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ABSTRACT

Many studies have documented a negative correlation between stock returns and both expected and unexpected inflation. One set of theories about this correlation holds that it is coincidental, resulting from exogenous shocks to real output that are positively correlated with stock returns but negatively correlated with inflation. A competing view is that the negative correlation between stock returns and inflation is directly causal resulting from the existence of nominal contracts between firms and other economic agents including the government. This paper develops a general equilibrium model of asset prices that implies a set of restrictions on the data that enable us to test the competing theories of the inflation asset return correlation. Our emprical results reject the hypothesis that this correlation is due to real shocks.

I. Introduction

The traditional view that stocks, being the claims to the income from real assets, should provide a hedge against inflation has been overturned by many studies that document a negative correlation between the return on stocks and both expected and unexpected inflation. One view of this surprising correlation is that it is coincidental, resulting from exogenous shocks to real output that are positively correlated with stock returns but negatively correlated with inflation. A competing view is that the negative correlation between stock returns and inflation is directly causal resulting from the existence of nominal contracts between firms and other economic agents including the government. The literature contains empirical evidence in support of both views but no direct attempts to test them against each other. In this paper we develop a general equilibrium model of asset prices that, under the assumption that markets are complete, implies a set of restrictions on data that enable us to test the competing theories of the inflation stock-return correlation.

The idea that the negative correlation between inflation and stock returns is mainly an artifact is due to Fama (1981). Fama argues that a negative shock to output will cause the incomes of corporations to fall and in turn cause stock prices to fall. The corresponding rise in the general price level is the result of a "combination of money demand theory and the quantity theory of money" (p. 545). With output lower and the money supply constant the price level must rise to satisfy the quantity equation. Thus, a negative shock to output causes both a fall in stock prices and a rise in the general price level. While Fama's argument is somewhat informal, LeRoy (1984) shows that the reasoning holds true in a simple general equilibrium model of money. Geske and Roll (1983) proposed an explanation very similar to Fama's. Again, a negative shock to real output is the factor causing stock prices to fall but in their explanation the link to the price level is more elaborate. The fall in real output causes a fall in tax collections and a corresponding rise in the federal deficit. The Federal Reserve acts to monetize the deficit by expanding the money supply which raises the price level. In both of these explanations it is a negative shock to output that causes a fall in stock prices. Consequently we will refer to this as the "real shocks" hypothesis in the subsequent discussion.

An alternative explanation of the inflation stock-return correlation holds that it results because inflation alters the values of nominal contracts to which firms are committed. An often cited example is the corporate tax system which can be viewed as a nominal contract between the firm and the government. Feldstein and Summers (1979) and Feldstein (1980) have argued that the use of historic cost depreciation and FIFO inventory accounting means that inflation will increase the effective tax rate on corporate income. They also argue that the cost of capital financed by equity increases with inflation because nominal capital gains on stock are taxed at the individual level without any offsetting deduction at the corporate level.[1] For our purposes, the feature that distinguishes the nominal contracting hypothesis from the real shocks hypothesis discussed above is that the effects are inflation induced and operate directly on stock returns. There have been many attempts to estimate the magnitude and importance of these nominal contracting effects but the results differ widely.

In this paper we develop a general equilibrium model of asset prices that describes the relation between macroeconomic shocks and asset returns. The model is a stochastic, overlapping-generations model with complete markets in

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[1] Of course the cost of debt finance will also decrease with inflation if the marginal tax rate at the individual level is less than that at the corporate level. The final magnitude of this effect depends on the value of the marginal tax rates and the mix of debt and equity finance.

contingent claims. The completeness of markets turns out to be an essential ingredient in differentiating the two theories we seek to test.

The model economy is described in the next section of the paper, and its stationary equilibrium is described in some detail. Our theoretical model implies restrictions on the behavior of asset returns that enable us to distinguish the real shocks hypothesis and the nominal contracting hypothesis. We derive expressions for asset prices in our model economy and show how changes in prices are determined by changes in yields, changes in the real interest rate, the underlying impatience of investors and the degree of persistence in the aggregate shocks faced by the economy.

In Section 3 we show that the real shocks hypothesis and the nominal contracting hypothesis are distinguished by the fact that there is an interest rate effect associated with the former that is not present under the latter if markets are complete. We illustrate the magnitude of changes in yields and interest rates that would be necessary to explain observed changes in stock returns under various assumptions about impatience and persistence. Section 4 of the paper derives an econometric test of the restrictions implied by our theoretical model. We test the competing hypotheses using annual data on after-tax returns on three assets: common stocks, government bonds and housing. The results of this test reject the Fama, Geske-Roll view that the inflation-return correlation is due to real output shocks. The Feldstein-Summers nominal contracting hypothesis could not be rejected by our tests. Nevertheless the model calibration exercises suggest that the observed variations in asset returns are unlikely to be explained solely by yield changes due to tax increases or other nominal contracts. In the final section of the paper we discuss some of the implications of our model and results.

II. A Model of Asset Prices

In this section we describe a model economy that is subject to exogenous macroeconomic shocks. Because investors are assumed to know the probability distributions and consequences of the exogenous shocks, the resulting rational expectations equilibrium is one in which asset prices reflect the stochastic process governing the shocks. In this respect our model follows the recent literature on asset prices stemming from Lucas (1978) and Brock (1982).

Like much of that literature, ours is a model of real asset prices; nominal prices and money supply play no role in the analysis. We chose a real model largely on pragmatic grounds. The most important consideration was the nature of the two hypotheses we seek to test. Each posits a well defined real impact which can be adequately represented in a real economy. By modeling those impacts as the exogenous shocks, we are able to derive testable restrictions that differentiate the two explanations. Similar restrictions hold in explicitly monetary models such as Dantine and Donaldson (1986) and LeRoy (1984) , but the choice of a monetary framework is itself a subject of controversy that seems best avoided here because it has no direct bearing on the alternative explanations we are testing. A real model yields all the restrictions we need.

A. The Economic Environment

Our model is a stochastic version of the overlapping generations model of Samuelson (1958). The time horizon is infinite, and time is partitioned into discrete periods indexed by the subscript $t = 0,1,\ldots$. A generation is born at the beginning of each period and lives for two periods. There is one perishable commodity, and each generation is endowed with quantities of that good. There is no production.

The shocks to this economy are real, but are assumed to be perfectly correlated with inflation. They are represented by a random variable s_t, which is referred to as the state of the economy in period t. This random variable assumes just two values, zero and unity. If $s_t = 1$, inflation is "on" in period t. If $s_t = 0$, inflation is "off". The state of the economy follows a Markov process. The probability that state j will occur next period if state i occurs this period is denoted by π_{ij}. We assume that $\pi_{11} = \pi_{22} = \pi$. The probability of either state in the initial period is 1/2.

An event at time t, denoted by e_t, is a specification of the history of this random process up through time t; $e_t = (s_0, s_1, \ldots, s_t)$. The set of all possible events at time t is denoted by E_t.

The first generation is endowed with a bundle of capital goods which yields amounts of the consumption good in each period after the initial period. To represent the impact of the shock on these yields, these yields are made a function of the history of the economy. The aggregate yield or income from all capital goods in period t if e_t occurs is denoted by $k(e_t)$.

All generations are endowed with amounts of the consumption good during each period of their lives. This endowment represents the net income from a generation's labor, and we assume that it is also a function of the state of the economy. Let $\ell(e_t)$ and $m(e_t)$ be the endowments of the young and the old in period t if e_t occurs, $t \geq 1$. In the initial period, $t = 0$, we assume that the young's endowment is fixed at $\bar{\ell}$ for both states.

