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ABSTRACT

Dynamic Real Trade Models: New Directions for Open Economy Macroeconomics

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Many central problems in open economy macroeconomics concern the dynamic responses of interacting economies to shifts in technologies, factor endowments, or government policies. This paper outlines a strategy for studying these interactions within dynamic versions of neoclassical real trade models. Our starting point for model development is a dynamic version of the basic "2x2x2" model: two countries, two final goods, and two factor inputs. Capital, but not labor, is assumed to be internationally mobile. We begin by deriving restrictions on preferences and technologies necessary for the world economy to display steady state growth, and study the dynamic response to sector–specific shocks in the context of a closed economy. A surprising characteristic of the model's steady state is that the long run production possibility frontier (PPF) for each country is linear. The slope of a country's long run PPF is determined by technology parameters, tax rates, and individuals' rate of discount. This model therefore displays indeterminacy of the national location of production in the case where the two countries have identical production technologies and identical tax rates. However, minor international differences in technologies or tax rates will lead to a situation in which at least one country specializes. The paper discusses possible routes for resolving the indeterminacy.

1. Introduction

The "pure theory of international trade" has a long and distinguished tradition of studying applied problems within the context of small-scale general equilibrium models. Static general equilibrium models of the type developed by Jones (1965) have been a rich source of theoretical and empirical insights for two decades. Although very simple, these models have had much to say about the international patterns of specialization, trade, factor migration, and the possibilities for equalization of factor returns. In the past fifteen years, extensions and modifications of the basic model have been undertaken in several directions: in the direction of a more explicit stochastic structure as in Helpman and Razin (1978); in the direction of a more complete dynamic structure as in Uzawa (1961, 1963), Oniki and Uzawa (1965), and Inada (1968); and in the direction of a richer industrial structure, as in Helpman and Krugman (1985). These models share with the earlier static models the feature that they are all equilibrium models, though not necessarily competitive equilibrium models.

This paper continues the traditional approach of real trade theory by studying a small-scale general equilibrium model of production and trade. What is not traditional about this work is that the model is a small-scale, dynamic stochastic model with endogenous capital accumulation and endogenous labor supply decisions. Until recently, technological constraints on our ability to solve models of this type have meant that a full analysis of these models' equilibria was not possible. Typically, the characteristics of equilibrium were investigated by means of comparative statics exercises. In the absence of rather strict, and often unrealistic, restrictions on the economic environment, unambiguous predictions of even a qualitative nature were rarely obtained.[1] However, recent advances in numerical solution methods have been fruitfully applied in the area of macroeconomics and these methods promise to be at least as fruitful in the analysis of problems in international trade.

The purpose of this paper work is therefore twofold: first, to develop a general class of stochastic, dynamic, equilibrium models of international trade, a class which includes as special cases many commonly-used trade models; and second, to develop appropriate methods for quantitative evaluation of these models. These two goals are viewed as inseparable. The approach taken here builds on methods and models from the best recent work in macroeconomics, growth theory, and econometrics, adapting and extending them for application to international trade theory.

Development of a class of dynamic stochastic real trade models affords two important advances over earlier work in this area. First, we can investigate the ways in which the response to exogenous shocks differs between dynamic and static models, due to response of forward-looking agents in the dynamic models. In particular, agents living in a dynamic environment have intertemporal substitution possibilities not available in a static model. Second, we can evaluate the dynamic effects of policy changes in a way that correctly reflects the optimal reactions of private agents to changes in the policy environment. Using these methods, we can study such markedly applied problems as (i) the economy's response to sector-specific technology shocks, (ii) the effects of government expenditure financed by

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[1] A large body of work has been devoted to determining conditions under which one can derive unambiguous qualitative predictions. A recent contribution is Helpman (1988).

distortionary taxation, (iii) the effects of productive externalities, (iv) the dynamic implications of short run factor immobility or factor specificity, and (v) the welfare effects of a tariff.

1.1 Model development

This paper takes as its starting point for model development a stochastic two-sector model. While it shares important features with traditional two-sector models, it is perhaps best viewed as a two-sector extension of the one-sector stochastic growth model developed by Brock and Mirman (1972), further modified to allow variable labor supply and various types of distortions. The models developed here have the traditional "2x2x2" structure, with two final goods, two factors, and two countries. (Generalization to more goods/factors/countries is straightforward.) Individuals in the two countries choose consumption of the two produced goods and their labor input to maximize utility subject to their resource constraints. The models is explicitly stochastic, with shocks stemming from stochastic technological change and from shocks to government policy. Model solution and simulation techniques are developed which permit investigation of the effects of departures from optimal equilibria. Two examples of suboptimal competitive equilibria considered here are (i) government spending financed by distortionary taxation and (ii) increasing returns to scale in production.

1.2 Model Evaluation

Perhaps the major departure of the proposed work from traditional work in this field is the emphasis on quantitative evaluation of the models under study. By evaluation we mean the comparison of the model's dynamic response to shocks with the dynamic behavior of actual economic aggregates. This quantitative approach is obviously important for judging the empirical adequacy of a model. But it seems especially important in trade theory for, as is well known, many well-specified trade models do not yield unambiguous theoretical predictions about patterns of specialization or trade.

In thinking about evaluation of purely real models of the sort developed in this paper, one is led immediately to wonder whether the behavior of real variables in the world economy depends in a crucial way on nominal variables such as national money supplies or the exchange rate. In a recent pair of papers, Baxter (1988b) and Baxter and Stockman (1988) investigate whether the behavior of real variables depends in a systematic way on the choice of exchange rate regime. These papers ask two related questions. First: do means, variances, and covariances of aggregate variables like output, consumption, investment, and trade flows behave differently in the fixed rate period (pre-1971) than in the floating rate period (post-1973)? Second: within the post-1973 period, do real variables in countries on fixed rate regimes (or managed exchange rate regimes like the EMS) behave differently from countries adopting a freely floating exchange rate policy?

The results from these papers are surprising. There is little evidence that the exchange rate regime matters for the behavior of real variables, with the well-known exception of the real exchange rate which is much more variable in the post-1973 period. These results suggest that purely real models of the type developed here have a good chance of capturing important features of the international determination of output, consumption, investment, and trade flows, without explicitly modelling exchange rate or monetary policy regimes.

The paper discusses two complementary strategies for numerical and simulation of dynamic models. The first strategy involves linearly approximating the economy around its steady state, and studying its dynamic response to shocks. This method has become standard for studying small-scale equilibrium models; it has been used by Kydland and Prescott (1982), Hansen (1985), Christiano (1988), Hansen and Sargent (1988), and King, Plosser and Rebelo (1988a,b), among others. The second strategy uses the method developed in Baxter (1988b) to generate equilibrium decision rules for dynamic, possibly suboptimal economies. This method does not rely on linearization as in the first method, and it does not impose certainty equivalence as does that method. While computationally more burdensome, its use is necessary in models where equilibrium decision rules are likely to be nonlinear (for example, in the models with irreversible investment discussed below.)

1.3 Outline of Paper

The paper is organized as follows. Section 2 lays out the basic two sector model for the pre-trade (autarky) economy. Within the context of the basic model, we discuss methods for studying approximate dynamics for the linearized economy, and apply the method to studying the economy's dynamic response to sector-specific shocks. Section 2.4 discusses several extensions of the basic model. Section 2.5 discusses an alternative strategy for computing equilibrium decision rules. Section 2.6 discusses methods for econometric evaluation of these models.

Section 3 presents the two country, two sector model. With identical constant returns to scale production functions and endogenous capital accumulation, and so long as there is perfect international and intranational mobility of capital, the model exhibits indeterminacy in terms of the (national) location of production of the two goods, even though worldwide quantities are pinned down. This section discusses potential routes for resolving this indeterminacy. Within the context of the open economy version of the two sector model developed in section 2, the long run indeterminacy can only be resolved by introducing specific factors. The assumption of factor specificity is intended to capture the immobility of certain types of factors, for example, human capital that is specific to a particular industry. Section 3 also discusses ways to make this model more realistic, for example by viewing the specific factors as factors that are only temporarily immobile, and which may move to more profitable sectors if they are willing to bear certain adjustment costs. However, long run indeterminacy of production location is still a likely characteristic of the model's equilibrium. The section concludes with a discussion of fruitful avenues for future research.

2. A Two-Sector Model of a Closed Economy

This section develops a two sector modethe traditional approach of real trade theory, we study first an economy operating in isolation and then study the effects of opening that economy to trade. Clearly, understanding the character of equilibrium in the closed economy is a necessary precondition to understanding the more complicated dynamics of an open economy. Before turning to a description of the model, however, we first present an empirical motivation for preferring a two-sector model to a one-sector model. Tables 1 and 2 presents summary statistics on quarterly and annual growth rates for investment, GNP, and three categories of consumption in the U.S. postwar economy. As is well known, purchases of consumer durables are much more volatile than either services or nondurables.[2] Second, the volatility in consumer durables is closer to the volatility in investment than to the volatility of other consumption categories or aggregate output. Figures (1a) and (1b) illustrate this effect; figure (1a) plots postwar annual growth rates of investment and consumer durables, and figure (1b) plots postwar annual growth rates of nondurables, services, and GNP on the same scale as figure (1a).

Figure (2a) plots producer price indices for consumer durables and nondurables (including food). The behavior of these two price indices is quite different, especially since about 1970. If a one-good model were adequate to explain aggregate consumption, the relative price of nondurables

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[2] Ideally, one should study the flow of services from the durable consumption good, rather than purchases of stocks. But if the flow of services is proportional to the stock of the durable consumption good, then it will still be the case that consumption of the services of consumer durables exhibits differential mean growth rates and volatility relative to other consumption categories.

to durables should remain close to one. This relative price is plotted in Figure (2b); clearly the relative price is not constant over time.

These facts suggest that a two sector model should outperform a one sector model in explaining the response of the actual economy to exogenous shocks. Further, the data suggest that one sector should produce consumer nondurables and services, and the other sector should be a capital-producing sector, producing both investment goods and consumer durables.

This section is divided into several subsections. The first subsection describes the basic model. In the model economy, there are two factors of production: labor and capital. These homogeneous factors are both used in production of each of the two final goods in the economy; a nonstorable consumption good ("food") and a capital good ("machines"). The model differs from traditional two-sector models in two important ways: first, we incorporate variable aggregate labor supply and second, the capital good may be allocated to use in consumption as a second consumption good. The second subsection describes the linearization method used to study the approximate dynamics of the model near its steady state. In the third subsection, we study the dynamic response of the model economy to sector-specific shocks in technology. The focus is on the quantitive effects of the shocks, and the way in which the patterns of response depend on the parameters of preferences and technology.

In the fourth subsection, we discuss three extensions of the basic two sector model. The main feature shared by all three extensions is that the modified models limit factor mobility in some way. Recent work by Grossman and Levinsohn (1988) suggests that short-run immobility of factors can be quantitatively important. Grossman and Levinsohn investigate the response to terms of trade shocks of the equity returns of import-competing firms, and find that there are significant effects of these shocks. They interpret these results as implying the existence of a reasonable degree of short run factor specificity.

The first extension investigates the effects of irreversibility of capital investment on the dynamic response to sector-specific shocks. The assumption of the basic model—that all capital is instantaneously perfectly mobile—seems patently unreasonable in light of both casual observation and the Grossman/Levinsohn evidence. This extension investigates the effect of assuming the opposite extreme: that new capital, once put in place in a particular sector, cannot be moved. Clearly, irreversibility of investment will damp responses to shocks and, if labor is also immobile in the short run, can lead to situations in which the rate of return to capital can differ across sectors.

The second extension involves a model which is an intermediate case between the basic model with completely mobile capital, and the model with completely irreversible investment. This extension assumes that capital can be transported between sectors, but that costs of adjustment must be borne. This idea has been extensively used in both the macroeconomics and the theoretical international trade literature (an example is Mussa (1978)). Having developed these models, we proceed next to study the way in which the quantitative effects of sectoral shocks depend on the intersectoral mobility of factors.

The third extension involves a specific factors model, in which one factor (labor) is entirely sector-specific, while the other factor (capital) is costlessly and instantaneously mobile between sectors. This model is viewed as a modern reincarnation of the specific factors models of Jones (1971) and Amano (1977).

