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Employment and Hours Over the Business Cycle

Cho, Jang-Ok and Thomas F. Cooley

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Jang-Ok Cho

and

Thomas F. Cooley

University of Rochester
Rochester, NY 14627

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ABSTRACT

Approximately one quarter of the adjustment in total hours of employment over the business cycle represents adjustments in hours while the remainder is explained by changes in employment. Real Business Cycle theories based on representative agent models have abstracted from these facts by characterizing agents as either continuously adjusting their hours or making only labor force participation decisions about jobs with indivisible hours. In this paper we extend the representative agent framework in a way that is more in the spirit of the modern labor supply literature; workers decide on both participation and hours. The special feature of our model is that agents are assumed to have a fixed cost associated with labor supply that may depend on indiviudal or aggregate variables, in our example the employment rate. In particular the cost of participating in the labor force is assumed to be an increasing function of the employment rate.

We present some examples that illustrate how the aggregate labor supply elasticity depends on both margins of adjustment. Finally, we calibrate and simulate a dynamic version of the model and show that it is better able to mimic features of the aggregate data.

Introduction

A cornerstone of modern empirical research on labor supply is the recognition that it is essential to model both the participation decision and the hours of work decision. For example, the econometric techniques pioneered by Heckman and applied by many (eg. Cogan(1981)) involve estimating a participation equation as a prelude to obtaining unbiased estimates of labor supply. Aggregate data suggest that it may be important to make this distinction in business cycle studies: approximately one quarter of the adjustment in total hours of employment over the business cycle represents adjustments in hours while the remainder is explained by changes in employment. Real Business Cycle theories based on representative agent models have abstracted from these facts by characterizing agents as either continuously adjusting their hours or making only labor force participation decisions about jobs with indivisible hours. As a result they have not been entirely successful at explaining the fluctuations in hours worked relative to productivity. In this paper we extend the representative agent framework in a way that is precisely in the spirit of the modern labor supply literature; workers decide on both participation and hours. In addition, there are fixed costs associated with the decision to participate in employment. With this extension we are able to mimic successfully the behavior of hours, employment and productivity in the aggregate data.

One of the major challenges to equilibrium real business cycle theories has been the claim that they assume a degree of intertemporal substitution in labor supply that seems inconsistent with the available empirical evidence. It is difficult to reconcile the large fluctuations in aggregate hours of work and the fluctuations in hours relative to productivity with existing estimates of the elasticity of labor supply. Kydland and Prescott (1982) presented a model with time-to-build technology and non-time-separable preferences that implies substantial intertemporal substitution in labor supply: when wages are temporarily high workers increased their hours. This highly elastic labor supply behavior is viewed as inconsistent with both microeconomic evidence based on panel studies (Ashenfelter (1984)), and macroeconomic evidence [see Altonji (1982), Mankiw Rotemberg and Summers (1985)]. These empirical studies reveal insufficient intertemporal substitution to explain the observed fluctuations in hours worked. Moreover, the evidence indicates that much of the fluctuation in aggregate hours of work over the business cycle takes the form of fluctuations in employment, the extensive margin, rather than changes in hours by employed workers, the intensive margin, as is assumed in the model economy studied by Kydland and Prescott.

Richard Rogerson (1984) constructed a model economy in which labor supply is indivisible, that is, individuals either work a given number of hours or not at all. In this setting, all fluctuations in aggregate hours of work are due to fluctuations in employment. Hansen (1985) extended Rogerson's model to a growth setting and then calibrated it using the methods of Kydland and Prescott. His results demonstrated that such a model was capable of explaining the high variability in total hours worked even though individuals do not substitute across time. Such a model could thus reconcile low measurements of the intertemporal substitution elasticity with observed fluctuations in aggregate hours. It has the unfortunate feature that <u>all</u> fluctuations in aggregate hours are due to fluctuations along the extensive margin. Moreover, it implies a ratio of fluctuations in aggregate hours to productivity nearly twice that found in U.S. data.

Our goal in this paper is to extend the representative agent business cycle framework in a way that permits workers to adjust their labor supply along both the intensive and extensive margins. A model with adjustment along both margins has been developed by Cho and Rogerson (1988). They achieve this feature by introducing heterogeneity in the opportunity sets of household decision makers.[1] In this paper, we assume a continuum of agents with identical preferences and opportunity sets. The special feature of our model is that agents are assumed to have a fixed cost associated with labor supply that depends on aggregate as well as individual variables, in our example the employment rate and the probability of work. In particular the cost of participating in the labor force is assumed to be an increasing function of both variables. This feature might reflect, among other things, the difficulties of replacing home production. Individuals in this model economy must decide both whether to participate in the labor force and how many hours to work. This feature combined with an employment lottery of the sort introduced by Rogerson produces an equilibrium that displays fluctuations along both the extensive and intensive margins.

In the next section of the paper we describe a static version of our economy and discuss the decision problem facing the representative worker.. Section 3 describes the equilibrium and the effect of introducing employment lotteries. In the fourth section we present some examples and illustrate how the aggregate labor supply elasticity depends on both margins of adjustment.

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[1]Another alternative would be to introduce heterogeneity in preferences. To collapse such a problem into a representative agent framework requires weighting individual utilities. Rogerson (1987) reports a very interesting case of weight determination.

These examples illustrate the dramatic differences in aggregate labor supply elasticities that arise in different model economies. They may help to understand the differences in estimates of aggregate labor supply elasticity that have appeared in the literature. Section 5 extends the model to a dynamic setting and discusses calibration and simulation. Our results show that this model is able to replicate almost exactly the variability of hours relative to productivity that is found in the U.S. data.

2. The Economy

In this section we describe a model economy with a continuum of agents (or households) uniformly distributed on the unit interval [0,1]. Each agent has identical preferences and the same opportunity set. There are three goods: labor, capital, and output. We first describe a static single period model which we later extend to a dynamic setting. Capital and labor are inputs to the production function:

$$(2.1) \qquad f(K,N) : R_{+} x R_{+} \text{------>} R_{+},$$

where K and N are the aggregate capital stock and labor input. We will use uppercase letters to denote aggregate variables and lowercase letters to denote per capita variables. The production function is continuous and strictly monotonic in K and N, and concave in K and N separately. In addition, it is assumed to be homogeneous of degree one and $f(0,0)=0$. Anticipating the dynamic version, we introduce a multiplicative productivity shock ,λ, and write the production function $\lambda f(K,N)$. For the time being we will assume λ is fixed.

Each agent is endowed with one unit of time and one unit of capital. Time is completely divisible so there is no indivisibility in labor supply. The utility function is assumed to be separable between consumption and leisure;

$$(2.2) \qquad U(c,\ell) = u(c) - v(1-\ell),$$

where c and ℓ are consumption and leisure respectively and $n=1-\ell$ is labor supplied to the market. We further assume that:

(1) u and v are twice continuously differentiable and increasing.

