Interest on Reserves and Sunspot Equilibria: Friedman's Proposal Reconsidered

Smith, Bruce D.

Working Paper No. 119
February 1988

INTEREST ON RESERVES AND SUNSPOT EQUILIBRIA: FRIEDMAN'S PROPOSAL RECONSIDERED

Bruce D. Smith
University of Western Ontario and
Rochester Center for Economic Research

Working Paper No. 119

February 1988

I would like to thank Costas Azariadis for interesting me in this project. I alone am responsible for any errors.

Abstract

Friedman's (1960) proposal to pay interest on (required) reserves is considered in a setting that eliminates the indeterminacy discussed by Sargent and Wallace (1985). In an overlapping generations model where the real rate of interest is technologically determined, it is shown that payment of interest on reserves results in a determinate steady state equilibrium. However, interest payments on reserves reduce the steady state welfare of all young agents. Moreover, the payment of interest on reserves often creates a different indeterminacy. It is shown that, for many economies, paying interest on reserves results in the existence of stationary sunspot equilibria, even if such equilibria cannot exist when reserves earn less than market interest rates.

A classic question in monetary economics concerns whether "money and credit markets should be separated" via legal restrictions. Or, put differently, the question asks whether private agents should be prevented from issuing close money substitutes.[1] Advocates of the view that money and credit markets should be separated have often argued that a failure to do so opens an economy to the possibility of "excessive fluctuations" in the price level, and to the possibility of price level indeterminacy as well.[2]

Recent developments in monetary economics permit this argument to be re-interpreted and re-examined in a way that is faithful to many proposals in favor of legally separating money from credit markets. In particular, the development of models displaying sunspot equilibria[3] provides a way of thinking about price level indeterminacy and excessive fluctuations in the price level that is consistent with the arguments advanced in favor of various legal restrictions on liability issues. For instance, the advocates of Peel's Bank Act of 1844 quite clearly were concerned about the possibility of cyclical fluctuations driven by expectations if money and credit markets were not separated.[4] They equally clearly believed that a legal separation of money and credit markets could prevent such fluctuations. Thus models of sunspot equilibria are appropriate to use in analyzing whether economies with no legal restrictions are open to possible indeterminacies (sunspot equilibria), and whether appropriately structured legal restrictions can eliminate these indeterminacies and hence preclude the possibility of excessive fluctuations in the price level.

One of the most widely discussed legal restrictions in this context is the imposition of 100% reserve requirements. This legal restriction has been re-examined in recent literature by Sargent and Wallace (1982) and Smith (1988). Both of these works constructed environments in which the 100% reserve proposal eliminated the possibility of "excessive fluctuations" in the price level. However, it was also the case in those examples that the imposition of a 100% reserve requirement had no obvious welfare justification.

The fact that the imposition of 100% reserve requirements alone might have adverse consequences was, of course, noted by Friedman (1960), and presumably motivated his proposal that interest (at market rates) be paid on reserves. This addition to the 100% reserve proposal seemed to Friedman (1960, p.72) "to improve greatly the attractiveness of the 100% reserve plan." Moreover, Friedman also advocated that interest be paid on reserves (at market rates) even if reserve requirements were something less than 100%.

This paper undertakes an analysis of Friedman's proposal to pay interest on reserves. In doing so it uses a model similar to that employed by Sargent and Wallace (1985) to analyze the same proposal. However, the model here differs from that of Sargent and Wallace in that it presents agents with a productive investment opportunity. In the model below this permits real interest rates to be technologically determined, thus avoiding the indeterminacy of equilibrium problem analyzed by Sargent and Wallace.[5] Thus the model produces a determinate steady state equilibrium. The model is then used to ask two questions:

(a) does the payment of interest on reserves improve welfare in steady state equilibria?

(b) does this proposal achieve its intended affect of preventing price level indeterminacy and excessive fluctuations in the price level?

The answer to both questions is no. In response to question (a), it will be shown that paying market interest rates on reserves makes all young generations worse off than the payment of zero nominal interest on reserves. In response to question (b), it will be shown that, despite the existence of a determinate steady state equilibrium, the payment of interest on reserves can open the economy to the possibility of stationary sunspot equilibria. In particular, economies that have unique stationary equilibria if no interest is paid on reserves can have multiple stationary sunspot equilibria under a 100% reserve regime with interest paid on reserves. Thus there is no sense in which Friedman's proposal has desirable consequences.

The paper proceeds as follows. Section I discusses a model with productive storage and reserve requirements in which interest is not paid on reserves. Section II introduces interest payments on reserves (at market rates) that are financed by taxation. The welfare consequences of paying interest on reserves are analyzed. Section III discusses stationary sunspot equilibria under the two different set-ups. The model is structured in such a way that there are no stationary sunspot equilibria if interest is not paid on reserves. Stationary sunspot equilibria can exist when interest is paid on reserves, however. Section IV concludes.

I. A Reserve Requirement Economy With No Interest Payments[6]

The model consists of an infinite sequence of two period lived, overlapping generations. Time is indexed by t=1,2,... At t=1 there is a set of initial old agents, endowed with a per capita stock of fiat money of M=1 units. The per capita money supply is constant through time. Also, at $t \geq 1$ there is a set of young agents, all of whom are identical. All generations have equal numbers of agents. Young agents have the utility function $U(c_1) + V(c_2)$, where c_j is age j consumption; j=1,2. It is assumed that $U', V' > 0$, $U'' \leq 0$, $V'' < 0$, and that

$$(1) \quad \frac{cV''(c)}{V'(c)} \geq -1$$

$\forall\ c \in R_+$, which is the consumption set at each date. Each young agent has an endowment $y > 0$ of the single consumption good when young, and agents have no endowment of the good when old. Finally, there is a storage technology. One unit of the good stored at t returns $x > 1$ units of the good at time t + 1.[7]

Behavior of Young Agents

Let k_t denote the quantity of the good stored by young agents at t, and let z_t denote their holdings of real balances. All young agents face the reserve requirement $z_t \geq \lambda\ k_t$. Throughout the focus will be on equilibria in which this legal restriction is binding.[8] Finally, let p_t denote the time t price level.

Taking the price level sequence $\{p_t\}$ as given, young agents at each date t choose z_t and k_t ($k_t \geq 0$) to solve the problem

$$\max U(c_1) + V(c_2)$$

subject to

$$(2) \quad c_1 + k_t + z_t = y$$

(3) $$c_2 = xk_t + \left(\frac{p_t}{p_{t+1}}\right) z_t$$

(4) $$z_t \geq \lambda k_t.$$

Noting that the focus is on equilibria where (4) is binding, this problem can be transformed as follows. Let Q_t denote total savings of a young agent at t; $Q_t = (1+\lambda)k_t$. Then young agents can be viewed as solving the problem

$$\max_{Q_t \in [0,\ y]} \quad U(y-Q_t) + V\{[\phi x + (1-\phi)\frac{p_t}{p_{t+1}}]Q_t\}$$

where $\phi = (1+\lambda)^{-1}$. This problem has associated first order condition

(5) $$U'(y-Q_t) = [\phi x + (1-\phi)\frac{p_t}{p_{t+1}}] \; V'\{[\phi x + (1-\phi)\frac{p_t}{p_{t+1}}]Q_t\}.$$

Defining $R_t \equiv \phi x + (1-\phi)\frac{p_t}{p_{t+1}}$, (5) implicitly defines a savings function $Q_t = f(R_t)$. From (5)

(6) $$f'(R) = \frac{V'(c_2)\,[1 + \frac{c_2 V''(c_2)}{V'(c_2)}]}{-U''(c_1) - R_t^2 V''(c_2)} \geq 0,$$

$\forall R$, by the assumption of equation (1). The behavior of young agents is completely summarized by the function f(R).

Equilibrium

An equilibrium is a pair of sequences $\{Q_t\}$ and $\{p_t\}$ satisfying equation (5), $z_t = \lambda k_t = [\lambda/(1+\lambda)]Q_t$, and the money market clearing condition

$$(6)\quad 1/p_t = z_t = [\lambda/(1+\lambda)]Q_t$$

for all $t \geq 1$. Substituting (6) into (5) yields the equilibrium law of motion for Q_t:

$$(7)\quad U'(y-Q_t) = [\phi x + (1-\phi)\frac{Q_{t+1}}{Q_t}]\, V'[\phi x Q_t + (1-\phi)Q_{t+1}];\ t \geq 1.$$

This economy has a unique steady state equilibrium value of Q_t, denoted $\hat{Q}$. This value satisfies the condition

$$(8)\quad U'(y-\hat{Q}) = (\phi x + 1-\phi)V'[(\phi x + 1-\phi)\hat{Q}].$$

The steady state equilibrium level of utility for all young agents is $U(y-\hat{Q}) + V[(\phi x + 1-\phi)\hat{Q}]$.

II. Interest on Reserves

In this section an equilibrium is derived when reserve holdings earn the market rate of return x. Two comments are in order before setting out the model, however. First, the only alternative asset to money here is storage. Since Friedman's proposal involves paying interest on reserves equal to short term market rates, it may seem odd to have the government pay interest on reserves equal to the return on storage. However here all assets must have the same maturity, and Friedman's proposal clearly requires that the opportunity cost of holding reserves be zero. Hence faithfulness to Friedman's proposal requires reserves to earn the real return x. Second, it is well known that if all reserve holdings earn market interest rates, there is a problem with obtaining determinate equilibria. Hence interest is paid here only on required reserve holdings. According to Sargent and Wallace (1985), "folk wisdom" then asserts the existence of a determinate equilibrium. It will be seen that this wisdom is false, despite the fact that the indeterminacy problem discussed by Sargent and Wallace (1985) is avoided here. Finally, in this section only deterministic equilibria are considered. A discussion of stationary sunspot equilibria is deferred until Section III.

__Behavior of Young Agents__

The economic environment is as discussed above, except that (i) all savings earn the "market" real return x (which exceeds p_t/p_{t+1} by assumption) and (ii) interest payments on reserves are financed by a lump-sum tax τ_{t+1} levied on old agents at t+1. Then young agents at t choose total savings Q_t to solve the problem

$$\max_{Q_t \in [0,\, y]} U(y-Q_t) + V(xQ_t-\tau_{t+1}).$$

This problem has the associated first order condition

$$(9) \quad U'(y-Q_t) = xV'(xQ_t-\tau_{t+1}).$$

In addition, since interest is paid only on required reserve holdings, $z_t = \lambda k_t = (\lambda/1+\lambda)Q_t = 1/p_t$.

Equilibrium

The reserve holdings of young agents at t are $(\lambda/1+\lambda)Q_t \equiv (1-\phi)Q_t$. Absent interest payments, these holdings of real balances earn the (gross) real return p_t/p_{t+1}. Hence per capita government interest obligations at t+1 are $(1-\phi)[x-(p_t/p_{t+1})]Q_t$. Government budget balance requires that

$$(10) \quad \tau_{t+1} = (1-\phi)[x - (p_t/p_{t+1})]Q_t.$$

Substitution of (10) into (9) gives the equilibrium law of motion for Q_t:

$$(11) \quad U'(y-Q_t) = xV'[\phi x Q_t + (1-\phi)Q_{t+1}]$$

where $Q_{t+1}/Q_t = p_t/p_{t+1}$ has been used in (11). Notice that there is a determinate steady state value for Q_t, denoted Q^*, with Q^* given by the condition

$$(12) \quad U'(y-Q^*) = xV'[(\phi x + 1-\phi)Q^*].$$

Moreover, from equation (11),

$$(13) \quad \left.\frac{dQ_{t+1}}{dQ_t}\right|_{Q_t = Q^*} = \frac{-U''(y-Q^*) - \phi x^2 V''[(\phi x + 1-\phi)Q^*]}{(1-\phi)xV''[(\phi x + 1-\phi)Q^*]} < 0.$$

Then the steady state equilibrium is locally stable if

$$(14)\quad \frac{-\,U''(y-Q^*) - \phi x^2 V''[(\phi x + 1-\phi)Q^*]}{-\,(1-\phi)xV''[(\phi x + 1-\phi)Q^*]} < 1.$$

__Welfare Consequences of Interest on Reserves__

It is now possible to say something about the welfare justifications for paying interest on reserves. This is done by comparing welfare across the (unique) steady state equilibria with and without interest payments on reserves. It is useful to begin with a preliminary result. Recall that $\hat{Q}$ is the steady state equilibrium level of per capita savings __absent__ interest on reserves. Q^* is the same equilibrium level when interest is paid on reserves. Then

__Proposition 1__. If $\phi < 1$ (so there is a reserve requirement), then $Q^* > \hat{Q}$.

__Proof__. Suppose to the contrary that $\hat{Q} \geq Q^*$. Then from (8) and (12)

$U'(y-Q^*) = xV'[(\phi x + 1-\phi)Q^*] > (\phi x + 1-\phi)V'[(\phi x + 1-\phi)\hat{Q}] = U'(y-\hat{Q})$.