All generations have identical preferences which are represented by the utility function $u(x) + v(y)$ where x is the consumption of a generation when young and y is its consumption when old. We assume that $u' > 0$, $v' > 0$, $u'' < 0$, $v'' < 0$, $\lim_{x\to 0} u'(x) = \infty$, and $\lim_{y\to 0} v'(y) = \infty$.

B. The Market in Contingent Claims

Capital goods, initially owned by the first generation, are passed down from generation to generation in exchange for current consumption. To determine the price of these goods, we assume that an Arrow-Debreu market in contingent claims opens before the initial period and that equilibrium contracts are established. The state in the first period is then revealed, deliveries are made on the contracts that are due, and the market in contingent claims is reopened. Agents can now recontract based on their new knowledge of the initial state. This same process continues in subsequent periods. The price of a capital good in each period is the value of all the contingent consumption that it entails for future periods. Variations in this price reflect the effect of various macroeconomic shocks on real asset prices.

The markets in contingent claims are assumed to be complete in the usual Arrow-Debreu sense. In particular, we assume a market open at a moment outside of time in which all generations are present and free to trade in all contingent claims. This assumption seems unusual in stochastic, overlapping generations models because the model resulting is similar to a representative agent economy. We retain the overlapping generations framework because we wanted a model economy in which the redistributions arising from nominal contracting could be analyzed.[2] The assumption of market completeness is essential to distinguishing between the two explanations we are analyzing.

In the initial market, generations trade claims contingent on all possible events. They know the probabilities of all events, but nothing of the

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[2] In Cooley and Sonstelie (1985) we analyzed the effects of such redistributions when markets are not complete. The equilibrium concept assumed in that paper is the same as that in Labadie (1986), Huffman (1986) and Huberman (1984).

event that will actually occur. Let $p(e_t)$ be the price in this market of one unit of consumption at date t if e_t occurs. The value of generation t's endowment, its wealth, is denoted by w_t. The first generation owns capital goods with income in all future periods and also receives labor income in the first two periods. This generation has wealth

$$w_0 = \sum_{t=1}^{\infty} \sum_{e_t \varepsilon E_t} p(e_t)k(e_t) + \sum_{e_0 \varepsilon E_0} p(e_0)\bar{\ell} + \sum_{e_1 \varepsilon E_1} p(e_1)m(e_1) \tag{1}$$

The wealth of the generation born in period t, $t \geq 1$, is

$$w_t = \sum_{e_t \varepsilon E_t} p(e_t)\ell(e_t) + \sum_{e_{t+1} \varepsilon E_{t+1}} p(e_{t+1})m(e_{t+1}). \tag{2}$$

Each generation demands the bundle of consumption claims that maximizes its expected utility subject to its wealth constraint. Generations only demand positive consumptions in the periods in which they live so a consumption plan for generation t may be written as a pair of functions x_t and y_t defined on E_t and E_{t+1}. The value $x_t(e_t)$ is the amount generation t consumes when young if e_t occurs, and the value $y_t(e_{t+1})$ is the amount it consumes when old if e_{t+1} occurs. The plans are an equilibrium if

$$x_0(e_0) = \bar{\ell} \tag{3}$$

for all $e_0 \varepsilon E_0$, and

$$x_t(e_t) + y_{t-1}(e_t) = k(e_t) + \ell(e_t) + m(e_t) \tag{4}$$

for all $e_t \varepsilon E_t$, t=1,2,....

C. A Stationary Equilibrium

In what follows, we shall restrict our attention to stationary environments in which the incomes from capital and labor each period are functions of the state of the economy only in that period. In Appendix A, we show that there is at least one stationary equilibrium associated with each such environment. By a stationary equilibrium, we mean an equilibrium in which the consumptions of the young and the old in any period depend only on the state of the economy in that period. This section characterizes the prices that support such an equilibrium.

Let x_i and y_i denote the equilibrium consumptions of the young and the old if state i occurs. These consumptions must satisfy the first order conditions of the young and the old for consumption in period t, $t \geq 1$, which implies

$$\pi(e_t)u'(x_i) - \mu_t p(e_t) = 0 \tag{5}$$

$$\pi(e_t)v'(y_i) - \mu_{t-1} p(e_t) = 0 \tag{6}$$

for all $e_t \varepsilon E_t$ for which $s_t = i$. In these conditions, $\pi(e_t)$ is the probability of e_t, and μ_t is the marginal utility of income for the generation born in period t. These conditions imply a recursive relationship for the marginal utility of income which in turn implies the following form for prices:

$$p(e_t) = \pi(e_t)(1+r)^{-t}\, p_i, \tag{7}$$

where

$$(1+r) \equiv \frac{u'(x_0)}{v'(y_0)} = \frac{u'(x_1)}{v'(y_1)} \tag{8}$$

$$p_i \equiv \frac{u'(x_i)}{\mu_0} = (1+r)\,\frac{v'(y_i)}{\mu_0} \tag{9}$$

The parameter, r, is the interest rate in this initial market. The price of a unit of certain consumption in period t, a bundle of contingent claims for all possible events in period t, is $(1+r)^{-t} \frac{p_0+p_1}{2}$. This implies that the ratio of the price of certain consumption in period $t+1$ to certain consumption in period t is $(1+r)^{-1}$, which accords with the usual notion of an interest rate in certainty models.

In the initial period, $t=0$, the endowment is the same for both states and the initial generation is the only consumer. Initial prices must therefore induce that generation to demand the same amount in both states. Denote this amount by $\bar{x}$. Since the probability of each state in the initial period is ½, the price of consumption in each state must be equal. Let $\frac{1}{2}\bar{p}$ be this price.

We have shown that stationary prices are determined by the probabilities of events and the four parameters, $\bar{p}$, p_0, p_1 and r. Of these, p_0 and p_1 are particularly relevant for us because they are the mechanism through which aggregate endowment shocks affect asset prices. To see this, we transform the equilibrium conditions into something more familiar.

By using the formula for stationary prices and then imposing the equilibrium condition, $\bar{x} = \bar{\ell}$, the budget constraint of the initial generation can be reduced to

$$p_0 y_0 + p_1 y_1 = p_0(k_0 + m_0 + \frac{k_0}{r}) + p_1(k_1 + m_1 + \frac{k_1}{r}), \tag{10}$$

where k_i is the aggregate income in state i from the capital goods initially owned by the first generation and m_i is the old's labor income in state i. This equation and the formula for stationary prices may then be used to reduce the budget constraint of generation t, $t \geqq 1$, to

$$p_0 x_0 + p_1 x_1 = p_0(\ell_0 - \frac{k_0}{r}) + p_1(\ell_1 - \frac{k_1}{r}) \tag{11}$$

where ℓ_i is the young's labor income in state i.

The formula for stationary prices may also be used to simplify the first order conditions yielding

$$\frac{u'(x_0)}{u'(x_1)} = \frac{p_0}{p_1} \tag{12}$$

and

$$\frac{v'(y_0)}{v'(y_1)} = \frac{p_0}{p_1} \tag{13}$$

These four conditions in conjunction with the market clearing condition

$$x_i + y_i = k_i + \ell_i + m_i \tag{14}$$

are precisely the conditions that characterize a competitive equilibrium in the familiar two-person, two-good model of exchange. The two people are the young and the old. Their consumption bundles are (x_0,x_1) and (y_0,y_1), their utilities are $u(x_0) + u(x_1)$ and $v(y_0) + v(y_1)$, and their endowments are $(\ell_0-\frac{k_0}{r}, \ell_1-\frac{k_1}{r})$ and $(k_0+m_0+\frac{k_0}{r}, k_1+m_1+\frac{k_1}{r})$.

