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ABSTRACT

Nash Solution and Uncertain Disagreement Points

We analyze bargaining problems with known feasible sets but uncertain disagreement points. We investigate the existence of solutions such that, under reasonably restricted circumstances, all agents be as well off by reaching an agreement today as they would be by waiting until the uncertainty is resolved. We use this requirement, together with a few other commonly used conditions, to characterize the Nash solution.

1. Introduction.

A typical management-labor conflict can be described in the following general terms. There are a number of feasible wage-benefits packages under discussion, over which the two parties are negotiating. Some of them are more favorable to the management, while others are more favorable to the workers. If the parties cannot agree on a contract to sign, the workers will go on strike. Strikes are costly to both sides. The problem we will address is that of predicting the compromise that will be reached, or, depending upon the interpretation given to the model, of recommending a compromise to the two parties.

This problem has been the object of much attention. What distinguishes our approach from the previous literature is that we will do away with the usual assumption that the costs of the strike to the two parties are known. Although in practice, the set of feasible wage-benefits packages is bound to be somewhat uncertain too, mature industries where cost and demand conditions are fairly stable from year to year are not uncommon. By contrast, strikes are rare events and predicting their costs is often difficult.

Adopting the abstract formulation of the bargaining problem proposed by Nash [1950],¹ we investigate the existence of solutions satisfying a list of standard properties together with a new property concerning possible uncertainty in the conflict point. Specifically, we require that, under reasonably restricted circumstances,² the two agents would be as well off by reaching a contingent agreement now as they would be by waiting until the uncertainty is lifted and solving then whatever problem has come up. In practice, we indeed often encounter situations where some flexibility ex-

¹ See Thomson [1988] for a review of the literature that arose out of Nash's paper.

² For a formal presentation of this restriction, see Section 3.

ists as to when a compromise has to be reached. Our axiom is designed to eliminate this as a possible source of conflict. It is a very mild requirement, and yet, together with a few other minimal conditions, it suffices to characterize the solution proposed by Nash.³

This study is a complement to an earlier paper (Chun and Thomson [1987]) in which we used a requirement in the same spirit but intended to give agents a strong incentive to reach agreement before the uncertainty is resolved. That requirement turned out to have unfortunate implications: when used in conjunction with other very minimal requirements, not only is it incompatible with full optimality of the compromise but also it essentially implies that the compromise be defined by means of interpersonal comparisons of utility. Instead, the requirement used here allows us to achieve full optimality as well as saves us from having to make utility comparisons.

Taken together, these two papers help identify the tradeoff between the strength of incentives that solutions give agents to reach an agreement early on, their optimality properties and the comparability assumptions made on utilities in computing the agreement.

2. Notation - Basic Axioms - Solutions.

An n -person bargaining problem, or simply a *problem*, is a pair (S, d) , where S is a subset of \mathbb{R}^n and d is a point in S , such that (1) S is convex and closed, (2) S lies below some hyperplane with a positive normal, i.e., there exist $p \in \mathbb{R}_{++}^n$ ⁴ and $c \in \mathbb{R}$, such that for all $x \in S, px \leq c$, (3) S is *comprehensive*, i.e., for all $x \in S$ and for all $y \in \mathbb{R}^n$, if $y \leq x$, then $y \in S$, and (4) there exists $x \in S$ with $x > d$.

³ Although our example above involves only two agents, our result holds for an arbitrary number of agents.

⁴ Vector inequalities: given $x, y \in \mathbb{R}^n$, $x \geq y, x \leq y, x > y$. $\mathbb{R}_{++}^n \equiv \{x \in \mathbb{R}^n | x > 0\}$.

S is the *feasible set*. Each point x of S is a *feasible alternative*. The coordinates of x are the von Neumann-Morgenstern utility levels attained by the n agents through the choice of some joint action. d is the *disagreement point*. The intended interpretation of (S, d) is as follows: the agents can achieve any point of S if they unanimously agree on it. If they do not agree on any point, they end up at d . Let Σ be the class of all n -person problems.

A *solution* is a function $F: \Sigma \rightarrow \mathfrak{R}^n$ such that for all $(S, d) \in \Sigma$, $F(S, d) \in S$. $F(S, d)$, the value taken by the solution F when applied to the problem (S, d) , is called the *solution outcome of (S, d)* .