But this implies $\hat{Q} < Q^*$, a contradiction.

As a corollary, if $z^* \equiv (1-\phi)Q^*$ and $\hat{z} \equiv (1-\phi)\hat{Q}$, $z^* > \hat{z}$. The following proposition is then immediate.

__Proposition 2__. The payment of interest on reserves makes the initial old generation better off, and reduces the welfare of all other agents.

__Proof__. That the initial old are made better off follows from $z^* > \hat{z}$; i.e., the payment of interest on reserves increases savings, and hence real balances. That all young generations are made worse off is immediate. Their utility if no interest is paid on reserves is $U(y-\hat{Q}) + V[(\phi x + 1-\phi)\hat{Q}]$, while their utility when reserves earn interest is $U(y-Q^*) + V[(\phi x + 1-\phi)Q^*]$. But by definition $U(y-\hat{Q}) + V[(\phi x + 1-\phi)\hat{Q}] \geq U(y-Q) + V[(\phi x + 1-\phi)Q]$ $\forall\ Q \in [0,y]$. Moreover, the inequality is strict if $Q \neq \hat{Q}$, since V is a strictly concave function.

Thus the payment of interest on reserves has no obvious welfare justification.

III. Stationary Sunspot Equilibria

A. Interest not Paid on Reserves.

Despite the fact that there is no fundamental uncertainty here, suppose that at each date there is a random variable e_t drawn from a two element set; $e_t \in \{1,2\}$. e_t evolves according to a stationary, two-state Markov chain. Let $q(e) = \text{prob}(e_{t+1}=1 : e_t=e)$; $e = 1, 2$.

The focus of discussion will be on stationary sunspot equilibria; i.e., on equilibria where current values depend only on the current state. Let $p(e)$ denote the current price level if $e_t = e$, while $z(e)$, $k(e)$, and $Q(e)$ are real balances, storage, and total savings ($Q(e) \equiv (1+\lambda)k(e)$) by young agents born in state e. All of these variables are selected after the realization of e_t. Finally, all trading is as above, so that (as is always the case in discussing stationary sunspot equilibria) state contingent claims trading is ruled out.

Behavior of Young Agents

A young agent at t, experiencing the realization $e_t = e$, chooses $z(e)$ and $k(e) \geq 0$ to solve the problem

$$\max U[y-k(e)-z(e)] + q(e)V[xk(e) + \frac{p(e)}{p(1)} z(e)] + [1-q(e)]V[xk(e) + \frac{p(e)}{p(2)} z(e)]$$

subject to $z(e) \geq \lambda k(e)$, taking $p(e)$, $q(e)$, $p(1)$, and $p(2)$ as given, where $p(e)$ is the current price level. Again the situation of interest is that where the reserve requirement is binding, so henceforth equilibria with $z(e)=\lambda k(e)$ are considered.[9] Then, recalling that $Q(e) \equiv (1+\lambda)k(e)$, the maximization problem of young agents may be rewritten as

$$\max_{Q(e)\in[0,y]} U[y-Q(e)] + q(e)V\{[\phi x + (1-\phi) \frac{p(e)}{p(1)}]Q(e)\} +$$

$$[1-q(e)]V\{[\phi x + (1-\phi) \frac{p(e)}{p(2)}]Q(e)\},$$

where again $\phi \equiv (1+\lambda)^{-1}$. This problem has associated first order condition

$$(15) \quad U'[y-Q(e)] = q(e)[\phi x + (1-\phi) \frac{p(e)}{p(1)}]V'\{[\phi x + (1-\phi) \frac{p(e)}{p(1)}]Q(e)\}$$

$$+ [1-q(e)][\phi x + (1-\phi) \frac{p(e)}{p(2)}]V'\{[\phi x + (1-\phi) \frac{p(e)}{p(2)}]Q(e)\}; \; e = 1,2.$$

<u>Stationary Sunspot Equilibria</u>

Following Azariadis (1981), a stationary sunspot equilibrium is a set of values $Q(1)$, $Q(2)$, $p(1)$, $p(2)$, $q(1)$, and $q(2)$ satisfying (15), the money market clearing condition

(16) $1/p(e) = z(e) \equiv [\lambda/(1+\lambda)]Q(e);\ e = 1, 2$

and $0 \leq q(e) \leq 1$; $e = 1, 2$. Given the assumption of equation (1), it is straightforward to establish

Proposition 3. If interest is not paid on reserves, there does not exist a stationary sunspot equilibrium (s.s.e.) with $Q(1) \neq Q(2)$.

Proof. Suppose to the contrary that a non-trivial s.s.e. exists. Then, substituting (16) into (15), the values Q(1), Q(2), q(1), and q(2) must satisfy

$$(17)\quad U'[y-Q(1)] = q(1)[\phi x + (1-\phi)]V'[(\phi x + 1-\phi)Q(1)] + [1-q(1)][\phi x + (1-\phi)\frac{Q(2)}{Q(1)}]V'[\phi xQ(1) + (1-\phi)Q(2)]$$

and

$$(18)\quad U'[y-Q(2)] = q(2)[\phi x + (1-\phi)\frac{Q(1)}{Q(2)}]V'[\phi xQ(2) + (1-\phi)Q(1)] + [1-q(2)](\phi x + 1-\phi)V'[(\phi x + 1-\phi)Q(2)].$$

Define

$$H = \phi x + 1-\phi$$

$$H_{12} = \phi x + (1-\phi)\frac{Q(1)}{Q(2)}$$

$$H_{21} = \phi x + (1-\phi)\frac{Q(2)}{Q(1)}.$$

Then, solving (17) and (18) for q(1) and q(2) yields

$$(19)\quad q(1) = \frac{U'[y-Q(1)] - H_{21}V'[H_{21}Q(1)]}{HV'[HQ(1)] - H_{21}V'[H_{21}Q(1)]}$$

$$(20)\quad q(2) = \frac{U'[y-Q(2)] - HV'[HQ(2)]}{H_{12}V'[H_{12}Q(2)] - HV'[HQ(2)]}.$$

Now suppose that Q(1) > Q(2). Then $H_{12} > H > H_{21}$. Further, since $cV''(c)/V'(c) \geq -1$, $nV'[nQ(1)]$ and $nV'[nQ(2)]$ are non-decreasing functions of n. Therefore $HV'[HQ(1)] \geq H_{21}V'[H_{21}Q(1)]$ and $H_{12}V'[H_{12}Q(2)] \geq HV'[HQ(2)]$ hold. It follows that for $0 \leq q(1) \leq 1$ to hold

$$(21)\quad HV'[HQ(1)] \geq U'[y - Q(1)] \geq H_{21}V'[H_{21}Q(1)]$$

must be satisfied. But (21) implies that $Q(1) \leq \hat{Q}$ (recall that $\hat{Q}$ is per capita savings in the steady state equilibrium). Similarly, for $0 \leq q(2) \leq 1$ to hold it is necessary that

$$(22)\quad H_{12}V'[H_{12}Q(2)] \geq U'[y - Q(2)] \geq HV'[HQ(2)].$$

But (22) implies that $Q(2) \geq \hat{Q} \geq Q(1)$, contrary to assumption. An identical contradiction arises if it is assumed that Q(2) > Q(1) holds. Hence Q(1) = Q(2), proving the proposition.

The assumption of equation (1) implies that all young agents have savings functions that are non-decreasing in the rate of interest. This precludes the existence of non-trivial s.s.e.'s if interest is not paid on reserves. However, the situation is substantially different if required reserve holdings bear market interest rates.

B. Interest on Reserves

Again, suppose that the government pays interest on reserves (required reserves only) at the market rate of interest x. A young agent born in state e acquires total savings of Q(e); e = 1, 2, against which the agent holds required reserves of $[\lambda/(1+\lambda)]Q(e) = (1-\phi)Q(e) = z(e)$. Since the young period state is e, the young period price level is p(e); if the old period state is e', the ex post return on reserves (before receiving interest from the government) is p(e)/p(e'). Hence government interest payments to old agents in state e', if the young period state was e, are

$$(1-\phi)[x - \frac{p(e)}{p(e')}]Q(e).$$

As above, these interest payments are financed by a lump-sum tax on old agents of $\tau(e,e')$ if the young period state was e, and the old period state is e'. Then government budget balance requires that

$$(23) \quad \tau(e,e') = (1-\phi)[x - \frac{p(e)}{p(e')}]Q(e); \; e,e' = 1,2.$$

The focus of discussion will be on sunspot equilibria "near" the steady state equilibrium, so attention will be restricted to the case in which $x > p(e)/p(e') \; \forall \; e, e'$.

<u>Behavior of Young Agents</u>

A young agent born in state e faces lump-sum taxes $\tau(e,e')$ if next period's state is e'. Taking these values as given, this agent chooses total savings Q(e) to solve the problem

$$\max_{Q(e) \in [0,y]} \quad U[y-Q(e)] + q(e)V[xQ(e) - \tau(e,1)] + [1-q(e)]V[xQ(e)-\tau(e,2)].$$

Since only required reserve holdings earn interest, $z(e) = \lambda k(e) = (1-\phi)Q(e)$.

The first order condition associated with this problem is

$$(24) \quad U'[y-Q(e)] = q(e)xV'[xQ(e) - \tau(e,1)] + [1-q(e)]xV'[xQ(e)-\tau(e,2)];$$

$$e=1,2.$$

<u>Equilibrium</u>

Again the focus is on stationary sunspot equilibria evolving according to a two-state Markov chain. Then a stationary sunspot equilibrium with interest on reserves is a set of values p(1), p(2), Q(1), Q(2), q(1), q(2), and $\tau(e,e')$ satisfying (23), (24), the money market clearing condition

$$(25) \quad 1/p(e) = z(e) = (1-\phi)Q(e); \; e = 1,2,$$

and $0 \leq q(e) \leq 1$; e = 1, 2. A sufficient condition for non-trivial s.s.e.'s to exist is as follows.

<u>Proposition 4</u>. Suppose that the (unique) steady state equilibrium when interest is paid on reserves is locally stable; i.e., suppose that

$$(26)\qquad \frac{-(1-\phi)xV''[(\phi x+1-\phi)Q^*]}{-U''(y-Q^*)-\phi x^2 V''[(\phi x+1-\phi)Q^*]} > 1.$$

Suppose further that $\phi = (1+\lambda)^{-1} < (1+x)^{-1}$. Then there exist stationary sunspot equilibria with $Q(1) \neq Q(2)$ and $x > p(e)/p(e')$ $\forall$ e, e'.

Remark. It is straightforward to show that there are no stationary sunspot equilibria (except the trivial steady state equilibrium) if $\phi \geq (1+x)^{-1}$. Thus the existence of s.s.e.'s requires that the reserve requirement $1-\phi$ be sufficiently large. Of course Friedman's proposal is for 100% reserve requirements ($\phi = 0$), so this requirement should not be troublesome.

