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Introduction

The purpose of this note is to present an algorithm for calculating an efficiency bound among a class of estimators for a continuous time financial economics model examined by Grossman, Melino, and Shiller (1987), Hall (1987), and Hansen and Singleton (1987) and the martingale taxation model examined by Barro (1981). In these continuous time models there are two-period conditional moment restrictions that result from time averaging the underlying continuous time processes and then sampling these averages. These conditional moment restrictions can be used to estimate an unknown parameter vector using the *generalized method of moments (GMM)* procedure set out by Hansen (1982). There is a vast array of *GMM* estimators that can be used to estimate the parameter vector. Hence it is of interest to compare the performance of the alternative *GMM* estimators. We apply a method developed by Hansen, Heaton, Ogaki (1988) to calculate a greatest lower bound for the asymptotic covariance matrices of the alternative *GMM* estimators, i.e. an efficiency bound.[1]

In the continuous time models cited above, there is a martingale restriction. When only time-averaged discrete time data is available, the martingale restriction results in a two-period conditional moment restriction. This restriction leads to an econometric disturbance. Under certain assumptions this disturbance has a first-order autocorrelation which is known *a priori* as Working (1960) pointed out. This knowledge implies another two-period conditional moment restriction. The second moment restriction induces conditional heteroskedasticity in the disturbance. We provide an algorithm to calculate the efficiency bound for *GMM* estimators

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[1] In this paper we follow the notation of Hansen et al. (1988) as closely as possible.

when both of these conditional moment restrictions are used. Hansen and Singleton (1987) calculated the efficiency bound using our algorithm. They also calculated the efficiency bound when only the first moment restriction is used. The algorithm they used for this case exploits the analysis in an example of Hansen (1985) which assumes conditional homoskedasticity. The calculations of Hansen and Singleton showed that there is a notable efficiency gain from the imposition of the second moment restriction for their model.

Calculating Efficiency Bounds

Consider a q-dimensional random vector Gaussian process $\{x_t:-\infty<t<\infty\}$. Let $\{w_t:-\infty<t<\infty\}$ denote a q-dimensional random vector process that is fundamental for x_t in the sense of linear prediction theory. In other words, the entries of w_t are orthonormal, the process $\{w_t:-\infty<t<\infty\}$ is serially uncorrelated, and w_t and x_t are informationally equivalent from the vantage point of linear least squares prediction [e.g. see Rozanov (1967)].

We can view that the stochastic process w_t is generated from a random variable w on a probability space (Ω, A, Pr) in conjunction with a transformation S mapping Ω onto Ω in the following manner (see Breiman [1968] pp. 106-120). The stochastic process $\{w_t:-\infty<t<\infty\}$ is constructed from the random vector $w=w_0$ via $w_t(\omega) = w[S^t(\omega)]$ for $-\infty<t<\infty$. Here S^t is interpreted as the transformation S applied t times for positive values of t. In addition we can take the transformation S so that S is one-to-one, measurable, measure-preserving, and ergodic and S^{-1} is measurable. Since S is one-to-one $w[S^t(\omega)]$ is also well-defined for negative values of t. Suppose that h is a random vector on Ω. Then h in conjunction with S generates a stochastic process via $h_t(\omega) = h[S^t(\omega)]$. Throughout this note we find it convenient to index stochastic processes constructed in this fashion.

For notational convenience, we define $y' \equiv [x', x_{-1}']$. Let B_t be the sigma algebra generated by y_t, y_{t-1}, ... Since x is Gaussian, best linear predictors coincide with conditional expectations and w is normally distributed and independent of B_{-1}.

