Legal Restrictions, "Sunspots," and Cycles

Smith, Bruce

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Bruce D. Smith

University of Western Ontario
and
Rochester Center for Economic Research

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Introduction

This paper is related to topics that are discussed in three different sets of literature. The first of these is the literature on the relation between economies that display stationary sunspot equilibria and economies that display equilibria with 2 period, deterministic cycles (e.g., Azariadis-Guesnerie (1987) or Grandmont (1986)). The second is the literature discussing "legal restrictions" on asset trades, and the kinds of monetary equilibria that can arise when such legal restrictions are imposed (e.g., Bryant-Wallace (1984)). The third is the literature on whether it is desirable to "separate" money from credit markets via legal restrictions in order to prevent "excessive" fluctuations in the price level, or even price level indeterminacy (e.g., Friedman (1960), Sargent-Wallace (1982), Smith (1987)).

The objective of the paper is to demonstrate that legal restrictions meant to prevent private agents from creating "close money substitutes", of a form that is commonly observed in practice, can substantially enhance the scope for stationary sunspot equilibria to exist. Or, phrased differently, the paper demonstrates that economies exist in which there are no stationary sunspot equilibria absent legal restrictions. However, when legal restrictions intended to separate money from credit markets are imposed, stationary sunspot equilibria can be constructed if the legal restrictions take the proper form.

In some sense such a result should not be surprising, since legal restrictions on asset trades are simply a device for limiting the participation of economic agents in different markets. It is this type of limited market participation that permits sunspot equilibria to arise, as emphasized by Cass and Shell (1983). However, the result still seems that dominate it in rate of return will result in the existence of sunspot equilibria, as well as deterministic equilibria of the form that Bryant and Wallace consider. Hence such legal restrictions need not have exactly their intended effects.[3]

Third, the result that legal restrictions meant to separate money from credit markets (that is, meant to prevent private agents from creating close money substitutes) can cause sunspot equilibria to exist is very relevant to an old debate in monetary economics. In particular, economists of what Sargent and Wallace (1982) term the "quantity theory school" have long argued that a failure to prevent private agents from issuing close money substitutes would result in "excessive" fluctuations in the price level and the stock of inside money, and perhaps in an indeterminate price level.

One example of a legal restriction meant to prevent the creation of too close a substitute for money is the prevention of banks from issuing liabilities bearing competitive interest rates in amounts less than $100,000. This paper examines the consequences of exactly this legal restriction in a model that, absent legal restrictions, has no stationary sunspot equilibria. It is shown that, if the minimum denomination is selected appropriately, stationary sunspot equilibria will exist when the legal restriction is imposed. In this instance, then, following the quantity theory proposal with respect to separating money from credit markets opens the economy to "excessive" fluctuations in the price level and the stock of inside money, as well as to price level indeterminacy (even in stationary equilibria).[4]

The scheme of the paper is as follows. Section I lays out the economic environment to be considered, and examines its equilibria in the absence of legal restrictions. Section II introduces a legal restriction that

The nature of trading is as follows. The initial old agents are endowed with a fixed stock of fiat money. Let $M = 1$ be the (constant) per capita stock of money. Then at each date old agents can buy goods with money, and young agents can engage in borrowing and lending. As is conventional in the sunspot equilibrium literature, there are no markets in state contingent claims.

Notice that nothing in the environment of this economy is stochastic. Nevertheless, it is assumed that there are 2 possible states of nature that can occur at each date. Let e_t denote the current period state at t. Then $e_t \in \{1,2\}$. Moreover, it is assumed that the state evolves according to a stationary Markov chain denoted as follows: let $\text{prob.}[e_{t+1}=1:e_t=e] = q(e)$; $e = 1,2$.

Having described the environment, it is now possible to lay out the notation employed. Under the assumptions above, young savers can either sell goods in exchange for currency, or lend to young borrowers. Let p_t denote the price level at t, let z_t denote the per capita accumulation of real balances by young savers at t, and let x_t be the amount lent by a representative young saver at t. One unit lent at t repays R_t units at t+1; i.e., R_t is the gross (real) rate of interest. Also, let L_t be the amount borrowed by each borrower at t. Finally, when we wish to focus on sunspot equilibria we will let these variables depend on the current period state and use the following notation: $p_t(e)$ will be the time t price level if the time t state is e, etc.

