Monotonic Allocation Mechanisms

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by

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1. *Introduction*

The Walrasian correspondence has been shown to have the following undesirable properties: the transfer of part of some agent's initial resources to another may make the donor better off and the recipient worse off (this is the classic "transfer paradox"); an agent may be made worse off when the initial resources of another agent increase (Thomson, 1978); an agent may be made better off by withholding part of his initial resources prior to trading (Postlewaite, 1979); an agent may even be made better off by destroying part of his initial resources prior to trading (Aumann and Peleg, 1974).

The next two results pertain to situations in which agents are collectively entitled to all the goods available in the economy, and concern the Walrasian correspondence operated from equal division: the arrival of additional agents, unaccompanied by an increase in the aggregate resources, may make one of the agents initially present better off, (Chichilnisky and Thomson, 1987); an increase in the aggregate resources may make some agent worse off (Thomson, 1978).

The purpose of this paper is to investigate whether negative results of this sort are specific to the Walrasian correspondence. Our main conclusion is that unfortunately, many are not.

As in the examples described above, we consider two situations: first, the standard situation where each agent is initially endowed with some bundle of resources over which he has control. The issue here is to reallocate these endowments so as to achieve efficiency as well as some minimal incentive and fairness conditions. Second, we discuss the classical problem of fair division There, all agents are collectively entitled to some aggregate bundle and an efficient and equitable allocation of this bundle must be found.

What is a fair trade or an equitable allocation is, of course, subjective. The notion that has so far played the principal role in the literature is the no-envy notion, proposed by Foley (1967). Another important concept is egalitarian-equivalence, introduced by Pazner and Schmeidler (1978). We will discuss both concepts as well as others, but comprehensiveness was not our only objective. Indeed, as noted above, many of our results are negative and in situations where it might be particularly desirable to have one or several of the monotonicity conditions satisfied, we may not be able to afford the luxury of using the concept that we would prefer. Do some notions perform better than others? Yes, egalitarian-equivalence performs uniformly better than no-envy. As a by-product of our analysis, we will therefore obtain some information on the main equity notions that should help informing the on-going debate on their relative merits.

The monotonicity conditions we consider are motivated by a variety of incentive and fairness considerations. The requirement that no agent be ever made worse off by an increase in his own initial endowment is desirable if initial endowments result from efforts agents have exerted. A violation of the property might induce agents to sometimes destroy part of the resources they control. This may be a serious problem from the viewpoint of overall social welfare since the right to destroy an object or a resource usually comes with the ownership of that object or that resource. Similarly, manipulation by withholding part of what is ours is in violation of no ownership law. The requirement that no agent be made worse off by an increase in someone else's initial endowment is perhaps less compelling, but it should help agents resist the temptation to tamper with their neighbor's property. Also, when the increase in an agent's endowment is due not to his merit but to circumstances beyond his control (inheritance, luck), why should others be penalized for his good fortune? One might expect social stability to be negatively affected by the resentment felt by agents that lose when their neighbors' resources increase. The desirability of avoiding the transfer paradox has been abundantly discussed in the international trade literature; the transfer problem was perceived as a serious practical problem that arose in connection with the payment of war reparations at the end of World War I.

In the context where resources are owned collectively, a minimum amount of solidarity among agents seems to imply that an increase or a decrease in the resources available to them would affect them all similarly; or, that an increase (or a decrease) in their numbers, keeping resources fixed, would affect all agents initially present (or all remaining agents) similarly.

Our results, which should deepen our understanding of the "paradoxes" enumerated above, complement general results by the following authors: Postlewaite (1979), who showed that vulnerability to strategic behaviour through withholding holds not only for the Walrasian correspondence but in fact for a wide class of correspondences; Chun and Thomson (1984), who examined the monotonocity of bargaining solutions when applied to economic problems of fair division; Roemer (1985, 1986), in whose work monotonicity properties play a central role; and Moulin and Thomson (1987), where the incompatibility of certain criteria of monotonicity and equity is established.

We conclude this introduction with a comment on the method of proof of the negative results (the proofs of the positive results are all straightforward). These proofs are by way of counterexamples. The counterexamples are chosen as simple (they involve only two goods and two agents) and as well-behaved (they involve monotone, convex, and even homothetic preferences) as possible. It is natural to expect that sufficiently strong additional restrictions on preferences would lead to possibility theorems. Indeed, Polterovich and Spivak (1980, 1983) have been able to identify domain restrictions guaranteeing that the Walrasian correspondence is immune to some of the paradoxes discussed above. Much more work of this kind remains to be done to map out the boundary between what is possible and what is not for other correspondences. Moulin (1987) contains some results along those lines.

2. *Notation - Definitions*

We will consider both the problem of allocating gains from trade among agents starting out with possibly different initial endowments, and the problem of allocating an aggregate endowment among agents with equal claims on it. Our terminology is chosen so as to help the reader keep in mind the distinction between these two problems.

There are ℓ goods and n agents. Agents are indexed by the subscript i. Agent i's preferences are represented by the continuous utility function u_i: $\mathfrak{R}_+^\ell \to \mathfrak{R}$. Let $u \equiv (u_1,\ldots,u_n)$. An ***economy in an initial position*** is a pair (u,ω), where $\omega \equiv (\omega_1,\ldots,\omega_n) \in \mathfrak{R}_+^{\ell n}$ is a list of ***initial endowments***. An ***economy*** is a pair (u,Ω), where $\Omega \in \mathfrak{R}_+^\ell$ is an ***aggregate endowment***. $A(\omega) \equiv \{z \in$

$\mathbb{R}_+^{\ell n} | \Sigma z_i = \Sigma \omega_i\}$ is the ***feasible set*** of (u,ω), and $A(\Omega) \equiv \{z \in \mathbb{R}_+^{\ell n} | \Sigma z_i = \Omega\}$ that of (u,Ω). Given a domain E of economies in an initial position, a ***correspondence on*** E is a mapping associating with every $(u,\omega) \in E$ a non-empty subset of $A(\omega)$. A correspondence on E, φ, is ***essentially single-valued*** if for all $(u,\omega) \in E$, for all $z, z' \in \varphi(u,\omega)$, and for all i, $u_i(z_i) = u_i(z_i')$. The concepts of a correspondence on a domain of economies, (as opposed to a domain of economies in an initial position), and of a single-valued correspondence on such a domain, are defined in a similar way. An essentially single-valued subcorrespondence of a given correspondence is called a ***selection***. (To indicate that φ is a subcorrespondence of φ' we will write $\varphi \subset \varphi'$.) We will consider selections from the intersection of the Pareto correspondence with the correspondences that are the most widely discussed in the literature on the fair division problem:

P, the ***Pareto correspondence***: $P(u,\Omega) \equiv \{z \in A(\Omega) | \nexists\ z' \in A(\omega)$ with $u_i(z_i') \geqq u_i(z_i)$ $\forall i$ and $u_i(z_i') > u_i(z_i)$ for at least one $i\}$ ($P(u,\omega)$ is defined analogously).

D, the ***no-domination correspondence***: $D(u,\Omega) \equiv \{z \in A(\Omega) | \nexists\ i,j$ with $z_i \geq z_j\}$[1].

I, the ***individually rational correspondence from equal division***: $I(u,\Omega) \equiv \{z \in A(\Omega) | u_i(z_i) \geqq u_i(\Omega/n)\ \ \forall i\}$.

F, the ***envy-free correspondence*** (Foley, 1967): $F(u,\Omega) \equiv \{z \in A(\Omega) | u_i(z_i) \geqq u_i(z_j)\ \ \forall i,j\}$.

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[1]Vector inequalities: $x \geqq y$, $x \geq y$, $x > y$.

E^2, the *egalitarian-equivalent correspondence* (Pazner and Schmeidler, 1978): $E^2(u,\Omega) \equiv \{z \in A(\Omega) | \exists\, z_0 \in \Re^\ell_+$ such that $u_i(z_i) = u_i(z_0) \;\; \forall i\}$.

W, the Walrasian correspondence from equal division.

The following five correspondences are the counterparts of the above five for the problem of fair allocation of gains from trade. (The counterpart of φ is denoted $\bar{\varphi}$.)

$\bar{D}$, the *no-domination trade correspondence*: $\bar{D}(u,\omega) \equiv \{z \in A(\omega) | z = \omega + t$, where $t \in \Re^{\ell n}$ is such that $\nexists\, i,j$ with $t_i \geq t_j\}$.

$\bar{I}$, the *individually rational correspondence*: $\bar{I}(u,\omega) \equiv \{z \in A(\omega) | u_i(z_i) \geqq u_i(\omega_i) \;\forall i\}$.

$\bar{F}$, the *envy-free trade correspondence* (Kolm, 1972; Schmeidler and Vind, 1972): $\bar{F}(u,\omega) = \{z \in A(\omega) | z = \omega + t$, where $t \in \Re^{\ell n}$ is such that $\nexists\, i,j$ with $u_i(\omega_i+t_j) > u_i(z_i)\}$.

$\bar{E}^2$, the *egalitarian-equivalent trade correspondence*: $\bar{E}^2(u,\omega) \equiv \{z \in A(\omega) | \exists\, t_0 \in \Re^\ell$ with $u_i(z_i) = u_i(\omega_i + t_0) \;\; \forall i\}$.

$\bar{\mathbf{W}}$, the Walrasian correspondence.

Finally, we will need the following notation: $\pi(A,B)$ is the *symmetric image of A with respect to B*. (A may be a point or a set, and B may be a point or a straight line.) The intersection of two correspondences φ and φ' is denoted $\varphi\varphi'$. Given a correspondence φ and an agent i, φ_i is the projection of φ onto agent i's consumption space: $\varphi_i(u,\omega) \equiv \{z_i' \in \Re^\ell_+ | \exists z \in \varphi(u,\omega)$ with $z_i' = z_i\}$. Λ is the 45° line. $\Delta^{\ell-1}$ is the $(\ell-1)$-dimensional simplex. Given $z_i \geq 0$, $R(z_i)$ is the ray passing through the origin and z_i. In the figures illustrating the proofs, a small segment centered at a point z_i indicates a tangency line to agent i's indifference curve through z_i. The slope of that tangency line is indicated in parentheses next to z_i.

