Competitive Diffusion

Jovanovic, Boyan and Glenn MacDonald

Working Paper No. 160
September 1988

COMPETITIVE DIFFUSION+

by

Boyan Jovanovic*

and

Glenn M. MacDonald**

Working Paper No. 160

Revised
September 1988

+ This research was supported by the C.V. Starr Center for Applied Economics and the National Science Foundation (Jovanovic) and the Bradley Foundation (MacDonald).

* Department of Economics, New York University.

** Centre for Decision Sciences and Econometrics, University of Western Ontario; Center for Economic Research, University of Rochester; Economics Research Center/NORC, University of Chicago.

ABSTRACT

The advance of knowledge has long been thought to be a key source of economic growth. Roughly, previous research has focused either on innovation by individual firms without exploring the spread of ideas, or taken ideas as given and analysed their spread (often in a non–optimizing environment). In this paper both the production of new ideas—Invention/Innovation—and the activities that cause their diffusion—Imitation—are objects of choice and chosen optimally.

Innovation and Imitation are activities that differ over both time and across firms at a point in time. When all firms have access to the same knowledge, innovation is the prime source of new ideas, but when knowledge is diverse across firms, imitation plays an important role. Innovation generates heterogeneity in firm knowledge and behavior. Imitation can exagerate heterogeneity, but eventually eliminates it. Complex Schumpeterian–type behavior may arise: In one scenario, the heterogeneity spawned by innovative activities inspires imitation which eventually produces homogeneity, causing a substitution back towards innovation, and a consequent growth in heterogeneity, and so on.

The paper is divided into three parts. In the first, a general model is developed and several results provided. The dynamics of the general model are illustrated in a three–state example. Subsequently, a restricted model is analysed. Two interesting cases given special attention have, as the primary source of new ideas, imitation and innovation respectively. The former instance is closely related to diffusion models with given ideas, while the latter is similar to an R&D model. Data on the diffusion of diesel technology in the U.S. railroad industry, and the growth of mechanized loading techniques in U.S. underground coal mining, is studied in light of the two specialized models.

I. INTRODUCTION

Technological change became the focus of intense research effort following the influential work of Solow (1957) and others who tried to understand patterns and determinants of economic growth. It was found that the better part of growth could not be explained within a model involving growth of factors producing output subject to a fixed technology. Somehow, over time more output is obtained from given inputs: increasing "total factor productivity". Whether this change was treated as the exogenous advance of knowledge [e.g. Denison (1967)] or as endogenous quality change [e.g. Griliches (1963), Jorgensen and Griliches (1969)], or as "technical change in the broadest sense" [e.g. Solow (1988)], it became clear that the development, utilization and spread of new techniques of production were the key economic issues to be understood.[1]

This realization led to an explosion of research, both empirical and theoretical, on the production of new ideas (R&D) and the manner in which they spread ("diffusion"). Empirical studies number (literally) in the thousands; see Scherer (1970), Rogers and Shoemaker (1971), Davies (1979), Griliches (1979), Sahal (1981) and Gort and Klepper (1982). Theoretical work falls into three basic categories. One focuses on individual firms acting as perfect competitors who undertake production of new ideas in order to reduce production costs [Smith (1937, Book I, Chapter 11), Nelson (1982), Jensen (1982), Jovanovic and Rob (1987a)]. The second emphasizes the strategic advantages a firm obtains by improving its production processes [Scherer (1967), Kamien and Schwartz (1972, 1975), Telser (1982) and Spence (1984)]. The third body of work is less concerned with where new ideas and techniques come from, and more with how they spread [(Schumpeter(1934), Salter (1966), David (1969), Futia (1980),

---

[1] Also, see Schmookler (1966); and for a contrasting view, Becker (1988).

imitation, the idea is put to use: innovation. Otherwise the firm retains use of the existing techniques or imitates by utilizing what (if anything) it learned from others.[2]

Optimization by each firm in the industry generates a distribution of ideas in use at each date, along with a specification of output and resources devoted to R&D and imitation for each firm. These entities make up the model's equilibrium. A complicating factor here is that firms' strategies concerning innovation and imitation influence the set of ideas in use in the future. Firms take as given the sequence of distributions of ideas and decisions when making their choices. Equilibrium requires that the sequence of distributions firms use to make their plans is exactly that generated by their behaviour.

In the general model, studied in Section II, existence of equilibrium is proved. It is also demonstrated that under a mild restriction the equilibrium is "symmetric" in the sense that all firms currently using a given idea behave identically. It is then shown that i) observed behaviour in a cross–section of such firms will likely have some unexpected properties. For example, the firms most likely to innovate will be neither the largest nor smallest currently producing; ii) As a result of the way that innovation and imitation effect heterogeneity, the industry's development can be "uneven" in that it may have repeated periods of slow and rapid growth; iii) from the observer's viewpoint, when industry development is uneven, it is sometimes possible to learn a good deal about the possibilities for coming up with new ideas. Otherwise, the process of diffusion through imitation contaminates the data; iv) The price of output declines over time, and simple predictions on the intertemporal behavior of the distribution of output and ideas emerge. v) The substitutability of innovation and imitation implies that within an industry there will generally be a great deal of "mixing" over time: Firms that are currently faring poorly may well overtake those that are performing better,although the leading edge will be stable as the industry matures. These properties are illustrated in a "three–state" (i.e. a basic technology and two better ones) example.

---

[2]The invention/innovation/imitation distinction is a traditional Schumpeterian one; see Baumol (1970) for more discussion.

mechanical techniques, the primary obstacles are reasonably mine–specific in which case information from other mines is not of much use but the problems are solvable given appropriate inputs of engineers, etc.

The most basic claims of the restricted model — increasing hazard under pure imitation, and decreasing hazard under pure innovation — describe the diesel and mine data quite well. The other results are also exhibited in the data, although the most rapid diffusion of mechanization in mining is not as early as the pure innovation model would suggest.

As this Section has precised the paper, there is no concluding Section. Most technical material is confined to an Appendix.

II A GENERAL MODEL

This section describes a general competitive model of innovation and imitation and provides the results available in that environment.

The formal structure describes a competitive industry as an anonymous sequential game; see Schmeidler (1973), Green (1980, 1984), Shefrin (1981) and Jovanovic and Rosenthal (1988). Doing so provides a tractable analytical framework and gives access to useful results on existence of equilibrium. The primary restrictions introduced by proceeding in this way are *anonymity*—there is a continuum of agents, each of whose return at date t depends on the actions of others solely through the distribution of their actions at date *t*, and *sequentiality*—the game is a sequence of games in which the influence of all previous history on the game to be played at date t can be completely summarized by (at most) (i) the game played at t–1; (ii) actions taken at t–1; and (iii) t itself. Loosely, for any given distribution of actions, the identities of the agents choosing those actions are irrelevant as far as any individual is concerned, and, given period t–1 strategies and outcomes, each agent may ignore any earlier history when choosing an action for period t.

The active players in this game are firms. Consumer behavior gives rise to an inverse market demand function $p_t = D_t(Q_t)$, where p_t is the unit price of the product and Q_t is total where an individual firm's rate of output, q, is one component of its action vector α. Thus, through (1), $\{\tilde{F}_t\}_0^\infty$ generates a price sequence $\{p_t\}_0^\infty$, which is the more familiar, but here insufficient, manner in which the actions and characteristics of other agents can be summarized under competition. In what follows $\{p_t\}_0^\infty$ will be understood to be the price sequence induced by $\{\tilde{F}_t\}_0^\infty$ through (1).

Strategies. A firm's strategy specifies the action α to be taken at t given ϑ and $\{\tilde{F}_t\}_0^\infty$. There are four types of possible actions: production, imitation, invention, and innovation. The last two activities, although conceptually distinct in this model, will be collapsed into a single activity called innovation. More specifically, ideas are indexed by ϑ. The invention process at date t may yield a new value of ϑ—a new idea, say ϑ'—and innovation corresponds to making use of ϑ' at t+1. Not all ϑ' will be implemented, in which case the innovation versus invention distinction is not vacuous. However, in the Appendix it is shown that the invention structure studied below is such that any $\vartheta' > \vartheta$ will be put to use; i.e. there is a simple rule characterizing innovation given invention. Since a good deal of notation is eliminated by assuming this implementation policy at this point, invention and innovation will be collapsed into a single process—innovation. It is easy to show, moreover, that if ϑ' is not implemented at some date t, it will never be implemented. Thus the current value of ϑ is all that need be specified about the ideas the firm knows at each date.

The feasible set of actions is as follows. At date t, any firm may engage in production, innovation and imitation. The levels at which these activities occur make up the vector $\alpha_t \equiv (q_t, \eta_t, \mu_t)$ for that firm, where q_t is the rate of output at t, and η_t and μ_t are associated with innovation and imitation respectively and are given an interpretation in terms of probabilities. Thus the feasible set of actions is $A = [0, \bar{q}] \times [0,1] \times [0,1]$, with $\bar{q} < \infty$ but as large as desired.

At date t, any firm may select some other firm and, with probability μ, learn what this firm knows and how this information might be applied to the situation at hand.[7]

The choice of which firm to copy may result from a variety of structures. If the firm has no information whatsoever on the likely values of ϑ for any other given firm (other than knowledge of $\tilde{F}_t$), the selection may be treated as a random draw from the marginal distribution of ideas known at t:

$$F_t(\vartheta) \equiv \int_A \tilde{F}_t(d\alpha, \vartheta) \qquad (2)$$

In this case, the probability that an idea no better than ϑ' is learned via imitation is $(1-\mu) + \mu F_t(\vartheta')$.

The key feature of this specification is that, for given μ, the probability that by imitation the firm will learn an idea better than what it presently knows—$\mu[1-F_t(\vartheta)]$—rises with the number of firms that know better ideas. This basic feature is shared by other specifications in which the firm can actually identify which other firms have given ϑ, but still must invest resources in imperfect learning and implementation of others' ideas. Suppose, for example, that a firm may attempt to learn how to implement the idea known by any particular firm once in each period, but can examine as many firms as desired within that period. Assuming any single attempt is successful with probability μ, and that the firm begins its activities by examining firms having the greatest ϑ in the support of $F_t(\vartheta)$, and then proceeds to firms knowing lower ϑ's, the probability that an idea better than ϑ will be uncovered using

---

[7]The basic characteristic of information that is emphasized here is what is usually taken to differentiate it from a standard economic commodity; namely, its non–exclusive nature. Compare Holmes and Schmitz (1988). In constrast to a public good situation however, others who wish to use information currently available to another party must invest resources to obtain it.

In some cases what other firms are doing—e.g. buying some different input—may be obvious, the difficulty lying in figuring out how the input is to be used once purchased; wine–making is a good example. In other cases, the opposite may occur in that the firm would know what to do with inputs if it could learn what to use: "secret mixtures of herbs and spices", etc. The key to imitation is that other firms have relevant information that is not freely observable.