Proof. The proof is by construction. To begin, substitute (25) into (23), and then substitute the result into (24) to obtain the conditions

$$(27)\quad U'[y-Q(1)] = q(1)xV'[(\phi x+1-\phi)Q(1)] + [1-q(1)]xV'[\phi xQ(1) + (1-\phi)Q(2)]$$

and

$$(28)\quad U'[y - Q(2)] = q(2)xV'[\phi xQ(2) + (1-\phi)Q(1)] + [1-q(2)]xV'[(\phi x+1-\phi)Q(2)]$$

Solving (27) and (28) for q(1) and q(2) yields

$$(27')\quad q(1) = \frac{xV'[\phi xQ(1)+(1-\phi)Q(2)]-U'[y-Q(1)]}{xV'[\phi xQ(1)+(1-\phi)Q(2)]-xV'[(\phi x+1-\phi)Q(1)]}$$

$$(28') \quad q(2) = \frac{xV'[\phi xQ(2)+(1-\phi)Q(2)]-U'[y-Q(2)]}{xV'[\phi xQ(2)+(1-\phi)Q(2)]-xV'[\phi xQ(2)+(1-\phi)Q(1)]}$$

Now choose Q(1) and Q(2) such that $y > Q(1) > Q^* > Q(2) > 0$. Then

$$V'[\phi xQ(1)+(1-\phi)Q(2)] > V'[(\phi x+1-\phi)Q(1)]$$

and

$$V'[(\phi x+1-\phi)Q(2)] > V'[\phi xQ(2)+(1-\phi)Q(1)]$$

hold, since $V''<0$. Therefore $0 \leq q(1) \leq 1$ if

$$(29) \quad xV'[\phi xQ(1)+(1-\phi)Q(2)] \geq U'[y-Q(1)] \geq xV'[(\phi x+1-\phi)Q(1)]$$

holds, while $0 \leq q(2) \leq 1$ holds if

$$(30) \quad xV'[(\phi x+1-\phi)Q(2)] \geq U'[y-Q(2)] \geq xV'[\phi xQ(2) + (1-\phi)Q(1)].$$

However, $U'[y-Q(1)] > xV'[(\phi x+1-\phi)Q(1)]$ and $xV'[(\phi x+1-\phi)Q(2)] > U'[y-Q(2)]$ hold, as $Q(1) > Q^* > Q(2)$, and Q^* is defined by the condition $U'(y-Q^*) = xV'[(\phi x+1-\phi)Q^*]$. Then (29) and (30) are satisfied if

$$(31) \quad xV'[\phi xQ(1)+(1-\phi)Q(2)] \geq U'[y-Q(1)]$$

$$(32) \quad U'[y-Q(2)] \geq xV'[\phi xQ(2)+(1-\phi)Q(1)]$$

are satisfied.

It will now be convenient to depict conditions (31) and (32) diagrammatically. Consider the loci defined by equations (31) and (32) holding with equality. These are depicted in Figure 1. Both loci intersect the 45^{o} line at $Q(1)=Q(2)=Q^*$, since $U'(y-Q^*) = xV'[(\phi x+1-\phi)Q^*]$. Points on or below the locus labeled (31) satisfy equation (31), and points on or above the locus labeled (32) satisfy equation (32). Then, if the locus (31) is less steeply sloped than the locus (32) at the point $(Q(1),Q(2)) = (Q^*,Q^*)$, all points in the shaded region satisfy (31), (32), and $Q(1) > Q^* > Q(2)$. Hence there are non-trivial s.s.e.'s if the locus (32) is more steeply sloped than the locus (31) at (Q^*,Q^*). Moreover, points in the shaded region of Figure 1 satisfy $x > p(e)/p(e') \ \forall\ e, e'$ if $Q(1)$ and $Q(2)$ are chosen sufficiently close to Q^* (so that $x > Q(1)/Q(2)$).

A sufficient condition for the existence of s.s.e.'s, then, is that (32) be more steeply sloped at (Q^*,Q^*) than (31). From equations (31) and (32) these slopes are

$$(33) \quad \left.\frac{dQ(2)}{dQ(1)}\right|_{(31)} = \frac{-U''(y-Q^*)-\phi x^2 V''[(\phi x+1-\phi)Q^*]}{x(1-\phi)V''[(\phi x+1-\phi)Q^*]}$$

$$(34) \quad \left.\frac{dQ(2)}{dQ(1)}\right|_{(32)} = \frac{x(1-\phi)V''[(\phi x+1-\phi)Q^*]}{-U''(y-Q^*)-\phi x^2 V''[(\phi x+1-\phi)Q^*]}$$

Then (32) is more steeply sloped than (31) at (Q^*,Q^*) if

$$\frac{x(1-\phi)V''}{-U''-\phi x^2 V''} < \frac{-U''-\phi x^2 V''}{x(1-\phi)V''} \quad (< 0).$$

This condition is equivalent to

$$(35)\quad \frac{-x(1-\phi)V''}{-U''-\phi x^2 V''} > 1,$$

establishing the result.

The set of economies for which (35) holds is non-trivial. For instance, if $U''=0$, (35) reduces to $(1+x)^{-1} > \phi$. Then non-trivial stationary sunspot equilibria will exist whenever the reserve requirement is sufficiently high. It is the case, then, that for many economies that do not have s.s.e.'s when no interest is paid on reserves, s.s.e.'s will exist when reserves earn market interest rates. Thus Friedman's proposal that interest be paid on reserves can open economies to price level indeterminacy, and to the possibility of "excessive" price level fluctuations driven by expectations.

IV. Conclusions

Friedman's (1960) proposal to pay interest on reserves has generally been well received by economists. It has only recently been criticized by Sargent and Wallace (1985) for resulting in an indeterminate steady state equilibrium. However, the Sargent-Wallace indeterminacy disappears when real interest rates are technologically determined, as they are in the model above. Such a model seems more faithful to the monetarist view of real interest rates that are uninfluenced by monetary factors (at least in the "long-run") than does the Sargent-Wallace model.

In the analysis above, the payment of interest on reserves (at market rates) is consistent with the existence of a unique steady state equilibrium. Nevertheless, the analysis does not suggest that Friedman's proposal has any desirable properties. In a steady state equilibrium, the payment of interest on reserves reduces the welfare of all agents, except the initial old. Moreover, the model is structured in such a way that, when reserves do not earn interest, there is a unique stationary equilibrium.

When reserves do earn interest, on the other hand, indeterminacies are a real possibility, in the form of stationary sunspot equilibria. Thus the Friedman proposal may open an economy to the possibility of "excessive fluctuations" in the price level.

In closing, two potential objections to the analysis might be anticipated. First, one might question whether the type of model employed here captures interesting aspects of real world monetary policy issues. A partial answer is given by Azariadis and Farmer (1987), who use a reserve requirement model similar to the one employed above (and actually more similar to the model of Sargent and Wallace (1985)) to capture many features of U.S. monetary policy since the creation of the Federal Reserve System. Their analysis suggests that models of the type used here do shed light on practical issues of monetary policy formulation.

Second, one might question whether the results would be different if interest were paid on reserves at below market rates, and if reserve requirements were employed by the government to enhance seigniorage revenue. Without addressing this issue directly, it might be noted that Freeman (1987) has shown (in a model exactly like the one above, but with a government faced with the problem of financing a deficit) that the optimal set of government actions under such circumstances includes driving the real return on reserves to zero. Hence such considerations seem unlikely to make Friedman's proposal more attractive.

<u>Notes</u>

1. A weaker version of this question is, should agents operating in certain financial markets have their ability to create close money substitutes limited by the imposition of reserve requirements, for instance?
2. See Sargent and Wallace (1982) for a summary of this viewpoint, which they identify with the "quantity theory of money."
3. See Shell (1977), Azariadis (1981), and Cass and Shell (1983) for the original developments in this literature.
4. See Smith (1988) for an argument to this effect. Smith provides a summary of a more detailed argument on the same point by White (1984).
5. Having real interest rates be technologically determined is attractive for this reason, and also because it is probably more faithful to the monetarist view of long-run real interest rates that are not affected by monetary factors than is the Sargent-Wallace formulation.
6. The economy of this section is essentially the same as that in section VI of Wallace (1981).
7. Notice that the environment is specified in a way that precludes borrowing and lending. This is for simplicity only; it should be apparent that none of the results depend upon homogeneous young agents.
8. If money is not dominated in rate of return, then the proposal to pay interest on reserves is, of course, not very interesting.
9. It is straightforward to show that this economy has no (non-trivial) stationary sunspot equilibrium if the reserve requirement does <u>not</u> bind in either or both states.

References

Azariadis, Costas, "Self-Fulfilling Prophecies," Journal of Economic Theory, 25, Dec. 1981, p. 380-96.

Azariadis, Costas, and Roger E.A. Farmer, "Fractional Reserve Banking," manuscript, 1987.

Cass, David, and Karl Shell, "Do Sunspots Matter?" Journal of Political Economy, 91, April 1983, p. 193-227.

Freeman, Scott, "Reserve Requirements and Optimal Seigniorage," Journal of Monetary Economics, 19, 1987, p. 307-14.

Friedman, Milton, A Program for Monetary Stability, Fordham University Press, New York, 1960.

Sargent, Thomas J., and Neil Wallace, "The Real Bills Doctrine versus the Quantity Theory: a Reconsideration," Journal of Political Economy, 90, Dec. 1982, p. 1212-36.

Sargent, Thomas J., and Neil Wallace, "Interest on Reserves," Journal of Monetary Economics, 15, May 1985, p. 279-90.

Shell, Karl, "Monnaie et Allocation Intertemporelle," manuscript, 1977.

Smith, Bruce D., "Legal Restrictions, 'Sunspots', and Peel's Bank Act: the Real Bills Doctrine versus the Quantity Theory Reconsidered," Journal of Political Economy, 96, Feb. 1988, p. 3-19.

Wallace, Neil, "A Modigliani-Miller Theorem for Open Market Operations," American Economic Review, 71, June 1981, p. 267-74.

White, Lawrence H., Free Banking in Britain: Theory, Experience, and Debate, 1800-1845, Cambridge University Press, Cambridge, 1984.

