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Tax Analysis in a Dynamic Stochastic Model on Measuring Harberger Triangles and Okun Gaps

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TAX ANALYSIS IN A DYNAMIC STOCHASTIC MODEL:
ON MEASURING HARBERGER TRINAGLES AND OKUN GAPS

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ABSTRACT

A stochastic representative agent model is analyzed in which the allocations are not necessarily Pareto–optimal. It is shown that a variation on traditional dynamic programming techniques can be employed to obtain a solution of the model. The economy is then specialized to study the impact of various distortional government tax and subsidy schemes. The presence of these government policies makes the resulting allocations non–optimal. The dynamic impact of varying various tax or subsidy rates is analyzed. It is shown that the government can use its policy tools to stabilize the cyclical fluctuations, and this is done for the economy under study. The benefits of implementing such a policy are calculated and are compared with the size of the welfare gains realized by reducing various tax distortions.

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"It takes a heap of Harberger Triangles to fill Okun Gap"

Tobin (1977)

"Our task as I see it is to write a FORTRAN program that will accept specific economic policy rules as 'input' and will generate as 'output' statistics describing the operating characteristics of time series we care about, which are predicted to result from these policies"

Lucas (1980)

I. INTRODUCTION

There is a voluminous public finance literature that studies the macroeconomic effects of distortional taxation and subsidy schemes which are currently used by governments. This analysis is usually undertaken within the context of static models, or models in which agents have perfect foresight. This may seem strange to many because an important feature of observed economies is that they are truly dynamic and there is a great deal of uncertainty about future economic events. The fact that these important issues are not being analyzed within the context of dynamic stochastic models is surprising for two further reasons. First, recent advances in the development of these models have made it much easier to address a wide range of issues within the context of these types of economies. Second, there are important policy questions which can only be answered with a model with these features. For instance, it may be desirable to know how a proposed fiscal policy will impact on the variability of macro aggregates over the course of the business cycle, or for that matter, whether the business cycle could actually be stabilized by some proposed fiscal program. Therefore, this new line of inquiry represents an important advance over the existing public finance literature.

The pioneering work of Kydland and Prescott (1982) as well as that of Long and Plosser (1983) has been an important contribution to our understanding of business cycles. Nevertheless, it appears that there has been little investigation into what implications these frameworks have for issues related to public finance. This research strategy illustrates how artificial economies can be constructed which are capable of mimicking many of the dynamic properties of observed economies. The present paper will show that these dynamic models can be used to answer a wide range of policy–oriented questions in a manner which is novel, especially when contrasted with existing paradigms.

There has been a recent increase in the quantity of public finance analysis which has been undertaken within the context of models which may be considered as dynamic. Typically, however, these models do not exhibit any natural uncertainty, such as that produced by a business cycle, which forces agents to solve their optimization problems under uncertainty. The important work of Judd (1985) and that of Auerbach, Kotlikoff, and Skinner (1983), as well as many others, would fall into this category. Changes in the tax rate are analyzed but there is no uncertainty explicitly incorporated into the agent's optimization problem concerning the future level of taxes, or, say, the future level of technology disturbances. The incorporation of such features may constitute an important feature of observed economies and hence should be integrated in the structure of the model.

There has been other recent innovative work which attempts to measure the cost of certain phenomena within the context of dynamic stochastic models. For example, Lucas (1987) shows how the cost of fluctuations in consumption over the business cycle might be measured. Cooley and Hansen (1987) analyze the cost of the inflation tax in a dynamic cash–in–advance economy. The present paper is intended to contribute to this debate by analyzing the cost of distortional fiscal policies *relative* to the costs of fluctuations in aggregates over the business cycle.

Attention in this paper is directed toward a class of problems, those involving computing distorted general equilibriums, that cannot be solved as a straightforward solution to a planning problem, a technique exploited by Kydland and Prescott (1982), and Long and Plosser (1983). Briefly the problem is this. In an economy where the government levies distortional taxes on agents and rebates the resulting revenue to agents in the form of lump–sum transfers the allocations are not generally Pareto–optimal. Agents must treat the lump–sum transfers as a variable beyond their influence, even though the equilibrium transfer is determined by the tax payments of agents. In effect, an externality prevails where agents are unable to internalize the effects of their actions on aggregate tax receipts. The difficulty with computing such equilibrium is as follows: In order to solve individual agents' dynamic programs one needs to know the equilibrium law of motion governing lump–sum transfer payments, but to know this in turn requires knowledge of aggregate tax receipts which necessitates knowing the outcome of decision–making at the individual level. In this case the traditional tools of dynamic programming cannot be employed in a straightforward fashion. It is shown, however, that even though the allocations are not optimal, a modification of the traditional dynamic programming methods can be applied to produce an equilibrium.

The remainder of this paper is organized as follows. In Section II a general environment is described in which the actions of any group of agents indirectly influences the optimization problem of other agents in the economy. A special case of this would be where all agents receive an equal portion, in the form of a lump–sum transfer, of the total revenue which the government receives from levying a distortional tax. It is shown that a variation on conventional dynamic programming techniques can be used to prove the existence of a solution to the distorted competitive equilibrium. In Section III, a particular economy, of the type analyzed in Section II, is described in detail in which there are various distortional government tax and subsidy schemes that are financed through lump–sum taxes or subsidies. An algorithm for computing the equilibria of such tax distorted economies is proposed. A sample economy to be simulated is then set up. In Section IV some welfare costs of distortional government taxation are calculated. The tax and subsidy rates are varied to obtain a feel for how the dynamic properties of the economy change when the government policies are altered. In Section V a distortional tax program which stabilizes the output for the model economy is implemented and the welfare costs (gains) from undertaking such a policy are evaluated. Some final remarks are contained in Section VI.

II. A GENERAL ENVIRONMENT

It will be useful to begin by analyzing a rather general problem and subsequently specialize it to address particular issues. The economy under consideration is populated by a continuum of identical agents. The representative agent has preferences over the consumption good which are written as

$$E\{\sum_{t=0}^{\infty} \beta^t U(c_t)\},$$

where $E\{\cdot\}$ is the expectation operator, β is the discount factor ($0 < \beta < 1$) and c_t represents consumption of the good in period t. The momentary utility function $U(\cdot)$ is strictly increasing, strictly concave, and twice differentiable.

For expository purposes, it will be convenient to assume that each agent is endowed with the same production technology which is written in the form

$$y_t = F(k_t, K_t, \lambda_t).$$

Here y_t represents output of the consumption good in period t, and k_t is the agent's private input of capital into the production process which was chosen in period t–1. The variable λ_t is a random technology shock which is known at the beginning of period t, and is a realization from a set Λ. K_t will represent the average or per–capita quantity of capital supplied by all agents in the economy. Each agent behaves as though their choice of capital stock k_t has no influence on the average capital stock K_t. Output in each period must be used for either consumption or investment purposes,

$$c_t + k_{t+1} \leq y_t.$$

Here k_{t+1} is the amount of the consumption good used for investment, which will produce k_{t+1} units of the capital good. It is assumed that $F(\cdot, \cdot, \cdot)$ is continuous strictly increasing and concave in its first argument, and twice differentiable. In addition, the following assumptions are made:

Assumptions: $\forall\, \lambda \in \Lambda$

(i) $F(0, 0, \lambda) = 0, \quad F_1(0,K,\lambda) = \infty$

(ii) $\exists\, \bar{K} \ni F(\bar{K}, \bar{K}, \lambda) \leq \bar{K}$

(iii) $\forall\, k, K \in [0,\bar{K}]$

$$F_1(k, K, \lambda) + F_2(k, K, \lambda) > 0$$

$$F_{11}(k, K, \lambda) + F_{12}(k, K, \lambda) < 0$$

Assumption (i) is standard. The next assumption (ii) merely restricts aggregate output and capital to be bounded from above by some constants. Assumption (iii) limits the extent to which aggregate decisions can affect the individual production technology. This assumption may appear to be overly restrictive as it assumes differentiability in both arguments of $F(\cdot, \cdot, \cdot)$. This is unnecessary. The ensuing analysis could be conducted with the production function being differentiable in only its first argument, with Assumption (iii) being interpreted as finite differences instead of derivatives. Also, it is not necessary that Assumption (iii) hold $\forall\, k, K \in [0,\bar{K}]$. It is sufficient that it hold locally at equilibrium points. That is, it is sufficient that for each K, there exists an $\varepsilon > 0$ such that (iii) holds for all $k \in [0,\bar{K}]$ such that $|K-k| \leq \varepsilon$.

The technology shocks are assumed to be in some compact set $\Lambda \subset R_{++}$. These random variables have transition distribution function $G\colon\ \Lambda \times \Lambda \rightarrow [0,1]$, which is continuous in both arguments. Here $G(\lambda'|\lambda) = \text{Prob}\{\lambda_{t+1} \leq \lambda' \,|\, \lambda_t = \lambda\}$, for $\lambda', \lambda \in \Lambda$.

In a competitive equilibrium each agent receives an amount of income at the beginning of each period from the sale of their factors of production. In each period the agent must choose an amount to consume and to invest in capital, subject to satisfying his budget constraint. Since the agent's production technology $F(k, K, \lambda)$ is increasing and concave in its first argument, $\forall\, \lambda \in \Lambda$ and $K \in [0,\bar{K}]$, the economy can be decentralized in a manner similar to that used by Romer (1986). Instead of analyzing the competitive equilibrium, the primary focus of the present paper will be that of a planning problem which is equivalent, from an agent's point of view, to the problem he would solve in a competitive equilibrium. This is exactly the procedure employed by Prescott and Mehra (1980).

To this end, consider the dynamic programming problem

$$\max \sum_{t=0}^{\infty} \beta^t U(c_t)$$

subject to

$$c_t + k_{t+1} \leq F(k_t, K_t, \lambda_t),$$

with $k_o = K_o$ given.