Subsection five describes a new method for approximating equilibria of stochastic, possibly suboptimal, dynamic equilibria. This discussion is based on the work of Baxter (1988a). Because this method does not take a linear approximation but instead computes equilibrium rules over a discretization of the state space, it can easily handle the nonlinearity induced by irreversible investment. The equilibrium decision rules for capital accumulation in a model with irreversible investment are nonlinear since there are situations in which individuals would like to move existing capital from one sector to another. Their inability to do so will truncate the equilibrium decision rules at the zero gross investment point, leading to a potentially important nonlinearity. The linear approximation technique described above will not, therefore, generally be adequate for describing approximate dynamics to an exogenous shock. Finally, subsection six concludes with a discussion of methods for estimating these models.

2.1 The basic two-sector model

This section develops a basic two sector, two factor model of a single economy. Savings behavior is determined by intertemporal optimization by private agents. The two factors in the basic model are privately-supplied labor and capital. These homogeneous factors are both used in production of each of the two final goods produced by the economy: a nonstorable consumption good ("food") and a capital good ("machines").

So far, this model is reminiscent of the standard two sector model. However, we alter the standard analysis so that new output of the capital good may be used as a second consumption good. If new capital is used in consumption, it depreciates fully in one period.[3] If the new capital is invested (set aside for use in production in the subsequent period), it depreciates at a slower rate. The output of the capital good sector, "machines", can be used as an investment good to augment the capital stocks in the two industries, or it can be consumed. The output of the consumption good sector, "food", is nonstorable and therefore can only be consumed in the period in which it is produced.

The economy is populated by a single representative agent, who owns the capital stock and operates the production technologies directly.[4] He is endowed with a unit of time each period, which he allocates to leisure and work effort.

2.1.1 The representative agent

The representative agent receives utility from the two consumption goods, (food and machines) and leisure. The fact that leisure enters the utility function means that aggregate labor supply is not exogenous; this represents a major departure from traditional treatments of the two sector model. Letting β denote the representative individual's discount factor, individual's utility function is given by:

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[3] It is straightforward, although slightly messy, to alter this assumption so that the second consumption good depreciates at a rate less than 100% per quarter.

[4] All the economies studied here could be decentralized, allowing "firms" and "workers" to maximize their respective objective functions subject to appropriate constraints, and finding equilibrium prices and quantities by imposing market clearing. We choose to study the analogous "Robinson Crusoe" economy because its description is more compact.

$$U = \sum_{t=0}^{\infty} \beta^t \, u(C_{1t}, C_{2t}, L_t) \tag{1}$$

where C_{1t} denotes consumption of food, C_{2t} denotes consumption of machines, and L_t denotes leisure.

2.1.2 Production technology

The two final goods are produced according to linearly homogeneous production functions. Sector 1 is the consumption good (food) sector, and sector 2 produces capital (machines). Denote the production functions by:

$$Y_{1t} = F_{1t}(K_{1t}, N_{1t}) = A_{1t}F_1(X_{K1t}K_{1t}, X_{N1t}N_{1t}) \tag{2a}$$

$$Y_{2t} = F_{2t}(K_{2t}, N_{2t}) = A_{2t}F_2(X_{K2t}K_{2t}, X_{N2t}N_{2t}) \tag{2b}$$

where K_{jt}, N_{jt} denote capital and labor used in producing sector j output, and where X_{ijt} denotes the level of factor "i" augmenting technical change in sector j at time t. A_{jt} denotes the level of total factor augmenting technical change in sector j at time t. In order to have a growing economy that is nevertheless subject to shocks, we separate the two kinds of technical change into "growth" and "cyclic" components. The X_{ijt} will be allowed to grow at constant geometric rates, and the A_{jt} will be stationary Markovian random variables. Further restrictions on these variables are considered below.

2.1.3 Endowments

The representative agent is endowed with one nonreproduceable unit of time each period, which he allocates to leisure and to work in the two sectors. The time constraint faced by an individual is given by:

$$L_t + N_{1t} + N_{2t} \leq 1 \qquad (3)$$

2.1.4 Restrictions implied by steady state growth

It is desirable to restrict preferences and technologies so that the model is capable of generating the steady state growth which characterizes developed economies.[5] Let γ_Z denote gross growth rate of any variable Z, i.e.: $\gamma_Z \equiv Z_{t+1}/Z_t$.

The first set of restrictions considered are those which make steady state growth feasible; these are derived from the production functions and resource constraints. In the one sector model, a necessary restriction for steady state growth is that technical change be labor-augmenting.[6] Similarly, in the two sector model presented here, it is necessary that technical change in the machines sector be purely labor augmenting: $\gamma_{XK2} = 1$. Labor augmenting technical change in the capital good sector grows at the rate γ_{XN2}.

In the food sector, however, it is not necessary that capital augmenting technical change be absent. However, a necessary condition for steady state growth is the following:

$$\frac{\gamma_{XN1}}{\gamma_{XK1}} = \gamma_{XN2} \quad . \qquad (4)$$

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[5] These methods are implemented for the basic neoclassical model in King, Plosser, and Rebelo (1987a,b). Variations of the model are studied in King (1987) Baxter and King (1988a,b).

[6] The exception is when the production function has the Cobb-Douglas form, for in this case the production function can always be rewritten as if the technical change were labor-augmenting.

In the one-sector model, the steady state growth rate of capital is equal to the growth rate of labor-augmenting technical change. In this model, the steady state growth rate of capital in each sector is equal to the rate of labor-augmenting technical change in the capital good sector:

$$\gamma_{K1} = \gamma_{K2} = \gamma_{XN2} \quad .$$

As in the one-sector model, output in the capital good sector also grows at the rate of labor-augmenting technical change in that sector:

$$\gamma_{Y2} = \gamma_{XN2} \quad .$$

Under the above conditions for steady state growth, output in the food sector grows at the rate:

$$\gamma_{Y1} = \gamma_{XN1} = \gamma_{XK1}\gamma_{K1} = \gamma_{XK1}\gamma_{XN2} \quad .$$

Thus, output of the food sector may grow faster or slower than output of the capital good sector.

Finally, consumption in each sector grows at the same rate as output in that sector:

$$\gamma_{C1} = \gamma_{Y1} = = \gamma_{XN1} = \gamma_{XK1}\gamma_{K1} = \gamma_{XK1}\gamma_{XN2}$$
$$\gamma_{C2} = \gamma_{Y2} = \gamma_{XN2} \quad .$$

Because individuals' allocation of time is fixed at one unit per period, there cannot exist steady state growth in labor or in leisure. Thus another set of technological restrictions is

$$\gamma_{N1} = \gamma_{N2} = \gamma_{L} = 1 \quad .$$

While the above restrictions guarantee that steady state growth is feasible, they do not guarantee that individuals will find it optimal to behave in such a way that steady state growth will occur. In order that individuals find steady state growth to be optimal, it is necessary that the utility function be of the following form:

$$u(C_{1t},C_{2t},L_t) = \frac{1}{1-\sigma}\left\{\left[C_{1t}^{\theta}\, C_{2t}^{1-\theta}\, v(L)\right]^{1-\sigma} - 1\right\} \quad \text{for } 0<\theta<1 \text{ and } \sigma>0. \tag{5}$$

A useful special cases of this utility function is the logarithmic utility function obtained when $\sigma=1$.

2.1.5 Government

The role of the government is to levy taxes, to distribute lump-sum transfers, and to purchase output for its own use. Taxes take the form of a distortionary income tax. The tax rate on income in sector j at time t is denoted τ_{jt}; the tax rate may be a function of state variables such as the technology shock or the capital stock. Government purchases of the output of sector j is denoted G_{jt}; in the basic model government spending does not yield utility to individuals, nor is it productive in the sense that it shifts the production functions F_j.[7] Transfers to individuals of the output of sector j are denoted T_{jt}. The government budget constraints are:

$$G_{jt} + T_{jt} = \tau_{jt}Y_{jt} \qquad \text{for } j=1,2.$$

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[7] Baxter and King (1988) provide a closed economy analysis of the dynamic effects of productive and utility-producing government spending.

Steady state growth requires that government purchases and transfers be a constant fraction of output, i.e., that they grow at the same rate as output in each sector.

The resource constraints for the representative agent in this economy are:

$$(1-\tau_{1t})Y_{1t} + \tau_{1t} - C_{1t} = 0$$

$$(1-\tau_{2t})Y_{2t} + (1-\delta_1)K_{1t} + (1-\delta_2)K_{2t} + \tau_{2t} - C_{2t} - K_{1,t+1} - K_{2,t+1} = 0 \quad (6)$$

where Y_{1t}, Y_{2t} denote the outputs of the two sectors:

$$Y_{jt} = A_{jt}\, F_j(X_{Kjt}K_{jt},\, X_{Njt}N_{jt}) \qquad \text{for } j = 1,2.$$

Finally, it is useful to think of there being an economy-wide capital stock, K_t, which is allocated across the two sectors:

$$K_t = K_{1t} + K_{2t} \qquad (7)$$

2.1.6 Competitive equilibrium

The competitive equilibrium for this economy is found by maximizing the utility function subject to the resource constraints; the Lagrangian for this problem is given by:

$$
\begin{aligned}
\mathcal{L} = & \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma} \left\{ \left[C_{1t}^{\theta} C_{2t}^{1-\theta} v(L) \right]^{1-\sigma} - 1 \right\} \\
+ & \sum_{t=0}^{\infty} W_t \; [1-L_t-N_{1t}-N_{2t}] \\
+ & \sum_{t=0}^{\infty} Q_t \; [K_t-K_{1t}-K_{2t}] \\
+ & \sum_{t=0}^{\infty} P_t [(1-\tau_{1t})Y_{1t} + T_{1t} - C_{1t}] \\
+ & \sum_{t=0}^{\infty} \Lambda_t [(1-\tau_{2t})Y_{2t}+(1-\delta_1)K_{1t}+(1-\delta_2)K_{2t}+T_{2t}- C_{2t}-K_{1,t+1}-K_{2,t+1}] \qquad (8)
\end{aligned}
$$

This economy is one in which all variables except labor and leisure exhibit steady state growth. It is useful to transform the economy into one in which all variables are stationary in the steady state. Toward this end, define:

$c_{1t} = C_{1t}/X_{N1t}$

$c_{2t} = C_{2t}/X_{N2t}$

$k_{1t} = X_{K1t}K_{1t}/X_{N1t}$

$k_{2t} = X_{K2t}K_{2t}/X_{N2t}$

$k_t = (\psi K_{1t}+K_{2t})/X_{N2t}$

$y_{1t} = Y_{1t}/X_{N1t}$

$y_{2t} = Y_{2t}/X_{N2t}$

$g_{1t} = G_{1t}/X_{N1t}$

$g_{2t} = G_{2t}/X_{N2t}$

$t_{1t} = T_{1t}/X_{N1t}$

$t_{2t} = T_{2t}/X_{N2t}$

where $\psi = X_{N2t} \dfrac{X_{K1t}}{X_{N1t}}$; which is a constant since $\gamma_{XN1}=\gamma_{XK1}\gamma_{XN2}$. Since there is no capital-augmenting technical progress in sector 2, we can normalize X_{K2t} at the value 1 and rewrite ψ as $\psi = (X_{N2t}/X_{K2t})\div(X_{N1t}/X_{K1t})$. The parameter ψ represents the relative level of (relative) technological conditions in the two sectors, which is a constant because of the growth rate restrictions. The discount factor for the transformed problem is given by:

$$\tilde{\beta} = \beta\left[(\gamma_{XN1})^{\theta}(\gamma_{XN2})^{1-\theta}\right]^{1-\sigma} \quad . \tag{9}$$

The Lagrangian for the transformed economy is given by:

$$\begin{aligned}
\mathscr{L} = & \sum_{t=0}^{\infty} \tilde{\beta}^{t} \frac{1}{1-\sigma}\left\{\left[c_{1t}^{\theta}\, c_{2t}^{1-\theta}\, v(L)\right]^{1-\sigma} - 1\right\} \\
+ & \sum_{t=0}^{\infty} \tilde{W}_t\,[1-L_t-N_{1t}-N_{2t}] \\
+ & \sum_{t=0}^{\infty} \tilde{Q}_t\,[k_t-\psi k_{1t}-k_{2t}] \\
+ & \sum_{t=0}^{\infty} \tilde{P}_t[(1-\tau_{1t})Y_{1t} + t_{1t} - c_{1t}] \\
+ & \sum_{t=0}^{\infty} \tilde{\Lambda}_t[(1-\tau_{2t})Y_{2t}+(1-\delta_1)\psi k_{1t}+(1-\delta_2)k_{2t}+t_{2t}- c_{2t}-\psi k_{1,t+1}-k_{2,t+1}]
\end{aligned} \tag{10}$$

Finally, it is convenient to work with current-valued multipliers, which can be interpreted as utility-demonstrated shadow prices, defined as follows:

$w_t = \tilde{W}_t/\tilde{\beta}^t$;	wage rate
$q_t = \tilde{Q}_t/\tilde{\beta}^t$;	rental rate
$p_t = \tilde{P}_t/\tilde{\beta}^t$;	price of good 1 (food)
$\lambda_t = \tilde{\Lambda}_t\tilde{\beta}^t$;	price of good 2 (machines).

