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The ideas contained in this paper developed over the course of several conversations with Edward Prescott. I also received extremely helpful comments from Glenn MacDonald and an anonymous referee. In addition, comments from the conversations with Russell Cooper, Lars Hansen, Finn Kydland, Michael Rothschild, Thomas Sargent, Kenneth Singleton, and Sanjay Srivastava were most helpful. None of these individuals is responsible for any of the contents, however.

## Introduction

Following the work of Kydland and Prescott (1982) and Long and Plosser (1983), a large literature has developed that examines "real business cycle" models. Most such models involve competitive economies with stochastically disturbed technologies. Models in this class have been fairly successful in accounting for the kinds of fluctuations in aggregate output and employment that are observed in the postwar U.S. economy, for instance.

Nevertheless, most real business cycle models encounter difficulties in confronting certain kinds of business cycle phenomena. One problem common to many of these models is that they are not easily able to generate relative variability in hours and productivity on the order of magnitude of what is observed. Or, put differently, hours vary too little relative to productivity in several real business cycle contexts.[1]

A second problem encountered in most such business cycle models is that, being representative agent models, aggregate behavior of wages and hours and (average) behavior of wages and hours for individual agents coincide. This is a problem, in that it puts these models in conflict with evidence that indicates significant differences in the co–movements of wages and hours for individuals, on the one hand, and in the aggregate on the other. Some of this evidence is surveyed by Ashenfelter (1984, p. 150), who states that

> The average labor supply elasticity must apparently be quite large to square up these [aggregate] hours and wage rate movements, while the available estimates of its slope that I have surveyed are, in fact, very small. The basic empirical problem seems to be that within the life–cycle, the person–specific correlation between hours and wages is simply too small to explain the time series movements in average hours relative to the time–series movements in average wage rates. The intertemporal substitution hypothesis originally advanced by Lucas and Rapping was, of course, precisely this suspicion.

In practice, existing business cycle models tend to rely heavily on an intertemporal substitution mechanism. However, the empirical support for the importance of this mechanism is fairly weak.[2]

Third, being competitive equilibrium models, the analyses of Kydland and Prescott (1982) and Long and Plosser (1983) have no role for unemployed labor. This raises an obvious question: would the introduction of a friction giving rise to unemployment significantly alter the ability of these models to account for the aggregate behavior of hours and output? A conjectured answer to this question is given by Lucas (1987, pp. 67–8):

> I began this section with another question in mind as well: whether modeling *aggregative* employment in a competitive way as in the Kydland and Prescott model (and hence lumping unemployment together with 'leisure' and all other non–work activities) is a serious strategic error in trying to account for business cycles. I see no reason to believe that it is. If the hours people work – *choose* to work – are fluctuating it is because they are substituting into some other activity. For some purposes – designing an unemployment compensation scheme, for example – it will clearly be essential to break non–work hours into finer categories, including as one 'activity' unemployment. But such a finer breakdown need not substantially alter the problem Kydland and Prescott have tried to face of finding a parameterization of preferences over goods and hours that is consistent with observed employment movements.

This conjecture can, of course, be examined by considering models that contain a friction giving rise to the unemployment of labor, and seeing whether this friction does impact on the determination of total employment (and output).

One potential resolution to these problems has been to introduce "indivisible labor" into real business cycle constructs. More specifically, it is possible to consider close relatives of the Kydland–Prescott or Long–Plosser models, differing primarily in that agents' labor supply can take on only one of a finite set of values. This strategy has been pursued by Rogerson (1985), Hansen (1985), Greenwood and Huffman (1987), and Rogerson and Wright (1987). These authors view agents not as selling labor, but as trading lottery tickets. The outcome of these lotteries determines which value an individual's hours of work will assume, with the lotteries serving to convexify agents' decision sets. The indivisibility of labor in these settings, along with the trading of lotteries, can amplify the variability of hours relative to productivity, as in Hansen (1985). Moreover, the "losers" of these lotteries (or "winners", since often unemployed agents experience higher utility than employed agents) can be interpreted as "unemployed", in that they work fewer hours than identical employed agents.

While the introduction of indivisibilities increases the ability of real business cycle models to confront observations, such an introduction does not address the complete set of issues just discussed. First, while this introduction enhances the variability of hours, it does so in a way that prohibits these models from confronting micro data sets. In particular, since agents do not "sell labor" in these models (instead they trade lottery tickets), there is no obvious counterpart to a wage rate. This prevents even a discussion of the kind of evidence surveyed by Ashenfelter (1984), for instance. Second, there is no discussion in these contexts of whether micro evidence exists suggesting the importance of indivisibilities. Third, since the models mentioned above employ representative agent constructs, "on average" individual and aggregate behavior still look the same.

This paper adopts a different strategy for enhancing the ability of real business cycle models to confront the range of issues discussed above. It begins by taking a fairly standard real business cycle model and introducing private information about worker productivity. This approach is consistent with standard explanations for the existence of unemployed labor based on private information.[3] More specifically, the economy at each date consists of a set of heterogeneous workers, who differ in terms of their productivities (and in their preferences over consumption–leisure streams), and a set of firms. Each worker is privately informed about his own marginal product. Firms are imperfect Nash competitors in labor markets. Thus in the economy considered a fairly standard adverse selection problem arises in labor markets. This problem is "resolved" by having firms introduce "wage–employment contracts" that specify a wage rate, and a level of employment for workers governed by them. In equilibrium some workers will receive contracts specifying hours levels below their "desired levels" at the going wage rate. These workers will be (partially) unemployed (or underemployed).[4]

In addition to specifying a technology where workers can differ in terms of their productivities, the model produced below has a technology that is subject to random disturbances. These shocks to technology are the only source of aggregate uncertainty, and give rise to a "real business cycle". Employment levels, output, and unemployment rates will vary over the cycle, as will productivity and real wage rates. The model can then be used to examine the properties of cyclical movements in these series. It will be seen that introducing private information provides a significant channel for amplifying the variability of hours relative to productivity. Thus, in particular, the friction that gives rise to unemployment here also plays an important role in determining the level and time path of total employment.

In addition, the model permits a discussion of how co–movements in wages and hours for individuals are related to aggregate co–movements. In order to permit such a discussion, however, it is necessary to generate a "panel" data set for individuals, as well as a time series for the economy. This requires that the model be dynamic, and that each worker be in the work force more than once. Since this is a model with an incentive compatibility problem, anything more than a very simple construction will lead into difficult issues regarding multi–period incentive problems. In order to avoid these kinds of issues the following strategy is adopted. The economy consists of a sequence of two period lived, overlapping generations. Each worker is in the work force only twice. Thus a panel is generated for each worker, but it is possible to keep the incentive compatibility problem present here quite simple. Some issues about using an overlapping generations model to examine business cycle questions are discussed below.

Finally, the objective of the paper (as in the case of Kydland and Prescott) is to show that an economy can be parameterized, and parameter values can be chosen, so that relevant moments for artificial time series from the model roughly match the same moments for postwar U.S. time series. Thus the paper in large part consists of a pair of numerical examples (along with a "calibration" exercise). These examples demonstrate that the model just described can readily match the observed variability of output, hours, and productivity. Moreover, it can do so while generating empirically plausible values for the unemployment rate, relative wage rates, and per capita employment levels. It is also shown that the model can accommodate trends in a way that is consistent with observation. And finally, the following possibility is demonstrated. In a panel data set *every* individual can display a non–positive correlation between hours worked and wages. Nevertheless, in the aggregate, the elasticity of total hours worked with respect to the real wage can lie near the upper end of estimated values. Moreover, while these results are obtained in the context of numerical examples, it is shown below that the features of the examples used to obtain the results are fairly general.

Before proceeding, it should be stated that there are clearly other strategies that might permit these kinds of results to be obtained. One that is often discussed is the introduction of heterogeneous agents into real business cycle models who make decisions about entering into and exiting from the labor force. The choice of introducing private information into these models instead does not reflect a preference for avoiding this issue. However, at this point dynamic models with heterogeneous agents who make entry and exit decisions appear not to be operational. Also, the addition of an entry/exit decision by itself will not explain the unemployment of labor without the presence of some other friction, such as an informational asymmetry.

