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Equilibrium Marketing Strategies: Is There Advertising, in Truth? (Revised)

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EQUILIBRIUM MARKETING STRATEGIES: IS THERE ADVERTISING, IN TRUTH?*

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Abstract

This paper presents a two-period model in which a firm has private information on product quality, about which consumers can learn useful but imperfect information by consuming the good. The firm's marketing strategy includes an offer to sell at some price (that may vary over time) and costly advertising if desired. In a (refined) sequential equilibrium neither price nor advertising can ever be informative about product quality for newly introduced goods. Equilibria in which price or advertising are informative are ruled out because they depend on "incredible" belief formation. For more established goods, price, profits and advertising may convey information and are positively correlated with quality but the manner in which information is transmitted is inherently noisy--i.e. a separating equilibrium is not possible. Other predictions are obtained. Among them, goods that are advertised once they have become established must command a higher price than that at which they were introduced, and conversely for those that go unadvertised; an increase in variable costs lowers the price of advertised goods.

I. INTRODUCTION

It is common to observe new goods being advertised well beyond the introductory period. However, current theories of advertising based on informational asymmetries (following Nelson (1974)) fail to explain this observation. In these models, there is a simple separating equilibrium in which only high quality goods are advertised in an introductory period so that advertising reveals quality to consumers. Subsequently, with quality known (and confirmed by consumption experience) no further advertising is necessary.

This paper presents a new model of the firm's marketing decision when there are informational asymmetries. A key feature of the equilibrium strategy is that the firm often advertises after the introductory period. However, unlike previous models, the marketing strategy never involves wasteful or dissipative advertising (i.e. advertising beyond that required simply to inform consumers of the product's existence) in the introductory period. Rather, the firm uses an introductory price that is low relative to prices set in subsequent periods that also involve advertising. This price conveys no information to consumers, though, because the firm adopts the same pricing policy irrespective of quality. These results are independent of the size of the advertising costs.

The basic structure of the model is straightforward. A firm has developed a new product and in the process of doing so has learned the good's quality. This information is private to the firm. The firm may advertise as well as set an initial price at which it is willing to sell the good. Consumers draw what new inferences they can from the firm's behavior and purchase if it pays for them to do so. Consumption experience is informative about quality but only imperfectly so. Subsequently, the firm may advertise again and make a new price offer and consumers may repurchase if they desire.

The principal manner in which the present model differs from earlier work [Nelson, especially as formalized by Milgrom and Roberts (1986) and Kihlstrom and Riordan (1984)] is deceptively simple: a single consumption experience cannot, by itself, provide consumers with all the information relevant to subsequent purchase decisions. Therefore, while consumption is informative, it is at best a noisy signal of quality. This assumption appears plausible; consumption frequently involves a considerable amount of randomness either because of stochastic elements in the product itself or the circumstances in which consumption takes place. Moreover, without this specification it is technologically impossible for the firm's private information to remain private over time. In a multiperiod model in which private information plays a key role, to assume that it is impossible for it to persist over time seems unreasonably restrictive.

What is deceptive about this specification is the way that it fundamentally alters the equilibrium structure. Unlike the models of Milgrom and Roberts and Kihlstrom and Riordan in which advertised goods are revealed to be high quality, this outcome can never happen here; that is, there are no separating equilibria in this model. In addition, in many cases, advertising will be observed to occur beyond the initial period as an equilibrium response by the firm to consumers' persistent uncertainty about quality. In fact, such an outcome can occur even if there is no possibility of repurchase in the future. Thus, in a fundamental sense, advertising is not a capital good.

The details of the model and a discussion of the equilibrium concept are provided in section II. As is common in imperfect information environments, the model analyzed here has many sequential equilibria. The equilibrium selection method used here is a slightly strengthened version of Grossman and Perry's (1986) perfect sequential equilibrium. For those who are not familiar with equilibrium selection procedures, section II provides a discussion of the rationale behind such "refinements" of sequential equilibrium and explains how the one used here is employed.

Section III details the firm's equilibrium marketing strategy. It is shown that the strategy implies a unique outcome in the initial period. This outcome yields no new information on quality, involving no advertising and a price that is independent of the quality of the firm's product. Subsequently, two outcomes are possible. In one, the product price falls relative to the introductory price so that even consumers who had unfavorable consumption experience are still willing to buy. Again, the good remains unadvertised. In the other, should the firm have a high quality product, it advertises and sets a price above that in the initial period. Only those consumers who had a favorable consumption experience repurchase in this case. If the firm has a lower quality good to sell, it randomizes between the policy of the producer of the high-quality good and a policy that involves charging a low price and not advertising.

Section IV spells out the model's observable implications. One coarse prediction is that, should advertising occur after the good's introductory period, then product price must be higher than it was initially. This result may be tested easily since it refers to the behavior of a single firm over time.

A second set of empirical implications deals with outcomes that should be observed across firms. These propositions again focus on situations in which advertising is observed to occur after the introductory period; that is, for established products. In these situations, the model predicts that the familiar Nelson-type relationships (e.g. advertising expenditures, profits, product price and quality all being positively correlated) should be observed.

Finally, comparative statics experiments are considered. Some results are unusual. For example, an increase in production cost lowers the price of advertised goods.

II. THE ADVERTISING GAME

The Problem

The situation under consideration is one in which a single firm is producing a good whose quality (q) can take on one of two values: h (high) or l (low). Quality is exogenous, for simplicity. A more complicated setup would allow quality to be a choice variable and the firm to have private information on its cost for different qualities.[1]

High and low quality might be thought of in terms of durability. For example, a high-quality battery functions longer than a low-quality one. Alternatively, quality might be interpreted as "quality of match", so that, within a given consumer population, high-quality products produce a larger fraction of "satisfied customers" than do low-quality products. In either case, since information transmission is the issue at hand, it must be that information on quality (i.e. the value of q) is distributed asymmetrically. In particular, it must be that the firm has better information on quality, at least initially, than do consumers. More importantly, for it to be at least feasible for information transmission to persist over time, it must be possible for this state of asymmetric information to persist as well.

This informational setup can be captured as follows. It is assumed that the good is such that its performance in the hands of consumers is stochastic. Specifically, the good either "works", in which case the consumer gets a utility normalized to unity from a unit of consumption, or it "fails", providing the consumer a utility of $V \in (0,1)$. It is also assumed that in the process of designing, developing, and testing the product, the firm has learned the probability with which the product works (i.e. has learned quality). This probability is given by ψ_q $(q = h,l)$, with $0 < \psi_l < \psi_h < 1$. Initially, the consumer does not have access to this information. That is, while he knows the probability that the product he buys will work is one of ψ_h or ψ_l, he does not know which it is (i.e. $q = h$ or $q = l$).

Because these assumptions differ from those generally adopted and are important in what follows, it is worth digressing to elaborate somewhat. First, it is clear that, under these assumptions, consumption experience alone is at best imperfectly informative. A single consumption experience provides the consumer only with the information that the product either worked or failed. Since it is assumed that the probability with which the product works is correlated with quality, the outcome provides some information on quality. However, consumption experience by itself does not yield the same information on quality as the firm has obtained. For example, a single observation on a particular battery's durability yields only imperfect information concerning the entire distribution of death dates of batteries.

Second, consumption experience in a given period will be wholly uninformative about future consumption experience unless quality is correlated across periods. Additional complication is avoided by assuming that quality is fixed over time. In this way, consumption experience in the current period provides useful information about the quality of the good purchased in future periods.

Finally, if an individual consumer's consumption experience is publicly observable, then the possibility for information transmission through means other than price and costly advertising arises. To rule out these other possibilities, it is assumed that each consumer's consumption experience is his private information. Under these circumstances, entities such as warranties and money-back guarantees, that might under some circumstances substitute for the pricing/ad behaviour as information carriers, become ineffective. This assumption also makes the pooling of information on consumption experience unavailable (particularly if accurate reporting is costly). The impact of this assumption in conjunction with the previous ones is to make it possible for the firm's private information to remain private unless the firm chooses to reveal the information through its marketing behaviour.

In terms of application, the model is applicable to products having the property that the characteristic(s) about which there is asymmetric information are also ones for which individual consumption experience is costly to verify. Again, batteries are a good example. Checking durability before purchase is difficult, as is inferring it from a single dead battery. Also, it is hard for anyone, other than the consumer, to verify how well the battery performed.

In addition to using consumption experience, consumers may also use observations on the outcomes of the firm's decision-making process to draw inferences on quality. The firm is assumed to make two choices: a unit price, p, for the good and a level of advertising, a. The choice of advertising level is a binary one involving either no ad, $a = \phi$, or advertising at a level a=A, where the costs of these choices are given by zero and $\alpha > 0$ respectively. The choice a=A should be thought of as a decision on whether to advertise beyond the level necessary simply to inform customers of the product's existence, price, availability etc.: $a=\phi$.[2]

This latter point is an important one. The feature of the action called "advertising" that makes it potentially useful as a signal is that it is evidently costly to the firm (in the sense that, in a world of perfect information the costs outweigh any gains). Therefore what is important about advertising, from a signalling standpoint, is that part of the advertising expenditure is in excess of the amount needed simply to inform consumers of the product's existence and price. Milgrom and Roberts make a similar point. Therefore, it is this part of the overall promotion activity that is referred to as advertisement. This notion of advertising means that many things not commonly considered advertising in the everyday sense of the word, like a firm's support for the arts or amateur sports, or contributions to neighborhood parks etc., might usefully be included in the definition of advertising. It also provides a simple explanation for the observed characteristics of much of what is commonly referred to as "ads".[3]

The problem that the firm faces is one of choosing a sequence of prices and advertising levels (one pair per period), given the quality of its product, in order to maximize the expected value of its profit stream. The firm makes these decisions realizing that consumers gather information both from the observed price/ad outcome and consumption experience. Consumers are faced with the problem of trying to infer from all their experience (both consumption and price/ad observations) the quality of the firm's product and, then, to make an expected utility-maximizing consumption decision. For simplicity, the consumer's decision is assumed to involve buying either zero or one unit of the product each period. Also, all consumers are risk neutral, in which case an individual buys if he obtains non-negative expected utility when prices are netted out. The complete structure of the consumer-firm game and a discussion of equilibrium is provided in the subsection following.