Letting D_j denote the derivative with respect to the j^th argument, the first-order necessary conditions for this problem are:

$$D_1u(c_{1t},c_{2t},L_t) - p_t = 0 \tag{11}$$

$$D_2u(c_{1t},c_{2t},L_t) - \lambda_t = 0 \tag{12}$$

$$D_3u(c_{1t},c_{2t},L_t) - w_t = 0 \tag{13}$$

$$-w_t + p_tA_{1t}(1-\tau_t)D_2F_1(k_{1t},N_{1t}) = 0 \tag{14}$$

$$-w_t + \lambda_tA_{2t}(1-\tau_t)D_2F_2(k_{2t},N_{2t}) = 0 \tag{15}$$

$$-\tilde{\beta}\psi q_t + \tilde{\beta}p_tA_{1t}(1-\tau_{1t})D_1F_1(k_{1t},N_{1t}) + \tilde{\beta}\psi(1-\delta_1)\lambda_t = 0 \tag{16}$$

$$-\tilde{\beta}q_t + \tilde{\beta}\lambda_tA_{2t}(1-\tau_{2t})D_1F_2(k_{2t},N_{2t}) + \tilde{\beta}\psi(1-\delta_2)\lambda_t = 0 \tag{17}$$

$$\tilde{\beta}q_{t+1} - \gamma_{XN2}\lambda_t = 0 \tag{18}$$

$$1 - L_t - N_{1t} - N_{2t} = 0 \tag{19}$$

$$k_t - \psi k_{1t} - k_{2t} = 0 \tag{20}$$

$$y_{1t} - c_{1t} - g_{1t} = 0 \tag{21}$$

$$y_{2t}+\psi(1-\delta_1)k_{1t}+(1-\delta_2)k_{2t}-c_{2t}-g_{2t}-\gamma_{XN2}k_{t+1} = 0 \tag{22}$$

together with the "transversality condition":

$$\lim_{t\to\infty} \tilde{\beta}^t\lambda_tk_{t+1} = 0 \quad . \tag{23}$$

2.1.7 Characteristics of the model's steady state

Because of the endogeneity of the capital stock and the constant returns nature of the production functions, the long run production possibility frontier for this economy is linear. By this we mean that the long run social transformation curve between output of good 1 and output of good 2 is a straight line, rather than being the usual "bowed out" or "convex from below" social production possibility frontier. This is a surprising result, because it implies that the equilibrium relative prices of factors and outputs are determined independently of demand conditions. An implication is that fiscal policy shocks that involve variations in government consumption of either of the two goods will have no effect on the output prices, rental rates, or wage rates.

Let us explore the reasons for this surprising result. Note first that from equation (18) we have that $q=\gamma_{XN2}/(\tilde{\beta}\lambda)$, where variables without dates denote steady state values. This is a relationship between the steady state rental rate, q, and λ, the steady state price of good 2 (capital). Using this relationship in equation (17) yields:

$$A_2\Omega_2D_1F_2(k_2,N_2) = [\gamma_{XN2}-\tilde{\beta}(1-\delta_2)]/\tilde{\beta} \quad .$$

This condition pins down the marginal product of capital in sector 2, $D_1F_2(k_2,N_2)$, as a function of the parameters of technology, fiscal policy, and the transformed discount factor, $\tilde{\beta}$. The important thing to notice is that, given $\tilde{\beta}$, it does not depend further on preference parameters governing the allocation of consumption between goods 1 and 2 (θ) or on steady state levels of government consumption of the two goods (g_1 and g_2). With a constant returns to scale technology, the marginal product of capital is a function of the capital labor ratio, which is therefore also pinned down. The wage-rental ratio is simply a function of the capital-labor ratio; thus the wage-rental ratio, and by extension the capital-labor ratio in sector 1 are also pinned down.

Therefore, the productive structure of the economy in terms of factor composition in the two sectors is determined independently of demand conditions. Since there is a one-to-one relationship between relative output prices and the wage rental ratio under the assumptions of this model, relative output prices are also determined independently of demand conditions. The only conclusion possible, then, is that the long production possibility frontier for this economy is linear. A way to restate this result is the following: given $\tilde{\beta}$, and the production technologies in the two sectors, it is possible to determine the long run capital–labor ratios, the wage–rental ratio, and relative output prices independently of other parameters governing demand.

Thus we can see that shocks to the terms of trade faced by a small open economy of this type will generally lead the country to specialize in one or the other of the goods, with the possibility that the pattern of specialization will shift dramatically in response to the shocks. This result is very different from results obtained in static trade models.

Looking ahead to the open economy analysis we can easily see that there is only one relative price for outputs that will lead this economy to continue to produce both goods: the price given by the slope of the economy's production possibility frontier (PPF). And if the relative price in this open economy is equal to the slope of the PPF, the economy will be indifferent to producing any bundle of goods along the PPF since all bundles yield the same profit. The result is indeterminacy of production location in an open economy setting. We return to this potential problem in section 3 below.

2.1.8 Differential growth in steady state

As mentioned earlier, it is not necessary for steady state growth that outputs of the two sectors grow at the same rate. But if outputs in the food and machines sectors grow at different rates, what is the implication for the growth rate of aggregate output? Clearly, aggregate output must be defined in terms of a numeraire; suppose it is food. Recalling that λ is the price of a machine, and p is the price of food, define food–denominated GNP as:

$$GNP_t = Y_{1t} + (\lambda_t/p_t)Y_{2t}. \tag{24}$$

We derived earlier that the growth rate of Y_1 (food) is $\gamma_{Y1}=\gamma_{XK1}\gamma_{XN2}$, and that the growth rate of Y_2 (machines) is $\gamma_{Y2}=\gamma_{XN2}$. From the first-order conditions (11) and (12), the growth rate of (λ/p), denoted $\gamma_{\lambda/p}$ equals the ratio of the growth rates of marginal utilities of the two goods. Letting γ_{MU1} denote the growth rate of marginal utility of good 1, and γ_{MU2} denote the growth rate of marginal utility of good 2, we have $\gamma_{\lambda/p} = \gamma_{MU2}/\gamma_{MU1}$ which, after some algebra, can be shown to be $\gamma_{\lambda/p} = \gamma_{XK1}$. Thus the "machines" component of GNP, measured in food units, grows at the rate $\gamma_{XN2}\gamma_{XK1}$, and the food component grows at the same rate. Food-denominated GNP therefore grows at a constant rate even though the individual components grow at different rates.

2.2 Linear approximations to equilibrium decision rules

Because of the highly nonlinear character of this system of equations given by (11)-(22), it is difficult to characterize the equilibrium dynamics of the model. Except for a few special cases, the first-order conditions cannot be solved to yield analytic solutions for the endogenous variables of interest. And the time-honored method of comparative statics is unlikely to yield unambiguous predictions about even the qualitative nature of the economy's response to shocks. Therefore, an important component of research in this area is the development of numerical methods for exploring the qualitative and quantitative response of the economy to shocks, obviating the need for restrictive assumptions imposed solely to force the model to yield unambiguous predictions.[8] This paper discusses two complementary methods for

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[8] An early application of a numerical approach to international trade problems can be found in Helpman (1976), in which he applies the Scarf algorithm to solve for equilibrium prices in a static economy with many factors, intermediate goods, and final goods.

computing approximate equilibria. The first appraoch is discussed below; the second is discussed in section 2.5.

One approach to approximating equilibrium decision rules is to linearly approximate the problem's first-order conditions, and then to study the approximate dynamics of the system in response to shocks of various sorts (e.g., temporary shocks to government spending, technology, or the capital stock.) This approach is very much in the spirit of the "hat calculus" approach to performing comparative statics in traditional real trade models.

Before presenting the linear approximations to (11)-(22), some definitions are necessary. The elasticity of the marginal utility of good i, i={1,2,3} with respect to good j is denoted ξ_{ij}. Similarly, the elasticity of the marginal product of factor i with respect to factor j in sector k is denoted ξ_{ijk}, for i={N,K}; j={N,K}; k={1,2}. Steady state shares in sectoral output of consumption, invstment, and government spending are denoted s_{cj}, s_{ij}, s_{gj}, for j=1,2. Finally, a circumflex (a "hat") denotes the percentage deviation of a variable from its steady state level. With these definitions, the linear approximations to (19)-(30) are given by:

$$\xi_{11}\hat{c}_{1t} + \xi_{12}\hat{c}_{2t} + \xi_{13}\hat{L}_t - \hat{p}_t = 0 \tag{25}$$

$$\xi_{21}\hat{c}_{1t} + \xi_{22}\hat{c}_{2t} + \xi_{23}\hat{L}_t - \hat{\lambda}_t = 0 \tag{26}$$

$$\xi_{31}\hat{c}_{1t} + \xi_{32}\hat{c}_{2t} + \xi_{33}\hat{L}_t - \hat{w}_t = 0 \tag{27}$$

$$\hat{p}_t + \hat{A}_{1t} + \hat{\Omega}_{1t} + \xi_{NN1}\hat{N}_{1t} + \xi_{NK1}\hat{k}_{1t} - \hat{w}_t = 0 \tag{28}$$

$$\hat{\lambda}_t + \hat{A}_{2t} + \hat{\Omega}_{2t} + \xi_{NN2}\hat{N}_{2t} + \xi_{NK2}\hat{k}_{2t} - \hat{w}_t = 0 \tag{29}$$

$$\hat{p}_t + \hat{A}_{1t} + \hat{\Omega}_{1t} + \xi_{KN1}\hat{N}_{1t} + \xi_{KK1}\hat{k}_{1t} - (\gamma_{XN2}/(\gamma_{XN2}-\tilde{\beta}(1-\delta_1))\hat{q}_t - \tilde{\beta}(1-\delta_1)\hat{\lambda}_t = 0 \tag{30}$$

$$\hat{A}_{2t} + \hat{\Omega}_{2t} + \xi_{KN2}\hat{N}_{2t} + \xi_{KK2}\hat{k}_{2t} - (\gamma_{XN2}/(\gamma_{XN2}-\tilde{\beta}(1-\delta_2))\hat{q}_t - (\gamma_{XN2}/(\gamma_{XN2}-\tilde{\beta}(1-\delta_2))\hat{\lambda}_t = 0 \quad (31)$$

$$\hat{q}_{t+1} - \hat{\lambda}_t = 0 \quad (32)$$

$$\hat{L}_t + \hat{N}_{1t} + \hat{N}_{2t} = 0 \quad (33)$$

$$\hat{k}_t - \psi\hat{k}_{1t} - \hat{k}_{2t} = 0 \quad (34)$$

$$\hat{A}_{1t} + s_{K1}\hat{k}_{1t} + s_{N1}\hat{N}_{1t} - s_{c1}\hat{c}_{1t} - s_{g1}\hat{g}_{1t} = 0 \quad (35)$$

$$\hat{A}_{2t} + s_{K2}\hat{k}_{2t} + s_{N2}\hat{N}_{2t} - s_{c2}\hat{c}_{2t} - s_{g2}\hat{g}_{2t} - s_{i1}\hat{i}_{1t} - s_{i2}\hat{i}_{2t} = 0 \,. \quad (36)$$

There are two potential sources of exogenous shocks in this model: the technology shocks ($\hat{A}_{jt}$) and shocks to government spending ($\hat{g}_{jt}$). To simulate the dynamic response of the endogenous variables to a shock, it is necessary first to choose parameter values for the models: the elasticities, discount factors, growth rates of technological change, and so forth. Typically, these parameters are chosen so that the steady state behavior of the economy matches long run behavior of the actual economy (see, for example, the work of Kydland and Prescott (1982), Hansen (1985), Prescott (1986), and King, Plosser and Rebelo (1988a)). Next, it is necessary to solve the linearized system (32)-(43) to yield equations linking changes the endogenous variables (consumption, investment, etc.) to changes in the exogenous variables (technology shocks and government spending).[9]

These "reduced form" equations appropriately reflect the cross-equation restrictions characteristic of rational expectations models. Therefore, these models can be used to appropriately evaluate the effects of policy experiments; for example, the dynamic response to an innovation in government

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[9] See the Technical Appendix to King, Plosser and Rebelo (1988a) for details of the solution method.

spending of an economy which use deficit finance versus an economy with a balanced–budget law.[10]

2.3 Dynamic response to sector–specific shocks

In this section, we choose a particular parameterization for this model and study the economy's dynamic response to technology shocks in the two sectors: $\hat{A}_{1t}$ and $\hat{A}_{2t}$. The purpose of studying this example is to examine the differential dynamic effects of technical change in the two sectors. As we shall see, the character of the response can be very different for the two types of shocks. In this example, the technology shocks are assumed to be serially uncorrelated.

