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A Private Information Approach

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AND THE PERSISTENCE OF "DISTURBANCES":
A PRIVATE INFORMATION APPROACH

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Much recent literature has been concerned with generating models that are consistent with the existence of unemployed labor, and with the observed cyclical behavior of other aggregate quantities. Following the strategy pioneered by Kydland and Prescott (1982) and Long and Plosser (1983), many of these models explain cyclical variation as the result of shocks to technology. Unemployment of labor is generated via a non-convexity (usually indivisibility of labor), and an objective of the research strategy is to explain the observed behavior of quantities like aggregate output and hours over the cycle. This requires that the models be consistent with the observed high variability of hours relative to either measured productivity or real wages, and with the high positive serial correlation of deviations from trend in either output or hours.¹

While less concerned with explaining cyclical fluctuations,² there is of course a large literature that attempts to explain the unemployment of labor as resulting from the presence of private information in labor markets.³ This paper represents something of a synthesis of the two approaches. In particular, it explains unemployment as being the result of private information in labor markets, and does so in a way that is consistent with the high observed variance of hours relative to real wages (or productivity) and with high observed persistence. In doing so it makes two additional points: (i) the presence of private information creates channels for magnifying the effects on hours of shocks to productivity, and (ii) it also creates a channel for generating serially correlated deviations of hours and output from trend. Thus the presence of private information can not only help to explain why labor is unemployed, but can also aid in understanding the observed magnitudes of co-movements in various time series.

The latter point is important, since the existing real business cycle literature has relied heavily on intertemporal substitution to explain co-movements in hours and productivity. In particular, as has been widely remarked, existing micro evidence on the behavior of real wages and labor supply is largely inconsistent with the kinds of co-movements required in hours and real wages for real business cycle models to be consistent with aggregate data. As stated by Ashenfelter (1984, p. 150)

The average labor supply elasticity must apparently be quite large to square up these hours and wage rate movements, while the available estimates of its slope that I have surveyed are, in fact, very small. The basic empirical problem seems to be that within the life-cycle, the person-specific correlation between hours and wages is simply too small to explain the time series movements in average hours relative to the time-series movements in average wage rates. The intertemporal substitution hypothesis originally advanced by Lucas and Rapping was, of course, precisely the suspicion that this was not the case.

In addition, however, there is a good deal of evidence against the importance of intertemporal substitution mechanisms.⁴ Thus it is desirable to have other devices for generating co-movements between hours and real wages (or productivity).

The analysis here produces a model that displays unemployment of labor, a high variability of hours relative to real wages or productivity, and significant persistence of disturbances in hours and output. Moreover, it does so in the context of a model that displays none of these features under full information. Specifically, in the absence of any informational asymmetry, the model will be specified in such a way that hours are constant. This will serve to emphasize that all of the interesting cyclical behavior of hours is due to the presence of private information. Moreover, the model is consistent with micro evidence on observed individual correlations between hours and real wages, and with a number of other empirical regularities. To

emphasize consistency with the micro evidence, the model will be specified such that, for individuals, hours and real wages are uncorrelated.

Nevertheless, the model is consistent with the large observed aggregate variability of hours, with observed persistence, and with the following cyclical regularities.

- (i) There is a strong positive correlation between aggregate hours movements and real wages.
- (ii) Average productivity is procyclical.
- (iii) Wage dispersions (defined in almost any manner) decline at cyclical peaks.
- (iv) Changes in relative employment across "sectors" are an observed cyclical phenomenon.⁵

Also, the model is consistent with the secular observation that trends in real wages have not been associated with trends in hours per capita. It also suggests that relative wages across occupations are an important "determinant" of labor market behavior. Some have suggested [e.g., Dunlop (1950), Keynes (1936), and Solow (1980)] that this is a desirable feature of a macro model.

The model developed is as follows. In order to generate time series behavior a dynamic model is obviously required. However, tractability requires that models with multi-period incentive problems be avoided. This problem is resolved by the use of a two-period lived overlapping generations model.⁶ Within each generation there is a heterogeneous workforce and a set of firms with access to a technology for converting labor and capital into a single consumption good. Workers differ in terms of their ability to convert labor and capital into this good. Each worker is privately informed

about his own productive abilities. This, then, is a fairly standard adverse selection environment, except that firms must make decisions about how to allocate capital among workers who differ in ability.

As in Rothschild and Stiglitz (1976), firms are viewed as imperfect Nash competitors (here in labor markets). As is well known, in equilibrium firms induce workers of different abilities to self-select by offering a range of contracts for workers to choose among. In the sequel contracts specify a wage rate and a level of employment. If the model is structured appropriately, some workers will be unemployed (underemployed). In order to generate cyclical behavior, the technology of the economy is subject to some random (aggregate) disturbances. It is then shown that models in the class at hand can display all of the behavior discussed above.

The paper proceeds as follows. Section I describes the environment, and the behavior of individuals. Section II describes the equilibrium of the model under full information. To emphasize that private information is sufficiently powerful to generate all interesting cyclical behavior of hours here, the model is parameterized in such a way that under full information hours do not vary. Section III develops an equilibrium under private information. Section IV examines a numerical example to demonstrate that fairly rich dynamics and cyclical behavior can emerge from the model. Since the model is elaborately parameterized, Section V discusses model specification.

I. The Model

A. The Environment

The model consists of a sequence of two period lived, overlapping generations. Let time be indexed by $t=0,1,\dots$. At each date t there is a

young generation that sells labor, a retired old generation, and a set of firms that have access to technologies for converting labor and capital into a single consumption good. Agents (workers) accumulate capital when young, which is then rented to firms in old age. To emphasize, only young agents supply labor, so that each individual is in the workforce only once. Hence any considerations related to multi-period incentive problems are avoided.

Each generation is identical in size and composition. Therefore attention will be restricted to stationary equilibria in the sequel. In addition, while it is not necessary to be specific on this point, there is a fixed and countable set of firms in each generation, and a continuum of workers.⁷ Workers are divided into two types (described below), with type indexed by $i \in \{1,2\}$. All workers of type i are identical, and μ is the proportion of the workforce that is of type 1.

Prior to discussing technology and agent preferences, it will be useful to provide an overview of the modelling strategy. A highly parameterized economy is presented with two features. (i) Under full information the economy is incapable of generating non-trivial cyclical or dynamic behavior in labor markets. (ii) Under private information the model allows for fairly rich cyclical and dynamic behavior that is consistent with a variety of features of postwar U.S. business cycles. This serves to emphasize that the presence of private information alone is sufficient to generate significant cyclical behavior in labor markets.

Under more general specifications of preferences and technologies the model developed below would display non-trivial cyclical behavior even under full information. Thus it is useful to begin by presenting special versions of these economies that underscore the ability of informational frictions alone to generate high hours variability, and channels of persistence. A

number of the special features of the model below are meant to have this effect. Other features of preferences and technology are assumed for reasons of tractability. After the model is presented and its equilibrium analyzed, a number of the special assumptions employed in the analysis will be discussed in detail.

Technology

A type 1 worker produces $\pi_1(s)$ units of output per unit time, where s is a current period productivity shock, and $\pi_1(s)$ is a scalar. Thus capital does not augment the output of type 1 workers. Therefore, it will be convenient to measure the capital stock in terms of capital per type 2 worker. Let k be this quantity. Then a type 2 worker providing L units of labor, combined with k units of capital, produces output according to the production function $\pi_2(s)k^\theta L^{1-\theta}$; $\theta \in (0,1)$. $\pi_2(s)$ is a scalar productivity parameter that varies with the current period productivity shock s . The current state of the economy, then, is completely described by the vector (s,k) . Output is expressed in per worker terms, and is additively separable across workers.