(2) u is strictly concave, v is strictly convex, and $v(0)=0$.

(3) $\lim_{c\to 0} u'(c) = \infty$, $\lim_{c\to\infty} u'(c) = 0$.

(4) $\lim_{n\to 0} v'(n) = 0$, $\lim_{n\to 1} v'(n) = \infty$

(5) $u'(c+y) + u''(c+y)c \geq 0$ for all $c>0$ and $y\geq 0$.

These are all standard conditions except (5) which is imposed to guarantee that the labor supply curve is not backward bending.

In addition to the above noted standard features we assume that there is a cost in terms of utility associated with entry into employment. This cost is assumed to capture features of the psychic burden of working which are different in nature from the disutility associated with additional hours of labor. The cost of labor supply function will be represented as:

$$(2.3) \qquad \psi : \Omega \text{ -------> } R_+,$$

where Ω is the space of individual and/or aggregate variables. Specifically, ψ will be assumed to depend on the overall employment rate of the economy and the agent's own employment rate where by the latter we will mean the probability of employment. Therefore, $\Omega = [0,1]x[0,1]$ and ψ is specified to be increasing and twice differentiable in both arguments. In addition, we assume that if one of the two arguments of ψ is 0, then ψ asumes the value 0.

One way to justify this specification of the fixed cost of labor force participation is to imagine that labor force participation is a substitute for home production that must somehow be replaced. Suppose the utility cost of labor supply is incurred because the agent has children to be cared for. In addition, suppose there are costs associated with other home production that must be replaced. Then, an increase in the rate of employment will make it more difficult for the agent to find someone to replace this home production. As a result, an increase in the employment rate can raise the psychic burden of agents who choose to work. Similarly, we assume that the fixed costs of working are increasing in the workers own participation; that is, a worker who works six days per week faces a higher fixed cost than one who works one day. In the appendix we discuss how a utility function of the sort used here can be derived as an equilibrium on the basis of such considerations.

With the above specification, the representative agent's utility function can be rewritten as:

$$U(c,\ell:e,E)= u(c) - v(1-\ell) -\psi(e,E)I(n>0), \tag{2.4}$$

where e denotes the probability of employment for an agent, E is the employment rate of the economy, and $I(n>0)$ is 1 if $n>0$ and 0 otherwise. Note that the employment rate E is not a variable subject to an agent's control. The agent takes it as given in deciding on participation and hours. Therefore, given an employment rate $E\epsilon[0,1]$, $\psi(1,E)$ is a fixed cost for each agent. But, the aggregate employment rate is a variable that will be determined in equilibrium.

3. Equilibrium and Optimality

A competitive equilibrium can be defined in a purely standard way. Let X be a consumption set, $X = \{ (c,n,e,k) \epsilon R^4 : c\geq 0,\ 0\leq n\leq 1,\ e=0 \text{ or } 1\ 0\leq k\leq 1 \}$.

Definition: An allocation for the economy is a list $\{c(t), n(t), k(t), e(t), K, N, E, w, r\}$, where for each $t\epsilon[0,1]$, $(c(t), n(t), e(t), k(t))\epsilon X$, and $K, N\geq 0$.

Definition: A competitive equilibrium for the economy is a list $\{c(t), n(t), k(t), e(t), K, N, w, r\}$ such that

(i) for each $t\epsilon[0,1]$, $(c(t), n(t), e(t), k(t))$ is a solution to the consumer's problem

$$\max\ [\ u(c) - v(n) - \psi(e,E)\cdot I(n>0)]$$
$$\text{s.t.}\quad c \leq wn + rk$$
$$c \geq 0,\ 0 \leq n \leq 1,\ 0 \leq k \leq 1$$
$$e=0 \text{ or } 1.$$

(ii) N, K are solution to the firm's problem

$$\max \quad [\lambda f(K,N) - rK - wN]$$
$$\text{s.t.} \qquad K \geq 0, \quad N \geq 0.$$

(iii) $K = \int_0^1 k(t)dt$, $\lambda f(K,N) = \int_0^1 c(t)dt$, $N = \int_0^1 n(t)dt$,
$E = \int_0^1 e(t)dt$.

The competitive equilibrium defined above is standard except for the feature that there is a fixed cost associated with labor supply. Rather than proving existence, we will concentrate on analyzing the consumer's problem since the only non-standard feature appears there. The key issue here is the determination of the aggregate employment rate.

The representative agent has to make two distinct decisions: whether or not to participate in the market and conditional on participating, how many hours to work. This first problem can be approached by forming an index function (see Rosen (1983)) as

$$\begin{aligned} I(w,r,E) &= U(wn+r,n:1,E) - U(r,0:0,E) \\ &= u(wn+r) - v(n) - \psi(1,E) - u(r). \end{aligned} \tag{3.1}$$

The index function is simply the difference between the utility level when the agent works [i.e., $u(wn+r)-v(n)-\psi(1,E)$] and when he does not, [i.e., $u(r)$], given wage and rental rates. Consequently, given a wage rate, a rental rate, and an employment rate, an agent will participate in the market if $I(w,r,E)>0$, but will not if $I(w,r,E)<0$. If $I(w,r,E)=0$, the agent will be indifferent between participation and non-participation. We now establish three propositions:

<u>Proposition 1</u>: Let E^* be an equilibrium employment rate. Then $0 < E^* \leq 1$.

(<u>pf</u>). Suppose $E^*=0$. Then it is not appropriate to use the index function criterion since there is no cost of labor supply, i.e., $\psi(1,0)=0$. $E^*=0$ implies that $n^*=0$. But

$$wu'(r) > 0 = v'(0). \tag{3.2}$$

from the assumptions in Section 2. Therefore, it is always welfare improving for an agent to work when nobody works. Therefore, $E^* > 0$.//

<u>Proposition 2</u>: If $E^* = 1$, $I(w,r,1) \geq 0$.

(<u>pf</u>) It is obvious from the definition of index function.//

<u>Proposition 3</u>: If $0 < E^* < 1$, $I(w,r,E^*) = 0$.

(<u>pf</u>) Let n^* be the equilibrium hours worked and suppose $I(w,r,E^*) > 0$. Then

$$u(wn^*+r) - v(n^*) - \psi(1,E^*) > u(r),$$

which means that utility when an agent works is greater than when he/she does not work. But then everybody will participate in the market since every agent is identical. Therefore, E^* cannot be an equilibrium employment rate.

On the other hand, suppose $I(w,r,E^*) < 0$. Then

$$u(wn^*+r) - v(n^*) - \psi(1,E^*) < u(r),$$

which means that every agent prefers not to work and hence the employment rate tends to go to 0. Therefore, E^* cannot be an equilibrium employment rate either.//

From the above propositions, we know that at least somebody participates in the labor force. If everybody participates, then by assumption the utility must be greater for working. But, if not all agents work in the market, the utility level received by the agents who work has to be exactly equal to that of the agents who do not. Consequently, the equilibrium employment rate can be characterized by the condition:

(3.3) $I(w,r,E)\geq 0$ if $E=1$ and $I(w,r,E)=0$ if $0<E<1$.