The format of the paper is as follows. Section I sets out the model, defines an equilibrium, and states some properties of equilibria in this context. Section II lays out a set of observations, and examines the ability of two parametric examples to confront these observations. Section III examines the generality of the important features of the example economies. Section IV comments on several features of the model specification. Most discussion of individual assumptions is left to this section. Section V concludes.

# I. THE MODEL

## A. Description

A model economy is now constructed that contains a heterogeneous work force, and in which workers are privately informed about their own productivities. In addition, the model generates an aggregate time series data set, as well as a panel data set for individuals. Essentially the simplest possible model with these features is considered.

Time is discrete and indexed by $t = 0,1,\ldots$. The population consists of a sequence of two period lived, overlapping generations. Each generation is identical in size and composition, and contains three types of agents. One of these is referred to as entrepreneurs. Entrepreneurs are endowed with ownership of a technology, described below, for converting labor into a single consumption good. The other two types of agents are called workers, who are divided into two groups. Worker types are indexed by $i \in \{1,2\}$. In order to fix ideas, it is convenient to think of there being a countable set of entrepreneurs, and a continuum of workers.[5] Within each young generation a fraction $\theta \in (0,1)$ of workers is of type 1.

The technology for converting labor into goods is a linear one that is subject to random disturbances. There are two possible states of nature at any date, indexed by $s \in \{1,2\}$.[6] If the current period state is s, one unit of type i labor employed will produce $\pi_i(s)$ units of the consumption good. The scalars $\pi_i(s)$ satisfy $\pi_1(s) > \pi_2(s) \; \forall \; s$, so that type 1 workers are the "high productivity" type.

The structure of information is as follows. At each date the current period state s is common knowledge. Let $p_{s,s'} = \text{prob}\,[s_{t+1} = s' : \; s_t = s]$, where $s_t$ is the time t state. The values $p_{s,s'}$ are common knowledge as well. At time t each worker's type is fixed, and known only to the individual worker (ex ante). In order to prevent an individual worker's type from being fully ascertainable in the first period of a worker's life, type changes are allowed. More specifically, suppose a young worker is of type i. Then the objective probability, which is also common knowledge, of being a type j worker when old is $q_{ij}$. However, at time t not even young workers know what their future type will be.

The following notation is employed for consumption and hours worked. Let $L_{1i}(s)$ denote hours worked by a young type i agent in state s, and $C_{1i}(s)$ is young period consumption by this agent. A more elaborate notation is required for old agents because of the possibility that any given agent has changed type. Thus let $L^j_{2i}(\tilde{s},s)$ be hours worked by an old type i agent who was type j when young, and who has experienced the state $\tilde{s}$ when young and s when old. $C^j_{2i}(\tilde{s},s)$ is the old period consumption of this agent.

Any any date, a type i worker has the period utility function $U(C) + \phi_i V(L)$, where $\phi_i$ is a positive scalar constant.[7] The functions U and V are defined on the non–negative reals, are twice continuously differentiable, and are strictly concave. In addition, $U'(C) > 0$ and $V'(L) < 0$ hold for all non–negative (C, L) pairs. Lifetime consumption streams are ranked by young type i agents according to the expected utility they generate. Thus a type i agent consuming $C_{1i}(s)$ and working $L_{1i}(s)$ when young, and who faces the old age (contingent) allocations $[C^i_{2j}(s,s'), L^i_{2j}(s, s')]$ if next period's type is j, receives expected utility

$$U[C_{1i}(s)] + \phi_i V[L_{1i}(s)] + \beta_i \sum_{j,s'} q_{ij} p_{s,s'} \{U[C^i_{2j}(s,s')] + \phi_j V[L^i_{2j}(s,s')]\},$$

where $\beta_i$ is the subjective discount factor of a young type i agent, and where s′ denotes "next period's state". Note that this formulation makes both next period's productivity, and a parameter of next period's utility function a random variable from the point of view of a young type i agent.[8]

Finally, workers are endowed with a single unit of labor in each period. They have no endowment of the consumption good.

B. Behavior of Workers

The behavior of firms is described below, but to foreshadow, firms in this context offer young workers of type i contracts specifying a (real) wage rate $w_{1i}(s)$ in state s, and an hours level $L_{1i}(s)$. Old type i workers (whose previous announced type was j) are offered a contract specifying a pair $[w^j_{2i}(s), L^j_{2i}(\tilde{s},s)]$, where $\tilde{s}$ is the state in the previous period. Given the announced contract offers of firms, the behavior of workers is as follows.

Young workers choose a contract, from among the set of contracts announced by firms, and a level of young period savings (borrowing if negative). Their choice of contract and their savings level is observed publicly, and will also be known next period. Let $\Phi_i$ denote the young period saving of a worker of (announced) type i. Savings earn the gross rate of return R(s) if s is the young period state.

Old workers of type i, who were of (announced) type j when young, choose their most preferred contract from among the set of contracts offered by firms. Their most preferred contract may depend on the income that accrues from last period's savings, which is just $R(\tilde{s})\Phi_j$. If an old worker of type i takes a contract specifying $[w^j_{2i}(s), L^j_{2i}(\tilde{s},s)]$, his old age utility level is just

$$\text{(1)} \quad W^j_i(\tilde{s},s) = U[w^j_{2i}(s)L^j_{2i}(\tilde{s},s) + R(\tilde{s})\Phi_j] + \phi_i V[L^j_{2i}(\tilde{s},s)]$$

(where j is last period's announced type).

When young, a worker chooses a contract $[w_{1i}(s), L_{1i}(s)]$ and a savings level $\Phi_i$. The expected utility derived from this choice, given that the contract $[w^i_{2j}(s'), L^i_{2j}(s,s')]$ will be chosen next period (if next period's state is s′ and next period's type is j), is given by

$$\text{(2)} \quad \Psi_i(s) = U[w_{1i}(s)L_{1i}(s) - \Phi_i] + \phi_i V[L_{1i}(s)] + \beta_i \sum_{j,s'} q_{ij} p_{s,s'} W^i_j(s, s').$$

It will be convenient to think of young workers as first choosing a contract $[w_{1i}(s), L_{1i}(s)]$ and then choosing $\Phi_i$ to maximize (2) taking R(s) as given. However, contracts and savings levels can be viewed as chosen simultaneously. If a type i (young) worker opts to take a type j contract, however, he must set his savings level equal to $\Phi_j$ (since savings is observable).

All of the actions of workers (and firms) are assumed to be taken after the current period state is known. Thus current period state contingent claims trades cannot occur. For simplicity, markets in which young workers could trade state contingent claims for delivery of the good next period are ruled out.[9]

To conclude, it is now possible to motivate a final assumption on preferences. It is henceforth assumed that

(3) $\phi_1 > \phi_2$.

The content of this assumption is as follows. For both young (by the envelope theorem) and old agents the slope of an indifference curve in (current period) income–labor space is given by

$$\left.\frac{\partial y}{\partial L}\right|_{dU_i = 0} = \frac{-\phi_i V'(L)}{U'(-)}$$

where y is (current period) income. (3) implies that, at any common income–hours pair, type 1 agents require more compensation for an incremental unit of leisure foregone than do type 2 agents (along an indifference curve). This assumption, then, has the following plausible interpretation. If $U(C) + \phi_i V(L)$ is viewed as an indirect utility function derived from a model of home production, (3) implies that type 1 workers, who are more productive in the market place, are also more productive than type 2 workers at home. (3) also implies that any equilibrium in which incentive compatibility considerations are binding will involve underemployment of labor.

C. Behavior of Firms

Firms offer contracts specifying wage rates and hours levels. When they do so, they observe last period's (announced) type for old workers. They do not directly observe the current type of any worker, however. Suppose that a given firm wishes to induce workers of type i to accept a contract. If this contract is offered to old workers of previously announced type j, the contract offer is denoted $[w^j_{2i}(s), L^j_{2i}(\tilde{s},s)]$, where $\tilde{s}$ represents "last period's state". A contract offered to young type i workers is denoted $[w_{1i}(s), L_{1i}(s)]$.