Given any sequence of actions $\{\alpha_t\}_0^\infty \equiv \{q_t, \eta_t, \mu_t\}_0^\infty$ the value of ϑ actually in use, ϑ_t, follows a stochastic process. Let E denote the expectation operator associated with that process. It is assumed that, given $\{F_t\}_0^\infty$, the firm selects $\{\alpha_t\}_0^\infty$ to maximize the payoff

$$E\left\{\sum_{t=0}^{\infty} \beta^t \left[p_t q_t - c(q_t, \eta_t, \mu_t; \vartheta_t)\right]\right\} \tag{4}$$

where $\beta \in (0,1)$ is a fixed discount factor. Under the assumptions on $D_t(\cdot)$ and $c(\cdot)$, the current return $p_t q_t - c(\cdot)$ is bounded. Thus standard dynamic programming arguments [see, for example, Bertsekas (1976, Ch. 6); or Blackwell (1965)] imply the existence of a sequence of functions $\{V_t\}_0^\infty$, with V_t: $\Theta \rightarrow [0, \bar{\pi}/(1-\beta)]$, where $\bar{\pi} \equiv \max_q[\bar{D}q - c(q,0,0,\bar{\vartheta})]$, such that for each t and ϑ

$$V_t(\vartheta) = \max_{\alpha \in A} \left\{p_t q - c(q,\eta,\mu;\vartheta) + \beta\left[\int_\Theta \max[V_{t+1}(\vartheta), V_{t+1}(\vartheta')]\tau(d\vartheta'; \eta,\mu,\vartheta,F_t)\right]\right\}. \tag{5}$$

It is shown in the Appendix that $V_t(\vartheta)$ is strictly increasing in ϑ. $V_t(\vartheta)$ is also increasing in any first order stochastic dominance change in F_t provided demand is sufficiently elastic.[11]

__All Firms Together__. Consider a firm that has access to idea ϑ at t. The distribution function of the value of ϑ in use by that firm at t+1, ϑ', is

$$\Psi(\vartheta';\eta,\mu,\vartheta,F_t) = \begin{cases} 0 & \vartheta' < \vartheta \\ \tau(\vartheta';\eta,\mu,\vartheta,F_t) & \vartheta' \geq \vartheta \end{cases}.$$

The evolution of ideas of all firms therefore obeys

$$F_{t+1}(\vartheta') = \int_{A\times\Theta} \Psi(\vartheta'; \eta, \mu, \vartheta, F_t)\, \tilde{F}_t(d\alpha \times d\vartheta), \text{ with } F_0 \text{ given.} \tag{6}$$

---

[11] Strictly, the dependence of $\{V_t\}_0^\infty$ and $\{\alpha_t\}_0^\infty$ on $\{\tilde{F}_t\}$ should be made explicit. To simplify notation this dependence is suppressed.

(ii) $-\nabla^2 c + \beta \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \nu_t \\ 0 & \nu_t & 0 \end{bmatrix}$ *is negative definite for all* t *and* ϑ. *Then, there exists a unique maximizer* $\alpha_t(\vartheta)$ *of the right hand side of* (5) *for all* t *and* ϑ. *That is, the equilibrium is symmetric.*

PROOF: Given (3), the matrix of second derivatives of the right hand side of (5) is the matrix in (ii) above. Thus under the conditions (i) and (ii) the maximization in (5) has an unique solution for each ϑ and t. ||

Observe that condition (ii) in the theorem imposes only one extra condition, namely that $|-\nabla^2 c + \beta[\cdot]| < 0$ rather than merely $|-\nabla^2 c| < 0$.[13] The other conditions implied by strict convexity of $c(\cdot)$ need not be strengthened. Also, in the case studied below, in which $c(q,\eta,\mu;\vartheta) = c_q(q;\vartheta) + c_\eta(\eta) + c_\mu(\mu)$, $c''_\eta c''_\mu > \nu_t^2$ is sufficient for symmetry. In terms of primitives, using (3) it is not hard to check that if i) $N(\vartheta'|\vartheta)$ is such that

$$\frac{\partial E(\vartheta'|\vartheta)}{\partial\vartheta} \leq B < \beta^{-1}$$

for some constant B; and ii) $c(\cdot;\vartheta)$ is a Lipschitz function with Lipschitz constant C, a sufficient condition for symmetry of equilibrium actions is

$$c''_\eta c''_\mu > \left[\frac{\beta C(\bar{\vartheta}-\underline{\vartheta})}{1-\beta B}\right]^2$$

In what follows the conditions sufficient for symmetry are assumed.

<u>Properties of Equilibrium in the General Model</u>. The points to follow are illustrated in the subsection following.

1. Cross–section Properties of $q_t(\vartheta)$, $\eta_t(\vartheta)$ and $\mu_t(\vartheta)$. Consider some date t at which the distribution of ϑ, $F_t(\vartheta)$, is nondegenerate.[14] (Most dates will have this feature.)

---

[13] It is easy to verify that $\nu_t < 0$.

[14] Since the distribution of α given ϑ is degenerate in a symmetric equilibrium, $F_t(\vartheta)$, as opposed to $\tilde{F}_t(\alpha, \vartheta)$, is the focus of attention in what follows.

and

$$\nabla \equiv \nabla^2 c + \beta \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & v_t \\ 0 & v_t & 0 \end{bmatrix}$$

negative semi definite. The last condition is satisfied automatically given the restrictions of theorem 2. Note that $|\nabla| < 0$. Calculation gives

$$\frac{dq_t}{d\vartheta} = \frac{1}{|\nabla|}\frac{\partial^2 c}{\partial q \partial\vartheta}\left[\frac{\partial^2 c}{\partial\eta^2}\frac{\partial^2 c}{\partial\mu^2} - v_t^2\right] > 0,$$

$$\frac{d\eta_t}{d\vartheta} = \frac{-1}{|\nabla|}\frac{\partial^2 c}{\partial q \partial\vartheta}\left[v_\eta \frac{\partial^2 c}{\partial\mu^2} + v_\mu v_t\right],$$

and

$$\frac{d\mu_t}{d\vartheta} = \frac{-1}{|\nabla|}\frac{\partial^2 c}{\partial q \partial\vartheta}\left[v_\mu \frac{\partial^2 c}{\partial\eta^2} + v_\eta v_t\right],$$

where

$$v_t = -\beta \int_{\underline{\vartheta}}^{\bar{\vartheta}} \frac{\partial V_{t+1}(\vartheta')}{\partial\vartheta} \frac{\partial^2}{\partial\eta\partial\mu} \tau(\vartheta'; \eta, \mu, \vartheta, F_t) d\vartheta' < 0,$$

$$v_\eta = \beta \left\{ \frac{\partial V_{t+1}(\vartheta)}{\partial\vartheta} \frac{\partial}{\partial\eta} \tau(\vartheta'; \eta, \mu, \vartheta, F_t) - \int_{\underline{\vartheta}}^{\bar{\vartheta}} \frac{\partial V_{t+1}(\vartheta')}{\partial\vartheta} \frac{\partial^2}{\partial\eta\partial\vartheta} \tau(\vartheta'; \eta, \mu, \vartheta, F_t) d\vartheta' \right\},$$

and

$$v_\mu = \beta \left\{ \frac{\partial V_{t+1}(\vartheta)}{\partial\vartheta} \frac{\partial}{\partial\mu} \tau(\vartheta; \eta, \mu, \vartheta, F_t) - \int_{\underline{\vartheta}}^{\bar{\vartheta}} \frac{\partial V_{t+1}(\vartheta')}{\partial\vartheta} \frac{\partial}{\partial\mu\partial\vartheta} \tau(\vartheta'; \eta, \mu, \vartheta, F_t) d\vartheta' \right\}.$$

The term v_t is the effect of μ on marginal returns to innovative activity, $v_t < 0$ indicating the gross substitutes relationship between η_t and μ_t; v_η (resp. v_μ) is the impact of ϑ on marginal returns to innovative (imitative) activity. The intuition given above would indicate that $v_\eta > 0$

2. When Will Progress Be Slow? Slow progress occurs when knowledge is not improving: $F_{t+1}(\vartheta) \cong F_t(\vartheta)$. This situation occurs under three sets of circumstances. One is the trivial one in which the marginal cost of η and μ is high even at low values of η and ϑ. For then, η_t and μ_t will be small and the probability of discovering a better idea, $1 - \tau(\vartheta; \eta, \mu, \vartheta, F_t)$, is correspondingly small for any firm.

More interesting is the situation wherein refinements of an idea are not hard to achieve but breakthroughs are very difficult; for example, $\Theta = [0,1] \cup [2,3]$ with $N(1|1) = 1-\varepsilon$ for ε small. In this case imitation causes all firms to "pile up" at $\vartheta = 1$, and given the slim odds for greater success, relatively little innovative activity goes on: in (5), the final integral almost vanishes. Of course, once a breakthrough has occurred the whole industry moves forward. This uneven development seems to characterize the dynamics of some industries (computers are an obvious instance). Such industry wide cycles or "waves" were emphasized by Schumpeter (1939).

The final case where development will be slow occurs when there is not much left to learn: $F_t(\vartheta) \cong 0$ for $\vartheta \cong \bar{\vartheta}$. In such "mature" industries both η and μ will again be slight but positive (assuming marginal costs low for low η and μ) and the progress to $\vartheta = \bar{\vartheta}$ drawn out.

Note that for firms that have discovered $\vartheta \cong \bar{\vartheta}$, both η and μ will be approximately equal to 0. For lower ϑ values, both η and μ will be positive. In this gross sense then, ϑ must be negatively associated with both η and μ in a cross–section provided t is such that at least some firms have achieved nearly all the technological improvements that can reasonably be expected; and when most firms have achieved this state ϑ must be negatively related to η and μ. Since R&D budgets and sales are sometimes observed to be positively correlated (e.g. Bound (1984)), and output and probability of adoption not consistently negatively related (Rose and Joskow), the model suggests that industries in which technological opportunity is nearly exhausted must be uncommon. For related discussion of slow diffusion, see Harley (1971, 1973) and Davies et al. (1987).

quality in that the price of a standard unit of service is falling over time. These properties are in agreement with evidence on the evolution of output and prices offered in Gort and Klepper (1982). Declining price is also the reason that the distribution of output will not invariably inherit the properties of the distribution of ϑ. Indeed, it is easy to verify that if the support of $F_t(\vartheta)$ is Θ, $p_{t+1} < p_t$ implies the support of the distribution of output has lower greatest and smallest values over time. But under some mild restrictions, the distribution of $q_t(\vartheta)/q_t(\underline{\vartheta})$ does inherit the properties of $F_t(\vartheta)$.