But, if $I(w,r,E)>0$, then the equilibrium is quite standard: we can just ignore the existence of the utility cost of labor supply since its existence affects nobody's participation decision. Once we disregard the existence of utility costs of labor supply, the environment is convex. Note that an equilibrium in that case involves only labor market adjustment along the intensive margin.

Interesting cases are found when (3.3) holds with equality. If we eliminate the nonconvexity due to the cost of labor supply and instead add the condition $I(w,r,E)=0$ to the market clearing conditions, then the alternative economy will give us the same equilibria as the original economy. We shall elaborate on this but first we set forth some comparative static results.

Proposition 4: The hours worked do not depend directly on the employment rate. Moreover, they are an increasing function of wage rate but a decreasing function of the rental rate.

(pf) From the first order conditions to the consumer's problem, we have

(3.4) $$wu'(c)=v'(n),$$

where $c=wn+r$. Therefore, hours worked depend on wage rate and rental rate, but not on the employment rate directly.

By differentiating (3.4) with respect to the wage rate and rental rate, we have

$$(3.5) \qquad \partial n/\partial w = [u'(c)+wnu''(c)]/[v''(n)-w^2u''(c)] > 0$$

$$(3.6) \qquad \partial n/\partial r = wu''(c)/v''(n) < 0,$$

since $v''(n)>0$, $u''(c)<0$, and $u'(c)+wnu''(c)>0$ by assumption.//

<u>Proposition 5</u>: If the equilibrium employment rate is less than one, employment increases with the wage rate and decreases with the rental rate.

(<u>pf</u>) By differentiating the condition (3.3) with equality and using (3.4), we have

$$(3.7) \qquad \partial E/\partial w = nu'(c)/\psi_2(1,E) > 0$$

$$(3.8) \qquad \partial E/\partial r = [u'(c)-u'(r)]/\psi_2(1,E) < 0,$$

since $u'(c)<u'(r)$ and $\psi_2(1,E)>0$ by assumption.//

If $I(w,r,E)=0$ and $0<E<1$, then two identical agents in this economy can receive different allocations due to the assumed nonconvexity. There could be different allocations which assign the same consumption bundle to each agent and improve the welfare of all agents because the preferences are convex. But for those allocations to be achieved, the employment rate would have to be a choice variable. We resolve this problem by introducing lotteries into the model.

The technique of using lotteries in an equilibrium model to resolve problems like that just posed was employed by Rogerson (1984). In his model, two identical agents receive different allocations due to the indivisibility of labor supply. He introduced an employment lottery to make convex the non-convex environment that arise from the indivisibility of labor supply (in his model work is an all or nothing decision with no marginal adjustment of hours.) He then showed that the lotteries improve welfare. The situation in our model is very similar. Accordingly, we assume that agents can sell contracts that specify a probability of working in a given period rather than being constrained to participate with probability 0 or 1. The probability of working will be chosen to maximize the expected utility of the representative agent's utility. When the employment decision is randomized in this way, the welfare of all agents improves.

Proposition 6: Suppose (c^*,n^*) is part of an equilibrium allocation for the original economy, given w and r, and suppose $0<E^*<1$. Then, randomizing the employment decision directly improves the welfare of all agents.

(pf) Since $0<E^*<1$,

$$U^* = u(c^*) - v(n^*) - \psi(1,E^*) = u(r),$$

where $c^*=wn^*+r$. Suppose each agent takes his probability of working to be $e=E^*$, then

$$\begin{aligned} U^* &= E^* [u(c^*) - v(n^*) - \psi(1,E^*)] + (1-E^*)u(r) \\ &= [E^*u(c^*)+(1-E^*)u(r)] - v(n^*)E^* - \psi(1,E^*)E^* \\ &< u(E^*c^*+(1-E^*)r) - v(n^*)E^* - \psi(1,E^*)E^* \\ &= u(wn^*E^*+r) - v(n^*)E^* - \psi(1,E^*)E^* \qquad (3.9) \\ &\le u(wn^*E^*+r) - v(n^*)E^* - \psi(E^*,E^*)E^*, \end{aligned}$$

since the function u() is strictly concave in consumption and $\psi(e,E)$ is increasing in both arguments. Consider a new allocation with randomized employment, $(wn^*E^*+r,\ n^*, E^*)$. This allocation is feasible since

$$wn^*E^* + r = \int_0^{E^*}(wn^*+r)dt + \int_{E^*}^{1} r dt.$$

But (3.9) is just the expected utility when employment is randomized and hence randomization of the employment decision improves the welfare of all of the agents without disturbing the aggregates of the original economy.//

Note that proposition 6 says nothing about the allocation after the randomization of the employment decision but says that randomization itself improves the welfare without disturbing the aggregates of the original economy. Now with the employment lotteries, the consumption set can be redefined as $X = \{ (c,n,e,k) \in R^4 : c\geq 0,\ 0\leq n\leq 1,\ 0\leq e\leq 1,\ 0\leq k\leq 1 \}$.

With the employment lotteries, we rewrite the consumer's problem as:

$$\text{(3.10)} \qquad \max \ [u(c) - v(n)e - \psi(e,E)e]$$
$$\text{s.t.} \qquad c \leq wne + r$$
$$c \geq 0, \quad 0 \leq n \leq 1, \quad 0 < e \leq 1,$$

where we used the fact that consumption does not depend on whether an agent works or not. We will assume from now on that $\psi(e,E)e$ is convex. The firm's problem is not disturbed at all by the introduction of lotteries. Now, the condition (3.3) has to be replaced by a marginal condition from (3.10). Assuming $0<e<1$, the first order conditions for (3.10) are:

$$(3.11)\qquad wu'(wne+r) = v'(n)$$

$$(3.12)\qquad nv'(n) = v(n) + \psi(e,E) + \psi_1(e,E)e.$$

Therefore, equilibrium allocations with randomized employment decisions are different from the original allocations in so far as $0<e<1$ since (3.12) is distinct from (3.3). But, the following proposition which we state without proof shows that the comparative static results are not altered:

Proposition 7: After the randomization of the employment decision, hours worked and the employment rate respond positively to the wage rate but negatively to the rental rate.

