|y_{t+1} \ge 0] + x_{1t}$$

The difference between this equation and that of Table 4b is that the variables e_t and p_t^* are constrained to have their theoretically correct values. The value y=1.35 obtained in estimation of the unrestricted money demand equation was used here as the coefficient of z_t and in the construction of the variable $[pr_t\{y_{t+1}\geq 0\}]E_t[y_{t+1}|y_{t+1}\geq 0]$; this variable is called EXPDEV in the table. Theory predicts $a_1 = \alpha < 0$.

VARIABLE	THEORETICAL COEFFICIENT	ESTIMATED COEFFICIENT	STANDARD ERROR
constant	k	-12.09	. 03
EXPDEV	α	45	. 28

Number of observations: 36 Degrees of freedom: 34

 R^2 : .94, \overline{R}^2 : .93

Durbin-Watson statistic: .82

Table 6 Autoregression for the forward premium

(standard errors are in parentheses)

$$FP_{t} = .041 + .633 FP_{t-1} - .030 FP_{t-2} - .570 FP_{t-3} + u_{t}$$

Number of observations: 34 Degrees of freedom: 27

 R^2 : .28 \overline{R}^2 : .12 Durbin-Watson: 1.87

Table 7

Statistical properties of the risk premia: Autoregression for ex ante risk premium,

calculated from model's estimated exchange rate rule

(standard errors are in parentheses)

$$EARP_{t} = .015 + .218 EARP_{t-1} - .244 EARP_{t-2} + .234 EARP_{t-3} + u_{t}$$

$$(.021) (.193) (.193) (.198)$$

Number of observations: 32 Degrees of freedom: 27

 R^2 : .098 \overline{R}^2 : -.035 Durbin-Watson: 1.98

Table 8 Testing efficiency of forward market in levels

(standard errors in parentheses):

$$e_{t+1} = .03 + .99 f_t + u_t$$

number of observations: degrees of freedom: 33

 R^2 : .90 \bar{R}^2 : .89 Durbin-Watson: 2.00

Table 9

Testing unbiasedness of model's prediction of future spot rate:
Regression of realized future spot rate on expected future spot rate
computed from model

(standard errors in parentheses):

$$e_{t+1} = -.04 + 1.01 E_{t} e_{t+1} + u_{t}$$

number of observations: 35 degrees of freedom: 33

 R^2 : .95 \overline{R}^2 : .95

Table 10 Testing efficiency of forward market in differences

(standard errors in parentheses):

$$(e_{t+1}-e_t) = -.05 + 3.27 (f_t-e_t) + u_t$$

number of observations: 36 degrees of freedom: 34

 R^2 : .34 R^2 : .32 Durbin-Watson: 1.61

Table 11

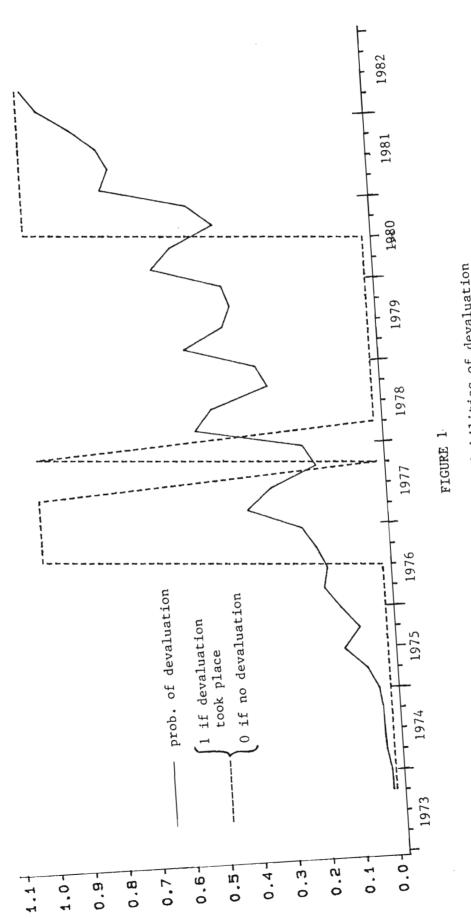
Testing unbiasedness of model's prediction of devaluations: Regression of realized devaluations on expected devaluations computed from model

(standard errors in parentheses):