3. *The Results*

The methodology we follow is that pioneered by Hurwicz (1972) when he showed that there is no selection φ from $\overline{I}P$ (the individually-rational and Pareto-efficient correspondence), such that no agent would ever gain by misrepresenting his preferences, assuming all other agents behave honestly. Hurwicz proved this by constructing a well-behaved two-good, two-person economy (u,ω) in which agents have identical preferences, and endowments that are symmetric with respect to the 45° line. Such an economy has a Walrasian allocation $\bar{z}$ with $\bar{z}_1 = \bar{z}_2$. Since φ is a subcorrespondence of the Pareto-efficient correspondence, then if $z \in \varphi(u,\omega)$, either (i) $u_1(z_1) \leqq u_1(\bar{z}_1)$ or (ii) $u_2(z_2) \leqq u_2(\bar{z}_2)$. Assuming (i) first, Hurwicz shows that agent 1 could misrepresent his preferences so as to be guaranteed to be better off than at $\bar{z}_1$, and therefore than at z_1. Then, assuming (ii), the symmetry of the construction implies that agent 2 could misrepresent his preferences so as to be guaranteed to be better off than at $\bar{z}_2$, and therefore than at z_2. As a consequence, it really suffices to compare to $\bar{z}$ what φ would produce after the change in preferences.

Here we consider changes of endowments, with fixed preferences, (for that reason, it is more convenient not to use the Edgeworth box), but, in our proofs of impossibilities, it is also sufficient to compare the allocation obtained by operating φ after the change in endowments to a similarly defined reference allocation $\bar{z}$.

We investigate the existence of well-behaved correspondences defined over domains of economies with an arbitrary number of commodities or agents. We establish our negative results by way of examples of economies with 2 commodities and 2 agents. In addition to the usual properties of convexity and monotonicity, the agents' preferences are homothetic. Of course, the simpler the economy used in a counterexample, the more serious is the problem that this counterexample illustrates. Since economies with homothetic preferences are quite well-behaved in general, this domain restriction strengthens our results. From a technical viewpoint, operating under this extra constraint has a cost, but this cost is somewhat compensated by two facts which simplify other aspects of the proof. First, we will need to specify only one indifference curve (since the others can be obtained by homothetic expansions and contractions). Second, and more importantly, the Pareto-efficient set has a much simpler structure[2] in such economies.

The counterexamples are specified mainly geometrically, and we have chosen to present them in such a way as to make as intuitive as possible the way they were arrived at. We do not give explicit analytical expressions for the utilities. Such expressions would be quite complicated and lengthy without shedding much additional light on the nature of our results.

This paper does contain some positive results, which of course are proved under standard assumptions (for arbitrary numbers of commodities and agents and without requiring homotheticity of preferences).

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[2]Indeed, under the additional assumption that the preferences of each agent are strictly convex in the cone of consumptions where his indifference curves admit lines of support that are neither vertical nor horizontal, the Pareto-efficient set is a curve (a one-dimensional manifold) connecting the origins of the Edgeworth box, and that curve is upward sloping. Therefore, its projection onto the consumption space of either one of the agents is an upward sloping curve having the origin and the aggregate endowment as end-points. This fact is well-known to international trade theorists.

We are now ready to state the results.

In Sections 3.1, 3.2, 3.3 and 3.4, we study the standard situation where each agent has control over his initial endowment. In Sections 3.5 and 3.6 we consider the case where agents are collectively entitled to all the goods available.

3.1. *Withholding*. First, we investigate the existence of correspondences that are immune to manipulation through withholding. Our starting point is the following negative result, due to Postlewaite (1979).

Theorem 1. There is no selection from $\bar{I}P$ such that no agent ever gains by withholding part of his initial endowment.

In the two-person, two-commodity example used by Postlewaite to prove this result, each agent i has preferences that are obtained by translation of a sample indifference curve parallel to the i^{th} axis. Theorem 2, which also has to do with withholding, differs from Theorem 1 in two ways. First, it is proved by way of an example with homothetic preferences, a more standard domain restriction, also used in our other results, with which comparison will therefore be facilitated.

Second, and more importantly, we show that it is possible to go beyond the qualitative result of Postlewaite's and to quantify the extent to which the agent who withholds can gain: suppose the agent is able to recover only the proportion $1 \geqq \epsilon \geqq 0$ of what he has withheld. *We show that no matter how small ϵ is, provided it is positive, there are economies in which manipulation through withholding is profitable*. If $\epsilon = 0$, then "withholding" is the same thing as destroying. Since there are subcorrespondences of $\bar{IP}$ that are immune to manipulation by destruction (see Theorem 5), Theorem 2 reveals the existence of a sharp discontinuity at $\epsilon = 0$.

Theorem 2. Let $1 \geq \epsilon > 0$ be given. There is no selection from $\bar{IP}$ such that no agent ever gains by withholding part of his initial endowment, even if he recovers only an ϵ percentage of what he withholds.

Proof: The proof is by way of an example of a two-good, two-person economy in which agents have homothetic preferences with $u_2(z_2) = u_1(\pi(z_2,A))$ for all $z_2 \in \mathfrak{R}^{\ell}_{+}$. Most of the proof is devoted to the construction of one of agent 1's indifference curves. The specification of agent 1's preferences is completed by subjecting th indifference curve to homothetic transformations, and agent 2's preferences are obtained by subjecting agent 1's preferences to a symmetry with respect tc

See Figure . Let $\varphi \subset \bar{IP}$ be given. Initially, $\omega_1 = \omega_2 \in A$. Given ω_1' with $\omega_{11}' = \omega_{11}$ and $\omega_{12}' < \omega_{12}$, let $\bar{z}_1 \in R(\omega_1')$ be such that $\bar{z}_{11} + \bar{z}_{12} = 2\omega_{11}$ and let $\tilde{z}_1 \equiv (\bar{z}_{11},\omega_{12}')$. Let ω_1' as above be such that in addition, $(\bar{z}_{12} - \tilde{z}_{12})/(\omega_{12} - \omega_{12}') = \epsilon/2$. This is possible since this ratio approaches 0 with ω_{12}'. Then, let $x_1 \in \mathfrak{R}^2_+$ be such that $x_{11} = \bar{z}_{11}$, $x_{12} < \tilde{z}_{12}$ and $x_{12} + \epsilon(\omega_{12} - \omega_{12}') > \bar{z}_{12}$. Also, let $\omega' \equiv (\omega_1',\omega_2)$.

We will now specify agent 1's indifference curve through x_1, I. Our objective is to make (i) x_1 the consumption of $\bar{IP}(u,\omega')$ that agent 1 likes the least and (ii) $\bar{z}_1$ the maximizer of u_1 on the line through ω_1 of slope -1. For (i) to hold, I should pass through ω_1'. Given that $\bar{z}_1 \in R(\omega_1')$, for (ii) to hold, I should admit at ω_1' a line of support of slope -1. This is by homotheticity of preferences. Let $x_2 \equiv \omega_1' + \omega_2 - x_1$. Let p be the slope of the segment $[\omega_1', x_1]$ (note that $p < 0$)and let $p' \in \mathfrak{R}$ be such that $\max\{-1, p\} < p' < 0$. We require p' to be the slope of I at x_1. Then, for $x \equiv (x_1, x_2)$ to be in $\varphi(u, \omega') \subset P(u, \omega')$, as required by (i), agent 2's indifference curve through x_2 should have at x_2 a line of support of slope p'. By symmetry of the preferences, agent 1's indifference curve through $\pi(x_2, \Lambda)$ has at that point a line of support of slope $1/p'$ and therefore, by homotheticity of preferences, I has a line of support of slope $1/p'$ at its point of intersection with $R(\pi(x_2, \Lambda))$. Note that $\pi(x_2, \Lambda)$ lies above $R(\omega_1')$ (in fact, $\pi_2(x_2, \Lambda) = \bar{z}_{12}$). Also, by the choice of p', $1/p' < -1$. All the constraints on I are compatible, as indicated by the strictly convex example represented in Figure 1.