In solving the consumers problem the aggregate employment rate E is taken as given. This is a sort of externality in the model so the competitive equilibrium is not necessarily Pareto optimal. To solve the first order conditions for the hours of work and the employment rate, we impose the equilibrium condition for the employment rate, say e=E. Once we obtain aggregate labor supply using this condition, we can get the equilibrium quantities by equating demand and supply. The solution cannot be represented as a solution to a planners problem because of the externality, but we can still find the competitive equilibrium allocation. First we get the first order conditions for n and e with the aggregate employment rate E as given in the following problem:

$$(3.13)\qquad \max\ [\ u(c) - v(n)e - \psi(e,E)e]$$

$$\text{s.t.}\quad c \le \lambda f(K,N)$$

$$K = \int_0^1 k(t)dt,\quad N = \int_0^e n(t)dt$$

$$c \ge 0,\quad 0 \le e \le 1,\quad 0 \le n \le 1.$$

The first order conditions for this problem are:

$$\lambda u'(c)f_2(K,ne) = v'(n) \tag{3.14}$$

$$v'(n)n = v(n) + \psi(e,E)e + \psi_1(e,E)e, \tag{3.15}$$

where $c=\lambda f(K,ne)$. Now if we solve (3.14) and (3.15) by imposing the conditions $e=E$, and $k=K=1$ then we obtain the competitive outcome of the hours of work and the employment rate.

Finally, suppose there is some mechanism through which each agent expects the aggregate employment rate to be same as his own probability of work say because of the market for employment lotteries. Then the equilibrium can be represented as the solution to a planners problem and the condition (3.12) has to be replaced with:

$$v'(n)n = v(n) + \psi(e,e) + [\psi_1(e,e)+\psi_2(e,e)]e. \tag{3.16}$$

Comparing (3.12) and (3.16), we see that overemployment takes place when there is the externality.

4. Examples

In this section we consider several examples based on different specifications of preferences. These serve to highlight some of the implications of this model for aggregate labor supply, employment and welfare. The examples are all based on versions of the folowing specification of preferences and technology:

$$(4.1) \qquad u(c) = (1/\sigma)c^{\sigma}$$

$$(4.2) \qquad v(n) = (a/(\gamma+1))n^{\gamma+1}$$

$$(4.3) \qquad \psi(e,E) = (b/(\tau+1))[\nu e+(1-\nu)E]^{\tau}$$

where it is assumed that $0<\sigma<1$, $\gamma>0$, $\tau>0$ and $0\leq\nu\leq1$. The technology is given by:

$$(4.4) \qquad \lambda f(N) = \lambda N^{\alpha},$$

(here we will abstract from capital stock to make the problem simple.)[2]

We first consider an example with externalities ($\nu=0$) where the competitive equilibrium is not Pareto optimal. The second example is the case $\nu=1$ where the competitive equilibrium is Pareto optimal. The third example considers the cases of pure fixed cost and no nonconvexities.

Example 1: Suppose $\nu=0$. The representative agent has to solve the problem:

$$(4.5) \qquad \max \ [(1/\sigma)c^{\sigma} - (a/(\gamma+1))n^{\gamma+1} - (b/(\tau+1))E^{\tau}]$$
$$\text{s.t.} \quad c \leq wn$$
$$c \geq 0, \quad 0 \leq n \leq 1, \quad 0 \leq E \leq 1,$$

where the choice variables for the agent are consumption, c, and hours of work, n. By plugging in the consumption constraint, we have the first order condition:

$$(4.6) \qquad w(wn)^{\sigma-1} - an^{\gamma} = 0.$$

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[2]To keep our examples simple we assume that the firm is owned by an agent whose only role is to dispose of the profits associated with this decreasing returns production function.

For an employment rate to be an equilibrium rate, the index function criterion has to be satisfied.

$$(4.7) \qquad (1/\sigma)(wn)^{\sigma} - (a/(\gamma+1))n^{\gamma+1} - (b/(\tau+1))E^{\tau} = 0.$$

These two conditions determine the supply of hours and the equilibrium employment rate as functions of the wage rate.

From (4.6), we have the supply of hours of an agent who participates;

$$(4.8) \qquad n^{s} = Aw^{\sigma/(\gamma+1-\sigma)},$$

where $A=a^{1/[\sigma-(\gamma+1)]}$. The proportion of workers who will participate in the market is obtained from (4.7) using (4.8).

$$(4.9) \qquad E^{s} = [(\tau+1)d/b]^{1/\tau}w^{(\gamma+1)\sigma/(\gamma+1-\sigma)\tau},$$

where $d=(1/\sigma)a^{\sigma/[\sigma-(\gamma+1)]}-(1/(\gamma+1))a^{1+(\gamma+1)/[\sigma-(\gamma+1)]}$. Therefore, the aggregate labor supply can be written as:

$$(4.10) \qquad \begin{aligned} N^{s} &= n^{s}E^{s} \\ &= AFw^{\sigma(\tau+\gamma+1)/\tau(\gamma+1-\sigma)}, \end{aligned}$$

where $F=[(\tau+1)d/b]^{1/\tau}$.

The demand for labor can be obtained from the firm's maximization problem:

$$(4.11) \qquad \max \quad [\lambda N^{\alpha} - wN] \quad \text{s.t.} \quad N \geq 0.$$

The first order condition for (4.11) is

$$(4.12) \qquad \lambda\alpha N^{\alpha-1} - w = 0,$$

and the demand for aggregate labor is

$$(4.13) \qquad N^{d} = Gw^{1/(\alpha-1)},$$

where $G=(\lambda\alpha)^{1/(1-\alpha)}$. Equating the aggregate demand and supply of labor, we can solve for the equilibrium wage rate as:

$$(4.14) \qquad w^{*} = (G/AF)^{1/Q},$$

where $Q=\sigma(\tau+\gamma+1)/\tau(\gamma+1-\sigma) + 1/(1-\alpha)$. If the equilibrium wage rate is determined by (4.14), the equilibrium hours of work per worker is determined by (4.8) and the equilibrium employment rate is determined by (4.9). Note that in this case the equilibrium employment rate is not necessarily equal to 1.

To get a concrete idea of the implications of this model for aggregate labor supply elasticities we assume the parameter values a=7.5, b=.8, σ=.8, γ=2, τ=.8, λ=1, and α=.64. These parameters imply the following labor supply and demand functions:

$$(4.8') \qquad n^{s} = 0.40w^{0.36}$$

$$(4.9') \qquad E^{s} = 0.99w^{1.36}$$

$$(4.10') \qquad N^{s} = 0.40w^{1.72}$$

$$(4.13') \qquad N^{d} = 0.29w^{-2.78}$$

$$(4.14') \qquad w^{*} = .93$$

Thus, the elasticity of hours with respect to the wage is 0.36, that of employment is 1.36 and the aggregate is the sum of these two, 1.72. With these parameters, the equilibrium allocation is $(c^*, n^*, w^*) = (.36, .39, .93)$ and the equilibrium employment rate is $E^*=.90$. The implied utility level is zero and is the same across all agents.