Equilibrium

Young savers at t solve the problem[6]

$$\max \ln(y - z_t - x_t) + \ln[R_t x_t + (p_t/p_{t+1})z_t]$$

Collapsing (1)-(3) into a single equation yields the equilibrium law of motion for μ_t:

$$(4) \quad \mu_{t+1} = 2(1-\lambda)w\mu_t/\lambda y(1-\mu_t); \; t \geq 0$$

There are 2 steady state equilibrium values for μ; $\mu_t = 0 \;\forall\; t$ and $\mu_t = \hat{\mu} \equiv 1 - 2\left(\frac{1-\lambda}{\lambda}\right)\left(\frac{w}{y}\right) \;\forall\; t$. Also

$$(5) \quad \frac{d\mu_{t+1}}{d\mu_t} = 2\left(\frac{1-\lambda}{\lambda}\right)\left(\frac{w}{y}\right)\left(\frac{1}{1-\mu_t}\right)^2$$

so that

$$(6) \quad \left.\frac{d\mu_{t+1}}{d\mu_t}\right|_{\mu_t=\hat{\mu}} = \frac{1}{2\left(\frac{1-\lambda}{\lambda}\right)\left(\frac{w}{y}\right)} > 1,$$

by the assumption of footnote 6. Then the steady state equilibrium with $p < \infty$ is locally unstable.

It is probably apparent that this economy has no stationary sunspot equilibrium. However, since this heterogeneous agent, multiple asset environment does not fit into existing discussions of when stationary sunspot equilibria exist, appendix A demonstrates formally that this economy does not possess a stationary sunspot equilibrium.

II. A Legal Restrictions Regime[8]

The following legal restriction on asset trades is now imposed on the economy of section I: loans can be made only in dollar denominations greater than or equal to $\$\bar{x}$. This restriction is meant to prevent private agents from issuing liabilities that are too close a substitute for money, Finally, since savers can voluntarily choose to make loans or hold money, but cannot do both, any saver must be indifferent between the two. A saver who holds money has $z_t = y/2$, $c_1 = y - z_t = y/2$, and $c_2 = (p_t/p_{t+1})z_t = (p_t/p_{t+1})(y/2)$. These decisions yield the utility level $\ln[(p_t/p_{t+1})(y/2)^2]$. On the other hand, a saver who makes loans has $c_1 = y-(\bar{x}/p_t)$ and $c_2 = R_t(\bar{x}/p_t)$. This results in utility $\ln[y - (\bar{x}/p_t)] + \ln[R_t(\bar{x}/p_t)]$. Then, since savers must be indifferent between making loans and holding money,

$$\ln[(p_t/p_{t+1})(y/2)^2] = \ln[y-(\bar{x}/p_t)] + \ln(R_t\bar{x}/p_t) \tag{9}$$

must hold $\forall$ t if money is to have value.

An equilibrium is a sequence of prices $\{p_t\}_{t=0}^{\infty}$, interest rates $\{R_t\}_{t=0}^{\infty}$, and values of μ_t, $\{\mu_t\}_{t=0}^{\infty}$, that satisfy (1)-(3) for all t such that $\bar{x}/p_t \leq y/2$, and that satisfy (7)-(9) for all t such that $\bar{x}/p_t > y/2$.

Equilibrium

Equations (7)-(9) can be collapsed into the single condition

$$\ln[(\mu_{t+1}/\mu_t)(y/2)^2] = \ln[y - \bar{x}\lambda(y/2)\mu_t] + \ln[(\frac{1-\lambda}{\lambda})(\frac{w}{1-\mu_t})]$$

which in turn can be written as

$$\mu_{t+1} = \phi\mu_t[\frac{1 - \bar{x}(\lambda/2)\mu_t}{1 - \mu_t}], \tag{10}$$