In this strictly atemporal model we may think of (ℓ_0,ℓ_1) and (k_0+m_0, k_1+m_1) as the fundamental endowments of the young and the old with $(\frac{k_0}{r}, \frac{k_1}{r})$ being an additional transfer from the young to the old. This transfer has a natural intertemporal interpretation. The young and old are entitled to their labor incomes each period which are (ℓ_0,ℓ_1) and (m_0,m_1). The old also owns a bundle of goods which entitles him to the income (k_0,k_1). In addition, the old must pass on ownership of the capital goods to the young. The value of the good is determined by the interest rate in the usual way, and thus the young gives up $(\frac{k_0}{r}, \frac{k_1}{r})$ units of its endowment in exchange for those goods. Once these initial arrangements are complete, both agents may then trade the two goods, 0 and 1.

The prices, p_0 and p_1, determine the terms of this trade. Equations (10) and (11) are the budget constraints of the two traders, and equations (12) and (13) are their first order conditions. In Figure 1, equilibrium prices and consumptions are represented in the familiar Edgeworth Box diagram. Since the two goods are consumption in two states of the economy, the interpretation is that equilibrium prices allocate the risky endowment between the two individuals. Using the Edgeworth Box diagram, it is easy to demonstrate that if the total endowment is the same in both states the prices of consumption in both states will also be equal. In that case, both individuals have certain consumption even though their endowments may be risky. It is also easy to demonstrate that if one state has a lower total endowment than the other it must also have a higher price. This higher price induces both the young and the old to consume less in that state, and thus the social risk is shared. These are familiar propositions from standard models of risk sharing.

This characterization of prices applies to the initial market. Suppose now that time advances, nature reveals the state in the first period, and consumers take delivery on claims contingent on that state. Then, the market reopens, allowing consumers to recontract based on new information. That new information is the event e_0, which is now the history of the economy, and it causes agents to update their probabilities of future events. Prices must change as a result, but new equilibrium prices can be easily derived from the old. Simply replace the probability $\pi(e_t)$ in the formula for prices with the updated probability $\pi(e_t|e_0)$. These new prices will satisfy the first order conditions with the same consumptions as before, and thus they will yield an equilibrium in which no consumer wishes to recontract.

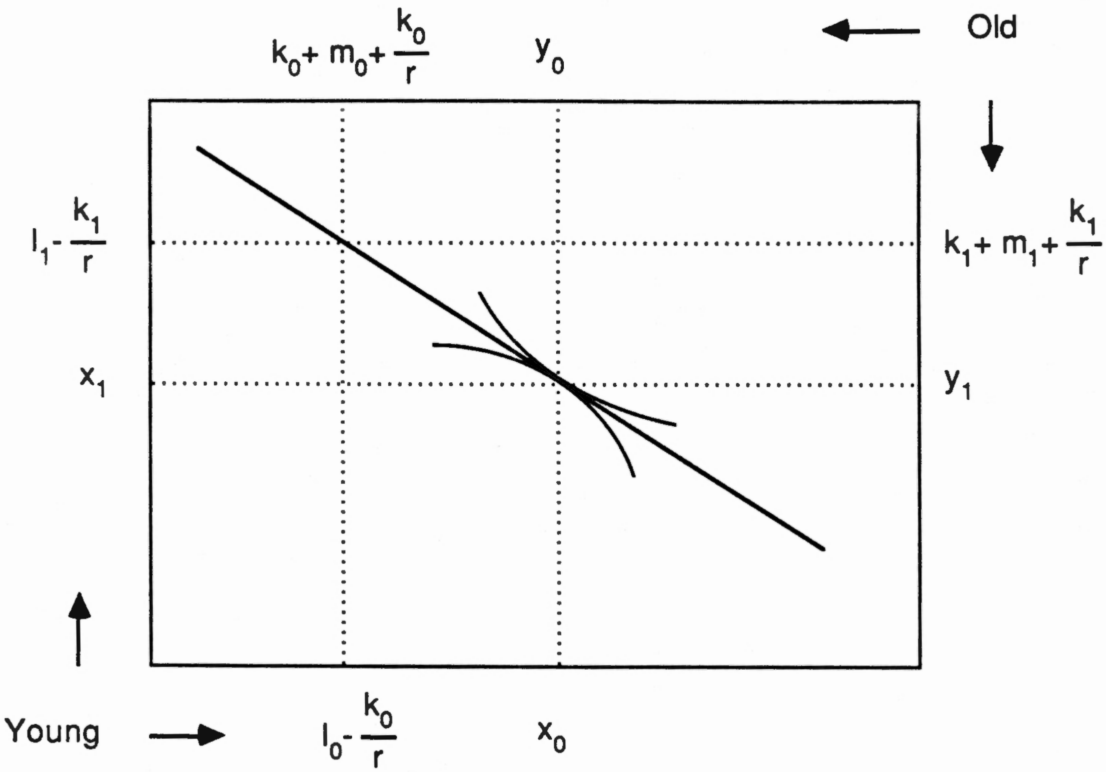

Figure 1

The Allocation of Risk

An analogous updating of prices works just the same way for any subsequent period, and thus equilibrium prices for all subsequent markets can be easily derived from the prices in the initial market. Since the convention is to express prices of capital goods in terms of current consumption, we make current consumption the numeraire in these markets. With that normalization, the price of consumption contingent on event e_t given event $e_{t'}$ at present is

$$p(e_t|e_{t'}) = \pi(e_t|e_{t'})(1+r)^{(t'-t)}\frac{p_j}{p_i} \qquad (15)$$

for $s_{t'} = i$ and $s_t = j$.

Interest rates are easily derived from these prices. The interest rate in state i is just the rate at which consumption today can be exchanged for certain consumption next period. To obtain consumption with certainty, a contingent claims must be obtained for both possible states next period implying that the interest rate is

$$r_i = (1+r)\frac{p_i}{\pi p_i + (1-\pi)\,p_j} - 1,\ i\neq j \qquad (16)$$

This equation for interest rates accords with intuition. If there is no aggregate uncertainty, $p_1 = p_0$, and the interest rate is r in both states. On the other hand, if aggregate consumption is scarcer in state 1 than in state 0, $p_1 > p_0$ and $r_1 > r > r_0$. Thus, the interest rate is higher in the state with the lower aggregate endowment. In what follows, we shall find it easier to work with p_0 and p_1 than with the interest rates r_0 and r_1. Nevertheless, we shall often refer to implications deriving from differences in p_0 and p_1 as "interest rate effects".