The production functions in this example are Cobb–Douglas and, following tradition, the consumption good sector has been chosen to be the capital intensive sector.[11] The parameter values chosen are given in Table 3; all share parameters are steady state shares. The discount factor, labor's share, and depreciation rates were chosen so that the model roughly matches the long run behavior of the U.S. economy. The time period is a quarter of a year.

2.3.1 Dynamic response to a technology shock in the food sector (sector 1)

The first experiment we consider is an unanticipated 1% increase in A_{1t} (total factor augmenting technical change in the consumption good sector).

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[10] See Baxter and King (1988b) for an analysis of these types of experiments in a one–sector, closed economy context.

[11] This assumption is the usual one because it typically is necessary to guarantee stability. However, we have experimented with several cases involving the nontraditional factor intensity assumption (i.e., capital goods are capital intensive), and have not found evidence of instability.

The positive technology shock makes all factors allocated to sector 1 more productive. Labor and capital move into sector 1, expanding output in that sector, while output of sector 2 contracts. The relative price of good 1 declines as a results of its lower real unit cost of production. The effect on capital can be seen in Figure 3, which plots the response of capital in the two sectors as well as the response of aggregate capital. Since aggregate capital is predetermined in this model, the immediate response to the unanticipated positive technology shock in sector 1 is to move capital out of sector 2 and into sector 1, leaving total capital unchanged. Capital in sector 1 rises almost 2% relative to its steady state level, and capital in sector 2 drops about 3.5%. Because sector 2 is the capital producing sector, this causes capital to be below its steady state level from period two onward.

Another notable feature of Figure 3 is the fact that capital in sector 1 drops dramatically in period two, moving into sector 2 in order to help build the capital stock back to its steady state level. In many parameterizations of the model, this implies that, in period 2, capital is actually taken out of sector 1 and put into use in sector 2. Given the length of the period under study, such rapid adjustment does not really seem plausible. A potentially important extension to this model would involve irreversible investment, as in Sargent (1980). Adjustment costs to investment represent an intermediate case between perfectly mobile and perfectly immobile capital. These extensions are discussed further in sections 2.4.3 and 2.4.4 below.

Figure 4 plots the impulse responses of consumption of good 1 (the consumption good), consumption of good 2 (the capital good), and leisure to a

1% innovation in A_1 which takes place in quarter 1. Consumption of all three goods rises in response to the positive technology shock. Consumption of good 1 rises by almost 3%, a result of reinforcing income and substitution effects. Consumption of good 2 and of leisure rise as well, but by smaller amounts. That consumption of good 2 and leisure rise in response to the shock is somewhat counterintuitive. Because there has been a positive, temporary technology shock, individuals should wish to work harder due to an intertemporal substitution effect, and should also consume less of good 2 since its relative price has risen. But in this case, the countervailing income effect of the positive technology shock dominates. This dominance occurs for values of σ greater than one (σ is the inverse of the elasticity of intertemporal substitution). Along the transition path, periods two through infinity, consumption of all three goods falls below steady state levels, reflecting the fact that aggregate capital is below its steady state level.

Figure 5 plots the impulse responses of output in the two sectors. In the period the shock takes place, the output of sector 1 goes up by about 2.5%, and output of sector 2 falls by nearly 4%. Because the shock to A_1 has no persistence, output responses in periods 2 through infinity mirror factor responses. Along the transition path both factors have moved into sector 2 (relative to long run factor allocations) because of the need to rebuild the capital stock. Thus along the transition path, output of sector 2 is above its long run level, and output of sector 1 is below its long run level.

Figure 6 plots the response of capital/labor ratios to the shock in A1; recall that the units are percentage deviations from steady state values. Even though steady state levels of K/L are different in the two sectors, the percentage deviations from these levels is the same in both sectors in response to a shock because the two sectors have identical elasticities of factor substitution. Since capital is predetermined and aggregate labor supply falls in response to the shock, capital/labor ratios in both sectors rise about .4% in the first period. Along the transition path, however, capital is below its steady state value and labor is above its steady state level, implying that capital/labor ratios in both sectors are below steady state levels along the transition path.

Figure 7 plots the response of utility-denominated prices to the 1% shock in A_1 (for example, the wage is the shadow price of leisure in utility terms.) The contemporaneous responses are a small (.2%) increase in the wage rate; even smaller increases in the rental rate and the price of good 2 (which is also the price of a unit of capital); and a decline of slightly more than 1% in the price of good 1. Along the transition path the wage is below its steady state value, reflecting the above-steady-state level of effort and the below-steady-state level of capital. In periods 2 through infinity, the rental rate and the prices of both final goods are above their steady state levels, reflecting the fact that the economy is rebuilding the capital stock.

Figure 8 plots the response of the relative price of good 2 in terms of good 1, together with the response of the wage-rental ratio. The relative price of good 2 is about 1.1% above its steady state level in period 1, and slightly below its steady state level along the transition path. Along the transition path, the wage-rental ratio is below its steady state level while the capital labor ratio is above its steady state level; this is in accord with the predictions of static real trade models. But in the impact period (the period in which the shock takes place), the capital labor ratio rises as does the wage-rental ratio. This effect is not what one would expect from static models, and is basically due to the fact that aggregate labor supply has declined in response to the shock.

2.3.2 Dynamic response to a technology shock in the capital goods sector

Now, consider an unanticipated 1% technology shock in sector 2, the capital-producing sector. Figure 9 plots the dynamic response of capital to this shock. The immediate effect of the technology shock is, as before, to reallocate capital into the more productive sector; relative to steady state levels, capital in sector 1 falls by about 4%, and capital in sector 2 rises by 7.5%. In the case of an A_2 shock, capital is moved out of sector 1 and into sector 2, leading to an increase in sector 2 output relative to its steady state level as seen in figure 11. The transitional dynamics, however, are very different compared with the effects of a shock to sector 1. Because sector 2 output was above its steady state level in period 1, the effect of the technology shock in sector 2 is to push the aggregate capital stock above its steady state level along the transition path.

Figure 10 plots the response of consumption of the two produced goods and leisure. With a shock to A_2, consumption of the two goods and leisure all exhibit substantial declines in the first period, and are above their steady state levels in all subsequent periods. Figure 11 plots the response of outputs of goods 1 and 2; output of good 2 rises about 9% in the period the shock takes place, while output of good 1 falls by nearly 4%. But along the transition path, while the economy is decumulating capital, (and while the aggregate capital labor ratio is above its long run level, as discussed below) output of good 1 is above its steady state level, while output of good 2 is below its steady state level.

Figure 12 plots the response of capital–labor ratios to the shock in sector 2; since effort rises in response to the shock, capital/labor ratios decline in the first period. But since the capital stock is above its steady state level along the transition path, while labor is below its steady state level, capital intensities are above steady state levels along the transition path.

Figure 13 plots the response of utility–denominated prices to the 1% technology shock in sector 2. In the first period, the price of good 1 rises about 1%; and there are slight decreases in the rental rate and the price of good 2, together with a small increase in the wage rate. Figure 14 plots the response of the relative price of good 2 and the wage rental ratio. The most interesting effect concerns the wage–rental ratio. With a shock to A_2,

2.4 Extensions of the basic two–sector model

This section discusses three extensions of the basic two–sector model developed above. These extensions are viewed as being important modifications to the basic model which are likely to be necessary before the model can be said to adequately mimic actual time series. These modifications all generally have the effect of limiting factor mobility, either in the short run, or in the long run, or both.

2.4.1 Irreversible investment

The first modification of the basic model involves irreversibility of investment: once capital is placed in use in a sector, it cannot be moved. New investment goods, however, can be placed in either sector. As we saw in section 2.3, the response to sector–specific shocks without irreversibility of investment meant that there were large shifts in capital between sectors in response to a shock; shifts that were reversed within one period (here, one quarter). We do not observe this level of volatility in capital allocated to a particular sector, and assuming that capital is immobile is one way to get the model to mimic this feature of the data. A one-sector model with irreversible investment was studied by Sargent (1980) in a paper investigating Tobin's q-theory of investment. Sargent argued that irreversibility of investment was one reason that the q-theory should not be expected to hold up empirically. However, in experimental versions of Sargent's model with technology shocks but no preference shocks (Sargent had both types of shocks), it has been found that the nonnegativity constraint on investment does not bind within the steady state distribution of capital, at least for reasonable parameter values and for processes on the technology shock which produce realistic output volatility.

But within the basic two sector model developed above, the irreversibility constraint seems likely to bind quite often. This has two main implications: first, a model with irreversible investment should fit the data better than a model without this constraint; second, the presence of the nonnegativity constraint on investment will induce an important nonlinearity into the decision rule for capital accumulation. The presence of the nonlinearity means that analysis of the system using linear approximate dynamics as discussed in section 2.2 will not be appropriate. A new technique for studying these problems—one which does not take a linear approximation to the equilibrium decision rule but works directly from the model's first order conditions—is described in section 2.6 below.

2.4.2 Adjustment costs

A less stringent way of modifying the model in the direction of greater realism is to introduce adjustment costs in capital. There are a variety of ways to do this; three are explored in Mussa (1978). Generally, however, adjustment cost models may be viewed as being of two broad types: models in which the adjustment uses final goods (output of at least one of the two sectors evaporates in the adjustment process) and models in which the adjustment process uses factor inputs (labor and/or capital) in a third, "capital moving," sector. In future work, we plan to explore the way introduction of adjustment costs improves the ability of the model to imitate actual time series, together with investigating the extent to which one or the other type of adjustment cost model provides a better fit to the data.

2.4.3 A specific factors model

A third way to limit the mobility of factors of production is by means of a specific factors model. In this model one type of factor, labor, is sector-specific in the sense that it is unproductive in the other sector. This approach views labor as having such large industry-specific human capital components that it is unproductive for the labor to move to the other sector. Capital is viewed as able to move instantaneously and costlessly between sectors. Yet in equilibrium, the mobility of capital is effectively reduced because the presence of the specific factor (labor) means that there is decreasing returns to the mobile factor. Thus we expect to see a damping of the response of the economy to shocks in the case where labor is sector-specific, relative to the case in which labor is also costlessly mobile.

The empirical existence of specific factors, at least in the short run, is indicated by some recent empirical work by Grossman and Levinsohn (1988). These authors found that the security prices of firms in import-competing firms responded strongly to terms of trade shocks, suggesting that the factors employed by these firms are not freely mobile in the short run. Clearly, the specific factors in an industry include some types of labor (management, for example) but not others (keypunching); and some types of capital (specialized machines that produce only Brand X Widgets) but not others (company cars). The purpose of the specific factors model developed here is to provide a starting point for evaluating the ways in which models with specific factors differ from frictionless in their predictions concerning the response of the economy to shocks.