5. Turnover and Growth in the Population of Firms. Unless $N(\vartheta'|\vartheta)$ becomes increasingly sensitive to increases in ϑ as ϑ rises, or $\eta_t(\vartheta)$ rises quickly with ϑ, firms having low current ϑ are likely to experience larger and more variable growth in ϑ. Imitation possibilities are more diverse for firms having low ϑ (thus leading to greater variation in the growth of ϑ), and under mild conditions, the expected zhange in ϑ,

$$\int_{\Theta} \vartheta' \Psi(d\vartheta'; \eta, \mu, \vartheta, F_t) - \vartheta,$$

is declining in ϑ. That is, on average, imitation possibilities are also better for smaller firms. These effects are exaggerated when expressed in proportional terms. Provided the impact of ϑ on $\partial c/\partial q$ does not increase greatly with ϑ, the same effects will generally occur in terms of output and/or profit as well: smaller, less profitable firms will experience comparatively large and variable growth in output and profit.[16] The general point is that the presence of imitation possibilities causes a phenomenon like regression to the mean in the evolution of firm size, and renders growth rates more variable for small firms.[17] These results are strikingly illustrated in the three state model studied below.

---

[16] Similar predictions, but from a very different model, may be found in Jovanovic (1982). Mansfield's (1962) observations are also relevant.

[17] Recent empirical documentation is contained in Evans (1987).

$$N(\vartheta'|\vartheta) = \begin{cases} 0 & \vartheta' < \underline{\vartheta},\ \vartheta = \underline{\vartheta} \\ 1-(\delta+\delta^2) & \underline{\vartheta} \le \vartheta' < \hat{\vartheta},\ \vartheta = \underline{\vartheta} \\ 1-\delta^2 & \hat{\vartheta} \le \vartheta' ,< \bar{\vartheta},\ \vartheta = \underline{\vartheta} \\ 1 & \bar{\vartheta} \le \vartheta'\ \vartheta = \underline{\vartheta} \\ 0 & \vartheta' < \hat{\vartheta},\ \vartheta = \hat{\vartheta} \\ 1-(\delta+\delta^2) & \hat{\vartheta} \le \vartheta' < \bar{\vartheta},\ \vartheta = \hat{\vartheta} \\ 1 & \bar{\vartheta} \le \vartheta',\ \vartheta = \hat{\vartheta}, \end{cases}$$

$$F_0(\vartheta) = \begin{cases} 0 & \vartheta < \underline{\vartheta} \\ 1 & \vartheta \ge \underline{\vartheta}, \end{cases}$$

and

$$g(x) \equiv 1.$$

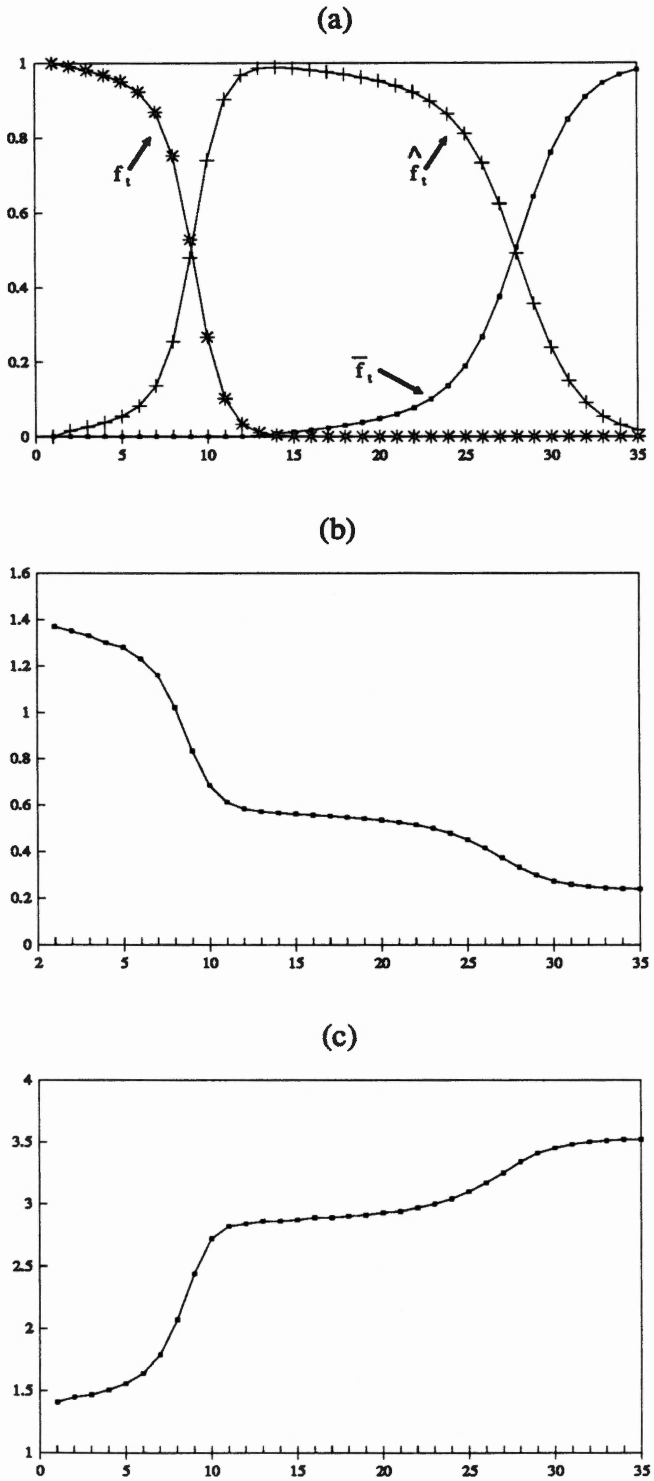
The first three requirements are self–explanatory, as is the fifth. As regards the fourth, the structure of $N(\vartheta'|\vartheta)$ is such that for either $\vartheta = \underline{\vartheta}$ or $\hat{\vartheta}$, a better idea occurs with probability $\delta + \delta^2$. However, given $\vartheta = \underline{\vartheta}$ any new idea yields $\vartheta = \bar{\vartheta}$ only with probability $\delta/1+\delta$, whereas $\sim = \bar{\vartheta}$ is the only possible better idea given $\vartheta = \hat{\vartheta}$. Finally, $g(x) = 1$ simply means that when attempting to imitate, the firm selects a target at random from the distribution of other firms.

The specific parameter values used are listed in the Appendix, section 6. They were chosen as if time periods were fairly short (for example, $\beta = .98$), and both innovation and imitation quite costly. The resulting evolution of the economy is comparatively smooth, which offers significant computational advantages. The parameters were also chosen so that new ideas are not common (eg. $\delta + \delta^2 = .035$). The reason for this specification is that it is close to the "breakthrough" environment discussed above, in which most issues became uncluttered.

Figure 1 illustrates some basic features of the equilibrium. $\underline{f}_t$, $\hat{f}_t$ and $\bar{f}_t$ ($\equiv 1 - \hat{f}_t - \underline{f}_t$) are the fractions of firms having access to $\vartheta = \underline{\vartheta}$, $\hat{\vartheta}$, and $\bar{\vartheta}$ respectively. As would be expected from general proposition no. 4, $F_{t+1} \le F_t$, $\underline{f}_t$ falls over time, $\hat{f}_t$ rises then declines, and $\bar{f}_t$ is rising, eventually reaching unity. Similarly, the product price declines over time, changing most rapidly when new ideas are spreading quickly and average output is rising.

Figure 1

a) $\underline{f}_t$, $\hat{f}_t$, $\bar{f}_t$; b) p_t ; c) Average Output.



picks up and becomes more variable. These, of course, are the periods during which $\vartheta = \hat{\vartheta}$ and $\vartheta = \bar{\vartheta}$ diffuse. The notable difference is that for these small firms, compared to those for which $\vartheta = \hat{\vartheta}$, growth is greater and becomes much more variable because there is more for them to learn and they try harder ($\mu_t(\underline{\vartheta}) > \mu_t(\hat{\vartheta})$).

These complicated dynamics are further illustrated in Figure 4. $Pr(\underline{\vartheta} \rightarrow \bar{\vartheta})$ is the probability with which a firm that currently has access to $\vartheta = \underline{\vartheta}$ will learn $\vartheta = \bar{\vartheta}$ in the current period. $Pr(\underline{\vartheta} \rightarrow \hat{\vartheta})$ and $Pr(\hat{\vartheta} \rightarrow \bar{\vartheta})$ are defined analogously. All transition probabilities involving learning $\vartheta = \bar{\vartheta}$ are very small early on since $\vartheta = \bar{\vartheta}$ is difficult to obtain by innovation and there are hardly any firms to imitate. $Pr(\underline{\vartheta} \rightarrow \hat{\vartheta})$ is small for low values of t for the analogous reason. However, once $\vartheta = \hat{\vartheta}$ has been learned by some firms, the imitative activity of firms that currently know only $\vartheta = \underline{\vartheta}$ produces a large change in $Pr(\underline{\vartheta} \rightarrow \hat{\vartheta})$. Similarly $Pr(\underline{\vartheta} \rightarrow \bar{\vartheta})$ and $Pr(\hat{\vartheta} \rightarrow \bar{\vartheta})$ rise once innovation yields $\vartheta = \bar{\vartheta}$ for some firms. Observe that $Pr(\underline{\vartheta} \rightarrow \bar{\vartheta}) > Pr(\hat{\vartheta} \rightarrow \bar{\vartheta})$: the greater reliance of small firms on imitation yields a greater likelihood that they will learn $\vartheta = \bar{\vartheta}$ —"mixing".

