Monotonic Allocation Mechanisms in Economies with Public Goods

Thomson, William

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by

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* University of Rochester. The author thanks NSF for its support under grant No. 85 11136.

1. *Introduction*

We investigate the existence of correspondences allocating resources in public good economies so as to satisfy the following requirements: (i) Pareto-efficiency, (ii) some minimal condition expressing that agents should not be discriminated against, (iii) a variety of monotonicity conditions with respect to changes in resources. We consider two situations; one is the standard situation where each agent starts out with a bundle of goods over which he may have special rights (his initial endowment); the other is when there is an aggregate bundle of goods to which agents are collectively entitled.

The main conditions of type (ii) are individual rationality: each agent is assigned a final consumption that he finds at least as desirable as his initial endowment; or, strong individual rationality: his final consumption is at least as desirable as any consumption that he could achieve, given the available technology to produce the public goods and given his initial endowment. Finally, we also consider the condition that no agent prefers someone else's consumption to his own. Then, the allocation is said to be envy-free.

The conditions of type (iii) are the following: no agent should ever benefit by destroying part of his initial endowment; no agent should ever benefit by withholding part of his initial endowment; no agent should ever be hurt by an improvement in someone else's initial endowment; no agent should ever benefit from transfering part of his initial endowment to someone else. In case resources are initially collectively owned, the conditions are: no agent should ever lose if the aggregate endowment increases; upon the arrival of additional agents unaccompanied by an expansion of resources, no agent should strictly lose if some other agent strictly gains.

First, we consider the Lindahl correspondence, which is perhaps the most important theoretical tool for the allocation of resources in public good economies, and we show that this correspondence satisfies none of the conditions of type (iii). Then, we ask the general question whether there are any correspondences satisfying our conditions.

The answers are mixed. Some properties are compatible. However many are not. Here is a short summary. There are selections from the strong individually rational and efficient correspondence such that an agent always benefits from an increase in his own initial endowment or such that a transfer of initial resources from one agent to another always benefits the recipient at the expense of the donor. However, there is no selection from the individually rational and efficient correspondence such that an agent never gains by withholding part of his initial endowment, nor such that an increase in an agent's initial endowment never hurts at least one of the others.

In the case of public ownership, we show that if there is only one private good, there are selections from the individually rational and Pareto-efficient correspondence but not from the envy-free and efficient correspondence, such that an increase in the aggregate endowment always benefits all agents. Also, there are selections from the strong individually rational and efficient correspondence such that an increase in the number of agents, unaccompanied by an increase in the aggregate resources, affect all agents initially present similarly.

We also present a result for economies where the only goods agents care about are public goods but each agent controls some amount of a good which can serve as input into the production of the public goods. The monotonicity property we consider for that case is whether an increase in the amount of the input provided by an agent necessarily makes him better off. We show the incompatibility of this condition with efficiency and strong individual rationality.

This study extends previous work by Moulin and Thomson (1987) and Thomson (1987c), which concerned economies with private goods only. Most of the conclusions of these two papers were negative.

The positive results presented here are established for general classes of economies, with arbitrary numbers of goods and agents, and arbitrary technologies. On the other hand, the impossibility results are established by way of examples of economies with one private good, and one or more public goods. Assuming that there is only one private good case is not a limitation for the negative results; in fact, it is really the interesting case. Indeed, in order to study the various ways in which the construction of the public goods can be financed, it is necessary to assume the existence of at least one private good, but, were we to allow for several private goods, the impossibility results obtained for private goods economies with at least two private goods would extend directly. We would simply consider "degenerate" public good economies with two private goods and with agents whose preferences are independent of the public good levels. (If one were to insist that preferences be strictly monotonic in all goods, then the conclusion would follow by an approximation argument.) An adaptation of our earlier impossibilities to the case of economies with only one private good is not possible however, which is why we had to develop separate proofs.

2. *Notation Definitions*

Let ℓ, m, n be the numbers of private goods, public goods, and agents respectively. $Y \subset \mathfrak{R}^{\ell+m}$ is the production set. Y is closed and bounded from above. We assume Y to be known and fixed and we ignore it in the notation. Each agent i is equipped with a utility function $u_i : \mathfrak{R}_+^{\ell+m} \to \mathfrak{R}_+$. Let $u \equiv (u_1, \ldots, u_n)$. A consumption for agent i is a pair $(x_i, y) \in \mathfrak{R}_+^{\ell+m}$ where $x_i \in \mathfrak{R}_+^{\ell}$ is the vector of private goods he consumes and $y \in \mathfrak{R}_+^m$ is the vector of public goods consumed by all agents. An ***economy in an initial position*** is a pair (u, ω) where $\omega = (\omega_1, \ldots, \omega_n)$ and $\omega_i = (\omega_{ix}, 0) \in \mathfrak{R}_+^{\ell+m}$ is agent i's initial endowment (that is, we assume that there are no public goods initially). A ***feasible allocation for*** (u, ω) is a list $(x, y) \in \mathfrak{R}_+^{\ell n+m}$ with $x = (x_1, \ldots, x_n) \in \mathfrak{R}_+^{\ell n}$, $y \in \mathfrak{R}_+^m$ and $(-\Sigma\omega_{ix} + \Sigma x_i, y) \in Y$. $A(\omega)$ is the set of feasible allocations.

An ***economy*** is simply a pair (u, Ω), where $\Omega \in \mathfrak{R}_+^{\ell}$ is the aggregate bundle of private goods initially present in the economy. Agents are collectively entitled to these goods, for consumption as private goods or for use as inputs in the production of the public goods.

P is the Pareto correspondence. A correspondence φ is ***essentially single-valued*** if its image in utility space is single-valued. An essentially single-valued subcorrespondence of a given correspondence is called a ***selection***.