It remains to describe the process generating s . The simplest possible specification is adopted here: s evolves according to a two state Markov chain.⁸ Hence $s \in \{1,2\}$. Letting s' denote "next period's state," $p(s)$ is the probability that $s'=1$ conditional on the current period state s . It will often be convenient to have notation for "last period's state" as well, which is denoted \tilde{s} .

Preferences and Endowments

Each worker is endowed with one unit of time when young to be allocated between labor and leisure. Workers have no endowment of the good at any date,

and no endowment of labor when old. Let L denote the fraction of time allocated to work, $L \in [0,1]$, and let c_j denote age j consumption; $j=1,2$. When it is necessary to distinguish between agents of different types the notation c_{ji} and L_i will be employed.

The preferences of type 2 workers are given by the utility function $U_2(c_1, c_2, L) = \ln c_1 + \ln(1-L)$. Hence type 2 workers care only about young period consumption. Type 1 agents have preferences described by $U_1(c_1, c_2, L) = c_1 + \beta c_2 + \phi \ln(1-L)$. ϕ satisfies $\phi \geq (\frac{1}{2})\max[\pi_1(1), \pi_1(2)]$. The content of this assumption is as follows. The marginal rate of substitution between young period consumption and leisure for type 1 agents is given by $\phi(1-L)^{-1}$. The marginal rate of substitution between young period consumption and leisure for type 2 agents is $c_1(1-L)^{-1}$, and young period consumption for any agent will be bounded above by $(\frac{1}{2})\max[\pi_1(1), \pi_1(2)]$ under assumptions to be made below. Thus $\phi \geq (\frac{1}{2})\max[\pi_1(1), \pi_1(2)]$ guarantees that for all relevant values of c_1 and L , type 1 agents have higher marginal rates of substitution than do type 2 agents at any (c_1, L) pair. This assumption is essentially the "single crossing property" of Cooper (1984), and guarantees that if any agents are "off of" their labor supply curves in equilibrium the equilibrium will be associated with unemployment (rather than overemployment) of labor.

Since the assumption that $\phi \geq (\frac{1}{2})\max[\pi_1(1), \pi_1(2)]$ plays an important role in the analysis, its plausibility should be defended. This assumption guarantees that at any relevant (c_1, L) pair, type 1 agents would value an incremental unit of leisure relatively more than would type 2 agents. Under assumptions made below, type 1 workers have higher productivity in the marketplace than do type 2 workers. It is then natural that type 1 agents value incremental leisure more highly than do type 2 agents if leisure is actually time employed in home production. Or, in other words, such an

assumption is plausible if workers who are more productive in labor markets are also more productive at home.

Trading

At each date there is a labor market open in which workers can sell labor to firms. There is also a rental market in capital in which firms can obtain capital from retired workers. Capital depreciates entirely in one period, which is not unreasonable in an overlapping generations context. Hence the return to holding capital is its rental rate, which is denoted by $r(s,k)$.

Young workers can accumulate capital. In doing so they take the function $r(s,k)$ as given. There is no storage of goods possible except via capital accumulation, however. Borrowing and lending is permitted, but will not occur in equilibrium. This is the case since type 2 agents do not care about old age consumption. As agent types are revealed in equilibrium, they cannot borrow, and hence there will be no agents for type 1 workers to borrow from or lend to.

It remains to discuss state contingent claims trading. It is assumed that all young agents are born after the realization of the current period state, so claims are not traded contingent on the current period state. In addition, type 2 workers care only about young period allocations, while type 1 workers are risk neutral with respect to old age consumption. All firms are risk neutral. Hence there need be no trading of claims to consumption contingent on s' . Thus, in equilibrium, state contingent claims trades are zero. This fact, along with the absence of borrowing and lending in equilibrium, is exploited by eliminating these possibilities in the notation that follows.

Information

Each worker knows his own type, but this is private information ex ante. Savings behavior is also assumed to be unobservable.⁹ All trades in labor markets are observed by all, as is the current period state (s,k) , and all current period prices are known at each date.

B. Behavior of Agents

Workers

There are two aspects to workers' behavior; an employment decision and a savings decision. Employment decisions are best described after a discussion of firm behavior. However, savings decisions are easily described. As discussed above, type 2 workers do not save (or borrow). Thus all capital accumulation is carried out by type 1 workers. Let $\phi(s,k)$ denote capital accumulation by a representative type 1 worker in state (s,k) . Then for any given level of current period income $y_1(s,k)$, $\phi(s,k)$ is chosen to solve the problem

$$\max c_1 + \beta E_s c_2(s',k')$$

subject to

$$c_1 + \phi(s,k) \leq y_1(s,k)$$

$$c_2(s',k') \leq \phi(s,k)r(s',k'),$$

where E_s denotes the conditional expectation taken with respect to s' .

Writing the maximand as a function of $y_1(s,k)$, $\phi(s,k)$, and $r(s',k')$, we see that $\phi(s,k)$ is chosen to maximize the expression

$$y_1(s,k) + \beta\{p(s)r[1,k(s)]+[1-p(s)]r[2,k(s)]\}\phi(s,k)-\phi(s,k).$$

Hence in equilibrium, we must have

$$(1) \quad p(s)r[1,k(s)]+[1-p(s)]r[2,k(s)] = \beta^{-1}$$

where $k(s) \equiv \phi(s,k)\left(\frac{1-\mu}{\mu}\right)$, i.e., $k(s)$ is the future capital stock (per type 2 worker) implied by $\phi(s,k)$.

Firm Behavior

Recall that firms have access to a technology for converting labor and capital into the consumption good. Moreover, there are constant returns to scale in a dual sense: the production function for each individual worker displays constant returns to scale, and output is additively separable across workers. This will make an equilibrium easy to characterize.

It is assumed that firms acquire capital in a competitive rental market, i.e., firms make a capital rental decision taking $r(s,k)$ as parametric. In addition, firms are assumed to be imperfect competitors in labor markets. Hence the analysis follows Hart (1982) and much subsequent literature in using a model of an imperfectly competitive labor market to examine macroeconomic issues. In the model here, however, the Nash equilibrium examined coincides with a competitive equilibrium in the absence of private information.

Firms, then, are viewed as operating in the following manner in labor markets. Each firm, taking the actions of other firms as given, announces a set of contracts with each contract consisting of a wage-hours pair. In particular, if a firm's contract offer is accepted by any workers one of the

following will occur: only type 1 workers accept the contract offered, only type 2 workers accept the contract, or workers of both types accept the contract. Hence each firm offers (at most) two contracts denoted $[w_i(s,k), L_i(s,k)]$; $i = 1, 2$. A contract specifies a wage rate to be paid any worker accepting it, and a number of hours the worker will be employed. Without loss of generality the contract $[w_i(s,k), L_i(s,k)]$ will be accepted (if at all) by type i workers. Hence if a firm announces a contract pair such that $(w_1, L_1) \neq (w_2, L_2)$, it hopes to induce self-selection of workers by contract accepted. If a contract announcement is not meant to induce workers of different types to accept different contracts, contract announcements satisfy $(w_1, L_1) = (w_2, L_2)$.