D. Asset Prices

In this model economy, we can construct and price any asset. An asset with a nontrival price involves an uncertain payoff over an infinite horizon. To find the factors that determine the value of an asset, consider a representative asset with income $z(e_t)$ in period t if e_t occurs. Assume the payoff is stationary so that $z(e_t) = z_i$ if $s_t = i$. The value of that asset in the market at time t' with event $e_{t'}$ is

$$v(e_{t'}) = \sum_{t=t'+1}^{\infty} \sum_{e_t \varepsilon E_t} p(e_t | e_{t'})\, z(e_t) \tag{16}$$

Since $p(e_t | e_{t'})$ depends only on the state at time t', the states at time t and t', and the length of time between those two periods, t-t', the value of the representative asset, written in general terms in equation (16), will only be a function of the current state. Using equation (15) and the formula for the transition probabilities of a Markov process,[3] the values of the asset in states 0 and 1 can be reduced to

$$\begin{aligned} v_o &= \frac{\lambda p_0 z_0 + (1-\lambda) p_1 z_1}{p_0 r} \\ v_1 &= \frac{\lambda p_1 z_1 + (1-\lambda) p_0 z_0}{p_1 r} \end{aligned} \tag{17}$$

where $\lambda = \frac{1}{2}(1+\theta)$ and $\theta = \dfrac{(2\pi-1)r}{2(1-\pi)+r}$. The parameter θ lies between -1 and 1,

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[3]
$$\pi_{ij}^t = \begin{cases} \frac{1}{2}(1 + (2\pi-1)^t) & \text{if } i=j \\ \frac{1}{2}(1 - (2\pi-1)^t) & \text{if } i \neq j \end{cases}$$

where π_{ij}^t is the probability that state j occurs in t periods given state i at present.

and thus λ lies between zero and unity. The numerator of the expression for asset prices is thus the weighted average of the value of the payoffs in both states.

This formula is easier to interpret after prices and yields are conveniently normalized. We choose the same normalization for both,

$$\frac{p_0 + p_1}{2} = 1$$
$$\frac{z_0 + z_1}{2} = 1 \quad , \tag{18}$$

and define

$$\Delta p \equiv p_1 - 1 = 1 - p_0$$
$$\Delta z \equiv z_1 - 1 = 1 - z_0 \quad . \tag{19}$$

The formula for asset values then becomes

$$v_0 = \frac{1 + \Delta p \Delta z - \theta(\Delta p + \Delta z)}{r(1 - \Delta p)}$$
$$v_1 = \frac{1 + \Delta p \Delta z + \theta(\Delta p + \Delta z)}{r(1 + \Delta p)} \tag{20}$$

The parameter θ is the weight attached to the deviation in income in the current state above its average. The parameter reflects both the degree of persistence in the Markov process and the degree of impatience implied by equilibrium prices. In the most extreme case of persistence (π=1) θ equals unity; while in the case of serial independence (π=½) θ equals zero. Other values of θ are given in Table 1 for representative values of π and r. The table demonstrates the tradeoff between π and r in determining θ. As the table also demonstrates, the high degree of patience embedded in typical interest rates implies that θ is quite small, unless π is very near unity. Therefore, differences in income between states will have a relatively modest effect on asset values, unless investors are nearly certain about the future course of events.

The role of θ is most graphically illustrated by considering the case of aggregate certainty in which $\Delta p = 0$. In that case the percentage change in the price of an asset from state 0 to state 1 is $(1 + \theta\Delta z)/(1 - \theta\Delta z)$. For the values of θ in Table 1 that seem realistic, this formula implies that asset prices are relatively stable. For example, assume an asset for which $\Delta z = 0.5$, which is a threefold increase in income from state 0 to state 1. For $\theta = 0.05$, the price of the asset increases only 5% from state 0 to state 1.

In contrast, differences in interest rates between states can have relatively dramatic effects on asset prices. To see this, consider the case of an asset with a certain return ($\Delta z = 0$). Then, the ratio of prices in states 1 and 0 is

$$\frac{v_1}{v_0} = \left(\frac{1 + \theta\Delta p}{1 - \theta\Delta p}\right)\left(\frac{1 - \Delta p}{1 + \Delta p}\right) \qquad (21)$$

Since θ is small, the effect of the difference in prices between states is translated at full force into differences in asset prices. The intuition is that the value of future income from the asset is nearly the same in both states (since θ is small), so what matters in determining the price of the asset is the value of the present consumption that must be sacrificed to obtain it. And this factor is translated directly into asset prices. If the price of consumption is half as much in state 1 as in state 0, the value of the asset will be nearly twice as high in state 1.

This discussion forms the background for our representation of the competing explanations of the relationship between inflation and asset prices. In our simple model, changes in asset values between states come from two different sources: changes in asset yields (Δz) and changes in the underlying interest rates (Δp). The competing explanations imply different patterns for these variables.

III. Inflation and Asset Prices

In the nominal contracts explanation, inflation causes the corporate tax rate to rise, which lowers corporate earnings. In our model, this implies that Δz is negative for a representative unit of corporate stock. No change in the interest rate is implied, however. The higher corporate tax rate increases tax revenue, which permits more public spending or lower taxes on other income. These fiscal activities redistribute income from investors in stock to others, and can be represented in our model as decreases in the income stream from capital and increases in labor income. The important point is that these redistributions net to zero. Investors can therefore insure themselves against them, nullifying their effect. Interest rates will be constant as a consequence.

In contrast, the real shock hypothesis does imply a change in interest rates. A negative shock to GNP simultaneously causes inflation and a decline in corporate income. In the terms of our model, Δz is negative for corporate stock, just as in the nominal contracts explanation. But, since the aggregate endowment is also lower in the inflationary state consumption is relatively scarce and Δp is positive. Investors can't completely insure themselves against lower corporate income, and the interest rate will be higher in the inflationary state.[4] In summary, the nominal contracts and the endowment

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[4] It is important to note that the interest rate consequences just discussed are different from those commonly assumed in the literature. Feldstein and Summers, for example, assume that real after-tax interest rates fall with inflation. Some of the recent empirical evidence supports the view that ex-post real rates of return on nominal assets, a measure quite different from the real interest rates discussed above, fall with inflation. The evidence on ex-ante real interest rates is less clear.

shock explanations share the same implications for Δz, but differ in their implications for Δp.

Both explanations must pass a difficult test because they must account for a very large decline in stock prices. From 1968 to 1979, the real value of the Standard and Poor's 500 fell by nearly 50%. Table 2 lists the percentage decline in stock prices that our model predicts for various combinations of Δp and Δz. With Δp equal zero, as implied by the nominal contracting story, a 50% decline in stock prices requires a value of Δz far less than -2.5. Plausible estimates of Δz are much higher than this. For example, Feldstein and Summers (1979) estimated that the effective tax rate on corporate income was 66% in 1977 and would have been 41% had there been no inflation. These tax rates yield a Δz of -0.30. A Δz as low as -2.5 requires a tax rate of 125%, assuming the same 41% without inflation. These calculations illustrate the difficulty of explaining the observed decline in stock prices through nominal contracting alone.

It is easier to explain, however, if the real interest rate rises with inflation, as is the case with an endowment shock. As Table 3 shows, if Δp is 0.3, a 50% decline in stock prices requires only a moderate decrease in corporate income. But, while a small value of Δp yields large changes in stock prices, it also implies large changes in the aggregate economy. To illustrate this point, the interest rates for various values of Δp are listed in Table 3. For ΔP equal to 0.2 or larger, the real interest rate exceeds 5% in the inflationary state and is negative in the non-inflationary state. By historical standards, these variations seem implausibly large.