Given a probability distribution over the realizations of $\{K_t\}_{t=1}^{\infty}$, the agent could solve this problem using standard dynamic programming tools. However, since agents are identical, the solution to such a problem will coincide with that of the problem faced by an agent in the economy under consideration only if the agent chooses a capital stock k_t is identical to that of the variable K_t in all states. That is, in a period in which per–capita and individual capital stocks are both equal $K_t = k_t$, and the technology shock is λ_t, the agent must choose capital stock k_{t+1} which coincides with what the per–capita capital stock will be in the subsequent period K_{t+1}.

The optimization problem facing the agent is the following: Given the distribution function $G(\cdot|\cdot)$ for the exogenous disturbances, and a law of motion for the aggregate per–capita capital stock ($K_{t+1} = Q(k_t, \lambda_t)$), the agent must, at each date t, choose a capital stock k_{t+1} as a function of the state variables for the agent (i.e. $k_{t+1} = q(k_t, K_t, \lambda_t)$) such that the agent solves the problem

$$\max E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(c_t) \right\} \tag{1}$$

subject to

$$c_t + k_{t+1} \leq F(k_t, K_t, \lambda_t), \tag{2}$$

with $k_0 = K_0$ given.

Furthermore, in equilibrium, it must be that the individual's decision–rule is consistent with the law of motion for the aggregate per–capita capital stock

$$q(K_t, K_t, \lambda_t) = Q(K_t, \lambda_t). \tag{3}$$

An equilibrium for such an economy is then described as follows:

<u>Definition</u>. A stationary equilibrium for the economy described above is a pair of functions $q(\cdot,\cdot,\cdot)$ and $Q(\cdot,\cdot)$ such that:

(i) the decision–rule for each agent ($k_{t+1} = q(k_t, K_t, \lambda_t)$) solves the optimization problem given by equations (1) and (2),

and

(ii) the individual's decision–rule is consistent with the law of motion for the aggregate per–capita capital stock, or $q(K_t,K_t,\lambda_t) = Q(K_t,\lambda_t)$.

<u>Proposition</u>. There exists a stationary equilibrium for the economy described above.

<u>Proof</u>. See Appendix A.

<u>Remark</u>. It is shown in Appendix A that if the objective function (1) is truncated to make the problem a *finite*–horizon programming problem, then the resulting equilibrium is ***unique***. It is also shown that the value function used by agents to solve this programming problem is concave in the individual capital stock, for both the finite and infinite–horizon problem.

The expression $F(k, K, \lambda)$ is a general formulation that could embody many environments in which an individual's opportunities are affected by aggregate variables. For example, if $f(k, \lambda)$ is the individual production function, and there is a government which levies a proportional tax on capital income at a rate θ and rebates this to agents through lump–sum transfers, then this problem could be solved using the previous technique by letting

$$F(k, K, \lambda) = (1-\theta)\, f(k, \lambda) + \theta\, f(K, \lambda).$$

Alternatively, output per agent could be written as $[f(k, \lambda) + (1-\delta)k]$, which allows capital to depreciate at the rate δ. Then if the government implemented a capital consumption allowance, at the rate μ, and financed it through lump–sum taxes, this could be written as

$$F(k, K, \lambda) = f(k, \lambda) + (1-\delta(1-\mu))\, k - (\mu\delta)K.$$

Finally, labor as an additional factor of production could also be included. Similarly, positive or negative externalities in production could also be studied, as for example if

$$F(k, K, \lambda) = \lambda\, k^{\alpha}\, K^{\beta},$$

where part (iii) of the Assumptions makes use of the restriction $0 < \alpha + \beta < 1$.

In the next section a particular environment is described in detail and a set of tax and subsidy policies is studied.

III. TAX DISTORTED ECONOMIES

Consider an economy inhabited by a representative agent, a firm, and a government. The firm produces output using factor inputs, namely capital and labor services, hired from the individual. The individual in turn uses his income derived from supplying these services to purchase either consumption or investment goods from the firm. Lastly, a set of distortional taxes is levied on private agents by the government with the tax revenue raised being rebated to the private sector via lump–sum transfer payments.

Output in any given period t, or y_t, is governed by the following constant–returns–to–scale production function

$$y_t = f(k_t h_t, \ell_t) \tag{4}$$

where $k_t h_t$ represents the input of capital services in this period and ℓ_t period–t labor input. Given a rental rate for capital services, r_t, and a wage rate for labor, w_t, the firm chooses $k_t h_t$ and ℓ_t so as to maximize profits, π_t. Specifically, the firm solves the following problem:

$$\max_{k_t h_t, \ell_t} \pi_t = f(k_t h_t, \ell_t) - r_t k_t h_t - w_t \ell_t, \tag{P1}$$

which has as first–order conditions

$$f_1(k_t h_t, \ell_t) = r_t \tag{5}$$

$$f_2(k_t h_t, \ell_t) = w_t. \tag{6}$$

The representative agent's goal in life is to maximize his expected lifetime utility as given by

$$\underset{0}{E}\left[\sum_{t=0}^{\infty} \beta^t \underline{U}(c_t, \ell_t)\right] \qquad 0 < \beta < 1,$$

where c_t and ℓ_t represent his period–t consumption and labor supply. For convenience let the adopted form of $\underline{U}(\cdot)$ be restricted to

$$\underline{U}(c_t, \ell_t) = U(c_t - G(\ell_t)), \tag{7}$$

with $U' > 0$, $U'' < 0$, $G' > 0$ and $G'' > 0$. The individual has three primary sources of income: first the income derived from selling capital services $r_t k_t h_t$, second, labor income $w_t \ell_t$ and third, the lump–sum transfer payment τ_t he receives from the government. Capital and labor income are taxed at the rates λ_k and λ_ℓ, respectively.

The agent faces a nontrivial decision concerning the supply of capital services. Specifically, a given stock of capital k_t can be utilized at a variable rate, h_t. On the one hand a higher rate of utilization h_t allows for a higher level of capital services $k_t h_t$ to be obtained from a given stock of capital k_t while on the other hand it causes the capital stock to depreciate at a higher rate of δ_t given by $\delta_t = \delta(h_t)$ with the depreciation rate function satisfying $0 \leq \delta \leq 1$, $\delta' > 0$ and $\delta'' > 0$.

The individual can either consume or save his after tax income. Savings is undertaken in the form of physical capital accumulation. The evolution of the individual's capital stock is described by

$$k_{t+1} = k_t(1 - \delta(h_t)) + i_t(1 + \varepsilon_t).$$

Here i_t units of output invested in period t increases the period–t+1 capital stock by $i_t(1 + \varepsilon_t)$ units, where ε_t is a disturbance term (known at time t) drawn from the distribution function

$\Phi(\varepsilon_t|\varepsilon_{t-1})$ defined on $\Lambda\times\Lambda$. The shock ε_t operates as technological shift factor affecting the efficiency of "newly" produced capital. As is discussed in Greenwood, Hercowitz and Huffman (1988), a high realization of ε_t operates to stimulate the formulation of "new" capital and promote the more intensive utilization and accelerated depreciation of "old" capital. Gross investment, i_t, is subsidized by the government at the rate λ_i. Additionally, they provide a capital consumption allowance at rate, λ_δ.

The representative agent's dynamic programming problem can now be cast. Formally, the problem is

$$V(k_t, s_t) = \max_{(c_t,\, k_{t+1},\, h_t,\, \ell_t)} \left\{U(c_t - G(\ell_t)) + \beta \int_Q V(k_{t+1}, s_{t+1})\, d\Psi\,(s_{t+1}|s_t)\right\} \quad \text{(P2)}$$

subject to

$$c_t + (1-\lambda_i)[k_{t+1} - k_t]/(1+\varepsilon_t) = (1-\lambda_k)\, r_t k_t h_t + (1-\lambda_\ell)\, w_t \ell_t$$
$$- (1-\lambda_i - \lambda_\delta)\, \delta(h_t) k_t/(1+\varepsilon_t) + \tau_t, \tag{8}$$

with the state vector $s_t \equiv (r_t, w_t, \tau_t, \varepsilon_t)$ being governed by distribution function $\Psi(s_t|s_{t-1})$. The upshot of the above optimization problem is summarized by the following set of efficiency conditions — in addition to the budget constraint (8):

$$\frac{(1-\lambda_i)\, U'(c_t - G(\ell_t))}{(1+\varepsilon_t)} = \beta \int U'(c_{t+1} - G(\ell_{t+1}))\Big[(1-\lambda_k) r_{t+1} h_{t+1}$$
$$+ \frac{(1-\lambda_i)(1-\delta(h_{t+1}))}{(1+\varepsilon_{t+1})} + \frac{\lambda_\delta\, \delta(h_{t+1})}{(1-\varepsilon_{t+1})}\Big] d\Psi(s_{t+1}|s_t) \tag{9}$$

$$(1-\lambda_k) r_t = (1-\lambda_i - \lambda_\delta)\, \delta'(h_t)/(1+\varepsilon_t) \tag{10}$$

$$(1-\lambda_\ell) w_t = G'(\ell_t). \tag{11}$$

The first equation (9) is a standard optimality condition governing investment. The left–hand side of this equation represents the loss in current utility which is realized when an extra unit of current investment is undertaken. Note that an efficiency unit of period $-\,t+1$

capital costs $1/(1+\varepsilon_t)$ units of current output to produce. Given the investment tax credit, the cost to private agents of acquiring an extra unit of period – t+1 capital is only $(1-\lambda_i)/(1+\varepsilon_t)$. The right–hand side portrays the discounted expected future utility obtained from an extra unit of investment today. Observe that the term in brackets represents the realized marginal after–tax return on investment. This realized return has three components. The first, $(1-\lambda_k)r_{t+1}h_{t+1}$, is the after–tax return earned on capital from production in period t+1. Second, the undepreciated portion, $(1-\delta(h_{t+1}))$, has a resale value then of $(1-\lambda_i)(1-\delta(h_{t+1}))/(1+\varepsilon_{t+1})$. Finally, the agent receives an allowance of $\lambda_\delta\ \delta(h_{t+1})$ from the government for the depreciated capital, $\delta(h_{t+1})$.