The Game Representation

The situation described above can be represented by an extensive form game with imperfect information. The main elements of this game and the necessary notation are provided below. In the interest of brevity, and to avoid the associated proliferation of notation, some of the fine details of the game have been suppressed.

There are two kinds of active players: the firm (or producer), $\wp$, and a continuum of consumers, uniformly distributed on $[0,N]$. Any particular consumer will be labelled C, with $C \in [0,N]$.

Time is indexed by t with $t=0,1$ representing the introductory and subsequent periods respectively. The quality (q) of $\wp$'s good is determined exogenously at $t = 0$ and is fixed for both periods. The outcome $q = h$ is assumed to occur with probability $\delta \in (0,1)$, which also gives the consumer's initial priors on the outcome $q = h$. δ may be thought of as the outcome of (unmodelled) costly and imperfect information gathering by C, including what can be learned by inspection prior to purchase. A firm having obtained the outcome q will be referred to as being of "type q"; q = h,l. Quality is assumed to be such that the unit cost of production, γ, is constant and independent of firm type.[4]

$\wp$'s action at t = 0 consists of a choice of a price offer for period 0, p_0, and an advertising choice, $a_0 = \phi$ or A, the choice of each potentially depending on $\wp$'s type. To allow for randomization possibilities, a typical strategy for $\wp$ is represented by a function $F_0(p_0,a_0|q)$, giving the probability with which a price less than or equal to p_0 along with the ad choice a_0 is observed, given $\wp$ is of type q.

C's strategy at t = 0 consists of either buying a single unit of the good, or not, given the observation (p_0,a_0) and the prior probability δ. For simplicity, C is restricted to pure strategy choices only. Should C purchase, consumption experience is as outlined previously. In particular, C's experience, c, may be either good (g) or bad (b), with $1 > Pr(c=g|q=h) \equiv \psi_h > Pr(c=g|q=l) \equiv \psi_l > 0$. An independent realization of either c=g or b is obtained by each C at t=0 and this realization is private information, as discussed previously. A realization on any subsequent purchase is independent as well.

In period t=1 the situation is similar to that in t=0 with two exceptions. One is that some (or indeed all) C may not have purchased at t=0 while others may have. It is easy to show, however, that there could not be an equilibrium in which $\wp$'s strategy results in any C not purchasing at t=0. Therefore, to simplify the exposition, the remainder of this section focuses only on the situation in which all C purchase at t=0. The other difference is that, at t=1, $\wp$'s strategy may specify an action (p_1,a_1) that results in only those *C* having had a good consumption experience in t=0 purchasing, only those having had a bad experience purchasing, or all *C* purchasing. Of course, while $\wp$ cannot observe who these various individuals are, there is enough information to calculate the expected number in each case (i.e. $\psi_q N$, $(1-\psi_q)N$, or N).

Otherwise, the specification of the problem for t=1 is entirely analogous to that at t=0. A typical strategy for $\wp$ is given by the function $F_1(p_1,a_1|q,p_0,a_0)$ and is interpreted as before with the addition that F_1 may depend on p_0 and a_0 as well as q. For brevity's sake, this dependence on p_0 and a_0 will be implicit subsequently. *C*'s strategy, again, consists of purchasing or not and may be conditioned on p_0,a_0,p_1,a_1 and consumption experience c.

The description of the game is completed by a specification of the payoffs. *C*'s payoffs need not be presented in any detail. Conveniently, the analysis below yields the necessary information as a by-product.

As for $\wp$'s payoff, consider, first, the expected profit earned from sales at t=1 given the strategy pair F_0,F_1. Since *C*'s strategy specifies a purchase/no purchase decision for every (p_1,a_1), it is possible to construct an acceptance set, B_1, defined as the set of (p_1,a_1) such that *C* purchases at t=1. This set is simply all (p_1,a_1) yielding *C* non-negative expected utility. B_1 therefore depends on the prior δ, the strategies F_0,F_1 and *C*'s consumption experience at t=0. If all *C* are treated symmetrically, in the sense that each draws identical inferences from the observation of off-equilibrium path (p_1,a_1) pairs regardless of consumption experience, then the acceptance set for those *C* with bad experience at t=0, $B_1(b)$, is a subset of the one for those with good experience, $B_1(g)$.[5]

It follows that $\wp$'s expected profit from sales at t=1 for some strategy pair F_0, F_1 is

$$\pi(p_0, a_0|q) \equiv \int_{B_1(g)-B_1(b)} [N\psi_q(p_1-\gamma) - \alpha(a_1)]dF(p_1, a_1|q)$$

$$+ \int_{B_1(b)} [N(p_1-\gamma) - \alpha(a_1)]dF(p_1, a_1|q)$$

where $\alpha(a_1) = 0$ if $a_1 = \phi$ and α if $a_1 = A$. The first term in this expression is the expected return to $\wp$ should it choose a (p_1, a_1) that results in a purchase only by those *C* with a good consumption experience at t=0. (i.e. (p_1, a_1) in the set $B_1(g) - B_1(b)$). Any one of these (p_1, a_1) is chosen with "probability" $dF(p_1, a_1|q)$. Since only $\psi_q N$ of the *C* are expected to have good experience, variable returns are proportional to that number. The second term is the expected return to $\wp$ should it choose a (p_1, a_1) that results in those *C* with bad experience in t=0 purchasing. Since those with good experience purchase if those with bad experience do, variable returns here are proportional to N.

$\wp$'s profits from sales at t=0 are defined similarly, with *C*'s strategy determining an acceptance set B_0 giving the (p_0, a_0) that result in a purchase by *C* at t=0. The only difference is that, for t=0, *C* may purchase at t=0 even though expected utility in t=0 is negative. That is, *C* will purchase as long as the expected utility stream from such a decision is higher than that from not purchasing in t=0 and then proceeding optimally.

$\wp$'s payoff -- total expected profit -- is then defined as (ignoring discounting)

$$\pi(q) \equiv \int_{B_0} [N(p_0-\gamma) - \alpha(a_0) + \pi(p_0, a_0|q)]dF_0(p_0, a_0|q). \qquad (1)$$

That is, given (p_0,a_0), $\wp$ earns $(p_0-\gamma)$ for each of the N units sold, and spends $\alpha(a_0)$ on advertising. This outcome is followed by the period 1 profit $\pi(p_0,a_0|q)$ defined previously. Under the strategy $F_0(\ )$, (p_0,a_0) occurs with probability $dF(p_0,a_0|q)$.

Additional Restrictions

It is possible to analyze the game set out above without further restrictions. However, discussion of a variety of trivial outcomes can be eliminated through the imposition of two parameter restrictions. Because these restrictions all involve the one period expected utility for *C*, it is useful to define the variable $\mu_q \equiv \psi_q + (1-\psi_q)V$. μ_q is *C*'s expected utility from consumption, given that quality is q, and provides information about *C*'s demand behavior (acceptance set). For instance, no *C* could credibly claim to be unwilling to purchase a unit of the good at a price μ_1 or less. Similarly, no *C* could ever credibly claim to be willing to purchase a unit of the good at a price beyond μ_h. Also define

$\bar{p} \equiv \delta\mu_h+(1-\delta)\mu_1$ as *C*'s unconditional expected utility from consumption.

Given these characteristics of *C*'s demand behavior, one obvious restriction is:

$$\mu_1 - \gamma \geq 0 \tag{2}$$

In the absence of (2), no action that reveals q = 1 could ever be part of an equilibrium. For were $\wp$ of type 1, ceasing production would be preferable to the claimed equilibrium action. Therefore, (2) is required to make such separation feasible, at least.

The next restriction is less obvious but needed to guarantee that advertising is at least possible at t=1:

$$N(\mu_h - \gamma)\psi_1 - \alpha > N(\mu_1 - \gamma). \qquad (3)$$

Equation (3) guarantees the existence of a p_1 with the property that a $\wp$ of type 1 would prefer to advertise and set price p_1, selling only to those *C* having good experience in t=0, to revealing itself as type 1 and selling to all *C*. If (3) does not hold, then, it will be shown that there can never be sufficient return to a $\wp$ of type 1 to induce it to advertise in t=1. $\wp$ would simply prefer to reveal itself as type 1 and sell to all *C*. Thus, (3) allows for the possibility that type 1 may be mimicking type h by advertising at t=1. For future reference, observe that (3) implies $N(\mu_h-\mu_1) > \alpha$ (let $\psi_1 \to 1$ in (3)).

Definition of Equilibrium

In the next section, the equilibria of the game set out above are constructed. It is shown that the game has at most two kinds of equilibria satisfying a strengthened version of Grossman and Perry's perfect sequential equilibrium (PSE) notion. This subsection discusses the nature of the refinements employed and tries to provide some justification for their use. The reader who finds refinements of sequential equilibrium objectionable may wish to skip this subsection and merely interpret the equilibria discussed below as two of a large set of Nash (or, if it is preferred, sequential) equilibria.

A Nash equilibrium (NE) of the marketing game is a pair of functions, $F_0(p_0,a_0|q)$ and $F_1(p_1,a_1|p_0,a_0,q)$, and a purchase rule by *C* such that:
i) given $F_0(\cdot)$ and $F_1(\cdot)$, no *C* can earn a higher expected utility by altering

the purchase rule (i.e. *C*'s strategy must be a best reply to F_0,F_1), and ii) given *C*'s purchase rule, *P* can earn no higher expected payoff by altering F_0 or F_1. As is now well known, games such as the one being analysed may have a large number of Nash equilibria, many supported by "incredible threats" by one (or both) of the players. For example, in this game, even though $\mu_1 > \gamma$, the outcome $a_t \equiv \phi$, $p_t \equiv \gamma$ can occur in a Nash equilibrium. This outcome is supported by *C* refusing to purchase at any $p_t > \gamma$ (independent of a_t).

The approach commonly adopted to deal with such outcomes is to use the sequential equilibrium (SE) concept of Kreps and Wilson (1982). Basically, it is required that each player's actions be ones that are in his interest to undertake if he is ever called upon to do so, and that respect the information structure of the game. In the proposed equilibrium above, for instance, *C* would not be permitted to adopt the action "no purchase" if he observed a p_t such that $p_t < \mu_1$ because purchasing instead is sure to yield positive expected utility irrespective of q. Therefore, the above Nash equilibrium would not be part of a sequential equilibrium.