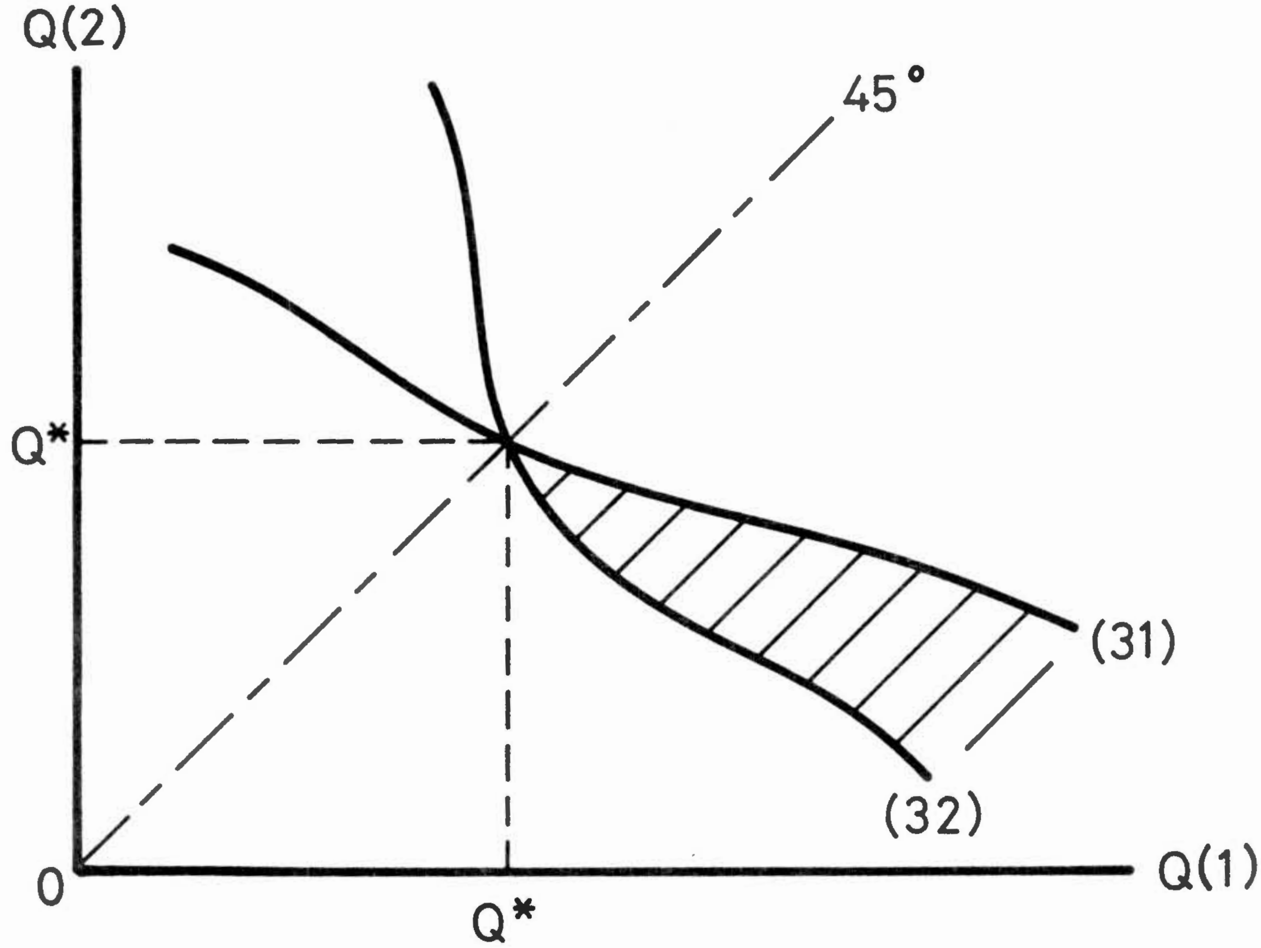

Figure 1

Rochester Center for Economic Research
University of Rochester
Department of Economics
Rochester, NY 14627

1987-88 DISCUSSION PAPERS

WP#68 RECURSIVE UTILITY AND OPTIMAL CAPITAL ACCUMULATION, I: EXISTENCE, by Robert A. Becker, John H. Boyd III, and Bom Yong Sung, January 1987

WP#69 MONEY AND MARKET INCOMPLETENESS IN OVERLAPPING-GENERATIONS MODELS, by Marianne Baxter, January 1987

WP#70 GROWTH BASED ON INCREASING RETURNS DUE TO SPECIALIZATION by Paul M. Romer, January 1987

WP#71 WHY A STUBBORN CONSERVATIVE WOULD RUN A DEFICIT: POLICY WITH TIME-INCONSISTENT PREFERENCES by Torsten Persson and Lars E.O. Svensson, January 1987

WP#72 ON THE CONTINUUM APPROACH OF SPATIAL AND SOME LOCAL PUBLIC GOODS OR PRODUCT DIFFERENTIATION MODELS by Marcus Berliant and Thijs ten Raa, January 1987

WP#73 THE QUIT-LAYOFF DISTINCTION: GROWTH EFFECTS by Kenneth J. McLaughlin, February 1987

WP#74 SOCIAL SECURITY, LIQUIDITY, AND EARLY RETIREMENT by James A. Kahn, March 1987

WP#75 THE PRODUCT CYCLE HYPOTHESIS AND THE HECKSCHER-OHLIN-SAMUELSON THEORY OF INTERNATIONAL TRADE by Sugata Marjit, April 1987

WP#76 NOTIONS OF EQUAL OPPORTUNITIES by William Thomson, April 1987

WP#77 BARGAINING PROBLEMS WITH UNCERTAIN DISAGREEMENT POINTS by Youngsub Chun and William Thomson, April 1987

WP#78 THE ECONOMICS OF RISING STARS by Glenn M. MacDonald, April 1987

WP#79 STOCHASTIC TRENDS AND ECONOMIC FLUCTUATIONS by Robert King, Charles Plosser, James Stock, and Mark Watson, April 1987

WP#80 INTEREST RATE SMOOTHING AND PRICE LEVEL TREND-STATIONARITY by Marvin Goodfriend, April 1987

WP#81 THE EQUILIBRIUM APPROACH TO EXCHANGE RATES
by Alan C. Stockman, revised, April 1987

WP#82 INTEREST-RATE SMOOTHING
by Robert J. Barro, May 1987

WP#83 CYCLICAL PRICING OF DURABLE LUXURIES
by Mark Bils, May 1987

WP#84 EQUILIBRIUM IN COOPERATIVE GAMES OF POLICY FORMULATION
by Thomas F. Cooley and Bruce D. Smith, May 1987

WP#85 RENT SHARING AND TURNOVER IN A MODEL WITH EFFICIENCY UNITS OF HUMAN CAPITAL
by Kenneth J. McLaughlin, revised, May 1987

WP#86 THE CYCLICALITY OF LABOR TURNOVER: A JOINT WEALTH MAXIMIZING HYPOTHESIS
by Kenneth J. McLaughlin, revised, May 1987

WP#87 CAN EVERYONE BENEFIT FROM GROWTH? THREE DIFFICULTIES
by Herve' Moulin and William Thomson, May 1987

WP#88 TRADE IN RISKY ASSETS
by Lars E.O. Svensson, May 1987

WP#89 RATIONAL EXPECTATIONS MODELS WITH CENSORED VARIABLES
by Marianne Baxter, June 1987

WP#90 EMPIRICAL EXAMINATIONS OF THE INFORMATION SETS OF ECONOMIC AGENTS
by Nils Gottfries and Torsten Persson, June 1987

WP#91 DO WAGES VARY IN CITIES? AN EMPIRICAL STUDY OF URBAN LABOR MARKETS
by Eric A. Hanushek, June 1987

WP#92 ASPECTS OF TOURNAMENT MODELS: A SURVEY
by Kenneth J. McLaughlin, July 1987

WP#93 ON MODELLING THE NATURAL RATE OF UNEMPLOYMENT WITH INDIVISIBLE LABOR
by Jeremy Greenwood and Gregory W. Huffman

WP#94 TWENTY YEARS AFTER: ECONOMETRICS, 1966-1986
by Adrian Pagan, August 1987

WP#95 ON WELFARE THEORY AND URBAN ECONOMICS
by Marcus Berliant, Yorgos Y. Papageorgiou and Ping Wang, August 1987

WP#96 ENDOGENOUS FINANCIAL STRUCTURE IN AN ECONOMY WITH PRIVATE INFORMATION
by James Kahn, August 1987

WP#97 THE TRADE-OFF BETWEEN CHILD QUANTITY AND QUALITY: SOME EMPIRICAL EVIDENCE
by Eric Hanushek, September 1987

WP#98 SUPPLY AND EQUILIBRIUM IN AN ECONOMY WITH LAND AND PRODUCTION
by Marcus Berliant and Hou-Wen Jeng, September 1987

WP#99 AXIOMS CONCERNING UNCERTAIN DISAGREEMENT POINTS FOR 2-PERSON BARGAINING PROBLEMS
by Youngsub Chun, September 1987

WP#100 MONEY AND INFLATION IN THE AMERICAN COLONIES: FURTHER EVIDENCE ON THE FAILURE OF THE QUANTITY THEORY
by Bruce Smith, October 1987

WP#101 BANK PANICS, SUSPENSIONS, AND GEOGRAPHY: SOME NOTES ON THE "CONTAGION OF FEAR" IN BANKING
by Bruce Smith, October 1987

WP#102 LEGAL RESTRICTIONS, "SUNSPOTS", AND CYCLES
by Bruce Smith, October 1987

WP#103 THE QUIT-LAYOFF DISTINCTION IN A JOINT WEALTH MAXIMIZING APPROACH TO LABOR TURNOVER
by Kenneth McLaughlin, October 1987

WP#104 ON THE INCONSISTENCY OF THE MLE IN CERTAIN HETEROSKEDASTIC REGRESSION MODELS
by Adrian Pagan and H. Sabau, October 1987

WP#105 RECURRENT ADVERTISING
by Ignatius J. Horstmann and Glenn M. MacDonald, October 1987

WP#106 PREDICTIVE EFFICIENCY FOR SIMPLE NONLINEAR MODELS
by Thomas F. Cooley, William R. Parke and Siddhartha Chib, October 1987

WP#107 CREDIBILITY OF MACROECONOMIC POLICY: AN INTRODUCTION AND A BROAD SURVEY
by Torsten Persson, November 1987

WP#108 SOCIAL CONTRACTS AS ASSETS: A POSSIBLE SOLUTION TO THE TIME-CONSISTENCY PROBLEM
by Laurence Kotlikoff, Torsten Persson and Lars E. O. Svensson, November 1987

WP#109 EXCHANGE RATE VARIABILITY AND ASSET TRADE
by Torsten Persson and Lars E. O. Svensson, Novmeber 1987

WP#110 MICROFOUNDATIONS OF INDIVISIBLE LABOR
by Vittorio Grilli and Richard Rogerson, November 1987

WP#111 FISCAL POLICIES AND THE DOLLAR/POUND EXCHANGE RATE: 1870-1984
by Vittorio Grilli, November 1987

WP#112 INFLATION AND STOCK RETURNS WITH COMPLETE MARKETS
by Thomas Cooley and Jon Sonstelie, November 1987

WP#113 THE ECONOMETRIC ANALYSIS OF MODELS WITH RISK TERMS
by Adrian Pagan and Aman Ullah, December 1987

WP#114 PROGRAM TARGETING OPTIONS AND THE ELDERLY
by Eric Hanushek and Roberton Williams, December 1987

WP#115 BARGAINING SOLUTIONS AND STABILITY OF GROUPS
by Youngsub Chun and William Thomson, December 1987

WP#116 MONOTONIC ALLOCATION MECHANISMS
by William Thomson, December 1987

WP#117 MONOTONIC ALLOCATION MECHANISMS IN ECONOMIES WITH PUBLIC GOODS
by William Thomson, December 1987

WP#118 ADVERSE SELECTION, AGGREGATE UNCERTAINTY, AND THE ROLE FOR MUTUAL INSURANCE COMPANIES
by Bruce D. Smith and Michael J. Stutzer, February 1988

WP#119 INTEREST ON RESERVES AND SUNSPOT EQUILIBRIA: FRIEDMAN'S PROPOSAL RECONSIDERED
by Bruce D. Smith, February 1988

WP#120 INTERNATIONAL FINANCIAL INTERMEDIATION AND AGGREGATE FLUCTUATIONS UNDER ALTERNATIVE EXCHANGE RATE REGIMES
by Jeremy Greenwood and Stephen D. Williamson, February 1988

WP#121 FINANCIAL DEREGULATION, MONETARY POLICY, AND CENTRAL BANKING
by Marvin Goodfriend and Robert G. King, February 1988

To order a copy of the above papers complete the attached form and return to Mrs. Ellen Bennett, or call (716) 275-8396. The first three papers requested will be provided free of charge. Each additional paper will require a $3.00 service fee which must be enclosed with your order.

Requestor's Name ______________________________

Requestor's Address ______________________________

____

____

Please send me the following papers free of charge.

WP# ____ WP# ____ WP# ____

I understand there is a $3.00 fee for each additional paper. Enclosed is my check or money order in the amount of $ ________. Please send me the following papers.

WP# ____ WP# ____ WP# ____
WP# ____ WP# ____ WP# ____
WP# ____ WP# ____ WP# ____
WP# ____ WP# ____ WP# ____

INTEREST ON RESERVES AND SUNSPOT EQUILIBRIA: FRIEDMAN'S PROPOSAL RECONSIDERED

Bruce D. Smith
University of Western Ontario and
Rochester Center for Economic Research

Working Paper No. 119

February 1988

I would like to thank Costas Azariadis for interesting me in this project. I alone am responsible for any errors.