3. *The Lindahl correspondence*

The natural counterpart for public good economies of the Walrasian correspondence is the Lindahl correspondence. It is probably the most important theoretical tool in the analysis of allocation in such economies. For that reason, we consider it first, but we mainly use this example to introduce in a concrete way the properties with which we will be concerned.

For simplicity, we define the Lindahl correspondence only for the case $(\ell,m) = (1,1)$.

Definition. The *Lindahl correspondence* associates with every (u,ω) the set $L(u,\omega)$ of allocations $z = (x,y) \in A(\omega)$ such that for each i, there exists $\pi^i \in \Delta^1$, the 1-dimensional simplex, with

(i) for each i, $z_i = (x_i,y)$ maximizes $u_i(z_i')$ in $z_i' \in B_i(\pi^i) \equiv \{z_i' \in \Re^2_+ | \pi^i z_i' \leqq \pi^i \omega_i\}$,

(ii) $(-\Sigma\omega_{ix}+\Sigma x_i,y)$ maximizes $(\Sigma\pi^i)(x_o',y')$ in $(x_o',y') \in Y$.

The pair (π,z), where $\pi \equiv (\pi^1,\ldots,\pi^n)$ is then said to be a *Lindahl equilibrium for* (u,ω).

We first show that the behavior of the Lindahl correspondence, in response to a variety of changes in initial endowments, is qualitatively identical to that of the Walrasian correspondence. Indeed, each of the following facts has a counterpart involving the Walrasian correspondence.

Fact 1. An agent may lose when his initial endowment increases. (This problem has been discussed for exchange economies under the name of the "destruction paradox". See Aumann and Peleg, 1974.)

Proof. See Figure 1a. Initially, $\omega_1 = \omega_2$. Then ω_1 increases to ω_1'. (π,z) and (π',z') are Lindahl equilibria for (u,ω) and (u,ω') where $\omega' = (\omega_1',\omega_2)$. Note that $u_1(z_1') < u_1(z_1)$.

Q.E.D.

Figure 1a gives the Kolm triangle of the initial economy. In the Kolm triangle of the economy after the increase in the aggregate resources, the points marked z_1' and z_2' would coincide. Figures 1b and 2a should be similarly interpreted.

Fact 2 is a direct consequence of Fact 1.

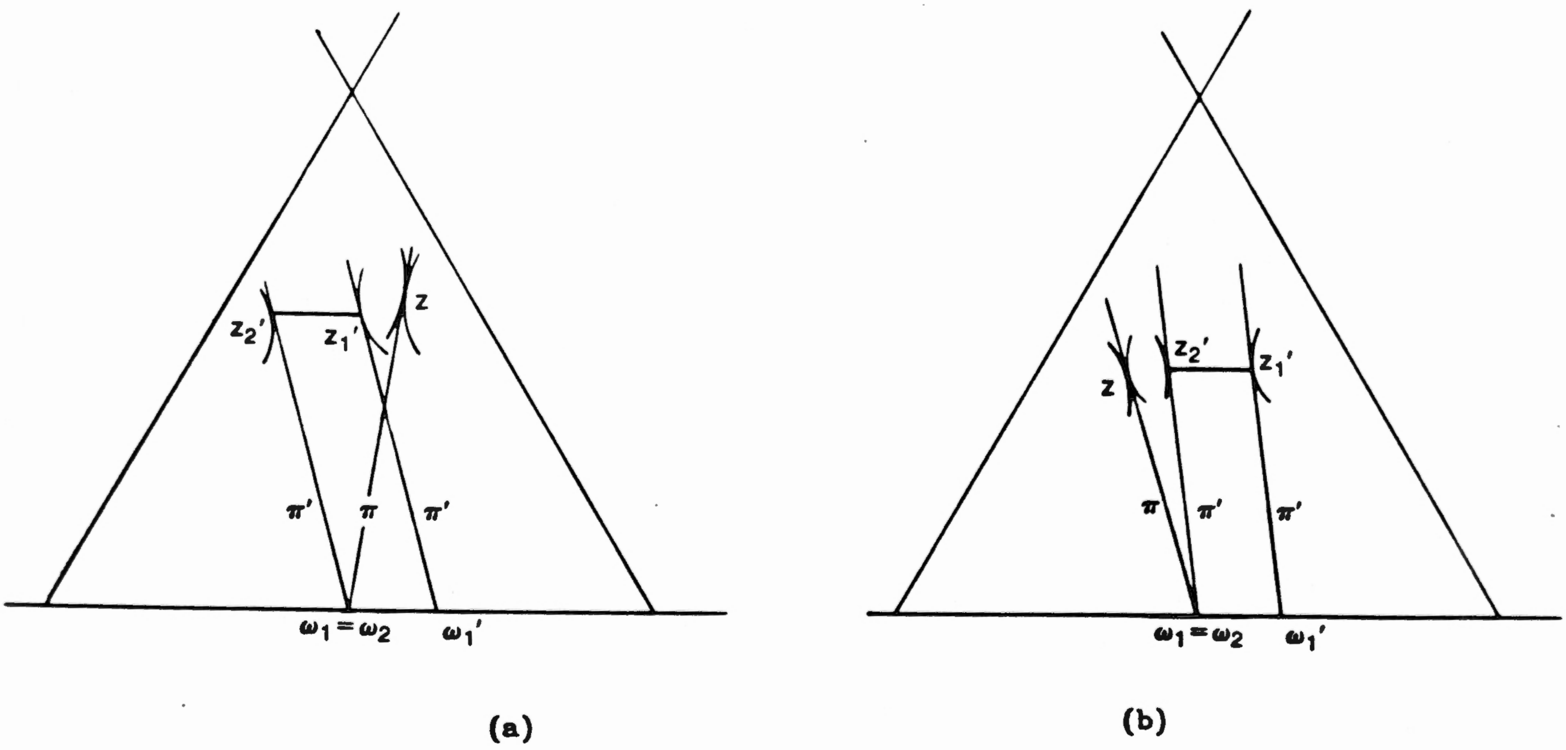


Figure 1

Fact 2. An agent may gain by withholding (that is, saving for later consumption) part of his initial endowment. (Postlewaite, 1979, discussed this phenomenon in private good economies.)