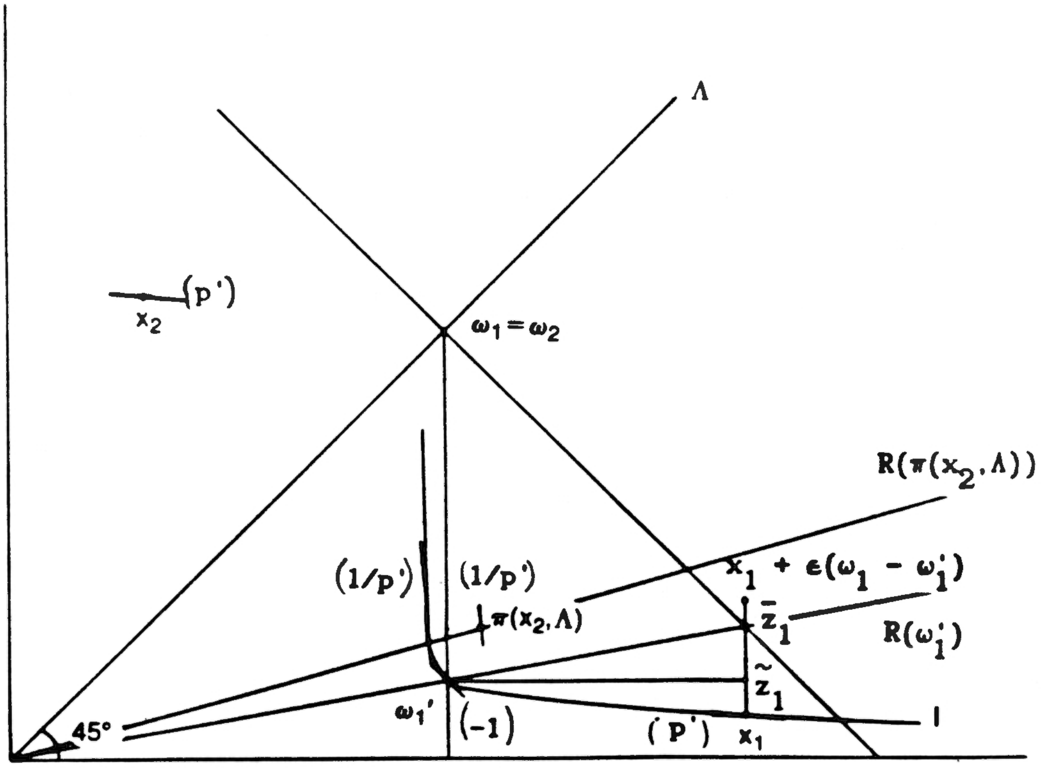

Figure 1

Now, let $z' \in \varphi(u,\omega')$ be given. As explained earlier (see our discussion of the Hurwicz result), it suffices to compare z' to $\bar{z} \equiv (\bar{z}_1, \pi(\bar{z}_1, \Lambda))$. Given (i) and since $\varphi \subset \bar{I}P$, $u_1(z_1') \geqq u_1(x_1)$. Since preferences are strictly convex and homothetic, $P_1(u,\omega')$ is an upward sloping curve (recall that $P_1(u,\omega')$ designates the projection of $P(u,\omega')$ onto agent 1's consumption space) so that if $z' \in \bar{I}P(u,\omega')$, then $z_1' \geqq x_1$, implying $z_1' + \epsilon(\omega_1 - \omega_1') \geqq x_1 + \epsilon(\omega_1 - \omega_1')$, and therefore $u_1(z_1' + \epsilon(\omega_1 - \omega_1')) > u_1(\bar{z}_1)$.

Q.E.D.

The proof of the next result, which concerns selections from $\bar{F}P$, is by way of a two-person example, just like the proof of the previous one. Since, in that case, $\bar{F} \supset \bar{I}$, the second proof is more general than the first. However, we have kept the direct proof of the first result since it is simpler and it might be a useful stepping stone to the proof of the second result. However, in order to simplify the Figures in this proof as well as in the proofs of the following results, we have not drawn strictly convex preferences.

***Theorem* 3.** There is no selection from $\bar{F}P$ such that no agent ever gains by withholding part of his initial endowment, even if he recovers only an ϵ percentage of what he withholds.

***Proof*.** The proof is carried out in the way specified in the first paragraph of the proof of Theorem 2. See Figure 2. Let $\varphi \subset \bar{F}P$ be given. Initially, $\omega_1 = \omega_2 \in \Lambda$. Given ω_1' with $\omega_{11}' = \omega_{11}$ and $\omega_{12}' < \omega_{12}$, let $\bar{z}_1 \in R(\omega_1')$ be such that $\bar{z}_{11} + \bar{z}_{12} = 2\omega_{11}$ and let $y_1 \equiv \pi(\bar{z}_1, \omega_1')$. Let ω_1' as above be such that in addition $(\bar{z}_{12} - y_{12})/(\omega_{12} - \omega_{12}') = \epsilon/2$. This is possible since this ratio approaches 0 with ω'_{12}. Then, let $x_1 \in \Re^2_+$ be such that $x_{11} = \bar{z}_{11}$ and $x_{12} = y_{12}/2$. Let $\omega' \equiv (\omega'_1, \omega_2)$.

We will now specify agent 1's indifference curve through x_1, I. Our objective is to make (i) x_1 the consumption of $\overline{FP}(u,\omega'_1)$ that agent 1 likes the least and (ii) $\bar{z}_1$ the maximizer of u_1 on the line through ω_1 of slope -1. For (i) to hold, I should go through $\pi(x_1,\omega'_1)$ and for (ii) to hold, I should admit at its point of intersection with $R(\bar{z}_1) = R(\omega'_1)$ a line of support of slope -1. Let p be the slope of $[y_1,x_1]$. I will have a line of support of slope p at x_1. If $\omega'_1 < 1/2$, as we also assume, $p > -1$. For x to be in $\varphi(u,\omega') \subset P(u,\omega')$, as required by (i), agent 2's indifference curve through $x_2 \equiv \omega'_1 + \omega_2 - x_1$ has a line of support of slope p at x_2. By the symmetry of preferences, agent 1's indifference curve through $\pi(x_2,\Lambda)$ has a line of support of slope 1/p at that point. Note that $\pi(x_2,\Lambda)$ is above $R(\omega'_1)$ (in fact, $\pi_2(x_2,\Lambda) = \bar{z}_{12}$) and that $1/p < -1$. Let $\tilde{x}_1$ be the intersection of the line of slope 1/p through $\pi(x_1,\omega'_1)$ with $R(\omega'_1)$ and x'_1 be the intersection of the line of slope -1 through $\tilde{x}_1$ with the line passing through x_1 and y_1. Note that $x'_1 \in [x_1,y_1]$. Finally, we take I to be the union of the segment

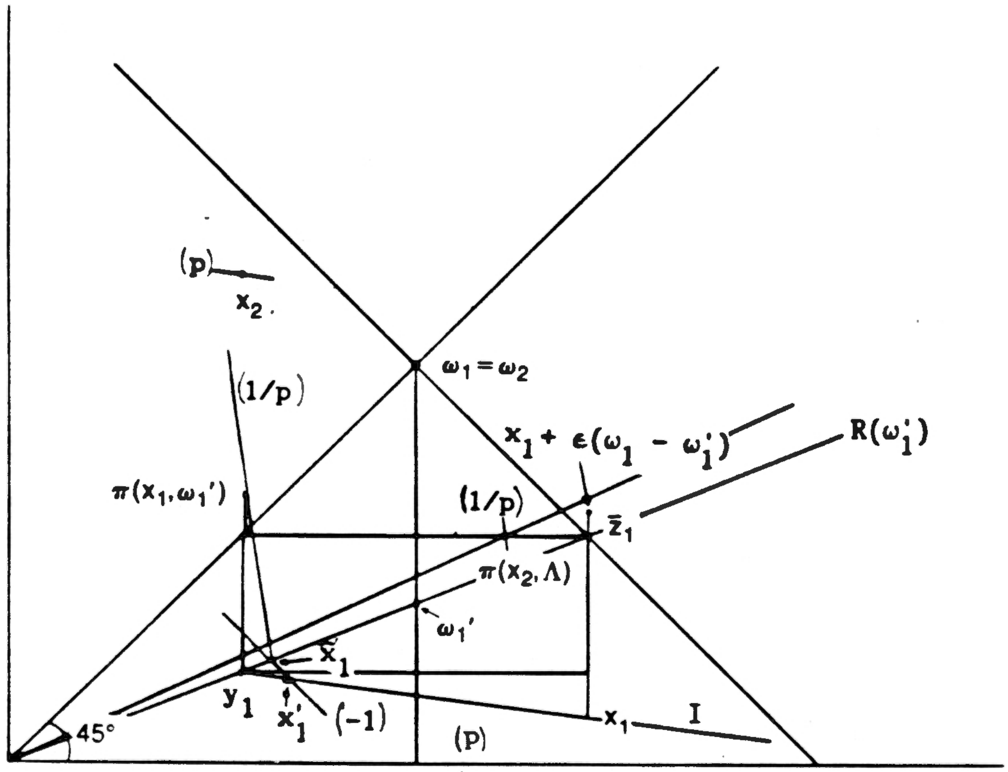

Figure 2

$[\tilde{x}_1, x_1']$ and of the linear extensions toward the horizontal and vertical axes respectively of the segments $[x_1', x_1]$ and $[\tilde{x}_1, \pi(x_1, \omega_1')]$.

It remains to take an appropriate strictly convex approximation to I has all the desired properties. If $z' \in \overline{F}P(u, \omega')$, then by (i) $u_1(z_1') \geqq u_1(x_1)$. Since preferences are strictly convex and homothetic, $P_1(u, \omega')$ is upward sloping and $z_1' \geq x_1$. Note that $z_1' + \epsilon(\omega_1 - \omega_1') \geq \bar{z}_1$, and $u_1(z_1' + \epsilon(\omega_1 - \omega_1')) > u_1(\bar{z}_1)$.

Q.E.D.

Theorem **4.** There is no selection from $\bar{\bar{E}}^2P$ such that no agent ever gains by withholding part of his initial endowment.

Proof. Here too, the first elements of the proof are specified as in the first paragraph of the proof of Theorem 2. See Figure 3. Let $\varphi \subset \bar{E}^2P$ be given. Initially, $\omega_1 = \omega_2 \in \Lambda$. $\bar{z}_1$ is the maximizer of u_1 on the line of slope -1 through ω_1, and $\bar{z}_2 \equiv \pi(z_1,\Lambda)$. Agent 1 withholds the amount $\omega_1 - \omega_1'$. Let $\omega' \equiv (\omega_1',\omega_2)$.

First, we identify the allocation $x \in \varphi(u,\omega')$ that agent 1 likes the least. Since $\varphi \subset E^2$, (i) agent 2's indifference curve through x_2, J, translated by the amount $\omega_1' - \omega_1$, is tangent to agent 1's indifference curve through x_1, I, and lies above it. Also, since $\varphi \subset P$, (ii) I and J admit parallel lines of support at x_1 and x_2 respectively, of slope denoted p. By the symmetry of the construction, agent 1's indifference curve through $\pi(x_2,\Lambda)$ admits there a line of support of slope 1/p. Note that $\bar{z}_1$ lies below $R(\pi(x_2,\Lambda))$ and above $R(x_1)$. The indifference curves I and J of Figure 5 satisfy all the constraints. $J + \{\omega_1' - \omega_1\}$ lies above I but the two curves have a point of contact, x_0. It remains to take an appropriate strictly convex approximation to I.

