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ABSTRACT

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The centerpiece of Keynesian macroeconomic theory is the multiplier process: the mechanism by which changes in government purchases are transmitted to real output and other components of real activity. This paper investigates whether equilibrium models of economic fluctuations can produce large multiplier effects of government purchases. Three types of government purchases are studied: "basic purchases," which are not valued in utility or production; "productive purchases," which affect the marginal products of privately owned factors of production; and "government services," which provide utility directly to private individuals. We study the effects of government purchases under several rules for taxation, including a distortionary income tax and a balanced budget requirement. We find that equilibrium business cycle models are certainly capable of producing large multipliers. However, the predictions of the equilibrium model of fluctuations often differ markedly from the predictions of standard Keynesian analyses.

1. Introduction

The centerpiece of Keynesian macroeconomic theory is the multiplier process: the mechanism by which changes in government purchases are transmitted to real output and other components of real activity. In the atemporal Keynesian models developed by Hicks (1937) and Modigliani (1944), the multiplier was necessarily a static concept, although the verbal descriptions of the multiplier process had an unmistakable dynamic character. During the 1950s, formal dynamic multiplier concepts were developed by Klein (1950), Goldberger (1959) and Theil and Boot (1962), with separate multiplier definitions applied to various horizons and time paths of disturbances.

For several decades thereafter, a central issue in Keynesian macroeconomics was the size (i.e., importance) of various multipliers. However, researchers in the newer area of neoclassical macroeconomics have neither defined nor investigated multipliers in the context of equilibrium business cycle models. Partly this reflects the youth of the field, which has until recently been mainly concerned with the development of methods for solving, simulating, and estimating dynamic rational expectations models. Thus, the emphasis on getting quantitative answers to specific policy questions has been less strong. In addition, the well-known Lucas (1976) critique of econometric policy evaluation implies that the concept of a "multiplier" which is invariant to changes in policy is potentially misleading.

Recently, however, a particular class of equilibrium macroeconomic model—real business cycle models—has been criticized because these models are said to lack important multipliers.[1] The purpose of this paper is to reopen consideration of multiplier issues within the context of an equilibrium business cycle model that incorporates several types of government purchases.

The organization of the paper is as follows. Section 2 provides an overview of traditional multiplier terminology and its relation to the language of modern time series analysis. The main conclusion from Section 2 is that, in order to perform valid policy analysis for an arbitrary intervention, we must first carefully specify the economic circumstances in which the displacement occurs. Expectations play a central role in this specification. In particular, as stressed by Lucas (1976), we must draw a distinction between policy rules and policy events. Under rational expectations, the policy rule fixes the character of expectation formation. Given the policy rules, we can then develop the dynamic multipliers that pertain to a specific shock to government policy, e.g., the event that government purchases are unexpectedly high or low.

Section 3 presents the equilibrium macroeconomic model that we employ in subsequent sections to study the dynamic effects of various policy events. In section 4, we use this model to study the effects of "basic government purchases", defined as government purchases which are not valued in utility or production. We study the effects of government purchases under several rules for taxation, including a balanced budget provision. In subsequent sections, we analyze alternative forms of government purchases: section 5 investigates government purchases that are productive in the sense that they expand society's production possibility frontier and section 6 considers purchases that provide utility directly. Section 7 contains a summary of issues and evidence presented, as well as concluding comments.

2. Dynamic Multipliers

A central component of the Keynesian revolution was the focus on propagation mechanisms in the transmission of policy changes or other shocks. Generally, propagation mechanisms in Keynesian models were introduced by incorporating dynamic elements to aggregate demand, such the accelerator. While there were important precursors to this focus on propagation mechanisms, notably in the work of Slutsky (1937) and Frisch (1933), the Keynesian revolution made explicit consideration of dynamic responses part of the regular activities of macroeconomists.

There were two main ways in which this alteration in thought occurred. First, the dynamic analysis of Samuelson (1939) formalized the general intuition that the Keynesian system could produce cyclical outcomes, in the sense of a linear dynamic system with complex roots. Second, beginning with Tinbergen (1937, 1939), builders of econometric models of macroeconomic activity found that empirical performance of the Keynesian models was enhanced by the addition of lags of the model's variables. Because of the presence of lagged endogenous variables in these macroeconometric models, simulations of policy changes were dynamic in character: a one-time change in a government variable generally affected output, consumption, investment, etc. for many periods into the future. Further, Allen (1940) showed that cyclical outcomes arose in empirical Keynesian models, in that Tinbergen's reduced form displayed the cyclical outcomes whose possibility Samuelson had illustrated.

Subsequently, the work of Klein (1950), Goldberger (1959) and Theil and Boot (1962) provided formal definitions of multipliers for linear dynamic macroeconomic models. Isolation of multipliers was taken by Theil and Boot (1962) to be equivalent to "[a systematic analysis of the] time path of response of an endogenous variable to a unit shock in a single period [which is not sustained] of an exogenous variable."

2.1 Theil–Boot Multipliers

The analysis of Theil and Boot (1962) provides a tripartite classification of multipliers. Let y_t be the value of an endogenous variable at date t and x_t the value of an exogenous variable at date t. Then, the Theil–Boot multipliers are defined as follows:

The *impact multiplier* measures the effect of a change in an exogenous variable at date 1, Δx_1, on an endogenous variable at date 1, y_1, i.e., $\Delta y_1/\Delta x_1$. We denote this multiplier by $m_{yx}(1)$.

The *interim multipliers* measure the effect of a change in an exogenous variable at date 1 on the endogenous variable at date $s > 1$, i.e., $\Delta y_s/\Delta x_1$. We denote these multipliers by $m_{yx}(s)$.

The *total multiplier* measures the cumulative effect of a change in an exogenous variable at date 1, Δx_1, on the entire time path of the endogenous variable, y; it is equal to the sum of the impact and interim multipliers: $\sum_{s=1}^{\infty} \Delta y_s/\Delta x_1 = \sum_{s=1}^{\infty} m_{yx}(s)$. Appendix A reviews the calculation of Theil and Boot (1962) procedures for the calculation of multipliers in a linear dynamic system.

Multiplier Definitions in Our Analysis

In our investigation of dynamic responses of the basic equilibrium macroeconomic model, we will consider changes in government purchases that have the potential to be sustained over time,

$$\Delta g_t = \rho^{t-1} \Delta g_1 \quad \text{with } 0 \leq \rho < 1.$$

The initial shock, Δg_1 is assumed to be unexpected, but once the shock is realized, agents expect that the shock will have persistent effects if $\rho > 0$. The addition of this sustained element to shifts in policy complicates the definitions of multipliers, since (i) the cumulative change in purchases through period t will be larger than Δg_1 (it will be equal to $((1-\rho^t)/(1-\rho))\Delta g_1$); and (ii) in a model with rational expectations and intertemporal choice the magnitude of agents' responses to Δg_1 will generally depend on ρ. For these reasons, we find it necessary to extend and modify the Theil-Boot definitions.

Impact and Interim Multipliers: As in the Theil-Boot definitions, our definitions of impact and interim multipliers are normalized by the size of the initial policy change: i.e., $\Delta y_s/\Delta g_1$ for s=1 (impact multiplier) and s=2,3,... (interim multipliers).

Cumulative and Total Multipliers: We define the date t cumulative multiplier of g on a variable y as as the sum of the changes—from date 1 to date t—in y ($\sum_{s=1}^{t} \Delta y_1$) divided by the sum of the changes in g ($\sum_{s=1}^{t} \Delta g_s$). Our total multiplier is then defined as the limit of the cumulative multiplier as t becomes arbitrarily large ($\lim_{t\to\infty} [(\sum_{s=1}^{t} \Delta y_s)/(\sum_{s=1}^{t} \Delta g_s)]$). This gives a measure of the total effect on the path of y that is induced by the sequence of Δg's, normalized by the scale of these changes.

2.2 Multipliers in the Samuelson Model

The simple dynamic Keynesian model due to Samuelson (1939) provides a useful point of reference for discussion of dynamic multipliers. This familiar model involves three equations. First, a simple dynamic consumption function:

$$c_t = \upsilon\, y_{t-1} \quad . \tag{1}$$

Second, the model includes a dynamic investment function of the "accelerator" form:

$$i_t = \kappa\, (y_{t-1} - y_{t-2}). \tag{2}$$

Using the Keynesian assumption that output is demand determined, the market clearing condition is:

$$c_t + i_t + g_t = y_t, \tag{3}$$

where g_t denotes the level of government purchases at date t.

Substituting for c_t and i_t, the model's reduced form contains a second order linear difference equation for output:

$$y_t = (\upsilon+\kappa)\, y_{t-1} - \kappa\, y_{t-2} + g_t. \tag{4}$$

The impact multiplier is unity ($m_{yg}(1) = 1$)and the first interim multiplier is $m_{yg}(2) = (\upsilon+\kappa)$. Subsequent interim multipliers can be computed recursively using the formula $m_{yg}(s) = (\upsilon+\kappa)\, m_{yg}(s-1) - \kappa\, m_{yg}(s)$, for $s \geq 3$. Alternatively, we can show that the impact and interim multipliers of government expenditure on output may be expressed as:[2]

$$m_{yg}(s) = [\mu_2^{s+1} - \mu_1^{s+1}]/[\mu_2 - \mu_1], \tag{5}$$

where the μ_i are roots of the polynomial $\kappa z^2 - (\upsilon+\kappa)z + 1$ and where we assume that $\mu_2 > \mu_1$. Under the assumption that υ and κ are such that this difference equation is stable, the total multiplier is just

$$\sum_{s=1}^{\infty} m_{yg}(s) = [(1-\mu_1)(1-\mu_2)]^{-1} = [1-\upsilon]^{-1}.$$

Note that this total multiplier is the same as it would be in the analogous static Keynesian model, i.e., one in which the consumption function is of the form $c = \upsilon\, y$.

Figure 1 plots the dynamic multipliers of the Samuelson model for assumed values of the marginal propensity to consume (ν=.25) and the accelerator coefficient (κ = .9). Given that the total multiplier must be $1/(1-\nu)$=4/3, there is still substantial variability in the time profile of the interim multipliers, with large swings from negative to positive values.

2.3 Multipliers and the Lucas Critique

In a dynamic environment, the expectations that agents have about the future course of economic events will generally influence their current actions. Lucas (1976) developed the far-reaching implications of this observation for econometric policy evaluation. The "Lucas Critique" implies that there is an important element missing in the analysis of Theil and Boot (1962).

To investigate this point, suppose that we alter the Samuelson investment function to incorporate expectations of the future. Expectation formation may be important since investment decisions must be initiated prior to the time when the new investment (for example, a new machine or building) will be ready for productive use.[3] Modifying the investment function to allow for this effect, we have:

$$i_t = \kappa\,(E_{t-2}y_{t-1} - y_{t-2}) \tag{6}$$

In this modified investment function, investment depends on the expected growth of output, rather than the actual growth rate.

Eliminating consumption and investment, as before, leads to a second order linear difference equation analogous to that studied by Samuelson, but with the addition of a new term involving the expectation error in forecasting output:

$$y_t = (\upsilon+\kappa)y_{t-1} - \kappa y_{t-2} + g_t - \kappa(y_{t-1}-E_{t-2}y_{t-1}) \quad . \tag{7}$$

Further, taking expectations of this difference equation conditional on information at date t-1, it follows that $(y_t-E_{t-1}y_t) = (g_t-E_{t-1}g_t)$. Thus, output evolves according to

$$y_t = (\upsilon+\kappa)y_{t-1} - \kappa y_{t-2} + g_t - \kappa(g_{t-1}-E_{t-2}g_{t-1}). \tag{8}$$

Solution of this difference equation yields:

$$y_t = \sum_{s=1}^{\infty} m_{yg}(s)\, g_{t+1-s} - \kappa \sum_{s=1}^{\infty} m_{yg}(s)\, [g_{t-s} - E_{t-s-1}g_{t-s}] \tag{9}$$

Equation (9) says that output at a point in time depends on the entire history of government purchases and, as well, on the entire history of expectation errors about government purchases. Thus, in contrast to the dynamic model (4), one cannot sensibly discuss the impact of a one-time change in government purchases without specifying additional information about the expectations of agents.[4] In addition, (9) makes it clear that there are potentially important implications of assumptions about expectations. When g_{t-s} and $E_{t-s-1}g_{t-s}$ coincide (e.g., under perfect foresight), then the expenditure multiplier corresponds to that in the basic

Samuelson model. But if there is an unexpected change in government purchases then the interim multipliers will be altered, because some portion of the amplification attached to the accelerator is eliminated.[5] Figure 2 shows the quantitative difference between the multipliers on unanticipated and anticipated changes in the current version of the Samuelson model, using the same parameter values as in the last section.

2.4 Naive Multipliers

We have seen that anticipated and unanticipated policy actions generally differ in terms of the time path of the response of output. Thus traditional macroeconometric models of the type discussed by Theil and Boot (1962) are unlikely to provide correct answers to questions of policy evaluation when expectations depend in a systematic way on the policy environment (i.e., if expectations are rational). This is true even if the models are relatively good at fitting historical data and at short-term forecasting during conditions of unchanged policy rules.

To make the discussion more concrete, we shall suppose that government purchases are well-represented by a first-order autoregressive process:

$$g_t = \rho g_{t-1} + \epsilon_{gt} \tag{10}$$

with $E(\epsilon_{gt}) = 0$ and $E(\epsilon_{gt}^2) = \sigma_\epsilon^2$. We view g_t as representing deviations from a constant average level of purchases, and suppress the constant term for expositional simplicity. Under rational expectations, the model's reduced form contains a dynamic linear relation analogous to (4) above:

$$y_t = (\upsilon+\kappa)y_{t-1} - \kappa y_{t-2} + g_t - \kappa\, g_{t-1} + \kappa\rho\, g_{t-2}. \tag{11}$$

Thus, if this representation were taken as the estimated macroeconomic model, it would be possible to calculate impact and interim multipliers. We call these multipliers naive multipliers, as they would be the outgrowth of fitting the forecasting equation (11) to data without understanding the expectational structure of the model. These naive multipliers are

$$
\begin{aligned}
m^n_{yg}(1) &= 1 \\
m^n_{yg}(2) &= [(\mu_2-\kappa)\mu_2 - (\mu_1-\kappa)\mu_1]/(\mu_2-\mu_1) \qquad (12)\\
m^n_{yg}(s) &= [(\mu_2^2-\mu_2\kappa+\kappa\rho)\mu_2^{s-2} - (\mu_1^2-\mu_1\kappa+\kappa\rho)\ \mu_1^{s-2}]/[\mu_2-\mu_1] \text{ for } s\geq 3
\end{aligned}
$$

The total multiplier (using the Theil-Boot definition) is given by

$$\sum_{s=1}^{\infty} m^n_{yg}(s) = [1-\kappa+k\rho]/[1-\upsilon].$$

Figure 3 shows the difference between naive multipliers and the Samuelson multipliers of Figure 1, with ρ=.9 and the previous values for ν and κ.

There are a number of important, general points that are illustrated by the preceding discussion. First, the multipliers given in equations (12) are not those associated with the effect of government purchases under perfect foresight. Second, only under very special conditions do these multipliers provide the correct answers to either of two commonly posed policy questions: (i) What would be the effect of an unanticipated, temporary change in government purchases today on the time profile of economic activity?; and (ii) What would be the ultimate effect of a permanently higher level of government purchases? The correct answer to (i) is given by the impact and interim multipliers with the parameter ρ set to 0 and the correct answer to (ii) is given by the total multiplier with the parameter ρ set to 1. But, obviously, the empirical estimate of the parameter ρ typically will not be either zero or one, implying that multipliers estimated from the data are not useful in answering these questions. Third, and most important to our subsequent investigation, the foregoing analysis implies that correct analysis of multipliers in rational expectations models requires careful specification of the environment and of the experiment being undertaken.

2.5 Multipliers and Vector Autoregressions

Despite the issues raised in the preceding section, Sims (1980, 1985, 1986) has argued that vector autoregressions (empirical macroeconomic models subject to few theoretical restrictions) are useful for quantitative policy evaluation. To investigate the potential for policy evaluation within such a model, we construct the vector autoregressive representation for the preceding economy.[6] Defining the vector $z_t = (y_t, g_t)'$, the system (10,11) can be written in the form

$$z_t = A_1 z_{t-1} + A_2 z_{t-2} + \epsilon_t \tag{13}$$

where $A_1 = \begin{bmatrix} \upsilon+\kappa & \rho-\kappa \\ 0 & \rho \end{bmatrix}$, $A_2 = \begin{bmatrix} 0 & \kappa\rho \\ 0 & 0 \end{bmatrix}$ and $\epsilon_t = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \epsilon_{gt}$.

A central focus of Sims' analyses is the vector moving average representation, which is defined as

$$z_t = \Pi(L)\, \epsilon_t = \sum_{s=0}^{\infty} \Pi_s\, \epsilon_{t-s}, \tag{14}$$

where in our context Π_s is a 2 x 2 matrix, and the moving average polynomial satisfies $[I - A_1 L - A_2 L]\Pi(L) = I$, where I is the 2 x 2 identity matrix. The coefficients in the moving average representation of a vector autoregession look at first glance like the Theil-Boot multipliers since they measure the response to a shock in government purchases. In the simple case under study, the moving average representation of output takes the form

$$y_t = \sum_{s=0}^{\infty} \pi_{g\epsilon}(s)\ \epsilon_{g,t-s}$$

where the $\pi_{g\epsilon}(s)$ coefficient—which is the (1,1) element of the Π_s matrix above—is given by

$$\pi_{yg}(0) = 1$$

$$\pi_{yg}(1) = \left[\frac{\mu_2^3(\mu_1-\rho) - \mu_1^3(\mu_2-\rho) + \rho^3(\mu_2-\mu_1)}{(\mu_2-\mu_1)(\mu_1-\rho)(\mu_2-\rho)} - \kappa\right] \tag{15}$$

$$\pi_{yg}(s) = \left[\frac{(\mu_2^2-\kappa\mu_2+\kappa\rho)(\mu_1-\rho)\mu_2^s - (\mu_1^2-\kappa\mu_1+\kappa\rho)(\mu_2-\rho)\mu_1^s + \rho^2(\mu_2-\mu_1)\rho^s}{(\mu_2-\mu_1)(\mu_1-\rho)(\mu_2-\rho\ )}\right],$$

for $s \geq 2$.

The moving average representation of government purchases takes the analogous form, $g_t = \sum_{s=0}^{\infty} \pi_{gg}(s)\epsilon_{t-s}$, with

$$\pi_{gg}(s) = \rho^s. \tag{16}$$

The coefficients in the moving average representation provide the information necessary to analyze the impulse response characteristics of the dynamic system, i.e., how a single shock ϵ_{gt} impinges on the time profile of y_{t+s} and g_{t+s}.

Figure 4 provides the impulse response profiles for output and government purchases using the parameter values assumed above (ρ=.9, ν=.25, κ=.5). The impulse response for output differs substantially from the dynamic multipliers of the preceding figures because there is a sustained, rather than a one time, shock to government purchases.

But the presence of a parameters of government policy (ρ) in the moving average coefficients which govern the impulse response function implies that there are a limited number of policy experiments that can be meaningfully conducted in the context of this structure. Roughly, the impulse response function provides the answer to the following question: "What will be the effect of an unanticipated shock to government purchases on the time profile of output, if this shock affects the future profile of government purchases in the same manner that has historically described government purchases decisions?". This does not generally correspond to the Theil–Boot question in a rational expectations environment. That is, it is not equivalent to "[a systematic analysis of the] time path of response of an endogenous variable to a unit shock in one single period [which is not sustained] of an exogenous variable."

2.6 Implications

The experiments with the expectations augmented version of the Samuelson model provide several results that will be important for our subsequent analysis. First, we find that one must distinguish between multipliers for anticipated and unanticipated changes in government purchases. In the experiments we conduct in section 4 below, we choose to focus our discussion on unexpected changes, since this approach conforms mostly closely to standard policy experiments. Second, under rational expectations, the parameters of the process governing policy variables enter in the reduced form for endogenous variables, thus influencing the system's multipliers. This limits the policy experiments that can sensibly be performed within the context of a the model's reduced form (or a vector autoregression). Third, when changes in government purchases are persistent, then the natural policy experiment to consider is not a one-time shock, but rather involves changing government purchases in a way that conforms to the average behavior of policy. This type of experiment is captured in the notion of a "system impulse response" of the form examined by Sims (1980). We take these lessons with us as we turn to analysis of government purchases in an equilibrium business cycle model.