Abstract

Friedman's (1960) proposal to pay interest on (required) reserves is considered in a setting that eliminates the indeterminacy discussed by Sargent and Wallace (1985). In an overlapping generations model where the real rate of interest is technologically determined, it is shown that payment of interest on reserves results in a determinate steady state equilibrium. However, interest payments on reserves reduce the steady state welfare of all young agents. Moreover, the payment of interest on reserves often creates a different indeterminacy. It is shown that, for many economies, paying interest on reserves results in the existence of stationary sunspot equilibria, even if such equilibria cannot exist when reserves earn less than market interest rates.

A classic question in monetary economics concerns whether "money and credit markets should be separated" via legal restrictions. Or, put differently, the question asks whether private agents should be prevented from issuing close money substitutes.[1] Advocates of the view that money and credit markets should be separated have often argued that a failure to do so opens an economy to the possibility of "excessive fluctuations" in the price level, and to the possibility of price level indeterminacy as well.[2]

Recent developments in monetary economics permit this argument to be re-interpreted and re-examined in a way that is faithful to many proposals in favor of legally separating money from credit markets. In particular, the development of models displaying sunspot equilibria[3] provides a way of thinking about price level indeterminacy and excessive fluctuations in the price level that is consistent with the arguments advanced in favor of various legal restrictions on liability issues. For instance, the advocates of Peel's Bank Act of 1844 quite clearly were concerned about the possibility of cyclical fluctuations driven by expectations if money and credit markets were not separated.[4] They equally clearly believed that a legal separation of money and credit markets could prevent such fluctuations. Thus models of sunspot equilibria are appropriate to use in analyzing whether economies with no legal restrictions are open to possible indeterminacies (sunspot equilibria), and whether appropriately structured legal restrictions can eliminate these indeterminacies and hence preclude the possibility of excessive fluctuations in the price level.

One of the most widely discussed legal restrictions in this context is the imposition of 100% reserve requirements. This legal restriction has been re-examined in recent literature by Sargent and Wallace (1982) and Smith (1988). Both of these works constructed environments in which the 100% reserve proposal eliminated the possibility of "excessive fluctuations" in the price level. However, it was also the case in those examples that the imposition of a 100% reserve requirement had no obvious welfare justification.

The fact that the imposition of 100% reserve requirements alone might have adverse consequences was, of course, noted by Friedman (1960), and presumably motivated his proposal that interest (at market rates) be paid on reserves. This addition to the 100% reserve proposal seemed to Friedman (1960, p.72) "to improve greatly the attractiveness of the 100% reserve plan." Moreover, Friedman also advocated that interest be paid on reserves (at market rates) even if reserve requirements were something less than 100%.

This paper undertakes an analysis of Friedman's proposal to pay interest on reserves. In doing so it uses a model similar to that employed by Sargent and Wallace (1985) to analyze the same proposal. However, the model here differs from that of Sargent and Wallace in that it presents agents with a productive investment opportunity. In the model below this permits real interest rates to be technologically determined, thus avoiding the indeterminacy of equilibrium problem analyzed by Sargent and Wallace.[5] Thus the model produces a determinate steady state equilibrium. The model is then used to ask two questions:

(a) does the payment of interest on reserves improve welfare in steady state equilibria?

(b) does this proposal achieve its intended affect of preventing price level indeterminacy and excessive fluctuations in the price level?

The answer to both questions is no. In response to question (a), it will be shown that paying market interest rates on reserves makes all young generations worse off than the payment of zero nominal interest on reserves. In response to question (b), it will be shown that, despite the existence of a determinate steady state equilibrium, the payment of interest on reserves can open the economy to the possibility of stationary sunspot equilibria. In particular, economies that have unique stationary equilibria if no interest is paid on reserves can have multiple stationary sunspot equilibria under a 100% reserve regime with interest paid on reserves. Thus there is no sense in which Friedman's proposal has desirable consequences.

The paper proceeds as follows. Section I discusses a model with productive storage and reserve requirements in which interest is not paid on reserves. Section II introduces interest payments on reserves (at market rates) that are financed by taxation. The welfare consequences of paying interest on reserves are analyzed. Section III discusses stationary sunspot equilibria under the two different set-ups. The model is structured in such a way that there are no stationary sunspot equilibria if interest is not paid on reserves. Stationary sunspot equilibria can exist when interest is paid on reserves, however. Section IV concludes.

I. A Reserve Requirement Economy With No Interest Payments[6]

The model consists of an infinite sequence of two period lived, overlapping generations. Time is indexed by $t=1,2,...$ At $t=1$ there is a set of initial old agents, endowed with a per capita stock of fiat money of $M=1$ units. The per capita money supply is constant through time. Also, at $t \geq 1$ there is a set of young agents, all of whom are identical. All generations have equal numbers of agents. Young agents have the utility function $U(c_1) + V(c_2)$, where c_j is age j consumption; j=1,2. It is assumed that $U', V' > 0$, $U'' \leq 0$, $V'' < 0$, and that

$$\text{(1)} \quad \frac{cV''(c)}{V'(c)} \geq -1$$

$\forall\, c \in R_+$, which is the consumption set at each date. Each young agent has an endowment $y > 0$ of the single consumption good when young, and agents have no endowment of the good when old. Finally, there is a storage technology. One unit of the good stored at t returns $x > 1$ units of the good at time $t + 1$.[7]

Behavior of Young Agents

Let k_t denote the quantity of the good stored by young agents at t, and let z_t denote their holdings of real balances. All young agents face the reserve requirement $z_t \geq \lambda\, k_t$. Throughout the focus will be on equilibria in which this legal restriction is binding.[8] Finally, let p_t denote the time t price level.

Taking the price level sequence $\{p_t\}$ as given, young agents at each date t choose z_t and k_t ($k_t \geq 0$) to solve the problem

$$\max U(c_1) + V(c_2)$$

subject to

$$\text{(2)} \quad c_1 + k_t + z_t = y$$

(3) $c_2 = xk_t + (\frac{p_t}{p_{t+1}})\, z_t$

(4) $z_t \geq \lambda k_t$.

Noting that the focus is on equilibria where (4) is binding, this problem can be transformed as follows. Let Q_t denote total savings of a young agent at t; $Q_t = (1+\lambda)k_t$. Then young agents can be viewed as solving the problem

$$\max_{Q_t \in [0,\, y]} \quad U(y-Q_t) + V\{[\phi x + (1-\phi)\frac{p_t}{p_{t+1}}]Q_t\}$$

where $\phi = (1+\lambda)^{-1}$. This problem has associated first order condition

(5) $U'(y-Q_t) = [\phi x + (1-\phi)\frac{p_t}{p_{t+1}}]\, V'\{[\phi x + (1-\phi)\frac{p_t}{p_{t+1}}]Q_t\}$.

Defining $R_t \equiv \phi x + (1-\phi)\frac{p_t}{p_{t+1}}$, (5) implicitly defines a savings function $Q_t = f(R_t)$. From (5)

(6) $f'(R) = \dfrac{V'(c_2)\,[1 + \dfrac{c_2 V''(c_2)}{V'(c_2)}]}{-U''(c_1) - R_t^2 V''(c_2)} \geq 0,$

$\forall R$, by the assumption of equation (1). The behavior of young agents is completely summarized by the function f(R).

<u>Equilibrium</u>

An equilibrium is a pair of sequences $\{Q_t\}$ and $\{p_t\}$ satisfying equation (5), $z_t = \lambda k_t = [\lambda/(1+\lambda)]Q_t$, and the money market clearing condition

$$(6)\quad 1/p_t = z_t = [\lambda/(1+\lambda)]Q_t$$

for all $t \geq 1$. Substituting (6) into (5) yields the equilibrium law of motion for Q_t:

$$(7)\quad U'(y-Q_t) = [\phi x + (1-\phi)\frac{Q_{t+1}}{Q_t}]\, V'[\phi x Q_t + (1-\phi)Q_{t+1}];\ t \geq 1.$$

This economy has a unique steady state equilibrium value of Q_t, denoted $\hat{Q}$. This value satisfies the condition

$$(8)\quad U'(y-\hat{Q}) = (\phi x + 1-\phi)V'[(\phi x + 1-\phi)\hat{Q}].$$

The steady state equilibrium level of utility for all young agents is $U(y-\hat{Q}) + V[(\phi x + 1-\phi)\hat{Q}]$.

II. <u>Interest on Reserves</u>

In this section an equilibrium is derived when reserve holdings earn the market rate of return x. Two comments are in order before setting out the model, however. First, the only alternative asset to money here is storage. Since Friedman's proposal involves paying interest on reserves equal to short term market rates, it may seem odd to have the government pay interest on reserves equal to the return on storage. However here all assets must have the same maturity, and Friedman's proposal clearly requires that the opportunity cost of holding reserves be zero. Hence faithfulness to Friedman's proposal requires reserves to earn the real return x. Second, it is well known that if all reserve holdings earn market interest rates, there is a problem with obtaining determinate equilibria. Hence interest is paid here only on required reserve holdings. According to Sargent and Wallace (1985), "folk wisdom" then asserts the existence of a determinate equilibrium. It will be seen that this wisdom is false, despite the fact that the indeterminacy problem discussed by Sargent and Wallace (1985) is avoided here. Finally, in this section only deterministic equilibria are considered. A discussion of stationary sunspot equilibria is deferred until Section III.

Behavior of Young Agents

The economic environment is as discussed above, except that (i) all savings earn the "market" real return x (which exceeds p_t/p_{t+1} by assumption) and (ii) interest payments on reserves are financed by a lump-sum tax τ_{t+1} levied on old agents at t+1. Then young agents at t choose total savings Q_t to solve the problem

$$\max_{Q_t \in [0,\, y]} U(y-Q_t) + V(xQ_t-\tau_{t+1}).$$

This problem has the associated first order condition

$$(9) \quad U'(y-Q_t) = xV'(xQ_t-\tau_{t+1}).$$

In addition, since interest is paid only on required reserve holdings, $z_t = \lambda k_t = (\lambda/1+\lambda)Q_t = 1/p_t$.

Equilibrium

The reserve holdings of young agents at t are $(\lambda/1+\lambda)Q_t \equiv (1-\phi)Q_t$. Absent interest payments, these holdings of real balances earn the (gross) real return p_t/p_{t+1}. Hence per capita government interest obligations at t+1 are $(1-\phi)[x-(p_t/p_{t+1})]Q_t$. Government budget balance requires that

$$(10) \quad \tau_{t+1} = (1-\phi)[x - (p_t/p_{t+1})]Q_t.$$

Substitution of (10) into (9) gives the equilibrium law of motion for Q_t:

$$(11) \quad U'(y-Q_t) = xV'[\phi x Q_t + (1-\phi)Q_{t+1}]$$

where $Q_{t+1}/Q_t = p_t/p_{t+1}$ has been used in (11). Notice that there is a determinate steady state value for Q_t, denoted Q^*, with Q^* given by the condition

$$(12) \quad U'(y-Q^*) = xV'[(\phi x + 1-\phi)Q^*].$$

Moreover, from equation (11),

$$(13) \quad \left.\frac{dQ_{t+1}}{dQ_t}\right|_{Q_t = Q^*} = \frac{-U''(y-Q^*) - \phi x^2 V''[(\phi x + 1-\phi)Q^*]}{(1-\phi)xV''[(\phi x + 1-\phi)Q^*]} < 0.$$

Then the steady state equilibrium is locally stable if

$$(14)\qquad \frac{-\,U''(y-Q^*) - \phi x^2 V''[(\phi x + 1-\phi)Q^*]}{-\,(1-\phi)xV''[(\phi x + 1-\phi)Q^*]} < 1.$$

__Welfare Consequences of Interest on Reserves__

It is now possible to say something about the welfare justifications for paying interest on reserves. This is done by comparing welfare across the (unique) steady state equilibria with and without interest payments on reserves. It is useful to begin with a preliminary result. Recall that $\hat{Q}$ is the steady state equilibrium level of per capita savings __absent__ interest on reserves. Q^* is the same equilibrium level when interest is paid on reserves. Then

__Proposition 1__. If $\phi < 1$ (so there is a reserve requirement), then $Q^* > \hat{Q}$.

__Proof__. Suppose to the contrary that $\hat{Q} \geq Q^*$. Then from (8) and (12)

$$U'(y-Q^*) = xV'[(\phi x + 1-\phi)Q^*] > (\phi x + 1-\phi)V'[(\phi x + 1-\phi)\hat{Q}] = U'(y-\hat{Q}).$$

But this implies $\hat{Q} < Q^*$, a contradiction.