Then it is easy to check that $T(\mu)$ is a continuous function (and in particular is single valued) on $[0, 2(\bar{x}\lambda)^{-1}]$, and that $T(\mu)$ is continuously differentiable on $(0, 2(\bar{x}\lambda)^{-1})$ except at $\mu = (\bar{x}\lambda)^{-1}$. Moreover, the equilibrium law of motion for μ is given by

$$(14) \quad \mu_{t+1} = T(\mu_t); \; t \geq 0.$$

Steady State Equilibria

Suppose that there is a steady state equilibrium level of μ, μ^*, such that the legal restriction is binding; i.e., such that $\bar{x}\lambda\mu^* > 1$. Then, solving (10) for μ^* yields

$$(15) \quad \mu^* = \frac{\phi - 1}{\phi\bar{x}(\lambda/2) - 1}.$$

In Section III, it will be shown that stationary sunspot equilibria exist only if $\bar{x}(\lambda/2) > 1$ holds. This condition is henceforth assumed. Then, given that $\bar{x}(\lambda/2) > 1$, μ^* satifies $0 < \mu^* < 1$ iff $\phi > 1$, so that $\phi > 1$ is henceforth assumed.[10] (The assumption of footnote 6 implies that $\phi < 2$.) Finally, in the construction of μ^*, it was assumed that $\bar{x}\lambda\mu^* > 1$. Using (15), this condition is satisfied if

$$(16) \quad \frac{1}{2-\phi} > \bar{x}(\lambda/2).$$

If $\phi > 1$ and $(2-\phi)^{-1} > \bar{x}(\lambda/2) > 1$ hold, then, there is a steady state equilibrium in which the legal restriction is binding. Also, $\mu_t = 0$

From (17), $G'(\mu^*) > 0$ is equivalent to the condition $H(\mu^*) > -\mu^*H'(\mu^*)$. It is straightforward but tedious to verify that $H(\mu^*) > -\mu^*H'(\mu^*)$ if $\bar{x}(\lambda/2) > 1/\phi(2 - \phi) \geq 1$. Thus a locally stable steady state equilibrium exists if

$$(18) \quad \frac{1}{2-\phi} > \bar{x}(\lambda/2) > \frac{1}{\phi(2-\phi)},$$

where the first inequality in (18) is simply condition (16), and where it will be recalled that $\phi > 1$.

Non-existence of 2 period cycles

It is now demonstrated that, if the legal restriction $\bar{x}$ satisfies (18), there are no 2 period cycles for this economy. This is shown in two steps. First, it is shown that satisfaction of (18) implies that $T(\mu)$ is a nondecreasing function for $\mu\epsilon[0,\mu^*]$. Then it will be apparent that this economy has no 2 period cycle.

Lemma. If (18) holds then $T(\mu)$ is non-decreasing for $\mu\epsilon[0,\mu^*]$, where μ^* is the steady state equilibrium value given by equation (15).

Proof. Clearly for $\mu\epsilon[0,(\bar{x}\lambda)^{-1}]$, $T(\mu)$ is an increasing function. For $\mu\epsilon((\bar{x}\lambda)^{-1},2(\bar{x}\lambda)^{-1})$, $T'(\mu)$ exists and is a continuous function of μ. Then if $T'(\mu) \geq 0$ for all $\mu\epsilon((\bar{x}\lambda)^{-1}, \mu^*]$ the lemma is proved.

If (18) holds, then $T'(\mu^*) = \phi G'(\mu^*) > 0$. Moreover,

$$(19) \quad \lim_{\substack{\epsilon\to 0\\ \epsilon>0}} T'[(\bar{x}\lambda)^{-1} + \epsilon] = \left(\frac{\phi}{2}\right) \frac{\bar{x}\lambda}{(\bar{x}\lambda-1)^2} > 0.$$

Proposition. There is no equilibrium displaying a 2 period cycle; i.e., there are no values μ_1 and μ_2 satisfying $\mu_1 \neq \mu_2$ and $\mu_2 = T(\mu_1)$, $\mu_1 = T(\mu_2)$.

III. Stationary Sunspot Equilibria

We now construct stationary sunspot equilibria in which the legal restriction binds in each possible state; i.e., in which $\bar{x}/p(e) > y/2$; $e = 1,2$. This equilibrium is constructed under the assumption that any individual saver either accumulates money or makes loans, but does not do both. After the equilibrium is constructed it is then verified that no agent wishes to make loans and hold money simultaneously.