3. An Equilibrium Model of Fluctuations

The influential papers of Lucas (1980,1986) have provided cogent arguments for conducting macroeconomic analysis under the postulate that agents exhaust perceived gains from trade. A growing branch of the macroeconomics literature explores the implications of this postulate in the context of simple dynamic equilibrium models. Since the apparent cyclical influence of money is difficult to introduce into equilibrium theories, attention has principally directed toward understanding the dynamic response of model economies to real shocks of a technological nature (Kydland and Prescott (1982), Long and Plosser (1983)). These models have thus earned the name "real business cycle models."

The real business cycle program has at times been criticized (e.g., McCallum (1987)) for ignoring government and distortionary taxation. In our view, this omission derives not from any hypothesis that such factors that are unimportant but, rather, from technological constraints on model solution. Linear approximation methods developed in King (1987) make it feasible to study suboptimal dynamic equilibria with roughly the same amount of conceptual and computational difficulty as optimal equilibria.[7] In this paper, we use these methods to analyze the dynamic implications of government purchases for real activity. These purchases may be financed by lump sum taxes or by distortionary income taxes.

3.1 The Basic Neoclassical Model

The building blocks of an equilibrium business cycle model are specification of environmental characteristics (preferences, technologies, and endowments) and policy rules. The economy is populated by identical agents of sufficient number that each perceives his influence on aggregate quantities to be insignificant.

Preferences. Each agent has preferences

$$U = \sum_{t=1}^{\infty} \beta^{(t-1)} [u(C_t, L_t)], \qquad \beta < 1 \tag{17}$$

where the amount of consumption is C_t and the amount of leisure is L_t.

Production Technology. For each agent, output at a point in time is the result of operating a constant returns to scale production function:

$$Y_t = F(K_t, X_{Nt} N_t) \tag{18}$$

where K_t is the capital stock (predetermined at date t), N_t is the quantity of labor input, and X_{Nt} is labor-augmenting technical change, which is assumed to grow at the gross constant rate γ_X (i.e., $X_{Nt}/X_{N,t-1} = \gamma_X$).

Accumulation Technology. In this simple neoclassical economy, there is one commodity that can either be consumed or invested. The evolution of capital is specified as

$$K_{t+1} = [(1-\delta_K)K_t + I_t] \tag{19}$$

where I_t is gross investment (i.e. the amount of current output stored to be used in next period's production) and δ_K is the rate of depreciation of capital.

Individual Resource Constraints. In each period, an individual agent faces two resource constraints: (i) that the sum of time devoted to work and leisure not exceed his endowment of one unit of time per period, and (ii) that his total uses of goods (for consumption and investment) not exceed his disposable income, which equals output less his net transactions with the government. These constraints are written:

$$L_t + N_t \leq 1 \tag{20}$$

$$C_t + I_t \leq Y_t^d = (1-\tau_t)\ Y_t + T_t, \tag{21}$$

where τ_t is the tax rate on output and T_t is the level of transfer payments at date t.

Policy Rules. The government specifies a path for the per capita level of government purchases ($\underline{G}_t$) and taxes output at a rate τ_t according to a rule of the form (per capita quantities are distinguished by an underbar):

$$\tau_t = \tau(\underline{G}_t, \underline{K}_t, \underline{N}_t) \tag{22}$$

Thus, the tax rate (τ_t) may depend on the level of exogenous variables such as the level of per capita government purchases ($\underline{G}_t$) and on the level of endogenous variables such as per capita labor and capital.

The government follows a budget policy which specifies that the difference between government expenditures and the output tax revenue is financed by lump sum taxes or transfers (T):

$$\tau_t \underline{Y}_t = \underline{G}_t + \underline{T}_t. \tag{23}$$

Per Capita Resource Constraints. The per capita resource constraints follow from the combination of private and government constraints:

$$\underline{L}_t + \underline{N}_t \leq 1 \tag{24}$$

$$\underline{C}_t + \underline{I}_t + \underline{G}_t \leq \underline{Y}_t \quad . \tag{25}$$

The representative agent, dynamic competitive equilibrium involves sequences of quantities that are consistent with optimal private decisions given individual resource constraints; market clearing (imposition of aggregate resource constraints); and rational expectations. We discuss computation of equilibrium in Appendix C.

3.2 Specification of the Relation Between Taxes and Purchases

Because the central focus of this paper is the effect of shocks to government policy variables on the rest of the economy, we must carefully specify the range of options open to the government. In particular, we must specify the way in which an innovation to government purchases is financed. As we shall see, the method of financing can have an enormous impact on the private sector response to the shock in question.

Our baseline case assumes that shocks to government purchases will be financed by lump sum taxes or, equivalently, by a reduction in transfer payments. Reasoning along the lines of Barro (1974) suggests that this change in transfer payments should be equivalent to deficit financing of a certain form. We shall also investigate "balanced budget" changes in purchases, in which tax rates are adjusted to the state of the economy so as to keep transfer payments fixed at zero.

3.3 Dynamic Multipliers in An Equilibrium Business Cycle Model

In order to undertake a quantitative examination of the size of dynamic multipliers in our equilibrium business cycle model, we need to specify values of the parameters of preferences, technologies and policy rules. In this paper, we adopt the parameter values for preferences and technologies in the baseline example of King, Plosser and Rebelo (1988a). The full set of parameter values used here is given in Table 1.

The next three sections present results on the multipliers associated with three types of government purchases: (i) purchases valued neither in production nor in consumption, (ii) productive purchases, and (iii) utility-producing purchases.

4. Dynamic Multipliers for Basic Purchases

We use the terminology "basic purchases" to refer to government purchases that are not utility producing and which are not productive in the sense of shifting the economy's production possibility frontier; this is the type of purchases studied in most Keynesian analyses of multipliers.[8]

4.1 Serially Uncorrelated Basic Purchases

We consider first a situation in which shocks to basic government purchases are serially uncorrelated. The precise intervention considered is an unexpected 1 unit increase in the level of basic purchases, denoted g^b. Figure 5 plots the multipliers for output, consumption, and investment that are associated with this intervention. Previously we defined the "impact multipliers" to be the change in a variable associated with the 1 unit contemporaneous innovation in g^b. Here and below, the innovation is assumed to take place in quarter one. Thus the impact multipliers are the multipliers at quarter one in Figure 5.

The impact multiplier for output is positive but small; the 1 unit rise in g^b leads to a contemporaneous increase in output of less than one tenth of a unit. There is a small negative multiplier on consumption. The largest effect is on investment, with an impact multiplier of about -.9, i.e., there is nearly one-for-one contemporaneous crowding out of private investment. It is the influence of this decline in investment on capital accumulation that means that there are dynamic multipliers in the current model.

In the case of serially uncorrelated g^b, the interim multipliers are all very small: recall that these multipliers are defined as $\Delta y_s/\Delta g_1$ for $s \geq 2$. The period 2 interim multiplier on output is negative; output has fallen to a level slightly below its steady state level, since capital has declined due to the drop in investment in the previous period. Consumption is also below its steady state level, and investment is above its steady state level. The interim multipliers do not change sign after period two, and converge to zero as the economy makes a smooth transition back to its (unchanged) steady state.

Figure 6 plots the cumulative response of consumption, investment, and output to the innovation in g^b. Recall that the cumulative multiplier for output in period t is defined as the sum from period one to period t of the the deviation of output from its steady state level, divided by the sum from periods one to t of the deviation in purchases (g) from its steady state level. Further, in section 1 above, we defined the total multiplier for a variable as the limit of the cumulative multipliers. Figure 6 shows that the total multiplier (or cumulative effect) on output is negative: about -.5.

The total multipliers for consumption and investment are also negative, with the total multiplier for consumption greater than one in absolute value.

4.2 Serially Correlated Basic Purchases

We consider next the effects of an unexpected innovation to basic purchases in a situation where basic purchases are serially correlated: $g^b_{1t}=\rho g^b_{1,t-1} + \epsilon^b_{gt}$. We set the serial correlation coefficient ρ equal to .9 so that shocks to purchases are very persistent. Note that our impact and interim multipliers measures simply involve scaling by Δg_1, despite the fact that the cumulative displacement to government purchases through date s will be $[(1-\rho^s)/(1-\rho)]\ \Delta g_1$. By contrast, the cumulative and total multipliers normalize by the sum of the deviations of g^b from its steady state level.

As before, we consider a unit innovation to g^b. The impact and interim multipliers for this case are plotted in Figure 7. Relative to the zero serial correlation case, the impact multipliers for output and consumption are larger. The impact multiplier for output in this case is about .4, and the impact multiplier for consumption is about −.3. Both of these multipliers are larger (in absolute value) than in the zero serial correlation case. The impact multiplier for investment is negative, about −.35, compared with a value of nearly −1.0 in the zero serial correlation case.

These results are of special interest because they relate to previous work by Barro (1981) and Hall (1980) investigating the differential effects of permanent versus temporary government purchases. In their theoretical analyses, Barro and Hall stress that permanent and temporary changes differ in two ways. First, there is a larger wealth effect associated with more permanent purchases. Second, increased persistence of government spending limits the differences in opportunities between adjacent periods that motivate intertemporal substitution. These authors assume that the substitution effect is quantitatively more important than the wealth effect, and argue that temporary changes in government purchases (associated mainly with wars in an empirical context) should exert larger output effects than permanent changes in purchases. Temporary changes should have these larger effects, according to the Barro—Hall argument, since these induce intertemporal substitutions which do not arise with permanent changes. But because the Barro and Hall analyses are qualitative, they cannot determine the relative importance of the wealth and substitution effects.

In contradistinction to the Barro and Hall analyses, our quantitative analysis reveals that the wealth effect dominates; increased persistence of government purchases leads to <u>larger</u> multipliers on output and consumption, not smaller ones as predicted by Barro and Hall.

Reasoning along the lines of Barro and King (1984) suggests that this result will hold in any version of our model, in which preferences are time separable. The argument hinges on two characteristics of the basic neoclassical model. First, consumption obeys the permanent income hypothesis in general equilibrium, falling by more the more persistent is the shock to unproductive government purchases. (Because government purchases are not valued, they can be thought of as a negative technology shock of an additive form.) Second, consumption and labor obey an intratemporal efficiency condition requiring that the utility—denominated value of the marginal product of labor equals its utility cost in terms of foregone leisure. Because consumption falls by more with more persistent shocks, satisfying the intratemporal efficiency condition means that effort must also rise by more.

Since capital is predetermined when the shock occurs, the multiplier on output is necessarily larger the more persistent is the shock to purchases.[9]

Figure 8 plots the cumulative multipliers for the case of persistent government purchases (ρ=.9). As before, the total multipliers for consumption and investment are negative. The total multiplier (the limit of the cumulative multiplier) for output is positive in this case, as compared with a negative value for serially uncorrelated purchases. This can be understood by noting that if government purchases were fully permanent (increasing by one unit for all time) the equilibrium response would be a steady state path with lower consumption and higher effort, which translates into a higher steady state flow of output.

4.3 Balanced Budget Multipliers: Serially Uncorrelated Basic Purchases

Next, we consider "balanced budget multipliers": multipliers associated with the requirement that $\tau_t y_t = g_t$.[10] This requirement means that changes in basic purchases must be financed by distortionary taxes levied in the same period. Further, changes in the tax base y_t, which arise when capital or labor varies over time, must be offset by contemporaneous changes in τ_t. Figure 9 plots impact and interim multipliers for this experiment in the case where basic government purchases are serially uncorrelated. The impact multipliers in this case are very large: the impact multiplier for output is −1.6, and the impact multiplier for investment is −2.5. The multiplier on consumption is about −.2. The large negative multipliers on output and investment are due to the double whammy of a rise in government purchases under the balanced budget requirement. In this fiscal setting, a rise in government purchases requires an increase in tax rates; the double whammy is due to the necessity of raising a fixed amount of revenue while labor input declines in response to the rise in tax rates. Thus, the increase in the tax rate associated with the balanced budget requirement can be substantial. Because of the temporary nature of the disturbance, individuals substitute intertemporally, consuming more leisure in the period in which the tax rate is high.

The balanced budget requirement also has implications for transition path dynamics. The interim multipliers in Figure 9 show that output is below its steady state level along the transition path, as is consumption and investment. However, effort is above its steady state level in periods 2 through infinity, reflecting the necessity to invest to offset the period 1 drop in the capital stock. The cumulative multipliers for the balanced budget experiment are plotted in Figure 10. The cumulative multipliers and the total multipliers are all negative.

4.4 Balanced Budget Multipliers: Serially Correlated Basic Purchases

Finally, we consider a balanced budget experiment for serially correlated basic purchases, in order to examine the effects of an innovation to purchases that is close to permanent. The impact and interim multipliers for this experiment are plotted in Figure 11. Compared with the zero serial correlation case, the impact multipliers on output and investment are smaller. The difference in these multipliers is attributable to the fact that there is a much smaller opportunity for intertemporal substitution when innovations to g^b are close to permanent. This is also the reason for the difference in the impact multipliers on investment: now is a bad time to invest, but later isn't much better. The cumulative multipliers for this case are plotted in Figure 12. The total multipliers are all negative, as in the zero serial correlation case, but are not as large in absolute value.

5. Dynamic Multipliers for Government Purchases

This section examines multipliers associated with government purchases that are productive in the sense that they shift the economy's production possibility frontier; we use the notation g^P to refer to this type of government purchases. We consider two types of productive government purchases: (i) purchases that can be modeled as augmenting factor productivity in the period in which the purchase takes place, and (ii) investment in durable government capital which enters the production function as a third factor, and which depreciates in the same way as private capital.

5.1 Serially Uncorrelated, Productivity-Augmenting Government Purchases

In order to allow the flow of government purchases to be productive, we rewrite the production function as:

$$Y_t = F(K_t, X_{Nt}N_t; G_t^P). \tag{26}$$

For the economy to exhibit steady state growth it must be the case that G^P has the same growth rate as labor augmenting technical change and output: γ_X. Define g^P=G^P/X_N. Dividing by X_{Nt} yields the transformed production function:

$$y_t = F(k_t, N_t; g_t^P) \quad . \tag{27}$$

Three new parameters need to be specified. The first is the elasticity of output with respect to G^P:

$$\eta \equiv (\partial Y/\partial G_2)(G_2/Y). \tag{28}$$

The other two parameters are the elasticities of the marginal products of labor and private capital with respect to G^P; call these ζ_{KG} and ζ_{NG}. But only two of these three parameters can be chosen independently, since differentiation of (47) yields

$$\eta = \zeta_{NG} s_N + \zeta_{KG} s_K. \tag{29}$$

Fundamentally, these productive public purchases act like technical shifts from the standpoint of private decisions. We draw on this similarity to construct two types of productivity enhancing government actions. First, we consider government purchases that augment the productivity of all factors (i.e., act like total factor augmenting technical change). Second, we consider government purchases that augment the productivity of private capital (i.e., act like capital augmenting technical change). In terms of the parametric specification in (28) and (29), these represent alternate choices of parameter values. The first case corresponds to $\eta = \zeta_{NG} = \zeta_{NG}$. The second case corresponds to setting $\zeta_{KG}=[1-(s_K/\zeta_{KN})][\eta/s_K]$. Given η and ζ_{KG}, ζ_{NG} is then obtained from (29).

When government purchases are productive, there are two counterbalancing effects at work. An increase in purchases uses resources, leading to effects similar to the basic purchases case studied above. But because purchases act to augment factor productivity, they lead to increases in output at unchanged factor inputs. The net effect on resources available for private use depends on the relative sizes of the parameters η and s_g^P; holding inputs constant: $\Delta y - \Delta g^P = ((\eta - s_g^P)/s_g^P)\ \Delta g^P$.

When $\eta > s_g^P$, an innovation in g^P leads to a net increase in output available for private consumption and investment. When $\eta = s_g^P$, the marginal unit of government purchases use up exactly the additional output available because of the purchases. Thus it is easily shown that the level of government purchases which maximizes net output available to the private sector is found by setting $s_g^p = \eta$.

Figure 13 plots multipliers for the case of zero net resource use ($\eta = s_g^p = .05$) and zero serial correlation in g^p. There are dramatically different responses from those encountered in the basic purchases case studied above. The impact multiplier on output is about 1.8, which reflects both the direct productivity-enhancing effects of the purchases and the increase in effort in response to the temporary productivity shock. Most of the increase in output is used for investment, which has an impact multiplier of .8. The interim multipliers are all very small individually, which reflects the relatively weak internal propagation mechanism for effects of technology shocks within the basic neoclassical model.[11] However, the cumulative effects of these mechanisms is substantial—over 80 quarters the total multiplier is about 2.4. The consumption effects of the productive purchases shock are spread out over time in accord with consumption smoothing—the total multiplier on consumption in Figure 14 is close to unity.

5.2 Serially Correlated, Productivity-Augmenting Government Purchases

Figures 15 and 16 plot the multipliers for a unit shock to productivity-augmenting government purchases where the shock process has a serial correlation coefficient of .9. Compared to the serially uncorrelated case, the impact multipliers are identical. However, the interim multipliers are substantial in the case of serially correlated shocks (Figure 15), and are nearly nonexistent in the case of serially uncorrelated shocks (Figure 13).

The cumulative and total multipliers are the same in the uncorrelated and correlated cases.

5.3 Government Purchases Which Enhance the Productivity of Private Capital

In the previous two subsections we studied the effects of government purchases which acted like neutral technical change. However, government purchases need not affect the productivity of all factors in a symmetric way. In this subsection we explore the effects of government purchases which act to augment the productivity of private capital.

Table 2 presents multipliers for government purchases which act as capital-augmenting technical change, under a variety of assumptions about (i) the elasticity of substitution between capital and labor, and (ii) the relative size of η and s_g^P. In all cases, the shock to purchases is serially uncorrelated and the share of government purchases in output, s_g^P, is held fixed at .05. For each combination of the elasticity parameter and the productivity parameter, the table reports the contemporaneous impulse response for output, the impact multiplier for output (which is the impulse response divided by s_g^P), and the total multiplier for output.

The second column of the table gives the direct effect on output of the innovation in government purchases. For example, with $\eta=s_g^P=.05$, we know that there is zero net resource use from the innovation in government purchases. Thus, a one unit innovation in purchases leads to a one unit (gross) increase in output. The second column of Table 2 therefore shows a direct effect of 1.0 for the case $\eta=s_g^P$. The table illustrates that the impact and total multipliers on output are larger (i) the larger is η (the productivity parameter for government purchases), and (ii) the lower is the elasticity of substitution between capital and labor. The columns of the table for a unit elasticity of substitution can also be interpreted as multipliers for a shock to neutral technical change.

5.4 Public Capital

In this section we study the effect of government provision of a public capital good, which again enters the production function as a third factor,

$$y_t = F(k_t, N_t; k_t^p), \tag{30}$$

where k_2^p is a second capital good which depreciates at a rate $\delta_g<1$.

A key difference between this example and the example of section 5.3 is in the timing. In section 5.3, a unit increase in the flow of productive government purchases (g^p) at time t enhanced the productivity of private capital in place at time t. Since we assumed that shocks to g^p were serially uncorrelated, there were no productivity effects of this shock at any future dates.

In the case of government provided capital, the shock considered is a shock to <u>investment</u> in public capital. In period t, there is a one unit increase in public investment. The new capital arising from this investment will not become productive, however, until period t+1. (This timing convention matches the timing convention for production of additional private capital.) In period t, therefore, the shock to purchases appears unproductive since the government uses resources to invest in public capital, but the productivity-enhancing effects of the capital have not yet been realized. However, the private response to the shock differs from a shock in unproductive purchases (studied in section 4) since individuals know that in future periods, private factor productivity will be enhanced because of a larger stock of public capital.

Figure 17 plots the impulse response of public capital to a one unit innovation in public investment at time t (time period 1 on the graph). Because capital is predetermined at time t, the contemporaneous response to the shock is zero. The new government capital becomes productive in period 1, and depreciates at the rate δ_g=.025 per period. (A one unit innovation in investment relative to the steady state level leads to only about a .29% deviation from the steady state level of the government capital stock since steady state investment is a small proportion of the existing capital stock.)