The next equation (10) characterizes efficient capital utilization. It states capital should be utilized at the rate h_t which sets the marginal benefit of capital services equal to the marginal user cost. The marginal user cost of capital is made up of two components. Specifically, $\delta'(h_t)$, represents the marginal cost in terms of increased depreciation from utilizing capital at a higher rate, while $(1-\lambda_i)/(1+\varepsilon_t)$ is the current replacement cost of old in terms of new capital. Finally, equation (11) sets the after–tax marginal product of labor equal to the marginal disutility of working, measured in terms of consumption.

The government, like any other entity in the economy, must satisfy a budget constraint. Specifically,

$$\begin{aligned}\tau_t &= \lambda_k r_t k_t h_t + \lambda_\ell w_t \ell_t - \lambda_i(k_{t+1}-k_t)/(1+\varepsilon_t) - (\lambda_i + \lambda_\delta)\delta(h_t)k_t/(1+\varepsilon_t) \qquad (12)\\ &= \lambda_k f_1(k_t h_t, \ell_t)k_t h_t + \lambda_\ell f_2(k_t h_t, \ell_t)\,\ell_t - \lambda_i(k_{t+1}-k_t)/(1+\varepsilon_t) - (\lambda_i + \lambda_\delta)\delta(h_t)k_t/(1+\varepsilon_t)\end{aligned}$$

The description of the tax distorted economy is now completed by substituting (12) into (8) to obtain the aggregate resource constraint

$$c_t + \frac{k_{t+1}}{(1+\varepsilon_t)} = f(k_t h_t, \ell_t) + \frac{(1-\delta(h_t))k_t}{(1+\varepsilon_t)}. \qquad (13)$$

Equations (5), (6), (9), (10), (11), and (13) completely describe the model's general equilibrium in the sense that they implicitly determine solutions for k_{t+1}, h_t, ℓ_t, r_t and w_t as time – invariant functions of the current state–of–the–world (k_t, ε_t). Thus, using upper case letters to denote the *equilibrium* or *aggregate* values of the representative agent's decision variables, this allows for the model's general equilibrium to be described as follows: $K_{t+1} = K(K_t, \varepsilon_t)$, $H_t = H(K_t, \varepsilon_t)$, $L_t = L(K_t, \varepsilon_t)$, $r_t = r(K_t, \varepsilon_t)$, $w_t = w(K_t, \varepsilon_t)$ and $\tau_t = \tau(K_t, \varepsilon_t)$. Finally, in Appendix C it is shown how the tax distorted economy presented in this section can be modelled as a special case of the general economy presented in Section II.

Computation of Equilibrium

Observe from (5), (6), (8), (9), (10), (11) and (12) that the representative agent's dynamic programming problem (P2) can be cast equivalently as

$$V(k_t, K_t, \varepsilon_t) = \max_{(c_t,\, k_{t+1},\, h_t,\, \ell_t)} \left\{ U(c_t, \ell_t) + \beta \int_Q V(k_{t+1}, K_{t+1}, \varepsilon_{t+1}) d\Omega(\varepsilon_{t+1} | \varepsilon_t) \right\} \quad \text{(P3)}$$

subject to

$$c_t + (1-\lambda_i)[k_{t+1}-k_t]/(1+\varepsilon_t) = (1-\lambda_k)f_1(K_tH_t, L_t)k_th_t + (1-\lambda_\ell)f_2(K_tH_t, L_t)\,\ell_t$$
$$+ (1 - \lambda_i - \lambda_\delta)\delta(h_t)k_t/(1+\varepsilon_t) + \tau_t,$$

with the aggregate capital stock K_t following the law of motion $K_t = K(K_{t-1}, \varepsilon_{t-1})$, and where $H_t = H(K_t, \varepsilon_t)$, $L_t = L(K_t, \varepsilon)$ and $\tau(K_t, \varepsilon)$ are defined by

$$(1-\lambda_k)f_1(K_tH_t, L_t) = (1-\lambda_i - \lambda_\delta)\, \delta'(H_t)/(1+\varepsilon_t),$$

$$\underline{U}_1(f(K_tH_t, L_t) - [K(K_t, \varepsilon_t) - (1-\delta(H_t))K_t]/(1+\varepsilon_t), L_t)(1-\lambda_\ell)f_2(K_tH_t, L_t)$$
$$= - \underline{U}_2(f(K_tH_t, L_t) - [K(K_t,\varepsilon_t) - (1-\delta(H_t))K_t]/(1+\varepsilon_t), L_t),$$

and

$$\tau_t = \lambda_k f_1(K_tH_t, L_t)K_tH_t + \lambda_\ell f_2(K_tH_t, L_t)L_t - \lambda_i(K_{t+1}-K_t)/(1+\varepsilon_t) - (\lambda_i+\lambda_\delta)\delta(H_t)K_t/(1+\varepsilon_t).$$

Note that the tax distorted economy's general equilibrium cannot be directly computed from the representative agent's problem (P3) using traditional dynamic programming algorithms. Solving this programming problem requires knowing the equilibrium law of motion for the aggregate capital stock, but this in turn requires knowing the individual's decision–rule governing capital accumulation. The following iterative procedure, which is a variation on the standard dynamic programming algorithm, is proposed here: To begin with, an initial guess is made for *both* the value function on the right–hand side of (P3) and the equilibrium law of motion for the capital stock. Denote these guesses by $V^0(k_{t+1}, K_{t+1}, \varepsilon_{t+1})$ and $K^0(K_t, \varepsilon_t)$. Next problem (P3) is solved using these guesses. The optimized value of the maximand, which represents the lefthand side of the functional equation, is used as a revised guess for the value function, or $V^1(\cdot)$. As part of the solution to this problem, the individual's decision–rule for capital accumulation is obtained; it has the form $k_{t+1} = k^0(k_t, K_t, \varepsilon_t)$. Since in *equilibrium* capital accumulation at the individual and aggregate leads must coincide, or $k_t = K_t$, this decision–rule forms the basis for the revised guess for the law of motion for aggregate capital stock, $K^1(K_t, \varepsilon_t)$. Specifically, $K^1(K_t, \varepsilon_t) = k^0(K_t, K_t, \varepsilon_t)$. These revised guesses for $V(\cdot)$ and $K(\cdot)$ are used as the foundation for the next round in the iterative scheme, the procedure being repeated until the decision sales have converged. Finally, Baxter (1988), Coleman (1988), Cooley and Hansen (1988), Danthine and Donaldson (1988) and Kydland (1987) also tackle the problem of computing sub–optimal equilibria.

Quantitative Analysis

A quantitative analysis of the macroeconomic ramifications and welfare costs of distortional taxation and business cycle stabilization will now be undertaken. To undertake such an investigation, both tastes and technology must suitably parameterized. To this end, let

$$\underline{U}(c, \ell) = \frac{1}{1-\gamma}\left[\left[c - \frac{\ell^{1+\theta}}{1+\theta}\right]^{1-\gamma} - 1\right],$$

$$f(kh, \ell) = (kh)^{\alpha}\, \ell^{1-\alpha},$$

and

$$\delta(h) = \frac{1}{\omega} h^{\omega},$$

with $\gamma = 2.0$, $\theta = 0.6$, $\alpha = 0.29$, $\omega = 1.42$ and $\beta = 0.96$ — a complete discussion of this parameterization for tastes and technology is contained in Greenwood, Hercowitz and Huffman (1988).

The technology shock is assumed to follow a two–state Markov process. Specifically,

$$\varepsilon_t \epsilon E \equiv \{e^{\xi_1} - 1, \varepsilon^{\xi_2} - 1\},$$

with

$$\text{prob}\ [\varepsilon_{t+1} = e^{\xi_s} - 1 \,|\, \varepsilon_t = e^{\xi_r} - 1] \equiv \pi_{rs},$$

and where $0 \leq \pi_{rs} \leq 1$, $\pi_{r1} + \pi_{r2} = 1$ and $r, s = 1,2$. It is additionally assumed that $\pi_{11} = \pi_{22} = \pi$ and $\xi_1 = -\xi_2 = \xi > 0$. Given this representation, it is easy to show that the technology shock's stochastic structure is conveniently summarized by the standard deviation, σ, and autocorrelation coefficient, ρ, of the random variable $\log(1 + \varepsilon)$; in particular $\sigma = \xi$ and $\rho = 2\pi - 1$. The parameters σ and ρ are chosen such that the sample economy generates the same standard deviation and first–order serial correlation for output as is observed in the postwar US data.

The individual and aggregate capital stocks for the economy are constrained to be elements of the finite time invariant set $\Upsilon = \{K_1, \ldots, K_n\}$. Thus the aggregate state space of the economy, $\Upsilon \times E$, is discrete.[1] This discretization of the model's state space allows for the exact stationary joint distribution of the sample economy's state variables — the aggregate capital stock and technology shock — to be numerically computed in a straightforward manner. (See Appendix B for further details.) Once the joint stationary distribution governing the model's state variables is obtained, it is easy to calculate population moments of interest for the model's various endogenous variables since these are all functions of the current state of the world. An evenly spaced grid of capital stock values is chosen for the set Υ such that further subdivision does not affect the values of the population moments under study. A grid of 120 evenly spaced points spanning the interval [.103, .165] turns out to sufficient for the benchmark artificial economy under study. Similar discretization procedures are employed in

Sargent (1980), Danthine and Donaldson (1988), and Greenwood, Hercowitz and Huffman (1988).

IV. WELFARE COSTS OF DISTORTIONAL TAXATION

In this section the tax rates of the economy are specified and an analysis is conducted into the impact on economic aggregates of changing these rates. The initial tax rates serve as a benchmark so that comparisons can be made between this benchmark and the resulting economy when the tax structure is changed. These experiments are meant to serve as only illustrative exercises of the effect of changes in tax parameters.

The capital and labor income tax rates are both initially chosen at a rate of 35%. This is roughly in the range of rates quoted by Auerbach, Kotlikoff, and Skinner (1983, p. 97). The capital consumption allowance was chosen at a rate of 0.4%, which is the effective rate quoted by Fullerton and Gordon (1983, p. 372). The investment subsidy has varied a great deal in the post–war era. It has ranged from zero, to 10% after 1975 [see Fullerton and Gordon (1983), p. 384]. A rate of 7% was chosen as the benchmark for this parameter.