Since firms do not observe workers' current types, their contract offers must be incentive compatible. This requires that the contract offers $[w^j_{2i}(s), L^j_{2i}(\tilde{s},s)]$ satisfy

(4) $W^j_1(\tilde{s},s) \geq U[w^j_{22}(s) L^j_{22}(\tilde{s},s) + R(\tilde{s})\Phi_j] + \phi_1 V[L^j_{22}(\tilde{s},s)]$; $\tilde{s},s,j = 1,2$

(5) $W^j_2(\tilde{s},s) \geq U[w^j_{21}(s)\, L^j_{21}(\tilde{s},s) + R(\tilde{s})\Phi_j] + \phi_2 V[L^j_{21}(\tilde{s},s)];\ \tilde{s},s,j = 1,2$

(4) implies that old type 1 workers (of previously announced type j) will not opt to take type 2 contracts, while (5) is the converse. Notice that workers are not tied to firms for more than one period. Then, since old workers are mobile, incentive constraints must be satisfied on a period–by–period basis for each worker. Similarly, the contract offers $[w_{1i}(s), L_{1i}(s)]$ must satisfy

(6) $\Psi_1(s) \geq U[w_{12}(s)L_{12}(s) - \Phi_2] + \phi_1 V[L_{12}(s)] + \beta_1\, E\, W^2_i(s,s') \;\forall\, s$ .

(7) $\Psi_2(s) \geq U[w_{11}(s)L_{11}(s) - \Phi_1] + \phi_2 V[L_{11}(s)] + \beta_2\, E\, W^1_i(s, s') \;\forall\, s,$

where the expectations in (6) and (7) are taken over both future possible types and future states of technology. Notice that if $[w_{11}(s), L_{11}(s)] = [w_{12}(s), L_{12}(s)]$, for instance, then (6) and (7) are satisfied trivially.

Finally, for simplicity, it is convenient to impose the assumptions of Rothschild and Stiglitz (1976) and Wilson (1977) that require each offered contract to earn non–negative profits given the workers accepting it. Then if contracts for young workers induce self–selection, they must satisfy

(8.a) $w_{1i}(s) \leq \pi_i(s) \;\forall\, s$

Similarly, if contracts for old workers (of previous announced type j) induce self–selection, they must satisfy

(8.b) $w^j_{2i}(s) \leq \pi_i(s) \;\forall\, s.$

If self–selection fails for young workers, then all young workers receive a contract specifying a common wage rate $w_1(s)$. This value must satisfy

(8.c) $w_1(s) \leq \theta\pi_1(s) + (1 - \theta)\pi_2(s) \;\forall\, s.$

Finally, if self–selection fails for old workers at any date, there are two possibilities. One is that self–selection did occur when young. In this case a fraction $q_{j1}$ of workers of previous type j are now of type 1. Then the common wage rate received by old workers (of young type j) must satisfy

(8.d) $$w_2^j(s) \leq q_{j1}\pi_1(s) + q_{j2}\pi_2(s) \ \forall \ s.$$

If self–selection did not occur when young, then the common wage rate $w_2(s)$ received by all old workers must satisfy

(8.e) $$w_2(s) \leq [\theta q_{11} + (1-\theta)q_{21}]\pi_1(s) + [\theta(1-q_{11}) + (1-\theta)(1-q_{21})]\pi_2(s) \ \forall \ s.$$

It is assumed that each firm offers its contracts taking the contract offers of other firms as given.

D. Equilibrium

A standard Nash equilibrium notion is now imposed on this environment, making it a straightforward variant of the Rothschild and Stiglitz (1976) adverse selection model.

*Definition:* A (stationary) Nash equilibrium is a set of contract announcements $\{[w_{1i}(s), L_{1i}(s)]\}$ and $\{[w_{2i}^j(s), L_{2i}^j(\tilde{s},s)]\}$, and a set of values R(s) such that

(a) contract announcements satisfy (4)–(8)

(b) no firm has an incentive to offer a new set of contracts (that satisfy (4)–(8)), given the announcements of other firms.

(c) the loan market clears, i.e.,

$$\theta\Phi_1 + (1-\theta)\Phi_2 = 0.$$

It will prove convenient in constructing examples to rule out the existence of loan markets. When borrowing and lending is not permitted, (c) is replaced by $\Phi_i \equiv 0$; i = 1,2. Finally, it will be noted that attention has been restricted to equilibria in pure strategies.

The properties of a Nash equilibrium here are essentially identical to those demonstrated by Rothschild–Stiglitz (1976). First, no Nash equilibrium in pure strategies need exist. However, as in Rothschild–Stiglitz, the values $\theta$ and $q_{ij}$ can be chosen to guarantee that there are "enough" type 2 agents for an equilibrium to exist. In the sequel parameter values are always chosen so as to imply the existence of an equilibrium.

Second, as in Rothschild–Stiglitz, any equilibrium has the property that, for every state and for every past labor market history, self–selection of type by contract selected occurs. To see this, notice that when workers are old the argument given by Rothschild–Stiglitz as to why self–selection occurs applies to workers of common previously announced type. Since previously announced type is observable, old workers must self–select in any equilibrium.[10] Then, given this fact, it is clear that an analog of the Rothschild–Stiglitz argument applies to young workers as well.

In contrast to the situation analyzed by Rothschild–Stiglitz, however, the incentive compatibility conditions need not "bind" in the determination of equilibrium contracts. However, since the situation of interest is one where there is some unemployment (or underemployment) of labor, attention will be focused on equilibria where the incentive constraints (4)–(7) bind.

In light of these observations, it is straightforward to verify that equilibrium contracts can be characterized as follows. Competition among firms for workers (along with the fact that any equilibrium displays self–selection) implies that (8.a) and (8.b) always hold with equality in equilibrium. Moreover, $L^j_{22}(\tilde{s},s)$ maximizes $W^j_2(\tilde{s},s)$ subject to (8.b) $\forall$ j,s,$\tilde{s}$, while $L_{12}(s)$ maximizes $\Psi_2(s)$ subject to (8.a) $\forall$ s. Thus type 2 workers receive contracts that are unaffected by considerations of self–selection.

The hours worked by old type 1 agents (of previously announced type j) must also be maximal for these agents (given that $w^j_{21}(s) = \pi_1(s)$) among the set of contracts that are consistent with self–selection. Thus $L^j_{21}(\tilde{s},s)$ maximizes $W^j_1(\tilde{s},s)$ subject to (8.b) and (5). Similarly, $L_{11}(s)$ maximizes $\Psi_1(s)$ subject to (8.a) and (7), given the value R(s). R(s) must, of course, adjust so that $\theta\Phi_1 + (1-\theta)\Phi_2 = 0$.[11]

Determination of equilibrium values is depicted in Figure 1 under the assumption that the self–selection conditions (5) and/or (7) hold with equality. Figure 1 can be interpreted as depicting hours determination for either old agents (of announced type j when young), or young workers who know the probability distribution of contracts that they will receive when old. In Figure 1 hours worked appear on the horizontal axis, and current period income is on the vertical axis. The loci labelled $\bar{U}_i$ are type i indifference curves in this space, and the rays $y = \pi_i L$ are zero profit loci. Assumption (3) implies that the indifference curve of a type 1 worker through any point is steeper than the indifference curve of a type 2 worker through the same point.

The income–hours pair received by type 2 workers occurs at a tangency of a type 2 indifference curve with the relevant zero–profit locus (point A). The income–hours pair received by type 1 workers must be maximal for them among the set of all pairs that earns non–negative profits (when a firm employs a type 1 worker), and that is consistent with self–selection. The point in this set that is most preferred by type 1 workers is point B in the Figure. Notice that the "notional labor supply" of type 1 workers at the wage rate $\pi_1$ is $L_1^*$, so that for the preference maps depicted this economy will have unemployed (underemployed) labor.

## II. TWO EXAMPLES

The objective of this paper is to show that the kind of economy just described can display aggregate fluctuations in hours and output of the magnitude observed in the postwar U.S., and can do so without violating the observation that most individuals display little correlation between hours worked and real wages earned. Such a demonstration clearly requires that the model be parameterized, and that some numerical examples be computed. Two such examples are now considered. However, the model contains enough free parameters that it will be desirable to impose further discipline on the choice of parameter values. Thus it will be convenient to begin by laying out a set of observations for the model to confront.