The following notation and terminology will be used frequently. Given $(S, d) \in \Sigma$, $IR(S, d) \equiv \{x \in \mathfrak{R}^n | x \geq d\}$ is the set of *individually rational points of (S, d)* . $PO(S) \equiv \{x \in S | \text{for all } x' \in \mathfrak{R}^n, x' \geq x \text{ implies } x' \notin S\}$ is the set of *Pareto-optimal points of S* . Similarly, $WPO(S) \equiv \{x \in S | \text{for all } x' \in \mathfrak{R}^n, x' > x \text{ implies } x' \notin S\}$ is the set of *weakly Pareto-optimal points of S* . $(S, d) \in \Sigma$ is *smooth at $x \in S$* if there exists a unique hyperplane supporting S at x . Finally, given $A \subset \mathfrak{R}^n$, $comp\{A\}$ is the smallest *comprehensive* set containing A .

Next, we define the solution introduced by Nash [1950].

Definition. The *Nash solution*, N , is defined by setting, for each $(S, d) \in \Sigma$, $N(S, d)$ to be the maximizer of the product $\prod_{i=1}^n (x_i - d_i)$ in $IR(S, d)$.

We are interested in solutions satisfying the following axioms.

Pareto Optimality (P.O). For all $(S, d) \in \Sigma$, $F(S, d) \in PO(S)$.

Symmetry (SY). For all $(S, d) \in \Sigma$ and for all permutations $\pi: \{1, \dots, n\} \rightarrow \{1, \dots, n\}$, if $S = \pi(S)$ and $d = \pi(d)$,⁵ then $F_i(S, d) = F_j(S, d)$ for all $i, j = 1, \dots, n$.

⁵ $\pi(S) \equiv \{x' \in \mathfrak{R}^n | x'_i = x_{\pi(i)} \text{ for some } x \in S \text{ and for all } i = 1, \dots, n\}$ and $\pi(d) \equiv$

A *positive affine transformation* is a function $\lambda: \mathfrak{R}^n \rightarrow \mathfrak{R}^n$, given by $a \in \mathfrak{R}_{++}^n$ and $b \in \mathfrak{R}^n$, such that for all $x \in \mathfrak{R}^n$, $\lambda(x) \equiv (a_1x_1 + b_1, \dots, a_nx_n + b_n)$.

Scale Invariance (S.INV). For all $(S, d) \in \Sigma$ and for all positive affine transformations $\lambda: \mathfrak{R}^n \rightarrow \mathfrak{R}^n$, $F(\lambda(S), \lambda(d)) = \lambda(F(S, d))$.

Independence of Non-Individually Rational Alternatives (I.N.I.R). For all $(S, d) \in \Sigma$, $F(S, d) = F(\text{comp}\{IR(S, d)\}, d)$.

Pareto Continuity (P.CONT). For all sequences $\{(S^\nu, d^\nu)\} \subset \Sigma$ and for all $(S, d) \in \Sigma$, if $PO(S^\nu)$ converges to $PO(S)$ in the Hausdorff topology and $d^\nu = d$ for all ν , then $F(S^\nu, d^\nu)$ converges to $F(S, d)$.

P.O requires that the solution outcome should exhaust all gains from cooperation. *SY* says that since the only information available on the conflict situation is contained in the mathematical description of (S, d) , there is no ground for favoring one agent at the expense of others if (S, d) is a symmetric problem. *S.INV* says that the solution outcome is defined only up to positive affine transformations of the utilities. It implies that the compromise is reached without interpersonal comparisons of utility being made. *I.N.I.R*⁶ states that the non-individually rational alternatives are irrelevant to the determination of the solution outcome. It is a natural requirement since both agents can guarantee themselves their utilities of the disagreement point. Finally, *P.CONT* requires that a small change in the Pareto-optimal set cause only a small change in the solution outcome. These are very weak conditions satisfied by most well-known solutions.

⁶ $(d_{\pi(i)})_{i=1, \dots, n}$.
 Introduced by Peters [1986b].