5.4.1 Case 1: $\eta=s_g^p$: Zero Net Resource Use

We begin by exploring the case in which $\eta=s_g^p$, interpreting s_g^p as the share of public investment, with a unitary elasticity of substitution between private capital and labor. In contrast to productive government purchases of the sort studied in sections 5.1 and 5.2, setting $\eta=s_g^p$ does not mean that the direct *current* resource effect of an innovation to government purchases is zero. Rather, it means that a permanent increase in the public capital stock would generate enough output to just offset the required steady state replacement investment. Figures 18 and 19 plot the multipliers for a unit innovation in public investment. The impact multipliers are very close to those for serially uncorrelated, unproductive government purchases (plotted in Figure 5). In both cases, the dominant effect is the resource use by the government. Although in the present experiment government purchases produces capital which will come on line in the next period, the magnitude of the effect on the government capital stock is too small to exert much of an influence on private decisions. While the behavior of consumption and investment is broadly similar over a 20–quarter horizon in the two examples, the interim multipliers for output are notably different. With public capital, output is above its steady state level along the transition path. This is in contrast to the transition path for unproductive government purchases, for which the interim multipliers on output are all negative. The cumulative multipliers in Figure 19 show that there are important long run differences between public capital and unproductive purchases. The cumulative multiplier for consumption is positive with public capital, in contrast to a cumulative multiplier of about –1 obtained when purchases are unproductive. In fact, in the case of an innovation to public capital, consumption is above its steady state level from about quarter 20 onward.

5.4.2 Case 2: $\eta > s_g^p$: Positive Direct Output Effects

Increasing the productivity of public capital heightens the differences between productive and unproductive purchases. In this example, we have set $\eta = .42$, which equals private capital's share of output. Also, the value is close to Aschauer's (1988) estimate of the productivity parameter of .39. Increasing the productivity parameter while holding constant the government investment share s_g^p implies that a change in public investment has larger substitution and wealth effects on the private sector. In the period the shock takes place, the representative consumer knows that future periods will be times of relatively high productivity. In Figure 20, it is clear that the positive wealth effect dominates in period 1; private consumption rises in response to the public investment shock and labor input falls implying that output also falls. After period 1, with the public capital on line, there is a substantial increase in the productivity of private factors of production, leading to increases in both investment and labor supply relative to their steady state levels. The combined result of these increases is a substantial increase in output. The cumulative multipliers for this experiment are plotted in Figure 21. The effects of the shock to public capital are both large and very persistent; consumption and output both exhibit important deviations from steady state levels 100 quarters after the shock takes place. Finally, Figure 22 plots the impulse response for public capital and private labor input. As noted above, labor input falls in the period that the shock takes place (quarter 1) and is above its steady state level for about twenty quarters subsequently. However, this graph demonstrates that there are relatively small movements in labor input induced by a relatively large shock to government capital.

6. Multipliers for Publicly Provided Consumption Services

In this section, we consider government purchases of a third type, which we view as provision of public consumption services, denoted by G^S. These public consumption services yield utility directly: examples might be public parks, school lunch programs, and Fourth of July fireworks. For the economy to exhibit steady state growth, provision of these services must grow at the same rate as output. There are also restrictions on the manner in which public consumption services enter the utility function. In particular, the utility function must be of the following form:

$$u(C,G,L) = \frac{1}{(1-\sigma)}[\omega(C,\theta G^S)\ v(L)]^{1-\sigma} + \phi(G^S) \qquad \text{if } 0<\sigma<1 \text{ or } \sigma>1 \qquad (31)$$

$$u(C,G,L) = \log(\omega(C,\theta G^S)) + v(L) + \phi(G^S) \qquad \text{if } \sigma = 1 \qquad (32)$$

where ω is a constant returns to scale function which aggregates C and G^s into a composite good. The function ϕ is unrestricted, since it has no implications for steady state behavior. (However, if the government were choosing G^s optimally, an additional restriction necessary for steady state growth is the restriction $\phi(G)=0$.) Following Bailey (1962) and Barro (1986), the parameter θ indexes the effectiveness of the government in transforming output units into goods that enter the utility function. We study two examples of utility producing government purchases; first, the case in which private consumption and public consumption services are perfect substitutes, and second, the case in which they are complements.

6.1 Case 1: Perfect Substitutes

In this example, private consumption and public consumption services are assumed to be perfect substitutes, implying a utility function of the form:

$$\omega(C,G^s) = C + \theta G^s \quad . \tag{33}$$

This specification has been studied previously by Aschauer (1985), Barro (1986), and Kormendi (1983). Based on their results, we set $\theta=.4$; an increase in G^s of one unit results in a net loss to the economy of $1-\theta=.6$ units. Figure 23 and 24 plot multipliers for this policy experiment. Figure 23 resembles Figure 5 (multipliers for serially uncorrelated basic spending) scaled by $1-\theta$ (.6), except for the behavior of private consumption in quarter 1, which is lower by θ (.4) compared with Figure 5. This reflects the perfect substitutability of private and public consumption.

6.2 Case 2: Complements

If the level of public consumption services raises the level of the marginal utility of private consumption ($D_2D_1u(C,\theta G^s,L)>0$) then the two types of consumption are complements in a Hicks-Allen sense. Figures 25 and 26 plot the multipliers for parameters implying this complementary property; unitary elasticity of intertemporal substitution (σ=1), and a Cobb-Douglas form for the aggregate function ω, $\omega(C,\theta G^s)=C^{\nu}(\theta G^s)^{1-\nu}$, where ν is pinned down by the parameters of preferences and technology already specified. To allow comparison with Case 1, we continue to set θ=.40. The positive (and substantial) impact multiplier for consumption reflects the fact that an increase in public consumption services enhances the desirability of private consumption.

7. Summary and Conclusions

This paper has investigated the conditions under which equilibrium models of economic activity possess sizable "multipliers," where this term is taken to mean the effect on real variables such as consumption, output, and investment, of shocks to government purchases or financing decisions. In section 2 of the paper we presented several Keynesian multiplier concepts and discussed their relation to concepts familiar from modern time series analysis. There are two main lessons from this section. First, when analyzing the dynamic response of the economy to fiscal policy shocks, it is important that one distinguish between anticipated and unanticipated changes in policy. Second, under rational expectations, the parameters of the stochastic process governing policy variables enter the reduced form for endogenous variables, limiting the range of policy experiments that can sensibly be performed with reduced forms, such as that of our model economy as with vector autoregressions.

Proper analysis of policy interventions requires careful specification of all aspects of the economic environment, including the assumed process for expectation formation. Section 3 provided a simple rational expectations equilibrium model of fluctuations which incorporates government purchases and in which distortionary taxation is one potential source of government finance.

In sections 4–6 we analyze the dynamic response of the model economy to shocks to three types of government purchases, under a variety of assumptions concerning the persistence of the shocks and the means of financing.

Basic purchases were defined to be purchases both (privately) unproductive and unvalued in consumption. We found that more persistent changes in this type of purchases lead to larger multipliers, due to the larger wealth effect associated with increased permanence. This finding stands in sharp contrast to the predictions of Barro (1981) and Hall (1980) about temporary wartime movements in government purchases. In a recent paper on this subject, Wynne (1987) employed a model very similar to the one used here to study the effects of wartime government purchases in the United States. Wynne interprets this wartime variation as a temporary, privately unproductive movement in government purchases ("basic purchases" in our terminology). He finds that this simple model does a good job of tracking the dynamic response of output and investment to this type of shock, although his model predicts larger variation in consumption than actually observed in the data.

Still within the context of basic purchases, we considered the effects of a balanced budget rule. The balanced budget rule has very strong effects on the dynamic response of the economy: effects that are stronger the more temporary is the shock to purchases. This happens because a temporary increase in tax financed government purchases induces individuals to substitute intertemporally, working less in the period in which the shock occurs. But the balanced budget requirement means that tax rates must rise in order to keep revenue constant while the tax base (output) is falling. This tax movement induces further substitution away from labor effort, and a further rise in the tax rate. The more temporary is the disturbance, the stronger is the intertemporal substitution effect. In the zero serial correlation case, the impact multiplier on output was −1.7 (an increase in basic purchases of one unit leads to a contemporaneous decline in output of 1.7 units). The impact multiplier on investment in this case is −2.5. When the shock is more persistent, with a serial correlation coefficient of .9, the impact multipliers on output and investment are −1.35 and −2.0. These are still very substantial effects.

In section 5, we studied government purchases that are productive in the sense that they either (i) augment factor productivity in the period the purchases takes place, or (ii) government purchases take the form of increased investment in the stock of public capital which acts as a third factor of production. Productive purchases can produce large multipliers. This is particularly true for government purchases of public capital goods in the case where government capital is underprovided. Using an estimate for the productivity of government capital close to that of Aschauer (1988), together with a relatively small output share for government investment, we find that shocks to government purchases of this type can have very substantial effects on aggregate output, consumption, and investment. These results complement Aschauer's empirical results. In fact, Aschauer has suggested that the "productivity slowdown" of the past fifteen years may be explainable by the fact that the stock of government-provided capital goods has declined markedly in this period. However, some have suggested that reverse causation is another possible explanation for this observed correlation. Our analysis lends support to the view that the productivity slowdown could in fact have been caused by the decline in public capital.

Finally, in section 6, we studied government purchases that yield utility directly to private individuals. We considered two cases: the case in which public and private consumption goods are substitutes, and the case in which they are complements. The parameter for the utility-producing "efficiency" of government purchases of this type was chosen to match empirical estimates by Aschauer (1988) and others. We found that government purchases of this sort generally have large impact multipliers on investment and consumption, as well as having very persistent effects on all variables.

In conclusion, we wish to stress two points. First, it is definitely the case that equilibrium business cycle models <u>can</u> produce large multipliers: certain types of government purchases and certain financing policies can have very important effects on the economy. This is not to say, however, that the results of this paper accord with the standard predictions of Keynesian analyses. We have highlighted several cases in which they do not.

Second, equilibrium models of the type developed here are viable laboratories for carrying out policy experiments which appropriately account for the rational forecasting of private agents in the economy. These methods do not require the optimality of equilibrium: otherwise we could not have studied the effects of distortionary income taxation. Thus we view this paper as a first step toward quantitative evaluation of policy. By demonstrating the feasibility of quantitative model evaluation, we wish to inspire other researchers to insist on quantitative (instead of merely qualitative) evaluation of the models which we use to think about policy.

Footnotes

[1]See, for example, the remarks of the author of a notable undergraduate text on macroeconomics (Barro (1986)).

[2]See Appendix B for derivation of the multipliers. Other dynamic multipliers are readily computed but will not be studied here. These follow directly from the structure of the consumption and investment functions. Thus, for example, it is easy to show that $m_{cg}(1) = 0$ and that $m_{cg}(s) = \nu\, m_{yg}(s-1)$ for $s \geq 2$ and that $m_{ig}(1)=0$, that $m_{ig}(2) = \kappa\, m_{yg}(1)$ and that $m_{ig}(s) = \kappa\, [m_{yg}(s) - m_{yg}(s-1)]$ for $s>2$.

[3]See Kydland and Prescott (1982). Taylor (1982) analyzes the effects of stabilization polic in a rational expectations Keynesian model with such investment lags.

[4]Technically, this statement is not quite correct for this particular model, for here one can discuss the impact effect of a change in government spending without specifying expectations. This is due to the very simple structure of the model, in which current actions—those dated t—do not depend on expectation about the future. (Consumption is simply a function of past income, and investment of actual and expected income in previous periods). But most macroeconomic models currently in use specify that consumption, via the permanent income theory, and investment, via dynamic specifications such as adjustment costs or time-to-build, depend on expectations about the future.

[5]In this case, we can construct multipliers for unanticipated shocks to government spending, which we label $m^u_{yg}(s)$. Then, one can show that $m^u_{yg}(1)=m_{yg}(1)=1$, so that there is no difference on impact. But at subsequent lags, it turns out that the multipliers for unanticipated shocks differ from those that were previously described for the Samuelson model or, equivalently, for anticipated changes in government spending:

$$m^u_{yg}(1) = m_{yg}(1)-\kappa m_{yg}(s-1) = [(\mu_2-\kappa)\mu_2^{s-1} - (\mu_1-\kappa)\mu_2^{s-1}]/[\mu_2-\mu_1].$$

There is also a smaller total multiplier, which is given by $[1-\kappa]/[1-\upsilon]$.

[6]To keep the theoretical example easy to understand, we have simplified in two ways. First, we have left out consumption and investment. Although these would be generally incorporated in a VAR, we may omit them here because becuase they do not add to the dynamics of the model. Second, we have abstracted from any random shifts in the consumption or investment schedules; the results is that there is perfect correlation between the "innovations" to

output and government expenditures. Most estimated VARs exhibit low, rather than high correlation of innovations. To increase the realism of the model, one might introduce other sources of shocks. Elaboration of the model along these lines would not alter the points discussed in the text.

[7]We review these methods in Appendix C. Baxter (1988) provides a method for computing suboptimal dynamic equilibria that does not require a linear approximation of the sort used by King (1987) and King, Plosser, and Rebelo (1988a).

[8]Wynne (1987) has studied the dynamic response to wartime purchases of an economy very similar to that of section 3. Wynne interprets this type of expenditure as temporary, unproductive purchases. He finds that the model mimics the response of the actual economy in terms of output and investment responses to shocks, but finds that consumption was less variable than predicted by the simulated model.

[9]In recent work, Christiano and Eichenbaum (1988) also point out that permanent government purchases have a larger output effect than temporary movements. In work undertaken subsequent to our analysis, King (1988) constructs quantitative measures of substitution and wealth components within the basic neoclassical model.

[10]An economy operating under this regime may display more than one steady state, since—using a "Laffer curve" perspective—multiple tax rates are consistent with a given level of spending. However, our analysis is local near a dynamically stable point, so we need not deal with these complexities.

[11]See the transitional dynamics to temporary shocks in King, Plosser and Rebelo, (1988a), pp. 206-211.

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Appendix A:

Theil-Boot Multipliers

A simple linear dynamic macroeconomic model of the class analyzed by Theil and Boot (1962) has the form

$$A_0 Y_t = A_1 Y_{t-1} + B_0 X_t + \tilde{\epsilon}_t \qquad \text{(A-1)}$$

where Y_t is a vector of endogenous variables, X_t is a vector of exogenous variables and $\tilde{\epsilon}_t$ is a vector of disturbances. Assuming that A_0 is invertible and defining $A = A_0^{-1} A_1$ and $B = A_0^{-1} B_0$, we can follow Theil and Boot in writing the reduced form of the model as $Y_t = A Y_{t-1} + BX_t + \epsilon_t$, where $\epsilon_t = A_0^{-1} \tilde{\epsilon}_t$. Then, the "final form" of the dynamic model is

$$Y_t = \sum_{s=0}^{\infty} A^s BX_{t-s} + \sum_{s=0}^{\infty} A^s \epsilon_{t-s} \qquad \text{(A-2)}$$

under the assumption that $\lim_{h\to\infty} A^h$ is a zero matrix, i.e., the eigenvalues of A have real part strictly less than unity.

This "final form" representation provides a ready means of defining and calculating dynamic multipliers of three sorts, using the terminology of Theil and Boot (1962). The <u>impact multiplier</u> measures the effect of a change in an exogenous variable, x_t, on an endogenous variable, y_t. This multiplier is given by the relevant element of the matrix B.[1]

[1]By the relevant element, we mean the following. Let x_t be the jth element of the vector X_t. Let y_t be the kth element of the vector Y_t. Then, the relevant element is the (j,k) element of the matrix B.

The <u>interim multipliers</u> measure the effect of a change in an exogenous variable at date t, x_t, on an endogenous variable at date t+s, y_{t+s}. This multiplier is given by the relevant element of the matrix A^sB.

The <u>total multiplier</u> measures the cumulative effect of a change in an exogenous variable at date t, x_t, on an endogenous variable, y; it is equal to the sum of the impact and interim multipliers. Theil and Boot (1962) show that the matrix of total multipliers is given by $[(I-A)^{-1}B]$, so long as $(I-A)$ is invertible.

For systems with higher order lags in Y and X, it is possible to use the familiar trick of expanding the dimension of the Y and X vectors in order to preserve the first-order autoregressive representation. Thus, we write the component of the system relevant for multiplier analysis as

$$Y_t^* = A^* \, Y_{t-1}^* + B^* \, X_t^* \qquad \text{(A-3)}$$

with $Y_t^* = [Y_t', Y_{t-1}', \ldots Y_{t-q}']'$ and $X_t^* = [X_t', X_{t-1}', \ldots X_{t-r}']'$ where q and r are the orders of maximum lags of Y and X. Theil and Boot (section 4) provide formulas for multipliers for this transformed system that are closely related to those for the simple linear system.

Appendix B:

Derivation of Multipliers and Impulse Responses

1. The Samuelson Model

The relationship between output and spending can be written as $(1-\mu_1 L)(1-\mu_2 L)y_t = g_t$ or (assuming $|\mu_i| < 1$)

$$y_t = \frac{1}{(1-\mu_1 L)(1-\mu_2 L)} g_t = M(L)g_t.$$

To calculate multipliers, we then need to express

$$\frac{1}{(1-\mu_1 L)(1-\mu_2 L)} = \frac{a}{1-\mu_1 L} + \frac{b}{1-\mu_2 L}$$

where a, b are to be determined. Equivalently,

$$1 = a(1-\mu_2 L) + b(1-\mu_1 L).$$

This implies the two conditions $1 = a + b$ and $a\mu_2 + b\mu_1 = 0$. Hence, it follows that

$$a = \frac{-\mu_1}{\mu_2-\mu_1} \qquad b = \frac{\mu_2}{\mu_2-\mu_1}$$

Thus, the polynominal M(L) can be written as

$$M(L) = \left(\frac{\mu_2}{1-\mu_2 L} - \frac{\mu_1}{1-\mu_1 L}\right)/(\mu_2-\mu_1) = \sum_{s=0}^{\infty} \left(\frac{\mu_2^{s+1}-\mu_1^{s+1}}{\mu_2-\mu_1}\right)L^s.$$

2. <u>The Samuelson Model with Expectations.</u>

The relationship between output and government spending can be written as $(1-\mu_1 L)(1-\mu_2 L)\ y_t = (1-\kappa L)g_t + \kappa L E_{t-1}g_t$. (Note that L shifts both conditioning date and variable date by one period, e.g., $LE_{t-1}g_t = E_{t-2}g_{t-1}$.) As in the previous model, we can write

$$[(1-\mu_1 L)(1-\mu_2 L)]^{-1} = [\mu_2(1-\mu_2 L)^{-1} - \mu_1(1-\mu_1 L^{-1})]/(\mu_2-\mu_1).$$

Further, $(1-\kappa L)(1-\mu_i L)^{-1} = [1 + (\mu_i-\kappa)L(1-\mu_i L)^{-1}]$, so that the multipliers for unanticipated government spending shocks are found as follows:

$$y_t = \frac{1-\kappa L}{(1-\mu_1 L)(1-\mu_2 L)} g_t + \frac{\kappa L}{(1-\mu_1 L)(1-\mu_2 L)} E_{t-1}g_t$$

$$= \left[\frac{\mu_2}{\mu_2-\mu_1}\ \frac{(1-\kappa L)}{(1-\mu_2 L)} - \frac{\mu_1}{\mu_2-\mu_1}\ \frac{(1-\kappa L)}{(1-\mu_1 L)}\right] g_t$$

$$+ \left[\frac{\mu_2}{\mu_2-\mu_1}\ \frac{\kappa L}{1-\mu_2 L} - \frac{\mu_2}{\mu_2-\mu_1}\ \frac{\kappa L}{1-\mu_1 L}\right] E_{t-1}g_t$$

$$= \left[\frac{\mu_2}{\mu_2-\mu_1}\left[1 + \frac{(\mu_2-\kappa)L}{1-\mu_2 L}\right] - \frac{\mu_1}{\mu_2-\mu_1}\left[1 + \frac{(\mu_1-\kappa)L}{1-\mu_1 L}\right]\right] g_t$$

$$+ \left[\frac{\mu_2}{\mu_2-\mu_1}\ \frac{\kappa L}{1-\mu_2 L} - \frac{\mu_1}{\mu_2-\mu_1}\ \frac{\kappa L}{1-\mu_1 L}\right] E_{t-1}g_t$$

$$= \left[1 + \sum_{s=1}^{\infty} \frac{(\mu_2-\kappa)\mu_2^s - (\mu_1-\kappa)\mu_1^s}{\mu_2-\mu_1} L^s\right] g_t$$

$$+ \left[\kappa \sum_{s=1}^{\infty} (\frac{\mu_2^s - \mu_1^s}{\mu_2-\mu_1}) L^s\right] E_{t-1} g_t$$