A. Observations

The observations to be confronted consist of a set of stylized facts, as well as some empirical observations from aggregate time series.

(i) Labor is (sometimes) unemployed (underemployed). Postwar U.S. unemployment rates range between 4 and 10 percent.

(ii) Labor force participants work about a third of available time. Male heads of households work roughly forty percent of available time.

(iii) The average productivity of labor is procyclical, as is the average real wage.

(iv) In aggregate time series there is a strong positive co–variation between hours worked (per capita) and the real wage. Lucas and Rapping (1969) report a value that, when converted into levels of hours and real wages, delivers an estimated elasticity of 2.12 for hours worked with respect to the real wage.

(v) Despite (iv), trends in real wages in the twentieth century have not been associated with similar trends in hours.

(vi) Workers who earn high real wages on average work low levels of hours.

(vii) Sectoral shifts in (relative) employment are an important feature of recent business cycles.

(viii) Wage dispersions decline at cyclical peaks.

(ix) Relative wages across occupations seem to play a significant role in labor market behavior.

(x) Hodrick and Prescott (1981) report a 2 percent standard deviation about trend of average hours per capita, and a 1 percent standard deviation about trend of average productivity. The percentage standard deviation for postwar U.S. GNP about trend is 1.8 percent.[12]

Finally, since the model contains two types of workers, some guidance is required on the choice of relative productivity parameters. For simplicity, the Economic Report of the President has been followed: non–agricultural, non–military employees not engaged in wholesale or retail trade have been divided into two categories— manufacturing and construction.

(xi) In 1947 average hourly earnings in manufacturing divided by average hourly earnings in construction was .79. In 1980 this number was .73. The ratio is largely in this range for the postwar period. Also, average weekly hours in manufacturing always exceed average weekly hours in construction.[13]

B. Parameterization

In order to simplify computation, loan markets are ruled out in the examples ($\Phi_i \equiv 0$; $i = 1,2$). This is not really restrictive, since this could be accomplished as an equilibrium outcome by appropriate choices of the values $\beta_i$. However, ruling out borrowing and lending avoids the calculations required to compute the necessary discount rates.

With $\Phi_i = 0 \ \forall\ i$, it is easy to verify that in equilibrium hours worked by old agents do not depend on their labor market history. Thus young and old workers of the same type will (in equilibrium) work the same number of hours. By the arguments of the previous section,

$$(9) \qquad L_{12}(s) = L^j_{22}(\tilde{s},s) = \operatorname{argmax} U[\pi_2(s)L(s)] + \phi_2 V[L(s)] \ \forall\ s,\tilde{s},j,$$

while $L_{11}(s)$ and $L^j_{21}(\tilde{s},s)$ solve the problem

$$(10) \qquad \max U[\pi_1(s)L(s)] + \phi_1 V[L(s)]$$

subject to

$$(11) \qquad U[\pi_1(s)L(s)] + \phi_2 V[L(s)] \le U[\pi_2(s)L_2(s)] + \phi_2 V[L_2(s)] \ \forall\ s,\tilde{s},j.$$

When the self–selection constraint (11) holds with equality, the solution to this problem is

$$(12) \qquad L_{11}(s) = L^j_{21}(\tilde{s},s) = \min\{L\colon\ U[\pi_1(s)L] + \phi_2 V(L) = U[\pi_2(s)L_2(s)] + \phi_2 V[L_2(s)]\}$$

as in Figure 1.[14]

It remains only to specify the forms of agents' utility functions. For the purpose of computing examples it is assumed that

$$U(C) + \phi_i V(L) = \ln C + \phi_i \ln (1 - L)$$

with $\phi_2 = 1$ and $\phi_1 = 2$. The reasons for choosing this specification will be discussed below.

C. Example 1

It is assumed that population demographics are the same at each date. If $q_{ij} \equiv q \; \forall \; i,j$ is imposed, this assumption requires that $\theta = q$. Then choose parameter values as follows: $\theta = q = .5$, $p_{11} = p_{12} = p_{21} = p_{22} = .5$, $\pi_1(1) = 8.5$, $\pi_1(2) = 8.6$, $\pi_2(1) = 7$, and $\pi_2(2) = 6.8$. Notice that, at each date, half the population is of each type.[15]

1. Full information

As a benchmark, it is useful to begin by considering the Nash equilibrium of this economy under full information (that is, when type is publicly observed). This equilibrium coincides with the competitive equilibrium. Wage rates obey $w_i(s) = \pi_i(s)$, and hours levels are given by $L_i(s) = \text{argmax}\{U[\pi_i(s)L] + \phi_i V(L)\}$. Then $L_1(s) = 1/3$ and $L_2(s) = 1/2 \; \forall \; s$ (where $L_i(s)$ is the common hours level for both young and old workers of type i). Relative wages are

$$\frac{w_2(1)}{w_1(1)} = \frac{\pi_2(1)}{\pi_1(1)} = .81$$

$$\frac{w_2(2)}{w_1(2)} = \frac{\pi_2(2)}{\pi_1(2)} = .79$$

which are roughly in the appropriate range.

Since half the population is of each type at each date, average per capita hours in the population is constant. Thus the examples have been parameterized in such a way that, under full information, the model is incapable of generating any cyclical variation in hours, or any co–movements in aggregate hours and real wages (productivity).

From the perspective of individuals, on the other hand, a non–trivial panel of observations can be generated because half the agents in this example experience type changes. Any worker who is type 1 when young and type 2 when old will experience falling wages and rising hours (and conversely). Any worker who does not change type between periods has constant hours, although some will experience real wage variation. Thus, in the panel data set generated by this economy, all agents will display non–positive correlations between hours worked and real wage rates.

## 2. Private Information

Under private information it continues to be the case that $L_2(s) = 1/2\ \forall\ s$. It is readily verified that it is not incentive compatible to set $L_1(s) = 1/3$, however, which is the desired hours level of all type 1 workers. Hence, by (12), $L_1(s)$ must be the smallest solution to

$$\ell n[\pi_1(s)L_1(s)] + \ell n[1 - L_1(s)] = \ell n[\pi_2(s)/4],$$

since $w_1(s) = \pi_1(s)$. The smallest solution to this equation is

$$L_1(s) = .5 - .5\left[1 - \frac{\pi_2(s)}{\pi_1(s)}\right]^{.5} = .5 - .5\left[1 - \frac{w_2(s)}{w_1(s)}\right]^{.5}. \tag{13}$$

Notice that $L_1(s)$ depends only on $w_2(s)/w_1(s)$, which is the relative wage across "occupations". This is consistent with Dunlop's (1950) observations on the importance of relative wages in labor markets. It is also apparent that $L_1(s)$ will be high when $w_2(s)/w_1(s)$ is high. Then, since $L_2(s)$ is constant, aggregate hours will be high when "wage dispersions" have declined. This is consistent with Reder's (1962) observations.

From (13), $L_1(1) = .29$ and $L_1(2) = .27$. Since type one workers would like to work a third of available time, the unemployment rate in state s, u(s), is given by

$$u(s) = 100\,\frac{[.33 - L_1(s)]\theta}{(.33)\theta + (.5)(1 - \theta)}.$$

Thus u(1) = 4.8 percent and u(2) = 7.1 percent. Also, letting average per capita hours be denoted by $\hat{L}(s)$, $\hat{L}(s) \equiv \theta L_1(s) + (1 - \theta)L_2(s)$. Therefore $\hat{L}(1) = .395$ and $\hat{L}(2) = .385$. This is consistent with the Ghez–Becker observation on average per capita hours worked as a fraction of available time.