The plausibility of these interest rate variations may also be gauged by calculating the change in the aggregate endowment necessary to yield various values of Δp. Assuming a utility function of the constant relative risk aversion type, equilibrium conditions imply

$$\frac{e_0}{e_1} = \left(\frac{1 + \Delta p}{1 - \Delta p}\right)^{1/\alpha} \tag{22}$$

where α is the coefficient of relative risk aversion and e_i is the aggregate endowment in state i. With α equal to unity, a value of 0.2 for Δp means that the aggregate endowment must be 50% higher in the non-inflationary state. For α equal 2, this difference shrinks to 22%, still implausibly large. Approaching this issue from the other direction, we find that with Δp equal 0.2 a 5% decline in the aggregate endowment requires an α of nearly 8. Empirical studies of risk attitudes typically find coefficients of relative risk aversion well below this magnitude.

From these calculations we conclude that it is difficult to account for the magnitude of the decline in stock prices with either explanation. The endowment shock explanation has the best chance since it combines a rise in the interest rate with a decline in corporate income, but these effects still must be quite large.

IV. Empirical Findings

In our model, stock prices may be negatively correlated with inflation for two reasons: net corporate income is negatively correlated with inflation ($\Delta z<0$) or the interest rate is positively correlated with inflation ($\Delta p>0$). The nominal contracts explanation invokes the first of these two reasons. The endowment shock explanation invokes both. A test of the two hypotheses comes down to estimating income and interest rate effects.

Our estimation procedure is motivated by the following considerations. If income effects were the sole source of the negative correlation between stock prices and inflation, the decrease in stock prices associated with a rise in the inflation rate would be less, in percentage terms, than the decrease in net income itself. For even though inflation may be high at present, investors place a positive probability on a return to low inflation, and higher corporate income, in the future. Expected future income would therefore be higher than its current realization implying that the price of stock would not be written down by the full amount of the decline in income. This implies that stock returns during an inflationary period would be lower than during a noninflationary period. On the other hand, if interest rate effects were the sole source, stock returns would be higher in an inflationary period than in a noninflationary period since the income from stock would be unaffected by the level of inflation and the price of the stock would be lower in the inflation period. These two polar cases show that income and interest rate effects have different implications for the relationship between stock returns and the level of inflation.

These implications hold for periods during which the inflation rate is stable and stock prices have fully adjusted to that rate. Periods of transition from one level of inflation to another are a different matter, however, because they involve capital gains and losses as the price of stock adjusts to its new level. If inflation rises during a period, for example, investors will experience capital losses which will lower returns below the rate that would have prevailed if the inflation rate had stayed constant throughout the period. Thus, stock returns will be affected by the change in the inflation rate as well as its level.

This suggests the following regression equation for stock returns:

$$r_t = a_0 + a_1 I_t + a_2(I_t - I_{t-1}) \tag{23}$$

where r_t is the rate of return on stock in period t and I_t is the inflation rate in period t. The coefficient a_1 represents the effect of the level of inflation on stock returns. If the correlation between inflation and the price of stock were due solely to an income effect, a_1 would be negative. If it were due solely to an interest rate effect, a_1 would be positive. The coefficient a_2 would be negative in either case.

Fama and Schwert (1977), Fama (1981), and French, Ruback and Schwert (1983) have estimated similar regressions. In those papers, stock returns are regressed on measures of expected and unexpected inflation. They generally find a negative coefficient for both inflation variables. To see the relationship between those results and the regression above, interpret the current inflation rate as a measure of expected inflation, and interpret the change in the inflation rate as a measure of unexpected inflation. Following the interpretation, a_1 is the coefficient on expected inflation, and a_2 is the coefficient on unexpected inflation. The finding that both coefficients are negative is then evidence against the hypothesis that the stock-price/inflation correlation is due strictly to interest rate effects. It is not evidence against the endowment shock explanation, however, because that explanation implies both $\Delta p>0$ and $\Delta z>0$. Thus we cannot distinguish between the two explanations through the signs of the regression alone. To make that distinction, those coefficients must be directly connected with the deep parameters of our model, a task to which we now turn. This connection is the primary difference between our approach and others.

In our model, the state variable is the realization of a two-state, Markov process which generates real shocks coincident with inflation. The first step in implementing our tests is to relate actual inflation rates to this state variable by fitting such a process to realizations of inflation. For the two state process, we assume that inflation takes the values I_0 in the low inflation state and I_1 in the high inflation state. The transition probability is denoted by π, the mean by μ, and the standard deviation by δ. For the realizations of inflation, we assume an autoregressive process of the form

$$I_t = \phi_0 + \sum_{i=1}^{\ell} \phi_i I_{t-i} + \varepsilon_t \tag{24}$$

where I_t is the actual inflation rate in period t and ε_t is a white noise error term. The two-state process is then fitted to the realizations of inflation by equating the moments of this process with the estimated moments of equation (24). We estimated the autoregressive model (24) using annual inflation rates based on the CPI for the period 1953-1985. A second order process seemed adequate and produced the following parameter estimates (standard errors in parentheses):

$$I_t = \underset{(.006)}{0.01476} + \underset{(.166)}{1.099}\, I_{t-1} - \underset{(.163)}{0.4012}\, I_{t-2} \tag{25}$$

$$\hat{\sigma} = 0.01989$$

The Box Pierce statistic for twenty lags was Q = 12.04 which is below the critical value. The parameters of the two state representation and the corresponding estimates of the high and low inflation states are

$$\hat{\mu} = 0.0488 \qquad \hat{\delta} = 0.035 \qquad \hat{I}_0 = 0.014 \qquad \hat{I}_1 = 0.084 \tag{26}$$

The implied estimate for the transition probability is $\hat{\pi} = 0.82$.

We use the estimates of μ and δ to define the inflation state variable

$$I_t^* = \frac{(I_t - \mu)}{\delta}. \tag{27}$$

If $I_t^* = 1$, inflation is high, and if $I_t^* = -1$, inflation is low. This makes the connection between actual inflation rates and the two states of our model.

The next step in implementing our tests is to define rates of return in terms of the parameters of the model. Let ρ_{ij} denote the return on an asset held throughout a period in which the initial inflation state is i and the terminal state is j. In the notation of our model,

$$1 + \rho_{ij} = \frac{v_j + z_j}{v_i} \tag{28}$$

This equation defines four returns: ρ_{00}, ρ_{11}, ρ_{01}, and ρ_{10}. To simplify the estimation we use the approximaton x = log(1+x) which is quite reasonable for the small numbers that we are dealing with. Asset returns can then be written as:

$$\rho_{00} = r(1-\Delta p)(1-\Delta z) \tag{29}$$

$$\rho_{11} = r(1+\Delta p)(1+\Delta z)$$

$$\rho_{01} = \rho_{11} + 2(\theta-1)\ \Delta p + 2\theta\Delta z$$

$$\rho_{10} = \rho_{00} - 2(\theta-1)\ \Delta p - 2\theta\Delta z$$

Consider the regression equation (23) with I_t replaced by I^*_t as defined in (27). If $I^*_t = I^*_{t-1} = -1$, inflation is low in both initial and terminal states and the asset returns are determined as

$$a_0 - a_1 = \rho_{00} = (r+r\Delta p\Delta z) - (r\Delta p+r\Delta z). \tag{30}$$

If the inflation state in both t and t-1 is high then

$$a_0 + a_1 = \rho_{11} = (r+r\Delta p\Delta z) + (r\Delta p+r\Delta z). \tag{31}$$

If there is a transition from the low to the high inflation state or from high to low then the coefficient a_2 enters into the determination of returns:

$$r_t = r(+\Delta p\Delta z) + r(\Delta p+\Delta z)\ I^*_t + ((\theta-1)\Delta p+\theta\Delta z)(I^*_t-I^*_{t-1}). \tag{32}$$