3. <u>The Naive Econometric Model</u>

The relationship between output and government spending takes the form

$$y_t = \frac{[1-\kappa L+\kappa\rho L^2]}{(1-\mu_1 L)(1-\mu_2 L)} g_t$$

$$= \left[\left[\frac{\mu_2}{\mu_2-\mu_1}\right] \frac{(1-\kappa L+\kappa\rho L^2)}{1-\mu_2 L} - \frac{\mu_1}{\mu_2-\mu_1} \frac{(1-\kappa L+\kappa\rho L^2)}{1-\mu_1 L}\right] g_t$$

$$= \left[\frac{\mu_2}{\mu_2-\mu_1} \left[1+(\mu_2-\kappa)L + \frac{(\kappa\rho+\mu_2(\mu_2-\kappa))L^2}{1-\mu_2 L}\right]\right.$$

$$\left. - \frac{\mu_1}{\mu_2-\mu_1} \left[1 + (\mu_1-\kappa)L + \frac{(\kappa\rho+\mu_1(\mu_1-\kappa))L^2}{1-\mu_1 L}\right]\right] g_t$$

$$= \left[1 + \frac{\mu_2(\mu_2-\kappa) - \mu_1(\mu_1- \kappa)}{\mu_2-\mu_1} L\right.$$

$$\left. + \sum_{s=2}^{\infty} \left\{\frac{(\mu_2^2-\kappa\mu_2+\kappa\rho)\mu_2^{s-1} - (\mu_1^2-\kappa\mu_1+\kappa\rho)\mu_1^{s-1}}{\mu_2-\mu_1}\right\} L^s\right] g_t$$

4. <u>The Moving Average Representation.</u>

The relationship between output and shocks to government spending can be written as

$$y_t = \left[\frac{[1-\kappa L+\kappa\rho L^2]}{(1-\mu_1 L)(1-\mu_2)}\right]\left[\frac{1}{1-\rho L}\right]\epsilon_{gt}.$$

This requires that we first express

$$\frac{1}{(1-\mu_1 L)(1-\mu_2 L)(1-\rho L)} = \frac{a}{1-\mu_1 L} + \frac{b}{1-\mu_2 L} + \frac{c}{1-\rho L}$$

which requires that

$$1 = a(1-\mu_2 L)(1-\rho L) + b(1-\mu_1 L)(1-\rho L) + c(1-\mu_1 L)(1-\mu_2 L).$$

Thus we have the system:

$$1 = a + b + c$$

$$0 = a(\mu_2+\rho) + b(\mu_1+\rho) + c(\mu_1+\mu_2)$$

$$0 = a(\mu_2\rho) + b(\mu_1\rho) + c(\mu_1\mu_2)$$

Some tedious algebra demonstrates that

$$a = \frac{-\mu_1^2}{(\mu_2-\mu_1)(\mu_1-\rho)}$$

$$b = \frac{\mu_2^2}{(\mu_2-\mu_1)(\mu_2-\rho)}$$

$$c = \frac{\rho^2}{(\mu_1-\rho)(\mu_2-\rho)}.$$

Note that expressions for a and b match those derived above when $\rho = 0$. Hence, the moving average representation is given by

$$y_t = (1-\kappa L+\kappa\rho L^2)\left[\frac{a}{1-\mu_1 L} + \frac{b}{1-\mu_2 L} + \frac{c}{1-\rho L}\right]\epsilon_{gt}$$

$$= \left\{ a\left[1 + (\mu_1-\kappa)L + \frac{(\mu_1^2 - \kappa\mu_1+\kappa\rho)L^2}{1 - \mu_1 L}\right]\right.$$

$$+ b\left[1 + (\mu_2-\kappa)L + \frac{(\mu_2^2-\kappa\mu_2+\kappa\rho)L^2}{1-\mu_2 L}\right]$$

$$\left. + c\left[1 + (\rho-\kappa)L + \frac{\rho^2 L^2}{1-\rho L}\right]\right\} \epsilon_{gt}$$

$$= 1 + \left[\frac{\mu_2^3(\mu_1-\rho) - \mu_1^3(\mu_2-\rho) + \rho^3(\mu_2-\mu_1)}{(\mu_2-\mu_1)(\mu_1-\rho)(\mu_2-\rho)} - \kappa\right] L$$

$$+ \sum_{s=2}^{\infty} \left[\frac{(\mu_2^2-\kappa\mu_2+\kappa\rho)(\mu_1-\rho)\mu_2^s - (\mu_1^2-\kappa\mu_1+\kappa\rho)(\mu_2-\rho)\mu_1^s + \rho^2(\mu_2-\mu_1)\rho^s}{(\mu_2-\mu_1)(\mu_1-\rho)(\mu_2-\rho)}\right] L^s$$

Appendix C:

Calculation of Dynamic Equilibrium

Since all agents are identical, in competitive equilibrium there will be no intertemporal trade, so we can focus on the decision problem for an individual agent facing a sequence of resource constraints. The agent seeks to maximize lifetime utility subject to the sequence of constraints, given sequences of tax rates and transfers. The Lagrangian associated with the optimization problem is

$$\begin{aligned}\mathcal{L} = & \sum_{t=0}^{\infty} \beta^t \, [u(C_t, 1-N_t)] \\ & + \sum_{t=0}^{\infty} \Lambda_t \, [(1-\tau_t) \, F(K_t, N_t) + \underline{T}_t + (1-\delta_K)K_t - C_t - \gamma_X K_{t+1}] \qquad \text{(C-1)}\end{aligned}$$

where K_0 is treated as given and Λ_t is the multiplier attached to the t period resource constraint.

Using the notation $D_n f$ to denote the partial derivative of the function f with respect to its nth argument, the efficiency conditions (for an interior optimum) are the following four equations,

$$\beta^t \, D_1 u(C_t, 1-N_t) \quad -\Lambda_t \qquad = 0 \qquad \text{(C-2)}$$

$$-\beta^t \, D_2 u(C_t, 1-N_t) \quad + \Lambda_t (1-\tau_t) D_2 F(K_t, N_t) \qquad = 0 \qquad \text{(C-3)}$$

$$\Lambda_{t+1} [(1-\tau_{t+1}) D_1 F(K_{t+1}, N_{t+1}) + (1-\delta_K)] - \gamma_X \Lambda_t \qquad = 0 \qquad \text{(C-4)}$$

$$(1-\tau_t) F(K_t, N_t) + \underline{T}_t + (1-\delta_K) K_t - \gamma_X \, K_{t+1} - C_t \qquad = 0, \qquad \text{(C-5)}$$

for t = 0,1,2,...∞ and the "transversality condition", $\lim_{t\to\infty} \Lambda_t K_{t+1} = 0$.

The preceding optimal decisions are valid for arbitrary specifications of tax and spending sequences. In perfect foresight competitive equilibrium, given the policy specification, tax and transfer sequences depend on individual decisions, which in turn depend on tax rates. As in Brock (1972) and Romer (1986), equilibrium sequences can be obtained by combining the individual's efficiency conditions with aggregate consistency conditions, which in this case are the constraints of the government. Equilibrium sequences $[\{\underline{C}_t\}_{t=0}^{\infty}, \{\underline{N}_t\}_{t=0}^{\infty}, \{\underline{K}_t\}_{t=0}^{\infty}$ and $\{\underline{\Lambda}_t\}_{t=0}^{\infty}]$ satisfy,

$$\beta^t \, D_1 u(\underline{C}_t, 1-\underline{N}_t) \quad - \underline{\Lambda}_t \qquad = 0 \tag{C-6}$$

$$\beta^t \, D_2 u(\underline{C}_t, 1-\underline{N}_t) \quad + \underline{\Lambda}_t \Omega_t(\underline{G}_t, \underline{K}_t, \underline{N}_t) D_2 F(\underline{K}_t, \underline{N}_t) \qquad = 0 \tag{C-7}$$

$$\underline{\Lambda}_{t+1}\{\Omega_{t+1}(\underline{G}_t, \underline{K}_{t+1}, \underline{N}_{t+1}) D_1 F(\underline{K}_{t+1}, \underline{N}_{t+1}) + (1-\delta_K)\} - \gamma_X \, \underline{\Lambda}_t = 0 \tag{C-8}$$

$$F(\underline{K}_t, \underline{N}_t) + (1-\delta_K)\underline{K}_t - \gamma_X \, \underline{K}_{t+1} - \underline{C}_t - \underline{G}_t \qquad = 0, \tag{C-9}$$

for $t = 0,1,2,\ldots\infty$ and the "transversality condition", $\lim_{t\to\infty} \underline{\Lambda}_t \underline{K}_{t+1} = 0$. The process of computing an (approximate) dynamic equilibrium will be discussed further below.

<u>Restrictions on Preferences</u>

Lucas (1980) argues that it is important to use information from long run growth experience to restrict preference specifications in equilibrium business cycle models. The existence of a steady state growth path—which includes consumption growing at a constant rate and constancy of hours—implies that the utility function must take the form

$$u(C,L) = \frac{1}{(1-\sigma)} C \quad v(L)^{1-\sigma} \qquad \text{if } 0<\sigma<1 \text{ or } \sigma>1 \tag{C-10}$$

$$u(C,L) = \log(C) + v(L) \qquad \text{if } \sigma = 1 \qquad \text{(C-11)}$$

where σ is a parameter and v(L) is a function.[2]

There is economic content to these preference restrictions. Time is bounded, so that in a steady state it cannot grow. Thus the utility function must be such that there are exactly offsetting income and substitution effects of the changes in real wages. This is necessary because, in the steady state, there is growth in the real wage (and in output) associated with the (exogenous) steady state growth in labor productivity.

Approximate Dynamics

In order to investigate multiplier concepts in the neoclassical model described above, we use the methods of linear approximate dynamics described by King (1987) and King, Plosser, and Rebelo (1988a, Technical Appendix). This method involves log-linear approximation of (C-6) through (C-9) near the stationary levels of k, N, c, g, and i. These linear approximations are interpreted as expressions for percentage deviations from steady state levels, which we denote by a circumflex (^). Approximating (C-6)and (C-7) yields:

$$\xi_{cc}\hat{c}_t - \xi_{c\ell}\{N/(1-N)\}\hat{N}_t = \hat{\lambda}_t \qquad \text{(C-12)}$$

---

[2]To insure that consumption and leisure are goods and that utility is concave, we need to impose some additional structure. When momentary utility is additively separable, all that we must require is that v(L) is increasing and concave. When momentary utility is multiplicatively separable, then it is necessary that we require that (i) v(L) is increasing and concave if $\sigma_c < 1$ and decreasing and convex if $\sigma_c > 1$. Further, we require that $-\sigma_c[LD^2v(L)/Dv(L)] > (1 - \sigma_c)[LDv(L)/v(L)]$ to assure overall concavity of $u(\cdot)$.

$$\xi_{\ell c}\hat{c}_t - \xi_{\ell\ell}\{N/(1-N)\}\,\hat{N}_t = \{\hat{\lambda}_t + \xi_{Nk}\hat{k}_t - \xi_{NN}\,\hat{N}_t\} + \{\omega_k\,\hat{k}_t + \omega_N\,\hat{N}_t + \omega_g\,\hat{g}_t\} \qquad \text{(C-13)}$$

In these expressions, there are a number of elasticities, all of which are evaluated at the stationary point (k,N,c,y,etc.). First, the ξ's on the left-hand side of (C-12) and (C-13) are elasticities of marginal utility.[3] For example, if the utility function is additive in the logarithms of c and L, then it follows that $\xi_{cc} = \xi_{\ell\ell} = -1$ and that $\xi_{\ell c} = \xi_{c\ell} = 0$. Second, the ξ's on the right hand side of (41) are elasticities of the marginal product of labor. If the production F is Cobb-Douglas, $Ak^{1-\alpha}N^{\alpha}$, then these are $\xi_{Nk} = (1-\alpha)$ and $\xi_{NN} = \alpha - 1$. Third, the ω's are elasticities of the wedge function, $\Omega_t = (1-\tau_t)$, with respect to its arguments.

Differentiation of the intertemporal efficiency (C-8) condition implies that

$$\{\hat{\lambda}_{t+1} + \eta_k\,\hat{k}_{t+1} + \eta_N\,\hat{N}_{t+1} + \eta_G\,\hat{g}_{t+1}\} = \hat{\lambda}_t \qquad \text{(C-14)}$$

where η_K is the elasticity of the net after-tax marginal product of capital, $[\Omega(k,N,A,g)AD_1F(k,N) + (1-\delta_K)]$, with respect to K evaluated at the steady state (η_g and η_N are defined in an analogous way). The elasticities are given by:

$$\eta_k = [\gamma_X - \beta^*(1-\delta_K)][\omega_k + \xi_{KK}]/\gamma_X$$
$$\eta_N = [\gamma_X - \beta^*(1-\delta_K)][\omega_N + \xi_{KN}]/\gamma_X$$
$$\eta_g = [\gamma_X - \beta^*(1-\delta_K)][\omega_g]/\gamma_X,$$

---

[3]When the utility function is additively separable, it follows that $\xi_{cc} = -1$, $\xi_{c\ell} = \xi_{\ell c} = 0$ and $\xi_{\ell\ell} = LD^2v(L)/Dv(L)$. When the utility function is multiplicatively separable, it follows that $\xi_{cc} = -\sigma_c$, $\xi_{c\ell} = LDv(L)/v(L)$, $\xi_{\ell c} = 1-\sigma_c$ and $\xi_{\ell\ell} = LD^2v(L)/Dv(L)$.

where, again, the ω's are the elasticities of the tax wedge and ξ_{KK} and ξ_{KN} are elasticities of the marginal product of capital with respect to capital and labor. (Under the Cobb-Douglas structure $\xi_{KK} = -\alpha$ and $\xi_{KN} = \alpha$).

Finally, differentiation of the resource constraint implies

$$\hat{y}_t = s_N \hat{N}_t + s_K \hat{k}_t = s_c \hat{c}_t + s_g \hat{g}_t + s_i \phi \hat{k}_{t+1} - s_i (\phi-1) \hat{k}_t, \qquad \text{(C-15)}$$

where s_K and s_N are shares of capital and labor in national income and s_c, s_g and s_i are shares of consumption, investment and government in national output, and where $\phi = \gamma_X/[\gamma_X-(1-\delta_K)] > 1$. The shares of output accruing to each of the two factors, s_N and s_K, take on simple forms under the Cobb-Douglas production structure, $s_N=\alpha$ and $s_k=(1-\alpha)$.

We briefly review the solution algorithm for this economy; it is discussed in more detail in King (1987) and King, Plosser and Rebelo (1988a). Expressions (C-16)-(C-17) can be combined to eliminate consumption, effort, and output flows, yielding a difference equation system in capital stock $(\hat{k})$ and shadow price $(\hat{\lambda})$. This difference system can then be solved, subject to the transversality condition, to produce a unique solution sequence for the capital stock $(\hat{k})$ and shadow price $(\hat{\lambda})$, given specification of an exogenous sequences for $(\hat{G})$. This capital solution sequence can be compactly summarized as follows

$$\hat{k}_{t+1} = \theta_1 \hat{k}_t + \psi_1 \hat{g}_t + \psi_2 \sum_{s=1}^{\infty} \theta_2^{-s} \hat{g}_{t+s},$$

(C-18)

where the parameters $\theta_1, \theta_2, \psi_1, \psi_2$ are generically functions of all of the parameters of preferences, technologies, and policy rules (i.e., the ω elasticities). This illustrates that optimal accumulation generally depends on government policy in two ways: (i) through operating rules (Ω) that link the levels of the taxes that agents face to endogenous variables; and (ii) through the expectations that agents hold about the future course of government spending.

With the solution sequence for $\hat{\lambda}$ and $\hat{k}$ in hand, it is straightforward to compute solution sequences for the flow variables $\hat{N}_t$ and $\hat{c}_t$. Further, given these solutions and the following expressions, expressions for other variables of interest can be constructed. In particular, the approximate solutions for output and investment are:

$$\hat{y}_t = \hat{A}_t + s_N \hat{N}_t + s_K \hat{k}_t \tag{C-19}$$

$$\hat{i}_t = \frac{1}{s_i} \hat{y}_t - \frac{s_c}{s_i} \hat{c}_t - \frac{s_g}{s_i} \hat{g}_t \tag{C-20}$$

Table 1

Baseline parameter values for the experiments in section 4
Time period is a quarter of a year

Parameters of preferences:

$\beta = (1+(.065/4))^{-1}$	discount factor; implying a steady state real interest rate of 6.5% per year.
$\bar{N} = .20$	share of total time devoted to market activities
$\sigma = 1$	$1/\sigma$ is the elasticity of intertemporal substitution
$\theta = .40$	parameter governing utility value of government provided consumption services

Parameters of production technology:

$s_N = .58$	labor's share
$s_K = .42$	capital's share
$\zeta_{KN} = 1$	unitary elasticity of substitution between k and L
$\delta_k = \delta_g = .025$	depreciation rates of private and government capital
$\eta = .05$	government productivity parameter
$\gamma_X = 1.004$	gross growth rate of labor augmenting technical change, implying a steady state growth rate of 1.6% per year.
$\Omega = .70$	Steady state income tax rate of 30%.

Steady state shares of government spending:

$s_g^b = .05$	share of basic spending in total output
$s_g^p = .05$	share of productive spending in total output
$s_g^c = .10$	share of gov't consumption services in total output

All other parameter values and shares used in simulating the model are obtained as functions of the parameters set out above.