Fact 3. An agent may lose when some other agent's initial endowment increases. (Thomson, 1978, noted a similar fact for the Walrasian mechanism.)

Proof. See Figure 1b. Initially, $\omega_1 = \omega_2$. Then ω_1 increases to ω_1'. (π, z) and (π', z') are Lindahl equilibria for (u, ω) and (u, ω'), where $\omega' = (\omega_1', \omega_2)$. Note that $u_2(z_2') < u_2(z_2)$.

Q.E.D.

Fact 4. An agent may gain by transferring part of his initial resources to another agent, and the recipient may lose. (This is the counterpart of the transfer paradox well known to trade theorists.)

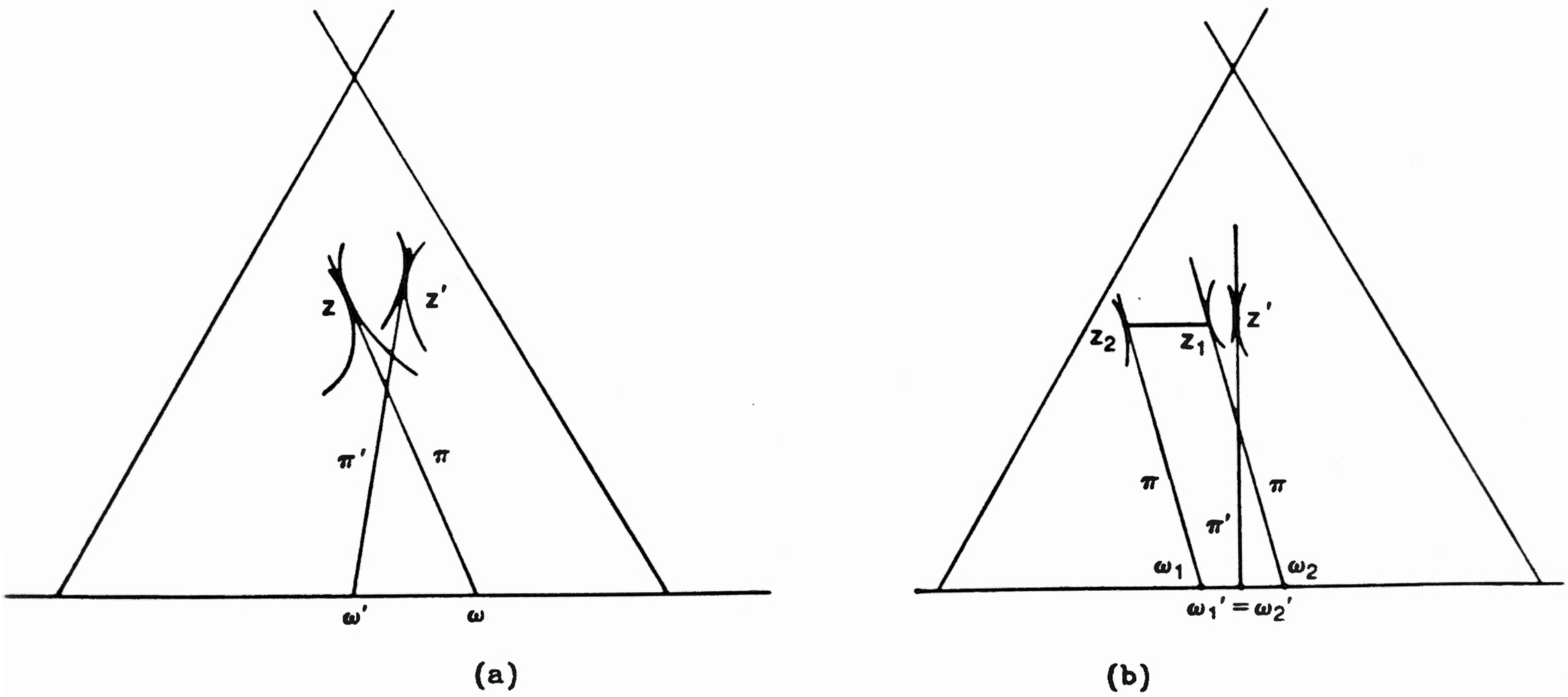


Figure 2

Proof. See Figure 2a. (π,z) and (π',z') are Lindahl equilibria for (u,ω) and (u,ω'), where $\omega_1' \leq \omega_1$, $\omega_2' \geq \omega_2$ and $\omega_1' + \omega_2' = \omega_1 + \omega_2$. Note that $u_1(z_1) <$ $u_1(z_1')$ and $u_2(z_2) > u_2(z_2')$.

Q.E.D.

In the statement of Facts 5 and 6, it should be understood that the Lindahl correspondence is operated from the point of equal division at which there are no public goods at all.

Fact 5. An agent may lose when aggregate resources increase. (This is a violation of resource monotonicity, a property which has been the object of much recent attention by Thomson (1978, 1987b), Chun and Thomson (1984), Roemer (1985, 1986a, 1986b), Moulin (1987a, 1987b) and Moulin and Thomson (1987).)

Proof. See Figure 2b. Initially, $\omega_1 = \omega_2 = \Omega/2$. Then Ω increases to Ω' and the Lindahl correspondence is operated from $\omega_1' = \omega_2' = \Omega'/2$. (π,z) and (π',z') are Lindahl equilibria for (u,ω) and (u,ω'). Note that $u_1(z_1') < u_1(z_1)$.