In many circumstances, a player is required to specify an action as part of a sequential equilibrium when he is uncertain about the payoff that the action will produce. *C*, for instance, may have to make a purchase decision without knowing q. In order to be sure that the action specified is one that is in a player's interest to carry out ("sequential rationality"), sequential equilibrium dictates that a player's beliefs about his position in the game be specified, so that expected utility may be computed. In the current context, doing so requires that functions $\rho_0(p_0,a_0)$, $\rho_1(p_0,a_0,p_1,a_1,c)$ be specified, defining *C*'s beliefs about the outcome q as a function of all observables. The function $\rho_0(\cdot)$, for instance, yields the probability *C* attaches to the outcome $q = h$ prior to the purchase at t=0 and given observation of any

(p_0,a_0). $\rho_1(\cdot)$ is interpreted similarly for t=1, with c being the consumption experience in t=0; c = g or b. In a sequential equilibrium, for (p_0,a_0) and (p_1,a_1) that could be the outcome of equilibrium play, $\rho_0(\cdot)$ and $\rho_1(\cdot)$ must be constructed from the equilibrium strategies F_0, F_1 and δ using Bayesian updating. Of course, since players must specify actions that they would be willing to take but may in fact not have to take in the equilibrium, beliefs must also be specified for (p_0,a_0) and (p_1,a_1) that may not be observed outcomes of the equilibrium play. In these situations, players' beliefs are effectively unrestricted.

While Nash equilibria supported by incredible threats are ruled out by the sequential equilibrium concept, many equilibria still remain that meet the sequential equilibrium requirements. None of these can be supported by incredible threats but many are supported by what has come to be described as "incredible beliefs". To illustrate, the outcome $a_t \equiv \phi$, $p_t \equiv \mu_1$ can be the result of a sequential equilibrium in which *C* refuses to purchase at any price above μ_1 (independent of a_t). This situation is supported by *C* believing that, if $p_t = \mu_1$ and $a_t = \phi$, then q = h with probability δ at t = 0 or δ updated using c = g or b at t = 1, while for any other observed (p_t,a_t) pair, q = l with certainty. With these beliefs, it is in *C*'s interest to buy only if $p_t \leq \mu_1$ and, with this strategy for *C*, it is in *P*'s interest, regardless of type, to set $p_t = \mu_1$, $a_t = \phi$. Further, when $p_t = \mu_1$, $a_t = \phi$ is observed, beliefs are consistent with *P*'s equilibrium strategy and Bayesian updating of the prior δ.

In this example, the threat by *C* not to buy if $p_t > \mu_1$ is observed is credible because, given the belief that $p_t > \mu_1$ implies q = l, it is not in *C*'s interest to purchase when $p_t > \mu_1$. Moreover, there is nothing in *P*'s equilibrium actions ($p_t = \mu_1$ and $a_t = \phi$) to contradict these beliefs. Nevertheless, if *C* could be given some good reason to interpret the observations $p_t > \mu_1$ as not implying $q = 1$, then *C*'s insistence on interpreting such observation as implying $q = 1$ could be viewed as incredible (in contrast to the threat being incredible). Suppose, for instance, that *C* interprets $a_t = \phi$ and $p_t > \mu_1$ as containing no new information and acts accordingly. Then $\wp$ can earn a payoff higher than that which he earns in the proposed equilibrium by having both types set $a_t = \phi$, $p_t \in (\mu_1, \bar{p})$, an action that would, in fact, confirm *C*'s supposed beliefs. In this way, *C* is compelled to believe that $a_t = \phi$ and $p_t > \mu_1$ indeed contains no new information. *C*'s holding the beliefs that $p_t > \mu_1$ implies $q = 1$ are, therefore, incredible, and $p_t = \mu_1$ and $a_t = \phi$ arise in a sequential equilibrium only because of these incredible beliefs.

As sequential equilibrium reduces the set of Nash equilibria by ruling out incredible threats, so many "equilibrium refinements" have been proposed to reduce the set of sequential equilibria by ruling out incredible beliefs in a structured manner [See Grossman and Perry (1986), Cho and Kreps (1987), Banks and Sobel (1987), Farrell (1985).] The refinement adopted here is a variant of Grossman and Perry's perfect sequential equilibrium (PSE). To understand how the equilibrium concept employed here works, it is first necessary to understand what a PSE is in the context of the marketing game.

The aspect that distinguishes a PSE from an SE is embodied in the nature of a credible updating rule. To see how this concept operates, consider a strategy for $\wp$ at t=1, $F_1(p_1, a_1 | q)$, which is part of an SE, and proposed as part of a PSE. (Similar reasoning applies at t=0 as well.) Now, let $\wp$

consider an alternative strategy $G(p_1,a_1|q)$. $G(\cdot)$ is called a "signalling strategy". It may involve $G \equiv F_1$ for some types q, and may be such that the support of G intersects that of F_1 for every q. However, because the point of considering G is to restrict beliefs about off-equilibrium (p_1,a_1), $G(\cdot)$ <u>must</u> allow some (p_1,a_1) not possible under F_1; ie. off-equilibrium (p_1,a_1). Also, assume that in assessing $G(\cdot)$, $\wp$ imagines that for any (p_1,a_1) in the intersection of $G(\cdot)$ and $F_1(\cdot)$'s supports, C's beliefs are as specified by the SE beliefs constructed from F_1. That is, C uses the proposed equilibrium strategy F_1 to construct beliefs whenever possible. For (p_1,a_1) in the support of G but not F_1, C's beliefs are instead constructed from G using Bayes' rule. These beliefs may be different than those in the SE of which $F_1(\cdot)$ is a part. (Beliefs attached to all other (p_1,a_1) may be ignored for this discussion.) Suppose, also, that C's actions, given these beliefs, are sequentially rational for C.

Given this setup, assume $G(\cdot)$ to have the properties that i) any randomization for some type q involves only (p_1,a_1) that are equally good for that type; ii) for q such that the support of $G(\cdot|q)$ is a subset of the support of $F_1(\cdot|q)$, type q's payoff to following $G(\cdot|q)$ is at least as great as would be obtained by following $G(\cdot|q')$, for $q' \neq q$; iii) for some q, following $G(\cdot)$ is strictly preferred to following F_1. Then, a credible updating rule for the strategy pair F_0, F_1, must specify that beliefs adopted by C for (p_1,a_1) in the support of G but not F_1 be consistent with $G(\cdot)$ and Bayesian updating. If there is more than one $G(\cdot)$ that satisfies this requirement, then credible updating allows a choice from among the $G(\cdot)$. If there are no $G(\cdot)$ that satisfies the requirement, then credible updating implies no restriction beyond those given by the SE.

The impact of this credibility restriction comes from the fact that the beliefs implied by credible updating for those (p_1, a_1) that will not occur given F_1, may be incompatible with F_1 being part of an equilibrium. To illustrate, suppose that the ad game were a single-period game, so that $\wp$'s strategy would be a function $F(p,a|q)$ and C's information prior to observing (p,a) simply that $\Pr(q = h) = \delta$. Then, as previously, $(p,a) = (\mu_1, \phi)$ occurring with certainty is a sequential equilibrium of this game, supported by the beliefs for C that $\rho(p,a) = 0$ unless $(p,a) = (\mu_1, \phi)$, in which case $\rho = \delta$. However, it is not a PSE. To see why, note that any $(\hat{p}, \phi)$ with $\hat{p} \in (\mu_1, \bar{p}]$ earns $\wp$ higher profits for any q, should C purchase, than does (μ_1, ϕ). Further, since $\bar{p}$ is C's expected utility given only the prior information, C will purchase given $(\hat{p}, \phi)$ if $\rho(\hat{p}, \phi) = \delta$. Therefore, a signalling strategy always open to $\wp$ is to choose a $G(\cdot)$ such that both types pick $(\hat{p}, \phi)$ with probability one. Bayesian updating given this G will imply that $\rho(\hat{p}, \phi) = \delta$, C will purchase at price $\hat{p}$, and both types of $\wp$ will be better off. Thus $(p,a) = (\mu_1, \phi)$, supported by the beliefs given above, cannot be part of a PSE because ρ is not produced from δ via a credible updating rule. Indeed, it turns out that there is no signalling strategy that can result in $\rho(\hat{p}, \phi) = 0$ for all $\hat{p} \in (\mu_1, \bar{p}]$. Thus, credible updating implies the outcome (μ_1, ϕ) cannot be a part of any PSE.

The equilibrium concept used below is a further refinement of PSE. The additional restrictions arise from the fact that the ad game lasts more than one period and that the equilibrium outcomes at t=1, conditional on t=0 outcomes, are often not unique.

The first restriction is concerned with the way that *C*'s beliefs at t=1 can be conditioned on t=0 outcomes. At the beginning of t=1, *C* has observed (p_0,a_0) and obtained consumption experience c = g or b. At this point, *C* can construct a belief $\rho_c(p_0,a_0,c)$ from $\rho_0(p_0,a_0)$ and the outcome c using Bayesian updating. The restriction that is imposed on second-period beliefs $\rho_1(\cdot)$ is that $\rho_1(p_1,a_1,p_0,a_0,c)$ can be written as $\tilde{\rho}_1[p_1,a_1,\rho_c(p_0,a_0,c)]$. This restriction simply requires that any first-period outcomes that result in the same information about quality for *C* at t=1 must yield the same beliefs at t=1 when some outcome (p_1,a_1) is observed, and is equivalent to Bayesian updating when (p_1,a_1) occurs with positive probability. This restriction prevents outcomes that have identical informational consequences from being followed by drastically different equilibrium outcomes at t=1, and is usually referred to as a "Markov" restriction.