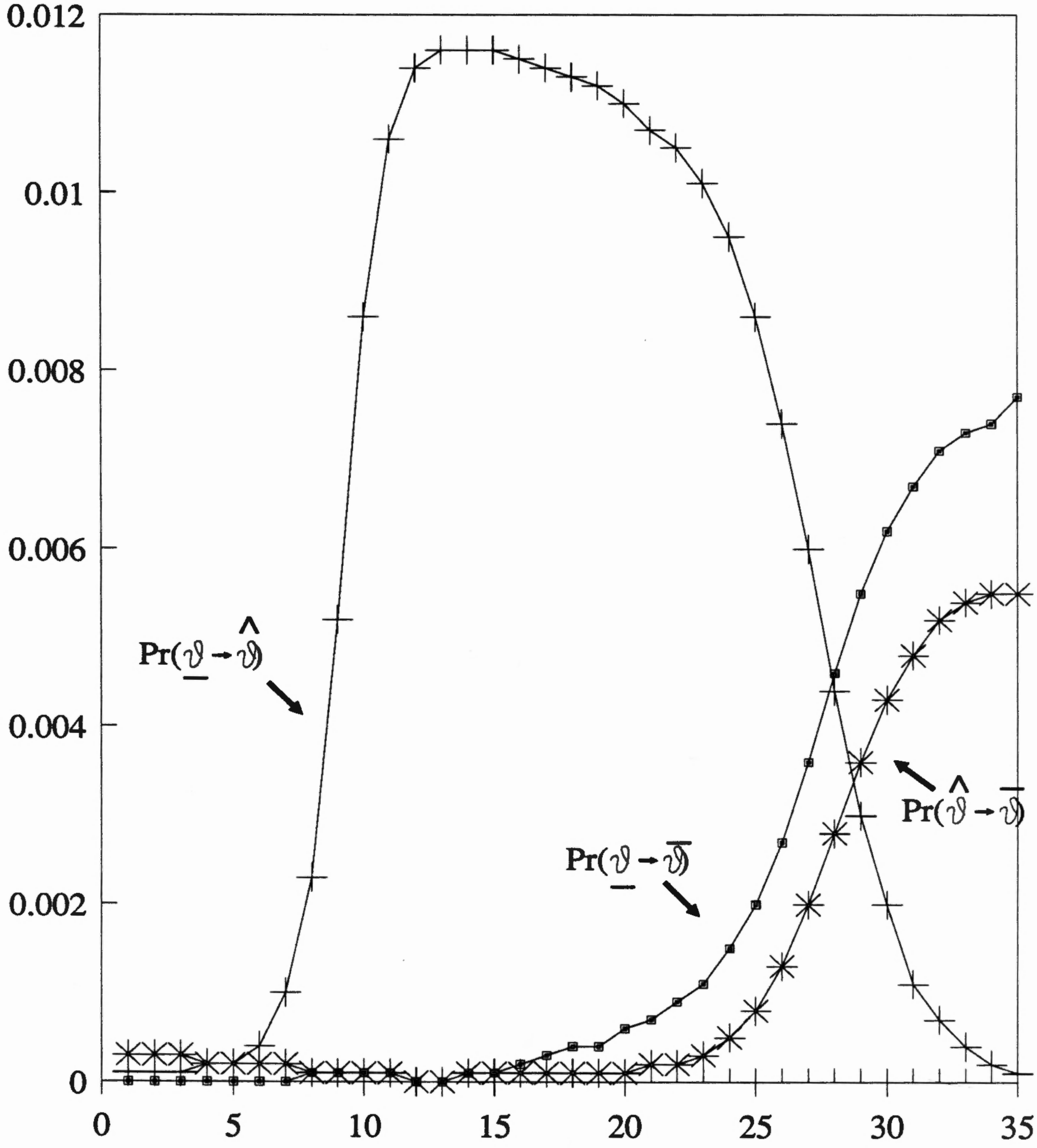
Finally, Figure 5 displays the heterogeneity in the industry over time as measured by the variance of industry output. The diversity generated by innovation, along with the eventual homogeneity–producing role of imitation, are readily apparent.

III A SPECIFIC MODEL

The previous section focused on a quite unrestricted model of innovation and imitation, and obtained some general results on existence and features of equilibrium in that environment. In this Section, a great deal of extra structure is imposed and much more specific conclusions obtained.

<u>Restrictions</u>. The key elements of the economic environment are product demand $D_t(Q_t)$, costs $c(q, \eta, \mu; \vartheta)$, the space of ideas Θ, possibilities for new ideas given existing ones $N(\vartheta' | \vartheta)$, the endowed ideas $F_0(\vartheta)$, and the impact of sampling procedures, $g(\cdot)$. These elements are restricted as follows: (i) for all t, $D_t(Q_t)$ is infinitely elastic at price p; (ii) $c(q, \eta, \mu; \vartheta) =$

Figure 4



$c_q q^2/2\vartheta + c_\eta \eta^2/2 + c_\mu \mu^2/2$, where c_q, c_η and c_μ are positive constants; (iii) $\Theta = \{\underline{\vartheta}, \bar{\vartheta}\}$ with $\bar{\vartheta} > \underline{\vartheta}$; (iv) $N(\underline{\vartheta}|\underline{\vartheta}) = 1-\delta$ for $\delta \in (0,1)$; (v) $F_0(\underline{\vartheta}) = 1$; and (f) $g(x) = 1 - (1-x)^{1/2}$.

The impact of these restrictions is as follows. Given $\Theta = \{\underline{\vartheta}, \bar{\vartheta}\}$, there are just two possible technologies. $F_0(\underline{\vartheta}) = 1$ requires that all firms have access only to $\vartheta = \underline{\vartheta}$ at the outset. Thus $\underline{\vartheta}$ will be referred to as the "old" technique, and $\bar{\vartheta}$ as the "new" one. The structure of g is such that if the fraction f of firms have access to $\vartheta = \bar{\vartheta}$, the probability of imitation given μ is is $\mu f^{\frac{1}{2}}$.

Fixing $p_t = p$ forces the environment to become "stationary"; in particular, it simplifies by causing the value of a firm that has access to the new technique to be constant over time: $V_t(\bar{\vartheta}) \equiv \bar{V}$. That is, given $\bar{\vartheta}$ there is no reason for the firm to choose η or μ not equal to zero [Section (II)], in which case the value of the firm is the discounted net revenue from production, the one period component of which is fixed for $p_t = p$.

The restrictions on $c(\cdot)$ are in line with, but stronger than, those that yielded $dq_t/d\vartheta > 0$ in Section II, the strengthening being purely for analytical tractability.

Finally, the structure placed on $N(\vartheta'|\vartheta)$ indicates that if a firm having access to the old technology $\underline{\vartheta}$ succeeds in inventing, the new idea will improve over the old with probability δ. The notion here is that there may be many technologies, but all yield production costs equal to one of $c_q q^2/2\underline{\vartheta}$ or $c_q q^2/2\bar{\vartheta}$. δ then indexes how difficult it is to come up with a cost reducing idea.

<u>Development of the Restricted Model</u>. Before proceeding, extra simplification is possible. Since $p_t = p$, profit maximization at t implies $q_t(\vartheta) = \vartheta p/c_q$ for any t. $q_t(\vartheta)$ will then be suppressed in what follows. Define

$$\bar{\pi} \equiv pq_t(\bar{\vartheta}) - c_q q_t^2(\bar{\vartheta})/2\bar{\vartheta} = \bar{\vartheta}p^2/2c_q$$

$$\underline{V}_t(f_t) = \underline{\pi} + \max_{(\eta,\mu)\in[0,1]^2} \left\{ -c_\eta \eta^2/2 - c_\mu \mu^2/2 + \beta \left[(1-\eta\delta)(1-\mu f_t^{\frac{1}{2}})\, \underline{V}_{t+1}(f_{t+1}) \right.\right.$$

$$\left.\left. + (\eta\delta + \mu f_t^{\frac{1}{2}} - \eta\delta\mu f_t^{\frac{1}{2}})\bar{V} \right] \right\}. \tag{8}$$

Notice that for a fixed value of $f_t \equiv f$ the environment is stationary over time. Thus the policies $\eta_t(\underline{\vartheta})$ and $\mu_t(\underline{\vartheta})$ ($\eta_t(\bar{\vartheta}) = \mu_t(\bar{\vartheta}) \equiv 0$ is taken as given, from Section II) can be written as the stationary policies $\eta(f)$ and $\mu(f)$. Moreover, the value of the firm also depends on f alone: $\underline{V}_t(f_t)$ can be written V(f). For given η and μ, let

$$f' \equiv f + (1-f)(\eta\delta + \mu f^{\frac{1}{2}} - \eta\delta\mu f^{\frac{1}{2}}) \tag{9}$$

be the "successor" to f. Then (7) becomes

$$V(f) = \underline{\pi} + \max_{(\eta,\mu)\in[0,1]^2} \left\{ -c_\eta \eta^2/2 - c_\mu \mu^2/2 + \beta \left[(1-\eta\delta)(1-\mu f^{\frac{1}{2}}) V(f') \right.\right.$$

$$\left.\left. + (\eta\delta + \mu f^{\frac{1}{2}} - \eta\delta\mu f^{\frac{1}{2}})\bar{V} \right] \right\}. \tag{10}$$

<u>Equilibrium</u>. In this stationary environment, an equilibrium is a triple of functions $\eta(f)$,and V(f) such that $\eta(f)$ and $\mu(f)$ yield the maximum on the right hand side of (10) when f' is given by (9) and $f \in [0,1]$.

An equilibrium exists, by Theorem 1. Moreover, in what follows the analysis will focus on interior solutions for η and μ, a sufficient condition for which—$\min\{c_\eta, c_\mu\} > \beta\bar{V}$—is imposed. This condition is also sufficient to guarantee that the equilibrium is symmetric (Theorem 2). Finally, it is tedious but not difficult to show the intuitive result that

It is easy to check that if (not only if) $f^{\frac{1}{2}}\Delta(f')$ is rising in f, $\eta(f)$ is monotonically decreasing in f, and $\mu(f)$ increasing. Referring back to (12) and (13), marginal returns to increasing μ are rising in $f^{\frac{1}{2}}\Delta(f')$, while marginal returns to η are falling. Intuitively, greater f makes it more likely that imitation rather than innovation will turn up $\vartheta = \bar{\vartheta}$. Provided the gain from learning $\vartheta = \bar{\vartheta}$—$\Delta(f') = \bar{V} - V(f')$—is not falling too quickly in f, greater f yields a substitution of imitation for innovation as the number of firms using $\vartheta = \bar{\vartheta}$ rises over time.

Must $f^{\frac{1}{2}}\Delta(f')$ increase in f? Since $f^{\frac{1}{2}}\Delta(f') = 0$ for $f = 0$ and $\Delta(1) > 0$, $f^{\frac{1}{2}}\Delta(f')$ must be increasing in f "on average." The Appendix provides a simple sufficient condition guaranteeing that $f^{\frac{1}{2}}\Delta(f')$ is increasing in f for all f. The difficulty is as follows. For $f \cong 0$ or $f \cong 1$, changes in f have little effect on the likelihood of imitation, in which case $V(f')$, and hence $\Delta(f')$, varies little with f. But for less extreme f, $\Delta(f')$ may be quite sensitive to f. The sufficient condition limits this sensitivity.[19] In what follows, to emphasize the dependence of $\Delta(f')$ on f, through (9), $\Delta(f')$ will be written $\tilde{\Delta}(f)$. Assuming differentiability of $\tilde{\Delta}$, $f^{\frac{1}{2}}\tilde{\Delta}$ increasing in f is equivalent to $f\tilde{\Delta}'/\tilde{\Delta} \geq -\frac{1}{2}$.