Table 1 illustrates the dynamic behavior of the model with these tax and subsidy rates imposed, as well as corresponding statistics from the U.S. data from 1948 to 1985, the latter numbers taken from Greenwood, Hercowitz, and Huffman (1988). The standard deviation of the technology shock σ and the serial correlation coefficient of this shock ρ were chosen so as to mimic the percentage standard deviation of output, and the serial correlation of output respectively, as observed in the U.S. postwar data. As a result, for this benchmark the percentage standard deviation of the shock was chosen as $\sigma = 4.6\%$, while the serial correlation of the shock was $\rho = 0.35$. That these numbers were appropriate for mimicking the actual U.S. data may be surprising given that this model *without* taxes was employed by Greenwood, Hercowitz, and Huffman (1988) to mimic the same data. However, they used a percentage standard deviation of the shock of 5.15% and a serial correlation of the shock of 0.51. Hence, the model with the tax (subsidy) schemes in place seems to require *less* variability and serial correlation in the exogenous disturbances in order to mimic the data. Put another way, it would appear that the tax structure seems to *magnify* rather than dissipate the variability and serial correlation effects of the exogenous disturbances. As will be illustrated below, this would lead one to conclude that the tax system imposed tends to amplify the aggregate fluctuations of the business cycle.

As in Greenwood, Hercowitz, and Huffman (1988), the statistics generated by the model, shown in Table 1, are largely consistent with the corresponding statistics garnered from the data. For example, consumption is less volatile than is total output whereas investment is much more volatile than is output. No corresponding statistics for the capital stock for the data are presented because the capital stock in the model is measured in efficiency units. No such corresponding measure is available for the data.

Table 2 presents comparisons of statistics from the benchmark economy with those from an otherwise identical economy where the tax (subsidy) parameters have been changed. Panel I reports statistics for the benchmark economy. Statistics for an economy which is identical to the benchmark except for the fact that the capital income tax is cut from 35% to 25%, are presented in Panel II. The average level of output increases 17% due to this policy and total hours increases by 10%. Productivity of labor increases by 6% and the average capital stock increases by 34%. As an illustration of the volatility induced by the tax system, the percentage standard deviations of the aggregates are all at least as large for Panel I as for Panel II. Additionally, this policy has the effect of increasing the correlation of investment with the output and decreasing the correlation of consumption with output.

Panel III illustrates the effects on the economy of cutting the labor income tax rate from 35% to 25%, while holding all other tax (subsidy) rates at their benchmark levels. In this case output and hours both increase by 27%. Average labor productivity actually declines slightly, but this is due to the fact that there is a very large increase in the quantity of labor employed. The capital stock increases by 26%. Again, Panel III illustrates that a cut in the

TABLE 1

Variables	I Annual U.S. date 1948–1985 (1)	(2)	(3)	II Benchmark Model (1)	(2)	(3)	III Benchmark Model – Stabilized (1)	(2)	(3)
Output	3.5	.66	1.00	3.5	.66	1.00	2.0	.56	1.00
Consumption	2.2	.72	.74	2.5	.95	.62	1.6	.73	0.15
Investment	10.5	.25	.68	19.2	.33	.78	15.5	.31	0.78
Hours	2.1	.39	.81	2.2	.65	1.00	0.1	.47	0.91
Productivity	2.2	.77	.82	1.3	.65	1.00	1.2	.60	0.94

Compensating Variation: 1.21%

Note: The U.S. original data was divided by the 16+ population, then logged and detrended by a linear–quadratic time trend. Output is GNP, and consumption and (gross) investment are the totals from the national income accounts, all in 1982 dollars. Hours data are from the *Current Population Survey* (which is a survey of households) and was calculated by multiplying total employment by average weekly hours.

(1) = standard deviations, measured in percent.
(2) = first–order autocorrelations.
(3) = correlations with output

TABLE 2

	Benchmark Model			Capital Income Tax cut from 35% to 25%			Labor Income Tax cut from 35% to 25%			Investment Tax Credit Increased from 7% to 14%			Capital Consumption Allowance increased from 0.4% to 1.4%		
	(I)			(II)			(III)			(IV)			(V)		
	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
Output	.0914	3.5	1.00	.1066	3.3	1.00	.1159	3.4	1.00	.0995	3.4	1.00	.0921	3.5	1.00
Consumption	.0784	2.5	.62	.0891	2.3	.55	.0994	2.4	.66	.0841	2.4	.58	.0789	2.5	.62
Investment	.0130	19.2	.78	.0175	16.8	.82	.0165	18.1	.80	.0153	17.8	.80	.0133	19.0	.78
Hours	.1380	2.2	1.00	.1522	2.1	1.00	.1753	2.1	1.00	.1457	2.1	1.00	.1389	2.2	1.00
Productivity	.6611	1.3	1.00	.7004	1.2	1.00	.6607	1.3	1.00	.6823	1.3	1.00	.6629	1.3	1.00
Capital	.1326	8.0	.64	.1772	7.0	.60	.1607	7.6	.63	.1552	7.4	.62	.1331	8.0	.64
Compensating Variation				12.1%			16.8%			6.5%			0.5%		

(1) mean
(2) % standard deviation
(3) correlation with output

labor income tax rate lowers the volatility of almost all the aggregates. This policy also raises the correlations with output of both consumption and investment.

It should be noted that the responses of output and labor to experiments such as the change in labor and capital income taxes may seem high. This may be partly attributable to the fact that there is no wealth effect on labor in the utility function in equation (7). Were leisure a normal good, it is possible that these tax cuts would lead to less of an increase in output and employment.

Panel IV shows the impact of increasing the investment tax credit from 7% to 14%. This raises average output by 9% and labor by 6%. Productivity and capital both increase by less than 1%. This policy change also appears to have the effect of lowering the variability of most of the aggregates.

The impact of raising the capital consumption allowance from 0.4% to 1.4% is illustrated in Panel V. The resulting changes in the aggregates are small because this is a small change in the policy parameter itself. Output increases by only 0.8% and labor changes by a similar magnitude.

The row at the bottom of Table 2, entitled compensated variation, is an attempt to measure the gain (or loss) in welfare from these policy changes. This measure is constructed as follows. The solution to the dynamic programming problem for the benchmark has an associated solution for the value function. For the utility function displayed in equation (7), one could also construct a level of *constant* consumption which, if the agent consumed this amount in every period, would yield the same discounted level of utility (or level of the value function) as was produced by the benchmark economy.[2] One could also construct such a measure for the economy used to produce the statistics in Panel II of Table 2, in which the capital income tax is cut from 35% to 25%. This is likely to be a higher level of consumption since the distortion from one of the taxes has been abated. The percentage change in this measure resulting from the economy of Panel II, relative to that of Panel I, is termed the compensating variation. This is a unit–free measure of the change in welfare which results from such a policy. Note that although it is likely that a cut in a distortional tax will increase the level of welfare, this need not always be the case. This is because the benchmark economy is one in which there are many distortions and, in such a second–best situation, it is well–known that relaxing any one of the distortions need not necessarily be welfare–improving. As can be seen from Table 2, cutting labor and capital income taxes substantially improve welfare. On this account the Harberger triangles associated with distortional taxation would appear to be quite large. For instance, a cut in the capital income tax rate from 35% to 25% operates to increase welfare by an equivalent of about 12.1% of GNP. Now it is interesting to note that increases (decreases) in the investment tax credit and capital consumption allowances have beneficial (detrimental) effects on welfare. This occurs because raising these subsidies operates to counteract the depressing effects on capital accumulation exerted by (relatively) high rates of labor and capital income taxation.

V. WELFARE GAINS FROM BUSINESS CYCLE STABILIZATION

Few topics in macroeconomics have had as much attention devoted to them as business cycle stabilization. Two questions at the heart of the debate are: (1) Is it desirable to pursue business cycle stabilization policies and (2) is it feasible, both theoretically and practically speaking, to stabilize economic fluctuations? Advocates of business cycle stabilization feel that the potential benefits are large with Tobin (1977) asserting: "It takes a heap of Harberger Triangles to fill an Okun Gap." Opponents feel that either the potential benefits are small [Lucas (1987)] or that business cycle stabilization is either infeasible or impractical [Friedman (1953)]. The artificial economy developed can be used to cast some further light on these issues.

To begin with, is business cycle stabilization feasible? The notion of business cycle stabilization must be formalized somehow, so suppose that the government desires to stabilize output at some target level $\bar{y}(K, \varepsilon)$ in state (K, ε). How should the government do this? The standard prescription would be that the government should attempt to control the flow of economic activity at the point where maximum influence can be exerted on the production of output. A tax or subsidy on the firm's production of output satisfies this prescription.