As in the full information case, this economy generates a panel data set for its members. For workers who are of type 2 throughout their lives, and who experience $\tilde{s} \neq s$, real wage rates and hours worked will be uncorrelated. For agents who experience type changes, hours worked and real wage rates will always be negatively correlated (and in fact more strongly so than for the full information version of the economy). And for workers who are of type 1 throughout their lives, either $\tilde{s} = s$ or not. If $\tilde{s} \neq s$, state $s = 1$ will be the low wage state ($\pi_1(1) = 8.5$, $\pi_1(2) = 8.6$), but $L_1(1) > L_1(2)$. Thus *every individual* in this economy will display either no change in both wages and hours, or a *non–positive correlation between hours and real wage movements.*

It remains to consider the time series behavior of aggregate per capita wages and hours, denoted $\hat{w}(s)$ and $\hat{L}(s)$ respectively. $\hat{L}(1) = .395$ and $\hat{L}(2) = .385$ have already been computed. Average per capita wage rates (which coincide with average per capita productivities), weighted by hours worked are

$$\hat{w}(1) = \left[\frac{\theta L_1(1)}{\hat{L}(1)}\right](8.5) + 7\left[\frac{(1-\theta)L_2(1)}{\hat{L}(1)}\right] = 7.551$$

$$\hat{w}(2) = \left[\frac{\theta L_1(2)}{\hat{L}(2)}\right](8.6) + (6.8)\left[\frac{(1-\theta)L_2(2)}{\hat{L}(2)}\right] = 7.431.$$

Then, at an aggregate level, there would be two elasticities of hours worked with respect to real wages: one in moving from state 1 to state 2, and one in the reverse case. These elasticities are

$$\left[\frac{\hat{L}(2) - \hat{L}(1)}{\hat{L}(1)}\right]\left[\frac{\hat{w}(1)}{\hat{w}(1) - \hat{w}(2)}\right] = 1.61$$

$$\left[\frac{\hat{L}(1) - \hat{L}(2)}{\hat{L}(2)}\right]\left[\frac{\hat{w}(2)}{\hat{w}(2) - \hat{w}(1)}\right] = 1.57$$

While smaller than the Lucas–Rapping (1969) elasticity, these figures are consistent with elasticities used in successful real business cycle models with representative agents (e.g., Greenwood, Hercowitz and Huffman (1988)).

Clearly, then, the model is consistent with small individual–specific correlations between wages and hours, while in the aggregate wage (or productivity) and hours correlations can be quite large. Thus the effect of introducing private information is to increase the aggregate responsiveness of hours worked to real wage movements, even though such responsiveness is not increased for individuals. This is also a counterexample to Lucas' (1987) conjecture that frictions giving rise to unemployment will have minimal impact on the determination of total employment (or hours).

Since the values $\hat{w}(s)$ are merely hours—weighted average productivities of labor, it is apparent that the average productivity of labor here is procyclical, as observed. It is also possible to compute the exact percentage standard deviations for GNP, average per capita hours, and the average productivity of labor for this example. These values are 2.1 percent, 1.8 percent, and 1.1 percent, which can be contrasted with the Prescott (1983) and Hodrick–Prescott (1981) values of 1.8 percent, 2 percent, and 1 percent respectively.

Finally, two additional points can be made. First, if type 1 and 2 workers are viewed as employed by different "sectors" of the economy, cyclical variation in employment will be accompanied by shifts in the relative shares of the sectors in total employment. This is in accord with observation [Lilien (1982)]. Second, the economy just considered can readily accommodate "trends" in growth. Specifically, suppose that $\pi_i(s, t) = (1 + n)^t \pi_i(s)$, where n is a trend rate of growth. Then wages and output will grow over time. However, since $L_2(s) \equiv 1/2$, and since $L_1(s)$ depends solely on

$$\frac{\pi_2(s,t)}{\pi_1(s,t)} = \frac{\pi_2(s)}{\pi_1(s)},$$

there will be no trend in hours. This is also consistent with observation.

D. Diagrammatic Exposition

As an aid to intuition, it is possible to depict example 1 diagrammatically. This is done in Figure 2. As above, L denotes hours and y denotes income, and both young and old agents of the same type have the same preferences over income–hours combinations (since $\Phi_i = 0$). The rays $y = \pi_i(s)L$ are the values of y and L consistent with zero profits for firms employing type i workers in state s, and the loci labelled $U_2^*(s)$ are the (equilibrium) indifference curves for type 2 agents in state s.

Points A and C in the figure represent the maximal income–hours combinations for type 2 agents among the set of such combinations that result in non–negative firm profits, and as such represent the equilibrium levels of income and hours for these workers. Under the assumption of Cobb–Douglas preferences, $L_2(1) = L_2(2)$ as shown.

As in Figure 1, self–selection constraints bind on the determination of hours for type 1 agents in each state. Then, as indicated by the discussion above, the equilibrium levels of income and hours for these agents in each state must occur where the relevant type 2 indifference curve intersects the appropriate zero profit locus for firms employing type 1 workers. Then when s = 1 type 1 workers receive the income–hours combination associated with point B in the figure, while in state s = 2 their income and hours levels are at D.

Consistent with the example, $\pi_1(1) < \pi_1(2)$, but $\theta$ and the values $q_{ij}$ can be chosen (in a way that is consistent with the existence of an equilibrium) such that $\hat{\pi}(1) > \hat{\pi}(2)$. Then clearly s = 1 is the high average real wage (and average productivity) state, and is the state in which total employment (or hours) is relatively high. Thus aggregate wages and hours are positively correlated. However, cross–sectionally hours and wages are negatively correlated, since $w_1(s) > w_2(s) \ \forall s$, while $L_1(s) < L_2(s) \ \forall s$. And finally, all individuals will display non–positive correlations between wages and hours. This is true since workers who experience type changes will have wages and hours moving in opposite directions. Moreover, agents who have i = 1 in both periods, but $\tilde{s} \neq s$, will have $w_1(1) < w_1(2)$, but $L_1(1) > L_1(2)$. Thus positive aggregate correlations in wages (or productivity) and hours are obtained without requiring any positive correlations among individuals.

E. Example Two

For purposes of illustrating the ability of this model to generate large aggregate correlations between wages and hours, without having large correlations for individuals, example 1 had the extreme feature that all individuals display non–positive elasticities of hours worked with respect to real wages. It also had a fairly large aggregate elasticity. A less extreme example is now presented. Preferences are as above, and parameter values are as follows: $\theta = q = .4$, $\pi_1(1) = 8.5$, $\pi_1(2) = 8$, $\pi_2(1) = 7$, $\pi_2(2) = 6$. Notice that in this example $s = 1$ is the high productivity state for all workers.

Equilibrium values are computed as before. Then $L_1(1) = .29$, $L_1(2) = .25$, $u(1) = 5$ percent, $u(2) = 10$ percent, $\hat{L}(1) = .416$, $\hat{L}(2) = .4$, $\hat{w}(1) = 7.42$, and $\hat{w}(2) = 6.5$. Relative wages are $w_2(1)/w_1(1) = .82$ and $w_2(2)/w_1(2) = .75$. These values are consistent with the stylized facts listed previously.

As in the first example, all agents who are of type 2 in both periods display no responsiveness of hours worked to real wage changes. Also, all agents who experience type changes have negative correlations between wages and hours. For the parameter values of this example, these groups constitute 84 percent of the population. The remaining 16 percent of the population, which is type 1 in each period, displays an elasticity of hours worked with respect to real wages equal to

$$\left[\frac{L_{21}(1) - L_{11}(2)}{L_{11}(2)}\right]\left[\frac{\pi_1(2)}{\pi_1(1) - \pi_1(2)}\right] = 2.56$$

if the state is $s = 2$ when young, and is $s = 1$ when old. This is in accordance with the observation that some subsets of the working population do display quite large elasticities in panel data [e.g., Heckman and Macurdy (1980)].

At an aggregate level, as the economy transits from state 2 to state 1,

$$\left[\frac{\Delta L}{L}\right]\left[\frac{w}{\Delta w}\right] = .28,$$

and as the economy moves from state 1 to state 2

$$\left[\frac{\Delta L}{L}\right]\left[\frac{w}{\Delta w}\right] = .31$$

These values are roughly consistent with the more conservative aggregate elasticity of .46 obtained by Hall (1980, p. 20).

## III. GENERALIZING THE RESULTS

The examples of the previous section exploited several features of the model related to the equilibrium behavior of hours for type 1 workers:

(a) hours worked by type 1 agents respond negatively to changes in their own wage rate.