This specific factors model is also viewed as a first step toward a more general theory of factor mobility. For example, it would be desirable to endogenize the human capital accumulation decision, allowing individuals to retrain for another industry if they wished. Thus there could be a third, "retraining" sector (notice that this sector could easily be Mussa's (1978) "capital moving" sector, renamed). Another route would extend Grossman and Shapiro's (1982) theory of factor mobility to a more general dynamic context than the two-period model considered in that paper.

2.5 The Euler Equation Approach

In a recent paper, Baxter (1988b) has developed a new method for obtaining approximate equilibrium decision rules for economies of the type studied here. Because the method works directly from the first-order necessary conditions for the problem, it is not necessary that competitive equilibrium be Pareto optimal.[12] In Baxter (1988b), this method was applied to two problems: (i) a distorted one-sector model similar to the model of Section 2, and (ii) a cash-in-advance model with capital accumulation in which money is required for purchase of both consumption goods and investment goods.

Application of this method to the two-sector model developed here is straightforward. And because the method does not take a linear approximation to the decision rules, but instead computes the equilibrium decision rules over a discretized state space, it will produce correct decision rules (policy functions) in cases where there are important nonlinearities. This will be especially important in studying the model with irreversible investment, in which the nonnegativity constraint on investment means that the policy function for investment has a "kink" in it.

While the Euler equation approach is computationally rapid, it is not nearly as rapid as the linearization approach discussed above. Thus an important use of the Euler equation approach is as a check on the extent of the approximation error inherent in the linearization approach. In the applications presented in Baxter (1988b), it was found that the exact decision rules computed with the Euler equation approach were in fact nearly linear, meaning that the linearly approximate decision rules for those problems involved very little approximation error.

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[12] Probably the most commonly-used method for computing equilibrium decision rules works by iterating on the value function for the problem. This method requires that equilibrium be Pareto optimal, or that the problem can be rewritten as a modified planner's problem. The Euler equation approach places no restrictions of this type on the problem.

Finally, an "exact" approach such as the Euler equation aproach is necessary for studying models which do not possess a steady state, and which cannot be transformed into models which possess a steady state as was done in the model of sections 2.1 and 2.2. In models of this sort, the economy will steadily drift away from any point at which the linear approximation is taken, implying that misleading results would be obtained from analysis of the linearized economy. Often, the restrictions necessary for steady state growth may be stronger than we wish to impose. Especially in an open economy setting, these restrictions may rule out some interesting cases that we wish to investigate: for example, economies in which agents in different countries have different rates of time preference, or experience technological change at different rates. In order to study these types of economies, we need methods for studying nonstationary economies. The Euler equation approach is capable of handling these types of models.

2.6 Econometric evaluation of the two-sector model

In the previous six subsections, we have developed a basic two-sector, two factor model, and have presented two ways of simulating the equilibrium response of the economy to shocks: the linear approximation method and the Euler equation method. Since estimation and testing of these models is viewed as an integral part of the proposed research program, this section addresses the natural next question: how are we to quantitatively evaluate the empirical adequacy of this class of models?

There are two distinct methods currently in use for evaluation of models of this type. The first involves use formal statistical procedures such as maximum likelihood; a recent paper by Christiano (1988) uses this methodology to study inventory investment. A second method involves calibration of the model economy and has been used, for example, by Kydland and Prescott (1982), Hansen (1985), and Prescott (1986). In this method of model evaluation, the researcher restricts key parameters of his artificial economy to match long run growth statistics or parameters estimated from microeconomic data, and compares the moments generated by his artificial economy with the moments of actual time series. Typically, first and second moments are studied and, although this is not made explicit, it appears that it is more important to match some moments than others.

Recently, Kenneth Singleton (1988) has proposed an evaluation procedure that is grounded in formal statistical theory while remaining sympathetic to the point of view of the "calibration" approach. Singleton argues that formal econometric evaluation is the proper method of model evaluation, and that the method of calibration and "moment matching" can be interpreted as a special kind of econometric testing procedure. Singleton argues that the "moment matching" method as it is currently used is undesirable since there are no formal criteria for deciding whether a particular model "fits well." Thus these methods are not easily applied to modifications or extensions of the model, since it is not clear what it means for a model to "fit well."

Singleton's suggestions for formal econometric evaluation of these models involve making explicit the function implicitly being used to weight deviations of the model economy from the behavior of the actual economy. He suggests that the class of minimum discrepancy estimators be used to estimate models of this type; GMM estimators belong to this class. Singleton suggests using the most efficient minimum discrepancy estimator as a basis for estimation. GMM estimation based on linearized models as in section 2.2 is straightforward and computationally quite feasible. Estimation of the linearized models is therefore viewed as an important first step in evaluating equilibrium models of the type developed in this proposal.

GMM estimation based on the Euler equation method is also straightforward although computationally much more demanding. For estimation along these lines, access to supercomputer networks is likely to be important.

3. A Two Sector Model of International Trade

This section develops a two country, two sector model of international trade. As is well known to anyone who has ever worked with these models, studying an open economy is not just a matter of copying down a second set of equations, attaching a "star" to all foreign variables, and redoing the analysis. Important decisions must be made concerning (i) which national and international markets are assumed to be available, (ii) the relative size of the two countries, (iii) whether differences in tastes exist across the two countries, and (iv) whether endowments and technological opportunities differ across the two countries.

The approach taken here is to restrict preferences, technologies, and endowments only where necessary to ensure the existence of a balanced growth path in the absence of shocks. The relative sizes of the two countries is left as a free parameter which can be varied to investigate the way in which the economic impact of a shock changes as the relative sizes of the two countries changes. The benchmark model will assume that all markets are open, and that capital, but not labor, is internationally mobile. The reason for preferring a benchmark model with a full complement of markets is that it provides a natural reference point for studying the effects of selectively closing markets.

In section 3.1, we discuss a two country, two sector model with perfectly mobile capital and immobile labor. Given the constraints necessary for steady state growth and the risk-neutrality implied by the linearization method, it turns out that the national location of production of the two types of goods is indeterminate. Even if we are willing to specify steady state national output shares of the two goods, the short run dynamic response of the economy to exogenous shocks is still indeterminate. And even if we assume that factors cannot move instantaneously in response to exogenous shocks (short run factor immobility), long run indeterminacy is still a problem. This result is a major departure from static or two-period models in which indeterminacy does not crop up, and is due to the fact that, in the long run, capital is completely mobile so long as there is international trade in the investment good.

In sections 3.2 and 3.3 we discuss ways to resolve this indeterminacy. We choose to resolve the indeterminacy in the context of the linear model by resorting to a specific factors model, for reasons discussed more fully in section 3.2. Section 3.3 discusses why the indeterminacy will not generally arise with the Euler equation approach to model solution. Section 3.4 concludes with a discussion of potential applications of the open economy model, and directions for future research.

3.1 A two country, two sector model with perfectly mobile capital

Although it will turn out that this model does not "work" in the sense that it fails to produce a determinate pattern of production, it is instructive to study it briefly in order to understand the reasons for this indeterminacy.

Imagine that the world consists of two countries, each of whose economies are described by models of the type developed in Section 2. Each country can produce both goods via identical constant returns production functions, and individuals in each country value consumption of both final goods and leisure. In order to guarantee that there is steady state growth, we must restrict the relationship between the countries in two ways: first, the rates of growth of technological change in the two sectors must be identical in the two countries; and second, the transformed discount factor, $\tilde{\beta}$, must be the same in the two countries. This second requirement means that either the utility functions in the two countries are identical or that they differ in ways that keeps $\tilde{\beta}$ the same in the two countries (recall that $\tilde{\beta}=\beta[\gamma_{XN1}^{\theta}\ \gamma_{XN2}^{(1-\theta)}]^{1-\sigma}$). For simplicity, we shall assume temporarily that government expenditure is financed by lump sum taxation.

To compute the competitive equilibrium in the world economy, consider a planner who seeks to maximize the weighted sum of the utilities of the representative agents in the two economies, given the production technologies in the two countries, subject to a world-wide resource constraints, and subject to the constraint that labor cannot move internationally, although it can move between sectors within a country. Assume also that the planner (standing in for the representative agents) is free to reallocate factors within a given period in response to exogenous shocks occurring in that period.[13] This model is very similar to the closed economy two sector model in that it possesses only one controlled state variable: the worldwide capital stock. (There is also one costate variable: the price of capital.)

As discussed in section 2, the production possibility frontiers in each country are linear. And since the production functions for the two goods are assumed to be identical across countries, the two national PPF's have the same slope. This means that the world PPF is also linear as illustrated in Figure 15. Thus there is indeterminacy in terms of the national location of production.

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[13] Other researchers (for example, Eaton (1979)) have investigated the effects of uncertainty about the current shock on factors that cannot move after the shock is realized. The current model is easily adapted to handle economies of this type; the cost is that it adds a state variable to the system.

It is nevertheless the case that world production of each of the two goods is determined in this model, as is the amount of consumption going to the representative agent in each country. But with identical production functions in the two countries and no taxes, the amount of production of each of the two goods in each country is not pinned down. Because capital and final goods are instantaneously, costlessly transportable across sectors and countries, it is a matter of indifference where any particular unit of a good is produced. Assigning long run (steady state) shares of output of the two goods for each country is one way to solve the long run indeterminacy problem, but there is still a problem in that the short run dynamic response to shocks involves production location indeterminacy as well. As we saw in section 2, the response to a shock in one sector involves alterations in output and factor usage in all other sectors as well. And given that capital and final goods are so mobile, the planner is generally indifferent between an infinite number of ways of allocating production of the two goods in response even to a sector-specific shock in one country.

3.1.1 Solving the indeterminacy

Solving the short-run and long-run indeterminacies within this linear model requires restrictions on the mobility of final goods or factors, or both. In the extensions of the basic two-sector model discussed in section 2.4, three modifications involving restricted factor mobility were discussed. These modifications were (i) irreversible investment (immobility of capital once placed in use in a particular sector), (ii) costs of adjustment in moving capital between sectors, and (iii) a specific factors model, with labor being the sector-specific factor. We discuss in turn the potential for each of these modifications to solve the open-economy indeterminacy problem.

The assumption of irreversible investment is sufficient to pin down the short run response of the national and world economies to exogenous shocks, given an initial allocation of factors to each sector in each country and given long run output shares. But while irreversibility of investment solves the short-run indeterminacy problem, it does not solve the long-run indeterminacy problem. Since capital depreciates, it will all eventually have to be replaced, meaning that any feasible pattern of production can be reached from any other. Thus there is still nothing in the model to pin down long run shares of each nation's contribution to world output of each good. It is clear that costs of adjustment in capital will work the same way, solving the short-run indeterminacy problem but not the long-run indeterminacy problem. In order to pin down long run national output shares in models with irreversible investment or adjustment costs, it is necessary to add another friction; for example: differential costs of transporting goods nationally versus internationally.

Another route to solving the indeterminacy is to consider a specific factors model. This route is attractive since it solves both the short-run and long-run indeterminacy problems. The model is extreme, however, in its assumption that the specific factor is permanently, completely immobile, and the specificity is imposed exogenously. Nevertheless, it provides a fruitful starting point.

We shall assume that labor is the immobile factor, so that labor in the two sectors are different types of labor, completely unproductive if moved to the other sector. Thus labor is viewed as having entirely industry-specific skills, and no "general" skills. There is nothing particularly special about having labor as the specific factor; it could just as easily be capital. But Amano ((1977), page 137, discussing the work of Sohmen (1963) and others concerning the Leontief paradox) has pointed out that a specific factors model with labor as the specific factor is potentially able to explain part of the paradox. Specifically, Amano argues that it can rationalize the empirical observation that relatively low wage countries tend to have a comparative advantage in the industry whose elasticity of factor substitution is relatively low.

Although we do not present here a detailed analysis of the model sketched out above, it is useful to discuss the general properties such a model will have. First, by effectively imposing decreasing returns in each industry to the mobile factor (capital), we render determinate short-run and long-run production location. There is a sense in which this model looks like a decreasing returns to scale model with a single factor (with capital being the single factor). But such an interpretation would be misleading, because the specific factor—labor—will vary endogenously as the owner of the factor substitutes between labor effort and leisure in response to shocks to the economy. Allowing labor to respond in this way is critical to correctly evaluating the effects of a proposed policy intervention.