To conclude the proof, note that $x_1 + \omega_1 - \omega_1'$ lies above agent 1's indifference curve through $\bar{z}_1$. Since preferences are strictly convex and homothetic, $P_1(u,\omega')$ is an upward sloping curve. Then, if $z' \in \bar{E}^2P(u,\omega')$, $z_1' + \omega_1 - \omega_1' \geq x_1 + \omega_1 - \omega_1' \geq \bar{z}_1$, so that $u_1(z_1' + \omega_1 - \omega_1') > u_1(\bar{z}_1)$, and we are done.

Q.E.D.

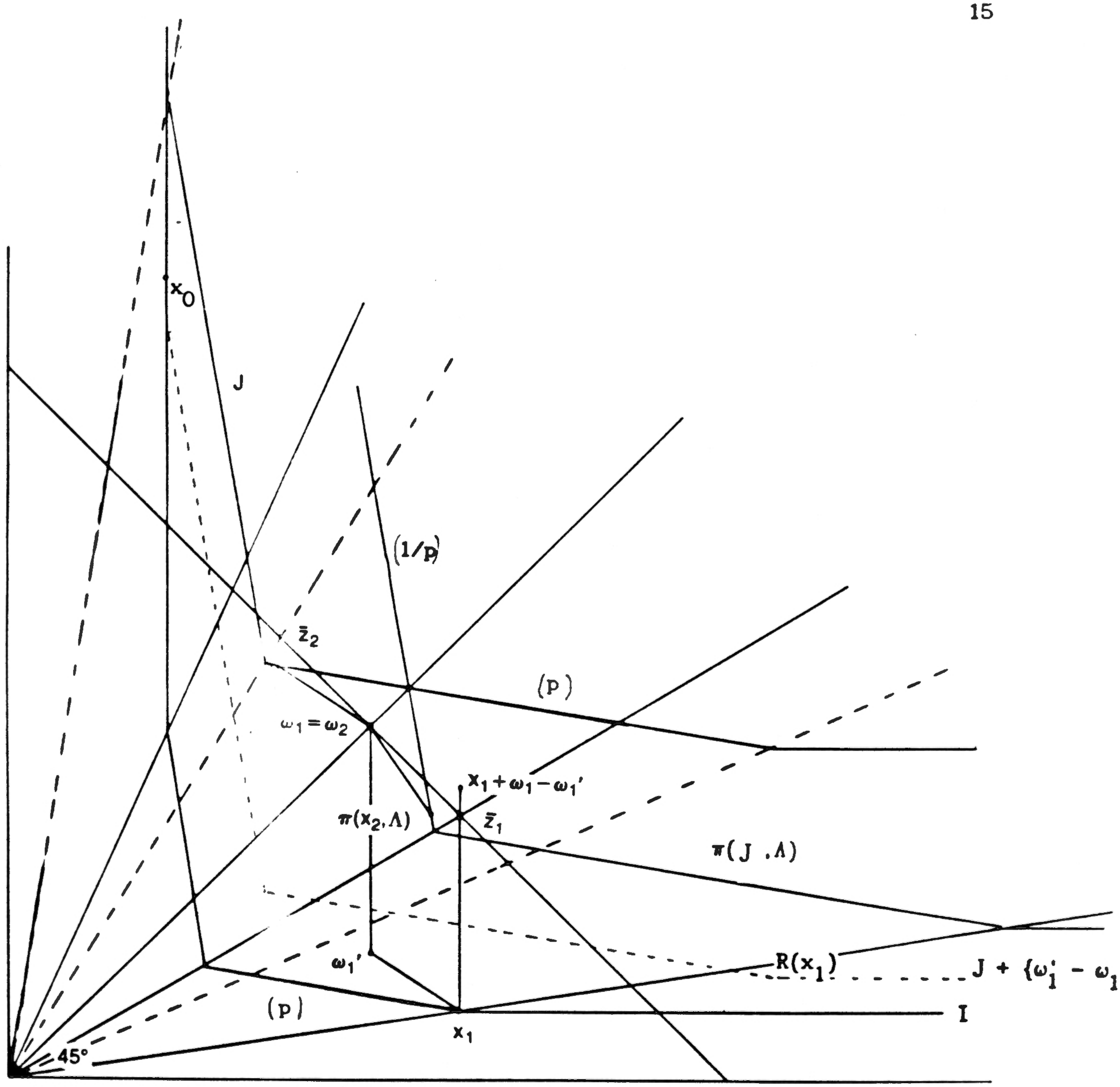

Figure 3

3.2. ***Destruction.*** Next, we investigate the existence of correspondences immune to manipulation through destruction of part of one's initial endowment. In other words, we would like agents to always benefit from increases in their initial endowments. Intuitively, it is less profitable to manipulate by destroying part of one's initial endowment than by withholding part of it, and one might hope for a positive result here. This hope is justified since our next result is indeed positive. To describe it, we need to present a few concepts of axiomatic bargaining theory.

Definitions. An ***n-person bargaining problem*** is a pair (S,d) of a subset S of $\mathfrak{R}^n$, the ***feasible set***, and of a point of S, d, the ***disagreement point***. Let Σ^n be the class of all problems (S,d) such that S is compact, convex and d-comprehensive (if $x \in S$ and $d \leq y \leq x$, then $y \in S$). A ***solution to the bargaining problem*** associates with every problem $(S,d) \in \Sigma^n$ a point of S. This point is interpreted as the compromise reached by the agents, or recommended to them, depending upon the context. The ***egalitarian solution***, E, is defined by setting E(S,d) equal to the maximal point of S at which the utility gains from d are equal across agents.

It is shown in Thomson (1987a) that the Egalitarian solution satisfies the following property, stated here for an arbitrary solution F.

strong d-monotonicity: For all (S,d) and $(S',d') \in \Sigma^n$, for all i, if $S = S'$, $d_i < d_i'$, and $d_j = d_j'$ for all $j \neq i$, then, (i) $F_i(S',d') \geqq F_i(S,d)$, and (ii) $F_j(S',d') \leqq F_j(S,d)$ for all $j \neq i$.

This property, which will be useful in the next section too, is also satisfied by the following solutions, which are generalizations of the egalitarian solution commonly discussed in bargaining theory.

Definition. Let G be a continuous, strictly monotone, and unbounded path in $\Re^n_+$ containing the origin. The ***monotone path solution relative to*** G, E^G, is defined by setting, for each $(S,d) \in \Sigma^n$, $E^G(S,d)$ equal to the maximal element of S on $G + \{d\}$.

Lemma 1. Monotone path solutions satisfy ***strong d-monotonicity.***

Proof: Let E^G be a monotone path solution with $\varphi:[0,\infty[\rightarrow \Re^n_+$ being a strictly increasing $(\lambda' > \lambda \Rightarrow \varphi(\lambda') > \varphi(\lambda))$ parametric representation of G. Given (S,d) and $(S',d') \in \Sigma^n$ with $S = S'$, $d_i < d'_i$ and $d_j = d'_j$ for all $j \neq i$, there exist $\lambda, \lambda' \in \Re$ such that $E^G(S,d) = d + \varphi(\lambda)$ and $E^G(S',d') = d' + \varphi(\lambda')$. We claim that $E^G_j(S,d) \geqq E^G_j(S',d')$ for all $j \neq i$. Otherwise, using the strict monotonicity of G, there is $j \neq i$ such that $E^G_j(S,d) = d_j + \varphi_j(\lambda) < d_j + \varphi_j(\lambda')$ $= d'_j + \varphi_j(\lambda') = E^G_j(S',d')$, so that $\varphi_j(\lambda) < \varphi_j(\lambda')$ and therefore, $\lambda < \lambda'$. This implies $E^G_k(S,d) = d_k + \varphi_k(\lambda) < d_k + \varphi_k(\lambda') = d'_k + \varphi_k(\lambda') = E^G_k(S',d')$ for all $k \neq i$. Also, $E^G_i(S,d) = d_i + \varphi_i(\lambda) < d_i + \varphi_i(\lambda') < d'_i + \varphi_i(\lambda') = E^G_i(S',d')$. Altogether, we have $E^G(S,d) < E^G(S',d')$ which is impossible since $S = S'$ and on Σ^n, E^G always selects weakly Pareto-optimal points. This concludes the proof that $E^G_j(S,d) \geqq E^G_j(S',d')$ for all $j \neq i$. The proof that $E^G_i(S,d) \leqq E^G_i(S',d')$ is similar.

Q.E.D.

This result is now applied to economic problems. It slightly generalizes Postlewaite's (1979) result that manipulation by destruction can be avoided by appropriately applying the Egalitarian solution.

Theorem 5. Suppose preferences are strictly monotone. Then, there are selections from $\bar{I}P$ such that an increase in an agent's initial endowment never hurts him.

Proof: Let E^G be a monotone path solution. Given an economy (u,ω), let $S \equiv \{(u_1(z_1),\ldots,u_n(z_n)) | z \in \mathbb{R}_+^{\ell n}, \Sigma z_i \leqq \Sigma\omega_i\}$ and $d \equiv (u_1(\omega_1),\ldots,u_n(\omega_n))$. Finally, let $\varphi(u,\omega) \equiv \{z \in A(\omega) | u(z) = E^G(S,d)\}$. φ satisfies all the desired requirements. That $\varphi \subset \bar{I}P$ is clear (it is to obtain Pareto-optimal, as opposed to only weakly Pareto-optimal, allocations, that we require strict monotonicity of preferences). That φ also satisfies the monotonicity property we claim it does is a consequence of part (i) of the strong d-monotonicity of E^G (for fixed feasible set (Lemma 1)) and of the fact, which is easily verified, that for d fixed, E^G causes all agents to gain from an expansion of the feasible set.[3]

Q.E.D.