Q.E.D.

The last property pertains to variations in the number of agents unaccompanied by any increase in the aggregate resources. If all goods are private goods, it seems normatively appealing that all agents initially present be negatively affected. In economies with public goods, however they may very well all benefit from the introduction of new agents, if these new agents have a strong preference for the same public goods; or, they may all lose if the new agents have little interest in these goods. Consequently, the natural monotonicity property to consider is not that all agents lose, or that all agents gain, but that they all be affected in the same direction: they all lose or they all gain. We show next that the Lindahl mechanism does not satisfy the property.

Fact 6. An increase in the number of agents, unaccompanied by an increase in the aggregate resources, may simultaneously benefit some agents and hurt some others.

Proof: (Figure 3.) Obviously, the simplest example needed to prove our claim has to involve at least three agents and the Kolm triangle representation cannot be used. Instead, we use standard rectangular axes. The initial economy has 2 agents.

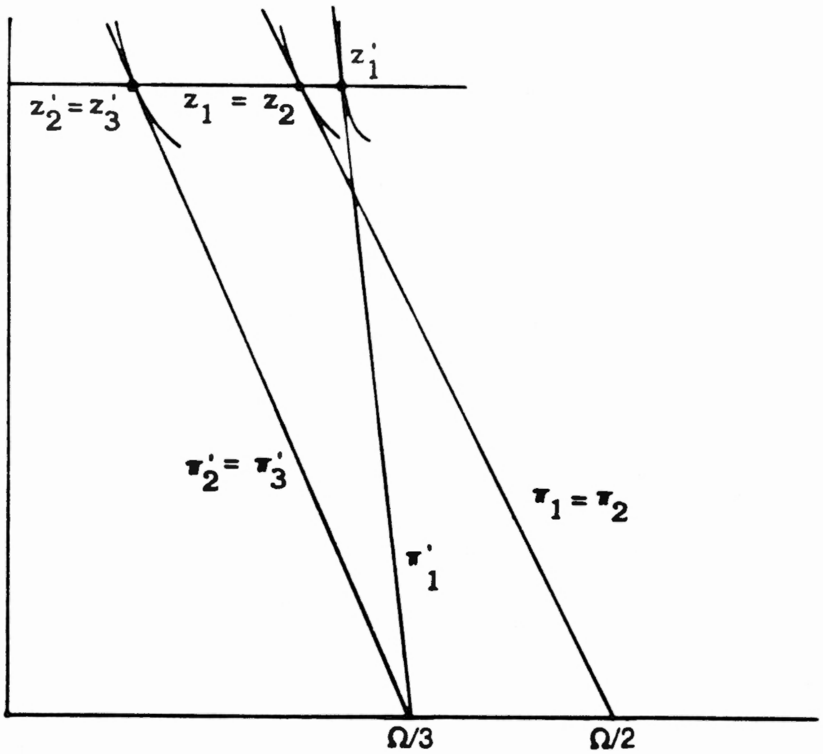


Figure 3

(π, z) is a Lindahl equilibrium with $\pi_{1y}/\pi_{1x} = \pi_{2y}/\pi_{2x} = 1/2$ and $z_1 = z_2$. Then, a third agent comes in and (π', z') is a Lindahl equilibrium with $\pi'_{1y}/\pi'_{2x} = 1/10$, $\pi'_{2y}/\pi'_{2x} = \pi'_{3y}/\pi'_{3x} = 4.5/10$. We note that $u_1(z'_1) > u_1(z_1)$ while $u_2(z'_2) < u_2(z_2)$.

Q.E.D.

In the last few years, a considerable amount of attention has been devoted to the analysis of precisely when the Walrasian correspondence violates the properties described above (although it is mainly the transfer paradox that has been studied; in fact some properties such as the one described in Fact 3 have not been studied in any detail). It was found that the phenomena we are concerned with can occur in otherwise very well behaved economies if the number of agents is greater than two. A similar analysis of the Lindahl correspondence seems to us to be long overdue but we will leave it to future research, pursuing instead the search of possibly other solution concepts that do behave well.

4. *General results*

In this section, we investigate the existence of essentially single-valued correspondences satisfying three types of conditions: (i) Pareto-efficiency; (ii) a minimal "non-discrimination" condition; and, (iii) one of several monotonicity conditions. The monotonicity conditions are really the main focus of this paper but the non-discrimination conditions deserve some discussion too. The condition that we will mainly use is *individual rationality*. In exchange economy, this phrase usually means that each agent is attributed a consumption that he finds at least as desirable as his initial endowment. Whenever production is possible, as is the case here, it may also mean that each agent is attributed a consumption that he finds at least as desirable as what he could achieve with his own endowment by exploiting the technology. The second condition is of course stronger than the first one, and for that reason, we will refer to it as strong individual rationality. We will prove our negative results (except for one) with the weak condition and the positive ones with the strong condition.

We close this section with formal definitions.

Given (u,ω), $I(u,\omega) \equiv \{z \in A(\omega) | u_i(z_i) \geqq u_i(\omega_i)$ for all $i\}$ is the set of ***individually rational*** allocations of (u,ω).

$I^*(u,\omega) \equiv \{z \in A(\omega) | u_i(z_i) \geqq u_i(z_i')$ for all $z_i' = (x_i' 'y_i')$ such that $(-\omega_{ix} + x_i', y_i') \in Y\}$ is the set of ***strongly individually rational*** allocation of (u,ω).

$F(u,\omega) \equiv \{z \in A(\omega) | u_i(z_i) \geqq u_i(z_j)$ for all $i,j\}$ be the set of ***envy-free*** allocations of (u,ω).

The intersection of two correspondences φ and φ' is denoted $\varphi\varphi'$.

4.1 *Destruction*. First, we examine the existence of correspondences immune to manipulation through destruction of one's own endowment. Our first result directly extends a positive result obtained for the private good case. It is included here mainly for completeness.