Table 2

Government spending which alters the productivity of capital:
The effects of variations in η and ζ_{KN}

	direct		$\zeta_{KN}=1$			$\zeta_{KN}=.50$			$\zeta_{KN}=2$	
η	effect	imp(0)	impact	total	imp(0)	impact	total	imp(0)	impact	total
.000	0.00	.0033	.0660	-.6266	.0029	.0580	-.8802	.0032	.0640	-.5077
.010	0.20	.0207	.4140	-.0197	.0223	.4460	-.1040	.0185	.3700	-.0161
.025	0.50	.0469	.9380	.8908	.0515	1.0300	1.0604	.0414	.8280	.7214
.050	1.00	.0904	1.8080	2.4081	.1001	2.0020	3.0010	.0795	1.5900	1.9505
.070	1.50	.1340	2.6800	3.9254	.1488	2.9760	4.9415	.1176	2.3520	3.1796
.100	2.00	.1776	3.5520	5.4428	.1974	3.9480	6.8821	.1557	3.1140	4.4087
.200	4.00	.3518	7.0360	11.5122	.3919	7.8380	14.6445	.3082	6.1640	9.3251

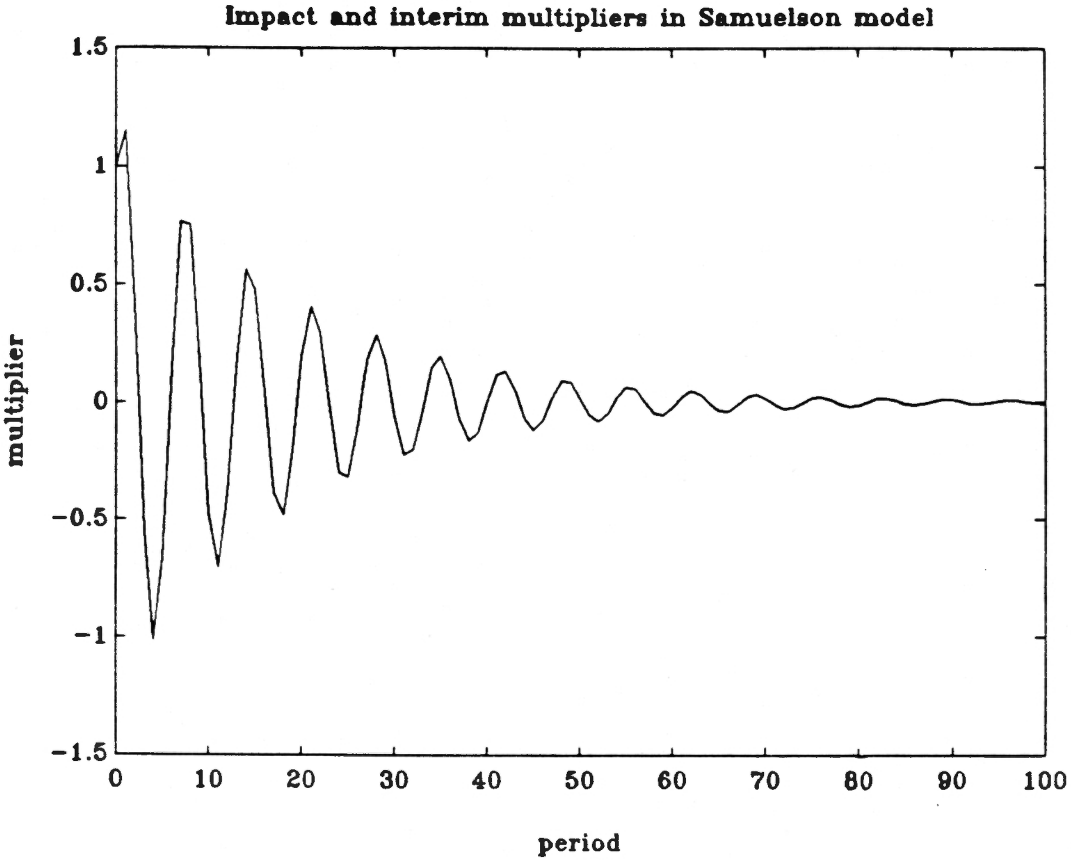
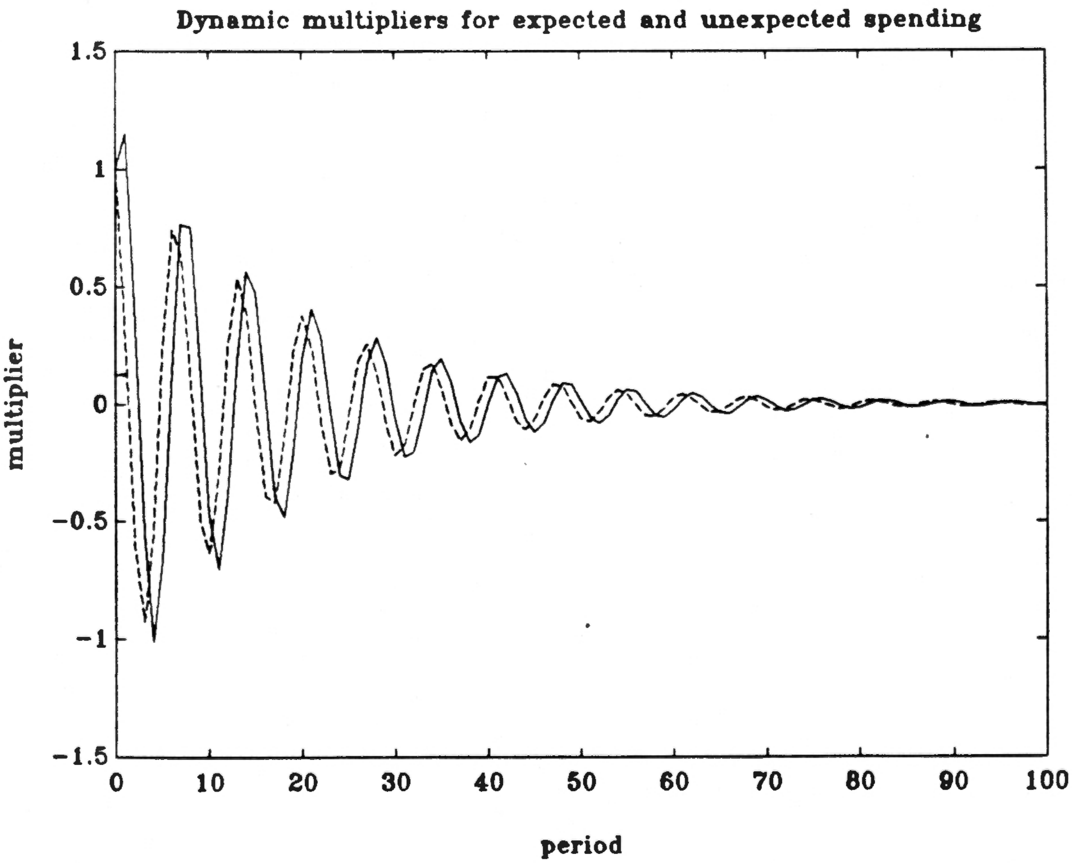


FIGURE 1



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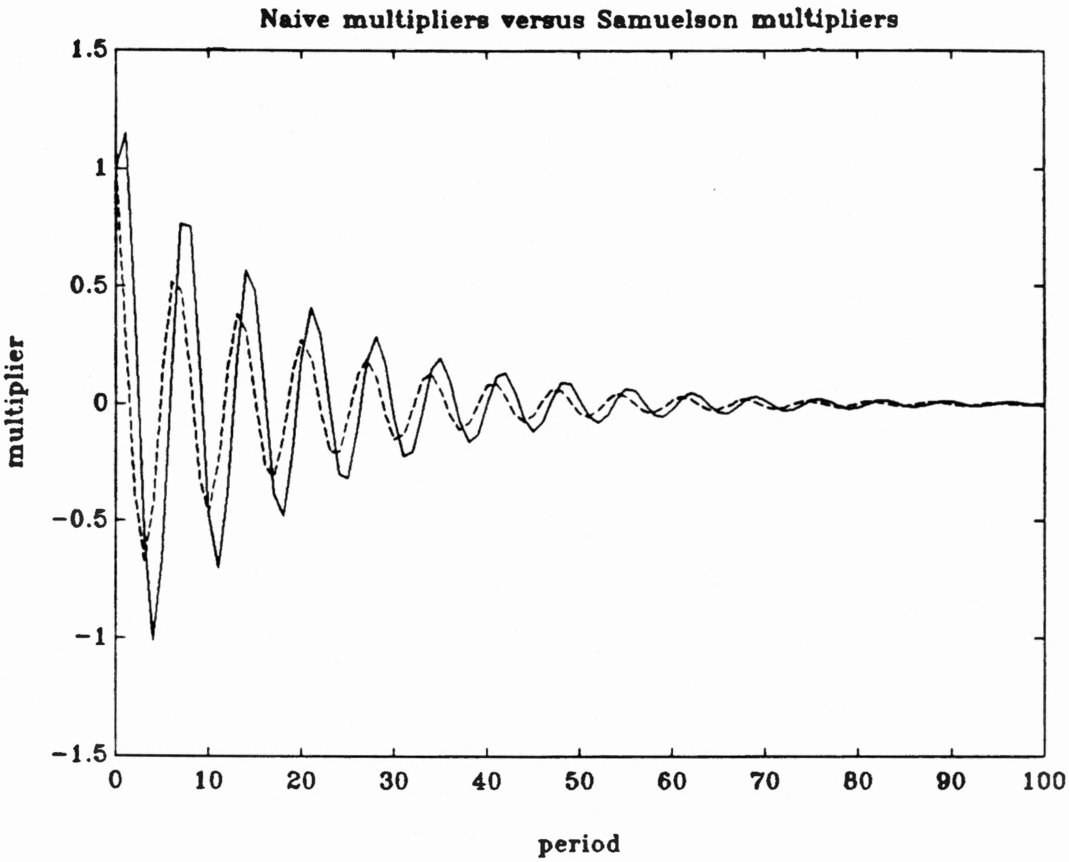

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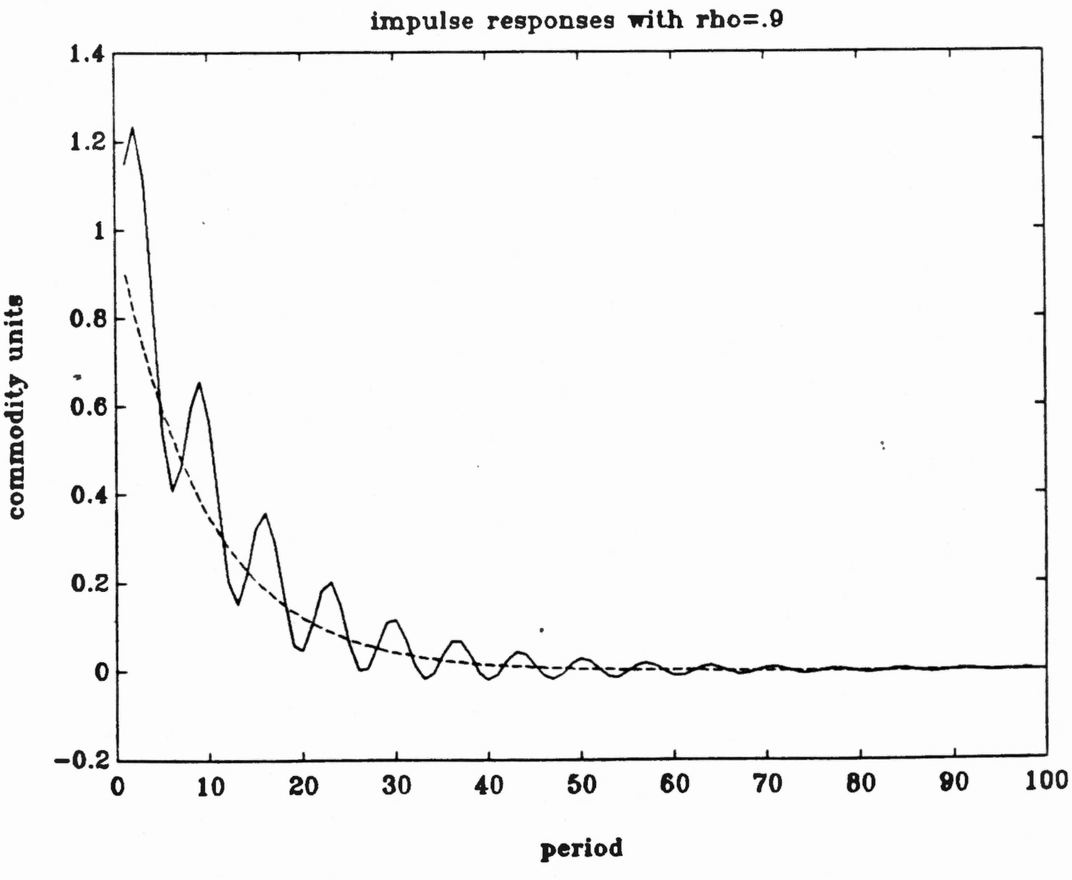

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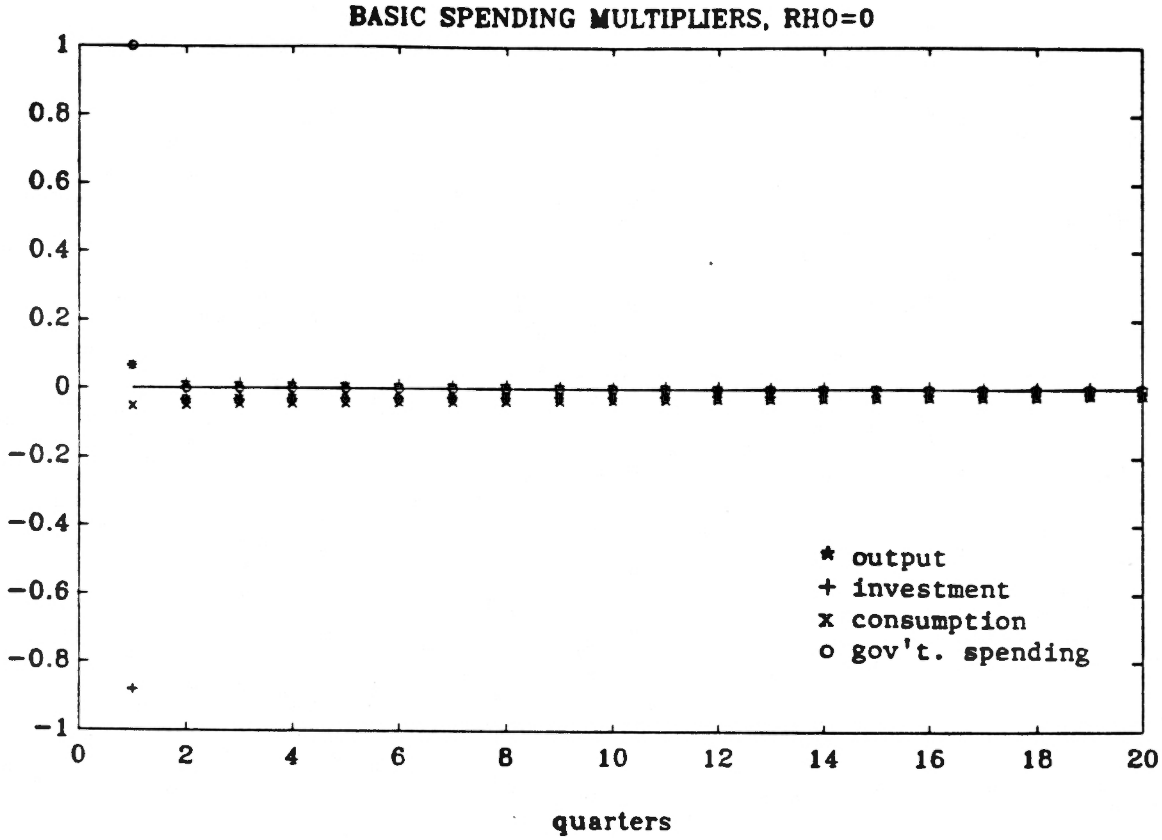

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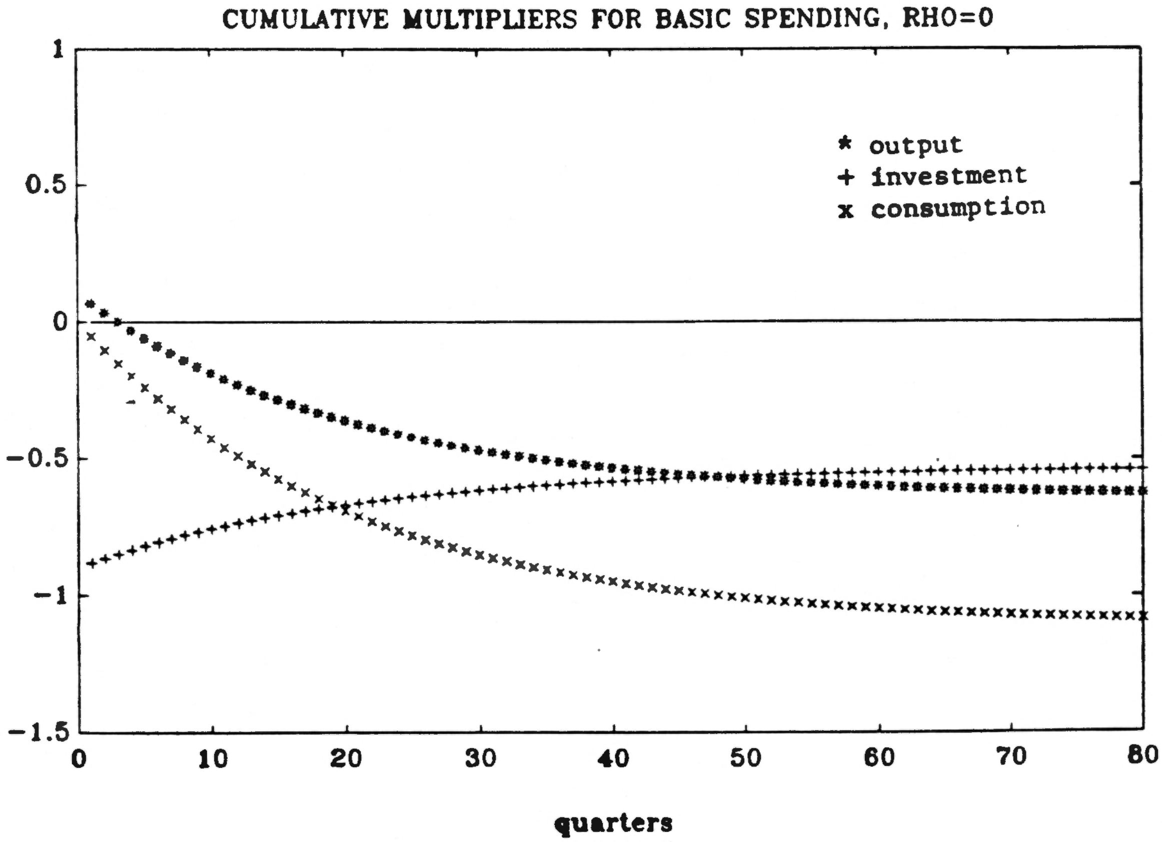

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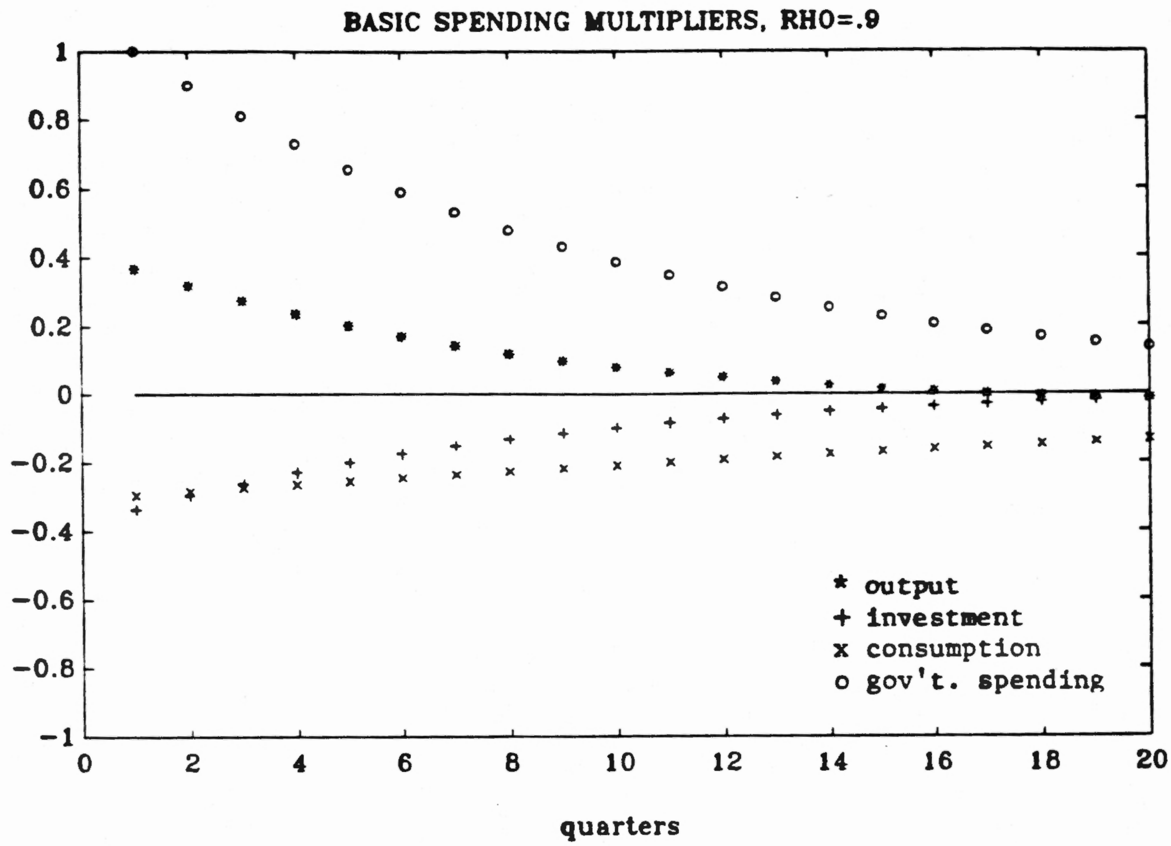