As a corollary, if $z^* \equiv (1-\phi)Q^*$ and $\hat{z} \equiv (1-\phi)\hat{Q}$, $z^* > \hat{z}$. The following proposition is then immediate.

__Proposition 2__. The payment of interest on reserves makes the initial old generation better off, and reduces the welfare of all other agents.

__Proof__. That the initial old are made better off follows from $z^* > \hat{z}$; i.e., the payment of interest on reserves increases savings, and hence real balances. That all young generations are made worse off is immediate. Their utility if no interest is paid on reserves is $U(y-\hat{Q}) + V[(\phi x + 1-\phi)\hat{Q}]$, while their utility when reserves earn interest is $U(y-Q^*) + V[(\phi x + 1-\phi)Q^*]$. But by definition $U(y-\hat{Q}) + V[(\phi x + 1-\phi)\hat{Q}] \geq U(y-Q) + V[(\phi x + 1-\phi)Q]$ $\forall\ Q \in [0,y]$. Moreover, the inequality is strict if $Q \neq \hat{Q}$, since V is a strictly concave function.

Thus the payment of interest on reserves has no obvious welfare justification.

III. Stationary Sunspot Equilibria

A. Interest not Paid on Reserves.

Despite the fact that there is no fundamental uncertainty here, suppose that at each date there is a random variable e_t drawn from a two element set; $e_t \in \{1,2\}$. e_t evolves according to a stationary, two-state Markov chain. Let $q(e) = \text{prob}(e_{t+1}=1:e_t=e)$; $e = 1, 2$.

The focus of discussion will be on stationary sunspot equilibria; i.e., on equilibria where current values depend only on the current state. Let p(e) denote the current price level if $e_t = e$, while z(e), k(e), and Q(e) are real balances, storage, and total savings ($Q(e) \equiv (1+\lambda)k(e)$) by young agents born in state e. All of these variables are selected after the realization of e_t. Finally, all trading is as above, so that (as is always the case in discussing stationary sunspot equilibria) state contingent claims trading is ruled out.

Behavior of Young Agents

A young agent at t, experiencing the realization $e_t = e$, chooses z(e) and $k(e) \geq 0$ to solve the problem

$$\max U[y-k(e)-z(e)] + q(e)V[xk(e) + \frac{p(e)}{p(1)} z(e)] + [1-q(e)]V[xk(e) + \frac{p(e)}{p(2)} z(e)]$$

subject to $z(e) \geq \lambda k(e)$, taking $p(e)$, $q(e)$, $p(1)$, and $p(2)$ as given, where $p(e)$ is the current price level. Again the situation of interest is that where the reserve requirement is binding, so henceforth equilibria with $z(e)=\lambda k(e)$ are considered.[9] Then, recalling that $Q(e) \equiv (1+\lambda)k(e)$, the maximization problem of young agents may be rewritten as

$$\max_{Q(e)\in[0,y]} U[y-Q(e)] + q(e)V\{[\phi x + (1-\phi) \frac{p(e)}{p(1)}]Q(e)\} +$$

$$[1-q(e)]V\{[\phi x + (1-\phi) \frac{p(e)}{p(2)}]Q(e)\},$$

where again $\phi \equiv (1+\lambda)^{-1}$. This problem has associated first order condition

$$(15) \quad U'[y-Q(e)] = q(e)[\phi x + (1-\phi) \frac{p(e)}{p(1)}]V'\{[\phi x + (1-\phi) \frac{p(e)}{p(1)}]Q(e)\}$$

$$+ [1-q(e)][\phi x + (1-\phi) \frac{p(e)}{p(2)}]V'\{[\phi x + (1-\phi) \frac{p(e)}{p(2)}]Q(e)\}; \; e = 1,2.$$

<u>Stationary Sunspot Equilibria</u>

Following Azariadis (1981), a stationary sunspot equilibrium is a set of values $Q(1)$, $Q(2)$, $p(1)$, $p(2)$, $q(1)$, and $q(2)$ satisfying (15), the money market clearing condition

(16) $1/p(e) = z(e) \equiv [\lambda/(1+\lambda)]Q(e); \; e = 1, 2$

and $0 \leq q(e) \leq 1$; $e = 1, 2$. Given the assumption of equation (1), it is straightforward to establish

<u>Proposition 3</u>. If interest is not paid on reserves, there does not exist a stationary sunspot equilibrium (s.s.e.) with $Q(1) \neq Q(2)$.

<u>Proof</u>. Suppose to the contrary that a non-trivial s.s.e. exists. Then, substituting (16) into (15), the values $Q(1)$, $Q(2)$, $q(1)$, and $q(2)$ must satisfy

$$(17) \quad U'[y-Q(1)] = q(1)[\phi x + (1-\phi)]V'[(\phi x + 1-\phi)Q(1)] + [1-q(1)][\phi x + (1-\phi)\frac{Q(2)}{Q(1)}]V'[\phi x Q(1) + (1-\phi)Q(2)]$$

and

$$(18) \quad U'[y-Q(2)] = q(2)[\phi x + (1-\phi)\frac{Q(1)}{Q(2)}]V'[\phi x Q(2) + (1-\phi)Q(1)] + [1-q(2)](\phi x + 1-\phi)V'[(\phi x + 1-\phi)Q(2)].$$

Define

$$H = \phi x + 1-\phi$$

$$H_{12} = \phi x + (1-\phi)\frac{Q(1)}{Q(2)}$$

$$H_{21} = \phi x + (1-\phi)\frac{Q(2)}{Q(1)}.$$

Then, solving (17) and (18) for q(1) and q(2) yields

$$(19)\quad q(1) = \frac{U'[y-Q(1)] - H_{21}V'[H_{21}Q(1)]}{HV'[HQ(1)] - H_{21}V'[H_{21}Q(1)]}$$

$$(20)\quad q(2) = \frac{U'[y-Q(2)] - HV'[HQ(2)]}{H_{12}V'[H_{12}Q(2)] - HV'[HQ(2)]}.$$

Now suppose that $Q(1) > Q(2)$. Then $H_{12} > H > H_{21}$. Further, since $cV''(c)/V'(c) \geq -1$, $nV'[nQ(1)]$ and $nV'[nQ(2)]$ are non-decreasing functions of n. Therefore $HV'[HQ(1)] \geq H_{21}V'[H_{21}Q(1)]$ and $H_{12}V'[H_{12}Q(2)] \geq HV'[HQ(2)]$ hold. It follows that for $0 \leq q(1) \leq 1$ to hold

$$(21)\quad HV'[HQ(1)] \geq U'[y - Q(1)] \geq H_{21}V'[H_{21}Q(1)]$$

must be satisfied. But (21) implies that $Q(1) \leq \hat{Q}$ (recall that $\hat{Q}$ is per capita savings in the steady state equilibrium). Similarly, for $0 \leq q(2) \leq 1$ to hold it is necessary that

$$(22)\quad H_{12}V'[H_{12}Q(2)] \geq U'[y - Q(2)] \geq HV'[HQ(2)].$$

But (22) implies that $Q(2) \geq \hat{Q} \geq Q(1)$, contrary to assumption. An identical contradiction arises if it is assumed that $Q(2) > Q(1)$ holds. Hence $Q(1) = Q(2)$, proving the proposition.

The assumption of equation (1) implies that all young agents have savings functions that are non-decreasing in the rate of interest. This precludes the existence of non-trivial s.s.e.'s if interest is not paid on reserves. However, the situation is substantially different if required reserve holdings bear market interest rates.

B. **Interest on Reserves**

Again, suppose that the government pays interest on reserves (required reserves only) at the market rate of interest x. A young agent born in state e acquires total savings of Q(e); e = 1, 2, against which the agent holds required reserves of $[\lambda/(1+\lambda)]Q(e) = (1-\phi)Q(e) = z(e)$. Since the young period state is e, the young period price level is p(e); if the old period state is e', the ex post return on reserves (before receiving interest from the government) is p(e)/p(e'). Hence government interest payments to old agents in state e', if the young period state was e, are

$$(1-\phi)[x - \frac{p(e)}{p(e')}]Q(e).$$

As above, these interest payments are financed by a lump-sum tax on old agents of $\tau(e,e')$ if the young period state was e, and the old period state is e'. Then government budget balance requires that

$$\tau(e,e') = (1-\phi)[x - \frac{p(e)}{p(e')}]Q(e); \quad e,e' = 1,2. \tag{23}$$

The focus of discussion will be on sunspot equilibria "near" the steady state equilibrium, so attention will be restricted to the case in which $x > p(e)/p(e') \; \forall \; e, e'$.

<u>**Behavior of Young Agents**</u>

A young agent born in state e faces lump-sum taxes $\tau(e,e')$ if next period's state is e'. Taking these values as given, this agent chooses total savings Q(e) to solve the problem

$$\max_{Q(e)\in[0,y]} U[y-Q(e)] + q(e)V[xQ(e) - \tau(e,1)] + [1-q(e)]V[xQ(e)-\tau(e,2)].$$

Since only required reserve holdings earn interest, $z(e) = \lambda k(e) = (1-\phi)Q(e)$.

The first order condition associated with this problem is

$$U'[y-Q(e)] = q(e)xV'[xQ(e) - \tau(e,1)] + [1-q(e)]xV'[xQ(e)-\tau(e,2)]; \quad e=1,2. \tag{24}$$

<u>Equilibrium</u>

Again the focus is on stationary sunspot equilibria evolving according to a two-state Markov chain. Then a stationary sunspot equilibrium with interest on reserves is a set of values p(1), p(2), Q(1), Q(2), q(1), q(2), and $\tau(e,e')$ satisfying (23), (24), the money market clearing condition

$$1/p(e) = z(e) = (1-\phi)Q(e);\ e = 1,2, \tag{25}$$

and $0 \leq q(e) \leq 1$; e = 1, 2. A sufficient condition for non-trivial s.s.e.'s to exist is as follows.

<u>Proposition 4</u>. Suppose that the (unique) steady state equilibrium when interest is paid on reserves is locally stable; i.e., suppose that

$$(26)\qquad \frac{-(1-\phi)xV''[(\phi x+1-\phi)Q^*]}{-U''(y-Q^*)-\phi x^2 V''[(\phi x+1-\phi)Q^*]} > 1.$$

Suppose further that $\phi = (1+\lambda)^{-1} < (1+x)^{-1}$. Then there exist stationary sunspot equilibria with $Q(1) \neq Q(2)$ and $x > p(e)/p(e')\ \forall\ e, e'$.

Remark. It is straightforward to show that there are no stationary sunspot equilibria (except the trivial steady state equilibrium) if $\phi \geq (1+x)^{-1}$. Thus the existence of s.s.e.'s requires that the reserve requirement $1-\phi$ be sufficiently large. Of course Friedman's proposal is for 100% reserve requirements ($\phi = 0$), so this requirement should not be troublesome.