If we introduce the employment lotteries in this economy, the agent has to solve the following problem:

$$(4.15) \qquad \max \; [1/\sigma\, c^{\sigma} - a/(1+\gamma)n^{1+\gamma}\, e - b/(1+\tau)\, E^{\tau} e]$$
$$\text{s.t.} \quad c \leq wne$$
$$c \geq 0, \quad 0 \leq n \leq 1, \quad 0 \leq e \leq 1.$$

The first order conditions are:

$$(4.16) \qquad w(wne)^{\sigma-1} - \sigma\, n^{\gamma} = 0$$

$$(4.17) \qquad wn(wne)^{\sigma-1} - a/(1+\gamma)n^{1+\gamma} - b/(1+\tau)E^{\tau} = 0$$

Note that (4.16) is similar to (4.6), the first order conditions for the case without lotteries, but the probability of work and the hours of work are chosen simultaneously in the present case.

If we use the equilibrium condition for the employment rate, namely $e=E$, then we can solve the first order conditions for the supply of hours and employment rate. Using the same procedures and parameter values as above we derive the hour and employment elasticities:

$$(4.18) \qquad n^s = 0.41w^{0.27}$$

$$(4.19) \qquad E^s = e^s = 0.74w^{1.02}$$

$$(4.20) \qquad N^s = n^s e^s = 0.30w^{1.29}$$

Note that the elasticities are overall lower than in the previous case due to the changes in the decision rules. The firm's problem is the same and hence the equilibrium allocation can be obtained as $(c^*, n^*, e^*, N^*, w^*)=(.30, .41, .73, .30, .99)$. We see that introducing lotteries increases the hours of work per person but decreases the overall employment rate of the economy. The implied utility level is .094, which is greater than zero. This verifies that the introduction of lotteries improves the welfare of the economy.

Example 2. If $\nu=1$ then the cost of labor supply depends only on the probability of work of the agent but not on the aggregate employment rate. With the introduction of employment lotteries the representative agent has to solve the maximization problem:

$$(4.21) \qquad \max \left[(1/\sigma)\, c^{\sigma} - (a/(\gamma+1))n^{\gamma+1}e - (b/(\tau+1))e^{\tau+1}\right]$$
$$\text{s.t.} \quad c \le wne$$
$$c \ge 0,\ 0 \le n \le 1, \quad 0 \le e \le 1,$$

The first order conditions for (4.21) are:

$$(4.22) \qquad w(wne)^{\sigma-1} - an^{\gamma} = 0$$

$$(4.23) \qquad w(wne)^{\sigma-1}n - (a/(\gamma+1))n^{\gamma+1} - be^{\tau} = 0.$$

These first order conditions are different from (4.6) and (4.7). In solving the latter, we sequentially solved for n^* from (4.6) and then solved for e^* from (4.7) using n^*. Here we have to solve for n^* and e^* simultaneously.

Plugging $w(wne)^{\sigma-1}=an^{\gamma}$ in (4.23), we have

$$(4.24) \qquad e = Hn^{(\gamma+1)/\tau},$$

where $H=[a\gamma/b(\gamma+1)]^{1/\tau}$. If we substitute (4.24) into (4.22), we get the supply of hours of work.

$$(4.25) \qquad n^* = Jw^{\sigma/R},$$

where $R=\gamma+(\tau+\gamma+1)(1-\sigma)/\tau$ and $J=(H^{\sigma-1}/a)^{1/R}$. The employment rate is determined as

$$(4.26) \qquad e^* = Lw^{\sigma(\gamma+1)/\tau R},$$

where $L=HJ^{(\gamma+1)/\tau}$. Aggregate labor supply is simply $N^S = n^*e^*$. This is similar to (4.10) but different due to the randomization of the employment decision. As emphasized in the previous section, the decision rules in the economy with an employment lottery are from the marginal conditions but with a discrete employment decision they come from one marginal condition and the index function criterion. Hence, one expects to get a different aggregate supply function.

The firm's problem is the same as in Example 1. We obtain the equilibrium wage rate by equating (4.13) and N^S obtained above. If we substitute the equilibrium employment rate into (4.25) and (4.26), we obtain the equilibrium hours of work and the employment rate. Note that the elasticity of aggregate labor supply is the sum of the elasticity of hours of work and the elasticity of employment.

Using the same parameter values as in Example 1, we have the following:

$$(4.25') \qquad n^* = 0.43w^{0.27}$$

$$(4.26') \qquad e^* = 0.43w^{1.02}$$

$$(4.27') \qquad N^S = 0.18w^{1.29}$$

$$(4.28') \qquad w^* = 1.12$$

The elasticities of the labor supply variables are the same in this example as in the previous example implying that the elasticities are not sensitive to the presence or absence of the externality. The equilibrium allocation in this example is $(c^*, n^*, e^*, N^*, w^*) = (.24, .46, .48, .21, 1.12)$, which is somewhat different. The hours worked are greater but the employment rate is smaller in this example. The utility of the representative agent is .17 which is greater than that obtained in the previous example.

Example 3: Finally for comparison, we consider an economy with a simple fixed utility cost and an economy without any nonconvexities.

The economy with fixed utility cost of labor supply has

$$\psi(e,E) = b \tag{4.29}$$

for all e and E without changing other features of the previous examples. With the employment lotteries, the agent faces the problem:

$$\max[(1/\sigma)c^{\sigma} - (a/(\gamma+1))n^{\gamma+1}e - be] \tag{4.30}$$
$$\text{s.t.} \quad c \leq wne$$
$$c \geq 0, \quad 0 \leq n \leq 1, \quad 0 \leq e \leq 1.$$

The first order conditions are:

$$w(wne)^{\sigma-1} - an^{\gamma} = 0 \tag{4.31}$$

$$w(wne)^{\sigma-1}n - [a/(\gamma+1)]n^{\gamma+1} - b = 0. \tag{4.32}$$

Using (4.31), we can rewrite (4.32) as

$$[a\gamma/(\gamma+1)]n^{\gamma+1} = b,$$

so, the hours of work do not depend on the wage rate:

(4.33) $$n^* = [b(\gamma+1)/a\gamma]^{1/(\gamma+1)} = \bar{n}(\text{fixed}).$$

This latter feature is discussed in Grilli and Rogerson (1988). With separable preferences and employment lotteries, fixed time cost and fixed utility cost imply an indivisibility as in (4.31). Using the hours the employment rate can be obtained as:

(4.34) $$e^s = (K/\bar{n}^{\sigma})^{1/(\sigma-1)}w^{\sigma/(1-\sigma)},$$

where $K=b+(a/(\gamma+1))\bar{n}^{\gamma+1}$. The aggregate labor supply is obtained as $N^s = n^*e^*$. The firm's problem is again the same as in Example 1.