This regression has four deep parameters, Δz, Δp, r and θ, but only three coefficients or reduced form parameters. If we now consider an equation with two assets, say, stocks and housing, and define a dummy variable

$$D_t = \begin{cases} 1 \text{ if stocks} \\ 0 \text{ if housing} \end{cases} \tag{33}$$

then the equation for asset returns can be written as

$$r_t = a_0 + a_1 I_t^* + a_2(I_t^*-I_{t-1}^*) + a_3 D_t + a_4 D_t I_t^* + a_5 D_t(I_t^*-I_{t-1}^*), \qquad (34)$$

where r_t is a vector consisting of both stock and housing returns. The coefficients a_i have the interpretation:

$$\begin{aligned} a_0 &= r(1+\Delta p\Delta zhse) \\ a_1 &= r(\Delta p+\Delta zhse) \\ a_2 &= \theta(\Delta p+\Delta Zhse) - \Delta p \\ a_3 &= r\Delta p(\Delta zstk-\Delta zhse) \\ a_4 &= r(\Delta zstk-\Delta zhse) \\ a_5 &= \theta(\Delta zstk-\Delta zhse), \end{aligned} \qquad (35)$$

where $\Delta zstk$ and $\Delta zhse$ refer to the income streams of housing and stocks respectively. This version has five deep parameters and six reduced form parameters. Assuming that $\Delta zstk$ does not equal $\Delta zhse$, the deep parameters can now be identified.

This identification condition requires the income streams from housing and stock to have different correlations with inflation. We have strong prior reasons for believing this to be the case. The income from housing, particularly owner-occupied housing, is virtually free from tax and thus also free from the non-neutralities ascribed to the corporate income tax in the nominal contracts explanation. Nor do we expect housing income to be particularly sensitive to the business cycle fluctuations that are the causal factor in the endowments shock explanation. Thus the two leading candidates for the link between stock returns and inflation do not appear to provide a similar link for housing. We conclude that housing is likely to satisfy the identification condition. In fact, our prior expectation is $\Delta zhse = 0$.

In the version of the regression reported below, three assets are actually included - stocks, housing and government bonds. That version has nine reduced form parameters and six deep parameters. If the identification condition is satisfied for any pair of assets, all the deep parameters will be identified.

The rates of return used in the estimation of (44) are real, after tax rates of return on housing, corporate stocks and government bonds from 1953-1985. The year 1953 was chosen as the starting point because it is the earliest year for which some of the data used in our calculations of the income from housing is available. The estimated rates of return on housing combine data from the CPI on rent, property taxes, insurance, maintenance and energy costs with the Bureau of Census price index of new single family homes. The latter index is based on a hedonic price function that holds constant a standard bundle of characteristics. This index is available only since 1963 so we backcast the changes in the index using the Department of Commerce composite construction cost index.

Rates of return on stocks were calculated using data from Ibbotson and Sinquefeld (1984) and Standard and Poors on rates of appreciation and dividend yields for the Standard and Poors Composite index. Rates of return on long term government bonds were calculated using the CRSP indices on capital appreciation and income on U.S. Government Bonds. All of the returns were adjusted for individual taxes on capital gains and dividends to yield nominal net rates of return. The inflaton rate was subtracted from these nominal rates to yield net real rates of return. The details of the data construction are described in Appendix B.

The results of the maximum likelihood estimation of the model with three assets are shown in Table 4. The estimates were obtained using a combination of Newton-Raphson and Davidon-Fletcher-Powell numerical algorithms. A variety of starting values were tried and the estimates are not sensitive to these. Because the dependent variable is the net real rate of return on assets with very different variances there is some heteroskedasticity in the error structure. Accordingly, the standard errors are based on White's (1982) consistent estimator of the covariance matrix.[5]

The major conclusion that emerges from Table 4 is that we cannot reject the hypothesis that $\Delta p = 0$. This result constitutes rejection of the Fama-Geske-Roll hypothesis that the correlation between asset prices and inflation is the result of real shocks to output. A decline in the aggregate endowment would be reflected in the interest rates, implying $\Delta p \neq 0$. The results also show that the implied change in the income stream of corporate stock with respect to inflation (Δzstk) is huge. Recall that, based on Feldstein and Summers (1979), a reasonable estimate of Δzstk is about -0.3, and that a Δzstk of -2.5 implies a tax rate of 125% in the high inflation state. Our estimate of Δzstk has large standard errors, however, making it difficult to reject the Feldstein-Summers explanation with a high degree of confidence. The change in the income stream of government bonds (Δzgb) is also sizable, as expected, and significant. The change in the income stream of housing is considerably smaller.

The major effect of these estimates is shown in the bottom half of Table 4 where the implied rates of return in the four states of the world are shown. All three assets have relatively low returns in stable high inflation states with housing having the only positive return. The transition from low to high

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[5] Additional concerns are raised by the use of generated regressors in that the I^* are defined using estimated moments of the two state Markov process. This does not seem to be a major cause for concern because the residual variance of the latter is quite small relative to the residual variances of the returns equation.

inflation is disastrous for stocks and bonds reflecting the costs of nominal contracting. Housing performs the best in the low to high transition but its return is still lower than in the stable low inflation state. Stocks and bonds perform the best in the transition from high to low inflation states. The relative volatility of the returns on all three assets is clearly reflected in these results with stocks being the most volatile and housing being the least. These estimates of returns by state seem to accord well with intuition and with prior empirical research.

Table 5 presents a similar set of estimates but with θ constrained to take on the value implied by the estimate of the transition probability for the two state inflation process ($\hat{\pi}$ = 0.82) and the estimate of r. There are no striking differences in these results. The estimated change in the income stream of housing (Δzhse) is smaller and not significantly different from zero. The implied estimates of rates of return are largely similar, the only difference being housing.

The results presented here appear to be robust to changes in the assumptions about the tax rates. We estimated the models without adjusting for taxes and assuming larger tax rates with no significant changes in the conclusions. We have also considered different mixes of assets and different specifications of the inflation autoregression with no significant consequences.

V. Conclusions

In an economy with complete contingent claims markets there is an observable difference between changes in stock returns due to real output shocks and changes in returns due to the existence of nominal contracts in an inflationary setting. Real output shocks change the aggregate endowment of the economy. This implies that consumption will be relatively more scarce or plentiful depending on the shock and real interest rates must change as a consequence of the real income shock. In contrast, a decline in the income stream of stock due to nominal contracting reflects simply a redistribution among agents in the economy, not social risk. If markets are complete individuals are able to insure themselves against the effects of such redistributions and no interest rate effect should be apparent. The presence or absence of this effect is the basis for our tests.

The important consequence of the complete markets assumption is that individuals are able to insure themselves against being born in a bad state of the world. This is neither more nor less controversial than arguments advanced in favor of intergenerational altruism or Ricardian equivalence. All require (or imply) that individuals are somehow insured against a bad draw or that redistributions caused by nominal contracting are non-distorting. That allocations consistent with this view are possible is a topic that is discussed in many other literatures.

The empirical results of this paper are striking. They clearly reject the view that real output shocks are the source of inflation stock-price correlation. There is no significant interest rate affect associated wtih the relationship between asset returns and the inflation state. This also seems to be evidence (albeit indirect) in favor of the complete markets framework because, in the absence of that assumption, real interest effects would be associated with either of the hypotheses considered here.