3.3. *Transfer*. Next, we consider the transfer problem. We show that this problem can also be avoided by using monotone path solutions. This result generalizes an observation made in Thomson (1987a).

Theorem 6: There are selections from $\bar{I}P$ such that transfers of initial resources from an agent to another always benefit the recipient at the expense of the donor.

Proof. Again, we use the monotone path solutions E^G, as in Theorem 5 and we appeal to the fact that the E^G satisfy strong d-monotonicity (Lemma 1). Given (u,ω), let $\omega' \in A(\omega)$ with $\omega_i' \leq \omega_i$, $\omega_j' \geq \omega_j$, $\omega_i' + \omega_j' = \omega_i + \omega_j$, and $\omega_k' = \omega_k$ for all $k \notin \{i,j\}$. Let $d \equiv u(\omega)$, $d' \equiv u(\omega')$. Also, let d" be defined by $d_i'' \equiv u_i(\omega_i')$, $d_j'' \equiv u_j(\omega_j)$ and $d_k'' \equiv u_k(\omega_k)$ for all $k \notin \{i,j\}$. By part (ii) of the

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[3]This last property is known in the axiomatic theory of bargaining as "strong monotonicity".

strong d-monotonicity of E^G, $E_i^G(S,d'') \leqq E_i^G(S,d)$ and $E_k^G(S,d'') \geqq E_k^G(S,d)$ for all $k \neq i$; similarly, $E_j^G(S,d') \geqq E_j^G(S,d'')$ and $E_k^G(S,d') \leqq E_k^G(S,d'')$ for all $k \neq j$. Therefore, $E_i^G(S,d') \leqq E_i^G(S,d)$ and $E_j^G(S,d') \geqq E_j^G(S,d)$.

Q.E.D.

3.4. ***Negative effects on others.*** Here, we ask whether correspondences exist such that an increase in some agent's initial endowment never hurts the others.

We could prove that there is no selection from $\overline{I}P$, $\overline{F}P$ or $\overline{D}P$ such that an increase in an agent's initial endowment never hurts the others. However, we will state a stronger result directly. Just as in Theorems 2 and 3, in which we introduced a coefficient ϵ to measure the extent to which the desired property (that the correspondence be immune to manipulation through withholding) was violated, it is also possible here to quantify how bad things are. Let $1 \geqq \epsilon \geqq 0$ be given and let $\overline{I}_\epsilon(u,\omega)$, $\overline{F}_\epsilon(u,\omega)$ and $\overline{D}_\epsilon(u,\omega)$ respectively be the sets of allocations $z = \omega + t \in A(\omega)$ such that $u_i(z_i) \geqq u_i(\epsilon\omega_i)$ for all i, or $u_i(\omega_i + t_i) \geqq u_i(\epsilon(\omega_i + t_j))$ for all i,j or $\omega_i + t_i \geq\!\!\!\!/\;\; \epsilon(\omega_i + t_j)$ for all i,j. It is clear that for $\epsilon = 1$, $\overline{I}_\epsilon = \overline{I}$, $\overline{F}_\epsilon = \overline{F}$ and $\overline{D}_\epsilon = \overline{D}$, and that for all ϵ, ϵ' with $\epsilon' < \epsilon$, $\overline{I}_\epsilon \subset \overline{I}_{\epsilon'}$, $\overline{F}_\epsilon \subset \overline{F}_{\epsilon'}$, and $\overline{D}_\epsilon \subset \overline{D}_{\epsilon'}$; finally that for $\epsilon = 0$, the three inequalities become equivalent to the trivial requirement that each agent be allocated a non-negative consumption. For $\epsilon > 0$ but small, the requirements are very weak. Nevertheless, we have

***Theorem* 7.** There is no selection from $\overline{I}_\epsilon P$, $\overline{F}_\epsilon P$ or $\overline{D}_\epsilon P$ such that an increase in an agent's initial endowment never hurts the others.

The proof of the theorem is obtained by adapting the proof of a result of Moulin and Thomson (1987) reproduced here as Theorem 9.

Proof. See Figure 4. As for our previous impossibilities, the proof is by way of a two-person, two-commodity example. The agents have piecewise linear

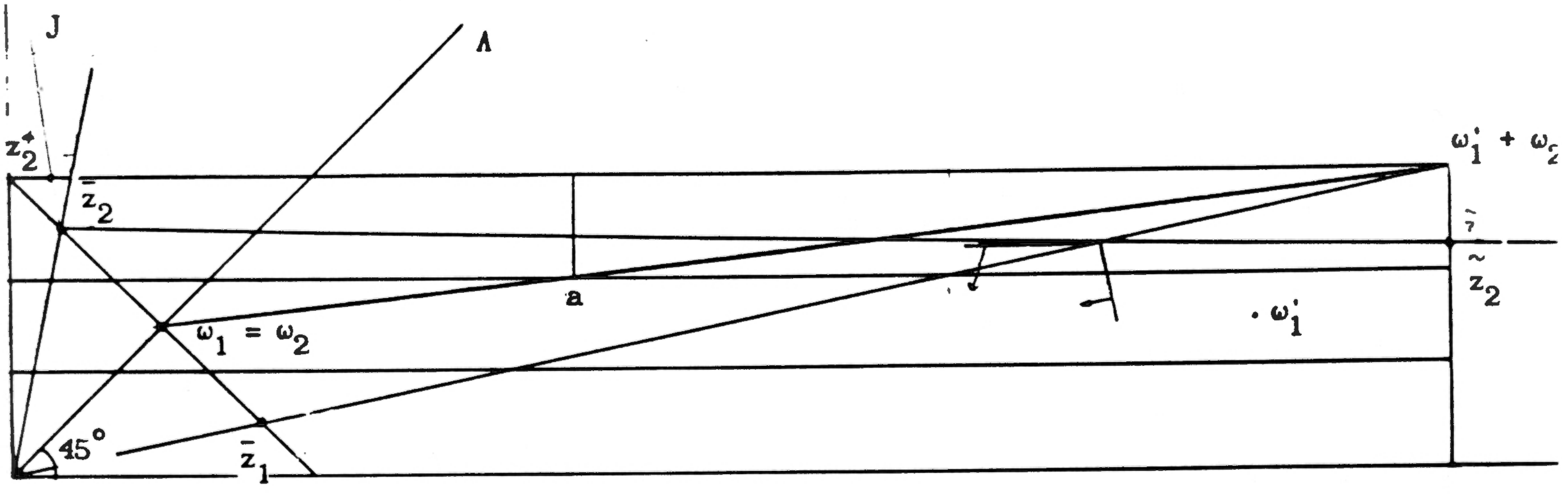


Figure 4

and homothetic preferences such that $u_2(z_2) = u_1(\pi(z_2, \Lambda))$ for all $z_2 \in \Re^2_+$. If n = 2, then given any ϵ, there exist ϵ' and ϵ'' so that $\bar{D}_\epsilon \supset \bar{F}_{\epsilon'}$ abd $\bar{D}_\epsilon \supset \bar{I}_{\epsilon''}$, so that it is enough to consider $\bar{D}_\epsilon$. Let then $\varphi \subset \bar{D}_\epsilon P$ be given.

Initially, $\omega_1 = \omega_2 \in \Lambda$. Let $\bar{z}_2 \equiv (\frac{\epsilon}{1 + \epsilon}\omega_{21}, \frac{2 + \epsilon}{1 + \epsilon}\omega_{22})$, $\bar{z}_1 \equiv \pi(\bar{z}_2, \Lambda)$, ω_1' such that $\omega_{12}' = \omega_{12}$ and $\omega_1' + \omega_2 \in R(\bar{z}_1)$, $\tilde{z}_2 \equiv (\omega_{11}' + \omega_{21}, \frac{4 + \epsilon}{2(1 + \epsilon)}\omega_{22})$ and z_2^+ such that $z_{22}^+ = 2\omega_{22}$ and the slope of the segment $[\bar{z}_2, z_2^+]$ be smaller than the inverse of the slope of the segment $[\bar{z}_2, \tilde{z}_2]$. Let agent 2's indifference curve through $\bar{z}_2$, J, be made up of the linear extensions towards the axes of these two segments. Agent 2's preference map is completed by homothetic transformations of J and agent 1's preferences are obtained by symmetry with respect to Λ.

Note that $\bar{z} \equiv (\bar{z}_1, \bar{z}_2) \in W(u,\omega)$. Let $\omega' \equiv (\omega_1', \omega_2)$ and $z' \in \varphi(u,\omega')$. As in the previous proofs, it suffices to compare z' to $\bar{z}$. Let $t \equiv z' - \omega'$. Since $z' \in \varphi(u,\omega') \subset \bar{D}_\epsilon(u,\omega')$, $\omega_1' + t_1 \not\geq \epsilon(\omega_1' + t_2)$ and since $t_2 = -t_1$, this inequality can be written as $\omega_1'(1 - \epsilon) \not\geq -t_1(1 + \epsilon)$, so that $z_1' = \omega_1' + t_1 \not\geq \omega_1' - \frac{1 - \epsilon}{1 + \epsilon}\omega_1' = \frac{2\epsilon}{1 + \epsilon}\omega_1'$.

We now construct the Edgeworth box of (u,ω') by placing agent 1's origin at the point $\omega_1' + \omega_2$. In Figure 4, the consumptions of agent 1 violating the inequality $z_1' \not\geq \frac{2\epsilon\omega_1'}{1 + \epsilon}$ are all the points in the quadrant to the North-West of the point a. However, for agent 2 not to be hurt by the increase in agent 1's endowment, z_2' should be about J. However, no such point z_2' is the second component of an allocation in $P(u,\omega')$. Indeed, $P(u,\omega')$ is the segment $[0,\omega_1' + \omega_2]$. This completes the proof.