Suppose that a particular linear combination of x is a two-period-ahead forecast error. More precisely, $E\{u| B_{-2}\} = 0$ where $u = [1\ \beta_0\ \ 0]x$ and β_0 is a scalar parameter to be estimated. In addition,

$$u = [1 \quad \beta_0 \quad 0]x \quad = \quad \nu_0 \cdot w \ + \ \nu_1 \cdot w_{-1} \tag{1}$$

where ν_0 and ν_1 are q-dimensional vectors of real numbers. A second restriction is that $E(u\ u_{-1}|B_{-2})/E(u^2|B_{-2}) \ = \ \nu_0 \cdot \nu_1/(\nu_0 \cdot \nu_0 + \nu_1 \cdot \nu_1) \ = \ \rho$, or equivalently that

$$E(\rho u^2 \ - \ u\ u_{-1}|B_{-2}) \quad = \quad 0 \tag{2}$$

where ρ is .25. The law of motion for the process x can be quite complicated and may depend on a large (or even infinite) number of auxiliary parameters. There is only one parameter of interest, namely β_0, and accordingly we focus on efficiency bounds for that parameter.

Define the disturbance vector

$$e = \begin{bmatrix} u \\ \rho u^2 \ - \ u\ u_{-1} \end{bmatrix}, \tag{3}$$

and a function

$$\phi(y,\beta) \quad = \quad \begin{bmatrix} [1\ \beta\ 0]x \\ \rho([1\ \beta\ 0]x)^2 \ - \ ([1\ \beta\ 0]x)([1\ \beta\ 0]x_{-1}) \end{bmatrix}. \tag{4}$$

The conditional moment restrictions take the form $E(e|B_{-2}) \ = \ 0$ where $e \equiv \phi(y,\beta_0)$ is the disturbance vector. Notice that the model is nonlinear in both the parameters and the variables even though the underlying process x is Gaussian. The entries of e are either normally distributed random variables or products of normally distributed random variables. Hence these entries have moments of all orders.

We construct estimators of β_0 as follows: Let z be an n × k matrix of random variables that are measurable with respect to the time -2 information set B_{-2}. In addition, suppose that $E|z|^{\eta}$ is finite for some η; then $E(z'e) = 0$. We refer to a consistent estimator $\{b_T: T\geq 1\}$ of β_0 for which $\{(1/T)^{1/2}\sum_{t=1}^{T}[z'_t\phi\ (y_t, b_T)]\ :\ T\geq 1\}$ is $o_p(1)$ (converges in probability to zero) as a *GMM* estimator with index z. Such estimator can also be viewed as an *M* estimator and as an instrumental variables estimator where z' is a matrix of instrumental variables.

Since there is great flexibility in the selection of z we have at our disposal a rich class of estimators of β_0. We find it convenient to introduce an index set for a family of such estimators: Let B^{τ} be the subsigma algebra of B_{-2} that is generated by $x, x_{-1}, \ldots, x_{-\tau}$, and

$$Z^{\tau} = \{z : z \text{ is an n} \times \text{k matrix of random variables that are measurable with respect to } B^{\tau} \text{ and } E(|z|^{\eta})<\infty\} .$$

Indexes in the space Z^{τ} are constructed using functions of the current and τ lags of the vector x. The specification of the finite lag τ is both arbitrary and inconvenient. For this reason, we consider the larger index set $Z = \text{U}_{\tau=1}^{\infty} Z^{\tau}$. This space is a linear subspace of $L^{\eta}(Pr)$ but is not necessarily closed.

Let $d = \partial\ \phi(y, \beta_0)/\partial\beta$, $B^* = B_{-2}$, and $d^* = E(d|B^*)$. The matrix of random variables d is given by

$$d = \begin{bmatrix} \dfrac{[0\ 1\ 0]x}{2\rho([1\ \beta_0\ 0]x)([0\ 1\ 0]x) - ([1\ \beta_0\ 0]x)([0\ 1\ 0]x_{-1}) \\ - ([0\ 1\ 0]x)([1\ \beta_0\ 0]x_{-1})} \end{bmatrix}.$$

The entries of both d and e are either normally distributed random variables or products of normally distributed random variables. Hence these entries have moments of all orders. Let η be any positive number greater than 2, then Assumption 4 of Hansen et al. (1988) is satisfied.