To present it, we need to introduce some concepts of bargaining theory. Given S, a compact, convex subset of $\mathfrak{R}^n$ and d, a point of S, the ***egalitarian solution*** outcome of the ***bargaining problem*** (S,d), denoted E(S,d), is the maximal point of S at which the agents' utility gains from d are equal.

***Theorem* 1.** Assume preferences are strictly monotone. There are selections from $I^{*}P$ such that an agent always benefits from an increase in his own initial endowment.

***Proof*:** Let $S \equiv u(A(\omega))$ be the image of the set of feasible allocations in utility space and for each i, let $d_i \equiv \max\{u_i(x_i,y) | (-\omega_{ix} + x_i,y) \in Y\}$. Then, let $\varphi(u,\omega) \equiv u^{-1}(E(S,d))$. We leave it to the reader to check that φ satisfies all the desired requirements.

Q.E.D.

4.2 *Withholding*. The next result is the counterpart of a result due to Postlewaite (1979) stating that there is no selection from IP in exchange economies that is immune to manipulation through withholding.

Before presenting it, we need to say a few words about the proofs of our negative results. These proofs are by way of examples. We have chosen to specify them geometrically only, without giving analytical expressions for the utilities. Such expressions would considerably lengthen the paper without yielding much additional insight into the phenomena under study.

***Theorem* 2.** There is no selection from IP such that no agent ever gains by withholding part of his initial endowment.

***Proof*:** See Figure 4. Let $\varphi \subset IP$ be given. There are two agents with identical preferences. Initially, $\omega_1 = \omega_2$ with $\omega_{1y} = 0$. $\bar{z} = (\bar{x}_1,\bar{x}_2,\bar{y})$ is a Lindahl allocation for (u,ω) with $\bar{z}_1 = \bar{z}_2$. Let $z \in \varphi(u,\omega)$. Since $\varphi \subset P$, then either $u_1(z_1) \leqq u_1(\bar{z}_1)$ or $u_2(z_2) \leqq u_2(\bar{z}_2)$. Without loss of generality, we assume the former. Then agent 1 withholds the amount $\omega_1 - \omega_1'$. Let $\omega' \equiv (\omega_1', \omega_2)$. Figure 4 represents the Kolm triangles of (u, ω) and (u, ω') with agent 1's origin kept fixed at 0_1 and agent 2's origin moved to the left from 0_2 to $0_2'$.

We identify the set $IP(u, \omega')$. It is a segment $[z^1, z^2] \equiv C$, where z^1 is the point that agent 1 likes the least and z^2 the point that he likes the most. z^1 is obtained by maximizing u_2 on agent 1's indifference curve through ω_1', which is chosen to be a straight line parallel to the public good axis; z^2 is obtained by maximizing u_1 on agent 2's indifference curve through ω_2 (measured from $0_2'$). The indifference curves through a typical point of $IP(u, \omega')$ are also represented. To each indifference curve of agent 2 is associated an indifference curve for agent 1 by symmetry with respect to the vertical line that divides the Kolm triangle of (u, ω') into two equal right triangles.

Let $z' \in \varphi(u, \omega')$ be given. Since $\varphi \subset IP$, $z' \in C$ and therefore $z_1' + \omega_1 - \omega_1' \in C + \{\omega_1 - \omega_1'\}$. Since any such point is preferred by agent 1 to $\bar{z}_1$, we are done.

Q.E.D.

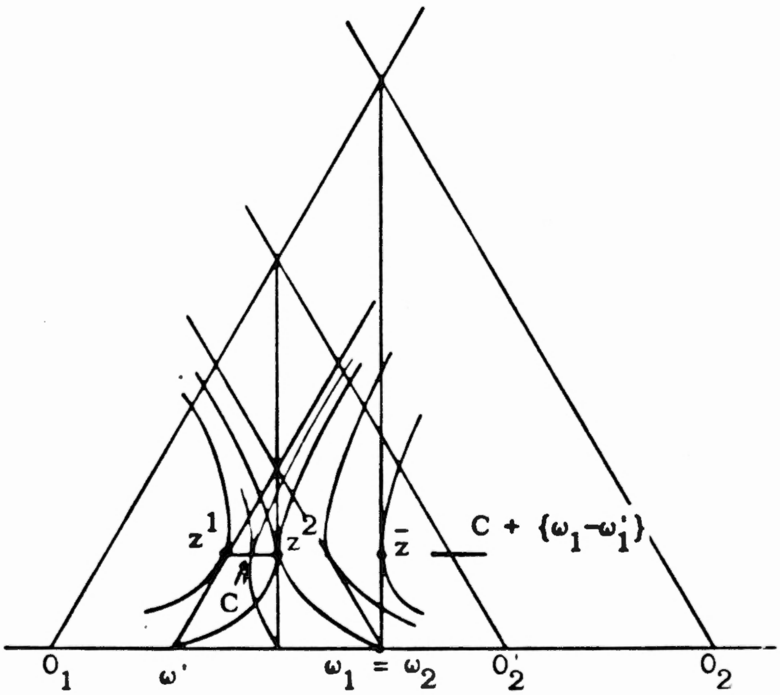


Figure 4

4.3 ***Negative effects on others.*** Next, we consider the possibility that some agent be negatively affected when someone else's initial position improves. Here too, we obtain a general impossibility result.

***Theorem* 3.** There is no selection from IP such that an increase in an agent's initial endowment never hurts the others.