The second restriction is in the nature of a continuity restriction. It arises because of the fact that, even with the above restriction, there are generally multiple PSE strategies for φ at t=1 given any $\rho_c(p_0,a_0,c)$. The beliefs that support each are, as one might expect, quite different, as are the $F_1(\cdot)$ themselves. This latter fact provides the motivation for the particular form that the restrictions take. Indeed, the impact of the restrictions is to require that if period 0 actions (p_0,a_0) are followed by a period 1 outcome having certain characteristics, then a signalling deviation, yielding information almost identical to that provided by (p_0,a_0), both can and will be followed by a period 1 outcome having nearly the same characteristics as that following (p_0,a_0). In particular, if it is impossible for some signalling deviation yielding information similar to (p_0,a_0) to be followed by a period 1 outcome having characteristics similar to that following (p_0,a_0), then (p_0,a_0) is not part of an equilibrium.

To understand the restriction more fully, consider the issue of whether some strategy pair $\tilde{F}_0,\tilde{F}_1$ is part of an equilibrium. Signalling strategies, G_0, followed by some F_1 that could be supported as part of a PSE for t=1, are examined, as above, to check whether the updating rule in use is credible. PSE reasoning supports $\tilde{F}_0$ and $\tilde{F}_1$ as long as there is <u>one</u> credible updating rule from which beliefs are constructed. Such a rule might <u>require</u> (i.e. no other rule will support $\tilde{F}_0,\tilde{F}_1$ as a PSE) that a signalling strategy G_0, yielding a value $\rho_c(p_0,a_0,c)$ arbitrarily close to the $\rho_c(\cdot)$ implied by $\tilde{F}_0$, be followed by F_1 supported by beliefs very different from those in $\tilde{F}_1$. This outcome is possible because of the multiple PSE in t=1. The continuity restriction would make such updating rules inadmissable. In formal terms, the updating rule is a function λ, associating with (p_0,a_0) and any value of ρ_c, a system of beliefs $\rho_1(p_1,a_1,\rho_c)$. The restriction is that it be possible to support the equilibrium with an updating rule λ that is continuous in ρ_c for (at least) ρ_c in a neighborhood of the value of ρ_c implied by $\tilde{F}_0$. An equilibrium supported by an updating rule obeying this and the preceding restriction will be referred to as Markov Perfect Sequential Equilibrium with Continuous Updating, or ME for short.

The intuition behind these restrictions is quite simple. Consider an action by $\wp$ at t=0 that results in only a small change in *C*'s assessment of the distribution of quality at t=1. The restrictions require that such an action not result in a large change in *C*'s purchase decision at t=1. In particular, if there is some p_1 at which *C* was previously willing to purchase in equilibrium, there must still be some price near p_1 at which *C* is willing to purchase. Again, if the reader views these requirements as too restrictive it should be recalled that the equilibria developed below are part of a larger class of equilibria (either NE, SE, or PSE). The ones considered can be viewed as particularly interesting elements of this class.

III. EQUILIBRIUM IN THE AD GAME

In this section the ME of the ad game are constructed. The procedure is as follows: A period 0 outcome (p_0, a_0) generates some initial assessment by *C* of $Pr(q = h|p_0, a_0) = \rho(p_0, a_0)$. This value, combined with his consumption experience, c, produces a period 1 assessment for *C*, $Pr[q = h|p_0, a_0, c] = \rho_c(p_0, a_0, c)$. Since t=0 outcomes affect *C*'s decisions only through $\rho_c(\cdot)$ the first step in the construction must be to consider the outcomes that can be supported as the t=1 portion of an ME for different values of $\rho_c(p_0, a_0, c)$. Having done so, the period 0 problem can be considered [i.e. what (p_0, a_0) outcomes can occur in a ME].

1. Period 1

A useful way to begin is to determine what *C* would be willing to pay in period 1 purely on the basis of information obtained in period 0, as summarized by $\rho_c(\cdot)$. Recall that *C* obtains expected utility of μ_q, given the good is of quality q. Because period 1 is the final consumption period, given no new information C is willing to pay at most

$$\rho_c(p_0,a_0,c)\mu_h + [1 - \rho_c(p_0,a_0,c)]\mu_1.$$

When $\rho(p_0,a_0) = 1$ or 0, $\rho(p_0,a_0) = \rho_c(p_0,a_0,c)$ for all c. In this case any C is willing to pay $\mu_h(\mu_1)$ when $\rho(p_0,a_0)$ takes on the value 1 (0). Otherwise, consumption experience provides additional information and $\rho_c \in (0,1)$. For those C having experience c=g (c=b), define p_H (p_L) as the highest price they would be willing to pay without further information. Sequential rationality implies that, should C receive no additional information in t=1, he cannot claim to be unwilling to purchase at some price less than his maximum willingness to pay (i.e. p_H for c=g, p_L for c=b).

Now consider $\wp$'s strategy at t=1 should period 0 somehow provide perfect information: $\rho(p_0,a_0) = 1$ or 0. When $\rho(\cdot) = 1$, any C is willing to pay μ_h. It follows that

$$F_1(p_1,a_1|q) = \begin{cases} 1 & p_1 \geq \mu_h,\ a_1 = \phi \\ 0 & \text{otherwise;} \end{cases}$$

i.e. $(p_1,a_1) = (\mu_h,\phi)$ holds with certainty, irrespective of q. The argument is that any $p_1 > \mu_h$ yields no sale, while any $p_1 < \mu_h$ could be improved upon by a deviation by either type of $\wp$ to $p_1 = \mu_h$.[6] Any nontrivial advertising could also be improved upon in a similar fashion. Analagously, when $\rho(\cdot) = 0$, $(p_1,a_1) = (\mu_1,\phi)$ must occur, with certainty.

When period 0 actions are not fully informative about quality, --$\rho_c \in (0,1)$-- the situation becomes more complicated. Nevertheless, it is easy to show that there are three kinds of potential equilibrium outcomes, depending on the value of ρ_c (more than one kind may exist for given ρ_c):

a) "High Pooling" (HP)--$F_1(p_1,a_1|q) = \begin{cases} 1 & p_1 \geq p_H,\ a_1 = \phi,\ q=h \text{ or } l \\ 0 & \text{otherwise} \end{cases}$

That is, independently of q, $\wp$ charges the maximum willingness to pay for those *C* who had favorable consumption experience in t=0, and $\wp$ does not advertise.

b) "Low Pooling" (LP)--$F_1(p_1,a_1|q) = \begin{cases} 1 & p_1 \geq p_L,\ a_1 = \phi,\ q=h \text{ or } l \\ 0 & \text{otherwise.} \end{cases}$

When LP occurs, $\wp$ invariably offers a price equal to the maximum willingness to pay of those *C* having bad experience in t=0. $\wp$, therefore, sells to all *C*. Again, no advertising occurs.

c) "Advertising" (AD)--$F_1(p_1,a_1|q) = \begin{cases} 1 & p_1 \geq \hat{p},\ a_1 = A,\ q=h \text{ or } l \\ f & \mu_1 \leq p_1,\ a = \phi,\ q=l \\ 0 & \text{otherwise,} \end{cases}$

for some $f \in (0,1)$. That is, for type h, $\wp$ charges price $\hat{p}$ and always advertises. For type l, $\wp$ randomizes between type h's actions and revealing his type by charging $p_1 = \mu_1$ and failing to advertise, this latter pair occurring with probability f. Only those *C* who had good consumption experience, c=g, purchase given $(p_1,a_1) = (\hat{p},A)$. All *C* would purchase having observed $(p_1,a_1) = (\mu_1,\phi)$. Because $(\hat{p},A)$ is more likely to be chosen if $\wp$ is of type h than of type l, the observation of this pair provides additional information to *C*. How likely *C* believes it is that q=h, given an observation of $(\hat{p},A)$, depends on the probability with which $\wp$ of type l chooses the pair. Therefore, a restriction on f is that it be sufficiently large that those *C* with good experience at t=0 are willing to purchase at $\hat{p}$. Also, $\hat{p}$ must be chosen so that φ is willing to randomize when q=1, which requires that

$$N(\hat{p} - \gamma)\psi_1 - \alpha = N(\mu_1 - \gamma),$$

or $$\hat{p} = \gamma + \frac{\mu_1 - \gamma + \frac{\alpha}{N}}{\psi_1}.$$

Note that (3) guarantees that $\hat{p} < \mu_h$, so that some f < 1 will succeed in generating a purchase when $(p_1,a_1) = (\hat{p},A)$.

The demonstration that if there are any ME, the t=1 portion must be of the form HP, LP or AD, is not difficult. First, consider possible pure strategy equilibria; that is, equilibria in which $F_1(p_1,a_1|q)$ yields some (q-dependent) pair (p^q,a^q) in t=1 with certainty, for each q. Suppose that the equilibrium is "separating"; that is $(p^h,a^h) \neq (p^l,a^l)$, so that q may be inferred from (p_1,a_1). If beliefs are consistent with such an equilibrium outcome, it must be that either all *C* purchase or no *C* purchase, regardless of consumption experience, which means that, for this outcome to occur, φ's t=1 payoff must be independent of q. Otherwise, it would pay φ to deviate to whichever of (p^q,a^q) offered the higher payoff. Further, since no *C* would be willing to pay more than μ_l for the good, having observed (p^l,a^l), it must be that the common payoff is no larger than $N(\mu_l - \gamma)$; that is, the payoff type l earns by setting $(p_1,a_1) = (\mu_l,\phi)$.

Consider, however, a signalling strategy, G, in which $(p_1,a_1) = (p_L,\phi)$ occurs with certainty for each q. Such a strategy yields φ a payoff strictly greater than that attained in the proposed equilibrium, irrespective of q. This outcome occurs because under G no customers are lost (due to *C* being required to have beliefs, having observed (p_L,ϕ), consistent with $G(\cdot)$, and $p_L > \mu_l$ if $\rho_c \in (0,1)$). Thus, beliefs that would result in $(p^h,a^h) \neq (p^l,a^l)$ as an equilibrium outcome cannot be consistent with credible updating, which means that only outcomes with $(p^h,a^h) = (p^l,a^l)$ can be part of a ME that specifies a pure strategy in t=1.

Given the absence of separation, it follows that a pure strategy F_1 can only yield HP or LP. Any outcome involving advertising is unsupportable due to the existence of the signalling strategy in which the same price is charged but no advertising is undertaken. Any price below $p_H(p_L)$ cannot be supported because of the availability of the signalling strategy in which $p_H(p_L)$ is charged. Whether (p_H,ϕ) or (p_L,ϕ) is an equilibrium outcome depends on a variety of factors, and is explored subsequently.