(b) hours worked by type 1 agents respond positively to changes in the wage rate of type 2 workers

(c) type 1 workers are unemployed (underemployed).

Also, a corollary of assumption (3) is that, if

$$\max_s\,[\pi_2(s)] < \min_s\,[\pi_1(s)],$$

then type changes within any agent's lifetime will be associated with opposite movements in hours and real wages if $\Phi_2$ is not too large in absolute value. Of course, all of these results depend on the assumption that the self–selection constraints bind in all states, and that an equilibrium exists. They also depend on the assumption of equation (3).

These are the important features of the examples. In this section it is demonstrated that, under the assumptions just described, these results will hold generally.

### A. Unemployment of Type 1 Workers

If the incentive constraints bind in each state, then the values $L^j_{21}(\tilde{s},s)$ for the hours of old type one workers solve the problem

$$\max\; U[\pi_1(s)L(\tilde{s},s) + R(\tilde{s})\Phi_j] + \phi_1 V[L(\tilde{s},s)]$$

subject to

(14) $U[\pi_1(s)L(\tilde{s},s) + R(\tilde{s})\Phi_j] + \phi_2 V[L(\tilde{s},s)] = U[\pi_2(s)\, L^j_{22}(\tilde{s},s) + R(\tilde{s})\Phi_j]$

$+ \phi_2 V[L^j_{22}(\tilde{s},s)]$

(15) $0 \leq L(\tilde{s},s) \leq 1 \;\forall\; \tilde{s},s,j,$

where $L^j_{22}(\tilde{s},s)$ is determined as above. If (14) has only one feasible solution then this must involve type 1 workers being underemployed, or in other words,

(16) $L^j_{21}(\tilde{s},s) < \operatorname{argmax}\{U[\pi_1(s)L(\tilde{s},s) + R(\tilde{s})\Phi_j] + \phi_1 V[L(\tilde{s},s)]\}$

will hold. This should be apparent from Figure 1, and the fact that (for any value $R(\tilde{s})\Phi_j$) the indifference curve of type 1 agents through any point will be steeper than the indifference curve of type 2 agents through the same point.

If (14) has more than one feasible solution (as in the examples), then (16) still holds. This can be seen as follows. Abbreviating equilibrium hours levels by $L_1(s)$ and $L_2(s)$,

(17) $U[\pi_1(s)L_1(s) + R(\tilde{s})\Phi_j] + \phi_1 V[L_1(s)] = U[\pi_1(s)L_1(s) + R(\tilde{s})\Phi_j] + \phi_2 V\,[L_1(s)]$

$+ (\phi_1 - \phi_2)\, V[L_1(s)] = U[\pi_2(s)L_2(s) + R(\tilde{s})\Phi_j] + \phi_2 V[L_2(s)] + (\phi_1 - \phi_2) V[L_1(s)]$

gives an expression for the utility level of type 1 workers. Then, letting $W^*_i$ denote the equilibrium utility level of an (old) type i worker,

$$W^*_1 = W^*_2 + (\phi_1 - \phi_2)\, V[L_1(s)]$$

As $\phi_1 > \phi_2$ and $V' < 0$, clearly the smallest value of $L_1(s)$ satisfying (14) is the equilibrium value. Thus (16) must hold. A similar argument applies for the hours levels worked by young type 1 agents.

The above argument also indicates that $L_{11}(s) < L_{12}(s)$ and $L^j_{21}(\tilde{s},s) < L^j_{22}(\tilde{s},s)$. Then if

$$\min_s[w_1(s)] > \max_s[w_2(s)],$$

type changes within an agent's lifetime will imply negative correlations between wage and hours movements if $L_{22}(s)$ and $L^j_{22}(\tilde{s},s)$ are sufficiently close $\forall\ \tilde{s},s,j$. This will be the case if $R(\tilde{s})\Phi_j$ is not too large in absolute value for any values of $\tilde{s}$ and j.

B. The Response of Type 1 Hours to Wage Changes

Under the assumption that the self–selection constraint (14) always binds on the determination of type 1 hours (for old agents), (14) implicitly defines $L_1(s)$ as a function of $w_1(s)$, $w_2(s)$, and $R(\tilde{s})\Phi_j$ (with an obvious abbreviation of notation). It is then straightforward to verify that

$$\text{(18)}\qquad \frac{\partial L_1(s)}{\partial w_1(s)} = \frac{-L_1(s)U'[\pi_1(s)L_1(s) + R(\tilde{s})\Phi_j]}{\pi_1(s)U'[\pi_1(s)L_1(s) + R(\tilde{s})\Phi_j] + \phi_2 V'[L_1(s)]}$$

The expression in the denominator on the right–hand side of (18) is positive, since $L_1(s) < \text{argmax}\ \{U[\pi_1(s)L(s) + R(\tilde{s})\Phi_j] + \phi_2 V[L(s)]\}$ (see Figure 1). Thus increases in $w_1$ (or equivalently, in $\pi_1$) reduce the hours worked by old type 1 agents. It is also possible to show, using an envelope theorem argument, that

$$\frac{\partial L_{11}(s)}{\partial w_{11}(s)} = \frac{-L_{11}(s)U'[\pi_1(s)L_{11}(s) - \Phi_1]}{\pi_1(s)U'[\pi_1(s)L_{11}(s) - \Phi_1] + \phi_2 V'[L_{11}(s)]}$$

Thus the same statement applies to young type 1 workers.

Similarly, implicitly differentiating (14) with respect to $w_2(s)$ gives

$$\frac{\partial L_1(s)}{\partial w_2(s)} = \frac{L_2(s)U'[\pi_2(s)L_2(s) + R(\tilde{s})\Phi_j]}{\pi_1(s)U'[\pi_1(s)L_1(s) + R(\tilde{s})\Phi_j] + \phi_2 V'[L_1(s)]}$$

so that increases in the wage rate received by type 2 workers result in an increase in the hours worked by old type 1 workers. Further, again using an envelope theorem argument,

$$\frac{\partial L_{11}(s)}{\partial w_{12}(s)} = \frac{L_{12}(s)U'[\pi_2(s)L_{12}(s) - \Phi_2]}{\pi_1(s)U'[\pi_1(s)L_{11}(s) - \Phi_1] + \phi_2 V'[L_{11}(s)]}$$

Thus the same statement applies with respect to the hours worked by young type 1 agents.

In summary, then, the features that were important in generating the behavior of type 1 hours in examples one and two will obtain generally in models of the kind described here. Such statements about the behavior of aggregate hours are not possible, however. This is simply because, in general, the response of hours worked by type 2 agents to movements in $\pi_2$ will be ambiguous. Such an ambiguity, of course, has nothing to do with the nature of the environment considered here, and will arise in any model. However, it should be apparent that the aggregate hours behavior generated by the examples is in no way pathological.

## IV. DISCUSSION

Several aspects of the analysis merit discussion. This section undertakes such a discussion of the roles played, and the limitations imposed, by several features of the modelling strategy employed above.

A first set of comments concerns the use of logarithmic utility in the construction of the examples. This utility function was chosen partly because it is a simple special case of utility functions commonly used in real business cycle models (e.g., Long and Plosser (1983), Kydland and Prescott (1982)) and in aggregate analyses of labor supply behavior (e.g., Altug (1983) or Eichenbaum, Hansen, and Singleton (1988)). The latter studies generally find empirical support for logarithmic utility. However, there are other features of this utility function that make its use attractive.

The assumption of logarithmic utility, along with the "weights" chosen for consumption and leisure for type 2 workers, permits a closed form solution for the model to be obtained. Moreover, this solution has the feature that only "relative wages" matter for the determination of hours worked by type 1 agents. This is consistent with the Dunlop–Keynes–Solow argument mentioned above. In addition, this aspect of the equilibrium, along with the fact that type 2 hours do not depend on type 2 wage rates, permits the model to accommodate (common) trends to productivity without inducing trends in hours.

Finally, the assumption of logarithmic utility, along with the assumed absence of borrowing and lending, implied that under complete information the model could produce *no* interesting cyclical behavior in aggregate hours. Thus all the interesting cyclical hours behavior in the examples is purely a consequence of the adverse selection problem, which is the friction giving rise to unemployment.