3.1.2 The likelihood of specialization

We explored in the previous two subsections the implications of endogenous capital accumulation and capital mobility. With constant returns to scale production and no taxes, the result was indeterminacy of the national location of production, since the world PPF was a straight line. This section explores the implications of small departures from the assumptions of identical (zero) tax rates and identical production functions. If the two production functions are not identical across countries, or if tax rates in the two sectors are not identical across countries, then the two countries' PPF's will still be linear, but they will not have the same slope. The world PPF will be as sketched in Figure 16; it will have a "kink" in it. In Figure 16, country 1 has a comparative advantage in producing good 1. Letting N denote a fixed amount of labor and K denote a fixed amount of labor, and letting stars denote country 2 values, we can define comparative advantage as follows. Country 1 has comparative advantage in producing good 1 if

$$\left[\frac{\Omega_1 A_1 (X_{K1}K)^{\alpha_1} (X_{N1}N)^{1-\alpha_1}}{\Omega_2 A_2 K^{\alpha_2} (X_{N2}N)^{1-\alpha_2}}\right] > \left[\frac{\Omega_1^* A_1^* (X_{K1}^* K)^{\alpha_1^*} (X_{N1}^* N)^{1-\alpha_1^*}}{\Omega_2^* A_2^* K^{\alpha_2^*} (X_{N2}^* N)^{1-\alpha_2^*}}\right]$$

(Recall that Ω_j is the tax "wedge" $1-\tau_j$.)

Thus we see that a country could have comparative advantage in a particular good simply because the tax rate is relatively lower in that sector compared with the other country, even though the production functions are identical. And if the world PPF has the shape drawn in Figure 16, at least one country will specialize in production of only one good. This will happen no matter how small are the differences in the tax rates or the production functions. Thus specialization must be viewed as the most likely outcome.

This result has a decidedly Ricardian flavor, since long run factor allocation and production patterns are determined completely by comparative advantage considerations. It has strong predictions about the response of the world economy to changes in fiscal policy: changes in the composition of government spending will have no effect on relative prices or specialization patterns (so long as the changes are sufficiently small to keep the economy on the same side of the "kink" as it was previously), but changes in tax rates could potentially lead to very large changes in world patterns of production, factor prices, and relative prices of output. These strong results obtain because changes in the tax rates alter the world PPF directly, while changes in government spending involve movements along an unchanged world PPF.

3.2 Diversification as a solution to the indeterminacy problem

So far, our discussion of open economy models has proceeded with an eye toward using the linear approximation method for simulation and estimation of these models. As was discussed in section 3.1, identical production functions in the two countries leads to indeterminacy in long run production locations; a problem not resolved by adding short run impediments to factor mobility. In that section, a specific factors model was suggested as a way to resolve both the short-run and long-run indeterminacies.

This section suggests resolving the indeterminacy in a way that requires abandoning the linear approximation technology and using instead the computationally more demanding Euler equation method. The chief difficulty with the linear approximation approach in this context is that, by imposing certainty equivalence it assumes agents are risk neutral—they care only about mean returns but not variances. But if two countries are subject to sector-specific technology shocks that are not perfectly correlated internationally, then risk-averse agents are <u>not</u> indifferent to the locational pattern of production.

In order to make the portfolio allocation problem meaningful, we assume that capital must be allocated to sectors and countries before realization of the technology shock. In this model, agents will allocate production of each good across the two countries in a way that optimally reduces the risk in production of each good. Because the Euler equation approach works directly from the first-order conditions and does not impose certainty equivalence, the indeterminacy problem discussed above will not arise. Exploration of the properties of the equilibria of the frictionless model with Euler equation methods is viewed as complementary to the model extensions discussed in the previous section.

3.3 Applications and directions for future research

Perhaps the most interesting applications of the open economy model involve analysis of fiscal policy shocks of various types. It hardly needs repeating that proper evaluation of governmental policy intervention requires a fully specified equilibrium model in which we can properly account for the endogenous response of private agents to the change in their environment. The chief barrier to policy evaluation in an international setting along the lines suggested by Lucas (1976) has, until now, been a technological barrier: we did not have methods to numerically evaluate the models' equilibria. This paper has outlined a broad class of equilibrium models that should be viewed as extensions of the traditional trade models to an explicitly dynamic stochastic setting. The paper has also discussed methods for numerical solution and econometric evaluation of the models. With these methods in hand, we are ready to take the first step toward policy evaluation in an open economy setting.

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Table 1

Summary statistics for U.S. postwar economy

Quarterly data
1947:2-1986:3

Growth rates: percent per quarter

	mean	standard deviation
services	.93	.57
nondurables	.61	.86
durables	1.25	4.20
investment	.83	5.88
GNP	.79	1.10

Correlations

	services	nondurables	durables	investment	GNP
services	1.00				
nondurables	.37	1.00			
durables	.13	.34	1.00		
investment	.21	.05	.25	1.00	
GNP	.35	.32	.42	.77	1.00

Table 2

Summary statistics for U.S. postwar economy:

Annual data
1948-1985

Growth rates: percent per year

	mean	standard deviation
services	3.81	1.43
nondurables	2.43	1.98
durables	4.59	7.99
investment	3.15	14.86
GNP	3.18	3.07

Correlations

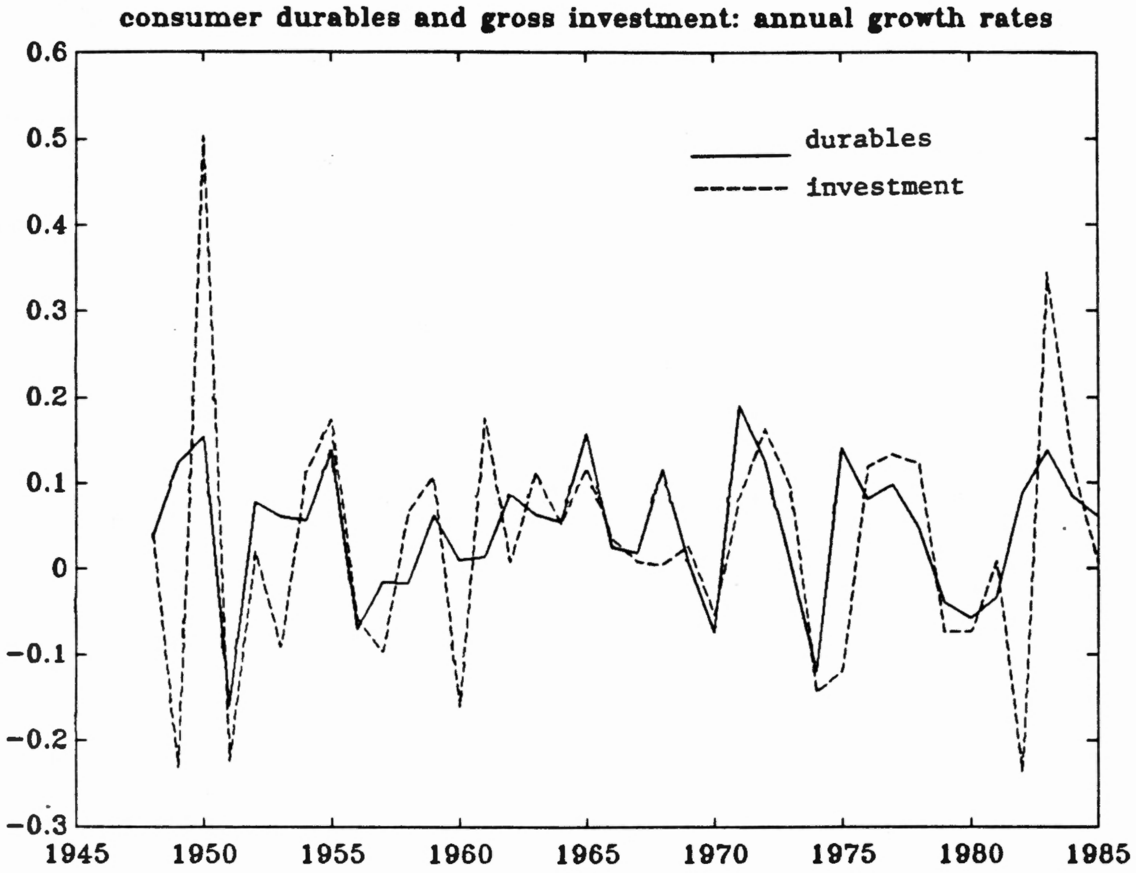
	services	nondurables	durables	investment	GNP
services	1.00				
nondurables	.52	1.00			
durables	.40	.40	1.00		
investment	.68	.34	.49	1.00	
GNP	.64	.54	.39	.77	1.00

Table 3

Parameter values for the experiments in section 2.3

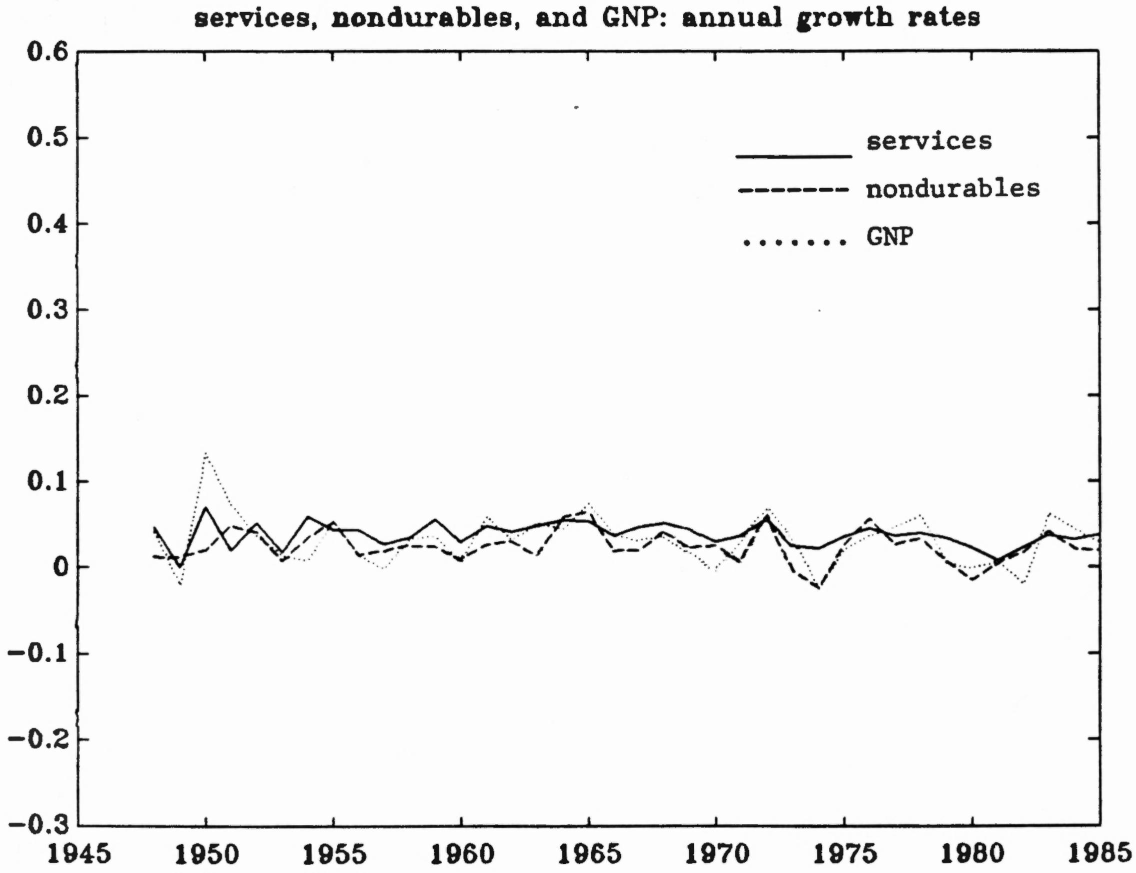
$s_{K1} = .6$	share of capital in sector 1
$s_{K2} = .4$	share of capital in sector 2
$\psi = 1$	
$\delta_1 = \delta_2 = .025$	depreciation rates of capital
$\sigma = .5$	$1/\sigma$ is the elasticity of intertemporal substitution
$\theta_1 = .89$	steady state consumption share of good 1: chosen to match postwar share of consumer services and nondurables in total consumption
$\theta_2 = .11$	steady state consumption share of good 2: chosen to match postwar share of consumer durables in total consumption
$\beta = (1+(.065/4))^{-1}$	discount factor; implying a steady state real interest rate of 6.5% per year.
$\overline{N} = .20$	share of total time devoted to market activities
$s_{g1} = .20$	share of sector 1 output purchased by government
$s_{g2} = .20$	share of sector 2 output purchased by government
$\gamma_{XN1} = 1.004$	gross growth rate of labor augmenting technical change in sector 1, implying a long run growth rate of 1.6% per year
$\gamma_{XN2} = 1.004$	gross growth rate of labor augmenting technical change in sector 2, implying a long run growth rate of 1.6% per year.
$\Omega = 1$	government spending financed by lump-sum taxation.