Next, consider strategies specifying nontrivial randomization in period 1. Since all the points over which randomization is to occur must yield the same payoff, given q, it is immediate that randomization for a given type can occur over at most four points. Of these four points, two can be such that all *C* purchase should the point be observed and two such that only those *C* for whom c=g purchase. In addition, two of the four can have advertising; however, these two cannot both be such that all *C* (or only those with c=g) purchase.

It is also simple to show that, in any equilibrium, type h cannot randomize over any points that are not possibly chosen by type l as well. That is, the only way that this situation could possibly arise in equilibrium is if the payoff to φ is independent of q. However, since some point chosen by both must be more likely to be observed if q=l than q=h and $\wp$'s payoff can be independent of q only if all *C* purchase at this point, that payoff must be strictly less than $(p_L-\gamma)N$. Therefore, a signalling strategy specifying (p_L,ϕ) with certainty yields a higher payoff for both types, implying that the beliefs supporting the proposed equilibrium strategy are not credible.

Next, it must also be that only one type randomizes in any equilibrium. Should an equilibrum involve $\wp$ randomizing for both types then either all points result in the same purchase behavior for *C* (in which case the randomization is over the two points at which all *C* purchase or the two at which those *C* for whom c=g purchase) or some points result in all *C* purchasing and others only those *C* having c=g purchasing. In the former case, the payoff to type q is bounded above by $(p_L-\gamma)N-\alpha$ $((p_H-\gamma)\psi_q N-\alpha)$. A signalling strategy specifying (p_L,ϕ) $((p_H,\phi))$ with certainty then would yield a larger payoff to both types. In the latter case, the (p_1,a_1) resulting in all *C* purchasing yield the same t=1 payoffs to both types. Those (p_1,a_1) resulting in purchase only by *C* for whom c=q yield a higher payoff for type h than for type l (since $\psi_h > \psi_l$). Therefore, $\wp$ must be unwilling to randomize for one type in this case.

Together with the previous result, the above implies that $\wp$ can only randomize if q=l. Because any point chosen only by l reveals quality as q=l, l's best option is to choose only the point (μ_l, ϕ). Any other point must yield a t=1 payoff lower than this point and so could be improved upon by a deviation. The (p_1,a_1) chosen with positive probability by $\wp$ for both types will be $(\hat{p},A)$ with $\hat{p}$ solving

$$N(\hat{p}-\gamma)\psi_l-\alpha = \mu_l-\gamma,$$

where this equation guarantees that $\wp$ is willing to randomize between $(\hat{p},A)$ and (μ_1,ϕ) for q=1. To see this, note that the only other alternative (given a mixed strategy equilibrium exists at all) is $(p_1,a_1) = (\tilde{p},\phi)$ with $\tilde{p}$ the solution to

$$N(\tilde{p}-\gamma)\psi_1 = \mu_1-\gamma.$$

However, a signalling strategy involving exactly the behavior given for the AD outcome results in type l being no worse off and type h strictly better off, implying that randomization over $(\tilde{p},\phi)$ and (μ_1,ϕ) cannot be supported by a credible updating rule. Thus, if randomization occurs in period 1, advertising must be involved should beliefs be constructed in a credible fashion. This outcome follows from the fact that advertising is profitable for $\wp$. Updating rules supporting any other randomizations do so by constraining *C* to ignore this fact even should he be presented with evidence to support it. As such, these rules commit *C* to ignoring the value of advertising as a signal of quality when good reason exists to pay attention to advertising.

The demonstration that the HP, LP and AD outcomes may in fact be part of a ME is much more tedious, although no more complicated, than the argument that there are no other possibilities. In particular, it must be checked that no signalling strategy exists that will undermine the credibility of the updating rule supporting these outcomes. There are, of course, many candidates for such signalling strategies. Nevertheless, it is possible to verify that under some conditions each of LP, HP and AD are part of an ME for some ρ_c. These conditions are provided in the Appendix. All the restrictions serve to prevent certain kinds of signalling strategies from destroying credibility of the updating rule.

As a final note, in what follows it will be assumed that ψ_1 is not so large as to violate

$$N(\mu_h-\gamma)\psi_1 < p_L-\gamma$$

when p_L is computed with $Pr(q=h) = \delta$. This condition is sufficient to guarantee that if $\rho_0 = \delta$ should occur as a result of t=0 actions, some t=1 behavior could be an equilibrium at t=1. Should this fail some signalling deviations are infeasible because they could not be followed by any t=1 actions supported by credible updating. The set of equilibria may expand in this case.

To summarize, then, period 1 outcomes depend on the information consumers have obtained in period 0. If consumers are certain about quality, in an ME there is no advertising in period 1 and price is the willingness to pay for the relevant q:μ_q. Otherwise, there are three possible outcomes that may be part of a ME: i) HP, in which there is no advertising and, irrespective of quality, price equals the willingness to pay for consumers who had good experience; ii) LP, identical to HP except that price equals the willingness to pay of consumers who had poor experience; or iii) AD, in which φ always advertises and charges a high price when the good is of high quality and, otherwise, randomizes between this same price-ad pair and a low price-no ad pair.

2. **Period 0**

There is exactly one strategy that may comprise the t=0 portion of a ME:

$$\text{"Uninformative" (U)--}F_0(p_0,q_0|q)\begin{cases} 1 & p_0 \geq \bar{p},\ a_0 = \phi,\ q=h \text{ or } l \\ 0 & \text{otherwise} \end{cases}$$

That is, $\wp$ always sets price equal to *C*'s willingness to pay given only prior information δ, and never advertises. Following this F_0, t=1 behavior is given by F_1 as specified in either of LP or AD above. The F_1 specified by HP cannot occur as part of an ME.

The demonstration that U-LP and U-AD describe the only possible equilibrium outcomes involves three steps. The first is comparatively straightforward and establishes that, if F_0 is such that (p_0,A) does not occur with positive probability for any p_0, --$a_0 = \phi$ is required to hold-- then only one of U-LP or U-AD can describe the equilibrium outcome (should there be one). Next, and much more difficult, is to show that a_0=A may never occur. The final step is then to prove that U-LP and U-AD do in fact describe equilibria under some conditions.[7]

At several points in the proof it is necessary to examine a number of potential outcomes and rule each out individually. In the interest of brevity these repetitive steps have been omittted.

a) Step 1: <u>If</u> a_0=A <u>can not occur in a ME, nothing but</u> U-LP <u>or</u> U-AD <u>can describe the outcome</u>.

Consider the various forms F_0 might take. Suppose first that no randomization occurs. Then $\wp$ chooses some possibly type-dependent pair (p_0^q,ϕ); q=h,l. If $p_0^h \neq p_0^l$, $p_0 = p_0^l$ reveals type l and $\wp$'s payoff may not exceed $2N(\mu_l-\gamma)$. Also, if $p_0^h \neq p_0^l$ is to be part of a SE, $\rho_0(p_0^h,\phi) = 1$ must hold. Thus to deter $\wp$ from selecting $p_0 = p_0^h$ when q=l in fact holds, $\wp$'s payoff for q=h must not exceed $2N(\mu_1-\gamma)$. Then, however a signalling deviation in which $(\bar{p},\phi)$ occurs with certainty, independent of q, increases profits for both types. This is because *C* will purchase given this deviation while $\wp$'s worst possible t=1 payoff is $N(\mu_1-\gamma)$. Therefore $\wp$ earns a total payoff of at least $N[(\bar{p}-\gamma) + (\mu_1-\gamma)] > 2N(\mu_1-\gamma)$. Thus this deviation destroys the credibility of the updating rule supporting $p_0^h \neq p_0^l$ as part of an ME so that $p_0^h = p_0^l = \tilde{p}_0$, for some $\tilde{p}_0$, must hold should no randomization occur.

A common value $\tilde{p}_0 < \bar{p}$ cannot arise as an ME either since it too is dominated by a signalling deviation in which both types choose $(\bar{p},\phi)$ with certainty. In this case, given the Markov assumption, the t=1 equilibrium following $(\bar{p},\phi)$ and $(\tilde{p},\phi)$ are identical profits are strictly larger for both types under the deviation. Thus only $p_0=\bar{p}$ can hold; i.e. the uninformative outcome $(\bar{p},\phi)$.

Finally should HP describe the t=1 outcome following $(\bar{p}_1,\phi)$ in t=0, there will also be a signalling deviation that makes $\wp$ better off. In this case, $\wp$ selects $(\bar{p}-\varepsilon,\phi)$ with certainty for q=h and randomizes between $(\bar{p}-\varepsilon,\phi)$ and $(\bar{p},\phi)$ for q=l; ε small. The randomization is chosen such that HP is still the t=1 outcome (possible, by continuity) and q=l receives the same payoff at both points. In this case, the payoff for q=h from

$(\bar{p}-\varepsilon,\phi)$ is strictly larger so that credibility of the beliefs is again destroyed. Therefore U-LP and U-AD are the only remaining possibilities not involving randomization.

Now consider arbitrary $F_0(p_0,a_0|q)$ with support s^q. An argument much like that given to eliminate $p_0^h \neq p_0^l$ yields $s^h \subset s^l$. Further, a check that the necessary indifference among elements of s^q needed to sustain randomization cannot occur if $s^h = s^l$ rules out this possibility. Indeed, such checking rules out any case in which s^h contains more than a single element. Thus, in equilibrium, if $a_0 = \phi$, $\wp$ may not randomize at t=0 for q=h. Finally, should $\wp$ randomize for type l only, there is always a deviation in which $\wp$ randomizes for both types that undermines credibility of updating irrespective of which of LP, HP or AD describes the subsequent outcomes. Therefore, if $a_0 = \phi$, $\wp$ may not randomize for q=l either.

Altogether then, when $a_0 = \phi$ there are no ME in which $\wp$ randomizes nontrivially at t=0, and U-LP and U-AD describe the only possible ME.

b) Step 2: a_0 = A <u>cannot occur with positive probability in any ME</u>. Suppose that a_0 = A occurs with positive probability in some ME strategy for $\wp$. What features must such an ME possess?