3. The Main Axiom.

Here, we introduce and discuss our main axiom which we will designate by the technical name:

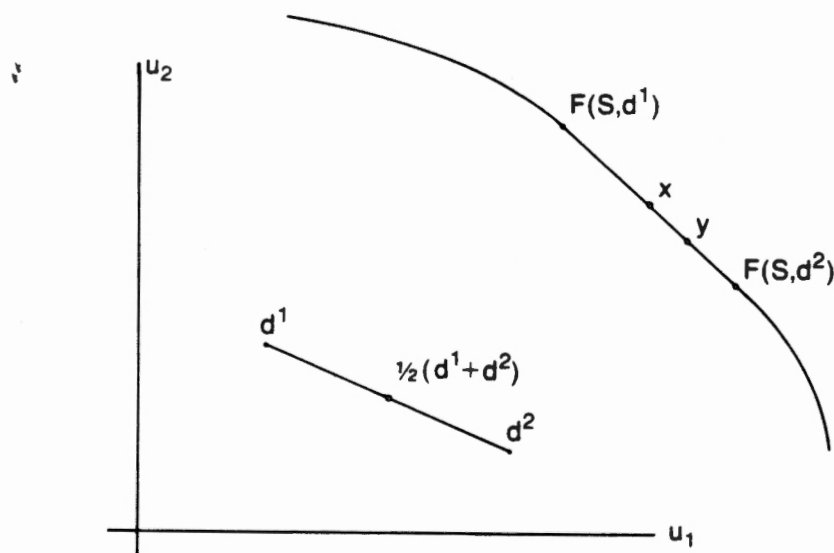
Restricted Disagreement Point Linearity (R.D.LIN). For all $(S^1, d^1), (S^2, d^2) \in \Sigma$ and for all $\alpha \in [0, 1]$, if $S^1 = S^2 \equiv S$, $\alpha F(S, d^1) + (1 - \alpha)F(S, d^2) \in PO(S)$ and S is smooth at both $F(S, d^1)$ and $F(S, d^2)$, then $F(S, \alpha d^1 + (1 - \alpha)d^2) = \alpha F(S, d^1) + (1 - \alpha)F(S, d^2)$.

(Note that $(S, \alpha d^1 + (1 - \alpha)d^2)$ is a well-defined element of Σ .)

This axiom can be motivated on the basis of considerations of timing of social choice. Consider agents *today*, who, *tomorrow*, will face one of two equally likely problems (S, d^1) and (S, d^2) , having the same feasible set, but different disagreement points. The agents have two options: either they wait until tomorrow for the uncertainty to be lifted and solve then whatever problem has come up, or they consider the problem obtained by taking as disagreement point the average of d^1 and d^2 and solve that problem today. Now let F be a solution satisfying *P.O.*, let $x \equiv \frac{1}{2}\{F(S, d^1) + F(S, d^2)\}$ and $y \equiv F(S, \frac{d^1 + d^2}{2})$ (see Figure 1). Furthermore, suppose that $x \in PO(S)$ and $x \neq y$. Then, if for some i , $x_i > y_i$, it follows that for some j , $x_j < y_j$, so that at least two agents have conflicting interests concerning when it is best to reach agreement. Imposing *R.D.LIN* prevents any such conflict of interests. (Note that if $x \notin PO(S)$, then $x \neq y$ might not cause disagreement, since $y \geq x$ would then be possible.)

The smoothness assumption, which may appear technical, has a very natural economic interpretation: it simply says that utility transfers are possible at the same rate in all directions (see Aumann [1985] and Peters [1986a]). If a solution outcome x is a Pareto-optimal point where $PO(S)$ is not smooth, then it could be a *forced* compromise: an agent who has conceded along $PO(S)$ until x might have been willing

to concede further at the same rate, but he cannot do so since the rate changes discontinuously. Suddenly, concessions become relatively more costly to him. Our axiom does not apply to situations where solution outcomes are so forced. (The situation is analogous to that which occurs in optimization theory when a corner solution is obtained.)



Motivation for the axiom of Restricted Disagreement Point Linearity.
Figure 1.