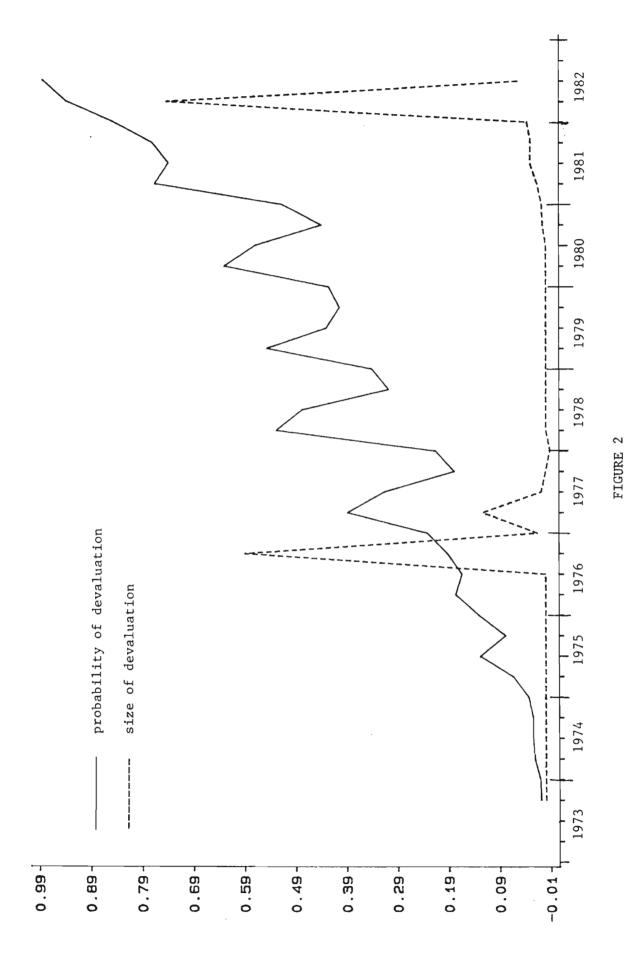
$$(e_{t+1}^{-e} - e_t) = -.0001 + .85 (E_t^{e} e_{t+1}^{-e} - e_t) + u_t$$