Suppose each individual saver either holds money or makes loans, but does not do both. Then young savers who are born in state e will accumulate real balances equal to $z(e) = y/2$; $e = 1,2$ if they accumulate money, or will make loans with a real value of $\bar{x}/p(e) > y/2$. Let $\mu(e)$ denote the fraction of savers who hold money in state e. Then money market clearing requires that

$$(21)\quad \lambda\mu(e)(y/2) = \frac{1}{p(e)};\ e = 1,2.$$

Since per capita loan demand is $(1-\lambda)w/R(e)$ in state e, loan market clearing requires that

$$(22)\quad \lambda[1-\mu(e)][\frac{\bar{x}}{p(e)}] = (1-\lambda)\ \frac{w}{R(e)};\ e = 1,2.$$

Finally, as before, savers must be indifferent between making loans and holding money. A young saver who lends $\bar{x}/p(e)$ when young receives

$$(25) \quad q(1) = \frac{\ln\{\phi H[\mu(1)][\frac{\mu(1)}{\mu(2)}]\}}{\ln[\frac{\mu(1)}{\mu(2)}]}$$

and

$$(26) \quad q(2) = \frac{\ln\{\phi H[\mu(2)]\}}{\ln[\frac{\mu(1)}{\mu(2)}]}$$

Remarks. Equations (25) and (26) are obtained by solving (23) for q(1) and q(2). It will be recalled that

$$H(\mu) \equiv \frac{1 - \bar{x}(\lambda/2)\mu}{1 - \mu}$$

Also, it will be noted that the theorem assets that (18), which is sufficient to rule out the existence of 2 period cycles in equilibrium, is also sufficient to guarantee the existence of stationary sunspot equilibria.

The theorem will be proved by construction. To begin, choose $\mu(1)$ and $\mu(2)$ satisfying $\mu(2) > \mu^* > \mu(1)$, where it will be recalled that μ^* is the steady state equilibrium value of μ when the legal restriction is binding. Since under the parameter restrictions of section II μ^* satisfies $1 < \bar{x}\lambda\mu^* < 2$, $\mu(1)$ and $\mu(2)$ satisfy (24) if they are chosen sufficiently close to μ^*. Thus the legal restriction is binding in each state.

Now select $p(e)^{-1} = \lambda\mu(e)(y/2)$ and $R(e) = (1-\lambda)wp(e)/\lambda[1-\mu(e)]\bar{x}$ for $e = 1,2$. Then, since the values q(1) and q(2) given by (25) and (26) satisfy (23), values constructed in this way satisfy (21)-(23).

It only remains, then, to verify that q(1) and q(2) are probabilities. Since $\mu(1) < \mu(2)$, $0 < q(1) < 1$ iff consider the expected utility of a saver who makes a loan with a real value of $\bar{x}/p(e)$, and acquires real balances in amount $m(e) \geq 0$. This is

$$V(e) \equiv \ell n[y - \frac{\bar{x}}{p(e)} - m(e)] + q(e)\ell n[R(e) \frac{\bar{x}}{p(e)} + \frac{p(e)}{p(1)} m(e)] +$$

$$[1 - q(e)]\ell n[R(e) \frac{\bar{x}}{p(e)} + \frac{p(e)}{p(2)} m(e)], \quad e = 1,2$$

where, for given values $\mu(1)$ and $\mu(2)$, $p(e)$ is given by (21), $R(e)$ by (22), and $q(1)$ and $q(2)$ by (25) and (26). Since from the point of view of an agent $q(e)$, $p(e)$, $R(e)$ and $\bar{x}$ are parameters, no person making a loan will acquire positive real balances if

$$(33) \quad \left. \frac{dV(e)}{dm(e)} \right|_{m(e) = 0} < 0; \; e = 1,2.$$

Condition (33) is equivalent to

$$(34) \quad [y - \frac{\bar{x}}{p(e)}]^{-1} > [R(e) \frac{\bar{x}}{p(e)}]^{-1} \{q(e) \frac{p(e)}{p(1)} + [1-q(e)] \frac{p(e)}{p(2)}\}; \; e = 1,2.$$