A stronger version of the axiom *R.D.LIN* was introduced by Chun and Thomson [1987] under the name of *disagreement point concavity (D.CAV)*.⁷ *D.CAV* requires, under much more general circumstances, unanimity of all agents in wanting to solve

⁷ *D.CAV*. For all $(S^1, d^1), (S^2, d^2) \in \Sigma$ and for all $\alpha \in [0, 1]$, if $S^1 = S^2 \equiv S$, then $F(S, \alpha d^1 + (1 - \alpha)d^2) \geq \alpha F(S, d^1) + (1 - \alpha)F(S, d^2)$.

the problem before the uncertainty is lifted. (Using the notation appearing in the statement of *R.D.LIN*, unanimity is required even if S is not smooth at either $F(S, d^1)$ or $F(S, d^2)$, and even if $\alpha F(S, d^1) + (1 - \alpha)F(S, d^2) \notin PO(S)$.) The reason why it is desirable that agents reach a contingent agreement today is that postponed agreement evaluated today, $\alpha F(S, d^1) + (1 - \alpha)F(S, d^2)$, will typically be Pareto-dominated, even if the solution satisfies Pareto-optimality.

Axiom *D.CAV*, in conjunction with very minimal requirements, characterizes a new family of solutions, which generalize the Egalitarian solution [Kalai, 1977]. Unfortunately, the members of this family suffer from the same two limitations as the Egalitarian solution. The first one is that they sometimes select weakly dominated outcomes, that is, they do not guarantee full optimality of the solution outcome. The second is that they are based on interpersonal comparisons of utility. Given that the axioms used together with *D.CAV* to obtain this characterization are very weak requirements, it follows that, if one wishes to recover full optimality and avoid the delicate conceptual and practical issues raised by utility comparisons, the requirement of disagreement point concavity has to be weakened. The main result of the present paper is that a theory compatible with full optimality and free of interpersonal comparisons of utility can indeed be developed without overly weakening the incentives that solutions give agents to reach an agreement today. Of all the well-known solutions satisfying full optimality that are free of interpersonal utility comparisons (the Nash solution, already mentioned, the Kalai-Smorodinsky [1975] solution, the Perles-Maschler [1981] solution), only the Nash solution satisfies our timing axiom together with the other minimal requirements listed earlier.

We close with two remarks indicating the robustness of our result. Indeed, we

are certainly not claiming that our *R.D.LIN* axiom is the only natural way to weaken *D.CAV*, and in the remarks following the theorem, we indicate two other appealing ways of doing this. However, in each case, we are led once again to a characterization of the Nash solution. Second, similar results can be obtained starting from the axiom of concavity with respect to the feasible set, dual to *D.CAV*, that Myerson [1981] had used to characterize the egalitarian and utilitarian solutions. Equally natural weakenings of the axiom once again have led to characterizations of that same solution. These results can be found in Peters [1986a] and Chun [1988].

4. The Results.

Next we present our main result.

Theorem. *A solution satisfies P.O, SY, S.INV, I.N.I.R, P.CONT and R.D.LIN if and only if it is the Nash solution.*

Proof. It is obvious that N satisfies the six axioms. The proof of the converse statement is divided into five steps.

Step 1. *S.INV* and *P.CONT* together imply the following property.

Continuity with Respect to the Disagreement Point (D.CONT). For all sequences $\{(S^\nu, d^\nu)\} \subset \Sigma$ and for all $(S, d) \in \Sigma$, if $S^\nu = S$ for all ν and $d^\nu \rightarrow d$, then $F(S^\nu, d^\nu) \rightarrow F(S, d)$.

P.O and *I.N.I.R* together imply

Individual Rationality (I.R). For all $(S, d) \in \Sigma$, $F(S, d) \in IR(S, d)$.

We omit the straightforward proofs of these results.

Step 2. Let F be a solution satisfying *P.O*, *I.R* and *R.D.LIN*. Also, let $(S, d) \in \Sigma$ be

a polygonal problem such that S is smooth at $F(S, d)$. Then for all $x \in [d, F(S, d)[$, $F(S, x) = F(S, d)$.