With the parameter values specified in Example 1 and with b=.33, we obtain the functions:

(4.33′) $$n^s = .40(\text{fixed})$$

(4.34′) $$e^s = .90w^{4.00}$$

(4.35) $$N^s = .36w^{4.00}$$

(4.36) $$w^* = .97.$$

The elasticity of the aggregate labor supply is 4.00, <u>much larger</u> than in the previous examples. In fact, the pure fixed cost case is identical to the models with indivisible labor studied by Rogerson (1984) and Hansen (1985). In such an economy, the labor market adjusts only along the extensive margin so the elasticity of the aggregate labor supply is necessarily greater than in the previous examples. The equilibrium in this example is $(c^*, n^*, e^*, N^*, w^*) = (.31, .40, .79, .32, .97)$.

If there is no cost of labor supply, $b=0$, the representative agent has to solve the problem:

$$\text{(4.37)} \qquad \max \quad [(1/\sigma)c^{\sigma} - (a/(\gamma+1))n^{\gamma+1}]$$

$$\text{s.t.} \qquad c \leq wn$$

$$c \geq 0, \quad 0 \leq n \leq 1.$$

The first order condition is,

$$\text{(4.38)} \qquad (wn)^{\sigma-1}w - an^{\gamma} = 0,$$

and the equilibrium labor supply is

$$\text{(4.39)} \qquad N^s = n^s = (1/a)^{1/(\gamma+1-\sigma)}w^{\sigma/(\gamma+1-\sigma)}.$$

Once again the firm's problem is the same. We can solve for the equilibrium wage rate and plug it into (4.39) to obtain the hours of work.

With the parameter values specified in Example 1, we get the labor supply function:

$$\text{(4.40)} \qquad N^s = n^* = .40w^{0.36},$$

$$\text{(4.41)} \qquad w^* = .90.$$

Thus, the elasticity of aggregate labor supply is .36, which is much smaller than in the previous examples. This is entirely due to the fact that labor market adjustment takes place only along the intensive margin. The equilibrium in this example is $(c^*, n^*, N^*, w^*) = (.35, .39, .39, .90)$.

Table 1 summarizes the labor supply elasticity in several examples. The pure fixed cost economy shows the greatest elasticity, while the economy without nonconvexities shows the smallest. The model economies studied in Examples 1 and 2 show an elasticity between these extremes.

5. Dynamics and Calibration

The static model characterized in Section 3 can be extended into a dynamic setting by incorporating capital accumulation and an information structure. Suppose there is a continuum of agents uniformly distributed over the closed interval [0,1] as was assumed in Section 2. Each individual is initially endowed with one unit of time and one unit of capital, and lives forever. There is one firm with technology which can be represented with the production function:

$$(5.1) \qquad Y_t = \lambda_t f(K_t, N_t),$$

where K_t, N_t, and Y_t are aggregate capital, aggregate labor, and aggregate output in period t respectively. We will abstract from population and technological growth. λ_t is a random shock which is assumed to be a realization of the AR(1) process:

$$(5.2) \qquad \lambda_{t+1} = \eta\lambda_t + \varepsilon_{t+1},$$

where ε_t's are assumed to be independently and identically distributed with distribution function F. It is assumed that the distribution has a positive support to guarantee that output is positive. Since we abstract from growth, λ_t will have an unconditional mean of 1 by assuming the mean of the distribution F to be $1-\eta$. Individuals are assumed to observe λ_t at the beginning of the period t.

Output can be either consumed or invested, and hence the following constraint has to be satisfied in the aggregate:

$$(5.3) \qquad C_t + I_t \leq \lambda_t f(K_t,N_t),$$

where C_t and I_t are aggregate consumption and investment in period t. The law of motion for the aggregate capital stock is given by:

$$K_{t+1} = (1-\delta)K_t + I_t, \tag{5.4}$$

where δ is the rate of capital depreciation and $0 \leq \delta \leq 1$. The stock of capital is assumed to be owned by the individuals who sell capital services to the firm. Thus, the aggregate law of motion for the capital stock, (5.4), arises from individual optimizing behavior. In this model, all agents are identical and are treated equally. From now on, uppercase letters will denote aggregate variables while lowercase letters will denote per capita variables. Anticipating the equilibrium, we use these interchangeably.

The representative agent will maximize the expected value of the discounted sum of temporal utilities. That is, the agent faces the lifetime expected utility given by

$$E \sum_{t=0}^{\infty} \beta^t \{u(c_t) - v(1-\ell_t)e_t - \psi(e_t, E_t)e_t\}, \tag{5.5}$$

and maximizes (5.5) subject to the constraints (5.1)-(5.4). As we noted earlier, the nonconvexity in each individual's preference is resolved by the introduction of the employment lottery. Competitive equilibrium can be shown to exist but will be suboptimal when the cost function ψ depends on the aggregate employment rate. As we showed by example in the previous section, the elasticities of labor supply do not depend on the particular specification of the cost of participation function. For that reason we consider only the Pareto Optimal allocations in this section where our primary interest is in dynamics.[3] Accordingly, we will eliminate the aggregate employment rate from the cost function $\psi()$.

The programming problem to be solved can be stated as follows:

$$\text{(5.6)} \qquad \max E\,[\sum_{t=0}^{\infty}\beta^t\{u(c_t) - v(n_t)e_t - \psi(e_t)e_t\}]$$

$$\text{s.t.} \quad c_t + i_t \le \lambda_t f(K_t, N_t),\ N_t = e_t n_t$$

$$K_{t+1} = (1-\delta)K_t + i_t$$

$$\lambda_{t+1} = \eta\lambda_t + \varepsilon_{t+1}$$

$$c_t \ge 0, \quad 0 \le e_t \le 1$$

$$0 \le n_t \le 1, \quad K_t \ge 0.$$

We are interested in the quantitative implications of this model since we have constructed it with the express purpose of being able to better understand the observed fluctuations in employment and productivity. We here follow the methods of Kydland and Prescott (1982) and Hansen (1985) because their approach to the quantitative study of these issues provides a convenient standard of comparison. We specify our model as follows:

$$\text{(5.7)} \qquad Y_t = \lambda_t K_t^{\alpha} N_t^{1-\alpha}$$

$$\text{(5.8)} \qquad u(c_t, \ell_t, e_t) = \log(c_t) - a/(1+\gamma)(1-\ell_t)^{1+\gamma} - b/(1+\tau)e_t^{1+\tau}.$$

With this specification, we can rewrite (5.6) as:

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[3] The introduction of externalities due to the presence of the aggregate employment rate creates some additional problems for simulation. These can be overcome using the approach in Cooley and Hansen (1988).

$$\text{(5.9)} \qquad \max \; E[\sum_{t=0}^{\infty}\beta^t\{\log(c_t) - a/(1+\gamma)n_t^{1+\gamma}e_t - b/(1+\tau)e_t^{1+\tau}\}]$$

$$\text{s.t.} \quad c_t + i_t \le \lambda_t K_t^{\alpha}, N_t^{1-\alpha}, \; N_t = e_t n_t$$

$$K_{t+1} = (1-\delta)K_t + i_t$$

$$\lambda_{t+1} = \eta\lambda_t + \varepsilon_{t+1}$$

$$c_t \ge 0, \quad 0 \le e_t \le 1$$

$$0 \le n_t \le 1, \quad K_t \ge 0.$$

Unfortunately, the problem (5.9) cannot be solved analytically for decision rules. Following Kydland and Prescott (1982), we approximate the model economy with a quadratic objective and linear constraints. The details of the approximation method are described in Kydland and Prescott (1982).