Now $p(e)$ in (34) is given by (21), $R(e)$ is given by (22), and $q(1)$ and $q(2)$ are given by (25) and (26). Each of these terms is a continuous function of $\mu(1)$ and $\mu(2)$ (in the neighborhood of the theorem), so define the continuous functions $P_e[\mu(1),\mu(2)]$ on the neighborhood of the theorem by

$$P_e[\mu(1),\mu(2)] = [y - \frac{\bar{x}}{p(e)}]^{-1} - [R(e) \frac{\bar{x}}{p(e)}]^{-1} \{q(e) \frac{p(e)}{p(1)} +$$

$$[1 - q(e)] \frac{p(e)}{p(2)}\}; \; e = 1,2.$$

also has a stationary sunspot equilibrium with $\mu(1) = .6$, $\mu(2) = .9$, $1/p(1) = .45$, $1/p(2) = .675$, $R(1) = 4/3$, $R(2) = 3.5556$, $q(1) = .449$, and $q(2) = .449$. (It is straightforward to check that these values satisfy (21)-(24). It is also straightforward to check that (33) is satisfied, so that no agent makes loans and holds money simultaneously.)

This economy also has an equilibrium with a 2 period cycle. To see this, compute

$$\left.\frac{d\mu_{t+1}}{d\mu_t}\right|_{\mu_t=.857} = \phi G'(.857) = -1.327.$$

Then consider the iterated map $\mu_{t+2} = T_2(\mu_t) \equiv T[T(\mu_t)]$ drawn in Figure 2. This map is continuous and clearly it intersects the 45° line at $\mu_t = \hat{\mu}$ and $\mu_t = \mu^*$. Moreover, $T_2'(\hat{\mu}) > 1$ and $T_2'(\mu^*) > 1$ both hold, as $T_2'(\hat{\mu}) = [T'(\hat{\mu})]^2 > 1$ and $T_2'(\mu^*) = [T'(\mu^*)]^2 = (1.327)^2$. Then, since $T_2(\mu)$ is continuous, it must cross the 45° line at least once from above, as at $\tilde{\mu}$ in the Figure. Thus there are distinct values $\tilde{\mu}$ and $T(\tilde{\mu})$ which constitute a 2 period cycle. It is not the case, then, that the absence of a 2 period cycle is necessary for the existence of a stationary sunspot equilibrium where the legal restriction binds in each state.

Aggregate Savings

As has been seen, the economy considered here can have a stationary sunspot equilibrium without displaying 2 period cycles in equilibrium. It also avoids the feature that the aggregate savings function is necessarily turn, that it is crucial to the results obtained here that legal restrictions state a minimum nominal value on loans, rather than a minimum real value. Appendix C demonstrates formally that, if legal restrictions specify a minimum real loan value, no stationary sunspot equilibrium exists in which an individual saver either holds money or makes loans, but does not do both. This appears to be the only tractable situation to consider here.

IV. Conclusion

In the presence of legal restrictions intended to separate money from credit markets, it is relatively straightforward to produce stationary sunspot equilibria despite the absence of 2 period cycles in equilibrium. It is also straightforward to produce stationary sunspot equilibria even though there is no obvious sense in which aggregate savings are decreasing (anywhere) in the rate of interest. These results may enhance the empirical plausibility of such equilibria.

In addition, these results call into question some traditional claims about why it is desirable to prevent private agents from creating "close" substitutes for money. Such claims, labelled the "quantity theory" by Sargent and Wallace (1982), assert that if money and credit markets are not separated, then the economy is exposed to the possibility of "excessive" fluctuations in the price level (presumably excessive relative to "fundamentals") and the stock of inside money, and possibly to price level indeterminacy.

The results of section III indicate that, contrary to this claim, legal restrictions intended to separate money from credit markets (of a form that is common in practice) can open the possibility of "excessive" price level fluctuations, fluctuations in the stock of inside money (which here equals

Appendix A

This appendix demonstrates that the economy of Section I (no legal restrictions) does not have a stationary sunspot equilibrium. To demonstrate this, the existence of such an equilibrium is supposed, and a contradiction derived.