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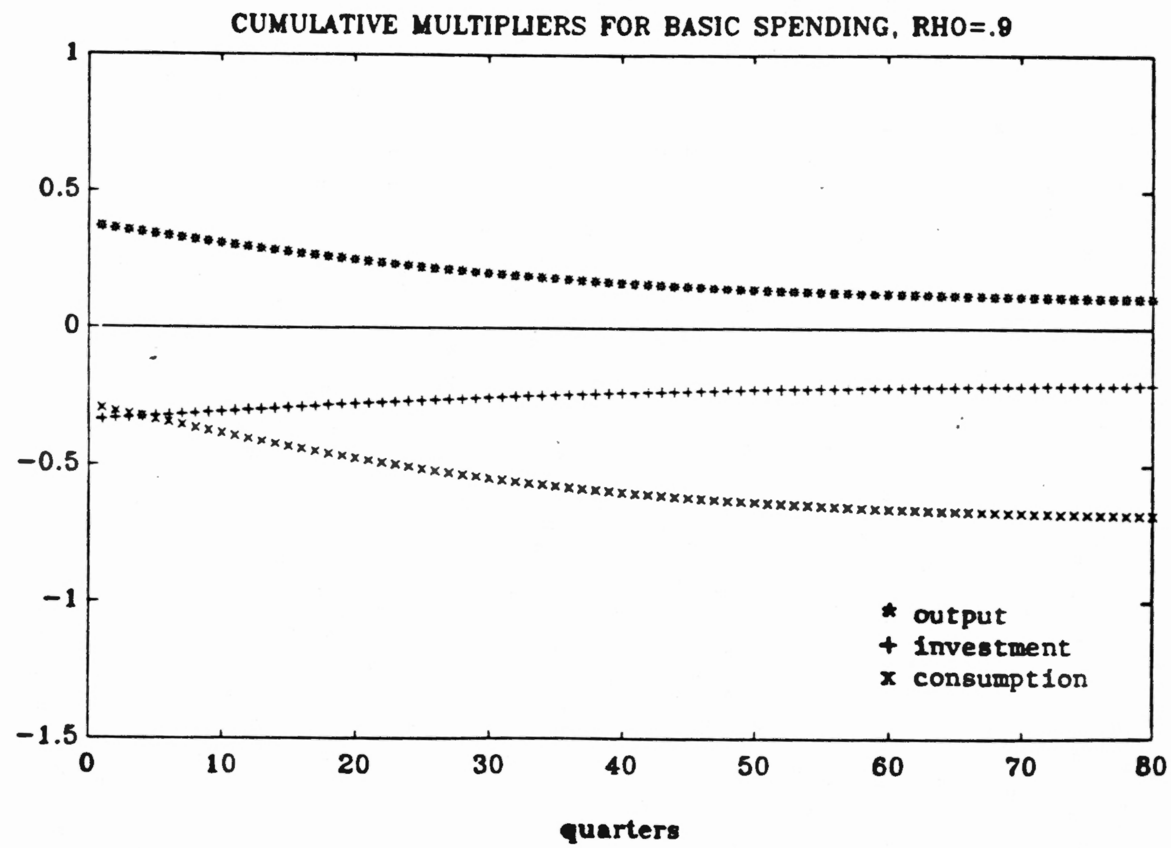

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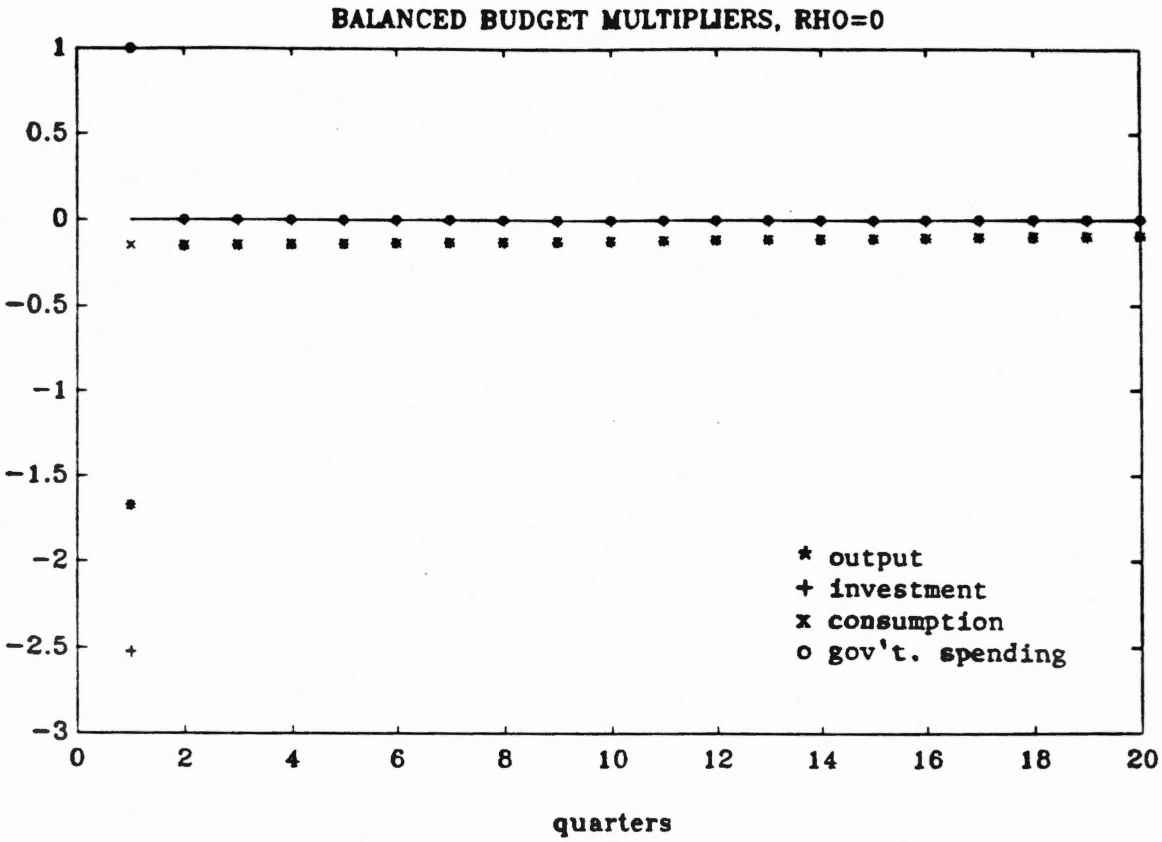

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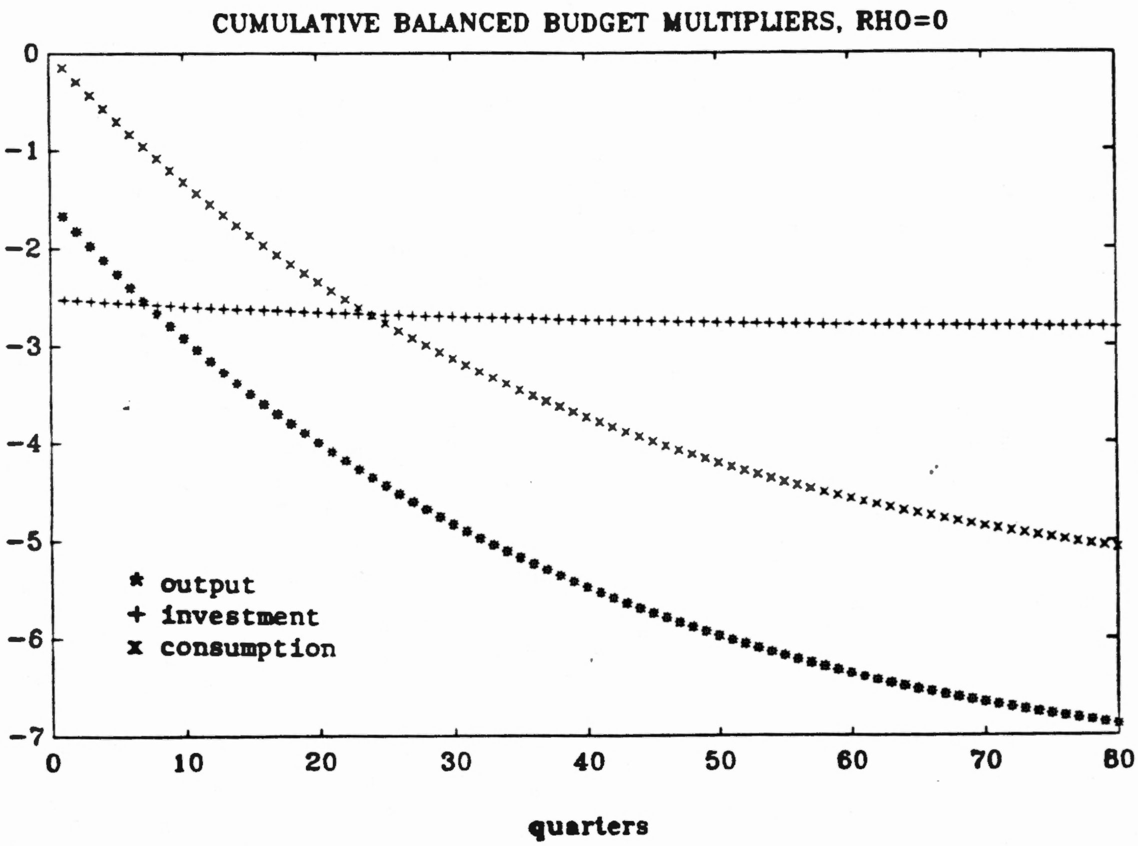

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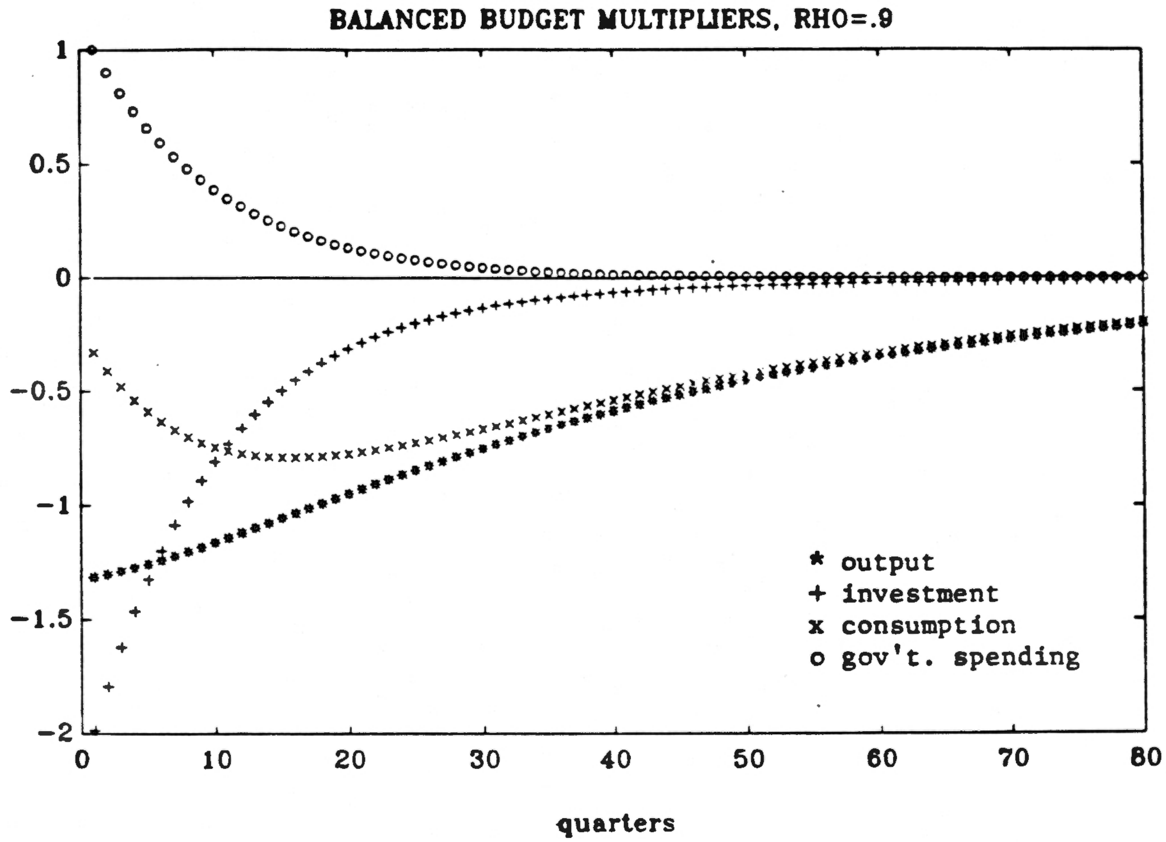

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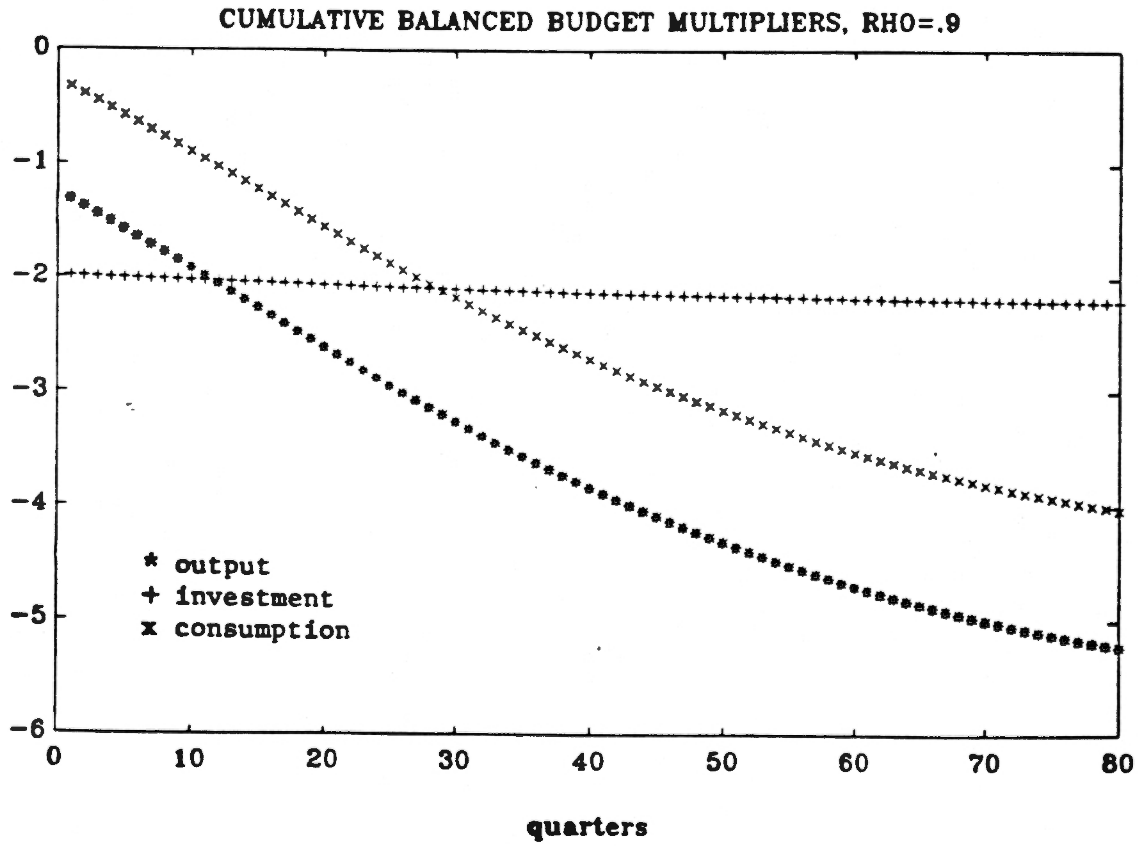

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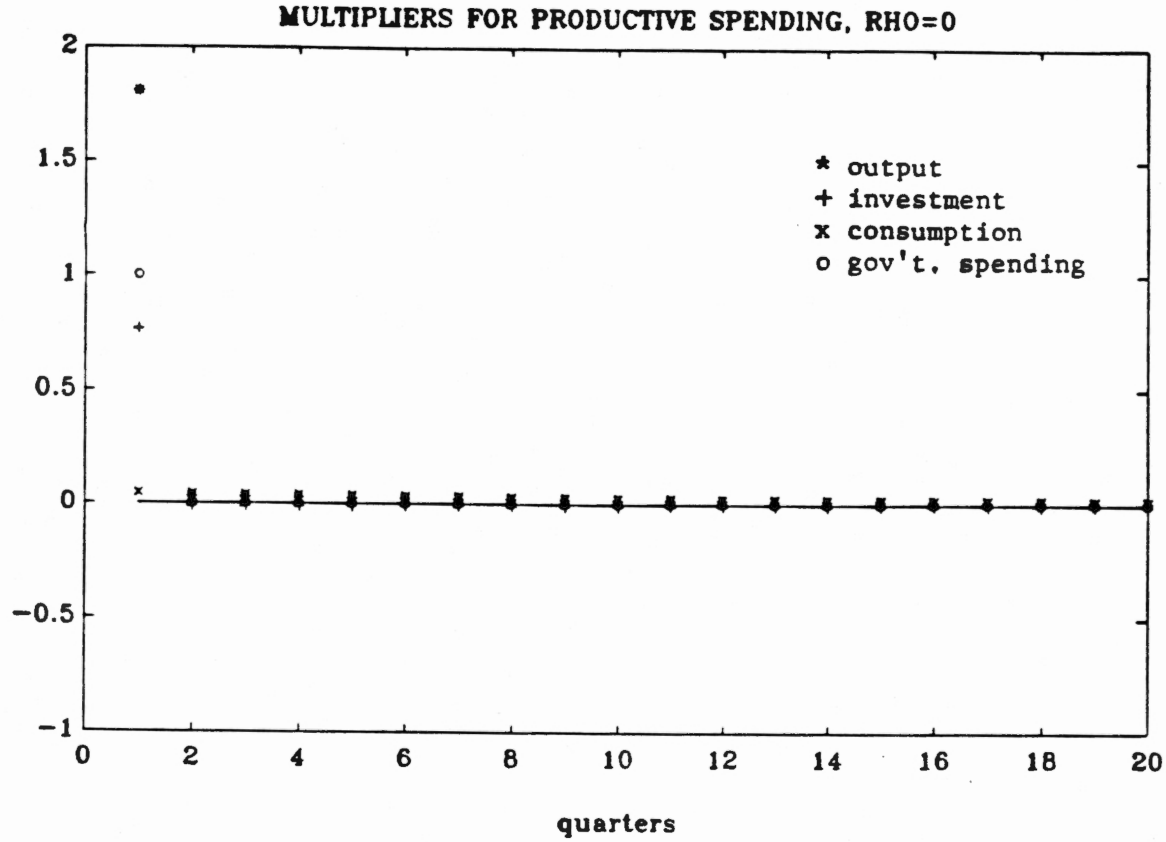