The basic result is then that, over time, firms for whom $\vartheta = \underline{\vartheta}$ devote more effort to imitation and less to innovation. Of course, those who have been successful ($\vartheta = \bar{\vartheta}$) do neither.

<u>Pure Imitation and Innovation</u>. Even in the very simple environment under consideration in this section, some entities of interest can behave in a relatively complicated manner; f_t is an important example. The sequence $\{f_t\}_0^\infty$ describes diffusion of new technology $\vartheta = \bar{\vartheta}$, a process that is of some interest. At this point all that can be said about diffusion is $f_{t+1} > f_t$, a direct consequence of $F_{t+1}(\vartheta) \leq F_t(\vartheta)$ and $\partial c/\partial\eta$ small for $\eta \simeq 0$ in the general model. The

---

[19] A great deal of numerical analysis of the two–state model has so far failed to yield an example where the sufficient condition is violated even though it seems that such a failure is a theoretical possibility.

1. Diffusion. How does new technology propagate in the population of firms? First, consider pure imitation. For $\delta \to 0$,[21]

$$\mu(f) \cong \frac{\beta f^{\frac{1}{2}}\tilde{\Delta}(f)}{c_\mu}, \tag{13}$$

$$\eta(f) \cong 0,$$

and

$$f' \cong f + (1 - f)f^{\frac{1}{2}}\mu. \tag{14}$$

Under pure imitation, only the small amount of innovation allows f ever to depart from zero. But subsequently, imitation forces f to unity.

The rate of adoption of new technology — $f' - f = (1-f)f^{\frac{1}{2}}\mu$ — is very small both early on ($f \approx 0$) and late ($f \approx 1$) in the industry's evolution. When is this rate a maximum? Let $\zeta = f\tilde{\Delta}'/\tilde{\Delta}$; $\zeta \in (-\frac{1}{2},0)$ for $f^{\frac{1}{2}}\tilde{\Delta}$ increasing in f, assumed earlier. If $f' - f$ is a maximum at some $\tilde{f}$, $\partial(f'-f)/\partial f = 0$ for $f = \tilde{f}$ must hold. Differentiation of (13) and (14) gives

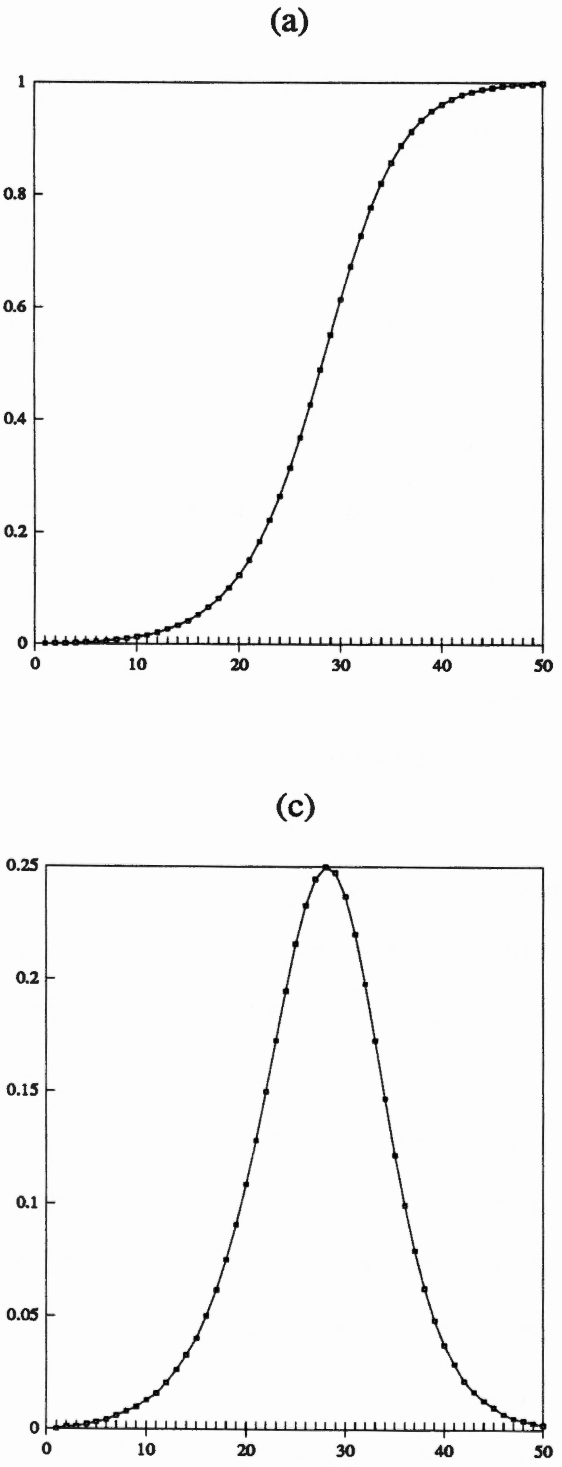
$$\tilde{f} = \frac{1+\zeta}{2+\zeta} \in (\tfrac{1}{3}, \tfrac{1}{2}).$$

That is, the maximum rate of new adoptions must occur when at least one third, but not a majority, of firms are already using the new technique: *Under pure imitation, diffusion must be "slow"* in this sense. To see why this result follows, suppose μ were fixed at some $\bar{\mu}$ for all f. Then the maximum rate of adoptions would occur at $\tilde{f} = \frac{1}{2}$. When μ is permitted to be a choice variable, additional forces come into play. New adoptions are $\tilde{f} - f = (1-f)f^{\frac{1}{2}}\mu = (1-f)f(\mu/f^{\frac{1}{2}}) \propto (1-f)f\tilde{\Delta}(f)$. The function $f(1-f)$ achieves a maximum at $f = \frac{1}{2}$, and $\tilde{\Delta}$ is declining in f, in which case $f' - f$ must peak before $f(1-f)$. (Equivalently, μ rises with f but at a rate

---

[21]The parameter ξ will be suppressed until some use is to be made of it.

Figure 6



captured in usual measures of R&D expenditures. However, it is not clear that this notion is correct. Indeed, the main empirical counterpart of expenditures on imitation is exactly that part of R&D associated with evaluation of products produced by others, in which case the measurement problem may well turn out to be more the difficulty of distinguishing between the two expenditure categories empirically. Assuming R&D expenditures include both costs of innovation and imitation, what does the model predict for such data?

Under pure imitation, industry expenditures on R&D are $(1-f)c_{\mu}\mu^2/2 = -f)\beta^2 f\tilde{\Delta}^2/2c_{\mu}$, which equals zero for f = 0 or 1. Differentiation with respect to f shows that this quantity does not vary with f for $f \cong 0$ (because $\mu \to 0$) and is strictly declining in f as f approaches unity. Moreover, it is at a maximum when

$$f = \frac{1+2\zeta}{2(1+\zeta)} \in (0, \tfrac{1}{2}).$$

$$\leq \frac{1+\zeta}{2+\zeta},$$

the latter being the value of f for which new adoptions are most rapid. Thus, maximum industry expenditure on R&D peaks before new adoptions. Moreover, industry expenditure on R&D, relative to variance of output, is falling in f.[22] Then, the f for which maximum expenditure occurs precedes that which maximizes variance of output. Over time then, industry heterogeneity, expenditures on R&D, and new adoptions all change very slowly at first, rise quickly, than fall to zero rapidly. However, expenditures peak first, then adoptions, then finally heterogeneity. These "ordering" results are somewhat sensitive to g. Indeed $g \equiv 1$ yields variance being maximized prior to expenditures, which peaks before adoptions. The invariant prediction is that expenditures peak before adoptions.

---

[22]That is

$$\frac{(1-f)\beta^2 f\tilde{\Delta}^2/2\, c_{\mu}}{f(1-f)(\bar{q} - q)^2} \propto \tilde{\Delta}^2.$$

and $\tilde{\Delta}$ is Falling in f.

The second issue pertains to the "productivity" of R&D expenditures. One index based on this concept is R&D expenditure per adoption subsequently generated. Under pure imitation, this value is proportional to $(1-f)\mu^2/[(1-f)f^{\frac{1}{2}}\mu] \propto \mu/f^{\frac{1}{2}}$ which declines over time (albeit slowly). Under pure innovation the same index is proportional to $(1-f)\eta^2/[(1-f)\eta\delta] \propto \eta$, which falls over time. The conclusion that expenditures per adoption falls under pure imitation depends on g. Indeed $g \equiv 1$ yields that index *rising* over time. The robust conclusion is that productivity falls off more quickly under pure innovation.

4. Parameter Changes

How do variations in underlying parameters affect the endogenous entities in the model?

For this question to be of interest it is necessary that equilibrium be unique. In the Appendix it is shown that equilibrium is always unique under pure innovation, and unique under pure imitation subject to a mild condition on $\mu(f)$. (The interpretation is thus that if there is a pure imitation equilibrium that satisfies the restriction, there is exactly one.) Since the results derived here are intuitive, their proofs, which are tedious, are left to the Appendix.

First, consider an increase in c_μ. Obviously, this change has no effect under pure innovation. So consider pure imitation. Therein, for all $f \in (0,1]$

$$\frac{\partial\mu(f)}{\partial c_\mu} < 0,$$

as would be expected. It then follows that f_t is lower for every t and that the maximum rate of new adoptions is lower and occurs later when c_μ rises. Moreover, the variance of output rises more slowly and also peaks later, although, due to the separability of costs, its maximum value does not depend on c_μ. Whether aggregate expenditures on R&D rise or fall with c_μ is generally indeterminate; i.e., whether the decline in μ more than offsets the increase in c_μ depends on parameter values.

too large. The difficulty is merely that since

$$\mu \equiv \frac{\beta f^{\frac{1}{2}}\xi\Delta(f')}{c_\mu},$$

if an increment to ξ has a large effect on f' (as may occur if f is moderate and μ not small), $\Delta(f')$ may decline greatly in response to a change in ξ. Generally, $\partial\mu/\partial\xi > 0$ is to be expected.

For pure innovation the situation is very similar. $\partial\eta/\partial\delta < 0$ is a possibility, but $\partial\eta/\partial\delta > 0$ is the leading case. Since Δ is almost constant for $\xi \to 0$ (yielding pure innovation) the only possible offset to $\partial\eta/\partial\delta > 0$ is the negative effect of δ on Δ.

Finally, as would be expected, in both pure models, greater β raises the investment in information (μ or η, as appropriate) with the obvious consequences for diffusion, R&D expenditures and heterogeneity.

IV TWO APPLICATIONS

The previous two sections developed a general model and then restricted it to obtain a series of sharper conclusions. This section presents two simple applications —diffusion of diesel locomotives in the U.S. railroad industry, and mechanization of loading coal in the U.S. underground mines — and illustrates some of the general issues on industry dynamics raised in the latter part of Section II.