Since firms do not directly observe the type of any worker, firm contract announcements must be incentive compatible, i.e., satisfy the self-selection conditions

$$(2) \quad U_2[w_2(s,k)L_2(s,k), 1-L_2(s,k)] \geq U_2[w_1(s,k)L_1(s,k), 1-L_1(s,k)]$$

$$(3) \quad w_1(s,k)L_1(s,k) + \phi \ln[1-L_1(s,k)] \geq w_2(s,k)L_2(s,k) + \phi \ln[1-L_1(s,k)],$$

$s \in \{1, 2\}$, for all equilibrium values of k , where (3) is the appropriate self-selection condition in light of the assumed form of type 1 preferences, and where use has been made of (1). Notice that contracts are announced with full knowledge by all parties of the current period state (s, k) . Finally, one additional restriction on announced contracts is imposed that is common in these contexts. In particular, each announced contract is required to at least break even given the set of workers accepting it. Hence contracts must satisfy

$$(4) \quad w_1(s,k) \leq \pi_1(s)$$

$$(5) \quad w_2(s,k)L_2(s,k) \leq \pi_2(s)k^\theta L_2(s,k)^{1-\theta} - r(s,k)k$$

if $(w_1, L_1) \neq (w_2, L_2)$. Possible pooling contracts are discussed below, but in equilibrium announced contracts must induce self-selection. Hence (4) and (5) are the relevant restrictions here.

Finally, it will be noted that firms are restricted to pure strategies and that firms are not permitted to offer contracts which consist of wage-employment lotteries. Neither of these assumptions is important to the results obtained.¹⁰

Finally, to complete the description of worker behavior, given the set of announced contracts each worker accepts the announced contract he most prefers within that set.

II. Equilibrium: Full Information

A. Definition

In order to provide a benchmark, the stationary Nash equilibrium of this economy under full information is now described. Behavior of all agents is as discussed above, except that firm contract offers need not satisfy (2) and (3).

As indicated above, firms acquire capital in competitive rental markets. Thus, given $r(s,k)$, each firm chooses a quantity of capital to be applied to each worker, and simultaneously chooses a set of contracts $\{[w_i(s,k), L_i(s,k)]\}$. Clearly given the specifications of technology no capital will be applied to type 1 workers. Let $\psi(s,r)$ be the quantity of capital applied to type 2 workers.

Definition. A (stationary) Nash equilibrium is a set of announced contracts $\{[w_i(s,k), L_i(s,k)]\}$ and a set of values $r(s,k)$, $k(s)$, and $\psi(s,r)$ such that

(i) $r(s,k)$ and $k(s)$ satisfy (1).

(ii) $\psi\{s, r[s, k(\tilde{s})]\} = k(\tilde{s}) \quad \forall s, \tilde{s}.$

(iii) given announced contracts and the current value $r(s,k)$, no firm has an incentive to announce a new contract or to change its per worker capital stock.

Conditions (i) and (ii) impose capital market clearing while (iii) is the standard Nash equilibrium condition.

B. Characterization

An equilibrium is now characterized under one additional assumption: that $\pi_1(s) = \pi_2(s) = \pi_1$. The purpose of this assumption will become apparent below. The content of the assumption is that the productivity of type 1 workers is non-stochastic.

As should be apparent, any Nash equilibrium must be characterized by a number of "no-surplus" conditions. Clearly $w_1(s,k) = \pi_1 V(s,k)$, and $L_1(s,k) = \operatorname{argmax}\{\pi_1 L_1(s,k) + \phi \ln[1-L_1(s,k)]\} V(s,k)$. Then $L_1(s,k) = 1 - (\phi/\pi_1) V(s,k)$.

Type 2 contracts and $\psi(s,r)$ are slightly more complicated. However, since firms compete for the labor services of type 2 workers, clearly in equilibrium $w_2(s,k)$, $L_2(s,k)$, and $\psi(s,r)$ must be chosen to maximize the utility of a type 2 worker subject to the constraint that their employer earns non-negative profits. Then $w_2(s,k)$, $L_2(s,k)$, and $\psi(s,r)$ must be chosen in equilibrium to solve the problem

$$\max U_2[w_2(s,k)L_2(s,k), 1-L_2(s,k)]$$

subject to (5) by choice of $w_2(s,k)$, $L_2(s,k)$ and $\psi(s,r)$, with $r(s,k)$ parametric. Given the assumed form of $U_2(-)$, and given that (5) holds with equality in any solution, the problem may be rewritten as

$$(6) \quad \max \ln[\pi_2(s)\psi^\theta L_2(s,k)^{1-\theta} - \psi r(s,k)] + \ln[1-L_2(s,k)].$$

This problem has the following solution:

$$(7) \quad L_2(s,k) = \frac{1}{2} \forall s,k$$

$$(8) \quad \psi(s,r)r(s,k) = \theta\pi_2(s)\psi(s,r)^\theta L_2(s,k)^{1-\theta} = \theta\pi_2(s)\left(\frac{1}{2}\right)^{1-\theta} \psi(s,r)^\theta.$$

Also, since capital markets clear, (8) may be rewritten as

$$(9) \quad \tilde{k}(s)r(s,k) = \theta\pi_2(s)\tilde{k}(s)^\theta \left(\frac{1}{2}\right)^{1-\theta}.$$

This, of course, is equivalent to the standard marginal productivity condition

$$(9') \quad r(s,k) = \theta\pi_2(s)\left(\frac{1}{2}\right)^{1-\theta} k^{\theta-1}.$$

Also, the wage rate received by type 2 agents can be inferred from the standard marginal productivity relation.

$$(10) \quad w_2(s,k) = (1-\theta)\pi_2(s)\left(\frac{1}{2}\right)^{-\theta} k^\theta.$$

It remains to describe requirements on equilibrium values imposed by equation (1). Using (9') in (1) yields

$$(11) \quad \tilde{k}(s)^{1-\theta} = \beta \Phi \{ p(\tilde{s}) \pi_2(1) + [1-p(\tilde{s})] \pi_2(2) \} \forall \tilde{s},$$

where $\Phi \equiv \theta 2^{-(1-\theta)}$. This and $\psi(s,r) = \tilde{k}(s)$ completes the description of an equilibrium.

Discussion

Given the choices $\{[w_1(s,k), L_1(s,k)]\}, \psi(s,r)$, and the values $r(s,k)$ and $k(s)$ above, no firm has an incentive to alter its announced contracts or its choices of capital inputs. Thus, so long as the values $k(s)$ implied by (11) are feasible (i.e., satisfy $\phi(s,k) \leq y_1(s,k) = \pi_1 L_1(s,k)$), the values derived above are equilibrium values. Moreover, the Nash equilibrium derived above coincides with the competitive equilibrium for this economy.

As is apparent, the specification of preferences and technology imply that no interesting labor market behavior is observed in this economy under full information. In particular $L_1(s,k) = 1 - (\phi/\pi_1)$ and $L_2(s,k) = \frac{1}{2} \forall(s,k)$, so hours worked in this economy do not vary over the cycle. This will serve to emphasize that all interesting cyclical behavior in labor markets in the economy are a result of the presence of private information in labor markets.

III. Equilibrium: Private Information

A. Definition

The equilibrium notion applied under private information is the same as that of section II, except that now announced contracts must satisfy (2) and (3).