Proof. The proof is by construction. To begin, substitute (25) into (23), and then substitute the result into (24) to obtain the conditions

$$(27)\quad U'[y-Q(1)] = q(1)xV'[(\phi x+1-\phi)Q(1)] + [1-q(1)]xV'[\phi xQ(1) + (1-\phi)Q(2)]$$

and

$$(28)\quad U'[y - Q(2)] = q(2)xV'[\phi xQ(2) + (1-\phi)Q(1)] + [1-q(2)]xV'[(\phi x+1-\phi)Q(2)]$$

Solving (27) and (28) for q(1) and q(2) yields

$$(27')\quad q(1) = \frac{xV'[\phi xQ(1)+(1-\phi)Q(2)]-U'[y-Q(1)]}{xV'[\phi xQ(1)+(1-\phi)Q(2)]-xV'[(\phi x+1-\phi)Q(1)]}$$

$$(28') \quad q(2) = \frac{xV'[\phi xQ(2)+(1-\phi)Q(2)]-U'[y-Q(2)]}{xV'[\phi xQ(2)+(1-\phi)Q(2)]-xV'[\phi xQ(2)+(1-\phi)Q(1)]}$$

Now choose Q(1) and Q(2) such that $y > Q(1) > Q^* > Q(2) > 0$. Then

$$V'[\phi xQ(1)+(1-\phi)Q(2)] > V'[(\phi x+1-\phi)Q(1)]$$

and

$$V'[(\phi x+1-\phi)Q(2)] > V'[\phi xQ(2)+(1-\phi)Q(1)]$$

hold, since $V''<0$. Therefore $0 \leq q(1) \leq 1$ if

$$(29) \quad xV'[\phi xQ(1)+(1-\phi)Q(2)] \geq U'[y-Q(1)] \geq xV'[(\phi x+1-\phi)Q(1)]$$

holds, while $0 \leq q(2) \leq 1$ holds if

$$(30) \quad xV'[(\phi x+1-\phi)Q(2)] \geq U'[y-Q(2)] \geq xV'[\phi xQ(2) + (1-\phi)Q(1)].$$

However, $U'[y-Q(1)] > xV'[(\phi x+1-\phi)Q(1)]$ and $xV'[(\phi x+1-\phi)Q(2)] > U'[y-Q(2)]$ hold, as $Q(1) > Q^* > Q(2)$, and Q^* is defined by the condition $U'(y-Q^*) = xV'[(\phi x+1-\phi)Q^*]$. Then (29) and (30) are satisfied if

$$(31) \quad xV'[\phi xQ(1)+(1-\phi)Q(2)] \geq U'[y-Q(1)]$$

$$(32) \quad U'[y-Q(2)] \geq xV'[\phi xQ(2)+(1-\phi)Q(1)]$$

are satisfied.

It will now be convenient to depict conditions (31) and (32) diagrammatically. Consider the loci defined by equations (31) and (32) holding with equality. These are depicted in Figure 1. Both loci intersect the 45° line at $Q(1)=Q(2)=Q^*$, since $U'(y-Q^*) = xV'[(\phi x+1-\phi)Q^*]$. Points on or below the locus labeled (31) satisfy equation (31), and points on or above the locus labeled (32) satisfy equation (32). Then, if the locus (31) is less steeply sloped than the locus (32) at the point $(Q(1),Q(2)) = (Q^*,Q^*)$, all points in the shaded region satisfy (31), (32), and $Q(1) > Q^* > Q(2)$. Hence there are non-trivial s.s.e.'s if the locus (32) is more steeply sloped than the locus (31) at (Q^*,Q^*). Moreover, points in the shaded region of Figure 1 satisfy $x > p(e)/p(e') \; \forall \; e, e'$ if $Q(1)$ and $Q(2)$ are chosen sufficiently close to Q^* (so that $x > Q(1)/Q(2)$).

A sufficient condition for the existence of s.s.e.'s, then, is that (32) be more steeply sloped at (Q^*,Q^*) than (31). From equations (31) and (32) these slopes are

$$(33) \quad \left.\frac{dQ(2)}{dQ(1)}\right|_{(31)} = \frac{-U''(y-Q^*)-\phi x^2 V''[(\phi x+1-\phi)Q^*]}{x(1-\phi)V''[(\phi x+1-\phi)Q^*]}$$

$$(34) \quad \left.\frac{dQ(2)}{dQ(1)}\right|_{(32)} = \frac{x(1-\phi)V''[(\phi x+1-\phi)Q^*]}{-U''(y-Q^*)-\phi x^2 V''[(\phi x+1-\phi)Q^*]}$$

Then (32) is more steeply sloped than (31) at (Q^*,Q^*) if

$$\frac{x(1-\phi)V''}{-U''-\phi x^2 V''} < \frac{-U''-\phi x^2 V''}{x(1-\phi)V''} \quad (< 0).$$

This condition is equivalent to

$$(35)\quad \frac{-x(1-\phi)V''}{-U''-\phi x^2 V''} > 1,$$

establishing the result.

The set of economies for which (35) holds is non-trivial. For instance, if $U''=0$, (35) reduces to $(1+x)^{-1} > \phi$. Then non-trivial stationary sunspot equilibria will exist whenever the reserve requirement is sufficiently high. It is the case, then, that for many economies that do not have s.s.e.'s when no interest is paid on reserves, s.s.e.'s will exist when reserves earn market interest rates. Thus Friedman's proposal that interest be paid on reserves can open economies to price level indeterminacy, and to the possibility of "excessive" price level fluctuations driven by expectations.

IV. Conclusions

Friedman's (1960) proposal to pay interest on reserves has generally been well received by economists. It has only recently been criticized by Sargent and Wallace (1985) for resulting in an indeterminate steady state equilibrium. However, the Sargent-Wallace indeterminacy disappears when real interest rates are technologically determined, as they are in the model above. Such a model seems more faithful to the monetarist view of real interest rates that are uninfluenced by monetary factors (at least in the "long-run") than does the Sargent-Wallace model.

In the analysis above, the payment of interest on reserves (at market rates) is consistent with the existence of a unique steady state equilibrium. Nevertheless, the analysis does not suggest that Friedman's proposal has any desirable properties. In a steady state equilibrium, the payment of interest on reserves reduces the welfare of all agents, except the initial old. Moreover, the model is structured in such a way that, when reserves do not earn interest, there is a unique stationary equilibrium.

When reserves do earn interest, on the other hand, indeterminacies are a real possibility, in the form of stationary sunspot equilibria. Thus the Friedman proposal may open an economy to the possibility of "excessive fluctuations" in the price level.

In closing, two potential objections to the analysis might be anticipated. First, one might question whether the type of model employed here captures interesting aspects of real world monetary policy issues. A partial answer is given by Azariadis and Farmer (1987), who use a reserve requirement model similar to the one employed above (and actually more similar to the model of Sargent and Wallace (1985)) to capture many features of U.S. monetary policy since the creation of the Federal Reserve System. Their analysis suggests that models of the type used here do shed light on practical issues of monetary policy formulation.

Second, one might question whether the results would be different if interest were paid on reserves at below market rates, and if reserve requirements were employed by the government to enhance seigniorage revenue. Without addressing this issue directly, it might be noted that Freeman (1987) has shown (in a model exactly like the one above, but with a government faced with the problem of financing a deficit) that the optimal set of government actions under such circumstances includes driving the real return on reserves to zero. Hence such considerations seem unlikely to make Friedman's proposal more attractive.

Notes

1. A weaker version of this question is, should agents operating in certain financial markets have their ability to create close money substitutes limited by the imposition of reserve requirements, for instance?

2. See Sargent and Wallace (1982) for a summary of this viewpoint, which they identify with the "quantity theory of money."

3. See Shell (1977), Azariadis (1981), and Cass and Shell (1983) for the original developments in this literature.

4. See Smith (1988) for an argument to this effect. Smith provides a summary of a more detailed argument on the same point by White (1984).

5. Having real interest rates be technologically determined is attractive for this reason, and also because it is probably more faithful to the monetarist view of long-run real interest rates that are not affected by monetary factors than is the Sargent-Wallace formulation.

6. The economy of this section is essentially the same as that in section VI of Wallace (1981).

7. Notice that the environment is specified in a way that precludes borrowing and lending. This is for simplicity only; it should be apparent that none of the results depend upon homogeneous young agents.

8. If money is not dominated in rate of return, then the proposal to pay interest on reserves is, of course, not very interesting.

9. It is straightforward to show that this economy has no (non-trivial) stationary sunspot equilibrium if the reserve requirement does not bind in either or both states.

References

Azariadis, Costas, "Self-Fulfilling Prophecies," Journal of Economic Theory, 25, Dec. 1981, p. 380-96.

Azariadis, Costas, and Roger E.A. Farmer, "Fractional Reserve Banking," manuscript, 1987.

Cass, David, and Karl Shell, "Do Sunspots Matter?" Journal of Political Economy, 91, April 1983, p. 193-227.

Freeman, Scott, "Reserve Requirements and Optimal Seigniorage," Journal of Monetary Economics, 19, 1987, p. 307-14.

Friedman, Milton, A Program for Monetary Stability, Fordham University Press, New York, 1960.

Sargent, Thomas J., and Neil Wallace, "The Real Bills Doctrine versus the Quantity Theory: a Reconsideration," Journal of Political Economy, 90, Dec. 1982, p. 1212-36.

Sargent, Thomas J., and Neil Wallace, "Interest on Reserves," Journal of Monetary Economics, 15, May 1985, p. 279-90.

Shell, Karl, "Monnaie et Allocation Intertemporelle," manuscript, 1977.

Smith, Bruce D., "Legal Restrictions, 'Sunspots', and Peel's Bank Act: the Real Bills Doctrine versus the Quantity Theory Reconsidered," Journal of Political Economy, 96, Feb. 1988, p. 3-19.

Wallace, Neil, "A Modigliani-Miller Theorem for Open Market Operations," American Economic Review, 71, June 1981, p. 267-74.

White, Lawrence H., Free Banking in Britain: Theory, Experience, and Debate, 1800-1845, Cambridge University Press, Cambridge, 1984.