The kinds of transactions that occur here are the same as in Section III, except that agents face no restrictions on their asset trades. In particular, if the current period state is e_t, savers choose a quantity of real balances $z(e_t)$ and a quantity of loans $x(e_t)$, taking $p(e_t)$, $p(e_{t+1})$, $q(e_t)$, and $R(e_t)$ as given, to maximize their expected utility; i.e., to solve the problem

$$\max \ln[y - x(e_t) - z(e_t)] + q(e_t) \ln[R(e_t)x(e_t) + \frac{p(e_t)}{p(1)} z(e_t)] + [1-q(e_t)] \ln[R(e_t)x(e_t) + \frac{p(e_t)}{p(2)} z(e_t)]$$

subject to $z(e_t) \geq 0$ and $x(e_t) + z(e_t) \leq y$. (Notice that a unit lent at t repays $R(e_t)$ at t+1; i.e., loan repayments are indexed.) The first order conditions associated with this problem are

$$\text{(A.1)} \quad [y - x(e_t) - z(e_t)]^{-1} = q(e_t)R(e_t)[R(e_t)x(e_t) + \frac{p(e_t)}{p(1)} z(e_t)]^{-1} + [1-q(e_t)]R(e_t)[R(e_t)x(e_t) + \frac{p(e_t)}{p(2)} z(e_t)]^{-1}; \ e=1,2,$$

and

(A.5) can be written as

$$\text{(A.5')}\quad \frac{\lambda p(1)R(1)}{\lambda p(1)R(1)y - \lambda\eta p(1) - R(1)} = q(1)\left[\frac{\lambda p(1)}{\lambda\eta p(1)+1}\right] + [1 - q(1)]\left[\frac{\lambda p(1)}{\lambda\eta p(2)+1}\right].$$

Solving (A.5') for q(1) yields

$$\text{(A.6)}\quad q(1)\left[\frac{\lambda p(1)}{\lambda\eta p(2)+1} - \frac{\lambda p(1)}{\lambda\eta p(1)+1}\right] = \frac{\lambda p(1)}{\lambda\eta p(2)+1} - \frac{\lambda p(1)R(1)}{\lambda p(1)R(1)y-\lambda\eta p(1)-R(1)}$$

Similar operations on (A.2) for $e_t = 2$ yield

$$\text{(A.7)}\quad q(2)\left[\frac{\lambda p(2)}{\lambda\eta p(2)+1} - \frac{\lambda p(2)}{\lambda\eta p(1)+1}\right] = \frac{\lambda p(2)}{\lambda\eta p(2)+1} - \frac{\lambda p(2)R(2)}{\lambda p(2)R(2)y-\lambda\eta p(2)-R(2)}$$

Now, since q(1) and q(2) are probabilities, $0 \leq q(1) \leq 1$ and $0 \leq q(2) \leq 1$. Then, from (A.6), for $0 \leq q(1) \leq 1$ it is necessary that

$$\text{(A.8)}\quad \frac{\lambda p(1)}{\lambda\eta p(2)+1} \geq \frac{\lambda p(1)R(1)}{\lambda p(1)R(1)y-\lambda\eta p(1)-R(1)} \geq \frac{\lambda p(1)}{\lambda\eta p(1)+1}$$

(since, by hypothesis, $p(1) > p(2)$). From (A.7), for $0 \leq q(2) \leq 1$, it is necessary that

$$\text{(A.9)}\quad \frac{\lambda p(2)}{\lambda\eta p(2)+1} \geq \frac{\lambda p(2)R(2)}{\lambda p(2)R(2)y-\lambda\eta p(2)-R(2)} \geq \frac{\lambda p(2)}{\lambda\eta p(1)+1}$$

In addition, since $p(1) > p(2)$, it is the case that

$$\text{(A.10)}\quad \frac{\lambda p(1)}{\lambda\eta p(1)+1} \geq \frac{\lambda p(2)}{\lambda\eta p(2)+1}\ .$$

Therefore, (A.8)-(A.10) imply that

$$\text{(A.11)}\quad \frac{\lambda p(1)R(1)}{\lambda p(1)R(1)y-\lambda\eta p(1)-R(1)} \geq \frac{\lambda p(2)R(2)}{\lambda p(2)R(2)y-\lambda\eta p(2)-R(2)}\ .$$

<u>Appendix B</u> (Proof of Proposition)

Suppose, contrary to the proposition, that there are distinct values μ_1 and μ_2 (say with $\mu_1 < \mu_2$) satisfying $\mu_1 = T(\mu_2)$ and $\mu_2 = T(\mu_1)$. Then there are three possible cases.