Specifically, let a state–contingent tax be levied on firms production in the amount $\lambda_s(K, \varepsilon)$ in state (K, ε). The firm's problem now becomes [cf.(P1)]

$$\max_{k_t h_t,\, \ell_t} \quad \pi = (1-\lambda(K, \varepsilon))\, f(k_t h_t, \ell_t) - r_t k_t h_t - w_t \ell_t,$$

with the associated first–order conditions

$$(1-\lambda_s(K_t, \varepsilon_t)) f_1(K_t H_t, L_t) = r_t$$

and

$$(1-\lambda_s(K_t, \varepsilon_t)) f_2(K_t H_t, L_t) = w_t$$

Using these conditions in conjunction with (10), (11) and the government's stabilization target leads to the following system of equations providing a joint determination of h_t, ℓ_t and $\lambda_{st} = \lambda_s(K_t, \varepsilon_t)$:

$$(1-\lambda_k)(1-\lambda_{st}) f_1(K_t H_t, L_t) = (1-\lambda_i-\lambda_\delta)\, \delta'(H_t)/(1+\varepsilon_t)$$

$$(1-\lambda_\ell)(1-\lambda_{st}) f_2(K_t H_t, L_t) = G'(L_t)$$

$$f(K_t H_t, L_t) = \bar{y}(K_t, \varepsilon_t)$$

Now suppose that the government seeks to keep output constant over the business cycle at some level $\bar{\bar{y}}$, ie $\bar{y}(K, \varepsilon) = \bar{\bar{y}}$ for all $(K, \varepsilon) \in \Upsilon \times E$. Not surprisingly this necessitates that the output tax must move procyclically in response to investment shocks so as to dissuade production in booms and entice it in recessions. By performing the standard comparative exercise on the above system of equation it is easy to demonstrate that

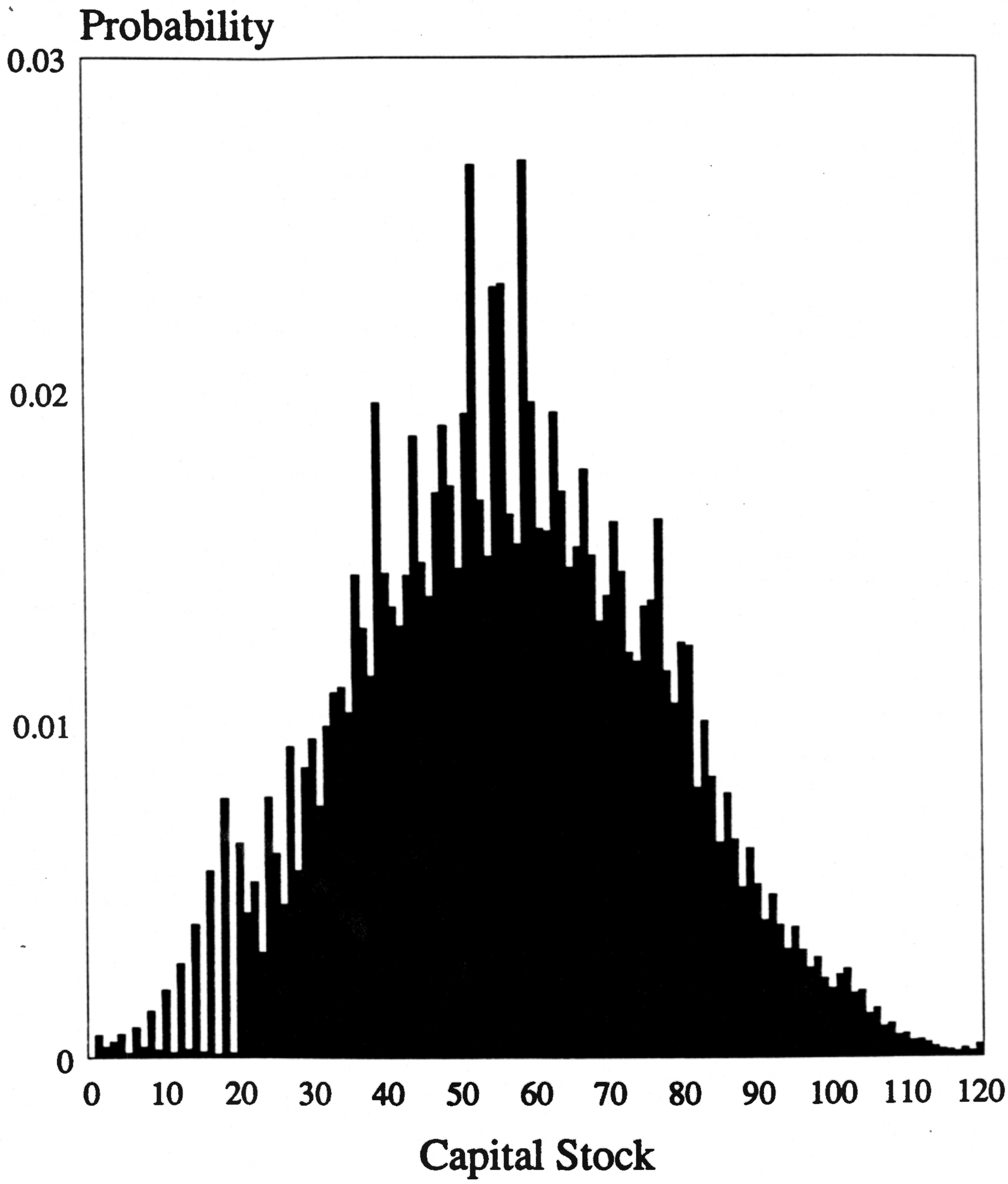
$$\frac{d\lambda_{st}}{d\varepsilon_t} > 0, \quad \frac{dH_t}{d\varepsilon_t} > 0, \text{ and } \frac{dL_t}{d\varepsilon_t} < 0.$$

Note that in response to a positive technological shock capacity utilization increases. Since "old" capital can be replaced with "new" capital it pays to depreciate off the existing capital stock faster. Ceteris paribus this rise in the utilization rate of capital stimulates production. This potential expansion in production is curtailed by an increase in the output tax which operates to stifle production by cutting labor input (as well as dampening the rise in capacity utilization).

In a distortion–free competitive equilibrium such a stabilization scheme can only reduce welfare, a fact demonstrated in Aschauer and Greenwood (1985). Given the presence of distortional taxation, however, the competitive equilibrium modelled above is not distortion free. As has been seen such taxation has a depressing effect on work effort, capital accumulation and output. These effects may be especially unwelcome during recessions. Now in order to measure the benefits from business cycle stabilization the notion of business cycle stabilization itself must first be operationalized. To do this assume that the government seeks to eliminate recessions from the economy, defined here simply as those realizations of income below the mean level of output for the benchmark economy. Eliminating recessions hopefully will operate to bring output closer on average to the ideal level that would prevail in a distortion–free environment.

Such a stabilization scheme operates to increase welfare by an equivalent of about 1.2% of consumption. If it is permissible to interpret this as an Okun Gap, then it seems small when compared with the Harberger Triangles for this economy. Such a stabilization policy also reduces the variance of output, consumption and investment, as can be seen by comparing column (3) in Panels II and III of Table 1. This latter fact is also reflected in Figures 1 and 2 which show the marginal distribution for the artificial economy's capital stock is condensed and skewed to the right by the stabilization scheme. Figure 3 illustrates the state contingent schedule of subsidies needed to implement the stabilization program. The mean and standard deviation of the required output subsidy rate over the business cycle are .0067 and .0086 respectively. These numbers are small, but this shouldn't be all that surprising. Postwar fluctuations in U.S. output have in fact been small with the standard deviation of the log of

Figure 1



Probability
0.03
0.02
0.01
0
0 10 20 30 40 50 60 70 80 90 100 110 120
Capital Stock