Q.E.D.

***Theorem* 8.** There is no selection from $\bar{E}^2P$ such that an increase in an agent's initial endowment never hurts the others.

***Proof*.** See Figure 5. We choose preferences to be homothetic and such that $u_2(z_2) = u_1(\pi(z_2,\Lambda))$ for all $z_2 \in \Re_+^\ell$. Initially, $\omega_1 = \omega_2 \in \Lambda$. Then agent 1's initial endowment increases to ω_1'. Let $\omega' \equiv (\omega_1', \omega_2)$. Let $\bar{z} \in W(u,\omega)$ with $\bar{z}_2 = \pi(\bar{z}_1,\Lambda)$ be given. First, we identify the allocation x of $\bar{E}^2P(u,\omega')$ that agent 2 prefers. Since $\varphi \subset \bar{E}^2$, (i) x is such that agent 2's indifference curve through x_2, J, translated by the amount $\omega_1' - \omega_1$ (this is the dashed line labelled $J + \{\omega_1' - \omega_1\}$), is tangent to agent 1's indifference curve through

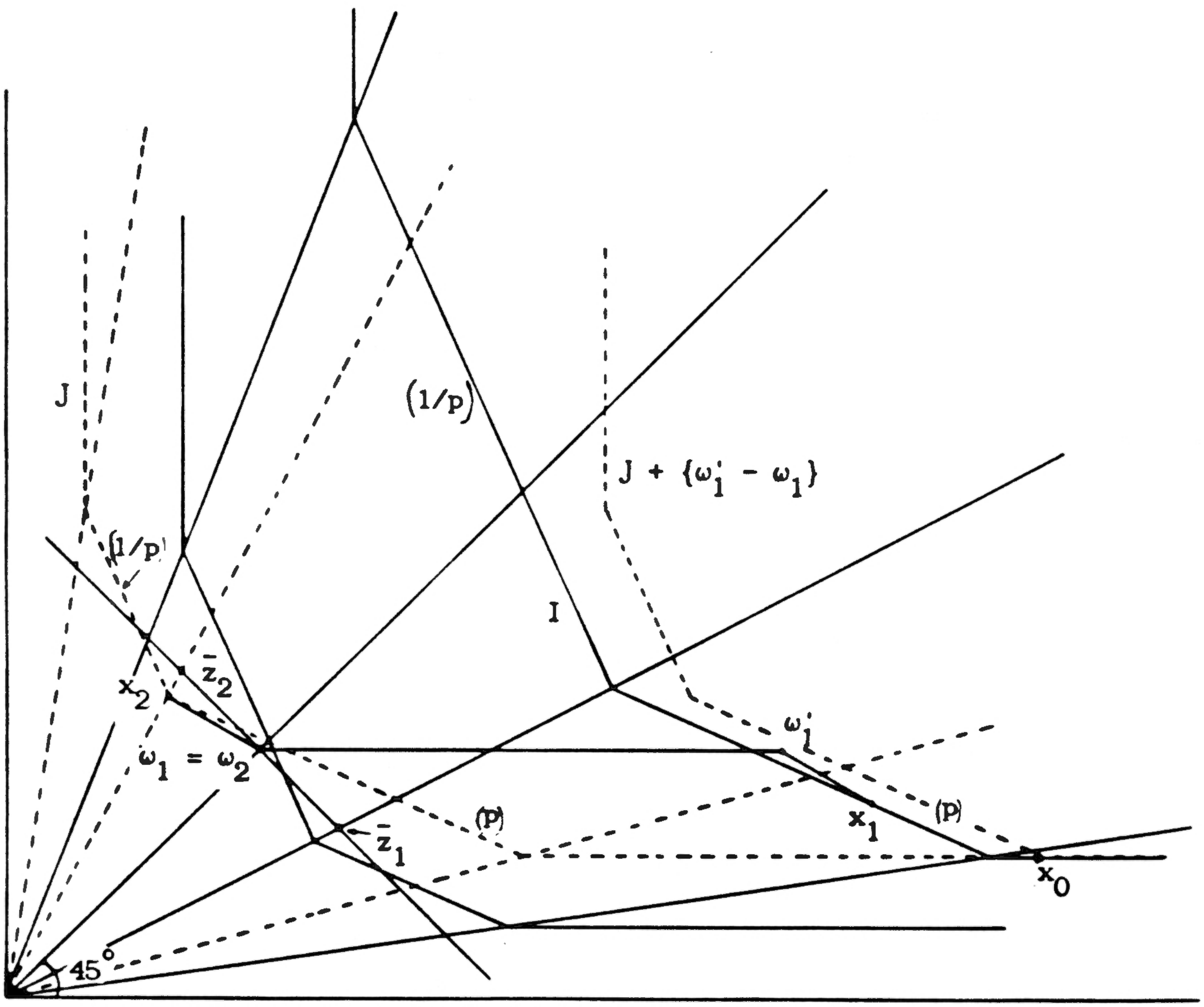


Figure 5

x_1, I, and lies above I. (x_0 is a point of contact of these two curves, and $t_0 \equiv x_0 - \omega_1'$ is such that $u_1(\omega_1' + t_0) = u_1(x_1)$ and $u_2(\omega_2 + t_0) = u_2(x_2)$.) In addition, since $\varphi \subset P$, (ii) I and J have parallel lines of support at x_1 and x_2 respectively. All of these conditions are met in Figure 5, and yet agent 2 strictly prefers $\bar{z}_2$ to x_2. Therefore, for all $z' \in \bar{E}^2P(u,\omega')$, $u_2(\bar{z}_2) > u_2(z_2')$ and we are done. (In Figure 5 we have represented $\pi(J,A)$ which can also be obtained by a homothetic transformation of I.)

Q.E.D.

In the final two sections, we assume agents to be collectively entitled to all goods available and we study two properties expressing their solidarity.

3.5. *Aggregate monotonicity.* First, we ask whether it can be guaranteed that all agents benefit when the resources at their disposal increase.

Given $\epsilon > 0$, let $I_\epsilon(u,\omega)$, $F_\epsilon(u,\omega)$ and $D_\epsilon(u,\omega)$ respectively be the sets of allocations $z \in A(\omega)$ such that $u_i(z_i) \geqq u_i(\epsilon\Omega)$ for all i, $u_i(z_i) \geqq u_i(\epsilon z_j)$ for all i,j and $z_i \geqq \epsilon z_j$ for all i,j. Moulin and Thomson (1987) prove that there is no selection from $D_\epsilon P$ such that an increase in the aggregate endowment always benefits all agents. This result directly implies the following.

***Theorem* 9.** There is no selection from $I_\epsilon P$, $F_\epsilon P$ or $D_\epsilon P$ such that an increase in the aggregate endowment always benefits all agents.

The next result however is positive. It involves a standard method of selecting from E^2, and is due to Hurwicz (1978) who was concerned with the existence of allocation mechanisms which would give agents the incentive to truthfully report their production capabilities. Hurwicz's positive answer is precisely by way of a selection from E^2.

Theorem 10. Assume preferences are strictly monotone. Then, there are selections from E^2P such that an increase in the aggregate endowment benefits all agents.

Proof: Let $G \subset \Re^{\ell}_{+}$ be a continuous and monotone path in the commodity space. For each i, let $v_i: \Re^{\ell}_{+} \rightarrow \Re_{+}$ be the utility representation of agent i's preferences obtained by setting $v_i(z_i)$ equal to the length of the curvi-linear segment on G connecting the origin to the point indifferent to z_i. Finally, let $\varphi(u,\Omega) \equiv \{z \in P(u,\Omega) | v_i(z_i) = v_j(z_j) \ \forall i,j\}$. φ satisfies all the desired requirements.

Q.E.D..

3.6 *Population monotonicity*. In contrast with all of the previous sections where resources varied but the number of agents remained fixed, we consider here the case of a variable number of agents with fixed resources.

Theorem 11. Assume preferences are strictly monotone. Then, there are selections from IP and E^2P such that an increase in the number of agents, unaccompanied by an increase in the aggregate endowment, never benefits any of the agents initially present.

Proof. Let $G \subset \Re^{\ell}_{+}$ be as in Theorem 10 except that in addition G passes through Ω, $\Omega/2$, $\Omega/3$,... . φ is then defined as in Theorem 10. φ satisfies all the desired requirements.

Q.E.D.

4. *Concluding Comments*

We conclude with a discussion of the robustness of our negative results.

(i) In some of the examples that we have used to prove the negative results, preferences are not strictly monotone. In all cases, approximations to these examples could be constructed exhibiting this property. Similarly, preferences could be drawn smooth. Finally, the indifference curves could be modified so as to be asymptotic to the axes.

(ii) It is intuitive that an agent is more likely to gain by withholding the good of which he is, before withholding, a net supplier. For instance, suppose that the Walrasian correspondence is being operated. Then such manipulative behavior with typically increase the equilibrium price of that commodity, and therefore his income. It is natural to conjecture that the same conclusion applies to other correspondences. In spite of this, we have been able to establish the non-existence of selections from $\bar{F}P$ or $\bar{E}^2P$ that are immune to manipulation through withholding, even assuming that the agent who manipulates is forced to withhold the "wrong" commodity. However, the preferences involved in the counterexamples constructed for these proofs are not homothetic. Whether homotheticity of preferences is incompatible with profitable manipulation by withholding the wrong commodity, is an open question. An answer to this question will help delineate the boundary between possibilities and impossibilities.

Appendix 1

Here, we give examples of selections from P that are such that all agents benefit from an increase in the aggregate endowment.

They are based on the following definition, which can be found in Thomson (1987b). This definition generalizes the concept of egalitarian equivalence.