The steady state of (5.9) can be solved from the following conditions:

$$\text{(5.10)} \qquad I = \delta K$$

$$\text{(5.11)} \qquad \lambda(1-\alpha)K^{\alpha}N^{-\alpha} = an^{\gamma}c$$

$$\text{(5.12)} \qquad \lambda(1-\alpha)K^{\alpha}N^{-\alpha}n = [a/(1+\gamma)n^{1+\gamma} + be^{\tau}]c$$

$$\text{(5.13)} \qquad \lambda\alpha K^{\alpha-1}N^{1-\alpha} = (\delta+\rho)$$

$$\text{(5.14)} \qquad c + I = \lambda K^{\alpha}N^{1-\alpha},$$

where the steady state of a variable is denoted by the variable's symbol without any script, $n=1-\ell$, $N=ne$, and $\rho=(1/\beta)-1$. Condition (5.10) is a standard one for a steady state. (5.11) and (5.12) equate the marginal benefits from adjustments along the intensive and extensive margins to the marginal costs of those adjustments respectively. Condition (5.13) requires the rental rate of capital to be equal to the marginal productivity of the capital stock, and (5.14) is the budget constraint.

We can solve these conditions for n, e, K, I, and c. For notational convenience, we define the following:

(5.15) $$\theta = \lambda(1-\alpha)[\lambda\alpha/(\delta+\rho)]^{\alpha/(1-\alpha)}$$

(5.16) $$\Omega = [\lambda\alpha/(\delta+\rho)]^{1/(1-\alpha)}$$

(5.17) $$\Gamma = (\delta(1-\alpha)+\rho)/\alpha.$$

Then the steady state values are given by;

(5.18) $$e = [\theta\gamma/((1+\gamma)b\Gamma\Omega]^{1/(1+\tau)}$$

(5.19) $$n = [\theta/a\Gamma\Omega e]^{1/(1+\gamma)}$$

(5.20) $$N = ne$$

(5.21) $$K = \Omega N$$

(5.22) $$I = \delta K,$$

and the value of consumption can be obtained from (5.14) using these.

We borrow most of the parameter values from Kydland and Prescott (1982), Hansen (1985), and Prescott (1986). The values used for calibration are: α=.36, β=.99, δ=.025, λ=1, and η=.95. The details of the justification for these parameter values except those for utility can be found in Prescott (1986). The utility parameters, α, τ and γ are determined to reflect three facts which can be observed in the U.S. economy. First, the model does make a distinction between people who are in or out of the labor force. Consequently, the data for the U.S. that corresponds to the employment rate in the model economy are formed by the product of the employment rate and the participation rate. For the U.S. economy the value of this product is about 65%. Second, about one-third of the time endowment is spent in labor market activity. This value may overestimate the true fraction, but it is not a bad estimate if we take into account the portion of time spent on commuting and preparing for work. Third, one of the features of the business cycle that we have stressed earlier is that 75% of the aggregate labor fluctuation is due to the fluctuation in employment and the remaining 25% is due to the fluctuation in hours of work per person. This ratio of fluctuation in hours per person relative to that in employment has been fixed at one third. We have determined the values of the utility parameters (a, b, τ, γ) to hit these three numbers. There is an additional degree of freedom to the utility parameters since there are four parameters and only three features to pin them down. We arbitrarily fixed γ=1 but the choice of this parameter affects both our business cycle features and the implied elasticity of intertemporal substitution.

Using the value function for the economy, we solve for the equilibrium decision rules as functions of the state variables, the technology shock and the capital stock. If we have the equilibrium decision rules, we can generate time series for the model economy. One hundred time series were generated and each of the time series was logged and detrended using Prescott-Hodrick filter.[4] Second moments were calculated from each of the time series and means of the one-hundred simulations were calculated. The results are reported in Table 1. The statistics for the model economy are

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[4] The choice of filter affects the resulting statistics. For example, King, Plosser and Rebello (1987) used the linear filter and the resulting statistics seems quite different from Kydland, Prescott (1982) and Hansen (1985). As large as the same filter is applied to the data and the model, this seems relatively unimportant.

computed with the standard deviation of the technology shock equal to .00825. This number, which lies in a range suggested by Prescott (1986), was chosen because it implies the mean of the standard deviation in output from the one-hundred simulations equal to the standard deviation in actual U.S. output.

6. Results

The results of the Monte Carlo experiments resemble the statistics from the actual U.S. economy with a few notable exceptions. The most important, from the standpoint of our objectives, is that the model economy shows less fluctuation than the actual economy. In other words, the standard deviations for all of variables in the model economy are less than those for the actual U.S. economy. Those discrepancies may be due to the aggregation bias (see Cho and Rogerson (1988)), measurement error (see Hansen and Sargent(1988)), the inclusion of an inventory component in the data on investment or other factors not present in the model economy. [5]

The correlations with output from the model economy are very close to those from the actual economy except that the hours, employment and productivity are correlated more highly in the model economy than in the actual economy. This result is due to the fact that the time series in the model economy were created by a single shock. In other words, we need more than one shock to the economy or we need to introduce measurement error to create time series having correlations close to those from the actual

[5] When different and perhaps more appropriate definitions of consumption, output, the capital stock and inventories are used the statistics look somewhat different and slightly improved.

economy. This stochastic singularity problem is common to real business cycle models.

In Table 1, we can see that the ratio of standard deviation of hours relative to that of employment is approximately one-third in the model economy. This ratio was fixed at that level by adjusting the utility parameters as explained in the previous section. But, note the ratio of aggregate hour variability relative to productivity variability in the model economy. This ratio is about 1.4, which is quite close to the ratio implied by the U.S. data. Kydland and Prescott (1982), Hansen (1985), Prescott (1986) and Bencivenga (1987) all focus attention on this key ratio. For the model economy studied by Kydland and Prescott this ratio turns out to be 1.17, while it is 2.70 for the indivisible labor economy studied by Hansen (1985). For the U.S. economy, the ratio is about 1.47 in physical units but 1.42 in efficiency units (see Hansen (1985)).

The implication of this discussion is that the ratio of fluctuations in hours to productivity is too low in a model economy with adjustment only along the intensive margin but too high in a model economy with adjustment along the extensive margin. It is not surprising that our model economy generates a ratio close to the actual one since it embodies both sources of labor market fluctuations.