The first feature of any such equilibrium is that it cannot be that $\wp$ always advertises; that is, $F_0(p_0,\phi|q) = 0$ for all q and p_0 cannot arise. To see why, consider a period 0 signalling strategy $G(p_0,\phi|q) = F_0(p_0,A|q)$ for all p_0. Here $\wp$ behaves exactly as under F_0 except $a_0 = \phi$ replaces a_0 = A. Under both F_0 and G any information revelation by $\wp$ is accomplished through variations in p_0. Thus *C*'s information upon entering period 1, $\rho_c(p_0,a_0,c)$, is the same irrespective of whether F_0 or G is used in period 1. Under the definition of ME then, it must be the case that F_0 and G are followed by the same second period outcome. Therefore, whether G is a profitable signalling strategy depends on its first period effect only. This effect clearly is to reduce φ's costs for each p_0. Thus G is a profitable deviation for both both types and any updating rule supporting φ always advertising could not be credible.

Second, φ cannot always advertise for one type and never for the other. To support this outcome, φ's payoff must be identical across types. Since type l is revealed by φ's actions, however, φ's payoff for type l (and hence h) cannot exceed $2N(\mu_l - \gamma)$. Under any updating rule, both types can improve via a signalling strategy implying $(p_0,a_0) = (\bar{p},\phi)$ with certainty.

Next, whenever φ chooses F_0 such that each type sets one price, p_0^q (which could vary with type), advertising cannot occur with positive probability for either type. The argument establishing this result is quite long due to the fact that even if there is no randomization over prices, there are still a number of signalling strategies to be considered, depending on both the type of randomization over ads hypothesized and the period 1 consequences. The key to the argument, however, is easy. Since φ neither always advertises nor always advertises for one type only, any advertising must be part of a randomized strategy. Further, the strategy must be one in which both types choose the same p_0. The proof shows that for any such strategy that could be part of a ME, there is a signalling strategy involving randomization over both price and ad that makes φ at least as well off for type l and better off for type h.

Taking these preliminary results together, if advertising in period 0 is to be part of ME, $\wp$ must randomize over both price and advertising for some type; i.e., there is no ME involving advertising at t=0 in which $\wp$ fails to act in this fashion. The form that this randomization takes must still be determined.[8]

One characteristic that is immediate is that the equilibrium strategy must have some (p_0,a_0) assigned positive probability for both types. If not, then the observed (p_0,a_0) always identifies quality. Given any beliefs consistent with this hypothesized equilibrium, $\wp$ could improve by imitating type h when q=l, in fact, holds.

Now suppose $\wp$'s randomization includes a pair (p_0^h,a_0^h) assigned positive probability for q=h but zero probability for q=l. Let (p_0^l,a_0^l) be chosen by both types, as required by remarks in the previous paragraph. To support the randomization, $\wp$'s payoff for type h must be the same irrespective of whether (p_0^h,a_0^h) or (p_0^l,a_0^l) occurs. Type l's payoff when (p_0^l,a_0^l) arises depends, among other things, on the equilibrium outcome in t=1. If (p_0^l,a_0^l) is followed by HP or AD, type l's payoff falls short of type h's given $\psi_h > \psi_l$. However, type h's payoff is available to type l via type l simply choosing (p_0^h,a_0^h); therefore, the hypothesized strategies are not equilibrium strategies. If (p_0^l,a_0^l) is followed by LP, both h and l earn the same payoff. This common payoff will be equal to $2N(\mu_l - \gamma)$ should $\wp$ choose some (p_0,a_0) for type l not chosen for type h. In this case the payoff can be improved upon for both types for any period 1 outcome by $\wp$ choosing $(\bar{p},\phi)$ with certainty. If there is no (p_0,a_0) chosen by type l but not type h, then there must be some (p_0,q_0) (possibly (p_0^l,a_0^l) itself) chosen with positive probability by both types that signals q=l. Then, $\wp$ can do better using a signalling strategy that raises all prices slightly, [i.e. (p_0^h, a_0^h) becomes $(p_0^h + \varepsilon(p_0^h), a_0^h)$], leaves advertising behavior unchanged and alters the randomization (again slightly) so that the highest price charged by both types signals q=l less strongly. By the assumed continuity of the updating rule in $\rho_c(\cdot)$, this signalling strategy results in LP again in period 1. However, since ρ_c is increased, the t=1 equilibrium price, p_L, is increased. Since this strategy also generates larger expected profits in period 0 for both types, it dominates the proposed equilibrium strategy. Therefore, the beliefs that support such a strategy as an equilibrium strategy cannot be credible.

The above result implies, then, that the (p_0, a_0) pairs possibly chosen if q=h must be a subset of those chosen for q=l in any equilibrium. Also, since randomization over both prices and advertising must occur for at least one type, the result just demonstrated implies such randomization occurs for type l at least. This leaves only four possible equilibrium configurations in t=0, distinguished one from the other by $\wp$'s strategy given type h; that is, it can either be that $F_0(p_0, a_0|h)$ involves no randomization, randomization over both price and advertising, randomization over price only or randomization over advertising only.

Take the case in which $\wp$ randomizes over both elements of (p_0, a_0) when q=h. For randomization to occur over some set of (p_0, a_0) pairs, it must be the case that the payoff associated with each such (p_0, a_0) is the same, given q. Taking into account the conditions needed to support the different t=1 outcomes, it is not difficult to show, given both types randomize over the same set, that all such (p_0, a_0) must be followed by AD at t=1. In any other case either the randomization cannot be supported, the conditions needed to support some t=1 outcome are violated, or a signalling deviation destroying credibility may be constructed. When AD occurs for t=1, $\wp$'s t=1 return is $\psi_q(\hat{p}-\gamma) - \alpha$, irrespective of (p_0,a_0). Thus the required indifference across the payoffs associated with distinct (p_0,a_0) must also involve indifference across returns during t=0 alone; i.e. the support of F_0 must be $\{(\tilde{p},\phi),\ (\tilde{p}+\alpha,A)\}$ for some $\tilde{p}$. Also, it is easy to show that F_0 must be chosen such that $\tilde{p}$ is maximized subject to i) C being willing to purchase given either $(\tilde{p},\phi)$ or $(\tilde{p}+\alpha,A)$ and ii) that F_0 assign both $(\tilde{p},\phi)$ and $(\tilde{p}+\alpha,A)$ positive probability for each type. If this is not the case, then a signalling deviation is readily available. A simple calculation reveals, however, that this programming problem has no solution: $\tilde{p}$ may always be raised slightly by changing F_0 in the direction of assigning lower probability to one of the elements of its support. Thus, $\wp$ may not randomize over both prices and advertising for q=h.

Very similar arguments establish that $\wp$ cannot randomize over either price or advertising alone for q=h. Thus, if there is a ME in which advertising occurs with positive probability at t=0, F_0 must be such that $\wp$ does not randomize for type h and randomizes over both price and advertising for type 1.

Within this final set of strategies, two possibilities occur. One is that $\wp$'s strategy for q=h involves no advertising. In this case, since advertising must then identify q=1, $\wp$'s payoff for type 1 is at most $2N(\mu_1 - \gamma) - \alpha$. This payoff can be improved upon for type 1 by choosing (μ_1,ϕ) instead, under any updating rule.

Therefore, all that is left is a period 0 strategy F_0 of the general form: $\wp$ always chooses a particular $(\tilde{p}_0, A)$ if q=h, and randomizes over both price and advertising for q=l, this randomization including $(\tilde{p}_0, A)$. It is immediate that $\wp$'s randomization for type l assigns positive probability to exactly one other pair--(μ_l, ϕ)--simply because the price can be no higher given $a_0 = \phi$. A calculation similar to the one that showed that, should $\wp$ randomize for both types AD must inevitably follow, establishes that when $\wp$ randomizes for l alone, AD must follow observation of $(\tilde{p}_0, A)$ (the price/ad pair assigned positive probability for both types by $\wp$). Since either AD or revelation of type l yields type l a return of $N(\mu_l - \gamma)$ during period 1, indifference across (μ_l, ϕ) and $(\tilde{p}_0, A)$ required to support randomization for type l must be due to t=0 returns alone: $\tilde{p}_0 = \mu_l + \alpha(< \mu_h)$. However, the credibility of the updating rule generating this outcome is undermined by a signalling deviation in which both types randomize over $(\mu_l + \varepsilon, \phi)$ and $(\mu_l + \varepsilon + \alpha, A)$, with ε small. Thus $\wp$ not randomizing for h may not arise as part of an ME either. This rules out the only remaining possibility for an ad outcome in period 0, thereby completing step 2.

c) <u>Step 3</u>: U-LP <u>and</u> U-AD <u>describe ME under some conditions</u>.

In order to prove step 3 it is necessary to state exactly what the ME corresponding to U-LP and U-AD are. Doing so was not required previously because the argument ruled out broad classes of potential equilibria sharing a common feature: U-HP for example. Here the task is to show that there exists a specific equilibrium with observable features described by U-LP or U-AD. Because U-AD is the focus of attention in what follows and the statement and analysis of U-LP is little different, only U-AD is presented in detail.

An ME comprises strategies for φ and a pair of updating rules, one generating ρ_0 from δ, and another producing ρ_1 from ρ_0 for each c=g or b and (p_1,a_1). Strategies in the U-AD ME are exactly those given in the description of U and AD, and so need not be repeated here. The updating rule yielding ρ_0 for each δ gives $\rho_0(p_0,a_0) \equiv \delta$; i.e. all (p_0,a_0) are treated as uninformative. The rule yielding ρ_1 from ρ_0 is more cumbersome to display explicitly. However, a compact and precise description of the manner in which it may be calculated is as follows. For any given ρ_0, the conditions in the Appendix determine which of LP, HP or AD is a possible t=1 outcome. If AD is a possible outcome for the given value of ρ_0, the update rule specifies as beliefs any one of the systems (there may be many) of beliefs supporting the AD outcome. If AD is not possible, beliefs supporting LP are selected. Failing that, beliefs supporting HP are used. (If no t=1 outcome is possible given ρ_0, the strategies and beliefs yielding ρ_0 are not equilibrium beliefs. Assumptions made above guarantee nonexistence does not occur for $\rho_0 = \delta$, the outcome of the updating rule under consideration here.)