Definition. A (stationary) Nash equilibrium is a set of announced contracts $\{[w_1(s,k), L_1(s,k)]\}$ satisfying (2) and (3), and a set of values $r(s,k)$, $k(s)$, and $\psi(s,r)$, such that

- (i) $r(s,k)$ and $k(s)$ satisfy (1).
(ii) $\psi\{s, r[s, k(\tilde{s})]\} = k(\tilde{s}) \quad \forall s, \tilde{s}$.
(iii) given announced contracts and the current value $r(s,k)$, no firm has an incentive to offer a new contract or to change its per worker capital stock, where any contract announcements must satisfy (2)–(5).

Before characterizing an equilibrium some remarks are appropriate. First, any equilibrium displays self-selection of workers by contract accepted. To see this, fix the choice of capital rentals for each firm and notice that a "single crossing property" on preferences has been imposed. Then the usual Rothschild–Stiglitz (1976) argument can be employed to show that any equilibrium must be a separating equilibrium.

Second, given the assumed form of agent preferences, it is a real restriction to rule out the use of wage–employment lotteries by firms. However, the remark in footnote 10 is relevant here.

Third, as in Rothschild–Stiglitz (1976), no equilibrium in pure strategies need exist. Existence issues are similar, but not identical, to those that arise in Rothschild–Stiglitz (1976). Thus conditions implying the existence of an equilibrium are discussed in an appendix.

B. Characterization of Equilibrium

Reasoning identical to that employed by Rothschild–Stiglitz (1976) implies that the incentive constraint (3) is never binding on the contracts received by type 2 workers in equilibrium. In particular, so long as

$$(12) \quad \pi_1 > (1-\theta)\pi_2 (s)^{-\theta} k(s)^{\theta} \quad \forall s, \tilde{s}$$

(with $\tilde{k}(s)$ chosen to satisfy (11)), the wage rates received by type 1 workers exceed those received by type 2 workers. Hence type 1 workers have no incentive to misrepresent themselves as type 2 workers. Condition (12) is henceforth assumed as a restriction on parameter values.

Since considerations of incentive compatibility do not impinge on the selection of contracts for type 2 workers, competition among firms for the labor of these agents implies the same "no surplus" conditions as applied under full information. Thus type 2 contracts and $\psi(s,r)$ --the per worker capital stock employed--are identical under full information and private information.

Now consider type (1) contracts. Again, a "no surplus" argument implies that $w_1(s,k) = \pi_1$ holds $\forall(s,k)$. Further, competition among firms for type 1 workers implies that $L_1(s,k)$ must be maximal for type 1 workers (given $w_1(s,k) = \pi_1$) among the set of contracts that are consistent with self-selection. Thus the equilibrium values $L_1(s,k)$ maximize $\pi_1 L_1(s,k) + \phi \ln[1-L_1(s,k)]$ subject to (2), where (1) has been used to simplify the maximand. Further, given the assumed form of preferences, (2) may be written as

$$(13) \quad \ln(\frac{1}{2})w_2(s,k) + \ln(\frac{1}{2}) \leq \ln[w_1(s,k)L_1(s,k)] + \ln[1-L_1(s,k)]$$

$\forall(s,k)$, where (7) has been used in (13), and where $w_2(s,k)$ is given by (10).

In equilibrium, then, $\pi_1 L_1(s,k) + \phi \ln[1-L_1(s,k)]$ is maximized subject to (13). (13) may or may not bind in this problem. Define L_1^* to be the equilibrium level of employment for type 1 agents under full information, so that $L_1^* \equiv 1 - (\phi/\pi_1)$. Then, using the definition of L_1^* , $w_1(s,k) = \pi_1$, and

(10) in (13), the self-selection constraint (13) binds in equilibrium iff

$$(14) \quad (\frac{1}{2})(1-\theta)2^{\theta} \pi_2(s)k(\tilde{s})^{\theta} < \phi - \frac{\phi^2}{\pi_1}$$

where $k(\tilde{s})$ is given by (11). (14) is henceforth assumed to hold $\forall s, \tilde{s}$, as otherwise equilibrium values coincide with the equilibrium that obtains under full information.

Since (14) holds, $[w_1(s,k), L_1(s,k)]$ satisfies $w_1(s,k) = \pi_1$, and (13) at equality. Solving (13) for $L_1(s,k)$ yields

$$(15) \quad L_1(s,k) = (\frac{1}{2}) \left\{ 1 \pm \left[1 - \frac{w_2(s,k)}{w_1(s,k)} \right]^{\frac{1}{2}} \right\}.$$

It is easy to check that if both values given by (15) are feasible, the one most preferred by type 1 workers is

$$(15') \quad L_1(s,k) = (\frac{1}{2}) \left\{ 1 - \left[1 - \frac{w_2(s,k)}{w_1(s,k)} \right]^{\frac{1}{2}} \right\}.$$

Since $w_1(s,k) = \pi_1$ and $L_1(s,k)$ are maximal for type 1 workers among all contracts earning zero profits and that are consistent with self-selection, and since self-selection must occur in equilibrium, these are equilibrium contracts for type 1 agents if an equilibrium exists. Conditions guaranteeing existence of an equilibrium are derived in the appendix.

Finally, capital market clearing continues to require that (11) hold, for the same reasons as previously,

Discussion

Recall that under full information, equilibrium employment is constant. When private information is present, the employment of type 2 workers continues to be independent of the current period state. However, the employment of type 1 workers, as described by (15'), depends on, and is positively related to the relative wage rate

$$(16) \quad \frac{w_2(s,k)}{w_1(s,k)} = \frac{(1-\theta)\pi_2(s) \left(\frac{1}{2}\right)^{-\theta} \tilde{\theta} k(s)}{\pi_1}$$

Hence in general the employment of type 1 workers will vary with the current period productivity shock s , and with last periods' productivity shock \tilde{s} as well (through its effect on the inherited capital stock). Thus the introduction of private information provides channels for the current period productivity shock to affect current employment, and also for the current period productivity shock to affect future employment as well. Since under full information hours do not vary in this economy, it is therefore clear that private information provides a device for amplifying hours variability, and also provides a channel for increasing the "persistence of disturbances" to hours and output.

Moreover, the equilibrium is consistent with a number of the observations discussed in the introduction. First, there does exist unemployed labor. In particular, the "notional labor supply" of type 1

workers is $L_1^* = 1 - (\phi/\pi_1)$ at the equilibrium real wage rate π_1 . However

$$L_1(s,k) = (\frac{1}{2}) \left\{ 1 - \left[1 - \frac{w_2(s,k)}{w_1(s,k)} \right]^{\frac{1}{2}} \right\} < 1 - (\phi/\pi_1)$$

by the assumption that $\phi \geq (\frac{1}{2})\pi_1$. There is obviously no unemployment among type 2 workers.¹¹ Then the equilibrium level of unemployment is

$$(17) \quad u(s,k) \equiv \mu[L_1^* - L_1(s,k)] / [\mu L_1^* + (1-\mu)L_2(s,k)].$$

Second, the only real wage that can vary here is the real wage of type 2 workers. When $w_2(s,k)$ is relatively high, inspection of (15') indicates that $L_1(s,k)$ will be relatively high (since $w_1(s,k) = \pi_1$), and that $u(s,k)$ will be relatively low. Hence high levels of observed real wages will be associated (in the aggregate) with high levels of hours worked and low levels of unemployment.

Third, this result is obtained without violating micro evidence on the low person specific correlation between wages and hours. In particular, $L_2(s,k)$ will be constant while $w_2(s,k)$ varies, while $L_1(s,k)$ will vary while $w_1(s,k)$ is constant. Therefore the person specific correlation between real wage and hours movements will be zero for all agents, despite a positive aggregate correlation.