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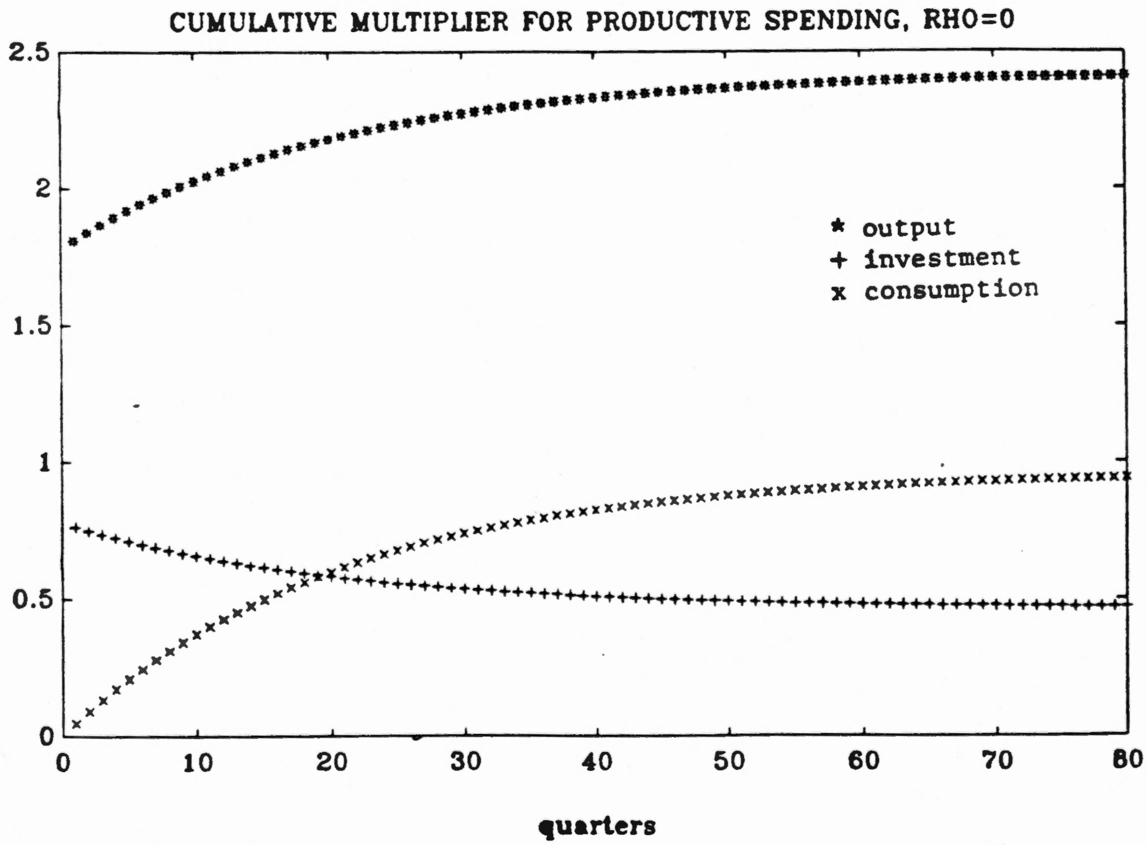

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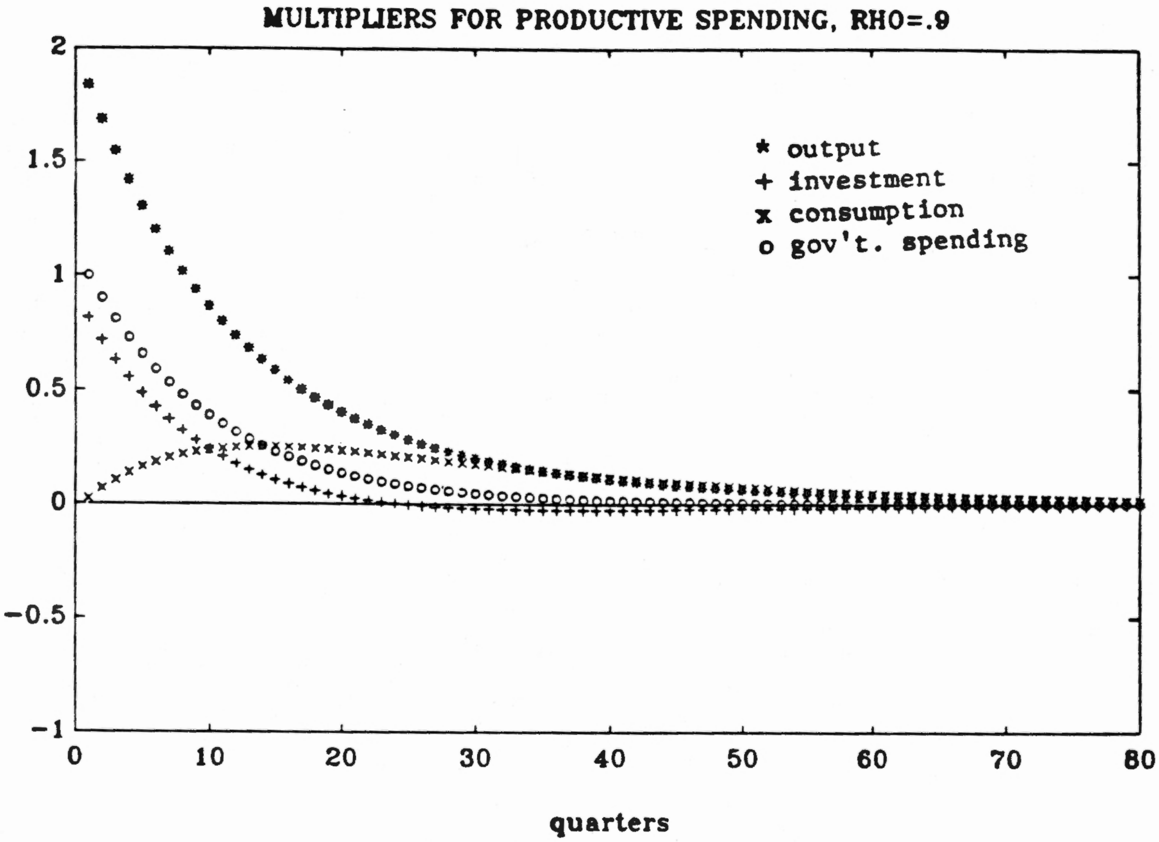

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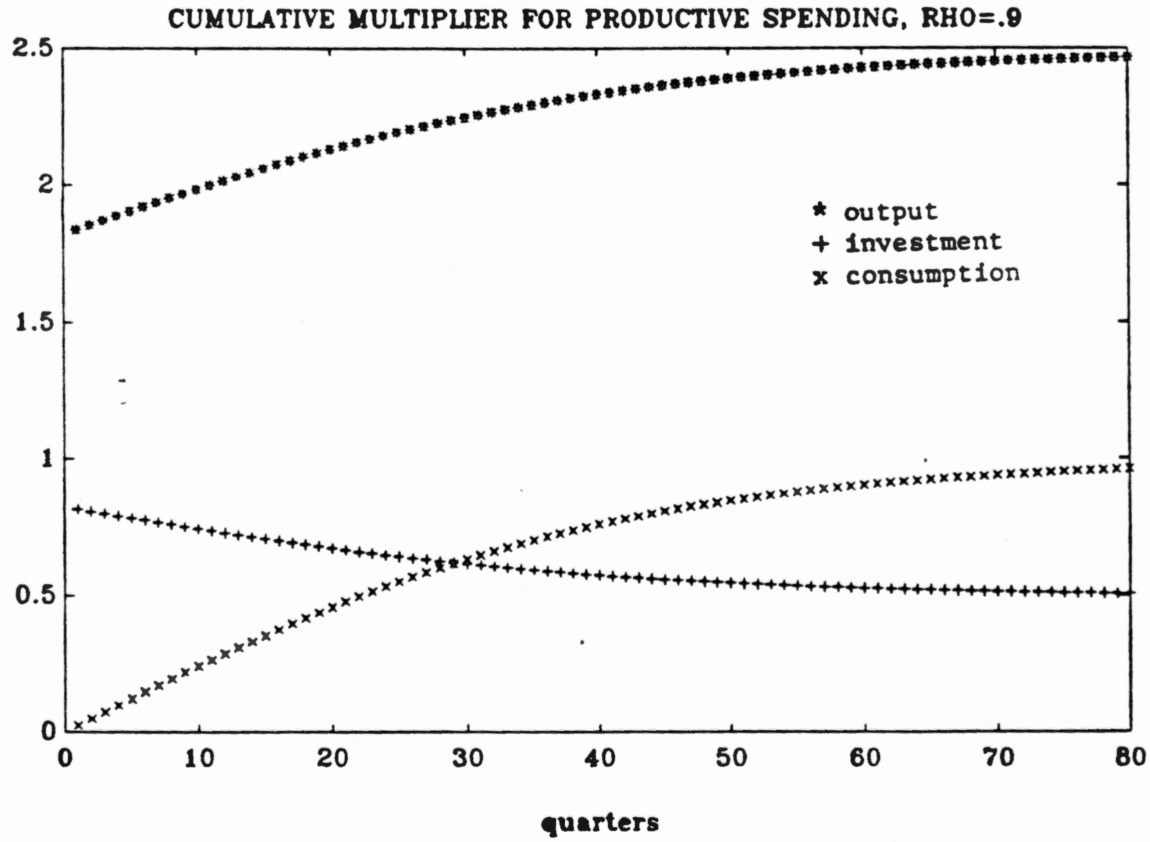

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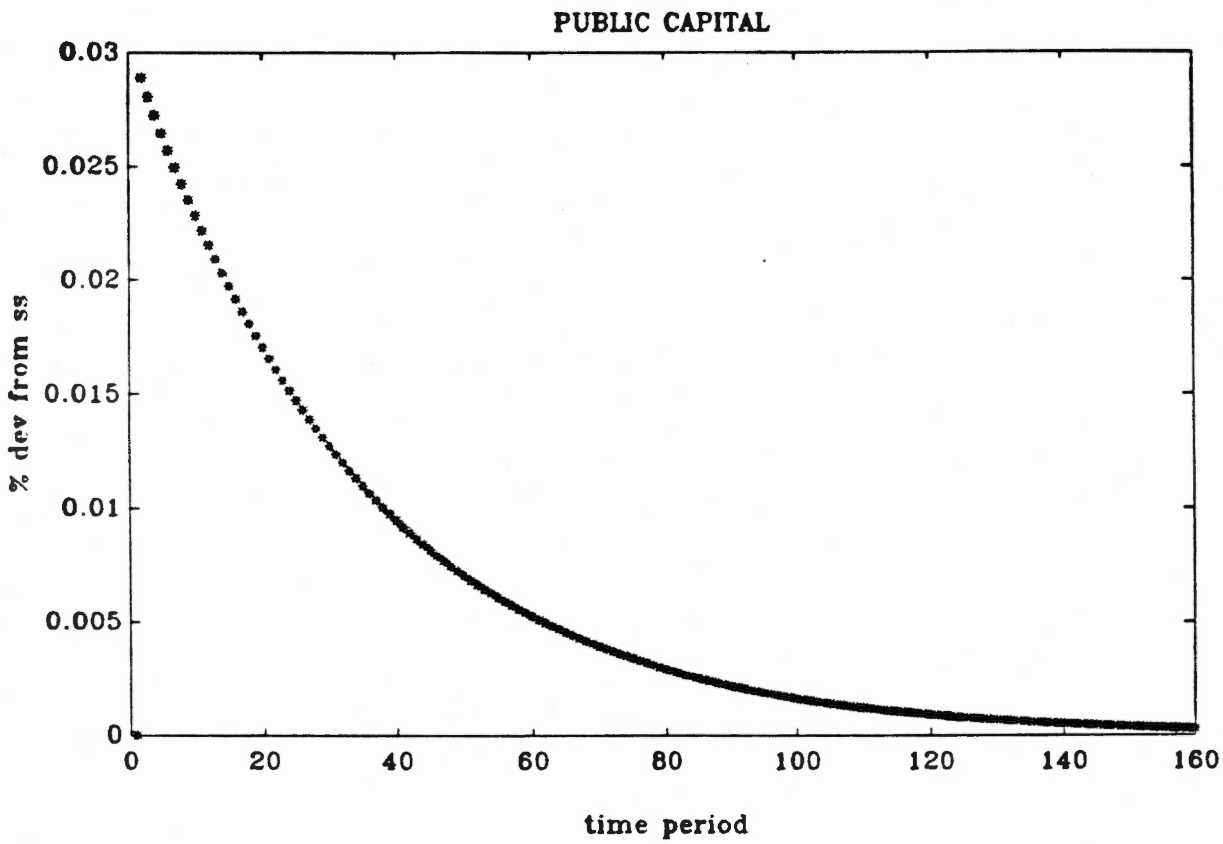

PUBLIC CAPITAL
0.03
0.025
0.02
0.015
0.01
0.005
0
% dev from ss
0
20
40
60
80
100
120
140
160
time period

FIGURE 17

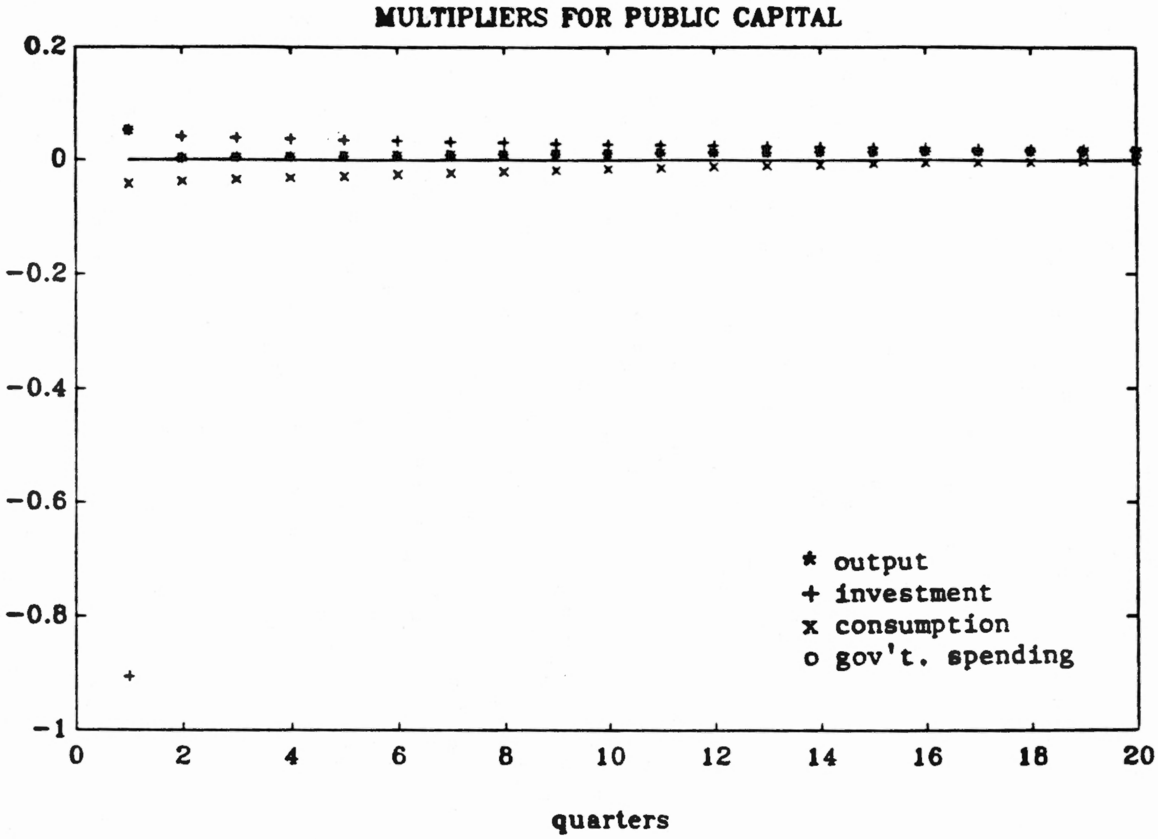
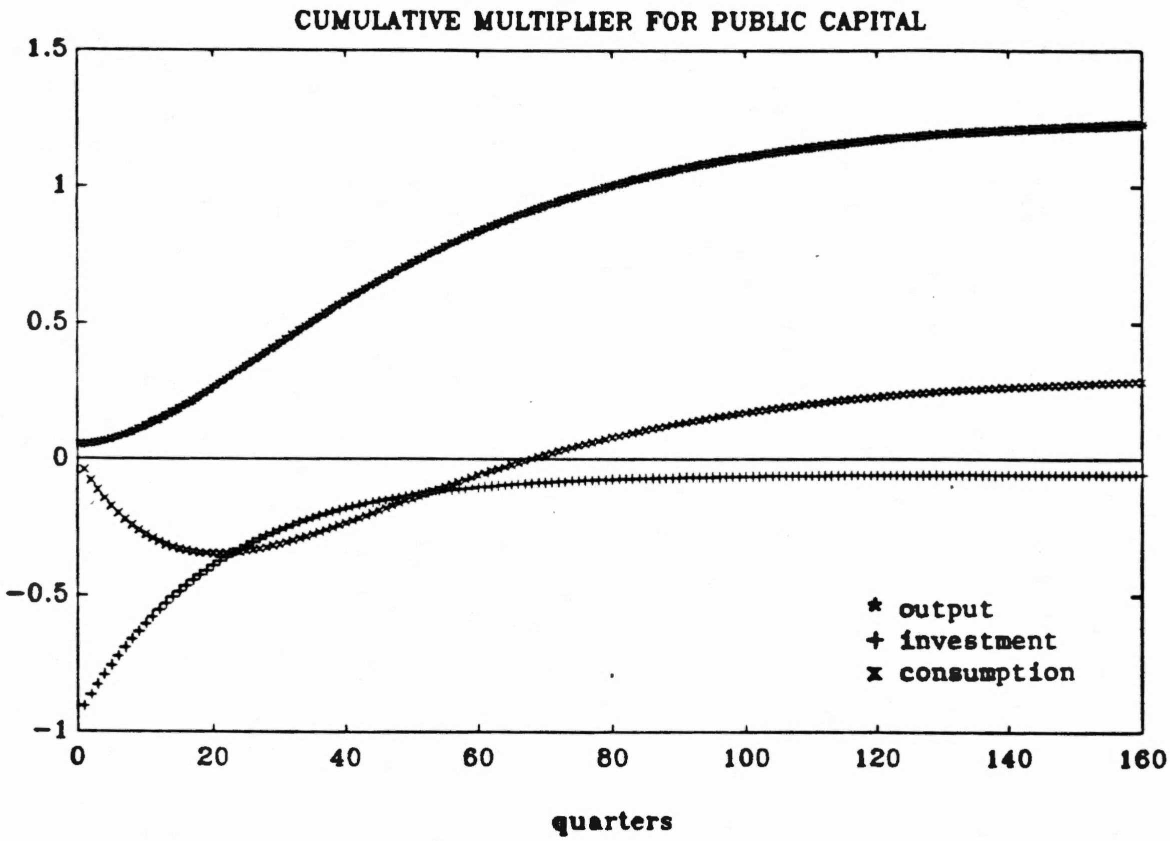


FIGURE 18



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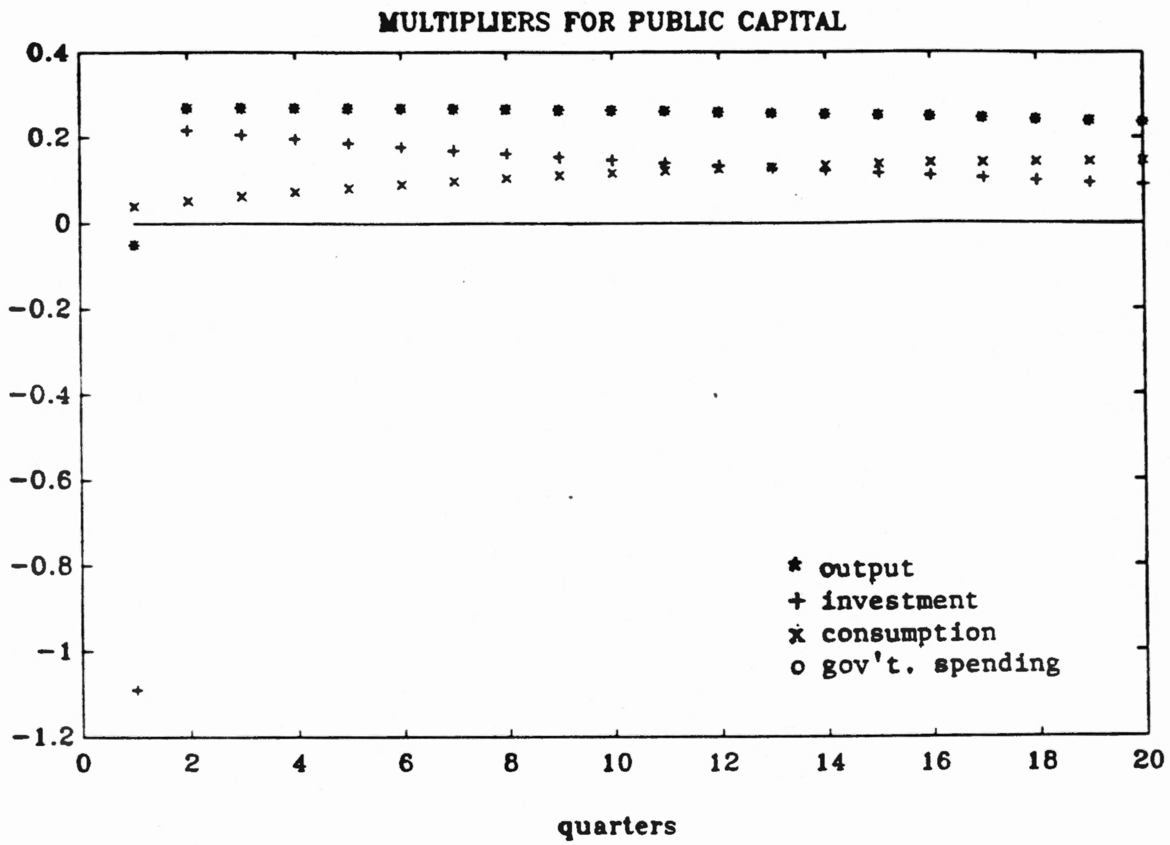