***Proof*:** See Figure 5. Let $\varphi \subset IP$ be given. There are two agents with identical preferences. Initially, $\omega_1 = \omega_2$ with $\omega_{1y} = 0$. Then, agent 1's initial endowment increases to ω_1'. Let $\omega' \equiv (\omega_1', \omega_2)$. $\bar{z} = (\bar{x}_1, \bar{x}_2, \bar{y})$ is a

Lindahl allocation for (u,ω) with $\bar{z}_1 = \bar{z}_2$. Let $z \in \varphi(u,\omega)$. Since $\varphi \subset P$, then either $u_1(z_1) \geqq u_1(\bar{z}_1)$ or $u_2(z_2) \geqq u_2(\bar{z}_2)$. Without loss of generality, we assume the latter. Figure 5 represents the Kolm triangles of (u,ω) and (u,ω') with agent 1's origin kept fixed at 0_1, and agent 2's origin moved to the right from 0_2 to $0_2'$.

Let J be agent 2's indifference curve through $\bar{z}_2$, and let $z' \in \varphi(u,\omega')$. In order for agent 2 not to be hurt by the increase in agent 1's initial endowment, z_2' should be above agent 2's indifference curve through z_2 and since $u_2(z_2) \geqq u_2(\bar{z}_2)$, z_2' should be above J. After the shift in agent 2's origin, this means that in the Kolm triangle of (u,ω'), z_2' should be above the curve labelled $J + \{\omega_1-\omega_1'\}$, obtained from J by the same translation that took 0_2 to $0_2'$. Note that agent 1's indifference curve through ω_1' lies to its right. Therefore, $u_1(z_1') < u_1(\omega_1')$ and the individual rationality condition is violated for agent 1.

Q.E.D.

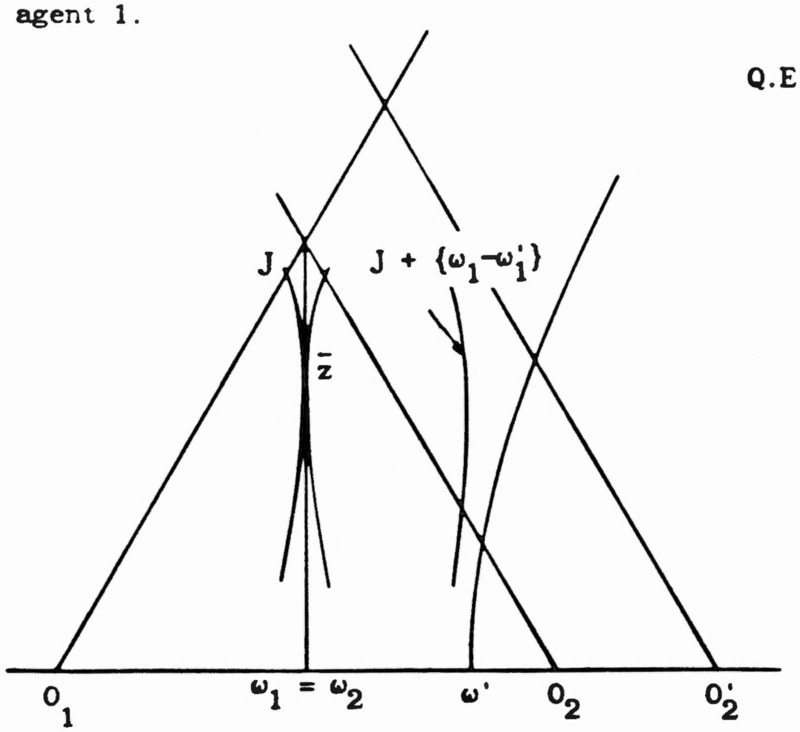


Figure 5

4.4 ***Transfer.*** The transfer problem can be avoided by the same egalitarian methods that helped us avoid the destruction paradox. This is as in the private good case and we need not elaborate. See Thomson (1987c) for details.

4.5. ***Aggregate monotonicity.*** In the next two sections, we turn to economies in which agents collectively own the aggregate resources. It is natural in private good economies to require the final allocation to Pareto dominate equal division. Since in public good economies there is not a unique point of equal division, we will focus here on the point of equal division at which the amount of the public good is zero. The individually rational correspondence from that point will be denoted I_{ed*}. There is no inclusion relation between $I_{ed*}P$ and FP: the set of envy-free allocations is the vertical segment through the top vertex of the Kolm triangle; its intersection with the Pareto set is usually made up of a finite number of points that may or may not Pareto-dominate ed*. (See Thomson, 1987b, for a discussion of these elementary facts.)

Theorem **4.** Suppose that there is only one private good and that for each i, and for each $z_i \in \mathfrak{R}^{n+1}_+$ there is $(\bar{x}_i,0) \in \mathfrak{R}^{\ell+1}_+$ such that $u_i(z_i) = u_i(\bar{x}_i,0)$. (This says that all indifference surfaces cross the private goods axis.) Then, there is a selection from $I^*_{ed*}P$ such that an increase in the aggregate endowment always benefits all agents.

Proof: This is the egalitarian-equivalent method (Pazner and Schmeidler, 1978) when the reference bundle is required to be proportional to the unit vector corresponding to the private good, i.e. $\varphi(u,\omega) \equiv \{z \in A(\omega) | \exists\, \bar{x}_o \geqq 0$ such that for all i, $u_i(z_i) = u_i(\bar{x}_o,0)\}$. Indeed, since it is assumed in the specification of ed* that there are no public goods, all initial endowments are proportional to that vector.

We omit the straightforward proof that φ satisfies all the desired requirements.

Q.E.D.

Remark. The importance of the one private good case should be emphasized. Indeed, in the private good case too, if it is known that the aggregate bundle varies but remains proportional to a given bundle, (or more generally increases along a one-dimensional path,) then the selection from the egalitarian-equivalent correspondence obtained by requiring the reference bundle to be proportional to that bundle is also such that a greater endowment benefits all agents.

Theorem 5. There is no selection from FP such that an increase in the aggregate endowment always benefits all agents.