1. Diffusion of Diesel Locomotives. The first usable diesel locomotive was invented by Rudolf Diesel in 1912 (Schmookler). Diesel locomotives were first used in the United States in 1925. By 1968 they had replaced steam locomotives entirely.[23]

This situation is an appropriate one for application of the two–state model of

---

[23]A few electric and "other" locomotives are ignored in what follows. As a group they never amounted to as much as 2% of the total.

Figure 7

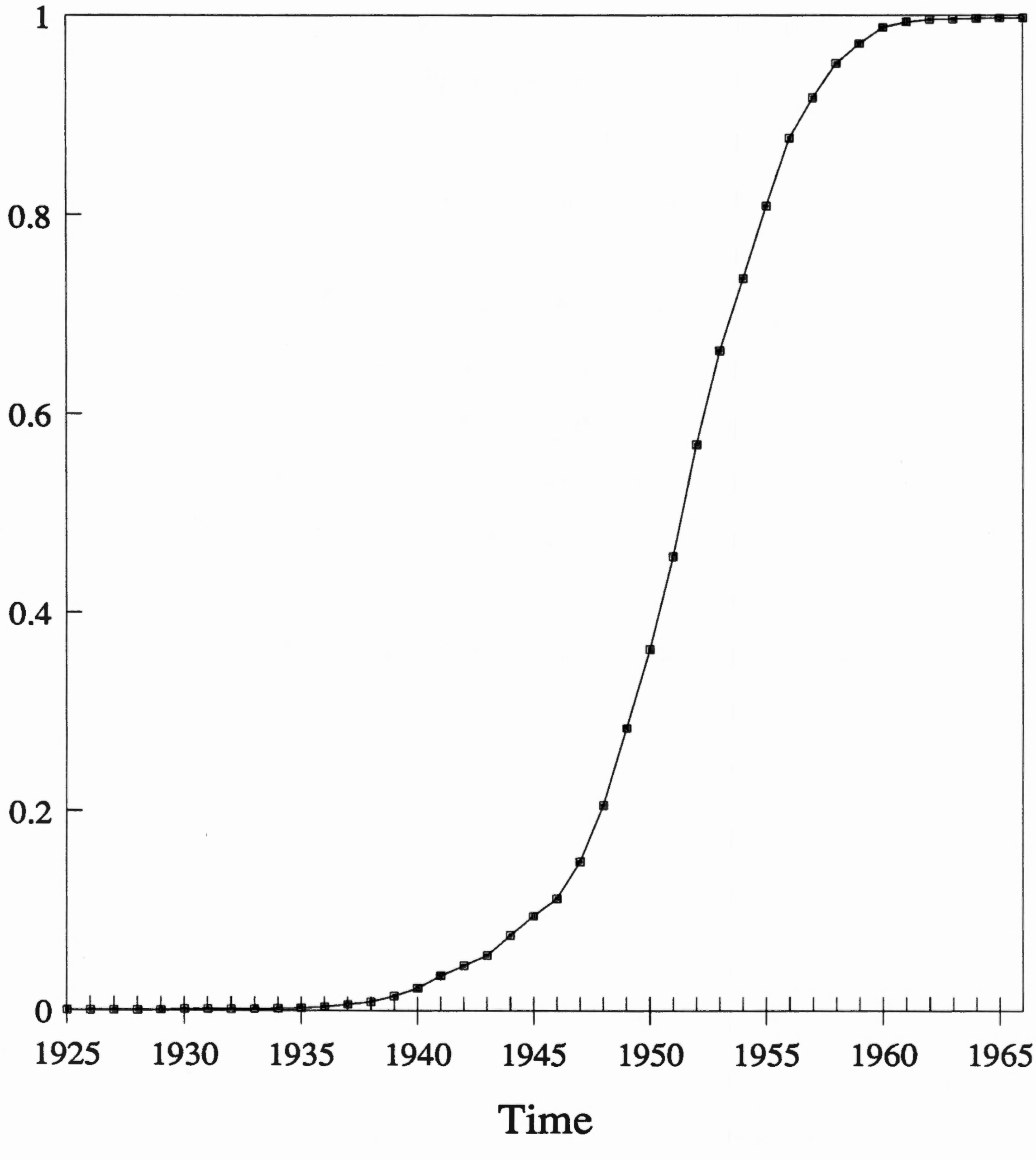
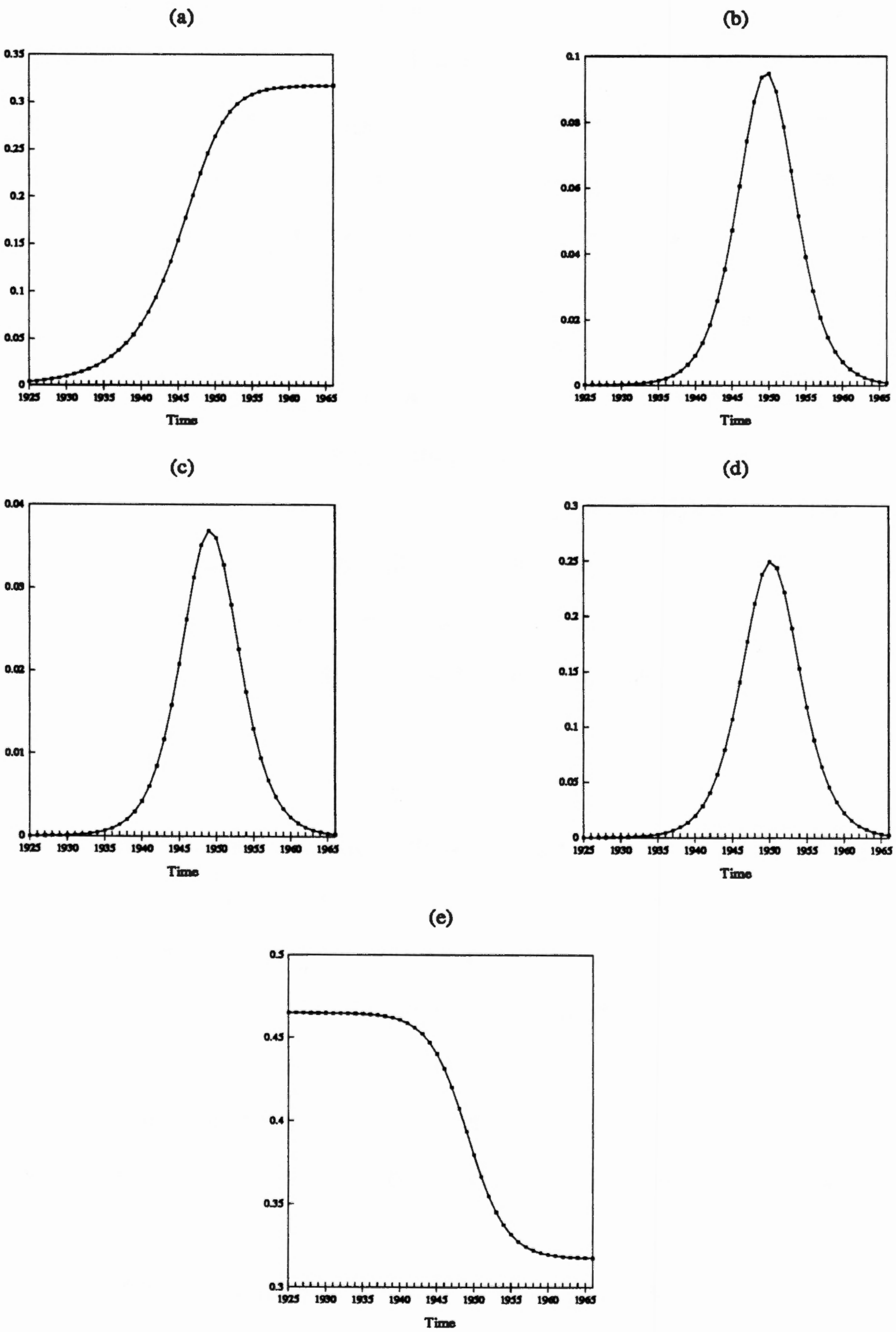

1
0.8
0.6
0.4
0.2
0
1925
1930
1935
1940
1945
1950
1955
1960
1965
Time

Figure 8


(a)
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
1925 1930 1935 1940 1945 1950 1955 1960 1965
Time
(b)
0.1
0.08
0.06
0.04
0.02
0
1925 1930 1935 1940 1945 1950 1955 1960 1965
Time
(c)
0.04
0.03
0.02
0.01
0
1925 1930 1935 1940 1945 1950 1955 1960 1965
Time
(d)
0.3
0.25
0.2
0.15
0.1
0.05
0
1925 1930 1935 1940 1945 1950 1955 1960 1965
Time
(e)
0.5
0.45
0.4
0.35
0.3
1925 1930 1935 1940 1945 1950 1955 1960 1965
Time

model—namely that differences across firms (here mines) renders it difficult to introduce new methods by simple imitation—appears a reasonable one in the mining context wherein there is substantial variation in the seams to be mined, etc.

Figure 9 provides the diffusion of mechanical loading of coal mined underground in the U.S. over 1924–69. In contrast to the markedly S–shaped diffusion of diesel locomotives, the mechanization of loading coal is slower and much more concave. (Compare figures 7 and 9.) This diffusion is not strictly stochastically increasing (i.e. $f_{t+1} > f_t$ fails five times over the sample period, two of the occasions during the Great Depression), but the departures are not major and mechanization does ultimately take over more or less completely.

As in the previous application, if all variation in these data is attributed to movements in the policy function η_t, the estimates of η_t very substantially. In contrast to the diesel case, in which measurement error appeared to be the major problem, in this case the Depression (1931–5 in particular) slows the rate of diffusion substantially, and there are less sizable slowdowns in both 1955–59 and 1962–64. As above, there are several methods for dealing with this issue. It turns out, again, that all yield basically the same conclusions. Only the results based on "smoothed" f_t are presented here.

The smoothing procedure uses the f_t predicted from the estimated equation

$$\ln[f_t/(1-f_t)] = -4.58 + \underset{(13.76)}{.3677t} - \underset{(6.30)}{.0086t^2} + \underset{(4.77)}{.00009\,t^3}$$

$$R^2 = .986$$

The hypotheses that may be confronted using these data are

1). Diffusion: $f_1 - f_0 > 0$, $f_t \rightarrow 1$, $f_{t+1} - f_t > 0$ and achieves a maximum at t=0;

2). Innovation: $\eta_{t+1} < \eta_t$ (Equivalent, under pure innovation, to falling productivity of R&D expenditures);

3). R&D Expenditures: achieve a maximum at t=0;

4). Heterogeneity: Variance of output achieves a maximum after both diffusion and R&D expenditures peak.