To prove that the strategies and update rule just specified form a ME, (it is clear that these form a sequential equilibrium) it must be checked that no arbitrary signalling strategy undermines credibility of the update rule.

Consider an arbitrary signalling strategy with support s^q, q=h or l. First, assume $(\bar{p},\phi) \notin s^l$, so that no signalling action by type l could be interpreted as equilibrium play. It is easy to show that $s^l \subset s^h$, for otherwise type l's payoff from the signalling strategy would be less than that earned in the proposed equilibrium. Suppose $s^l \neq s^h$. For some $(\tilde{p},\tilde{a}) \in s^l$, $Pr(q=h|\tilde{p},\tilde{a}) < \delta$ must hold, implying $\tilde{p} < \bar{p}$. If AD is the t=1 outcome subsequent to $(\tilde{p},\tilde{a})$ type l earns no t=1 benefit from the deviation and obtains less revenue at t=0. If LP or HP occur, the associated p_L or p_H following $(\tilde{p},\tilde{a})$ are lower than would have occurred if Pr(q=l) = δ obtained. But to support AD as the t=1 part of the equilibrium, the AD behavior ascribed to type h had to dominate HP or LP behavior using $\rho_0 = \delta$, which dominates what type h would obtain following $(\tilde{p},\tilde{a})$; see the Appendix. Thus if $(\tilde{p},\tilde{a})$ were followed by either LP or HP, type h is made worse off by the deviation. Therefore no deviation with $s^l \neq s^h$ can succeed in destroying credibility of the update rule. A parallel argument shows that no deviation with $s^l = s^h$ can succeed unless the signalling deviation is uninformative; i.e. independent of q. But the conjectured equilibrium is the most profitable uninformative outcome since all are followed by AD and $(\bar{p},\phi)$ is the most profitable uninformative t=0 behavior. Altogether, then, no signalling deviation having $(\bar{p},\phi) \notin s^l$ can destroy credibility of the updating rule.

Suppose $(\bar{p},\phi) \in s^l$. Observe that no deviation having $s^h - s^l \neq \phi$ can destroy credibility. If $s^h - s^l \neq \phi$, either type h does not gain from the deviation or type l will not deviate as specified. Thus $s^h \subset s^l$. Suppose $s^h \neq s^l$ and $(\bar{p},\phi) \in s^h$. Then type l earns a payoff at most $2N(\mu_l - \gamma)$, inferior to following the equilibrium path. Thus either $(\bar{p},\phi) \in s^h$ or $s^h = s^l$. Indeed, by a similar argument, either $s^l - s^h = (\bar{p},\phi)$ or $s^h = s^l$. When $s^l - s^h = (\bar{p},\phi)$, a calculation shows that either no type gains from the deviation or type l will not randomize as specified. When $s^l = s^h$ and both types choose $(\bar{p},\phi)$, then exactly the same argument as above for the case $s^l = s^h$ shows that this is not a profitable signalling deviation either. This, though, means that, there is no signalling deviation that destroys credibility of the updating rule given above; that is, the strategies and update rules given above form an ME.

To conclude, then, it has been shown that there are two possible ME strategies. Both involve $\wp$ selecting $(\bar{p},\phi)$ with certainty at the outset, irrespective of quality. The ME differ in terms of subsequent behavior. In one, $\wp$ selects a lower than introductory price and does not advertise; even *C* who had poor experience repurchase. In the other, $\wp$ selects a high price with advertising if a high quality product is for sale. If quality is low, however, $\wp$ randomizes between the high price and ad behavior that would occur if quality were high and a low price with no ad.

IV. IMPLICATIONS

What can be learned from this model of marketing strategy? Implications come in three kinds.

The first involves basic features of the model's equilibrium. Most important, advertising of new products beyond what is needed to inform consumers of the good's price, features and availability in an efficient manner should not be observed: <u>Advertising of new goods never signals quality</u>. In the context of advertising as the term is commonly used, testing this proposition may prove difficult. Obviously it is not easy to determine what expenditures constitute the appropriate empirical counterpart to the theoretical entities $a=\phi$ and $a=A$. A more straightforward route would be to give advertising the broader interpretation discussed above, including support for the arts, charities, amateur sports, etc. Under that interpretation, any expenditures at all may reasonably be regarded as $a=A$. The analysis then asserts that advertising of this kind should only be undertaken to promote more established products: <u>Advertising signals quality for established products even when there is no possibility of subsequent purchase</u>.

The model also generates a simple and robust new prediction: <u>whenever advertising is observed after a good is introduced</u> (i.e. advertising occurs in period 1), <u>the price at which the good is sold must exceed that at which it was sold when introduced</u>. That quantity sold declines is also implied. The argument is simply that because consumption experience $c=g$ is "goods news", $\bar{p} < p_H$, where p_H is computed using $Pr(q=h) = \delta$. But $p_H < \hat{p}$ is implied (see Appendix) for AD to occur at t=1. Intuitively, the point of φ's advertising is to signal quality and thereby obtain a higher price. But successful signalling is not possible at t=0 and may occur at t=1. Thus the indicated pattern of intertemporal prices given $a_1=A$ follows.

Another simple prediction is that when $a_1 = \phi$, φ's profits at t=1 must be lower than they were at t=0. Given $a_1 = \phi$, p_1 is either μ_1 or p_L, both less than $\bar{p}$, and the total number of units sold is unchanged. When $a_1 = A$, profits may be greater or lower than at t=0. If q=h, this occurs simply because fewer customers pay the higher price $\hat{p}$. For q=1, profits invariably fall (from $\bar{p}-\gamma$ to $\mu_1 - \gamma$).

These simple predictions are particularly useful because they involve few variables and are cast in a manner that renders testing comparatively straightforward. That is, they refer to time series observations on a given firm, in which case many difficult-to-measure entities ($\delta,\gamma,\ldots$) may be ignored so long as they may plausibly be assumed constant over time. Moreover, the predictions are sufficiently elementary that it may reasonably be expected that they are robust to a variety of relaxations of the model's restrictions. Also, they do not depend on which of the two ME occurs.

The second type of experiment involves cross-section comparison of firms. Focusing on established products, Nelson-type relationships should be observed: <u>Advertising</u>, <u>price</u>, <u>quality</u>, and <u>profits are all positively correlated, and the correlation must be imperfect</u>. (A separating equilibrium would require a deterministic relationship among these variables.) These correlations refer to games having identical parameters α, δ, γ, ψ_h, ψ_l and V, in which case empirical testing would have to control for parameter differences explicitly. Also, note that there is no need to discuss whether U-AD or U-LP is the equilibrium so long as the selection mechanism does not depend on q. Irrespective of the manner in which the equilibrium is determined in the data, the correlations are as stated, for given parameters. If parameter differences cannot be controlled for, the unconditional correlations are identical provided it is assumed that whatever mechanism selects the equilibrium does so independently of parameter values. Finally, observe that a variety of specific values of these correlations are consistent with the model because there are many possible equilibrium randomizations at $t=1$. All of course have the same fundamental structure. Thus, for given parameters, only bounds on the magnitudes of the covariances are implied by the theory. Checking whether the data are consistent with those bounds again requires some degree of measurement of the underlying parameters.

If estimates of the underlying parameters are available, the third type of proposition, comparative statics, may also be approached. Consider for instance a change in the technology of home consumption such that the utility of $c=g$, $c=b$, or both rises. μ_q thus increases and since price is always one of $\bar{p}$, p_L, $\hat{p}$ or μ_1, price and profits invariably rise with μ_q.

More interesting is the effect of an increase in γ on the price of advertized goods, $\hat{p}$: $d\hat{p}/d\gamma = 1 - 1/\psi_1 < 0$. $\hat{p}$ must fall because $\hat{p}$ is the price at which $\wp$ of type 1 is indifferent between selling to all N consumers at price $p_1 = \mu_1$ and selling to the $\psi_1 N$ consumers who had good period 0 consumption experience. In the latter case, when γ rises, the cost increase applies to fewer consumers, in which case the price obtained must fall to achieve indifference for $\wp$; i.e. the payoffs to the $(\hat{p},A)$ and (μ_1,ϕ) outcomes in period 1 must fall equally.

Also, note that when $a_1 = A$, nothing depends on δ. Period 1 price, profit and quantity sold are entirely determined by the requirement that $\wp$ is willing to randomize when $q=1$. Thus, a strong "zero restriction" is that all else constant, p_1, profits at $t=1$ and quantity sold are independent of δ if $a_1 = A$. Recall that δ may be treated as the outcome of P's investment in R&D, in which case the zero restriction applies with respect to such investments.

In regard to ψ_h and ψ_1, again main interest attaches to the situation given $a_1 = A$. $\hat{p}$ is independent of ψ_h. Thus, while advertising and quality are positively correlated, holding $\hat{p}$ constant and requiring $a_1 = A$, some quality variation should nevertheless be observed. On the other hand, changes in ψ_1 have an ambiguous effect given $a_1 = A$.

$$\frac{\partial \hat{p}}{\partial \psi_1} = (\gamma - V - \alpha/N),$$

which may take on either sign. Since profits at t=1 are $N(\mu_1 - \gamma)$ for type 1 and $\mu_1 = 1 + (1-V)\psi_1$, $\wp$ is always made better off by an increase in ψ_1 when $q = 1$. But for $q = h$, whether $\wp$ receives a greater payoff depends only on the sign of the change in $\hat{p}$. Thus, for example, in contrast to what might be expected, it may not be to $\wp$'s advantage to have ψ_1 low when $q = h$ in fact holds.

Finally, consider an increment to the cost of advertising, α. p_1 must increase for $a_1 = A$ so as to yield type 1 the same period 1 payoff as not advertising does. For $a_1 = \phi$, $p_1 = \mu_1$ holds, so p_1 is independent of α. For $q = h$, letting $\pi_1 \equiv \psi_h(\hat{p}-\gamma)-\alpha$

$$\frac{\partial \pi_1}{\partial \alpha} = \frac{\psi_h}{\psi_1} - 1 > 0,$$

in which case producers of high quality goods are made better off by an increment to ad costs. The reason is simply that if α is increased, a price adjustment must occur for advertising to continue serving its informational role. This adjustment is required for type 1 to be willing to advertise and still be no worse off than he would be were quality simply (and truthfully) announced. However, the price adjustment is large enough to yield φ a strictly higher payoff for q = h.