Figure 2

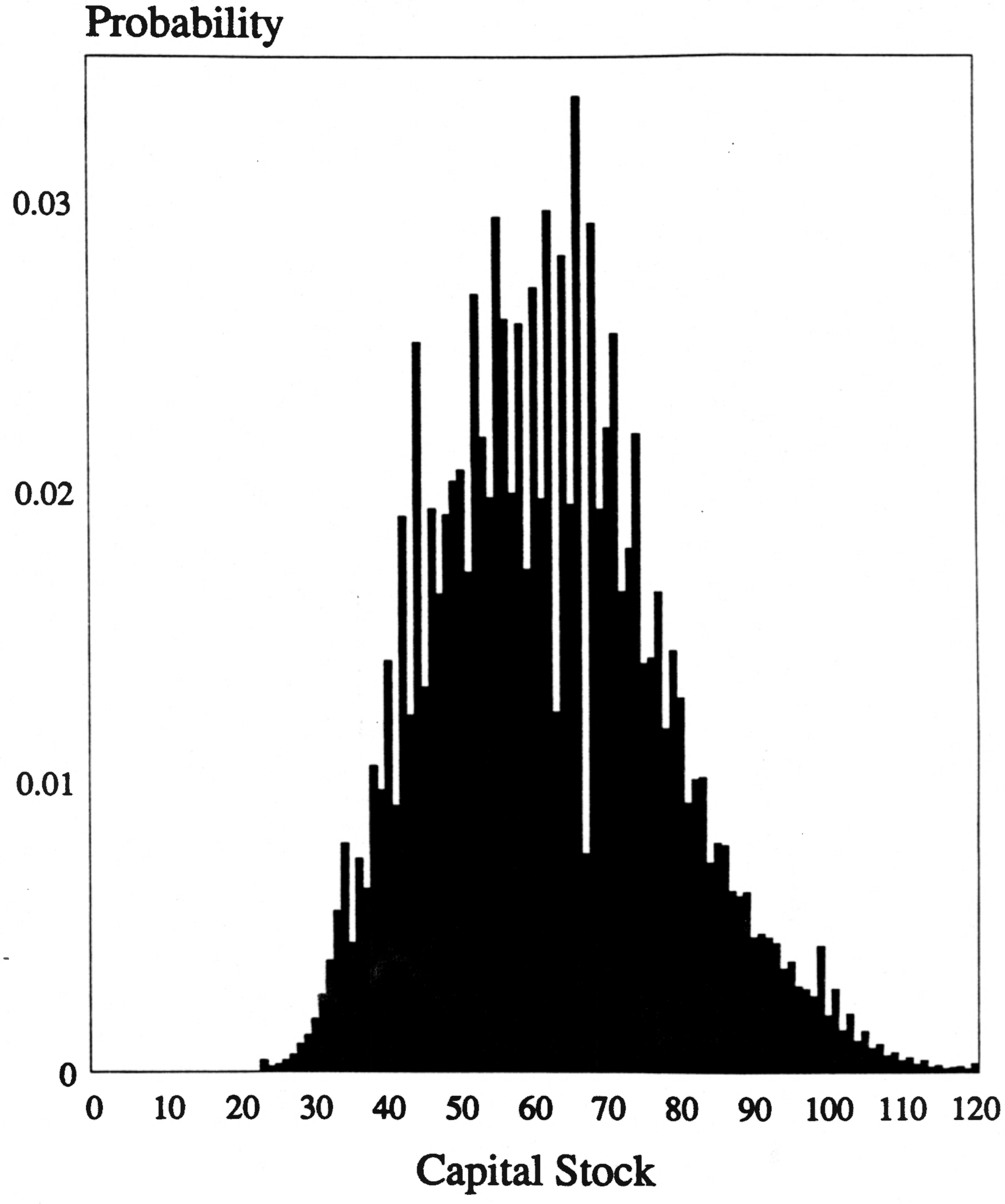
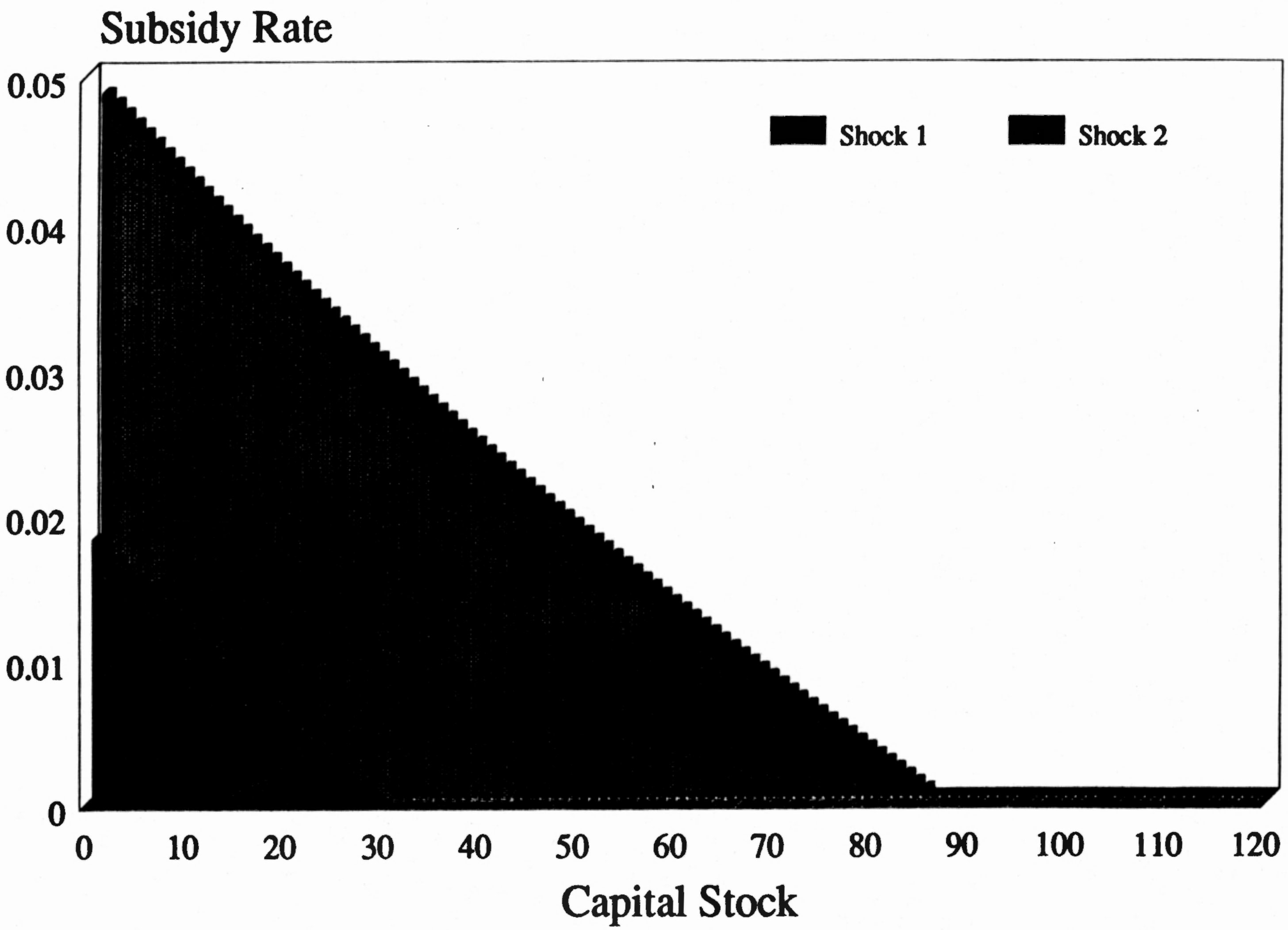

Probability
0.03
0.02
0.01
0
0 10 20 30 40 50 60 70 80 90 100 110 120
Capital Stock

Figure 3



Subsidy Rate
0.05
0.04
0.03
0.02
0.01
0
Shock 1
Shock 2
0
10
20
30
40
50
60
70
80
90
100
110
120
Capital Stock

U.S. output being only .035. The required subsidy for the worst case scenario in the model ($K = K_1$, $\varepsilon = e^{\xi_1} - 1$) is only .048 and the probability of this event occurring is infinitesimal. Also whether such plans would be practical in reality is questionable. If the execution of such plans requires that the government identify and adjust taxes quickly to the underlying state of nature, then achieving success could be problematic especially if mistakes are costly. Friedman [1953] argues against implementing discretionary stabilization exactly because of such practical considerations. Note that in the case under consideration taxes could not just be linked to output, say as under some traditional progressive income taxation scheme, because by design output remains constant over those states of the world where the stabilization scheme is in effect.

The benefits of business cycle stabilization reported here are somewhat large when compared with those that Lucas (1987) reports. (His number for a comparable economy would lie below 0.38% of GNP.) This is because business cycle stabilization operates here to ameliorate some of the deleterious effects of distortional taxation. In particular, it helps to counteract the depressing effects that distortional taxation has on work effort and capital accumulation. Unlike in Lucas (1987), business cycle stabilization raises the mean level of output. One might argue that the distortions should be attacked directly rather than indirectly through business cycle stabilization. This question rings just as loudly whether the source of the distortion is sticky prices, monopolistic competition, efficiency wages or distortional taxation to name a few possibilities. Clearly, any optimal monetary–cum–fiscal policy should be characterized by an optimal taxation program designed to simultaneously undertake government spending, raise revenue, and correct distortions in an efficient manner; in such a scheme there would be no role for business cycle stabilization, per se. The argument for business cycle stabilization must rest on improving on an initial set of non–optimal allocations in a world of imperfect government policymaking. If the notion of business cycle stabilization now seems somewhat imprecise and a little dubious, perhaps indeed it actually is so. The virtue (or some may say the vice) of the modelling approach adopted here is precisely that it forces the modeler to model explicitly the underlying distortion affecting the economy and explain why the proposed interventions dominate other schemes in the set of feasible government policies.

VI. CONCLUSIONS

The analysis of this paper has illustrated how a class of non–optimal equilibria could be analyzed within the context of a representative agent model. The analysis described here has been an extension of the techniques employed in the study of optimal allocations in this same framework. It was shown that distortional taxes, similar to those observed in the U.S. economy, appear to exacerbate the volatility and serial correlation properties of the aggregates of the economy, relative to what would happen if there were no such taxes. The model was also capable of analyzing how the average *levels* of the aggregates would change when the distortional taxes have changed. The experiments undertaken indicated that the "Harberger Triangles" associated with distortional taxation were of substantial size. Additionally, a set of state–contingent taxes was constructed which could stabilize output at any particular level. Such a policy of business cycle stabilization may or may not be welfare improving in general, but in the case of the model studied here it improved welfare. The benefits of eliminating "Okun Gap", however, were small when compared with those obtained from reducing "Harberger Triangles".

Given that in the framework employed here there was considerable disparity between the allocations in the economy with distortional taxation and the optimal allocations, there would appear to be some latitude for the government to minimize this distance. This observation brings to the fore the ultimate goal of analyses such as this. Ideally, one would like to calculate the properties of an optimal fiscal program designed to simultaneously provide government services, raise revenue and correct distortions in an efficient manner. This is a large and demanding task but progress, however slow, along this road is being made.

APPENDIX A

Consider the following mapping, which defines the function $V^j(\cdot)$, $j \geq 1$, by

$$V^j(k,K,\lambda) = \max_{\tilde{k}^j \in [0,\bar{K}]} \left\{ U(F(k,K,\lambda) - \tilde{k}^j) + \beta \int V^{j-1}(\tilde{k}^j, \tilde{K}^j(K,\lambda), \tilde{\lambda}) dG(\tilde{\lambda}|\lambda) \right\} \tag{A.1}$$

where $\tilde{K}^j(K,\lambda)) =$

$$\left\{ \underset{x}{\operatorname{argmax}} \left[U(F(K,K,\lambda) - x) + \beta \int V^{j-1}(x,y,\tilde{\lambda}) dG(\tilde{\lambda}|\lambda) \right] \right\} \Bigg|_{x=y} \tag{A.2}$$

Equations (A.1) and (A.2) will be used to define a sequence of functions $V^j(\ )$, $j \geq 0$ which lie in the space of bounded continuous functions defined on $[0,\bar{K}] \times [0,\bar{K}] \times \Lambda$ with the norm

$$\|f\| = \sup_{\substack{k,K \in [0,\bar{K}] \\ \lambda \in \Lambda}} |f(k,K,\lambda)|.$$

Let $V^0(k,K,\lambda) = 0 \quad \forall\ k,K,\lambda$. Equations (A.1) and (A.2) then imply $\tilde{K}^1(K,\lambda) = \tilde{k}^1(k,K,\lambda) = 0\ \forall\ (k,K,\lambda)$. Hence

$$V^1(k,K,\lambda) = U(F(k,K,\lambda)$$

and

$$V_1^1(k,K,\lambda) = U'(F(k,K,\lambda))F_1(k,K,\lambda) \tag{A.3}$$

$$V_{11}^1(k,K,\lambda) = U''(F(k,K,\lambda))[F_1(k,K,\lambda)]^2 + U'(F(k,K,\lambda))F_{11}(k,K,\lambda) < 0$$

$$V_{12}^1(k,K,\lambda) = U''(F(k,K,\lambda))[F_1(k,K,\lambda)F_2(k,K,\lambda)] + U'(F(k,K,\lambda))F_{12}(k,K,\lambda)$$

Note that Assumpion (iii) implies

$$V_{11}^1(K,K,\lambda) + V_{12}^1(K,K,\lambda) < 0.$$

Obviously $V_1^1(\cdot)$ is differentiable. For some j, let $V^j(\cdot)$ satisfy the conditions

$$V_{11}^j(K,K,\lambda) < 0$$

and

$$V_{11}^j(K,K,\lambda) + V_{12}^j(K,K,\lambda) < 0, \tag{A.4}$$

with it being assumed that $V_1^j(\cdot)$ is differentiable, which holds for j=1.