All other parameter values and shares used in simulating the model are obtained as functions of the parameters set out above.



consumer durables and gross investment: annual growth rates

Figure 1a



services, nondurables, and GNP: annual growth rates

Figure 1b

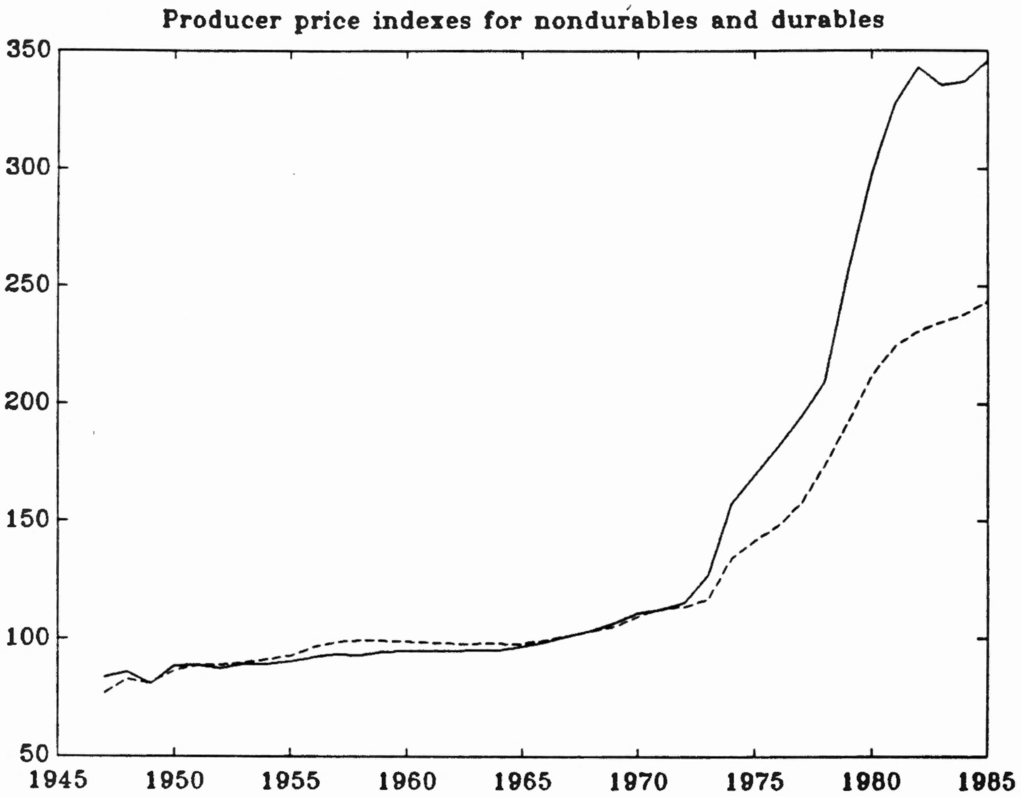


Figure 2a

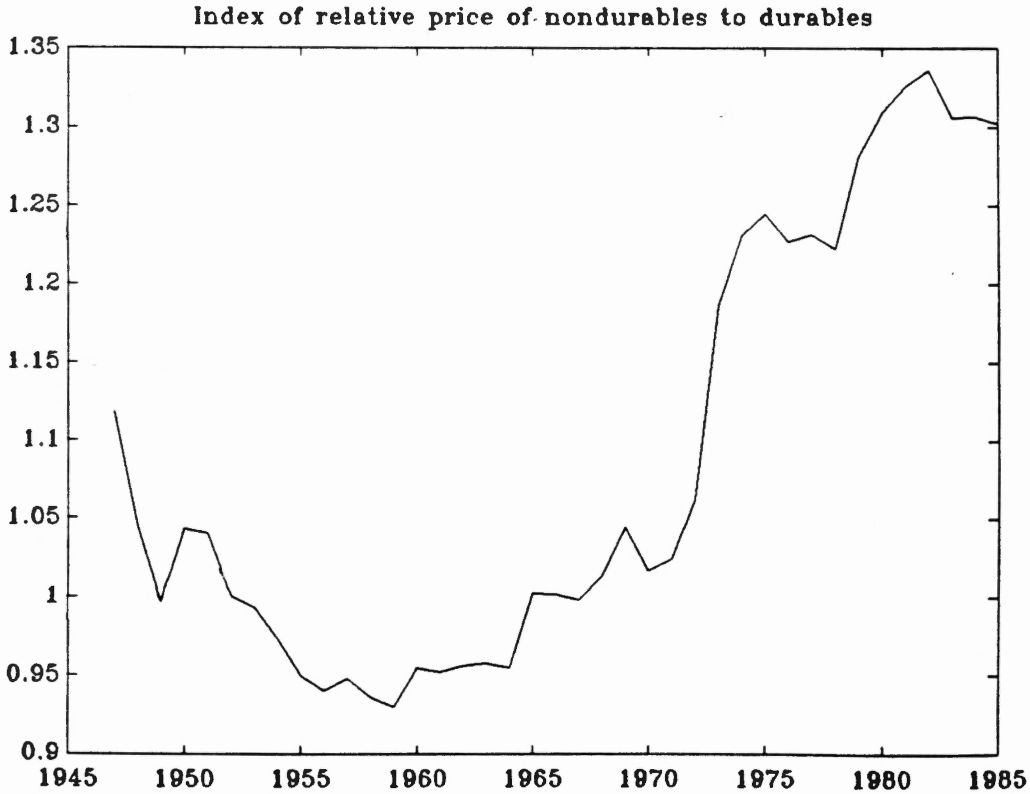


Figure 2b

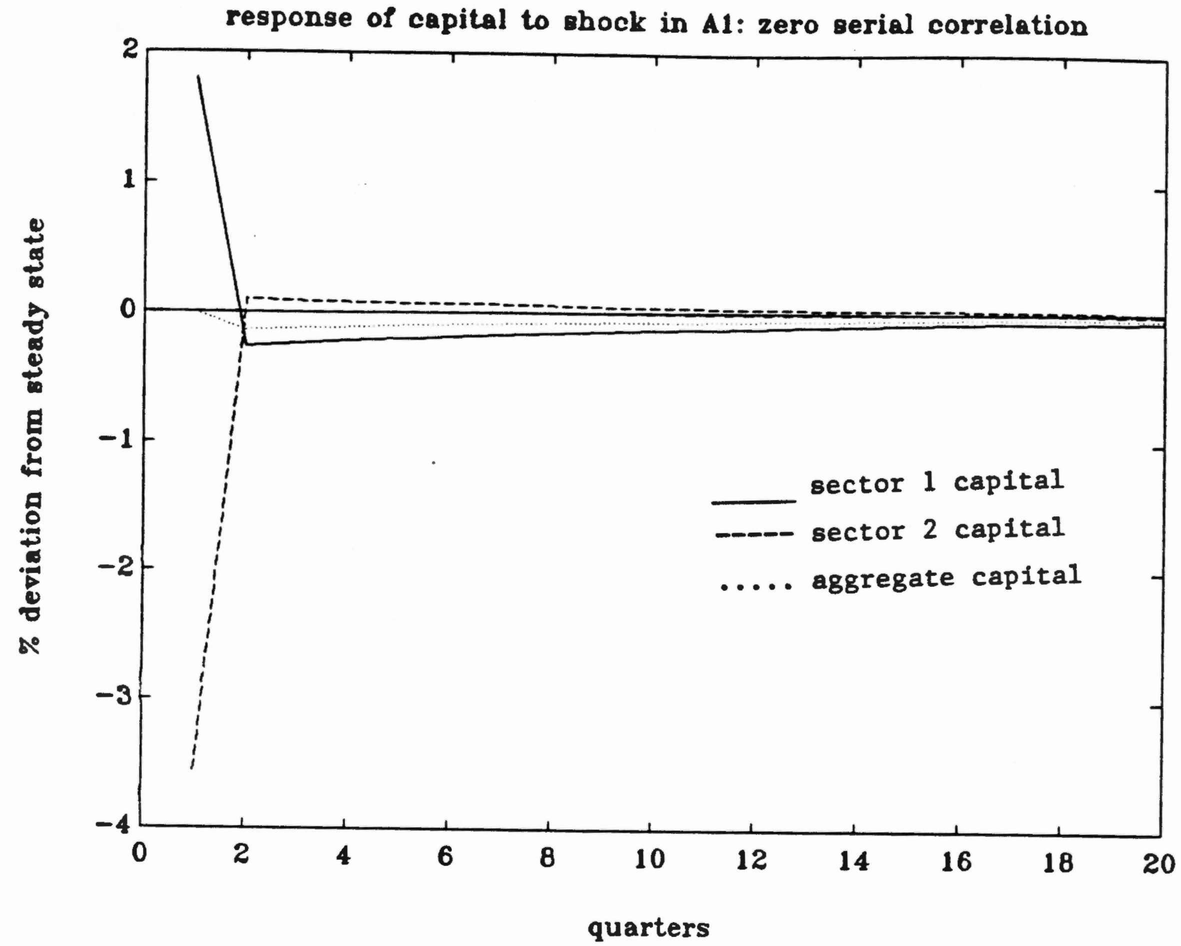