Definition. Let $\mathcal{B}$ be a family of subsets of $\Re^{\ell}$. $z \in A(\Omega)$ is an ***equal-opportunity-equivalent allocation relative to the family*** $\mathcal{B}$ ***for*** (u,Ω) if there is $B \in \mathcal{B}$ such that for each i, $u_i(z_i) = u_i(z_i^*)$, where z_i^* maximizes u_i over B. Let **EOE**$(\mathcal{B},u,\Omega)$ be the set of these allocations.

Theorem 12. Assume preferences are strictly monotone. Then, there are selections from P such that an increase in the aggregate endowment benefits all agents.

Proof. Let $\mathcal{B} \equiv \{B(t) \mid t \in \Re_+\}$ be a parametric family of subsets of $\Re_+^{\ell}$ with the following properties:

(i) $B(\cdot)$ is continuous,

(ii) $\forall t, t' \in \Re_+$, if $t' > t$, then $B(t') \supset B(t)$,

(iii) $d(B(0)) = 0$, where $d(A)$ is the diameter of the set A; $\forall r \in \Re_+$, $\exists$ t such that $B(t) \supset S(r) \cap \Re_+^{\ell}$, where $S(r)$ is the sphere of center 0 and radius r.

It is easy to check that the correspondence EOE$(\mathcal{B},.,.)$ is a selection from P satisfying the desired monotonicity property.

Q.E.D.

We can deduce from Theorem 12 the existence of selections from E^2P such that an increase in the aggregate endowment benefits all agents. Indeed the family $\mathcal{B}_1 \equiv \{B_1(t) | t \in \mathfrak{R}_+\}$ with $B_1(t) \equiv \{z \in \mathfrak{R}_+^{\ell} | z \leqq td$, for some fixed $d \in \mathfrak{R}_+^{\ell}, d \neq 0\}$ does satisfy (i), (ii) and (iii). Therefore, by Theorem 12, $EOE(\mathcal{B}_1, u, \Omega) \subset P(u, \Omega)$ for all (u, Ω). It is clear that in addition $EOE(\mathcal{B}_1, u, \Omega) \subset E^2 (u, \Omega)$ for all (u, Ω).

A natural example of a family $\mathcal{B}_2$ satisfying properties (i), (ii) and (iii) but such that the associated equal-opportunity-equivalent correspondence $EOE(\mathcal{B}_2, \cdot, \cdot)$ is not a subcorrespondence of E^2 is

$\mathcal{B}_2 \equiv \{B_2(t) | t \in \mathfrak{R}_+\}$ where $B_2(t) \equiv \{z \in \mathfrak{R}_+^{\ell} | pz \leqq t$, for some fixed $p \in \Delta^{\ell-1}\}$.

Appendix 2

The purpose of this appendix is to indicate a (perhaps surprising) direction in which some of our results can be strengthened. This result is briefly described in Section 4 (ii).

Consider a two-commodity, two-person economy in which under honest behavior, agent 1, say, trades away good 2 in exchange for good 1. If the Walrasian correspondence is being operated, manipulation by withholding will typically involve withholding good 2. This will make good 2 appear rarer, and, as a consequence, one would expect its equilibrium price to increase in relation to that of good 1, to the benefit of agent 1. It is natural to conjecture that the fact that manipulation involves withholding the good that one would supply under truthful behavior, holds for a wide class of correspondences, and not just for the Walrasian correspon- dence. It is therefore a strong indication of the seriousness of the difficulties under study that our theorem stating the non-existence of a selection from $\bar{F}P$ (the envy-free and efficient trade correspondence) that is immune from manipulation through withholding can be proved by forcing the agent who manipulates to withhold the "wrong" commodity. We also prove a similar impossibility for selections from $\bar{E}^2P$ (the egalitarian-equivalent and efficient trade correspondence).

***Theorem* 13.** There is no selection from $\bar{F}P$ such that no agent ever gains by withholding part of his initial endowment, even if he is forced to withhold the "wrong" commodity.

Proof: See Figure 6. Let $\varphi \subset \bar{F}P$ be given. Initially, $\omega_1 = \omega_2 \in \Lambda$. Also, $u_2(z_2) = u_1(\pi(z_2,\Lambda))$ for all $z_2 \in \Re^2_+$. Let $\bar{z} \in W(u,\omega)$ be given with $\bar{z}_2 \equiv \pi(\bar{z}_1,\Lambda)$. u is such that $\bar{z}_{11} > \omega_{11}$. This inequality identifies good 1 as the "wrong" good for agent 1 to withhold. Yet, ω'_{11} is such that $\omega'_{11} < \omega_{11}$ and $\omega'_{12} = \omega_{12}$ (that is, agent 1 withholds good 1). The example is constructed so that agent 1 gains from having withheld.

To show this, we identify the allocation $x \in \bar{F}P$ that agent 1 likes the least and the allocation $y \in \bar{F}P$ that he likes the most. x is such that $u_1(x_1) = u_1(\pi(x_1,\omega'_1))$, and agent 1's indifference curve through x_1 has at x_1 a line of support with the same slope p as the line of support at x_2 to agent 2's indifference curve through x_2. By symmetry of preferences, agent 1's indifference curve through $\pi(x_2,\Lambda)$ has at $\pi(x_2,\Lambda)$ a line of support of inverse slope 1/p. Similarly, $u_2(y_2) = u_2(\pi(y_2,\omega_2))$ and agent 1's indifference curve through y_1 has at y_1 a line of support of slope p inverse to that of the line of support at $\pi(y_2,\Lambda)$ to his indifference curve through $\pi(y_2,\Lambda)$. In the Figure p = q. The preference map is completed so that at any point of the form $z_1(\alpha) \equiv \alpha x_1 + (1 - \alpha)y_1$ for $\alpha \in [0,1]$, agent 1's indifference curve through $z_1(\alpha)$ admits a line of support of slope p and at any point of the form $\pi(z_2(\alpha),\Lambda)$, where $z_2(\alpha) = \omega'_1 + \omega_2 - z_1(\alpha)$, agent 1's indifference curve through $z_1(\alpha)$ admits a line of support of slope 1/p. Simple inspection of Figure 6 shows that all these requirements on agent 1's preference map can be jointly satisfied. Then, $\bar{F}P_1(u,\omega') = [x_1,y_1]$. Remembering that agent 1 has withheld the amount $\omega_1 - \omega'_1$, we conclude that if $z' \in \bar{F}P(u,\omega')$, then $z'_1 \in [x_1, y_1] + \{\omega_1 - \omega'_1\}$ and since then $u_1(z'_1) > u_1(\bar{z}_1)$, we are done.

Q.E.D.