A second feature of the analysis that merits comment is the assumed absence of state contingent claims trading. That the absence of such trading should not be upsetting is indicated by the fact that much of the analysis can be reproduced in settings where all agents have linear utility functions (Smith (1985a)). In such settings the absence of state contingent claims trades would, of course, be an equilibrium outcome. In the case considered above, however, where agents have strictly concave utility functions, it is desirable to rule out markets in state contingent claims. This is both for reasons of tractability, and because allowing state contingent claims markets would require that a stand be taken on whether trading in these markets is publicly observable or not.

An additional feature of the model that requires comment is the assumption that agents' types can change randomly between periods. This assumption is made for two reasons. First, it prevents each worker's type from being known when old, which would make the determination of hours levels for old agents uninteresting. Second, in conjunction with the assumption that workers are always mobile, it permits any consideration of multi–period incentive problems to be avoided. A consideration of such problems, even in a two period model, would greatly complicate the analysis.

A feature of the equilibrium that emerges from the model also requires discussion. In particular, all unemployment (underemployment) in the model is experienced by high productivity workers. Two comments are in order. First, there is no difficulty in principle in constructing a model with three or more types of workers. Since incentive constraints need not bind between adjacent types in this model, it would be possible to construct "several worker type" versions of the model in which unemployment is confined to the "next to lowest" type. For simplicity, a two type specification has been retained here.

However, at this level of aggregation, it is not even clear that it is inappropriate to have high productivity workers work low levels of hours and experience relatively high unemployment. For instance, in manufacturing and construction, construction workers earn high wage rates but work fewer hours and experience higher unemployment rates than do workers in manufacturing. Thus the implications of the model with two types of workers are not entirely counterfactual.[16]

Finally, the use of an overlapping generations model to study cyclical issues merits comment. It will be noted that attention has been confined to the levels of relevant variables and to their variances. In a stationary environment (with a large sample), frequency of sampling will not affect measured variances. Thus the use of a model that delivers predictions only about behavior over relatively long periods need create no real difficulty in matching statistics generated by the model with those observed in practice.

## V. CONCLUSIONS

The objective of this paper was to demonstrate that simple models with private information could confront a wide range of observations on the cyclical behavior of labor markets, and that they could do so without contradicting microeconomic evidence on labor supply behavior. As Section II indicates, this objective is fairly easy to accomplish. Moreover, it can be accomplished using artificial economies where the specification of preferences and parameter values is required to be consistent with a broad set of observations. And finally, as demonstrated in Section III, the important features of the parametric examples generalize fairly readily.

The analysis performed above is a natural extension to an economy with private information of the exercises performed by Kydland and Prescott (1982) or Long and Plosser (1983) for competitive economies with (essentially) complete information. The presence of private information allows for unemployed labor, however. As the model indicates, introducing a friction that permits unemployment to arise also has a profound impact on hours determination. A theory of hours determination is produced that is consistent with both macroeconomic and microeconomic evidence on the behavior of wages and hours. Moreover, this is achieved without any reliance on the empirically questionable intertemporal substitution hypothesis that underlies Kydland and Prescott (1982) or Hansen (1985).

To conclude, it is appropriate to mention some features excluded from the analysis. First, no issues related to "persistence" have been discussed. This is largely for simplicity, as even at this level the model contains mechanisms for generating persistence. One is simply to allow for persistent technology shocks, as in Kydland and Prescott (1982), which was not done in the examples. A second is to consider savings in more detail since with saving not ruled out, inherited savings will generally affect equilibrium hours levels for old agents. This mechanism for generating persistence was not considered in the examples. And finally, of course, it is possible to introduce capital into the production technology.

A second feature that has been omitted is the possibility of entry into and exit from the labor force. Such entry and exit can be incorporated into adverse selection models, as in Judd (1984). While as a first attempt at considering an adverse selection model of a labor market this complication was avoided, it would be interesting to extend the model so as to allow workers to enter or leave the labor force. This is left as a topic for future research.

Notes

[1]See, for instance, Kydland and Prescott (1982), Tables III and IV. In Kydland and Prescott's simulated model, the percentage standard deviation of hours and the percentage standard deviation of productivity are nearly equal. In postwar U.S. data hours are twice as variable as productivity.

[2]A partial survey of microeconomic evidence on this issue is provided by Ashenfelter (1984), who also gives references. Aggregate evidence against the intertemporal substitution hypothesis appears in Altonji (1982).

Kydland and Prescott (1982) enhanced the potential for intertemporal substitution to occur in their model by making agent preferences non–time–separable. Evidence against the specification employed by Kydland and Prescott appears in Altug (1983) and Eichenbaum, Hansen, and Singleton (1988).

[3]This literature is too large to permit even a partial list of efforts. A survey of the early literature on this topic as well as extensive references is given by Hart (1983).

[4]It bears mentioning that this strategy for introducing frictions into real business cycle models has certain complementarities with the "lottery" models of Rogerson (1985) and Hansen (1985). In particular, under more general specifications of preferences than employed below, lotteries can be used to partially (or sometimes fully) resolve the adverse selection problem present here. For an example illustrating this possibility, see Smith (1985b).

[5]This assumption makes any given worker's contribution to the total output of a firm negligible, and is attractive for this reason. However this assumption is not necessary to the analysis.

[6]The assumption that there are only two values for the "technology shock" is a convenient simplification. This assumption is also employed by Greenwood, Hercowitz, and Huffman (1988), and a close relative is employed by Mehra and Prescott (1985).

[7] The use of this utility function, rather than the more general $U_i(C, L)$, rules out the possibility that Nash equilibria for this economy involve firms offering lotteries over consumption–hours pairs. For a description of such lotteries see Prescott and Townsend (1984a, b).

[8] For an example of a competitive real business cycle model where workers have randomly changing preferences see Bencivenga (1987).

[9] Ruling out such trading provides both a technical and a conceptual simplification. In particular, if state contingent claims trades were allowed it would be necessary to take a stand on whether or not such trades were observable. It should be noted that results very much like the ones in the text can be obtained even if all agents have linear utility functions [Smith (1985a)]. Clearly the absence of these markets is easily justified in such a context.

[10] Since last period's actions by any old worker are observable, the labor market for old workers can be treated using the analysis of Hoy (1982).

[11] To see that the contracts described constitute an equilibrium (given that the possibility of preferred pooling contracts is implicitly ruled out by choice of $\theta$ and the $q_{ij}$), it is sufficient to observe three facts.

First, by assumption (3), old type 1 workers (who were type j when young) prefer the contract $[w_{21}^j(s), L_{21}^j(\tilde{s}, s)]$ described to the contract obtained by type 2 agents. Second, given this fact, young type 1 agents cannot prefer to take type 2 contracts. This is because of (3), and because if they fail to take type 1 contracts they will also be forced to make a suboptimal savings choice. Thus the contracts described satisfy (4)–(8). Finally, it is apparent that (taking R(s) as given) no firm can offer any worker a contract that will earn non–negative profits, that is incentive compatible, and that will be preferred by that worker to the contracts described.

[12] (ii) is essentially from Ghez and Becker (1975). (iii) is from Prescott (1983), Tables 2 and 3. On (iv) see also Hotz, Kydland and Sedlacek (1988), Ashenfelter (1984), and Prescott (1983). (vii) is from Lilien (1982), especially p. 779. On (viii) see Reder (1962). (ix) is

discussed by Dunlop (1950). Keynes (1936) and Solow (1980) have argued in favor of building this observation into macro models. On (x) see also Prescott (1983).

[13]These facts are from the Economic Report of the President (1981), p. 273.

[14]A formal argument that does not rely on $\Phi_i = 0$ is given in section III below.

[15]It is straightforward to check that these parameter values are consistent with the existence of a Nash equilibrium.