These hypotheses are confronted by noting that under pure innovation (normalizing δ to unity):

$$f'_t = f_t + (1-f_t)\eta_t,$$

in which case the time path of η_t, and variables proportional to R&D expenditures, diffusion and heterogeneity can be calculated. These data are illustrated in Figure 10 (panels (a)–(d)).

The basic diffusion hypotheses $f_{t+1} > f_t$ and $f_t \to 1$ have been discussed already. That diffusion is most rapid at the outset does not hold in these data, although it does peak early, when $f_t < \frac{1}{3}$. In particular, maximum diffusion occurs prior to maximum heterogeneity.

The most basic feature of the pure innovation model, $\eta_{t+1} < \eta_t$, is clearly evidenced in these data, as are the trivial (given $f_{t+1} > f_t$) additional implications that R&D expenditures should be strictly declining over time and that they should peak prior to maximum maximum heterogeneity.

APPENDIX

1. Monotonicity of $V_t(\vartheta)$, and the Reservation Value Property

Let ϑ be the current idea and $\mathcal{A}:\Theta \to \mathcal{B}(\Theta)$ be a correspondence from Θ to the Borel sets of Θ. $\mathcal{A}(\vartheta)$ is interpreted as the set of ideas that would be put to use at t+1 if discovered at t, when the current idea is ϑ. For each ϑ, $\mathcal{A}(\vartheta)$ is chosen by the firm . Let

$$T(V)(t,\vartheta) = \max_{q,\eta,\mu,\mathcal{A}} \left\{ p_t q - c(q,\eta,\mu;\vartheta) + \beta \left[\int_{\mathcal{A}(\vartheta)} V(t,\vartheta')\, \tau(d\vartheta';\eta,\mu,\vartheta,F_t) + V(t,\vartheta) \int_{\Theta\setminus\mathcal{A}(\vartheta)} \tau(d\vartheta';\eta,\mu,\vartheta,F_t) \right] \right\}$$

be an operator from the space of bounded functions of t and ϑ to itself. Define $T^k(V) = T[T^{k-1}(V)]$, for k=1,2,..., with $T^0(V) = T(V)$.

For any $V(t,\vartheta)$, $V_t(\vartheta) = \lim_{k\to\infty} T^k(V)(t,\vartheta)$. Let $V^0(t,\vartheta)$ be an arbitrary bounded function that is nondecreasing in ϑ. The maximization on the right hand side of T(V), for $V = V^0$, gives $\mathcal{A}(\vartheta) = \{\vartheta' \in \Theta \mid \vartheta' \geq \vartheta\}$. That is

$$\max_{q,\eta,\mu,\mathcal{A}} \left\{ p_t q - c(q,\eta,\mu;\vartheta) + \beta \left[\int_{\mathcal{A}(\vartheta)} V^0(t+1,\vartheta')\, \tau(d\vartheta';\eta,\mu,\vartheta,F_t) + V^0(t+1,\vartheta) \int_{\Theta\setminus\mathcal{A}(\vartheta)} \tau(d\vartheta';\eta,\mu,\vartheta,F_t) \right] \right\}$$

$$\leq \max_{q,\eta,\mu} \left\{ p_t q - c(q,\eta,\mu;\vartheta) + \beta \int_{\Theta} \max[V^0(t+1,\vartheta'), V^0(t+1,\vartheta')]\, \tau(d\vartheta';\eta,\mu,\vartheta,F_t) \right\}$$

$$= \max_{q,\eta,\mu} \left\{ p_t q_t - c(q,\eta,\mu;\vartheta) + \beta \left[\int_{\vartheta}^{\bar{\vartheta}} V^0(t+1,\vartheta')\, \tau(d\vartheta';\eta,\mu,\vartheta,F_t) \right.\right.$$

strictly improved on $\{\vartheta' \in \Theta \mid \vartheta' > \vartheta\}$, $V_t(\vartheta)$ is strictly increased. In particular V(f) is strictly rising in f.

3. A Sufficient Condition for $\mu(f)$ and $\eta(f)$ to be monotone: $f^{\frac{1}{2}}\Delta(f')$ Increasing in f.

Using the notation of Section III, define an operator on the space of bounded functions $V: [0,1] \to (-M,M)$, where $0 < M < \infty$, by

$$T(V)(f) \equiv \underline{\pi} + \max_{\eta,\mu}\Big\{-c_\eta\, \eta^2/2 - c_\mu\, \mu^2/2$$

$$+ \beta[(1-\eta\delta)(1-\mu f^{\frac{1}{2}})V(f')$$

$$+ (\eta\delta + \mu f^{\frac{1}{2}} - \eta\delta\mu f^{\frac{1}{2}})\bar{V}]\Big\},$$

where $f' = f + (1-f)(\eta\delta + \mu f^{\frac{1}{2}} - \eta\delta\mu f^{\frac{1}{2}})$ defines f' as a function of f. Also, define $\rho(f)$ as the precursor to f found by inverting the expression for f'; ie. $\rho(f') = f$. ($\rho(0) \equiv 0$, $\rho(f) < f$ and $d\rho/df > 0$). Then, with minor manipulation

$$\rho(f)^{\frac{1}{2}}[\bar{V} - T(V)(f)] = \rho(f)^{\frac{1}{2}}\Bigg[\bar{\pi} - \underline{\pi} - \max_{\eta,\mu}\Bigg\{-c_\eta\eta^2/2 - c_\mu\mu^2/2$$

$$- \frac{\beta(1-\eta\delta)(1-\mu f^{\frac{1}{2}})}{f^{\frac{1}{2}}} \cdot f^{\frac{1}{2}}[\bar{V} - V(f')]\Bigg\}\Bigg].$$

Next, define the operator $\tilde{T}(Y)(f)$ by

$$\tilde{T}(Y)(f) = \rho(f)^{\frac{1}{2}}\left[\bar{\pi} - \underline{\pi} - \max_{\eta,\mu}\left\{-c_\eta\eta^2/2 - c_\mu\mu^2/2 - \frac{\beta(1-\eta\delta)(1-\mu f^{\frac{1}{2}})}{f^{\frac{1}{2}}}\, Y(f')\right\}\right].$$

Observe that if $Y(f) = f^{\frac{1}{2}}[\bar{V} - V(f')]$ then $\tilde{T}(Y) = \rho(f)^{\frac{1}{2}}[\bar{V} - V(f)] > 0$, and for increasing V(f), $\rho(f) < f$ implies

$$\frac{\tilde{T}(Y)(f)}{\rho(f)^{\frac{1}{2}}} = \bar{V} - T(V)(f) > \bar{V} - T(V)(f') = \frac{Y(f')}{f^{\frac{1}{2}}} \qquad (*)$$

where, to simplify notation, it is taken as given that $\vartheta = \bar{\theta}$ implies $\eta_t = \mu_t = 0$. For each sequence $\{f_t\}$ the sequences of actions $\{\eta_t\}_0^\infty$ and $\{\mu_t\}_0^\infty$ are unique. Define

$$\lambda_t(\{f_t\}) \equiv \eta_t(\{f_t\})\delta + \mu_t(\{f_t\}) f_t^{\frac{1}{2}}[1-\eta_t(\{f_t\})\delta]$$

as the probability with which a firm currently knowing $\vartheta = \underline{\vartheta}$ discovers $\vartheta = \bar{\vartheta}$ at t, given $\{f_t\}$. Note that

$$f_{t+1} = f_t + (1-f_t)\lambda_t. \tag{**}$$

Let $\tilde{t} \equiv t - 1$ for $t > 1$. Let $\hat{\ell}_\infty[0,1]$ be the space of nondecreasing sequences converging to 1. Define the operator T^*: $\hat{\ell}_\infty[0,1] \to \hat{\ell}_\infty[0,1]$ by $T^*(\{f_t\})(t) = f_{\tilde{t}} + (1 - f_{\tilde{t}})\lambda_{\tilde{t}}$ for t = 1,2 ..., and $T^*(\{f_t\})(0) = 0$. Observe that for sequences $f^0 \equiv \{f_t^0\}$ and $f^1 = \{f_t^1\}$, $d_\infty(f^0, f^1) = \sup_t |f_t^0 - f_t^1|$ is a metric on $\hat{\ell}_\infty[0,1]$ and $(\hat{\ell}_\infty[0,1], d_\infty)$ is a complete metric space (completeness is straightforward, and the metric space property follows because it is a subset of $(\ell_\infty[0,1], d_\infty)$). Thus if T^* is a contraction mapping it has a unique fixed point. See for example, Harris (1987).

Since the sequences $\{f_t\}$ converge to unity, Blackwell's sufficient conditions for T^* to be a contraction mapping collapse to the requirements.

If $f_t^0 \geq f_t^1$ $\forall t$, $T^*(f^0)(t) \geq T^*(f^1)(t)$ $\forall t$. Again, see Harris. Under pure innovation, η_t, and hence λ_t, does not depend on f, in which case the sufficient condition is satisfied immediately. Under pure imitation, it is sufficient that sign $(f_t^0 - f_t^0)$ = sign $[\mu_t(\{f_t^1\})(f_t^0)^{\frac{1}{2}} - \mu_t(\{f_t^1\})(f_t^1)^{\frac{1}{2}}]$; i.e. that the parameters in the problem are such that in comparing f^0 to f^1, imitation is not so much easier under f^0, that μ_t not only falls (as it need not), but falls faster than $f^{\frac{1}{2}}$ rises.

5. Comparative Dynamics

The analysis of pure innovation is very similar to that of pure imitation. Consequently, only the latter is presented.

a) Increasing c_μ.

$\partial f'/\partial\xi = (1-f)f^{\frac{1}{2}}\mu > 0$ always obtains renders the result for general f ambiguous.

d) Increasing β.

The analysis of raising β is analogous to that of changing c_μ, and only very slightly different in detail (in contrast to the effect of changing the parameters in (b)). As indicated in the text $\partial\mu(f)/\partial\beta > 0$.

6. Parameter Values in Three State Example

a) Demand: $\beta_0 = 2$, $\beta_1 = .5$

b) Discount Factor: $\beta = .98$

c) Space of Ideas: $\vartheta = 1$, $\hat{\vartheta} = 5$, $\bar{\vartheta} = 15$

d) Distribution of New Ideas: $\delta = .0338$

e) Cost Function: $c_q = 1$, $c_\eta = c_\mu = 2100$.

References

Baumol, William J. *Economic Dynamics.* New York: Macmillan Co., 1970.

Becker, Gary S. "Family Economics and Macro Behavior," *American Economic Review*, 78 (March 1988), 1–13.