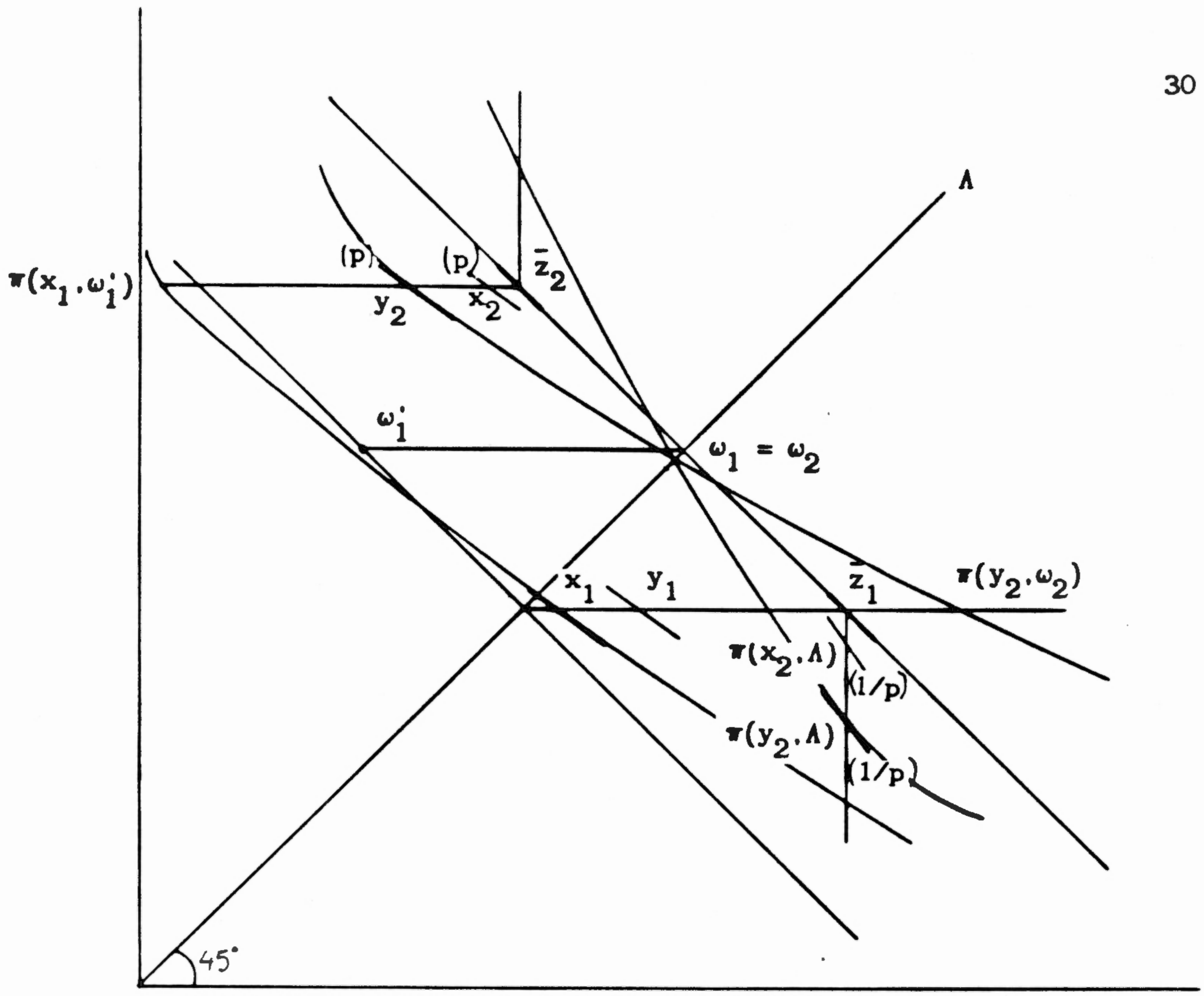
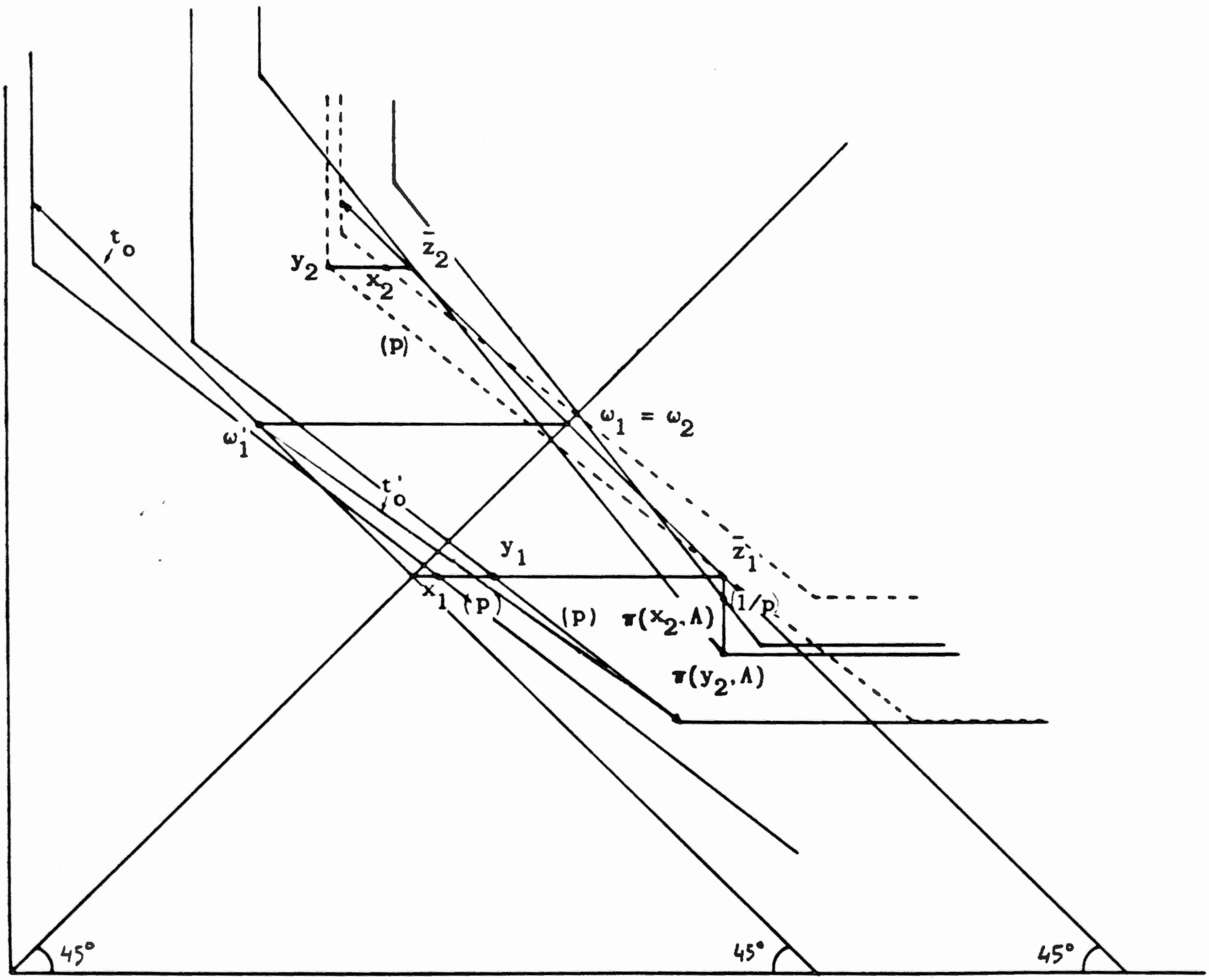


Figure 6

Theorem 14 There is no selection from $\bar{E}^2P$ such that no agent ever gains by withholding part of his initial endowment, even if he is forced to withhold the "wrong" commodity.

Proof. See Figure 7. The specification of the first elements of the proof is as in Theorem 13. Initially, $\omega_1 = \omega_2 \in A$. Also, $u_2(z_2) = u_1(\pi(z_2,A))$ for all $z_2 \in \mathbb{R}^2_+$. Let $\bar{z} \in W(u,\omega)$ be given with $\bar{z}_2 = \pi(\bar{z}_1, A)$. u is chosen so that $\bar{z}_{11} > \omega_{11}$. This means that good 1 is the wrong good for agent 1 to withhold. ω_1' is chosen so that $\omega_{11}' < \omega_{11}$ and $\omega_{12}' = \omega_{12}$. We identify the points x and y $\in$ $\bar{E}^2P$ that agent 1 likes the least and the most respectively. x is such that for some trade vector $t_o \in \Re^2$, $u_1(\omega_1' + t_o) = u_1(x_1)$ and $u_2(\omega_2 + t_o) = u_2(x_2)$, agent 2's indifference curve through x_2, translated by the amount $\omega_1'-\omega_1$ lies above agent 1's indifference curve through x_1, and finally agent 1's indifference curve through x_1 has at x_1 a line of support with the same slope p as the line of support at x_2 to agent 2's indifference curve through x_2. By symmetry of preferences, agent 1's indifference curve through $\pi(x_2,\Lambda)$ has at $\pi(x_2,\Lambda)$ a line of support of slope 1/p. Similarly, there exists $t_o' \in \Re^2$ such that $u_2(\omega_2 + t_o') = u_2(y_2)$ and $u_1(\omega_1' + t_o') = u_1(y_1)$, agent 1's indifference curve through y_1, translated by the amount $\omega_1-\omega_1'$ lies above agent 2's indifference curve through y_2, and finally agent 1's indifference curve through y_1 has at y_1 a line of support with a slope q equal to the inverse of the slope of the line of support at $\pi(y_2, \Lambda)$ of his indifference curve through $\pi(y_2,\Lambda)$. In Figure 7, p = q. We extend preferences over the intervals $[x_1, y_1]$ and $[\pi(x_2,\Lambda), \pi(y_2,\Lambda)]$ as in Theorem 13. To conclude, we note that if $z' \in \overline{E}^2P(u, \omega')$, then $z_1' \in [x_1, y_1] + \{\omega_1 - \omega_1'\}$ and since then $u_1(z_1') > u_1(\bar{z}_1)$, we are done.

Q.E.D.



t_o
y_2
x_2
$\bar{z}_2$
(p)
$\omega_1 = \omega_2$
ω_1'
t_o'
y_1
$\bar{z}_1$
x_1
(p)
(1/p)
(p)
$\pi(x_2, \Lambda)$
$\pi(y_2, \Lambda)$
45°
45°
45°

Figure 7

REFERENCES

Aumann, R. and B. Peleg, "A note on Gale's example," *Journal of Mathematical Economics*, 1 (1974) 209-211.

Chichilnisky, G. and W. Thomson, "The Walrasian mechanism from equal division is not monotonic with respect to variations in the number of consumers," *Journal of Public Economics*, 32 (1987) 119-124.

Chun, Y. and W. Thomson, "Monotonicity properties of bargaining solutions when applied to economies", University of Rochester Discussion Paper No. 5, (1984), forthcoming in *Mathematical Social Sciences*.

Foley, D., "Resource allocation and the public sector", *Yale Economic Essays*, 7 (1967) 45-98.

Hurwicz, L., "On informationally decentralized systems", Chapter 14 in *Decision and Organization*, (C.B. McGuire and R. Radner, eds), North-Holland, Amsterdam, (1972) 297-336.

__________, "On the interaction between information and incentives in organizations." in *Communication and Control in Society*, (K. Krippendorf ed), Scientific Publishers, Inc., New York, (1978) 123-147.

Kolm, S.C., *Justice et équité*, Editions du C.N.R.S. (1972).

Moulin, H., "Common property resources: Can everyone benefit from growth?" VPI discussion paper, (June 1987).

__________ and W. Thomson, "Can everyone benefit from growth? Two difficulties." University of Rochester Discussion Paper No. 87, (May 1987, revised October 1987).

Pazner, E., and D. Schmeidler, "Egalitarian-equivalent allocations: a new concept of economic equity", *Quarterly Journal of Economics*, 92 (1978) 671-687.

Polterovich, V. and V. Spivak, "Gross substitutability of point-to-set correspondences," *Journal of Mathematical Economics*, 11 (1983) 117-140.

__________ and __________, "The budgetary paradox in the model of economic equilibrium," *Matekon*, 16 (1980) 3-22.

Postlewaite, A., "Manipulation via endowments", *Review of Economic Studies*, 46 (1979) 255-262.

Roemer, J., "Axiomatic bargaining theory on economic environments", U.C. Davis DP No. 264 (1985), forthcoming in the ***Journal of Economic Theory***.

__________, "Equality of resources implies equality of welfare", ***Quarterly Journal of Economics*, 101** (1986) 751-784.

Schmeidler, D. and K. Vind, "Fair net trades", ***Econometrica*, 40** (1972) 637-642

Thomson, W., "Monotonic allocation mechanisms; preliminary results", University of Minnesota mimeo, (1978).

__________, "Monotonicity of bargaining solutions with respect to the disagreement point", ***Journal of Economic Theory*, 42** (1987a) 50-58.

__________, "Notions of equal opportunities", University of Rochester Discussion Paper No. 76, (1987b).

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