Proof. Let $(S, d) \in \Sigma$ be a polygonal problem satisfying the hypothesis of step 2 and $x \in]d, F(S, d)[$ be given. First, note that $(S, x) \in \Sigma$. Let $\bar{\alpha} \in]0, 1[$ be such that $x = \bar{\alpha}d + (1 - \bar{\alpha})F(S, d)$, and $\{\alpha^\nu\} \subset]0, 1[$ be such that $\alpha^\nu < \bar{\alpha}$ for all ν and $\alpha^\nu \rightarrow \bar{\alpha}$. Also, let $x^\nu \equiv \frac{x - \alpha^\nu d}{1 - \alpha^\nu}$ for all ν . Note that $x^\nu \in]d, F(S, d)[$, and therefore $(S, x^\nu) \in \Sigma$ for all ν . As $\nu \rightarrow \infty$, $x^\nu \rightarrow F(S, d)$, and by *I.R* and the fact that $F(S, d) \in PO(S)$, $F(S, x^\nu) \rightarrow F(S, d)$. Since S is a polygonal problem which is smooth at $F(S, d)$, for ν large enough, S is also smooth at $F(S, x^\nu)$ and $\alpha F(S, d) + (1 - \alpha)F(S, x^\nu) \in PO(S)$. Therefore, by *R.D.LIN*, $F(S, x) = F(S, \alpha^\nu d + (1 - \alpha^\nu)x^\nu) = \alpha^\nu F(S, d) + (1 - \alpha^\nu)F(S, x^\nu)$. Since $F(S, x^\nu) \rightarrow F(S, d)$ as $\nu \rightarrow \infty$, $F(S, x) = F(S, d)$.

Step 3. Let F be a solution satisfying *P.O*, *SY*, *S.INV* and *I.N.I.R*. Also, let $(S, d) \in \Sigma$ be a polygonal problem. Suppose that there exist $p \in \mathfrak{R}_{++}^n$ and $c \in \mathfrak{R}$ such that, for all $a \in WPO(\text{comp}\{IR(S, d)\})$, $ap = c$. Then $F(S, d) = N(S, d)$.

Proof. Let $(S, d) \in \Sigma$ be a problem satisfying the hypothesis of step 3. Let $\bar{S} \equiv \text{comp}\{IR(S, d)\}$ and $b^i(S, d) \equiv \max\{x \in S \mid x_j = d_j \text{ for all } j \neq i\}$. Then there exists a positive affine transformation λ such that $\lambda(d) = 0$ and $\lambda_i(b(S, d)) = e_i$, where e_i is the i^{th} unit vector, for all i . Since $\lambda(\bar{S}, d)$ is a symmetric problem, we have, by *P.O* and *SY*, $F(\lambda(\bar{S}, d)) = N(\lambda(\bar{S}, d))$. Now, by *S.INV*, $F(\bar{S}, d) = N(\bar{S}, d)$. By *I.N.I.R*, we conclude that $F(S, d) = F(\bar{S}, d) = N(\bar{S}, d) = N(S, d)$.

Step 4. If a solution F satisfies *P.O*, *SY*, *S.INV*, *I.N.I.R*, *P.CONT* and *R.D.LIN*, then for all polygonal problems $(S, d) \in \Sigma$, $F(S, d) = N(S, d)$.

Proof. The proof is divided into three cases.

Case (i). S is smooth at $F(S, d)$. Let $d' \in [d, F(S, d)[$ be such that (S, d') satisfies

the hypothesis of step 3. Since S is smooth at $F(S, d)$, such a d' always exists. By step 3, $F(S, d') = N(S, d')$, and by step 2, $F(S, d) = F(S, d')$. Since $\frac{F(S, d) - d}{\|F(S, d) - d\|} = \frac{N(S, d') - d'}{\|N(S, d') - d'\|} = \frac{d' - d}{\|d' - d\|}$, we conclude that $F(S, d) = N(S, d)$.

Case (ii). S is smooth at $N(S, d)$. Let $\{d^\nu\} \subset [d, N(S, d)[$ be a sequence of disagreement points such that $d^1 = d$ and $d^\nu \rightarrow N(S, d)$. Since S is smooth at $N(S, d)$, there is $\bar{\nu}$ such that for all $\nu \geq \bar{\nu}$, (S, d^ν) satisfies the hypothesis of step 3. Then, for all $\nu \geq \bar{\nu}$,
(a) $F(S, d^\nu) = N(S, d)$. On the other hand, from case (i), if $F(S, d) \neq N(S, d)$, then
(b) $F(S, d)$ should be a Pareto-optimal point of S , whose supporting hyperplane is not unique. (a) and (b) cannot be satisfied together without contradicting *D.CONT*.