Case 1. ($\mu^* \geq \mu_2 > \mu_1$). By hypothesis $\mu_2 = T(\mu_1) > \mu_1 = T(\mu_2)$. But $T(\mu)$ is non-decreasing for $\mu \leq \mu^*$. Thus this case is not possible.

Case 2. ($\mu_2 > \mu^* \geq \mu_1$). In this case $\mu_2 = T(\mu_1) \leq T(\mu^*)$, since $T(\mu)$ is non-decreasing for $\mu \leq \mu^*$. Since by hypothesis $\mu_2 > \mu^* = T(\mu^*)$, this case is also impossible.

Case 3. ($\mu_2 > \mu_1 \geq \mu^*$). For all $\mu_1 \geq \mu^*$, $(\mu_1, T(\mu_1))$ lies on or below the 45° line in Figure 1. Then $\mu_2 = T(\mu_1) \leq \mu_1$, contradicting the supposition. Thus this case is also not possible, establishing the proposition.

Now suppose that $\mu(2) > \mu(1)$. For $0 \leq q(1) \leq 1$ it is necessary that

$$(A.18) \quad 1 \geq 4\left(\frac{1-\lambda}{\lambda}\right)\left(\frac{w}{y^2}\right)(y-\hat{x})\left[\frac{\mu(1)}{\mu(2)}\right]\left[\frac{1}{1-\mu(1)}\right] \geq \frac{\mu(1)}{\mu(2)},$$

while for $0 \leq q(2) \leq 1$ it is necessary that

$$(A.19) \quad 1 \geq 4\left(\frac{1-\lambda}{\lambda}\right)\left(\frac{w}{y^2}\right)(y-\hat{x})\left[\frac{1}{1-\mu(2)}\right] \geq \frac{\mu(1)}{\mu(2)}.$$

However (A.18) and (A.19) imply that

$$4\left(\frac{1-\lambda}{\lambda}\right)\left(\frac{w}{y^2}\right)(y-\hat{x})\left[\frac{1}{1-\mu(1)}\right] \geq 1 \geq 4\left(\frac{1-\lambda}{\lambda}\right)\left(\frac{w}{y^2}\right)(y-\hat{x})\left[\frac{1}{1-\mu(2)}\right],$$

which in turn implies that $\mu(1) \geq \mu(2)$, contradicting the original supposition. A similar contradiction arises if it is assumed that $\mu(1) > \mu(2)$. Thus this economy has no stationary sunspot equilibrium in which savers either make loans or hold money, but do not do both.

stream (0,2w), and to assume that borrowers (agents with net indebtedness when young) are precluded from holding money. This would be in the spirit of Woodford (1986), where "workers" are, in effect, precluded from holding capital. It is easy to check that the modification just described would produce equilibria identical to those constructed below.

6. As will be shown (Appendix A), absent legal restrictions there are no stationary sunspot equilibria here. Hence only deterministic equilibria are discussed in the text.

7. In order for there to be a steady state equilibrium with valued fiat money it is necessary that $\left(\frac{1-\lambda}{\lambda}\right)\left(\frac{w}{y}\right) < 1/2$ hold. This condition is henceforth assumed to be satisfied.

8. This section considers deterministic equilibria under the legal restriction discussed. Section III discusses sunspot equilibria in this economy.

9. In some respects this resembles the situation in Woodford (1986), where a subset of agents hold money while other agents hold higher yielding assets. In Woodford (1986), however, individuals are in effect exogenously divided into holders of money ("workers") and holders of capital ("capitalists"). Here it is the fact that agents must voluntarily make loans and hold money that enables sunspot equilibria to be constructed.