number of observations: 35 degrees of freedom: 33

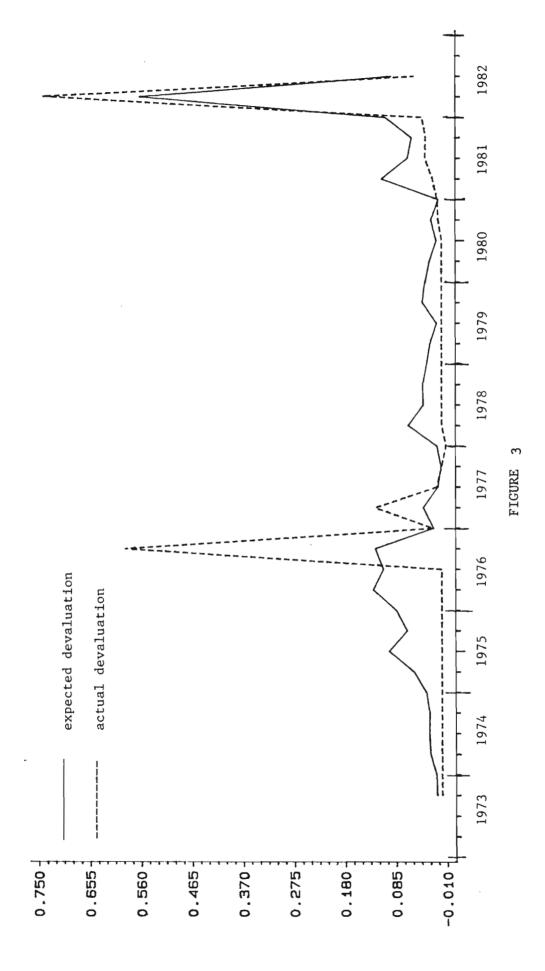
 R^2 : .60 \overline{R}^2 : .59



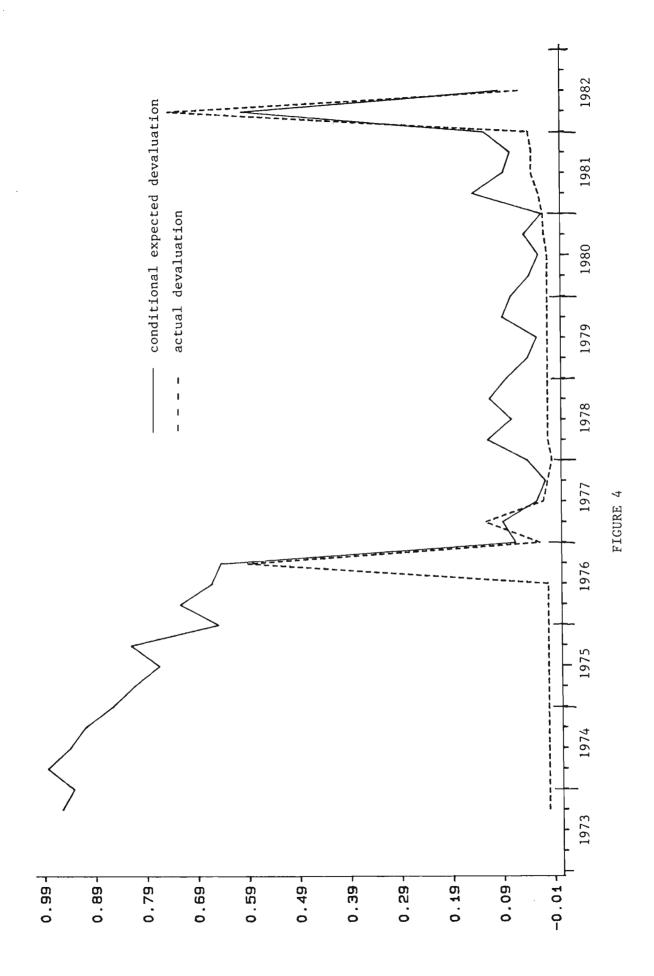
Devaluation "events" and probabilities of devaluation

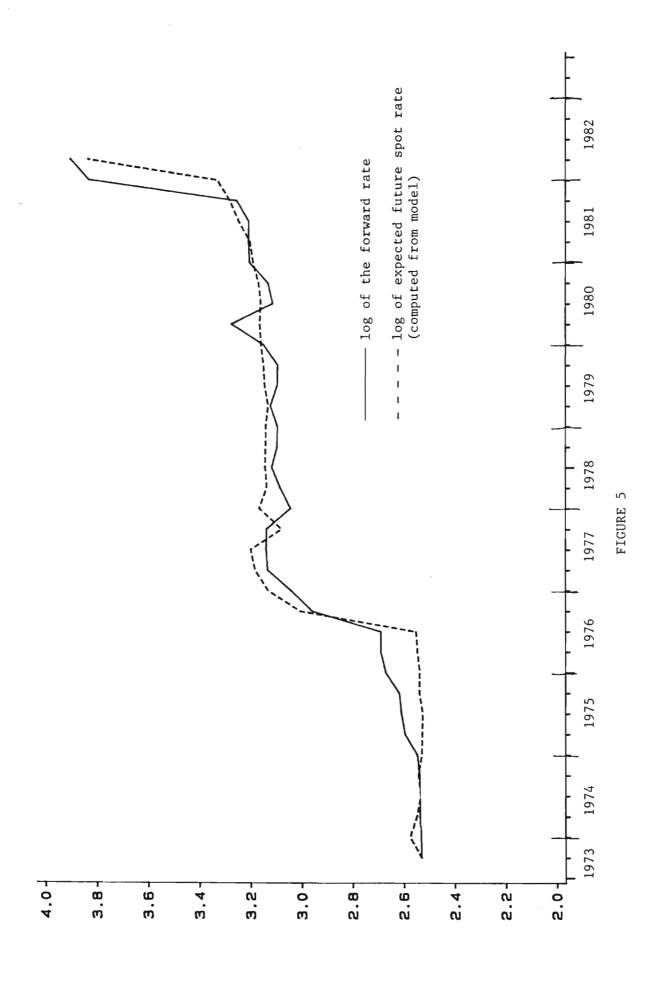


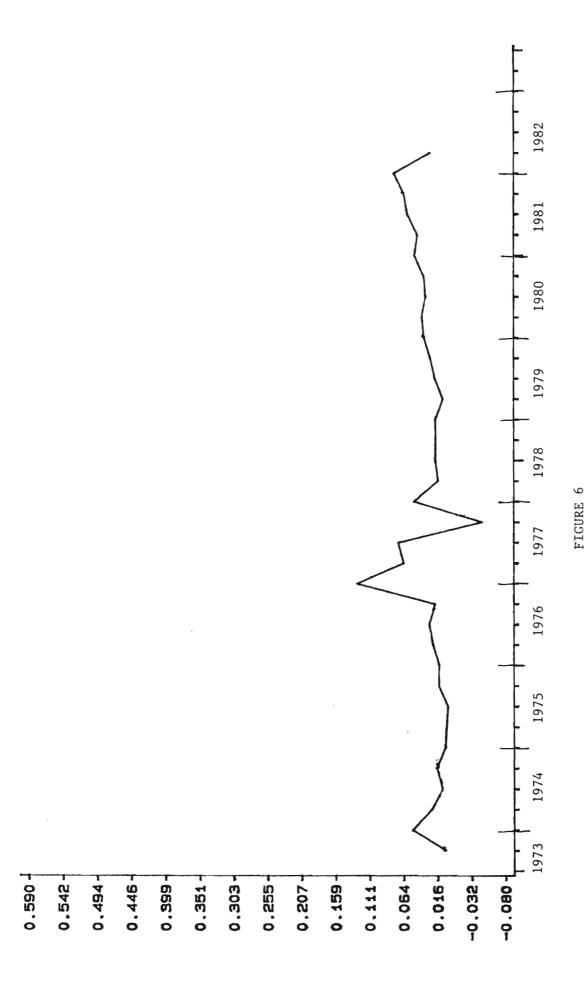
Size of devaluation and probability of devaluation