Bertsekas, Dimitri P. *Dynamic Programming and Stochastic Control.* New York: Academic Press, 1976.

Blackwell, David "Discounted Dynamic Programming," *Annals of Mathematical Statistics*, 36 (1965), 226–35.

Bound, J. et. al. "Who Does R&D and Who Patents?" in Griliches, Z. (ed) *R&D, Patents and Productivity*, Chicago,: University of Chicago Press, 1984.

David, Paul A. "A Contribution to the Theory of Diffusion," Stanford Research Center in Economic Growth, Memorandum No. 71, 1969.

Davies, S. *The Diffusion of Process Innovations.* Cambridge: Cambridge University Press, 1979.

Davis, Lance E., Gallman, Robert E., and Hutchins, Terese, D. "The Structure of the Capital Stock in Economic Growth and Decline," in Kilby, Peter (ed.) *Quantity and Quiddity: Essays in U.S. Economic History in Honor of Stanley Lebergott.* 1987.

Denison, E. *Why Growth Rates Differ.* Washington: The Brookings Institution, 1967.

Evans, D. "Tests of Alternative Theories of Firm Growth," *Journal of Political Economy*, 95 (August 1987), 657–73.

Futia, Carl A. "Schumpeterian Competition," *Quarterly Journal of Economics*, 94 (June 1980), 675–95

Green, Edward J. "Noncooperative Price Taking in Large Dynamic Markets," *Journal of Economic Theory*, 22 (1980), 155–82.

Green, Edward J. "Continuum and Finite–Player Noncooperative Models of Competition," *Econometrica*, 52 (July 1984), 975–95.

Griliches, Zvi "Hybrid Corn: An Exploration in the Economics of Technical Change," *Econometrica* 25 (1957), 501–22.

Griliches, Zvi "The Sources of Measured Productivity Growth: U.S. Agriculture 1940–1960," *Journal of Political Economy* 71 (1963), 331–46.

Griliches, Zvi "Issues in Assessing the Contribution of Research and Development to Productivity Growth," *Bell Journal of Economics*, 10 (Spring 1979), 92–116.

Ramey, Garey, "Preemption versus Learning in Product Innovation", 1988.

Reinganum, Jennifer F. "On the Diffusion of New Technology: A Game Theoretic Approach," *Review of Economic Studies*, 48 (July 1981).

Rogers, Everett M. and Shoemaker, F. Floyd *Communication of Innovations*. New York: Free Press, 1971.

Romer, Paul M. "Endogenous Technological Change" 1988.

Rose, Nancy L. and Joskow, Paul L. "The Diffusion of New Technologies: Evidence from the Electric Utility Industry," 1988.

Sahal, Devendra *Patterns of Technological Change*. London: Addison Wesley, 1981.

Salter, W.E.G. *Productivity and Technical Change*. Cambridge: Cambridge University Press, 1966.

Scherer, F.M. "Research and Development Resource Allocation Under Rivalry," *Quarterly Journal of Economics*, 81 (August 1967), 359–94.

Scherer, F.M. "Market Structure and Technological Innovation," Ch. 15 in Scherer, F.M. *Industrial Market Structure and Economic Performance*. Chicago: Rand McNally, 1970.

Schmookler, Jacob *Invention and Economic Growth*. Cambridge: Harvard University Press, 1966.

Schumpeter, Josef *The Theory of Economic Development*. Cambridge, Mass.: Harvard University Press, 1934.

Schumpeter, Josef *Business Cycles: A Theoretical, Historical and Statistical Analysis of the Capitalist Process*, 2 Vols. New York: McGraw Hill, 1939.

Schmeidler, D. "Equilibrium Points of Nonatomic Games," *Journal of Statistical Physics*, 7 (1973), 295–300.

Shefrin, H.M. "Games with Self–Generating Distributions," *Review of Economic Studies*, 48 (July 1981), 511–19.

Smith, Adam *The Wealth of Nations*. New York: Random House, 1937.

Solow, Robert M. "Technical Change and the Aggregate Production Function," *Review of Economics and Statistics*, 39 (1957), 312–20.

Solow, Robert M. "Growth Theory and After," *American Economic Review*, 78 (June 1988), 307–17.

Rochester Center for Economic Research
University of Rochester
Department of Economics
Rochester, NY 14627

1987-88 DISCUSSION PAPERS

WP#98 SUPPLY AND EQUILIBRIUM IN AN ECONOMY WITH LAND AND PRODUCTION
by Marcus Berliant and Hou-Wen Jeng, September 1987

WP#99 AXIOMS CONCERNING UNCERTAIN DISAGREEMENT POINTS FOR 2-PERSON BARGAINING PROBLEMS
by Youngsub Chun, September 1987

WP#100 MONEY AND INFLATION IN THE AMERICAN COLONIES: FURTHER EVIDENCE ON THE FAILURE OF THE QUANTITY THEORY
by Bruce Smith, October 1987

WP#101 BANK PANICS, SUSPENSIONS, AND GEOGRAPHY: SOME NOTES ON THE "CONTAGION OF FEAR" IN BANKING
by Bruce Smith, October 1987

WP#102 LEGAL RESTRICTIONS, "SUNSPOTS", AND CYCLES
by Bruce Smith, October 1987

WP#103 THE QUIT-LAYOFF DISTINCTION IN A JOINT WEALTH MAXIMIZING APPROACH TO LABOR TURNOVER
by Kenneth McLaughlin, October 1987

WP#104 ON THE INCONSISTENCY OF THE MLE IN CERTAIN HETEROSKEDASTIC REGRESSION MODELS
by Adrian Pagan and H. Sabau, October 1987

WP#105 RECURRENT ADVERTISING
by Ignatius J. Horstmann and Glenn M. MacDonald, October 1987

WP#106 PREDICTIVE EFFICIENCY FOR SIMPLE NONLINEAR MODELS
by Thomas F. Cooley, William R. Parke and Siddhartha Chib, October 1987

WP#107 CREDIBILITY OF MACROECONOMIC POLICY: AN INTRODUCTION AND A BROAD SURVEY
by Torsten Persson, November 1987

WP#108 SOCIAL CONTRACTS AS ASSETS: A POSSIBLE SOLUTION TO THE TIME-CONSISTENCY PROBLEM
by Laurence Kotlikoff, Torsten Persson and Lars E. O. Svensson, November 1987

WP#109 EXCHANGE RATE VARIABILITY AND ASSET TRADE
by Torsten Persson and Lars E. O. Svensson, Novmeber 1987

WP#110 MICROFOUNDATIONS OF INDIVISIBLE LABOR
by Vittorio Grilli and Richard Rogerson, November 1987

WP#111 FISCAL POLICIES AND THE DOLLAR/POUND EXCHANGE RATE: 1870-1984
by Vittorio Grilli, November 1987

WP#112 INFLATION AND STOCK RETURNS WITH COMPLETE MARKETS
by Thomas Cooley and Jon Sonstelie, November 1987

WP#129 POST-SAMPLE PREDICTION TESTS FOR GENERALIZED METHOD OF MOMENT ESTIMATORS
by Dennis Hoffman and Adrian Pagan, April 1988

WP#130 GOVERNMENT SPENDING IN A SIMPLE MODEL OF ENDOGENOUS GROWTH
by Robert J. Barro, May 1988

WP#131 FINANCIAL DEVELOPMENT, GROWTH, AND THE DISTRIBUTION OF INCOME
by Jeremy Greenwood and Boyan Jovanovic, May 1988

WP#132 EMPLOYMENT AND HOURS OVER THE BUSINESS CYCLE
by Jang-Ok Cho and Thomas F. Cooley, May 1988

WP#133 A REFINEMENT AND EXTENSION OF THE NO-ENVY CONCEPT
by Dimitrios Diamantaras and William Thomson, May 1988

WP#134 NASH SOLUTION AND UNCERTAIN DISAGREEMENT POINTS
by Youngsub Chun and William Thomson, May 1988

WP#135 NON-PARAMETRIC ESTIMATION AND THE RISK PREMIUM
by Adrian Pagan and Y. Hong, May 1988

WP#136 CHARACTERIZING THE NASH BARGAINING SOLUTION WITHOUT PARETO-OPTIMALITY
by Terje Lensberg and William Thomson, May 1988

WP#137 SOME SIMULATION STUDIES OF NON-PARAMETRIC ESTIMATORS
by Y. Hong and A. Pagan, June 1988

WP#138 SELF-FULFILLING EXPECTATIONS, SPECULATIVE ATTACKS AND CAPITAL CONTROLS
by Harris Dellas and Alan C. Stockman, June 1988

WP#139 APPROXIMATING SUBOPTIMAL DYNAMIC EQUILIBRIA: AN EULER EQUATION APPROACH
by Marianne Baxter, June 1988

WP#140 BUSINESS CYCLES AND THE EXCHANGE RATE SYSTEM: SOME INTERNATIONAL EVIDENCE
by Marianne Baxter and Alan C. Stockman, June 1988

WP#141 RENT SHARING IN AN EQUILIBRIUM MODEL OF MATCHING AND TURNOVER
by Kenneth J. McLaughlin, June 1988

WP#142 CO-MOVEMENTS IN RELATIVE COMMODITY PRICES AND INTERNATIONAL CAPITAL FLOWS: A SIMPLE MODEL
by Ronald W. Jones, July 1988

WP#143 WAGE SENSITIVITY RANKINGS AND TEMPORAL CONVERGENCE
by Ronald W. Jones and Peter Neary, July 1988

WP#144 FOREIGN MONOPOLY AND OPTIMAL TARIFFS FOR THE SMALL OPEN ECONOMY
by Ronald W. Jones and Shumpei Takemori, July 1988

To order copies of the above papers complete the attached invoice and return to Christine Massaro, W. Allen Wallis Institute of Political Economy, RCER, 109B Harkness Hall, University of Rochester, Rochester, NY 14627. Three (3) papers per year will be provided free of charge as requested below. Each additional paper will require a $5.00 service fee which must be enclosed with your order. For your convenience an invoice is provided below in order that you may request payment from your institution as necessary. Please make your check payable to the **Rochester Center for Economic Research. Checks must be drawn from a U.S. bank and in U.S. dollars**.

W. Allen Wallis Institute for Political Economy

Rochester Center for Economic Research, Working Paper Series

OFFICIAL INVOICE

Requestor's Name ______________________________

Requestor's Address ______________________________

____

____

____

Please send me the following papers free of charge (**Limit: 3 free per year**).

WP# ______ WP# ______ WP# ______

I understand there is a $5.00 fee for each additional paper. Enclosed is my check or money order in the amount of $__________. Please send me the following papers.

WP# ______ WP# ______ WP# ______

WP# ______ WP# ______ WP# ______

WP# ______ WP# ______ WP# ______

WP# ______ WP# ______ WP# ______