Fourth, "wage dispersions" (between type 1 and 2 workers) decline at "cyclical peaks", since $w_2(s,k)/w_1(s,k)$ will be high when total hours levels are high, and when unemployment rates are low.

Fifth, it is possible to generate all of these results while retaining the feature that trends in productivity induce no trends in hours worked. To see this, replace the technological specifications above with the following specifications: a type 1 worker employed at time t produces $(1+n)^t \pi_1$ units of output per unit time, while L units of type 2 labor and k units of capital at time t produce $(1+n)^t \pi_2(s)k^\theta L^{1-\theta}$ units of output. Letting $w_{it}(s,k)$ denote the time t real wage rate for type i workers in state (s,k) , clearly $w_{it}(s,k) = (1+n)^t w_i(s,k)$. Thus the trend rate of growth in productivity is also translated into a trend in real wages. However, it continues to be the case that $L_2(s,k) = \frac{1}{2} \forall s,k$ and that

$$L_1(s,k) = \left(\frac{1}{2}\right) \left\{ 1 - \left[1 - \frac{w_{2t}(s,k)}{w_{1t}(s,k)} \right]^{\frac{1}{2}} \right\}.$$

Since $w_{2t}(s,k)/w_{1t}(s,k) = w_2(s,k)/w_1(s,k)$, no trends in hours will be observed.

Sixth, since $L_1(s,k)$ is varying over the cycle, the relative shares of the "type 1 worker sector" and the "type 2 worker sector" in total employment will vary cyclically. This is also consistent with observation.

It remains to show that the model can generate a sufficiently large positive co-variation between aggregate (measured) productivity and hours to be consistent with observation. It also remains to show that significant "persistence" in hours variation can be generated by the model. The magnitude of co-variations between productivity and hours or the magnitude of serial correlation coefficients will clearly depend on parameter values. Thus

attention is now focussed on a numerical example to provide such a demonstration.

IV. An Example

A. Observations

An example is now presented in order to demonstrate that the fairly simple model of the previous sections can generate fairly rich dynamic and cyclical behavior. However, if the only objectives were to generate sufficient positive covariation between measured productivity and hours along with positive serial correlation in hours, this would be an easy exercise because of the relatively large number of free parameters in the model. Therefore, an example is constructed that is generally consistent with the following set of observations.

- (i) Individuals in the workforce on average work about one third of available time.
- (ii) Postwar U.S. unemployment rates range between 4 and 10 percent.
- (iii) Relative wages between construction and manufacturing range between .7 and .8.
- (iv) Capital's share in output is quite stable at approximately .3.
- (v) The ratio of gross private saving plus corporate saving to GNP is quite stable at about .15.
- (vi) The percentage standard deviation of the capital stock about trend is 1.2 percent.
- (vii) The percentage standard deviation of hours about trend is 2 percent.
- (viii) The percentage standard deviation of productivity about trend is 1 percent.

(ix) The percentage standard deviation of hours about trend is 1.8 percent.¹²
 A discussion of serial correlation parameters is deferred until after presentation of the example.

B. The Example

There are nine parameters, set as follows: $\beta = .579$, $\phi = 6$, $\pi_1 = 8.6$, $\pi_2(1) = 7$, $\pi_2(2) = 6.7$, $\theta = \frac{1}{4}$, $p(1) = \frac{2}{3}$, $p(2) = \frac{1}{3}$, $\mu = \frac{2}{3}$. β is the rate of discount. If the overlapping generations structure of the model is taken seriously, setting $\beta = .579$ corresponds to an annual rate of discount of .973.

It is now possible to compute all equilibrium quantities, as well as moments of the relevant stochastic processes for endogenous variables. From equation (11), $k(1) = 2$ and $k(2) = 1.942$. The implied percentage standard deviation of the capital stock about trend (which here is simply the mean of the capital stock) is 1.5 percent.

Having obtained equilibrium capital stocks it is easy to obtain equilibrium real wage rates. Obviously $w_1(s,k) = \pi_1 = 8.6$. Also, recalling that $w_2(s,k) = (1-\theta)\pi_2(s)k_2^\theta L_2^\theta(s,k)$, that $L_2(s,k) = \frac{1}{4}$, and defining $w_i(s, \tilde{s}) \equiv w_i[s, k(\tilde{s})]$, it is straightforward to compute the following relative wage rates:

$$\frac{w_2(1,1)}{w_1(1,1)} = .814$$

$$\frac{w_2(1,2)}{w_1(1,2)} = .802$$

$$\frac{w_2(2,1)}{w_1(2,1)} = .779$$

$$\frac{w_2(2,2)}{w_1(2,2)} = .768$$

Having computed equilibrium values of relative wage rates, (15') implies that

$$\begin{aligned} L_1(1,1) &= .284 & L_1(1,2) &= .278 \\ L_1(2,1) &= .265 & L_1(2,2) &= .259 \end{aligned}$$

where $L_i(s, \tilde{s}) \equiv L_i[s, k(\tilde{s})]$. It is also possible to compute (aggregate) per capita hours and real wages as follows. Letting $\hat{L}(s, \tilde{s})$ denote aggregate per capita hours, $\hat{L}(s, \tilde{s}) \equiv \mu L_1(s, \tilde{s}) + (1-\mu)L_2(s, \tilde{s})$. Letting $\hat{w}(s, \tilde{s})$ denote aggregate (hours weighted) real wages (and productivity),

$$\hat{w}(s, \tilde{s}) \equiv \mu \left[\frac{L_1(s, \tilde{s})}{\hat{L}(s, \tilde{s})} \right] w_1(s, \tilde{s}) + (1-\mu) \left[\frac{L_2(s, \tilde{s})}{\hat{L}(s, \tilde{s})} \right] w_2(s, \tilde{s}).$$

Finally, per capita output is given by

$$\hat{y}(s, \tilde{s}) \equiv \mu \pi_1 L_1(s, \tilde{s}) + (1-\mu) \pi_2(s) [k(\tilde{s})]^\theta L_2(s, \tilde{s})^{1-\theta}$$

and the unemployment rate is

$$u(s, \tilde{s}) = \frac{[L_1^* - L_1(s, \tilde{s})] \mu}{\mu L_1^* + (1-\mu)L_2(s, \tilde{s})},$$

since type 2 agents are not unemployed. Then the behavior of aggregate hours per capita in equilibrium is¹³

$$\begin{aligned} \hat{L}(1,1) &= .356 & \hat{L}(1,2) &= .352 \\ \hat{L}(2,1) &= .343 & \hat{L}(2,2) &= .339. \end{aligned}$$

It will be noted that, on average, people work roughly a third of available time. Also, the implied percentage standard deviation of per capita hours

about trend (mean) is 2.2 percent. (Contrast with an actual value of 2 percent.) Finally, it is possible to compute the first order autocorrelation of hours per capita, which is .547.

The behavior of average per capita wages is as follows:

$$\begin{array}{ll} \hat{w}(1,1) = 7.851 & \hat{w}(1,2) = 7.795 \\ \hat{w}(2,1) = 7.686 & \hat{w}(2,2) = 7.627. \end{array}$$

The implied percentage standard deviation about trend (mean) is 1.3 percent. (Contrast with an actual value of 1 percent.) Similarly, per capita output is

$$\begin{array}{ll} \hat{y}(1,1) = 3.961 & \hat{y}(1,2) = 3.894 \\ \hat{y}(2,1) = 3.752 & \hat{y}(2,2) = 3.686. \end{array}$$

The implied percentage standard deviation about trend (mean) is 3.2 percent, which is too large relative to the observed value of 1.8 percent. The first order autocorrelation coefficient of per capital output is .73. Finally,

$L_1^* = 1 - (\phi/\pi_1) = .302$, so that equilibrium unemployment rates are

$$\begin{array}{ll} u(1,1) = 3.3\% & u(1,2) = 4.3\% \\ u(2,1) = 6.7\% & u(2,2) = 7.9\% \end{array}$$

Hence unemployment rates lie in the appropriate range.