FIGURE 20

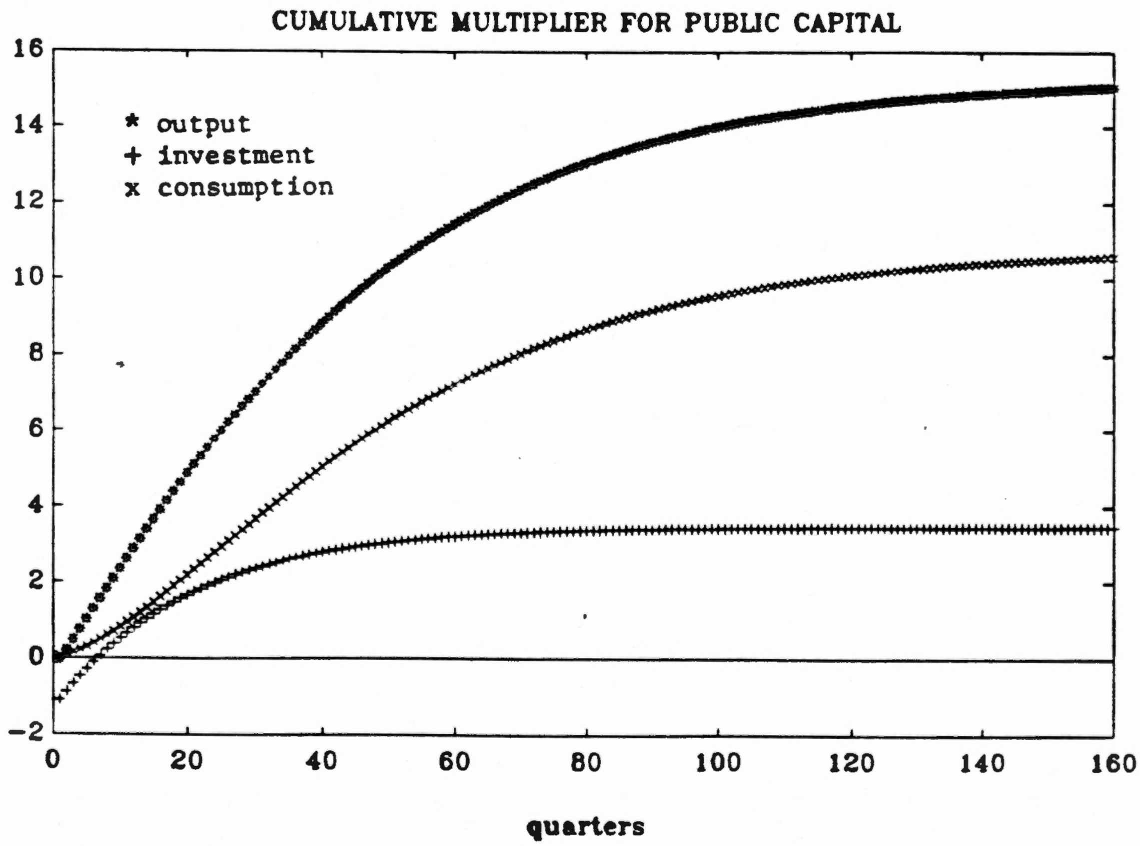

FIGURE 21

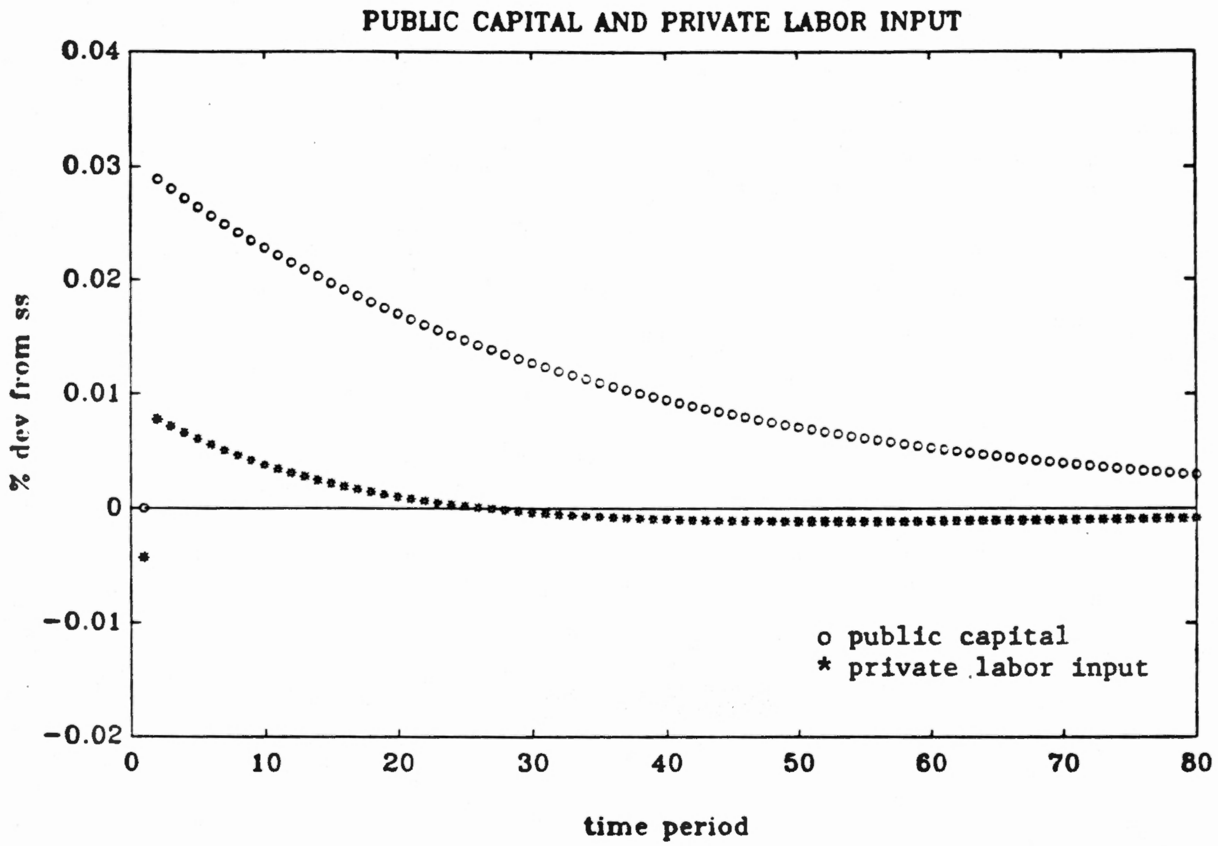
PUBLIC CAPITAL AND PRIVATE LABOR INPUT
0.04
0.03
0.02
0.01
0
-0.01
-0.02
% dev from ss
0
10
20
30
40
50
60
70
80
time period
o public capital
* private labor input

FIGURE 22

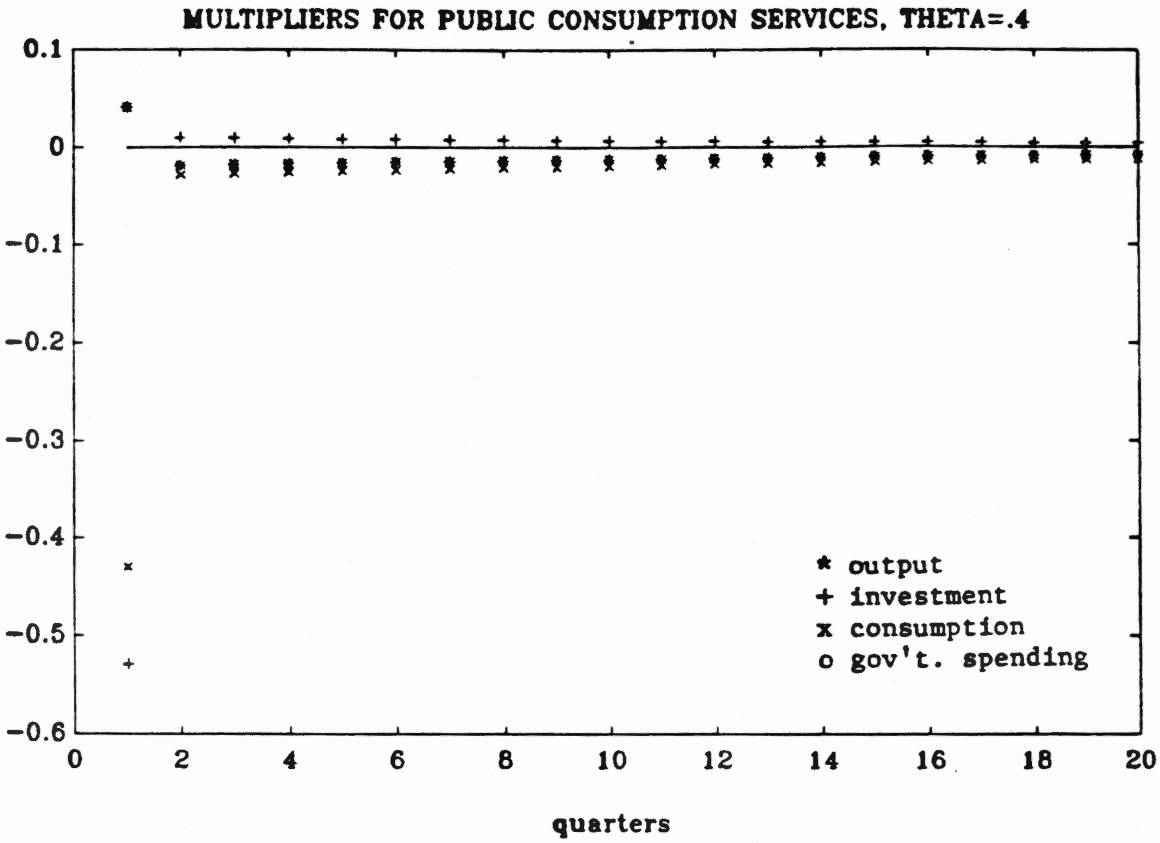

FIGURE 23

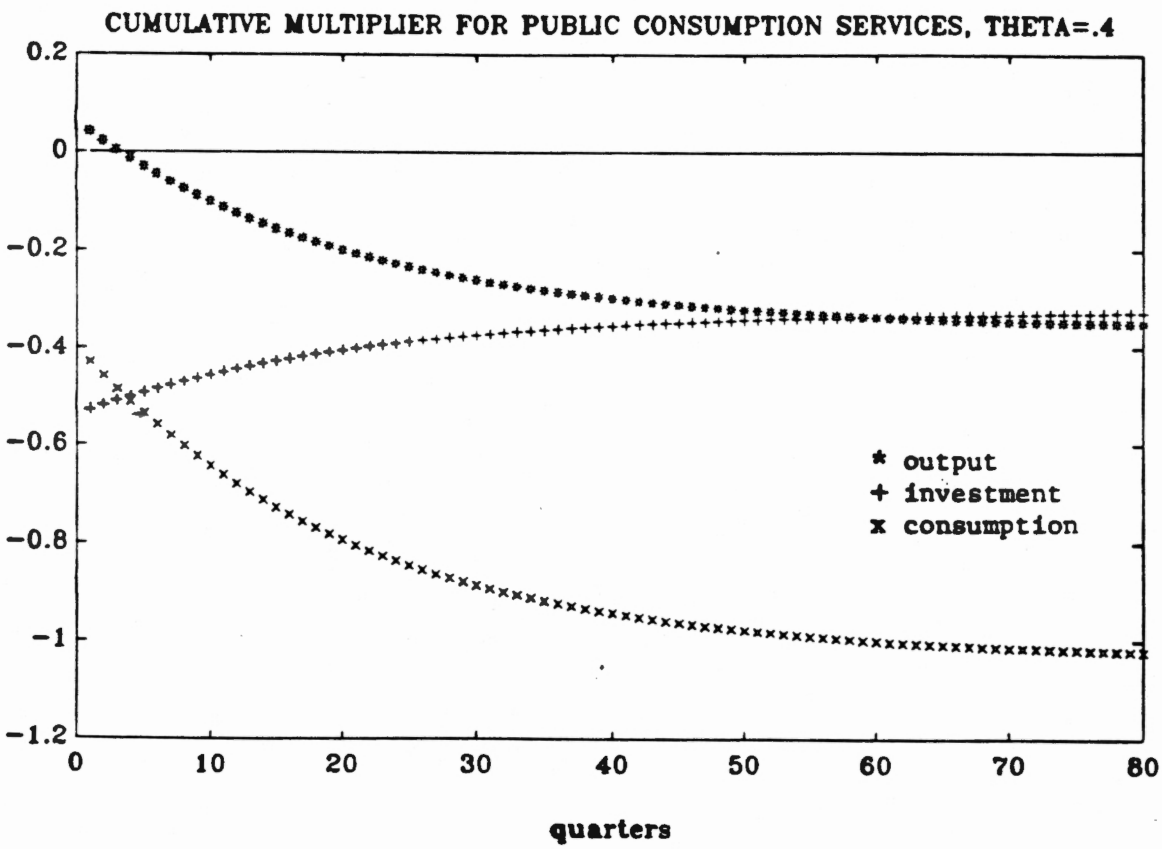

FIGURE 24

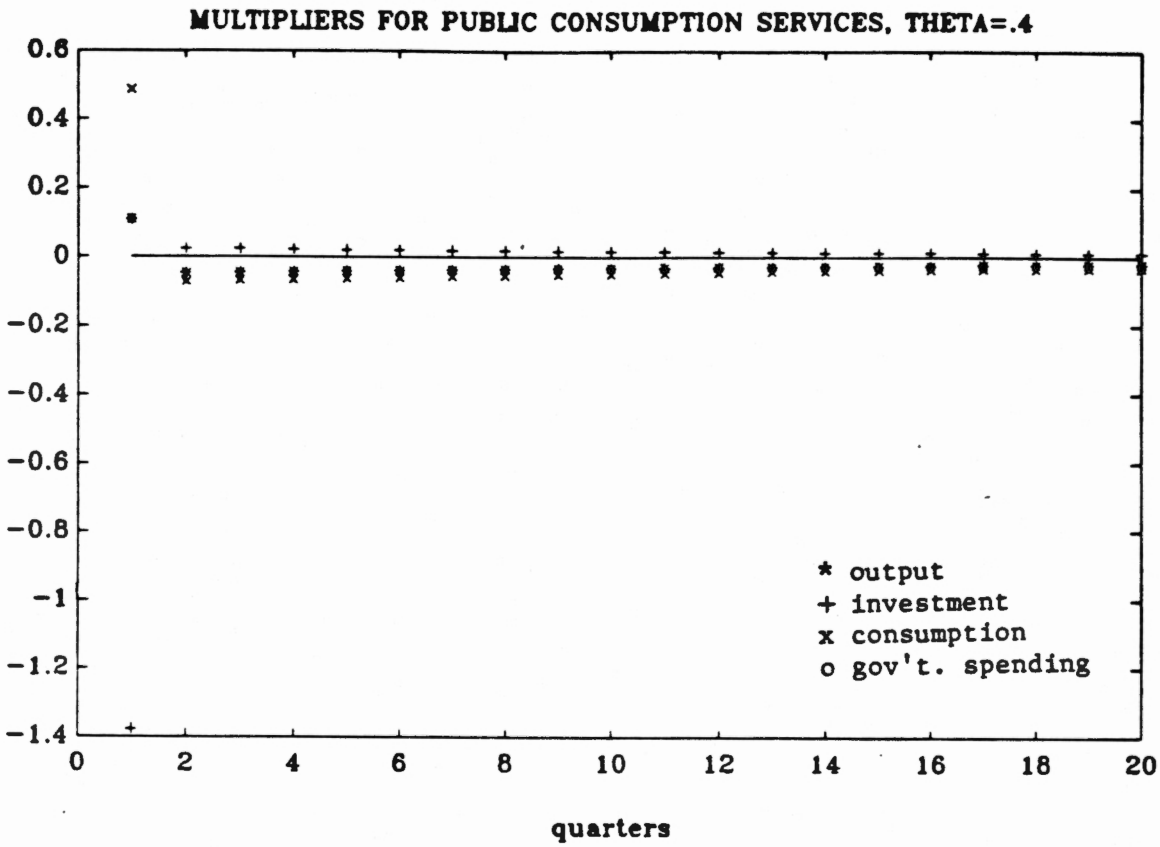

FIGURE 25

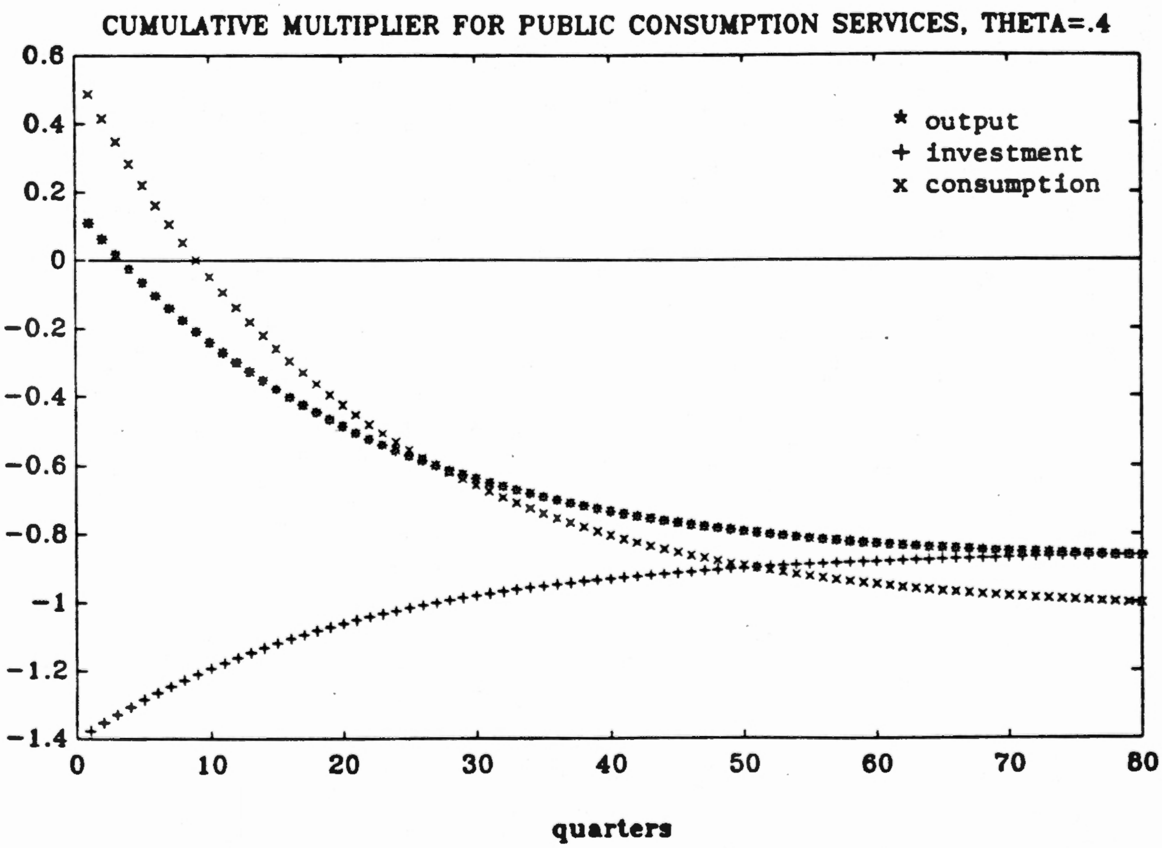

FIGURE 26

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