As discussed in Hansen et al. (1988), a *GMM* estimator b_T indexed by z in Z has a limiting normal distribution with asymptotic covariance matrix:

$$Cov\ (z) = [E(z'd^*)]^{-1}[\sum_{\tau=-1}^{1} z'ee_\tau{}'z_\tau][E(z'd^*)']^{-1} \tag{5}$$

as long as $E(z'd^*)$ is nonsingular. When $E(z'd^*)$ is singular, we define $Cov(z) = \infty$. Relation (5) gives a mapping *Cov* from the index set Z into the collection, *PSD*, of $k \times k$ positive semidefinite matrices augmented by the point infinity. The set *PSD* can be partially ordered as follows: THe inequality $c \le c^*$ is satisfied for c and c^* in *PSD* if c^*-c is in *PSD*. Let *LB* be the subset of *PSD* containing all matrices c that satisfy $c \le Cov(z)$ for all z in Z. The efficiency bound $Inf(Z)$ is defined to be the maximal element of *LB* assuming such a maximal element exists.

We calculate the efficiency bound in four steps. In the first step we obtain the conditional forward moving-average representation for e given in Lemma 4.1 of Hansen et al. (1988). Applying the lemma we know that e can be represented as $e = \lambda_0^0 e_0^+ + \lambda_1^1 e_1^+$, where λ_0^0 and λ_0^1 are 2×2 matrix of random variables that are measurable with respect to B^* and have finite second moments, λ_0^0 is nonsingular almost surely, and $E(e^+ e_j^+{}' | B^*) = 0$ for all $j \neq 0$.

It turns out that e is conditionally heteroskedastic. In particular, calculating from the definition of e and relation (1),

$$E(ee'|B^*) = \left[\begin{array}{c|c} \mu_{11} & \chi_{11}\cdot w_{-2} \\ \hline \chi_{11}\cdot w_{-2} & \mu_{12} + \mu_{11}(\nu_1\cdot w_{-2})^2 \end{array}\right], \text{and} \qquad (6)$$

$$E(ee'_{-1}|B^*) = \left[\begin{array}{c|c} \mu_{21} & \chi_{12}\cdot w_{-2} + \chi_{21}\cdot w_{-3} \\ \hline \chi_{21}\cdot w_{-2} & \mu_{22} + \chi_{22}(\nu_1\cdot w_{-2})^2 + \mu_{21}(\nu_1\cdot w_{-2})(\nu_0\cdot w_{-2}) \\ & + \mu_{21}(\nu_1\cdot w_{-2})(\nu_1\cdot w_{-3}) \end{array}\right]. \qquad (7)$$

The nonrandom μ's and χ's are functions of ν_0 and ν_1: $\mu_{11} \equiv (\nu_0\cdot\nu_0) + (\nu_1\cdot\nu_1)$, $\chi_{11} \equiv -[(\nu_0\cdot\nu_0) + (\nu_1\cdot\nu_1)]\nu_1$, $\mu_{12} \equiv (\nu_0\cdot\nu_0)^2 - (\nu_1\cdot\nu_0)^2 + (\nu_0\cdot\nu_0)(\nu_1\cdot\nu_1)$, $\mu_{21} \equiv \nu_1\cdot\nu_0$, $\chi_{12} \equiv 2\rho(\nu_0\cdot\nu_1)(\nu_1 - \nu_0)$, $\chi_{21} \equiv -(\nu_0\cdot\nu_1)\nu_1$, $\mu_{22} \equiv 2\rho\ [\rho\ (\nu_1\cdot\nu_0)^2 - (\nu_1\cdot\nu_0)(\nu_0\cdot\nu_0)]$, and $\chi_{22} \equiv -2\rho(\nu_1\cdot\nu_0)$.

Note that the second component of e introduces conditional heteroskedasticity. As a consequence, λ^0 and λ^1 depend on conditioning information. Suppose that λ^0 and λ^1 have the form

$$\lambda^0 = \begin{bmatrix} \xi_{11} & 0 \\ \alpha_{11}\cdot w_{-2} & \xi_{12} \end{bmatrix} \text{ and} \qquad (8)$$

$$\lambda^1 = \begin{bmatrix} \xi_{21} & 0 \\ \alpha_{21}\cdot w_{-2} + \alpha_{22}\cdot w_{-3} & \xi_{22} \end{bmatrix}. \qquad (9)$$