It remains to say something about capital's share in total output and about savings behavior for this economy. Capital's share varies between .294 and .299. Also the ratio of total savings (as defined above) to GNP here is given by $(1-\mu)k(s)/\hat{y}(s,\tilde{s})$, since $(1-\mu)k(s)$ is the aggregate level of capital accumulation. This ratio varies between .168 and .175, which is near the David-Scadding (1973) value of about .15.

It remains to say a word about the first order autocorrelations of output and hours reported above. Kydland and Prescott (1982) report a first order autocorrelation coefficient for output of .71, so this example generates the same kind of persistence in output as does the Kydland- Prescott model (if a period is taken to be a quarter). If the overlapping generations structure of the model is taken seriously, if a generation is taken to be twenty years, and if quarterly output follows a first order Markov process, the first order autocorrelation coefficient for output of .73 reported above corresponds to an autocorrelation coefficient of .996 for (detrended) quarterly output. Similarly, the first order autocorrelation coefficient for hours of .547 corresponds to an autocorrelation coefficient of .992 for (detrended) hours reported quarterly. Thus the example is consistent with quite high (and perhaps somewhat higher than observed) persistence in hours and output.

It remains to discuss existence issues for the parameters of the example. As discussed in the appendix, an equilibrium exists if the contract derived above for type 1 agents is preferred by them to any pooling contract which at least breaks even for the firm offering it. It is easy to check that the most preferred pooling contract for type 1 agents (among those that earn non-negative profits) sets hours equal to zero in all states. Since it is incentive compatible above to set $L_1(s,k) = 0 \forall (s,k)$, and since this is not done, type 1 agents prefer the contract derived above to any possible pooling contract. Hence there is no issue about the existence of an equilibrium here.

V. DISCUSSION

The model presented above demonstrates that the presence of private information creates channels for explaining both the high variability of hours

relative to average productivity (or measured real wages), and the serial correlation of disturbances. Moreover this can be done while explaining the presence of unemployed labor, and without contradicting micro evidence on individual specific correlations between wages (or productivity) and hours. Moreover, such an explanation is consistent with a range of other observations, as discussed in sections III and IV.

In order to emphasize the role of private information, the model was parameterized in such a way that hours do not vary over the cycle under full information. This objective dictated that several strong assumptions be made on the form of preferences and technology. Other strong assumptions were made in order to obtain closed form solutions for equilibrium quantities. The purpose of this section is to discuss some of these assumptions explicitly.

One feature of the analysis that merits discussion is the issue of "underemployment" versus "unemployment". The model of sections II-IV is specified in such a way that all workers are employed, although some workers work less than their "notional supply of labor". Thus the model generates "underemployment" rather than "unemployment" of labor. This feature of the model can be dispensed with, however. Smith (1985) presents a version of the model above in which employment can take an all or nothing form (say because of indivisibilities), and in which firms induce self-selection by offering contracts that specify wage-employment lotteries. The results of the analysis are qualitatively identical to those reported here. Thus there is no need to object to this model because it focuses on "intensive" rather than "extensive" margins with respect to employment choices.¹⁴

With respect to technology, two assumptions were made that merit discussion. First, type 1 workers had constant marginal products ($\pi_1(1) =$

$\pi_1(2) = \pi_1$). This assumption permitted hours of type 1 workers to be constant under full information. It also permitted real wages and hours to be uncorrelated for individual workers of this type.

Second, it was assumed that only type 2 workers could productively employ capital. This is merely a simplifying assumption. All that is necessary to the analysis is that a larger inherited capital stock, *ceteris paribus*, reduces the wage differential $w_1(s,k)/w_2(s,k)$. This reduction is in accordance with observation (see Reder (1962)). The simplest specification consistent with Reder's observation has been employed here.

With respect to preferences, several aspects of the specification merit comment. First, it was assumed that type 2 agents care only about young period consumption. This is an unnecessary simplification. However, this assumption, along with the linearity of type 1 agent preferences in c_1 and c_2 , permits an explicit solution for the equilibrium capital stock at each point in time. Second, the linearity of $U_1(c_1, c_2, 1-L)$ in c_1 and c_2 merits comment. In addition to permitting a closed form solution for equilibrium capital stocks, this assumption implies a constant *ex ante* real rate of interest. This is not strongly at variance with observation (see, e.g., Fama (1975)).

Third, type 1 and 2 agents have different functional forms for preferences. This again permits considerable simplification without being essential to the analysis. Smith (1985) presents a version of the model in which all agents have linear preferences, and which retains the qualitative features of the specification employed above.

Fourth, the reasons for assuming logarithmic utility for type 2 workers should be discussed. As is apparent from the discussion of section III, the utility function of type 2 agents is what matters in the determination of all hours levels. The assumption of logarithmic preferences permits type 2 agents

to have equilibrium hours levels that are independent of wage rates. This assumption also implies that the equilibrium employment levels of type 1 workers depend on relative wage rates only, in accordance with the arguments of Dunlop (1950), Keynes (1936), and Solow (1980). The assumption also permits equilibrium hours levels to be independent of trends in productivity. Finally, either logarithmic or linear preferences must be assumed for type 2 agents in order to generate explicit solutions to the model.

A word is also in order about informational assumptions, and in particular, assumptions about what is observable. It was assumed, for instance, that individual savings behavior is not observable here. This assumption can be relaxed without altering the content of the analysis, as demonstrated in Smith (1985).

Finally, it remains to discuss the use of the overlapping generations construct in studying cyclical phenomena. First, this assumption is made only to avoid the complexities introduced by having multi-period incentive problems. The use of an overlapping generations model permits such issues to be avoided in a first pass at looking at how private information can affect cyclical and dynamic behavior in labor markets.

Second, in order to obtain closed form solutions to the model, it is necessary to assume that capital depreciates completely in its period of use. For such an assumption to be reasonable it is clearly necessary that a "period" be fairly long, say on the order of a generation as assumed here.

Finally, a remark is in order about the use of such models to examine issues related to observed magnitudes of variances and covariances in quarterly data. Nothing prevents one from thinking of events unfolding over time in some arbitrary way within a period, while the model pins down only total employment, unemployment, income, etc. during the entire period.

Magnitudes of observed variances and covariances will not depend (in large samples) on how frequently data is sampled from an economy, and thus there is no logical problem with thinking about the economy in this way. Measured autocorrelations will, of course, depend on frequency of sampling, so interpretations of serial correlation parameters are more problematic.

APPENDIX

Existence of Equilibrium

The contracts described in section III are such that $[w_2(s,k), L_2(s,k)]$ (along with the choice $\psi(s,r)$) and $[w_1(s,k), L_1(s,k)]$ are the maximal contracts for type 2 and 1 agents (respectively) consistent with self-selection and with (4) and (5). Thus, there are no alternate contracts (or alternate choices of capital inputs plus contracts) that are consistent with self-selection, with (4) and (5), and that any workers prefer to the contracts derived above.