Case (iii). If neither case (i) nor case (ii) occurs, we approximate the problem (S, d) by a sequence of polygonal problems, $\{(S^\nu, d^\nu)\}$, such that S^ν is smooth at $N(S^\nu, d^\nu)$, $d^\nu = d$ for all ν and $PO(S^\nu) \rightarrow PO(S)$. By *P.CONT* and the fact that N satisfies *P.CONT*, we obtain the desired conclusion.

Step 5. If a solution F satisfies the six axioms, then $F = N$.

Proof. Since an arbitrary problem $(S, d) \in \Sigma$ can be approximated in the Hausdorff topology by a sequence of polygonal problems whose Pareto-optimal set converges to $PO(S)$, we conclude by *P.CONT* that $F(S, d) = N(S, d)$ for all $(S, d) \in \Sigma$.

Q.E.D.

Remark 1. In the Theorem, *R.D.LIN* can be replaced by the following axiom introduced by Livne [1986b]:

Weak Disagreement Point Linearity (W.D.LIN). For all $(S^1, d^1), (S^2, d^2) \in \Sigma$ and for all $\alpha \in [0, 1]$, if $S^1 = S^2 \equiv S$ and $F(S, d^1) = F(S, d^2) \equiv x$, then $F(S, \alpha d^1 + (1 - \alpha)d^2) = x$.

This says that if, for a fixed feasible set, two given disagreement points result in the same compromise, then any disagreement point obtained by randomizing between them also results in that compromise.

Remark 2. In the Theorem, *R.D.LIN* can be replaced for 2-person bargaining problems by the following axiom:

*Disagreement Point Quasi-Concavity (D.Q-CAV).*⁸ For all $(S^1, d^1), (S^2, d^2) \in \Sigma$, and for all $\alpha \in [0, 1]$, if $S^1 = S^2 \equiv S$, then $F_i(S, \alpha d^1 + (1-\alpha)d^2) \geq \min\{F_i(S, d^1), F_i(S, d^2)\}$ for all i .

This says that, for a fixed feasible set, the utility to each agent of the compromise reached for any disagreement point obtained by randomizing between any two given disagreement points is at least as high as that reached from the worse one of these disagreement points.

In fact, *P.O* and *D.Q-CAV* together imply *W.D.LIN*. However, it remains an open question whether the Nash solution satisfies *D.Q-CAV* for bargaining problems with more than 2 persons.

Remark 3. We noted earlier that concavity-type conditions on the feasible set have been used in the literature. One of those axioms used for the characterization of the Nash solution is a certain condition of *Restricted Additivity (R.AD)*.⁹ In fact, *R.AD* and *S.INV* together imply *R.D.LIN*. However, intuitively, *R.D.LIN* is a significantly

⁸ A related axiom was introduced by Peters [1986b] under the name of "linearity".

⁹ *R.AD.* For all $(S^1, d^1), (S^2, d^2) \in \Sigma$, if $d^1 = d^2 \equiv d$, $F(S^1, d) + F(S^2, d) \in PO(S^1 + S^2)$ and (S^1, d) and (S^2, d) are smooth at $F(S^1, d)$ and $F(S^2, d)$, respectively, then $F(S^1 + S^2, 2d) = F(S^1, d) + F(S^2, d)$.

This axiom, introduced by Peters [1986a], is closely related to Aumann's [1985] *Conditional Additivity*.

weaker condition than $R.AD$.¹⁰

5. Concluding Comment.

Most axiomatic studies of the bargaining problem have been concerned with the responsiveness of solutions to variations in the feasible set. Recently, however, several papers have focused on the role played by the disagreement point (Chun and Thomson [1987], Livne [1986a,b], Peters [1986b] and Thomson [1987]). This change of focus has been extremely enriching, since these papers have led to new characterizations of well-known solutions as well as to the introduction and the characterizations of new families of interesting solutions.

¹⁰ The relation between $R.D.LIN$ and $R.AD$ is similar to the relation between $D.CAV$ and its counterpart concerned with uncertain feasible sets. (See Chun and Thomson [1987].)

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