10. It can be verified by direct computation that $\phi > 1$ and $\bar{x}(\lambda/2) > 1$ imply that $\bar{x}\lambda\mu^* < 2$.

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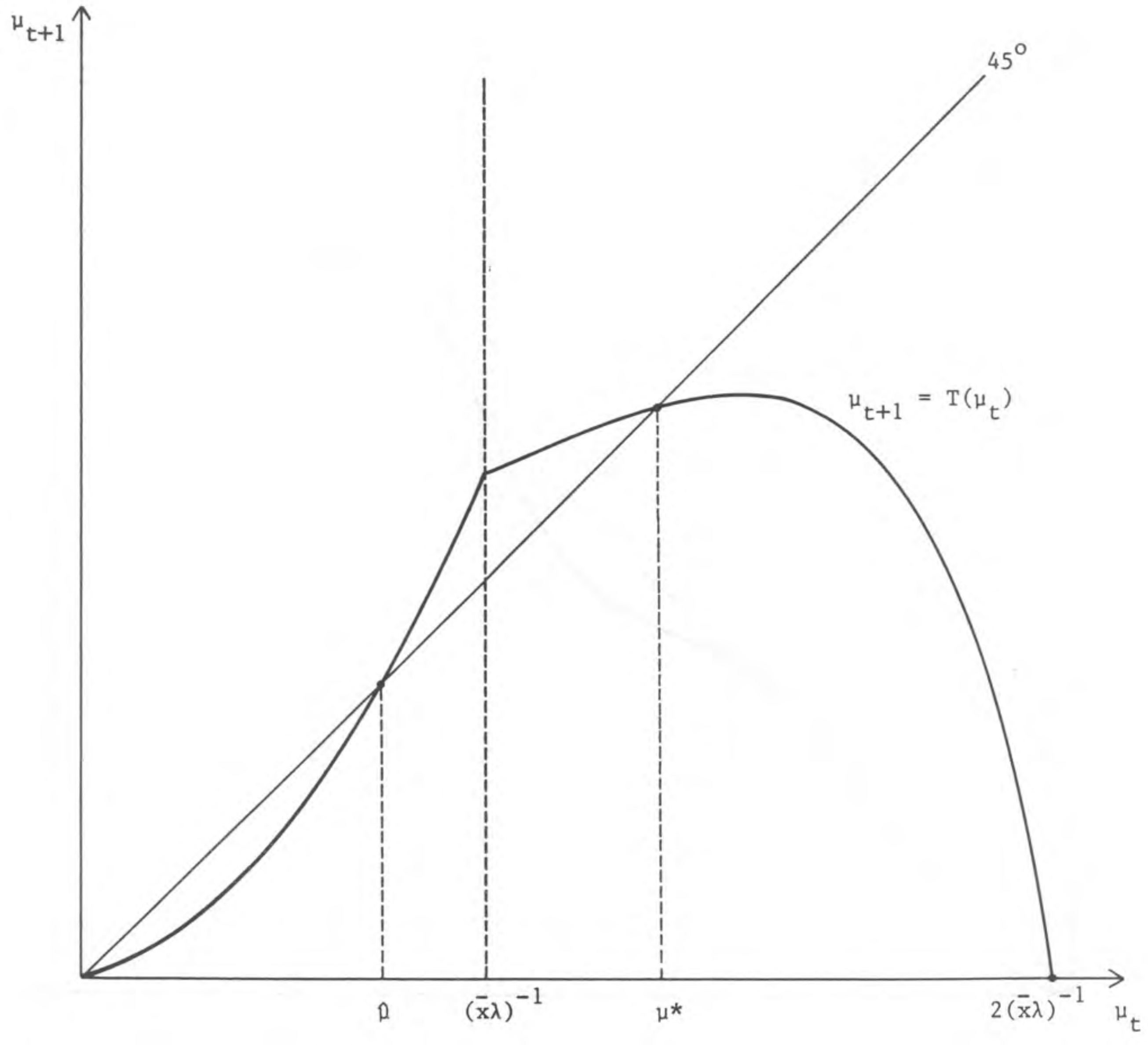
Figure 1



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