Although we do not test the nominal contracting hypothesis directly it seems clear from our analysis that, absent the associated interest rate changes, the implied changes in the income streams of stocks necessary to account for the change in returns seems too large to be explained as consequence of nominal contracting. Our analysis of the asset pricing equations implies, that for reasonable degrees of impatience on the part of asset holders, and reasonable degrees of persistence in the aggregate shocks, a 50% in stock prices would be difficult to explain either as the consequence of endowment shocks or nominal contracting.

The underlying reason for the large negative correlation between stock returns and inflation remains a puzzle. It may be that attempts to isolate systematic relations between stock returns and aggregate variables such as inflation and to estimate the deep parameters underlying those relationships are hampered by the excess volatility in stock returns that has been extensively documented elsewhere.

Table 1: Calculations of θ for Values of π and r

r	π					
	0.25	0.5	0.75	0.9	0.95	1.0
0.025	-0.0082	0	0.0238	0.0471	0.1800	1
0.05	-0.0161	0	0.0455	0.0889	0.3000	1
0.10	-0.0313	0	0.0833	0.1600	0.4500	1

$$\theta = \frac{(2\pi - 1)\ r}{2(1 - \pi) + r}$$

Table 2: Calculations of v_1/v_0 for Values of θ, Δz, and Δp

θ	Δp	Δz					
		0	-0.5	-1.0	-1.5	-2.0	-2.5
0.025	0	1.0	0.98	0.95	0.93	0.90	0.88
"	0.1	0.82	0.80	0.78	0.75	0.73	0.70
"	0.2	0.67	0.66	0.63	0.61	0.57	0.53
"	0.3	0.55	0.53	0.51	0.48	0.44	0.34
0.1	0	1.0	0.90	0.82	0.74	0.67	0.60
"	0.1	0.83	0.75	0.67	0.59	0.50	0.42
"	0.2	0.69	0.62	0.55	0.46	0.36	0.25
"	0.3	0.57	0.51	0.44	0.35	0.22	0.03

$$v_1/v_0 = \frac{(1 + \Delta p\Delta z + \theta(\Delta p + \Delta z))(1 - \Delta p)}{(1 + \Delta p\Delta z - \theta(\Delta p + \Delta z))(1 + \Delta p)}$$

Table 3: Calculations of r_1 and r_0 for Values of r, π and Δp

r	π	Δp	r_1	r_0
0.025	0.75	0.1	0.074	-0.029
"	"	0.2	0.118	-0.089
"	"	0.3	0.159	-0.156
0.025	0.90	0.1	0.044	0.003
"	"	0.2	0.060	-0.024
"	"	0.3	0.075	-0.056
0.05	0.75	0.1	0.100	-0.005
"	"	0.2	0.145	-0.067
"	"	0.3	0.187	-0.135
0.05	0.90	0.1	0.069	0.027
"	"	0.2	0.086	0.000
"	"	0.3	0.101	-0.033

$$r_1 = \frac{(1 + r)(1 + \Delta p)}{1 + (2\pi - 1)\,\Delta p} - 1$$

$$r_0 = \frac{(1 + r)(1 - \Delta p)}{1 - (2\pi - 1)\,\Delta p} - 1$$

Table 4: Maximum Likelihood Estimates

Regression

Parameter	**Estimate**	**Std. Error**	**t-Stat**
r	0.0256	0.0099	2.5644
θ	0.0678	0.0635	1.0673
Δp	-0.0599	0.0668	0.8958
$\Delta zstk$	-2.3694	1.6506	1.4355
$\Delta zhse$	-0.6257	0.4416	1.4170
Δzgb	-1.6349	0.8574	1.9069

Asset Returns

Inflation	Assets		
	Stocks	Housing	Gov't Bonds
Low-Low	0.091	0.044	0.072
High-High	-0.033	0.009	-0.015
Low-High	-0.242	0.036	-0.125
High-Low	0.301	0.017	0.182

Table 5: Maximum Likelihood Estimates with θ Constrained

Regression

Parameter	Estimate	Std. Error	t-Stat
r	0.0269	0.0104	2.584
Δp	-0.0194	0.2878	0.675
$\Delta zstk$	-2.5565	1.5013	1.703
$\Delta zhse$	-0.3793	0.4392	0.864
Δzgb	-1.6049	0.7884	2.036

Asset Returns

Inflation	Assets		
	Stocks	Housing	Gov't Bonds
Low-Low	0.098	0.038	0.071
High-High	-0.041	0.016	-0.016
Low-High	-0.232	0.020	-0.122
High-Low	0.288	0.034	0.177

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Appendix A

Existence of Stationary Equilibrium

The parameter r plays two different roles in equilibrium: it determines the allocation of the endowment between the young and the old in the Edgeworth Box diagram, and it determines the marginal utility of the young to that of the old through equation (9). In what follows, we prove that a stationary equilibrium exists by showing that there is a value of r that reconciles these two roles.

Consider the first role. We wish to construct a mapping from the contract curve of the Edgeworth Box to the value of r needed to yield the endowments necessary to support points on that curve as equilibria. We shall index the contract curve by the utility of the young, denoted by u. A point on that curve is a pair of consumptions for the young and the old, denoted by $(x_0(u),x_1(u))$ and $(y_0(u),y_1(u))$. Each point on the curve determines a unique ratio between the price of good 1 and the price of good 0. Let p(u) be this price ratio. To support the allocation associated with this point, the parameter r must set the value of the young's consumption equal to the value of its endowment. This requires

$$r^{-1} = \frac{\ell_0 - x_0(u) + p(u)(\ell_1 - x_1(u))}{k_0 + p(u)k_1} \tag{A1}$$

This equation defines a continuous function from the contract curve to r^{-1}, which we denote by f(u). Now consider the value of that function at the lower extreme of the contract curve at which the young consumes nothing and the old consumes the entire endowment. Denote this point by $\underline{u}$ and note that

$$f(\underline{u}) = \frac{\ell_0 + p(\underline{u})\ell_1}{k_0 + p(\underline{u})k_1} \tag{A2}$$

which is a positive number. At the other extreme of the contract curve, the young consumes the entire endowment and the old consumes nothing. Denote this point by $\bar{u}$ and note that

$$f(\bar{u}) = - \frac{k_0 + m_0 + p(\bar{u})(k_1+m_1)}{k_0 + p(\bar{u})k_1} \tag{A3}$$

which is a number less than -1. The function f is depicted in the first panel of Figure 2. Since $f(\underline{u})$ is positive, $f(\bar{u})$ is negative, and f is continuous, there is a point, u*, at which f is zero. The function f is depicted as monotonically decreasing, although this need not be the case.

The consequences for r of this relationship between u and r^{-1} are depicted in the second panel of Figure 2. When r^{-1} approaches zero from above, as when u approaches u* from the left, r must go to infinity. On the other hand, when r^{-1} approaches zero from below, as when u approaches u* from the right, r must go to negative infinity.

The second role for r is to determine the ratio between the marginal utility of the young and the marginal utility of the old. This is the equation

$$1 + r = \frac{u'(x_i)}{v'(y_i)} . \tag{A4}$$

This equation defines another mapping from the contract curve to r,

$$g(u) = \frac{u'(x_0(u))}{v'(y_0(u))} - 1 \tag{A5}$$

The function y is monotonically decreasing in u because an increase in u causes u' to decrease and v' to increase. Since we have assumed that $\lim_{x\to 0} u'(x) = \infty$ and $\lim_{y\to 0} u'(y) = \infty$, it follows that $\lim_{u\to \underline{u}} g(u) = \infty$ and $\lim_{u\to \bar{u}} g(u) = -1$. The function g is depicted in the second panel of Figure 2.