V. SUMMARY AND DISCUSSION

This paper has presented a simple model in which a firm has access to information on quality that is not perfectly discernable by consumers, either by prior inspection or consumption. Consumers draw inferences from the firm's marketing strategy which may include, along with a pricing policy, "dissipative signals" such as advertising. In equilibrium, there is nothing for consumers to learn from the marketing strategy of producers of new goods: New goods are not advertised and price does not reveal any new information about quality. For more established goods, price and advertising are jointly imperfect indicators of quality, and this is so even though the model does not permit subsequent purchases. Because advertising does not occur for new goods, and may arise for established goods, advertising is fundamentally not capital in this model.

Those who are familiar with the existing literature on advertising as a signal of quality (Nelson, Kihlstrom and Riordan, Milgrom and Roberts) may well find these conclusions surprising. The hypotheses offered therein are that there is a separating equilibrium in which advertising is a perfectly informative signal of quality for new goods and advertising beyond the introductory period does not occur.

While the details of the model analysed here differ from those in the earlier work in several respects, the main difference is that, in the present model, in no case can the consumer learn absolutely everything that is relevent for his future consumption decisions from a single consumption experience; $\psi_h < 1$ in particular. In earlier work a separating equilibrium arose because it was impossible for a producer of a low quality good to imitate a producer of a high quality good successfully over time. Consumption experience always gave such an attempt away. It then followed that by making advertising of new goods costly at first and rewarding later only if consumption experience verified high quality, separation could be supported. Advertising necessarily had a capital dimension. In the present model, consumption experience, while perhaps very informative, cannot by itself determine all elements of quality perfectly; separation is thus impossible.

Similarly, in the earlier literature advertising of established goods was pointless because consumers had access to all information relevant to their consumption decision having consumed once. In the present model consumption experience is an imperfect indicator of quality and produces heterogeneity among consumers that allows a marketing strategy involving advertising, and no other, to be somewhat informative in equilibrium. The return to advertising arrives immediately and ads are not capital.

Finally, it should be pointed out that the model can easily be given other interpretations, say as a job market signalling model. A worker performs some service for a firm and in so doing suffers disutility γ. The firm values the service at either 1 or V, with only the worker knowing the probability with which these outcomes occur: ψ_q and $1 - \psi_q$ respectively. The worker suggests an initial wage p_0 and may engage in dissipative signalling (schooling, for example) at cost α. The firm then hires the worker and learns the outcome for $t = 0$. No trade yields both a payoff of 0. Subsequently the worker may suggest another wage and signal further, say by engaging in yet more training, and the firm may accept the worker's offer or not. The equilibria of this game are precisely U-AD and U-LP as before. Young workers never signal but old workers might. Further, any signalling is inherently an imperfect indicator of talent.

This model of job market signalling departs more significantly from the earlier literature (Spence (1973), for example) than did its advertising counterpart studied above. In contrast to the advertising case, which focused on the capital aspects of advertising (through repeat purchases) and did not assume differences in signalling costs across types, the job market signalling literature has instead specified that signalling is actually cheaper for the more talented, and then ignored repeat purchases. The message to be obtained from the preceding discussion is thus that these exogenous cost differences are required if job market signalling by the young is to occur in equilibrium. The possibility of a long term attachment to the firm will not, by itself, permit schooling, for example, to serve as a signal of talent.

Two comments are in order. First, requiring that ad costs vary with quality may resurrect separation and dissipative signalling. However, in the marketing model this assumption appears implausible in the extreme.

Second, under any interpretation, the use of the PSE notion renders it much more difficult to support separation even with signalling costs depending on q ($\alpha^h \neq \alpha^l$). Not only must type ℓ be deterred from imitating type h -- which, as is familiar, can be ensured if $\alpha^l - \alpha^h$ is sufficiently large, -- it must also be the case that α^h is small enough that both types do not improve by a signalling deviation yielding $(\bar{p},\phi)$ with certainty.[9] This latter restriction is a new consequence associated with the use of refinements of the sequential equilibrium concept. Thus under the job market signalling interpretation, not only must the more reliable worker face lower signalling costs, those costs must also be low in an absolute sense.

Finally, even if α^h and α^l can be found that permit dissipative signalling, it is easy to show that signalling can survive only if suitable differences in γ (say $\gamma^h < \gamma^l$), depending on quality, are ruled out. Otherwise for an appropriate choice of (p_0, ϕ) by type h, type ℓ may prefer not producing to imitating type h. Again, α^l must be such that deviation to $(\bar{p}, \phi)$ does not dominate for both.

A general message of this work, then, is that signalling about quality, based solely on reputational considerations, is impossible for new goods or inexperienced workers, etc., unless agents' beliefs are allowed to be incredible. With credible beliefs such actions can never be purely reputational and, what is effectively equivalent, may only be informative when there is some <u>exogenous</u> relationship between the current (or sure future) return to signalling and quality. Thus, producers of higher quality goods can advertise only if previous experience yields them a larger number of satisfied consumers than a producer of low quality goods adopting an identical strategy would obtain, or if signalling costs vary with quality in just the right way. Absent some exogenous reason for attaching credence to such actions, they can never be informative.

In closing, it should be observed that not only has the requirement of credible belief formation reduced the set of sequential equilibrium -- what this and other refinements were designed to accomplish -- it has also succeeded in placing a variety of fairly strong restriction on the data. These range from coarse nonparametric restrictions on the path of product prices, to more specific assertions about the effect of various parameters on equilibrium outcomes. There is thus improved cause for optimism regarding the prospects for models allowing imperfect and asymetrically distributed information to be confronted empirically.

FOOTNOTES

[1]In particular, it is possible to allow the firm to invest in costly and imperfectly controllable R&D activities as follows. By investing \$k at the outset, the firm may invent a good that is high quality with probability $\delta(k)$ ($\delta'>0$, $\delta''<0$) and costs γ per unit to produce. So long as the function $\delta(k)$ is not private information, the analysis to follow is uncharged, the firm's choice of k yielding a $\delta(k)$ that replaces the exogenous value δ used below. Also, free entry at the outset can be accommodated. In this case, the situation studied is what occurs after the firm and consumers are initially paired off.

[2]Note also that the cost of advertising, α, is independent of quality, which is a reasonable specification for the problem at hand. However, in other interpretations (below), requiring α to depend on q can be justified. Provided the difference in ad costs for different q is not too large, the results desired below are basically unchanged. More information is provided in Section V.

[3]To illustrate, why does Pepsi hire Michael Jackson instead of simply setting a pile of cash ablaze? It is obvious (ie. verifiable at low cost) that Michael Jackson has excellent alternatives and so must actually be paid a large sum to perform for Pepsi. A stock of cash may more readily be faked in which case accompanying the fire with some costly verification procedure would be required.

[4]With some additional notation, production costs can be made positively related to quality. The situation envisaged here is one in which the invention process yields products that may be produced at a cost of γ, and q is the given quality of the product that an expenditure of γ will yield.

[5]$B_1(b) \subset B_1(g)$ follows from $\psi_h > \psi_1$, and must hold for (p_1,a_1) that occur with positive probability in equilibrium. Here it is required that the inclusion also holds off the equilibrium path; that is, that beliefs are not dependent upon consumption apart from its informational consequences.

[6]Here it is assumed that, should $\rho_c = 1$ or 0, no additional price or advertising signals alter this probability. This assumption is without loss of generality given the assumed structure of strategies and the equilibrium concept.

[7]It should be noted that the marketing game may possess no ME for certain parameter values. Indeed, nonexistence is an issue in general even for PSE (see Grossman and Perry). As an example, suppose that U in period 0 yields case 1 (Appendix) in period 1 and δ is such that the additional restriction needed to support AD in period 1 is not satisfied with $\rho(p_0,a_0) = \delta$. Then HP is the only possible outcome at t=1. Since HP cannot be part of an ME, none exist for the assumed parameter values. Examples have been produced which generate nonexistence. At the same time, examples have been produced which demonstrate that both kinds (U-LP and U-AD) of equilibria exist on an open set in the parameter space.

[8]Notice that the "separation" originally conjectured by Nelson, cannot occur here.

[9]That is, assuming for example that $\rho_o = \delta$ yields LP at t=1, and that type h chooses $(\tilde{p},A)$ and type 1 (μ_1,ϕ):

$$\tilde{p} - \gamma - \alpha^1 + \mu_h - \gamma \leq 2\,(\mu_1 - \gamma)$$

must hold to deter type ℓ from choosing type h's action $(\tilde{p},A)$, and

$$\tilde{p} - \gamma - \alpha + \mu_h - \gamma \geq \bar{p} - \gamma + p_L - \gamma$$

is required to ensure that a simple deviation to $(\bar{p},\phi)$ would not pay for type h. (It certainly would for type ℓ.)

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Appendix

Sign of Restrictions[†]

Case Number	R1	R2	R3	R4	R5	Outcome*
1	+	+	+	+	+	HP,AD
2	–	–	~	~	~	LP,AD
3	–	+	–	~	~	LP,AD
4	–	+	+	–	–	LP,AD
5	+	+	+	–	+	HP,AD
6	–	+	+	–	+	LP,HP,AD
7	+	+	+	+	~	AD
8	+	–	~	~	~	AD
9	+	+	–	~	~	AD
10	+	+	+	–	–	AD

Restrictions

R1: $(\mu_h - \gamma)\Psi_l - (p_L - \gamma)$

R2: $(p_H - \gamma)\Psi_h - (p_L - \gamma)$

R3: $(p_H - \gamma)\Psi_l - (\mu_l - \gamma)$

R4: $(p_H - \gamma)\Psi_l - (p_L - \gamma)$

R5: $N(p_H - \gamma\Psi_l - [N(\mu_h - \gamma)\Psi_l) - \alpha$

[†]Restrictions listed below Table. "+" denotes ≥ 0, "–" represents ≤ 0, and "~" is $\gtrless 0$.

*All AD outcomes require

$$N(\hat{p} - \gamma)\psi_h - \alpha \geq \max\{N(p_H - \gamma)\psi_h, N(p_L - \gamma)\}$$

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