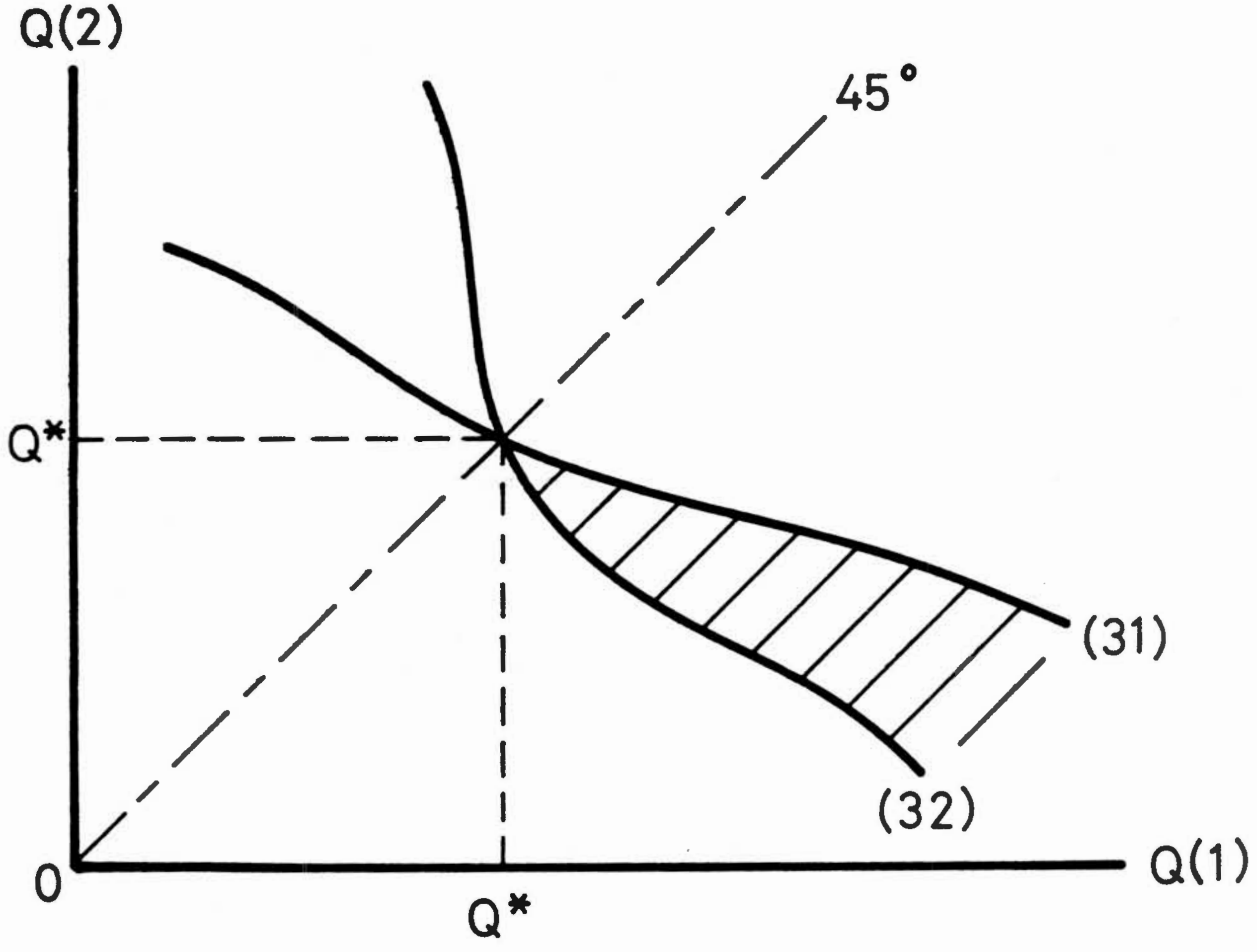

Figure 1

Rochester Center for Economic Research
University of Rochester
Department of Economics
Rochester, NY 14627

1987-88 DISCUSSION PAPERS

WP#68 RECURSIVE UTILITY AND OPTIMAL CAPITAL ACCUMULATION, I: EXISTENCE, by Robert A. Becker, John H. Boyd III, and Bom Yong Sung, January 1987

WP#69 MONEY AND MARKET INCOMPLETENESS IN OVERLAPPING-GENERATIONS MODELS, by Marianne Baxter, January 1987

WP#70 GROWTH BASED ON INCREASING RETURNS DUE TO SPECIALIZATION by Paul M. Romer, January 1987

WP#71 WHY A STUBBORN CONSERVATIVE WOULD RUN A DEFICIT: POLICY WITH TIME-INCONSISTENT PREFERENCES by Torsten Persson and Lars E.O. Svensson, January 1987

WP#72 ON THE CONTINUUM APPROACH OF SPATIAL AND SOME LOCAL PUBLIC GOODS OR PRODUCT DIFFERENTIATION MODELS by Marcus Berliant and Thijs ten Raa, January 1987

WP#73 THE QUIT-LAYOFF DISTINCTION: GROWTH EFFECTS by Kenneth J. McLaughlin, February 1987

WP#74 SOCIAL SECURITY, LIQUIDITY, AND EARLY RETIREMENT by James A. Kahn, March 1987

WP#75 THE PRODUCT CYCLE HYPOTHESIS AND THE HECKSCHER-OHLIN-SAMUELSON THEORY OF INTERNATIONAL TRADE by Sugata Marjit, April 1987

WP#76 NOTIONS OF EQUAL OPPORTUNITIES by William Thomson, April 1987

WP#77 BARGAINING PROBLEMS WITH UNCERTAIN DISAGREEMENT POINTS by Youngsub Chun and William Thomson, April 1987

WP#78 THE ECONOMICS OF RISING STARS by Glenn M. MacDonald, April 1987

WP#79 STOCHASTIC TRENDS AND ECONOMIC FLUCTUATIONS by Robert King, Charles Plosser, James Stock, and Mark Watson, April 1987

WP#80 INTEREST RATE SMOOTHING AND PRICE LEVEL TREND-STATIONARITY by Marvin Goodfriend, April 1987

WP#81 THE EQUILIBRIUM APPROACH TO EXCHANGE RATES
by Alan C. Stockman, revised, April 1987

WP#82 INTEREST-RATE SMOOTHING
by Robert J. Barro, May 1987

WP#83 CYCLICAL PRICING OF DURABLE LUXURIES
by Mark Bils, May 1987

WP#84 EQUILIBRIUM IN COOPERATIVE GAMES OF POLICY FORMULATION
by Thomas F. Cooley and Bruce D. Smith, May 1987

WP#85 RENT SHARING AND TURNOVER IN A MODEL WITH EFFICIENCY UNITS OF HUMAN CAPITAL
by Kenneth J. McLaughlin, revised, May 1987

WP#86 THE CYCLICALITY OF LABOR TURNOVER: A JOINT WEALTH MAXIMIZING HYPOTHESIS
by Kenneth J. McLaughlin, revised, May 1987

WP#87 CAN EVERYONE BENEFIT FROM GROWTH? THREE DIFFICULTIES
by Herve' Moulin and William Thomson, May 1987

WP#88 TRADE IN RISKY ASSETS
by Lars E.O. Svensson, May 1987

WP#89 RATIONAL EXPECTATIONS MODELS WITH CENSORED VARIABLES
by Marianne Baxter, June 1987

WP#90 EMPIRICAL EXAMINATIONS OF THE INFORMATION SETS OF ECONOMIC AGENTS
by Nils Gottfries and Torsten Persson, June 1987

WP#91 DO WAGES VARY IN CITIES? AN EMPIRICAL STUDY OF URBAN LABOR MARKETS
by Eric A. Hanushek, June 1987

WP#92 ASPECTS OF TOURNAMENT MODELS: A SURVEY
by Kenneth J. McLaughlin, July 1987

WP#93 ON MODELLING THE NATURAL RATE OF UNEMPLOYMENT WITH INDIVISIBLE LABOR
by Jeremy Greenwood and Gregory W. Huffman

WP#94 TWENTY YEARS AFTER: ECONOMETRICS, 1966-1986
by Adrian Pagan, August 1987

WP#95 ON WELFARE THEORY AND URBAN ECONOMICS
by Marcus Berliant, Yorgos Y. Papageorgiou and Ping Wang, August 1987

WP#96 ENDOGENOUS FINANCIAL STRUCTURE IN AN ECONOMY WITH PRIVATE INFORMATION
by James Kahn, August 1987

WP#97 THE TRADE-OFF BETWEEN CHILD QUANTITY AND QUALITY: SOME EMPIRICAL EVIDENCE
by Eric Hanushek, September 1987

WP#98 SUPPLY AND EQUILIBRIUM IN AN ECONOMY WITH LAND AND PRODUCTION
by Marcus Berliant and Hou-Wen Jeng, September 1987

WP#99 AXIOMS CONCERNING UNCERTAIN DISAGREEMENT POINTS FOR 2-PERSON BARGAINING PROBLEMS
by Youngsub Chun, September 1987

WP#100 MONEY AND INFLATION IN THE AMERICAN COLONIES: FURTHER EVIDENCE ON THE FAILURE OF THE QUANTITY THEORY
by Bruce Smith, October 1987

WP#101 BANK PANICS, SUSPENSIONS, AND GEOGRAPHY: SOME NOTES ON THE "CONTAGION OF FEAR" IN BANKING
by Bruce Smith, October 1987

WP#102 LEGAL RESTRICTIONS, "SUNSPOTS", AND CYCLES
by Bruce Smith, October 1987

WP#103 THE QUIT-LAYOFF DISTINCTION IN A JOINT WEALTH MAXIMIZING APPROACH TO LABOR TURNOVER
by Kenneth McLaughlin, October 1987

WP#104 ON THE INCONSISTENCY OF THE MLE IN CERTAIN HETEROSKEDASTIC REGRESSION MODELS
by Adrian Pagan and H. Sabau, October 1987

WP#105 RECURRENT ADVERTISING
by Ignatius J. Horstmann and Glenn M. MacDonald, October 1987

WP#106 PREDICTIVE EFFICIENCY FOR SIMPLE NONLINEAR MODELS
by Thomas F. Cooley, William R. Parke and Siddhartha Chib, October 1987

WP#107 CREDIBILITY OF MACROECONOMIC POLICY: AN INTRODUCTION AND A BROAD SURVEY
by Torsten Persson, November 1987

WP#108 SOCIAL CONTRACTS AS ASSETS: A POSSIBLE SOLUTION TO THE TIME-CONSISTENCY PROBLEM
by Laurence Kotlikoff, Torsten Persson and Lars E. O. Svensson, November 1987

WP#109 EXCHANGE RATE VARIABILITY AND ASSET TRADE
by Torsten Persson and Lars E. O. Svensson, Novmeber 1987

WP#110 MICROFOUNDATIONS OF INDIVISIBLE LABOR
by Vittorio Grilli and Richard Rogerson, November 1987

WP#111 FISCAL POLICIES AND THE DOLLAR/POUND EXCHANGE RATE: 1870-1984
by Vittorio Grilli, November 1987

WP#112 INFLATION AND STOCK RETURNS WITH COMPLETE MARKETS
by Thomas Cooley and Jon Sonstelie, November 1987

WP#113 THE ECONOMETRIC ANALYSIS OF MODELS WITH RISK TERMS
by Adrian Pagan and Aman Ullah, December 1987

WP#114 PROGRAM TARGETING OPTIONS AND THE ELDERLY
by Eric Hanushek and Roberton Williams, December 1987

WP#115 BARGAINING SOLUTIONS AND STABILITY OF GROUPS
by Youngsub Chun and William Thomson, December 1987

WP#116 MONOTONIC ALLOCATION MECHANISMS
by William Thomson, December 1987

WP#117 MONOTONIC ALLOCATION MECHANISMS IN ECONOMIES WITH PUBLIC GOODS
by William Thomson, December 1987

WP#118 ADVERSE SELECTION, AGGREGATE UNCERTAINTY, AND THE ROLE FOR MUTUAL INSURANCE COMPANIES
by Bruce D. Smith and Michael J. Stutzer, February 1988

WP#119 INTEREST ON RESERVES AND SUNSPOT EQUILIBRIA: FRIEDMAN'S PROPOSAL RECONSIDERED
by Bruce D. Smith, February 1988

WP#120 INTERNATIONAL FINANCIAL INTERMEDIATION AND AGGREGATE FLUCTUATIONS UNDER ALTERNATIVE EXCHANGE RATE REGIMES
by Jeremy Greenwood and Stephen D. Williamson, February 1988

WP#121 FINANCIAL DEREGULATION, MONETARY POLICY, AND CENTRAL BANKING
by Marvin Goodfriend and Robert G. King, February 1988

To order copies of the above papers complete the attached invoice and return to Christine Massaro, W. Allen Wallis Institute of Political Economy, RCER, 109B Harkness Hall, University of Rochester, Rochester, NY 14627. Three (3) papers per year will be provided free of charge as requested below. Each additional paper will require a $5.00 service fee which must be enclosed with your order. For your convenience an invoice is provided below in order that you may request payment from your institution as necessary. Please make your check payable to the **Rochester Center for Economic Research**. **Checks must be drawn from a U.S. bank and in U.S. dollars**.

W. Allen Wallis Institute for Political Economy

Rochester Center for Economic Research, Working Paper Series

OFFICIAL INVOICE

Requestor's Name ______________________________

Requestor's Address ______________________________

____

____

____

Please send me the following papers free of charge (**Limit: 3 free per year**).

WP# ______ WP# ______ WP# ______

I understand there is a $5.00 fee for each additional paper. Enclosed is my check or money order in the amount of $_________. Please send me the following papers.

WP# ______ WP# ______ WP# ______

WP# ______ WP# ______ WP# ______

WP# ______ WP# ______ WP# ______

WP# ______ WP# ______ WP# ______