We are now in a position to argue that there must be at least one value of u for which $g(u) = f(u)^{-1}$. Just to the right of $\underline{u}$, g(u) must exceed $f(u)^{-1}$, and just to the left of u^*, $f(u)^{-1}$ must exceed g(u). This implies that there must be a value of u between $\underline{u}$ and u^* that satisfies the equality. This will be a stationary equilibrium. Note further that the value of r must be positive in this equilibrium.

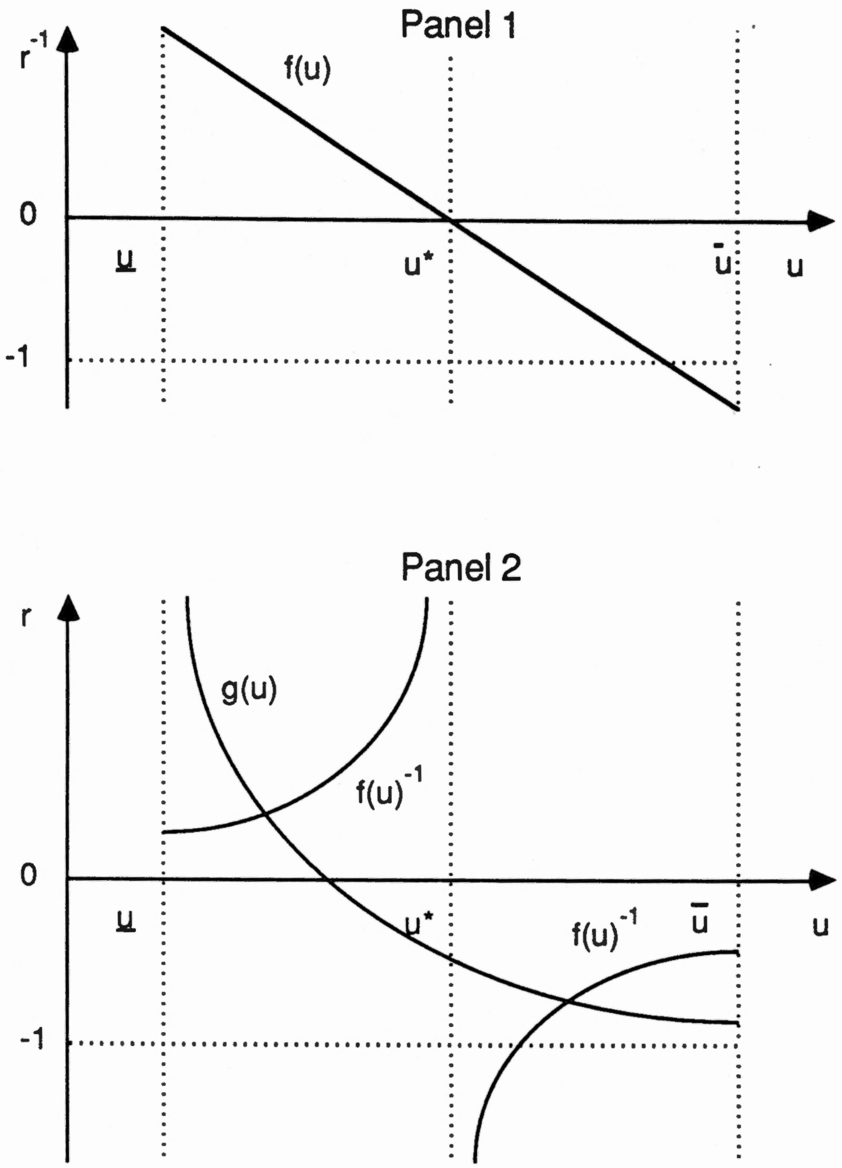

Figure 2

The Equilibrium Interest Rate

Appendix B
Calculation of Rates of Return

I. Description of Housing Variables

A. YEAR - year of observation

B. VALUE -

1. 1963-1983 - Bureau of Census Price Index for New Single Family Homes.
2. 1952-1962 - A regression of the real change in the Bureau of Census Price Index on the real change in the Department of Commerce composite construction cost index and the real change in the home purchase component of the Consumer Price Index was run using data from 1963-1973. This regression equation was then used to back cast the Bureau of Census Index for 1952-1962 using the Department of Commerce Index and the home purchase component for those years.

C. RENT - Consumer Price Index for Residential Rent

D. CPI - Consumer Price Index for All Commodities

E. MAINT - Consumer Price Index for Shelter Maintenance and Repairs

F. FUEL - Consumer Price Index for total fuel and other utilities

II. Normalizations

A. RENT - Rent series was normalized so that 1980 RENT divided by the 1980 VALUE is 0.07, which was the median rent to value of residential housing.[1]

B. MAINT - MAINT was normalized so that 1979 MAINT divided by 1979 VALUE equals 0.0066, which was the ratio of maintenance to the value of the median home in 1979.[2]

III. Calculation of Housing Rates of Return

A. Property Taxes - The property tax rate for the median-valued, new, one-family, sec. 203, house in 1979 was 1.4% of value.[2] Given a marginal federal income tax rate of 0.324 which is the rate assumed in calculating rates of return for stock,[3] this amounts to a net property tax rate of 0.9464.

B. Hazard Insurance - Hazard insurance was 0.33% of the value of the median home in 1979.[2]

C. The Rate of Return Calculation is thus:

$$RTOT_t = \frac{RENT_t}{VALUE_{t-1}} \qquad \text{(gross income)}$$

$$+ (VALUE_t - VALUE_{t-1})/VALUE_{t-1} \qquad \text{(nominal appreciation)}$$

$$- MAINT_t/VALUE_{t-1} \qquad \text{(cost of maintenance)}$$

$$- FUEL_t/VALUE_{t-1} \qquad \text{(cost of fuel)}$$

$$- (CPI_t - CPI_{t-1})/CPI_{t-1} \qquad \text{(inflation rate)}$$

$$- 0.009464 \qquad \text{(property tax rate)}$$

$$- 0.0033 \qquad \text{(hazard insurance rate)}$$

IV. Description of Stock Variables

A. PSRICE - The December value of the Standard and Poor's Composite Stock Price Index (500 stocks).[4]

B. YIELD - Percentage annual yield of the Standard and Poor's Composite Stock Price Index (500 stocks).[4]

V. Calculation of Stock Rates of Return

A. Taxes - The tax rate on dividend income is assumed to be 0.324 and the tax rate on nominal capital gains is assumed to be 0.0648.[3]

B. The calculation is

$$SRETURN_t = (1-0.324) \cdot YIELD_t \qquad \text{(net yield)}$$

$$+ (1-0.0648) \cdot (SPRICE_t - SPRICE_{t-1})/SPRICE_{t-1} \qquad \text{(net capital gains)}$$

$$- (CPI_t - CPI_{t-1})/CPI_{t-1} \qquad \text{(inflation)}$$

VI. Calculation of Government Bond Rates of Return

Capital gains and income returns series are taken from the CRSP Indices file. These are used to calculate net returns using the expression used for stocks.

Sources:

1. Census of Housing, 1980, Residential Finance, TAble 1a.
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