It will now be shown that this implies $V_{11}^{j+1}(K,K,\lambda) < 0$ and $V_{11}^{j+1}(K,K,\lambda)+V_{12}^{j+1}(K,K,\lambda) < 0$, with $V_1^{j+1}(\cdot)$ being differentiable.

To begin with, the condition for the solution of equation (A.2) is

$$U'(F(K,K,\lambda)-x) = \beta \int V_1^j(x,x,\tilde{\lambda})dG(\tilde{\lambda}|\lambda). \tag{A.5}$$

The implied solution for x can be seen to be *unique* by noting that the right side of (A.5) is decreasing in x [by (A4)] while the left side is strictly increasing. Hence equation (A.2) defines a *unique* continuous function $\tilde{K}^{j+1}(K,\lambda)$. Now from equation (A.4), letting $x = \tilde{K}^{j+1}$ yields

$$\frac{\partial \tilde{K}^{j+1}}{\partial K} = \left[\frac{U''(F(K,K,\lambda)-\tilde{K}^{j+1})[F_1(K,K,\lambda)+F_2(K,K,\lambda)]}{U''(F(K,K,\lambda)-\tilde{K}^{j+1})+\beta\int(V_{11}^j(\tilde{K}^{j+1},\tilde{K}^{j+1},\tilde{\lambda})+V_{12}^j(\tilde{K}^{j+1},\tilde{K}^{j+1},\tilde{\lambda}))dG(\tilde{\lambda}|\lambda)}\right]$$

(A.6)

$$\leq F_1(K,K,\lambda) + F_2(K,K,\lambda).$$

Next, the optimization condition for equation (A.1) is

$$U'(F(k,K,\lambda)-\tilde{k}^{j+1}) = \beta \int V_1^j(\tilde{k}^{j+1},\tilde{K}^{j+1}(K,\lambda),\tilde{\lambda})dG(\tilde{\lambda}|\lambda). \tag{A.7}$$

Differentiation of this expression yields

$$\frac{\partial \tilde{k}^{j+1}}{\partial K} = \left[\frac{U''(F(k,K,\lambda)-\tilde{k}^{j+1})F_2(k,K,\lambda)-\beta\int V_{12}^j(\tilde{k}^{j+1},\tilde{K}^{j+1}(K,\lambda),\tilde{\lambda})(\frac{\partial \tilde{K}^{j+1}}{\partial K})dG(\tilde{\lambda}|\lambda)}{U''(F(k,K,\lambda))+\beta\int V_{11}^j(\tilde{k}^{j+1},\tilde{K}^{j+1}(K,\lambda),\tilde{\lambda})dG(\tilde{\lambda}|\lambda)}\right]$$

and (A.8)

$$\frac{\partial \tilde{k}^{j+1}}{\partial k} = \left[\frac{U''(F(k,K,\lambda)-\tilde{k}^{j+1})F_1(k,K,\lambda)}{U''(F(k,K,\lambda)-\tilde{k}^{j+1})+\beta\int V_{11}^j(\tilde{k}^{j+1},\tilde{K}^{j+1}(K,\lambda),\tilde{\lambda})dG(\tilde{\lambda}|\lambda)}\right]. \tag{A.9}$$

The envelope theorem applied to (A.1), implies

$$V_1^{j+1}(k,K,\lambda) = U'(F(k,K,\lambda)-\tilde{k}^{j+1})F_1(k,K,\lambda). \tag{A.10}$$

Differentiating this yields

$$V_{11}^{j+1}(k,K,\lambda) = U'(F(k,K,\lambda)-\tilde{k}^{j+1})F_{11}(k,K,\lambda)$$

$$+ F_1(k,K,\lambda)\beta\int V_{11}^{j}(\tilde{k}^{j+1},\tilde{K}^{j+1}(K,\lambda),\tilde{\lambda})(\frac{\partial \tilde{k}^{j+1}}{\partial k})dG(\tilde{\lambda}|\lambda). \quad \text{(A.11)}$$

It is then easily shown that

$$V_{11}^{j+1}(K,K,\lambda) < 0 \quad \text{(A.12)}$$

by equation (A.9). Suppressing notation, it can also be shown that

$$V_{12}^{j+1} = U'F_{12} + F_1\beta\int V_{11}^{j}(\frac{\partial \tilde{k}^{j+1}}{\partial K})dG(\tilde{\lambda}|\lambda) + F_1\beta\int V_{12}^{j}(\frac{\partial \tilde{K}^{j+1}}{\partial K})\, dG(\tilde{\lambda}|\lambda)$$

$$= U'F_{12} + F_1\beta\int V_{11}^{j}\left[\frac{U''F_2-\beta\int V_{12}^{j}(\frac{\partial \tilde{K}^{j+1}}{\partial K})dG(\tilde{\lambda}|\lambda)}{U''+\beta\int V_{11}^{j}dG(\tilde{\lambda}|\lambda)}\right]dG(\tilde{\lambda}|\lambda)$$

$$+ F_1\beta\int V_{12}^{j}(\frac{\partial \tilde{K}^{j+1}}{\partial K})dG(\tilde{\lambda}|\lambda)$$

$$= U'F_{12}+F_1\beta\int V_{11}^{j}\left[\frac{U''F_2}{U''+\beta\int V_{11}^{j}dG(\tilde{\lambda}|\lambda)}\right]dG(\tilde{\lambda}|\lambda)$$

$$+ F_1\beta\int V_{12}^{j}\left[1-\frac{\beta\int V_{11}^{j}dG(\tilde{\lambda}|\lambda)}{U''+\beta\int V_{11}^{j}dG(\tilde{\lambda}|\lambda)}\right](\frac{\partial \tilde{K}^{j+1}}{\partial K})dG(\tilde{\lambda}|\lambda)$$

$$\leq U'F_{12}+F_1\beta\int V_{11}^{j}\left[\frac{U''F_2}{U''+\beta\int V_{11}^{j}dG(\tilde{\lambda}|\lambda)}\right]dG(\tilde{\lambda}|\lambda)$$

$$+ F_1\beta\int V_{12}^{j}\left[\frac{U''(F_1+F_2)}{U''+\beta\int V_{11}^{j}dG(\tilde{\lambda}|\lambda)}\right]dG(\tilde{\lambda}|\lambda).$$

by equation (A.5). It can then be seen that

$$V_{11}^{j+1}(K,K,\lambda) + V_{12}^{j+1}(K,K,\lambda) < 0. \quad \text{(A.13)}$$

Note that condition (A.13) ensures that the j+2 stage of the induction the aggregate law of motion for the capital stock, $\tilde{K}^{j+2} = \tilde{K}^{j+2}(K,\lambda)$ is *uniquely* determined — see (A.5).

Similarly, given this, (A.12) guarantees that the individual's decision–rule for capital accumulation, $\tilde{k}^{j+2} = \tilde{k}^{j+2}(k,k,\lambda)$, is also *unique*.

By construction $\tilde{K}^1(K,\lambda) = 0\forall(K,\lambda)$. Now (A.3) and (A.5) imply $\tilde{K}^2(K,\lambda) > \tilde{K}^1(K,\lambda)$ for all (K,λ). By (A.10) if at the j+1 stage of the induction

$$\tilde{K}^{j+1}(K,\lambda) > \tilde{K}^j(K,\lambda)$$

then

$$V_1^{j+1}(K,K,\lambda) > V_1^j(K,K,\lambda)$$

Using (A.5) and (A.12) this then implies

$$\tilde{K}^{j+2}(K,\lambda) > \tilde{K}^{j+1}(K,\lambda).$$

Hence this procedure produces a monotonically increasing sequence of aggregate laws of motion for the capital stock $\{K^j(K,\lambda)\}_{j=1}^{\infty}$ defined on $[0,\bar{K}] \times \Lambda$. Furthermore, this sequence is bounded above by Assumption (ii). Hence this sequence converges uniformly. Let

$$Q(K,\lambda) = \lim_{j\to\infty} \tilde{K}^j(K,\lambda)$$

Note that by construction the function $\tilde{K} = Q(K,\lambda)$ solves the Euler equation

$$U'(F(K,K,\lambda)-\tilde{K}) = \beta\int U'(F(\tilde{K},\tilde{K},\tilde{\lambda})-\tilde{\tilde{K}})F_1(\tilde{K},\tilde{K},\tilde{\lambda})dG(\tilde{\lambda}\,|\,\lambda), \tag{A.14}$$

where $\tilde{\tilde{K}} \equiv Q(\tilde{K},\tilde{\lambda})$.

It is now easy to that a competitive equilibrium of the sort described in Section II prevails. In this competitive equilibrium, the representative agent solves the following dynamic programming problem.

$$V(k,K,\lambda) = \max_{\tilde{k}}\{U(F(k,K,\lambda)-\tilde{k}) + \beta\int V(\tilde{k},\tilde{K},\lambda)dG(\tilde{\lambda}\,|\,\lambda)\} \tag{A.15}$$

with K evolving *exogenously* according to

$$\tilde{K} = Q(K,\lambda).$$

By the standard arguments [see Lucas, Stokey, Prescott (1985)] it is known that there exists a unique function $V(\cdot,\cdot,\cdot)$ which solves the functional equation (A.15); it is also strictly concave in its first argument. Thus there exists a *unique* decision–rule $\tilde{k} = q\,(k,K,\lambda)$ which solves the Euler equation

$$U(F(k,K,\lambda)-\tilde{k}) = \beta\int U'(F(\tilde{k},\tilde{K},\tilde{\lambda})-\tilde{\tilde{k}})F_1(\tilde{k},\tilde{K},\tilde{\lambda})dG(\tilde{\lambda}\,|\,\lambda), \tag{A.16}$$

where $\tilde{\tilde{k}} = q(\tilde{k},\tilde{K},\tilde{\lambda})$. Clearly, when $k = K$ it transpires that $\tilde{k} = \tilde{K}$ and $\tilde{\tilde{k}} = \tilde{\tilde{K}}$ since $q(K,K,\lambda) = Q(K,\lambda)$ [i.e., when $k=K$ the function $q(K,K,\lambda) = Q(K,\lambda)$ must solve (A.16) since $Q(K,\lambda)$ solved (A.14)].