where the ξ's and the α's are nonrandom. We require $|\xi_{21}|<|\xi_{11}|$ and $|\xi_{22}|<|\xi_{12}|$ so that $\lambda_0^0 e_0^+ + \lambda_1^1 e_1^+$ has the same informational content as e. We can calculate the conditional autocovariances of $e = \lambda_0^0 e_0^+ + \lambda_1^1 e_1^+$ as a function of the parameters of λ^0 and λ^1 in equations (8) and (9). Matching these autocovariances with the conditional autocovariances calculated by the definition of e and relation (1),

$$(\xi_{11})^2 + (\xi_{21})^2 = \nu_0\cdot\nu_0 + \nu_1\cdot\nu_1, \tag{10}$$

$$\xi_{11}\xi_{21} = \nu_0\cdot\nu_1, \tag{11}$$

$$\alpha_{11} = -\nu_1(\nu_1\cdot\nu_0)/\xi_{21}, \tag{12}$$

$$\alpha_{21} = (\nu_1\cdot\nu_0)(2\rho\nu_1 - \nu_0)/\xi_{11}, \tag{13}$$

$$\alpha_{22} = -\nu_1(\nu_1\cdot\nu_0)/\xi_{11}, \tag{14}$$

$$(\xi_{12})^2 + (\xi_{22})^2 = (\nu_0\cdot\nu_0)^2 + (\nu_0\cdot\nu_0)(\nu_1\cdot\nu_1) - (\nu_1\cdot\nu_0)^2 - (\xi_{21})^2(2\rho\nu_1 - \nu_0)\cdot(2\rho\nu_1 - \nu_0) \tag{15}$$

and

$$\xi_{12}\xi_{22} = 2\rho^2(\nu_1\cdot\nu_0)^2 - 2\rho(\nu_1\cdot\nu_0)(\nu_0\cdot\nu_0). \tag{16}$$

Equations (10), (11), (15) and (16) can be solved for the parameters ξ_{11}, ξ_{12}, ξ_{21}, and ξ_{22} subject to the restrictions that $|\xi_{21}| < |\xi_{11}|$ and $|\xi_{22}| < |\xi_{12}|$. Equations (12)-(14) then determine α_{11}, α_{21}, and α_{22}. Thus for these values of the parameters λ^0 and λ^1 in (8) and (9) give a conditional forward representation.

In the second step we calculate the operators Ψ and Ψ^- defined by Hansen et al. (1988). Let $\psi^0 = (\lambda^0)^{-1}$ and $\psi^j = -(\lambda^0_{-j})^{-1}\sum_{\tau=1}^{j}(\lambda^{\tau}_{\tau-j})^{-1}(\psi^{j-\tau})$ for j≥1 where $\lambda^j = 0$ for j≥2. Let D^+ be the set of all n × k matrices of random variables that are measurable with respect to B^*. Let D be the subset of D^+ for which $E[(\psi^j_j)z_j|B^*]$ is well defined for each j and $\Psi(z) = \sum_{j=0}^{\infty}E[(\psi^j_j)z_j|B^*]$ converges in $L^2(Pr)$. Similarly let D^- be the subset of D^+ for which $\Psi^-(z) = \sum_{j=0}^{\infty}(\psi^j)'z_{-j}$ converges in $L^{\eta}(Pr)$. Then D and D^- are the domains of the operators Ψ and Ψ^-, respectively. Using (8) and (9), ψ^0 and ψ^j are given by:

$$\psi^0 = \begin{bmatrix} 1/\xi_{11} & 0 \\ (\nu_1\cdot w_{-2})/\xi_{12} & 1/\xi_{22} \end{bmatrix}, \text{ and} \tag{17}$$

$$\psi^j = \left[\begin{array}{c|c} (\varphi_1)^j/\xi_{11} & 0 \\ \hline -\sum_{\tau=1}^{j} (\varphi_1)^{\tau-1}(\varphi_2)^{j-\tau}\ (\alpha_{21}\cdot w_{-\tau-1})/(\xi_{11}\xi_{12}) & (\varphi_2)^j/\xi_{22} \\ +\ (\varphi_2)^j(\nu_1\cdot w_{-2})/\xi_{12} & \end{array}\right] \tag{18}$$