Proof: Figure 6. Agent 1's preferences are piecewise linear and agent 2's preferences are linear. Let p be agent 2's uniform marginal rate of substitution. Agent 1's preferences are such that the broken line O_1 a z' O_2', where z' = $(\Omega'/2,\Omega'/2,0)$, is his expansion path at prices (1,1-p). In Figure 6, agent 1's marginal rate of substitution is constant and equal to p^1 above the broken line and it is constant and equal to p^2 below the broken line, with $1-p^1 < p < 1-p^2$. The aggregate resources increase from Ω to Ω'. The two resulting Kolm triangles are represented with agent 1's origin

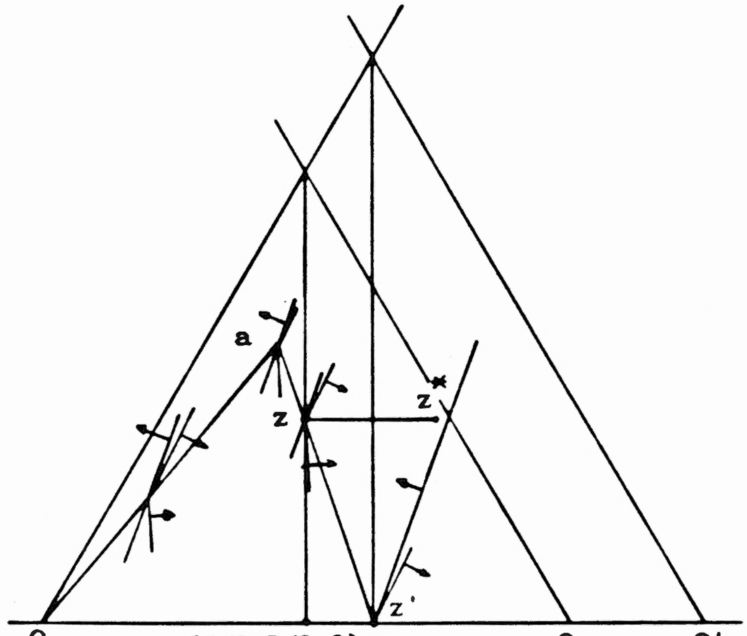


Figure 6

kept fixed at O_1 while agent 2's origin shifts to the right from O_2 to O_2'. $P(u,\Omega)$ is the broken line O_1 a z' O_2 and $P(u,\Omega')$ is the broken line O_1 a z' O_2'. In each case there is a single efficient and envy-free allocation. It is at the intersection of the efficient set with the vertical line through the top vertex. Let $\varphi \subset FP$. Then $\varphi(u,\Omega) = \{z\}$ and $\varphi(u,\Omega') = \{z'\}$. z_2^* is obtained by measuring z_2 from O_2'. Note that agent 2's indifference curve through z_2' passes below z_2^*. Agent 2 has lost as Ω increased to Ω'

Q.E.D.

4.6. *Population Monotonicity*. Here we have a positive result which is obtained by adapting a corresponding result for private good economies (Thomson, 1987c). This adaptation presents some difficulties which we discuss after the proof of the Theorem. Let $WP(u,\omega) \equiv \{z \in A(\omega) | \nexists\, z' \in A(\omega)$ with $u_i(z_i') > u_i(z_i)\ \forall i\}$ be the set of weakly Pareto-optimal allocations of (u,ω).

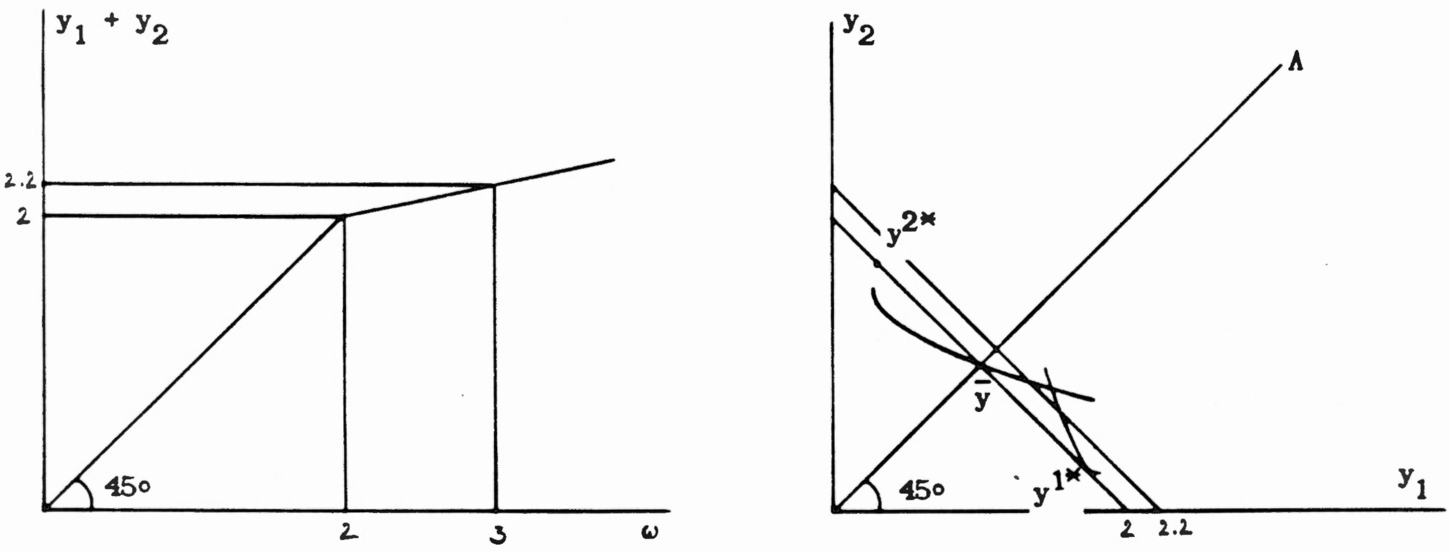
Theorem. Suppose utility is freely disposable. There are selections from $I^*_{ed^*}WP$ such that the agents initially present are all affected negatively, or they are all positively affected, when new agents come in while the aggregate resources remain the same.