APPENDIX B

The iterative scheme discussed in Section III is straightforward to operationalize. At the $t + 1$ stage of the algorithm the controller will be in possession of both a guess for the value function, $V^t(\cdot)$, and the aggregate law of motion for the capital stock, $K^t(\cdot)$. These guesses take the form of a value for $V^t(k_h, K_i, \xi_\tau)$ for each of the $2n^2$ possible combinations of (k_h, K_i, ξ_τ) in the state space $\Upsilon \times \Upsilon \times E$ and a value for $K^j(K_i, \xi_\tau)$ for each of the $2n$ potential combinations of (K_i, ξ_τ) in $\Upsilon \times E$. Problem (P3) is then solved using these guesses. The optimized value of the maximand, or the righthand side of the value function, is used for the updated guess of the value function, or $V^{t+1}(\cdot)$. As part of the solution the problem (P3), a decision–rule governing the individual agent's capital accumulation is computed. This rule, $k_j' = k^t(k_h, K_i, \xi_\tau)$, specifies an optimal value for the agent's capital stock next period $k_j' \in \Upsilon$ for each $(k_h, K_i, \xi_\tau) \in \Upsilon \times \Upsilon \times E$.

The revised guess for the aggregate law of motion K^{t+1} is given by $K_j' = K^{t+1}(K_i, \xi_\tau) = k^{t+1}(K_i, K_i, \xi_\tau)$: $\Upsilon \times E \rightarrow \Upsilon$. Thus, the algorithm forms a mapping T such that $K^{t+1} = TK^t$.

Ideally, this iterative scheme is repeated until the individual and aggregate laws of motion for capital accumulation have converged, or until $\max_{K_i,\xi_\tau} |k^t(K_i, K_i^t, \xi_\tau) - K^t(K_i, \xi_\tau)| = 0$. Given the discrete nature of the state space such a strict criteria may never be fulfilled. In particular the algorithm may exhibit cycling behavior between the individual and aggregate laws of motion governing capital accumulation. Specifically, it can happen that for some points $(K_i, \xi_r) \in \Upsilon \times E$, there exists a finite number T such that

$$k^t(K_i, K_i, \xi_r) = \begin{cases} k'_s & \text{for all odd } t \geq T \\ k'_{s\pm1} & \text{for all even } t \geq T \end{cases}$$

and

$$k^t(K_i, \xi_r) = \begin{cases} k'_s & \text{for all even } t \geq T \\ k'_{s\pm1} & \text{for all odd } t \geq T \end{cases}$$

That is, given an initial state of world characterized by (K_i, ξ_r), when the aggregate law of motion $K^t(K_i, \xi_r)$ dictate moving to K'_s it is optimal for the individual to choose one of the adjacent points K'_{s+1} or K'_{s-1}, but when the aggregate capital stock moves to either K'_{s+1} or K'_{s-1} the individual then picks K'_s. The algorithm should be terminated when such cycling is detected. The grid for the capital stock was chosen to be sufficiently fine so that the reported results were not sensitive to the point at which the algorithm was terminated.

Finally, the stationary joint distribution function for the technology shock and equilibrium capital stock is computed. [The discussion below parallels that in Greenwood, Hercowitz and Huffman (1988).] Note that the solution for next period's aggregate stock, K'_j, is such that given an initial aggregate capital stock, K_i, and a value for the technology shock, ξ_r, a unique value for $K'_j = K(K_i, \xi_r) \in \Upsilon$ is determined. Thus, the probability $\text{prob}[K' = K_j | K = K_i, \xi = \xi_r]$ will be equal to one for some $j \in \{1,\ldots,n\}$ and zero for the rest. Accordingly, the transition probability $p_{ir,js}$ of moving from the state characterized by capital stock k_i and shock ξ_r to the one represented by k_j and ξ_s can be expressed as

$$p_{ir,js} = \text{prob}[K' = k_j | k = k_i, \xi = \xi_r]\, \pi_{rs} \qquad \text{for } i, j = 1,\ldots,n \text{ and } r,s = 1,2.$$

Next the $2n \times 2n$ transition matrix P with elements $p_{ir,js}$ is formed. The asymptotic joint distribution function for the capital stock and technology shock is a $1 \times 2n$ vector, ρ, which specifies a probability ρ_{ir} that the long–run capital stock/technology shock is (K_i, ξ_r) for each i and r pair. The vector ρ solves the equation

$$\rho = \rho P,$$

subject to the constraints that $0 \le \rho_{ir} \le 1$ for all i, r and $\sum_{i=1}^{n} \sum_{r=1}^{2} \rho_{ir} = 1$.

Finally, note that in general equilibrium all of the model's endogenous variables can be expressed as functions of the aggregate state of the world. Thus, one may write $X = X(K, \xi)$ for $X = C, K', H, L$, and Y. Consequently, the stationary moments for Y, CY and $Y'Y$ can be written as

$$E[Y] = \sum_{r=1}^{2} \sum_{i=1}^{n} \rho_{ir} Y(K_i, \xi_r)$$

$$E[CY] = \sum_{r=1}^{2} \sum_{i=1}^{n} \rho_{ir} C(K_i, \xi_r) Y(K_i, \xi_r)$$

$$E[Y'Y] = \sum_{s=1}^{2} \sum_{j=1}^{n} \sum_{r=1}^{2} \sum_{i=1}^{n} p_{ir'js} \rho_{ir} Y'(K_j, \xi_s) Y(K_i, \xi_r).$$

APPENDIX C

It is easy to establish that the tax distorted economy outlined in Section III fits into the general framework presented in Section II. To do this consider the following "transformed" dynamic programming problem:

$$V(k_t, K_t, \varepsilon_t) = \max_{c_t^*, k_{t+1}} \left\{ U(c_t^*) + \beta^* V(k_{t+1}, K_{t+1}, \varepsilon_{t+1}) d\Phi(\varepsilon_{t+1} | \varepsilon_t) \right\} \tag{P4}$$

subject to

$$c_t^* + \frac{k_{t+1}}{(1+\varepsilon_t)} = F(k_t, K_t, \varepsilon_t), \tag{C.1}$$

where the aggregate law of motion for the capital stock, $K_{t+1} = Q(K_t, \varepsilon_t)$, is taken as given and $\beta^* = \beta/(1-\lambda_i)$. The adjustment to the discount factor has exactly the same effect on the agent's choice of k_{t+1} as an investment subsidy in the gross amount $(1-\lambda_i)$ does, a fact noted in Danthine and Donaldson (1986). Now define the function $F(k_t,K_t,\varepsilon_t)$ by

$$F(k_t,K_t,\varepsilon_t) \equiv \max_{h_t,\ell_t} \left\{(1-\lambda_k)f(k_t h_t,\ell_t) + \frac{(1-\lambda_i)k_t}{(1+\varepsilon_t)} - \frac{(1-\lambda_i-\lambda_\delta)\delta(h_t)k_t}{(1+\varepsilon_t)} - \frac{(1-\lambda_k)G(\ell_t)}{(1-\lambda_\ell)} + \tau^*(K_t,\varepsilon_t)\right\}.$$

Here the transfer payment term, $\tau^*(K_t,\varepsilon_t)$, has been modified to

$$\tau^*(K_t,\varepsilon_t) \equiv \lambda_k f(K_t H(K_t,\varepsilon_t),L_t) + \frac{\lambda_i K_t}{(1+\varepsilon_t)} - \frac{(\lambda_i+\lambda_\delta)}{(1+\varepsilon_t)}\,\delta(H(K_t,\varepsilon_t))K_t + \frac{\lambda_\ell - \lambda_k}{(1-\lambda_\ell)}G(L(K_t,\varepsilon_t)),$$

where the functions $H_t = H(K_t,\varepsilon_t)$ and $L_t = L(K_t,\varepsilon_t)$ solve

$$(1-\lambda_k)f_1(K_tH_t,L_t) = (1-\lambda_i-\lambda_\delta)\,\delta'(H_t)/(1+\varepsilon_t)$$

$$(1-\lambda_\ell)f_2(K_tH_t,L_t) = G'(L_t).$$

It is easy to check that the solution to the above dynamic programming problem (P4) generates, after imposing the equilibrium condition $k_t = K_t$, the competitive equilibrium described in Section III by equations (5), (6), (9), (10), (11) and (13). It is also straightforward, but somewhat tedious, to show that for all $k,K\epsilon[0,K]$

$$[F_1(k,K,\lambda) + F_2(k,K,\lambda]\Big|_{k=K} > 0$$

and

$$[F_{11}(k,K,\lambda) + F_{12}(k,K,\lambda)]\Big|_{k=K} < 0,$$

so that the assumptions imposed on $F(\cdot)$ in Section II are met.[3]

FOOTNOTES

[1]See Huffman (1988) for an analysis of the accuracy of this approximation.

[2]Throughout this paragraph, the term consumption is used to refer to the "effective" consumption bundle which the agent receives from the consumption good itself less the disutility from labor effort. Hence, effective consumption in any period is $(c_t - G(\ell_t))$.

[3]Strictly speaking, due to the presence of the disturbance term on k_{t+1} in the budget constraint (C.1), this is not a special case of the technology outlined in Section II. The line of argument presented in Appendix A can be trivially adapted to handle this situation, though.

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