for j = 1, 2, 3, ... where $\varphi_1 \equiv -\xi_{21}/\xi_{11}$ and $\varphi_2 \equiv -\xi_{22}/\xi_{12}$. Since $|\varphi_1|<1$ and $|\varphi_2|<1$, and the entries of w and d have moments of all orders, it follows that $E(d|B^*) = d^*$ is in D and Assumption 8 of Hansen et al. is satisfied. In addition $\Psi(d^*)$ has moments of all orders which implies that $\Psi(d^*)$ is in D^-. It follows from Corollary 4.1 of Hansen et al. that $Inf(Z) = 1/\{E[\Psi(d^*)'\Psi(d^*)]\}$.

In the third step we calculate $\Psi(d^*)$ which requires evaluating expectations of $\psi_j^j d_j$ conditioned on B^* for j=0,1,2,... . To perform these calculations requires more information about the process x than has been assumed so far. This computation is straightforward when x has a state space representation:

$$Y = A\,Y_{-1} + C\,w \quad \text{and} \quad x = H\,Y, \tag{19}$$

where Y is a p by 1 state vector, A is a p by p matrix with eigenvalues that have moduli that are less one, and C and H are p by q matrices of real numbers. In this case $\Psi(d^*)$ is given by:

$$\Psi(d^*) = \begin{bmatrix} Q\cdot Y_{-2} \\ R_1 + w_{-2}'R_2Y_{-2} + Y_{-2}'R_3Y_{-2} \end{bmatrix} \tag{20}$$

where Q is a p-dimensional vector of real numbers, R_1 is a scalar real number, and R_2 and R_3 are p by p matrices of real numbers. The definitions for these parameters are as follows. Let $H_1 \equiv [1\ \beta_0\ 0]H$ and $H_2 \equiv [0\ 1\ 0]H$.

Then

$$Q \equiv H_2(I - \varphi_1 A)^{-1}A^2/\xi_{11}$$

$$R_1 \equiv \nu_1\varphi_2 C' A^{3\prime}(I - \varphi_2 A')^{-1}H_2'/\xi_{12} - \alpha_{21}C' A^{2\prime}(I - \varphi_2 A')^{-1}H_2'/[\xi_{21}\xi_{11}(1-\varphi_1)]$$

$$+ \Big\{2\rho H_1 CC' H_2' + 2\rho H_1 \sum_{j=0}^{\infty} \varphi_2^j A^{j+1}CC' A^{j+1\prime} H_2'$$

$$- H_2 \sum_{j=0}^{\infty} \varphi_2^j A^{j+1}CC' A^{j\prime} H_1' - H_1 \sum_{j=0}^{\infty} \varphi_2^j A^{j+1}CC' A^{j\prime} H_2' \Big\}/[\xi_{12}(1-\varphi_2)],$$

$$R_2 \equiv \nu_1 H_2 A^2/\xi_{12} \text{ , and}$$

$$R_3 \equiv \sum_{j=0}^{\infty} (\varphi_2)^j \Big\{2\rho A^{2+j\prime} H_2' H_1 A^{2+j} - A^{1+j\prime} H_1' H_2 A^{2+j} - A^{1+j\prime} H_2' H_1 A^{2+j} \Big\}/\xi_{12}.$$

Note that the first entry of $\Psi(d^*)$ is a normally distributed random vector while the second entry is the translated sum of two quadratic forms of normally distributed random vectors.

In the final step we calculate $E[\Psi(d^*)'\Psi(d^*)]$. This involves computing up to fourth moments of Y and w using formulas for normally distributed random vectors. The efficiency bound is then given by $1/E[\Psi(d^*)'\Psi(d^*)]$. Thus the efficiency bound turns out to be insensitive to the choice of η, though the magnitude of η affects the size of the index set Z.

It is also possible to estimate β_o using just the first conditional moment restriction and ignoring relation (2). There is a corresponding loss in asymptotic efficiency since the resulting efficiency bound is the reciprocal of the second moment of the first entry of $\Psi(d^*)$. As discussed in the Introduction, Hansen and Sigleton (1987) found that this loss is substantial for their model.

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