Now consider why an equilibrium might fail to exist. There are two reasons. First, clearly the values $k(\tilde{s})$ which satisfy (11) must obey

$$\left(\frac{1-\mu}{\mu}\right)k(\tilde{s}) = \phi(s,k) \leq w_1(s,k)L_1(s,k). \quad \text{Otherwise required equilibrium savings}$$

levels would result which violate type 1 agents' budget constraints.

Parameter values are selected in the text to guarantee that this condition is satisfied. Second, given that all firms announce the contracts described in section III, some firm may have an incentive to offer some other set of contracts (and possibly choose a different capital stock). Since the contracts derived above were maximal for each type of agent among all contracts satisfying (4) and (5) and inducing self-selection, any contract that attracts workers must therefore be a pooling contract. Such a contract is now described, along with what would be necessary for such a contract to attract workers in a profitable manner.

Under a pooling contract obviously all workers work a common hours level $L(s,k)$ and receive a common wage rate $w(s,k)$. Moreover, as all workers appear identical from the point of view of a firm offering a pooling contract, all workers are allocated the same quantity of capital. Let ψ denote the firm's

choice for a level of capital input. Finally, suppose that all firms initially offer the contracts and make the capital input decisions described in section III. Then the prevailing rental rate on capital is given by $r(s,k) = \theta\pi_2(s)(1/2)^{1-\theta}k^{\theta-1}$. Since firms operate in competitive rental markets, any firm wishing to offer a different contract (and possibly to change its capital rental) takes this rental rate as given.

The issue of existence of an equilibrium is now quite similar to that discussed by Rothschild and Stiglitz (1976). However, the information generated by sorting permits capital to be allocated efficiently. If sorting does not occur then type 1 agents receive the same levels of capital inputs as do type 2 agents. In this context this amounts to a throwing away of resources. This will make existence easier to obtain here than it is in the Rothschild-Stiglitz setting.

Formally, then, consider a pooling contract $[w(s,k), L(s,k)]$ offered by a firm choosing per worker capital input ψ . Since workers are indistinguishable, ψ is allocated evenly among all workers. If the firm attracts workers in their population proportions, then fraction μ of this capital input is allocated to type 1 workers. This is completely unproductive. Fraction $(1-\mu)$ is allocated to type 2 workers. Hence with common hours level $L(s,k)$ and with a per worker capital stock of ψ , type 2 workers (per person) produce $\pi_2(s)[(1-\mu)\psi]^\theta L(s,k)^{1-\theta}$ units of output. Then per worker output is given by $\mu\pi_1(s)L(s,k) + (1-\mu)\pi_2(s)[(1-\mu)\psi]^\theta L(s,k)^{1-\theta}$, and firm profits are nonnegative iff

$$(A.1) \quad \mu\pi_1(s)L(s,k) + (1-\mu)\pi_2(s)[(1-\mu)\psi]^\theta L(s,k)^{1-\theta} - r(s,k)\psi - w(s,k)L(s,k) \geq 0.$$

Now a pooling contract will attract type 1 workers only if it is preferred to the separating contract derived in section III. Then consider the maximal pooling contract for type 1 agents. This contract solves the problem

$$\max_{\substack{0 \leq L(s,k) \leq 1 \\ 0 \leq \psi}} w(s,k)L(s,k) + \phi \ln[1-L(s,k)]$$

subject to (A.1), where this is the appropriate maximand since the values $r(s,k)$ satisfy (1). Substituting (A.1) (at equality) into the maximand, then, yields the unconstrained problem

$$(P) \quad \max_{\substack{0 \leq L(s,k) \leq 1 \\ 0 \leq \psi}} \mu \pi_1(s)L(s,k) + (1-\mu) \pi_2(s) \psi^{\theta} L(s,k)^{1-\theta} \\ - r(s,k)\psi + \phi \ln[1-L(s,k)].$$

The optimizing solution for ψ as a function of $L(s,k)$ and $r(s,k)$ is

$$(A.2) \quad \psi = L(s,k) \left[\frac{\theta(1-\mu) \pi_2(s) \frac{1}{1-\theta}}{r(s,k)} \right]$$

Substituting (A.2) into the problem (P) gives a problem defining the optimal value of $L(s,k)$:

$$\max_{0 \leq L(s,k) \leq 1} L(s,k) \left\{ \mu \pi_1(s) + [(1-\mu) \pi_2(s) r(s,k)]^{-\theta} \frac{1}{1-\theta} \left[\frac{\theta}{1-\theta} \frac{1}{1-\theta} \right] \right\} + \\ \phi \ln[1-L(s,k)],$$

with $r(s,k) = \theta \pi_2(s) (1/2)^{1-\theta} k^{\theta-1}$. Let $L^*(s,k)$ denote the maximizing value of $L(s,k)$. Then $L^*(s,k)$ is the optimal pooling contract for type 1 agents which

at least breaks even. Hence no pooling contract exists which can attract type 1 agents in a profitable manner if

$$(A.3) \quad \mu \pi_1(s) L_1(s, k) + \phi \ln[1 - L_1(s, k)] \geq L^*(s, k) \left\{ \mu \pi_1(s) + \left[\theta \left(\frac{\theta}{1-\theta} \right) - \theta \left(\frac{1}{1-\theta} \right) \right] \right. \\ \left. \cdot \left[(1-\mu)^{1+\theta} \pi_2(s) \right] \left(\frac{1}{1-\theta} \right) [r(s, k)] - \left(\frac{\theta}{1-\theta} \right) \right\} + \phi \ln[1 - L^*(s, k)]$$

$\forall s$ and for all values $k(\tilde{s})$ satisfying (1). Thus satisfaction of (A.3) by the values $L_1(s, k)$ given by (15') is sufficient for the existence of an equilibrium as derived in section III.

FOOTNOTES

1. Examples of such work include Hansen (1985), Rogerson (1985), Greenwood and Huffman (1986), Greenwood, Huffman, and Hercowitz (forthcoming), and Rogerson and Wright (1987).
2. Exceptions are Holmstrom and Weiss (1984), Smith (1984a,b), Townsend (1987), and Williamson (1987).
3. This literature is too large to permit a complete list of contributions here. Hart (1983) provides a survey of the early literature on this topic.
4. For aggregate evidence see Altonji (1983) and Altug (1984). Some micro evidence on this point is surveyed by Ashenfelter (1984). Some real business cycle models [e.g., Hansen (1985)] are able to generate strong positive co-variation in hours and productivity by having labor supply be "lumpy", while specifying individual preferences in such a way that conventional "intertemporal substitution parameters" are low. However, such models use representative agent specifications, and hence have observed individual correlations between hours and productivity (real wages?) that are as high as those in aggregate data. As pointed out above, this is inconsistent with observation.
5. On points (i) and (ii) see Prescott (1983). On point (iii) see Reder (1962), and on (iv) see Lilien (1982).
6. The use of such a model in interpreting time series behavior obviously creates some problems. Some of these are discussed below.
7. If each worker's contribution to output cannot be directly observed, then it also cannot be inferred by a firm since any individual has a negligible incremental effect on a firm's output.