We fixed the utility cost parameter τ so as to make the ratio of the standard deviation in hours worked to that in employment equal to one third given that we specified γ=1. In our model, this ratio of one third implies the value of 1.4 in the ratio of fluctuations in aggregate hours to productivity. If we increase the value of γ, then the value of the ratio of hours worked to employment variability decreases. But, there is nothing that guarantees the value of one third in the ratio of hours of work to employment variabilities will imply the value of 1.4 for the ratio of aggregate hour to productivity fluctuations. Therefore, the result in Table 1 has two very important quantitative implications for the equilibrium real business cycle view. First, an equilibrium business cycle model can explain the fact that aggregate hours vary more than the productivity and can produce the same ratio of fluctuations in the two variables as the actual economy. Second, in order to reproduce the ratio, we need the same ratio of fluctuations in both margins in the labor market as is observed in the economy. In this respect, equilibrium business cycle theory can be said to be consistent with observations from the actual economy.

TABLE 1

Elasticities in the Examples

Case	Hours	Employment	Aggregate
Pure Externality(ν=0)	σ/R (.27)	$\sigma(1+\gamma)/\tau R$ (1.02)	$\sigma(1+\gamma+\tau)/\tau R$ (1.29)
Pareto Optimal(ν=1)	σ/R (.27)	$\sigma(1+\gamma)/\tau R$ (1.02)	$\sigma(1+\gamma+\tau)/\tau R$ (1.29)
Middle Case(ν=.5)	σ/R (.27)	$\sigma(1+\gamma)/\tau R$ (1.02)	$\sigma(1+\gamma+\tau)/\tau R$ (1.29)
Pure Fixed Cost(τ=0)	0	$\sigma/(1-\sigma)$ (4.00)	$\sigma/(1-\sigma)$ (4.00)
Convex Environment(b=0)	$\sigma/(1+\gamma-\sigma)$ (.40)	0	$\sigma/(1+\gamma-\sigma)$ (.40)

NOTE: (1) The assumed utility function is:

$$U(c,\ell;e,E) = 1/\sigma\ c^{\sigma} - a/(1+\gamma)n^{1+\gamma} - b/(1+\tau)[\nu e+(1-\nu)E]^{\tau} I(n>0). \quad (2)$$

$R = \gamma + (\tau+\gamma+1)(1-\sigma)/\tau$.

(2) The employment lotteries are introduced in the cases of nonconvexities.

(3) The numbers in parenthesis are elasticities when σ=.8, γ=2 and τ=.8.

TABLE 2

Calibration Results

Series	U.S. STD DEV	U.S. CORR WITH OUTPUT	Model STD DEV	Model CORR WITH OUTPUT
Output	1.76	1.00	1.76(.17)	1.00(.00)
Consumption	1.29	.85	.53(.06)	.88(2.49)
Investment	8.60	.92	5.63(.57)	.98(.40)
Capital Stock	.63	.04	.47(.08)	.07(6.73)
Aggregate Hours	1.74	.77	1.06(.12)	.98(.56)
Hours	.46	.76	.25(.02)	.98(1.24)
Employment	1.50	.81	.81(.08)	.98(1.04)
Productivity	1.18	.35	.75(.08)	.96(.81)
Agg. Hrs/ Productivity				
in Physical Units	1.47		1.42	
in Efficiency Units	1.42		1.42	

Note: The data used are quarterly time series from the third quarter of 1955 to first quarter of 1984. Before the statistics were calculated, the data were logged and detrended by the Prescott-Hodrick filter described in Prescott (1986). STD = Standard Deviation, COR = Correlation With Output. Standard Deviations are in percentage terms. The statistics are means of 100 simulations. The numbers in parentheses are standard deviations of the 100 simulations in percentage term.

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Appendix

The specification of the utility function employed in the paper can be motivated by considering an explicit model of household production in the spirit of Becker (1965). Again, assume that each agent is endowed with one unit of time and one unit of capital. Time is completely divisible so there is no indivisibility in the supply of labor. We assume that each agent consume the final output from home production and that the production function is:

$$\text{(A1)} \qquad y = g(c, 1-n+n'),$$

where c represents goods purchased in the market as an input in home production (consumption in the usual sense), n is labor supplied to the market and n' is labor input purchased from other agents. We assume that $n=n'$ i.e that household work must be replaced one for one as must be true for things like child care and domestic services. As a result it must be the case that those who work in the household production sector (who are not counted as employed in this model economy) take care of the home production of more than one household. We further assume that if an agent chooses labor force participation and replaces his home production with that of others then he faces some output loss that depends on how intensely his household production is taken care of and his own probability of work. For example, if child care is important form of household production then "output" (nurturing) will be diminished as the scale of the childcare enterprise increases, which it must as employment increases. Suppose the output loss function can be expressed as

$$\text{(A2)} \qquad q[e,m/(1-E)],$$

where m is the number of household's home production that must be taken care of by a given agent e is the employment rate (or productivity of work) and E is the aggregate employment rate. For simplicity we assume m to be fixed. Combining yields the home production function;

$$\text{(A3)} \qquad y=g(c,1) - q[e,m/(1-E)]I(n>0),$$

where $I(n>0)$ is an indicator function which is 1 if $n>0$ but 0 otherwise. We assume that:

(i) $g(\ )$ is increasing, twice differentiable and strictly concave in c.

(ii) $\lim_{c\to 0} g_c(c,1) = \infty$ and $\lim_{c\to\infty} g_c(c,1) = 0$.

(iii) $g_c(c+y,1) + g_{cc}(c+y,1)c \geq 0$ for all $c \geq 0$ and $y \geq 0$.

(iv) q is continuously differentiable and increasing in both arguments.

(v) if either e or E = 0 then $q(\ )=0$.

The assumption (iii) guarantees that labor supply is not backward bending while assumption (iv) says that the function $q(\ )$ is an increasing function of the employment rate.

The utility function is assumed to be separable between the output of home production and market activity:

$$\text{(A4)} \qquad U(y,\ell) = y - v(n),$$

where y represents home production. We assume that the disutility of labor supply is characterized by:

(vi) v( ) is increasing, strictly convex and twice differentiable.

(vii) $\lim_{n \to 0} v'(n) = 0$ and $\lim_{n \to 1} v'(n) = \infty$.

Now, if we combine utility functions we have:

$$(A5) \qquad U(c,\ell) = u(c) - v(n) - \psi(e,E)\, I(n>0)$$

where $\ell = 1-n$, $u(c) = g(c,1)$ and $\psi(e) = q[e,m/(1-E)]$.

Now, if we assume if an agent works in the home production sector he incurs no disutility associated with work and if we fix the wage rate in home production at some level, say w_h, then we can characterize equilibrium in the home production sector exactly as in the text. The only additional constraint in this case is that the employment rate, E, can never be equal to 1 because then the output loss associated with participation would become infinite.

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