[16]An obvious objection to the argument just given might be as follows. Construction workers and workers in manufacturing may not be intrinsically different, as type one and two workers are in the model above. Rather unionization in construction might raise wages and increase unemployment in that sector, with the higher wages "compensating" construction workers for the unemployment they experience. However, this argument is incomplete. Abowd and Ashenfelter (1981) present estimates of "compensating wage differentials" of this type which never exceed 14 percent, and generally are more in the range of 6 to 8 percent. Wage differentials between manufacturing and construction are much larger than this, so even accepting the logic above, there is still much to be explained regarding the differences in behavior of workers in these two sectors.

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Figure 1

An Unemployment Equilibrium

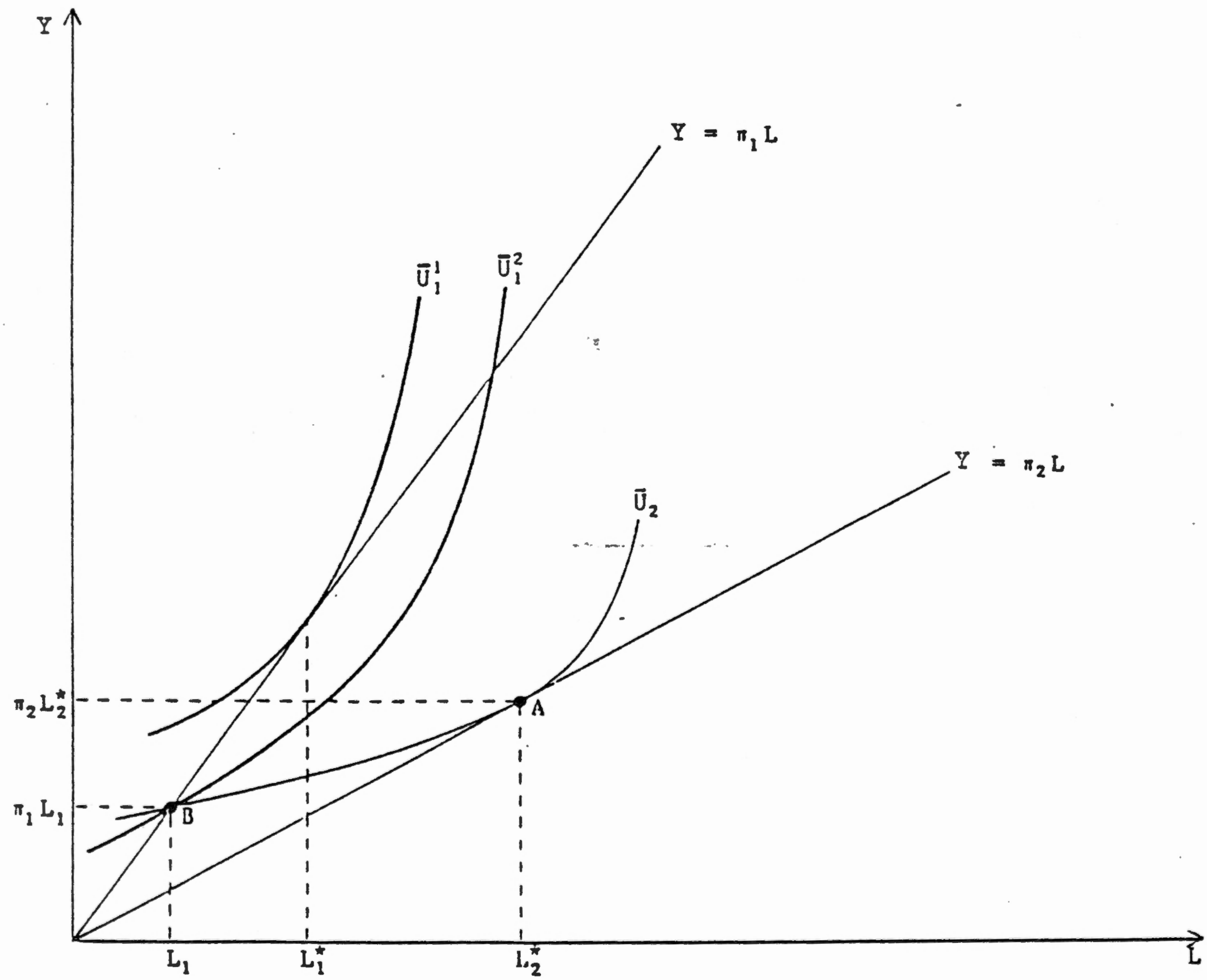
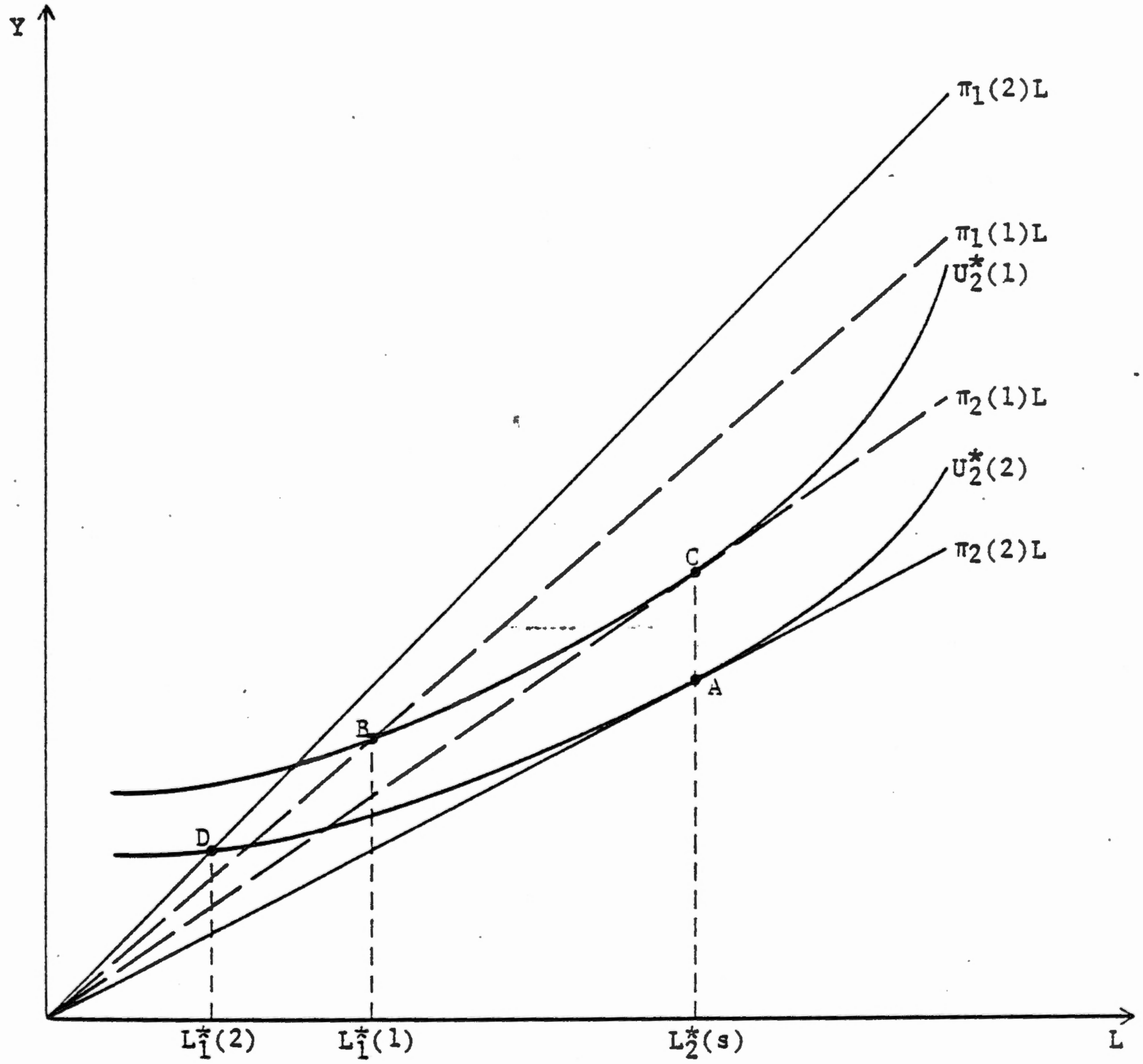


Figure 2



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