8. While the assumption that the aggregate shock evolves according to a two state Markov chain is restrictive, it is often used as a simplifying device. See, for instance, Mehra and Prescott (1985) or Greenwood, Hercowitz and Huffman (forthcoming).
9. Versions of the model can be produced that preserve all of the results, and that permit savings behavior to be observed. See Smith (1985).
10. Smith (1985) displays a different version of this model that permits firms to offer contracts consisting of wage-employment lotteries. The results are essentially the same as described here.
11. The result that unemployment occurs among "high wage" workers only may appear counterfactual. However, two comments are in order. First, it is entirely possible to have workers of three or more types, and to have unemployment among the "next to lowest" type only, where the ordering of types is with respect to productivity. It would be straightforward to construct such a model, since as pointed out above, incentive compatibility constraints may or may not bind between adjacent types. Such a construction would be consistent with the casual observation that there is no involuntary unemployment (or perhaps an excess demand for labor) in the most menial jobs, while an excess supply of labor exists for more attractive jobs. For simplicity, however, the two type construction of the text is retained.

Second, at the level of disaggregation of the model in the text, it is not clear that it is counterfactual to have high wage workers experience unemployment. In practice, when non-agricultural workers in non-service sectors are divided into two groups (as in the text), the

standard division is between construction and manufacturing (see, e.g., The Economic Report of the President). Construction workers earn relatively higher wages and experience higher rates of unemployment than workers in manufacturing.

12. On relative wages between manufacturing and construction, see the Council of Economic Advisers (1982), p. 276. On the relevance of this measure see the remark in footnote 11. On the ratio of saving to GNP see David and Scadding (1973). On the percentage standard deviation in the capital stock see Prescott (1983). The other figures cited are from Hodrick and Prescott (1981). The two-to-one ratio of variability in hours relative to productivity has been found in other studies of employment and real wages for the U.S. and other economies (see Geary and Kennan (1984)), and for the interwar U.S. economy in various sectors (see Bernanke and Powell (1984)).
13. Computation of serial correlation coefficients below requires that these values be computed to more than three decimal places. Only three decimal places are reported here for brevity.
14. For a discussion of this issue see Heckman (1984).

REFERENCES

- Altonji, Joseph G., "The Intertemporal Substitution Model of Labour Market Fluctuations: An Empirical Analysis," Review of Economic Studies, 49, 1982, p. 783-824.
- Altug, Sumru, "Gestation Lags and the Business Cycle: An Empirical Analysis," unpublished, 1984.
- Ashenfelter, Orley, "Macroeconomic and Microeconomic Analysis of Labor Supply," Carnegie-Rochester Conference Series on Public Policy, 21, 1984, p. 117-156.
- Bernanke, Ben S., "Employment, Hours, and Earnings in the Depression: An Analysis of Eight Manufacturing Industries," unpublished, 1984.
- Bernanke, Ben S., and James L. Powell, "The Cyclical Behavior of Industrial Labor Markets: A Comparison of the Pre-War and Post-War Eras," unpublished, 1984.
- Cooper, Russell, "On Allocative Distortions in Problems of Self-Selection," Rand Journal of Economics, 15, 1984, p. 568-577.
- Council of Economic Advisers, Economic Report of the President, U.S. Government Printing Office, Washington, 1982.
- David, Paul A., and John L. Scadding, "Private Savings: Ultrarationality, Aggregation, and 'Denison's Law'," Journal of Political Economy, 82, 1974, p. 225-249.
- Dunlop, John, Wage Determination Under Trade Unions, New York, A.M. Kelley, 1950.
- Fama, Eugene F., "Short-Term Interest Rates as Predictors of Inflation," American Economic Review, 65, 1975, p. 269-282.

- Geary, Patrick T., and John Kennan, "Intertemporal Substitution and the Phillips Curve: International Evidence," unpublished, 1984.
- Greenwood, Jeremy, Zvi Hercowitz, and Greg Huffman, "Investment, Capacity Utilization and the Real Business Cycle," American Economic Review, forthcoming.
- Greenwood, Jeremy and Greg Huffman, "A Dynamic Equilibrium Model of Inflation and Unemployment," Journal of Monetary Economics, 19, 1987, p. 203-28.
- Hansen, Gary, "Indivisible Labor and the Business Cycle," Journal of Monetary Economics, 16, 1985, p. 309-27.
- Hart, Oliver D., "A Model of Imperfect Competition with Keynesian Features," Quarterly Journal of Economics, 97, 1982, p. 109-138.
- Hart, Oliver D., "Optimal Labour Contracts Under Asymmetric Information: An Introduction," Review of Economic Studies, 50, 1983, p. 3-36.
- Heckman, James, "Comments on the Ashenfelter and Kydland Papers," Carnegie-Rochester Conference Series on Public Policy, 21, 1984, p. 209-224.
- Hodrick, Robert J. and Edward C. Prescott, "Post-war U.S. Business Cycles: An Empirical Investigation," unpublished, 1981.
- Holmstrom, Bengt, and Laurence Weiss, "Managerial Incentives, Investment, and Aggregate Implications," Review of Economic Studies, 1984.
- Keynes, John Maynard, The General Theory of Employment, Interest, and Money, Harcourt Brace and World, New York, 1936.
- Kydland, Finn E., and Edward C. Prescott, "Time to Build and Aggregate Fluctuations," Econometrica, 50, 1982, p. 1345-1370.
- Lilien, David M., "Sectoral Shifts and Cyclical Unemployment," Journal of Political Economy, 90, 1982, p. 777-793.

- Long, John B., and Charles I. Plosser, "Real Business Cycles," Journal of Political Economy, 91, 1983, p. 39-69.
- Mehra, Rajnish, and Edward C. Prescott, "The Equity Premium: A Puzzle," Journal of Monetary Economics, 15, 1985, p. 145-61.
- Prescott, Edward C., and Robert M. Townsend, "Pareto Optima and Competitive Equilibria with Adverse Selection and Moral Hazard," Econometrica, 52, 1984, p. 21-45.
- Prescott, Edward C., and Robert M. Townsend, "General Competitive Analysis in an Economy with Private Information," International Economic Review, 25, 1984, p. 1-20.
- Prescott, Edward C., "Can the Cycle be Reconciled with a Consistent Theory of Expectations?" unpublished, 1983.
- Reder, Melvin W., "Wage Differentials: Theory and Measurement," in Aspects of Labor Economics (H. Gregg Lewis, ed.), Princeton University Press, Princeton, 1962.
- Rogerson, Richard, "Indivisible Labor, Lotteries and Equilibrium," unpublished, 1985.
- Rogerson, Richard, and Randall Wright, "'Involuntary' Unemployment in Economies with Efficient Risk Sharing," unpublished, 1987.
- Rothschild, Michael, and Joseph Stiglitz, "Equilibrium in Competitive Insurance Markets: an Essay on the Economics of Imperfect Information," Quarterly Journal of Economics, 90, 1976, p. 629-650.
- Smith, Bruce D., "A Business Cycle Model with Private Information," unpublished, 1984a.

- Smith, Bruce D., "A Model of Nominal Contracts," unpublished, 1984b.
- Smith, Bruce D., "Unemployment, the Variability of Hours, and the Persistence of Disturbances: A Private Information Approach," unpublished, 1985.
- Solow, Robert M., "On Theories of Unemployment," American Economic Review, 70, 1980, p. 1-11.
- Townsend, Robert M., "Private Information and Limited Insurance: The Revelation Principle Extended," unpublished, 1987.
- Wilson, Charles, "A Model of Insurance Markets with Incomplete Information," Journal of Economic Theory, 16, 1977, p. 167-207.
- Williamson, Stephen D., "Financial Intermediation, Business Failures, and Real Business Cycles," Journal of Political Economy, 95, 1987, p. 1196-1216.

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