Proof. Without loss of generality, assume agent i's utility function to be normalized so that $u_i(0) = 0$. The number of potential agents is assumed to be countably infinite, and we index them by $\mathcal{N}$. Given a group $P \subset \mathcal{N}$ of agents, we denote by $\mathcal{R}^P$ its utility space. Let $d_{iP} \equiv \max \{u_i(x_i,y) | (-\frac{\Omega}{|P|} + x_i,y) \in Y\}$ be the maximal utility agent i can achieve, if given access to the technology and allocated the share $1/|P|$ of the aggregate endowment of the private good. For each $P \subset \mathcal{N}$, let G^P be a continuous, monotone and unbounded path in $\mathcal{R}^P$ to which $d_P \equiv (d_i | i \in P)$ belongs. Finally, let $G \equiv \{G^P | P \subset \mathcal{N}\}$ be a sequence of paths such that for all $P \subset Q$, the projection of G^Q onto $\mathcal{R}^P$ coincides with G^P. Then, given $P \subset \mathcal{N}$ and $(u,\omega) \equiv \{(u_i,\omega_i) | i \in P\}$, let $\varphi(u,\omega) = \{(z_i | i \in P) | (-\Omega + \Sigma x_i,y) \in Y,\ (u_i(z_i) | i \in P)$ is maximal along $G^P\}$. It is straightforward to verify that φ satisfies all the desired properties.

Q.E.D.

We discuss here why we only obtain weak efficiency, while in the private good case, by requiring preferences to be strictly increasing, a similar method actually guaranteed full efficiency. Here, the same domain restriction will not work, for the following reason: if there are public goods (or externalities), the image of the feasible set in utility space is not necessarily comprehensive (S is comprehensive if $t \in S$, $t' \leqq t \Rightarrow t' \in S$). Assuming free disposability of goods does not suffice to obtain this property since, starting from $t \in u(A(\omega))$, in order to reach $t' \leq t$, with for example $t'_1 \leq t_1$ and $t'_i = t_i$ for all $i \neq 1$, one might have to dispose of goods that other agents value too. Comprehensiveness of the feasible set in utility space can of course be obtained by assuming utility itself to be freely disposable: this means that starting from any feasible allocation, it is possible to decrease any agent's utility without affecting the utilities of the others. Without comprehensiveness of the feasible set in utility space, the monotone path solutions described in the proof of the Theorem may yield strictly dominated outcomes. One could recover efficiency by a lexicographic operation as is standard in bargaining theory. However, this will lead to a violation of Population Monotonicity. This is why we have to be content with the weak form of efficiency.

4.7. *Input monotonicity*. Here, we consider economies in which preferences depend only on the vector of public goods, these goods being produced from an input whose control is initially dispersed among the agents. In contrast with all of our analysis so far, the input is assumed not to be directly consumed. In the present context, the strong individual rationality correspondence is the correspondence associating with each economy the set of allocations at which each agent is at least as well off as he would be if he had access to the technology, given the input he controls. (The weaker condition, which was sufficient up to now to derive our negative results, would not be sufficient here.)



(a) (b)

Figure 7

Theorem 6. There is no selection from I^*P such that an increase in the input controlled by an agent never hurts the others.

Proof. There are two agents, one input and two public goods. The technology to produce the public goods is defined by $Y:\mathfrak{R} \to \mathfrak{R}^2$

$$Y(\omega) = \{y \in \mathfrak{R}^2_+ | y_1 + y_2 \leqq f(\omega)\},$$

where f is the function graphed in Figure 7a. We choose preferences so that $u_2(y) = u_1(\pi(y,A))$ for all $y \in \mathfrak{R}^2_+$ where $\pi(y,A)$ is the symmetric image of y with respect to A, the 45° line. Let $\omega_1 = \omega_2 = 1$, $\bar{y} \equiv (1,1)$, and $y1^*$ be the maximizer of u_1 on Y(2). Note that $\bar{y} \in P(u,\omega) = [y^{1*},y^{2*}]$ where y^{2*} is the symmetric image of y^{1*} with respect to the 45° line. Let $y \in \varphi(u,\omega)$. Since $\varphi \subset P$, either $u_1(y) \geqq u_1(\bar{y})$ or $u_2(y) \geqq u_2(\bar{y})$. Without loss of generality, we suppose that $u_2(y) \geqq u_2(\bar{y})$. Then, let ω_1 increase to $\omega_1' = 2$, and let $y' \in$

$\varphi(u,\omega')$ be given. Since $y' \in \varphi(u,\omega') \subset I^*(u,\omega')$, $u_1(y') \geqq u_1(y^{1*})$. Note that no point $y' \in A(\omega')$ satisfying this inequality also satisfies $u_2(y') \geqq u_2(\bar{y})$, so that agent 2 is negatively affected by the increase in the input controlled by agent 1.

Q.E.D.

5. *Concluding comment.*

(i) It is easy to see that all of the negative results could be modified so as to give all agents strictly monotone preferences. Similarly, smooth preferences could be used.

(ii) Since production possibilities have to be explicitly introduced in order to deal with public goods, monotonicity properties with respect to improvements in the technology could be formulated. Such properties play a central role in recent work by Moulin (1987a), Moulin and Roemer (1986) and Roemer (1986a). As shown by Moulin, this property can even be used to provide a characterization of a selection from the core. This selection is the egalitarian-equivalent correspondence obtained by requiring the reference bundle to be proportional to the unit vector corresponding to the (unique) public good.

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