Expected and actual devaluations







The forward premium

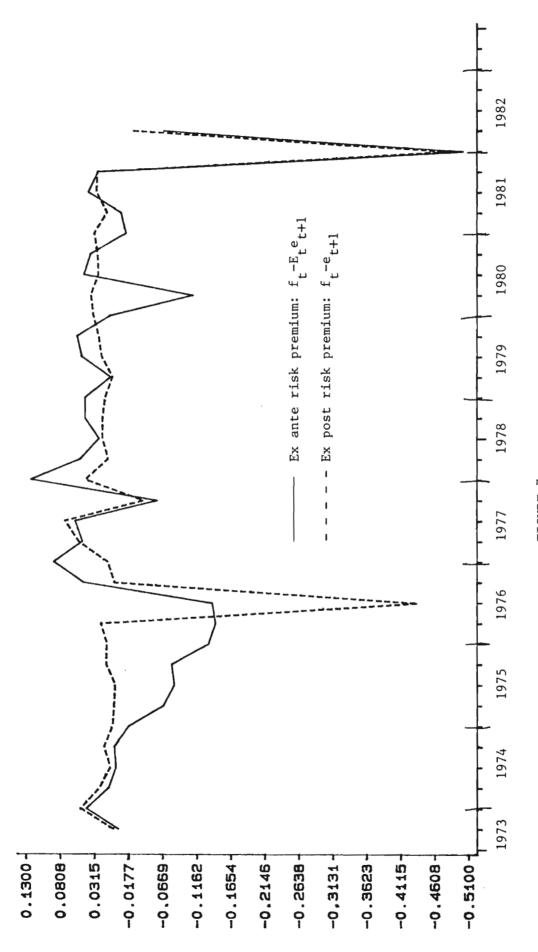
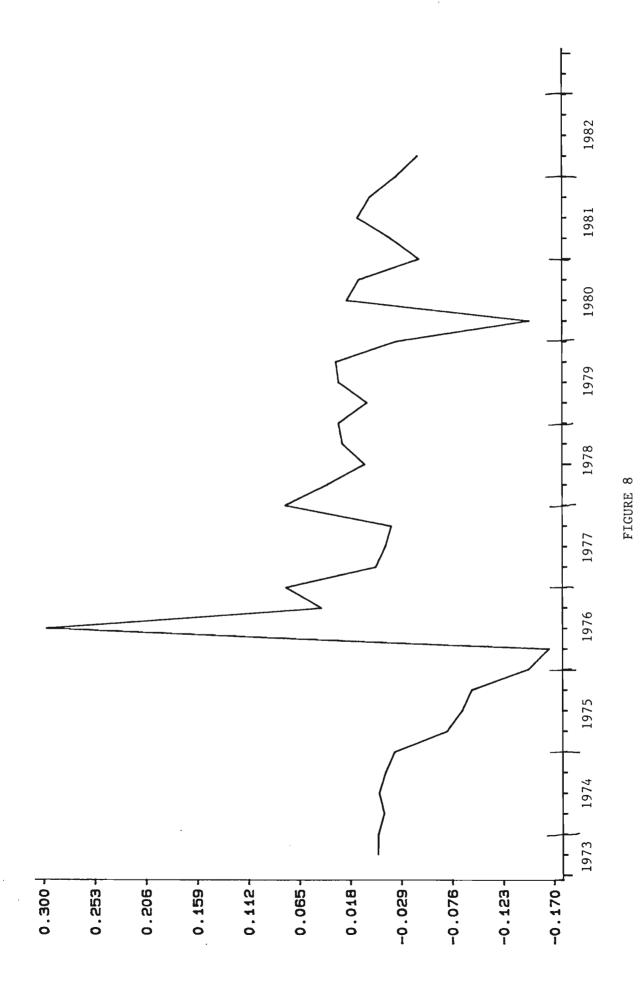


FIGURE 7



Expectation error: $e_{t+1} - E_t e_{t+1}$

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