response of capital to shock in A1: zero serial correlation

Figure 3

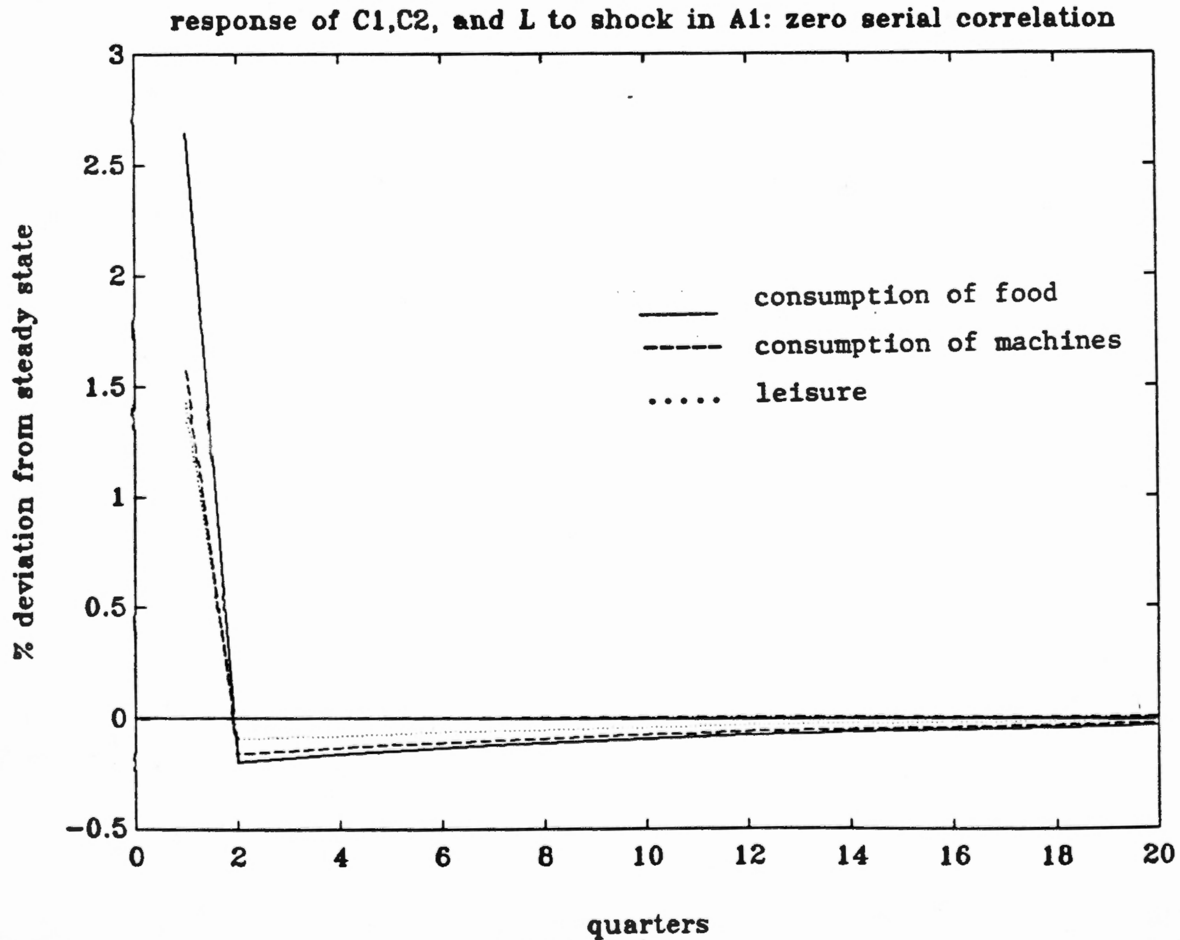
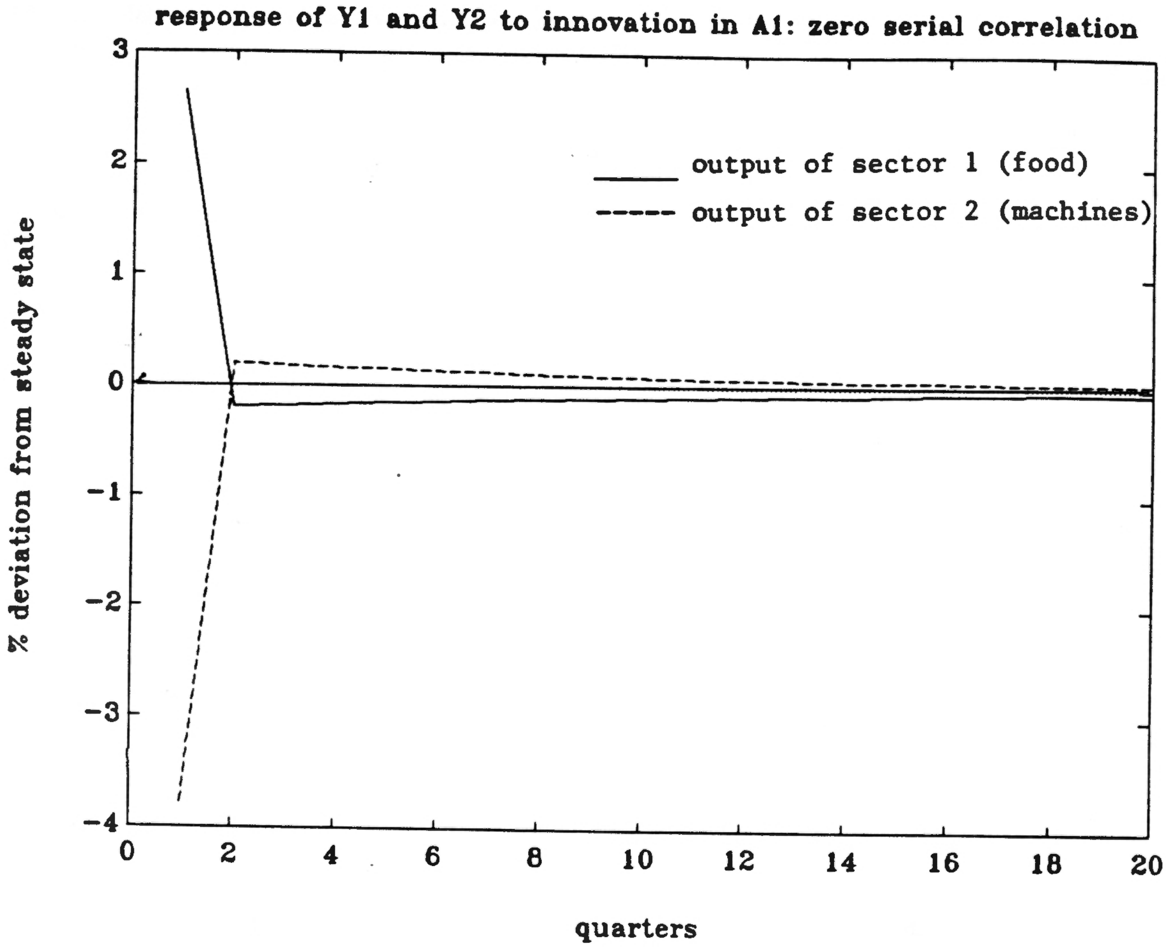
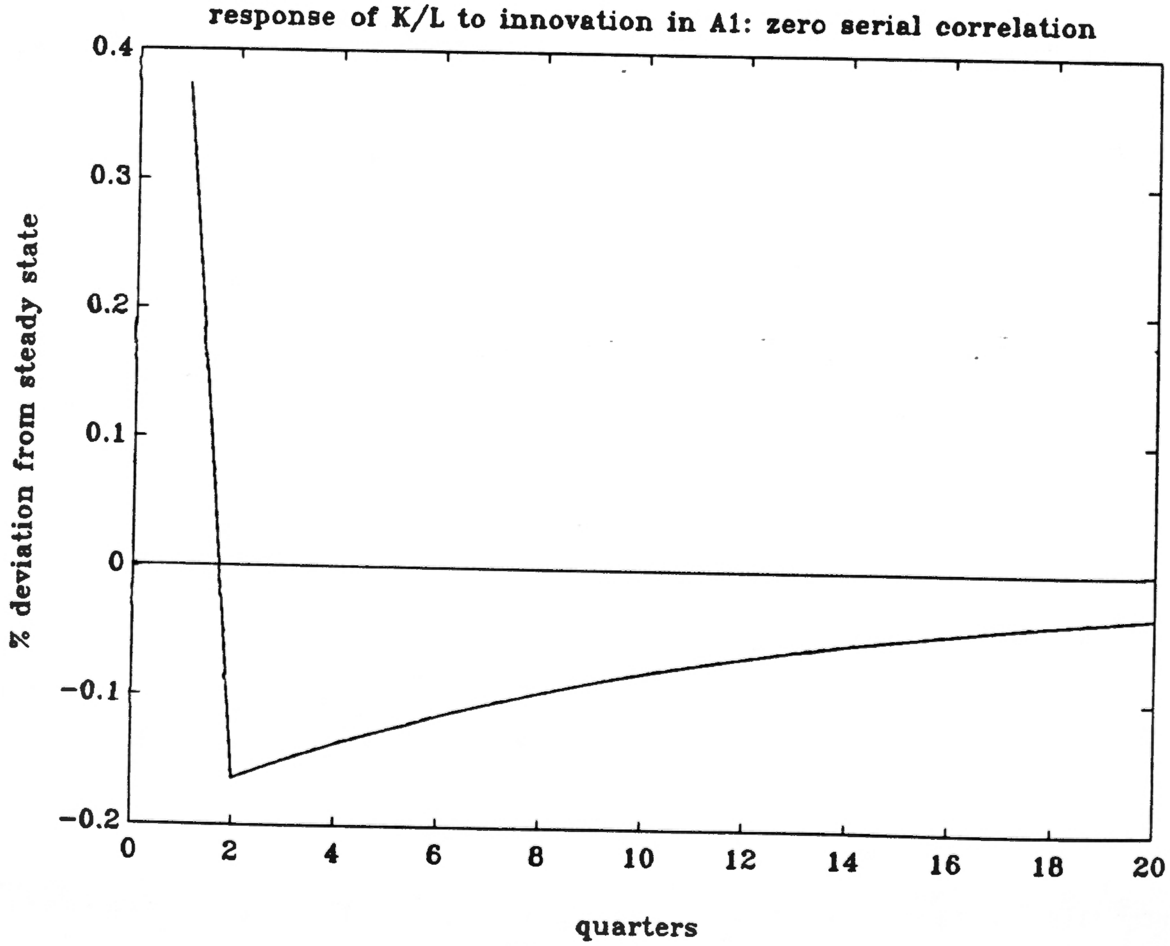

response of C1,C2, and L to shock in A1: zero serial correlation

Figure 4



response of Y1 and Y2 to innovation in A1: zero serial correlation

Figure 5



response of K/L to innovation in A1: zero serial correlation

Figure 6

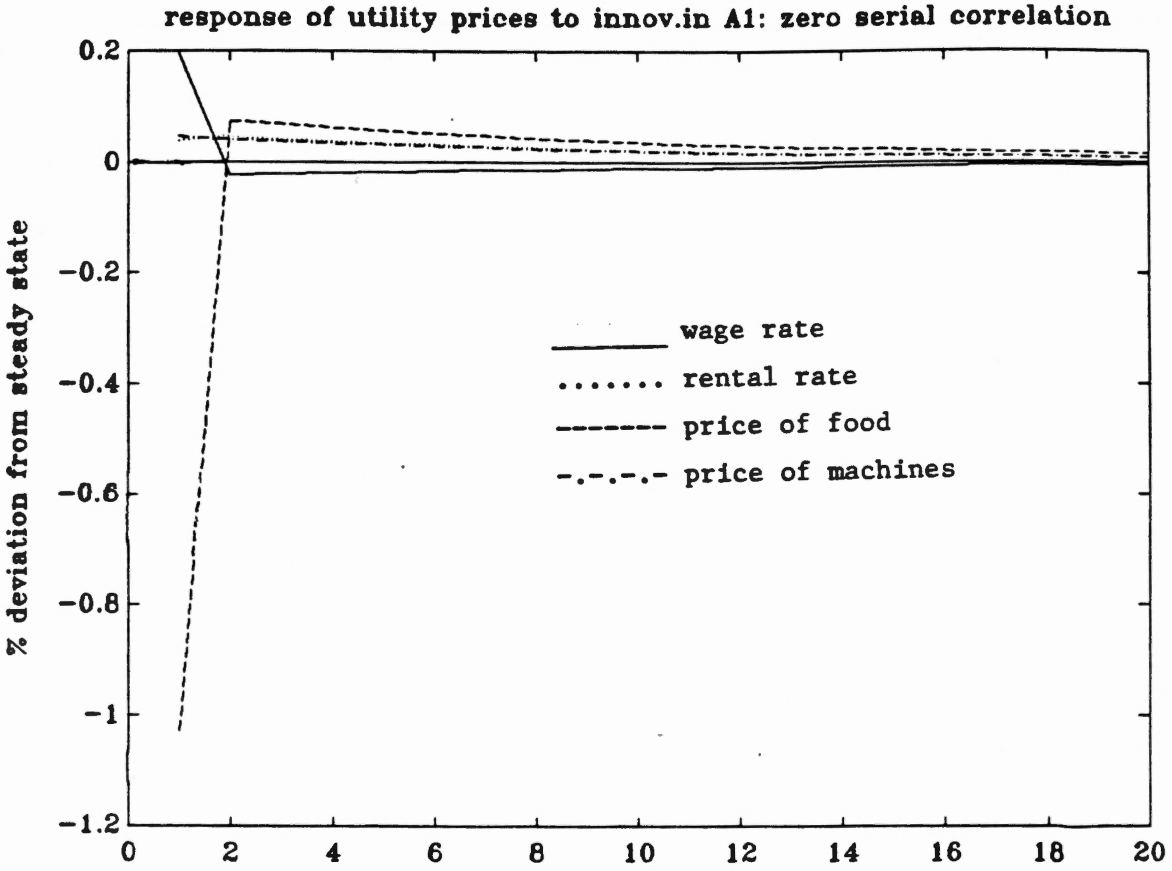


Figure 7

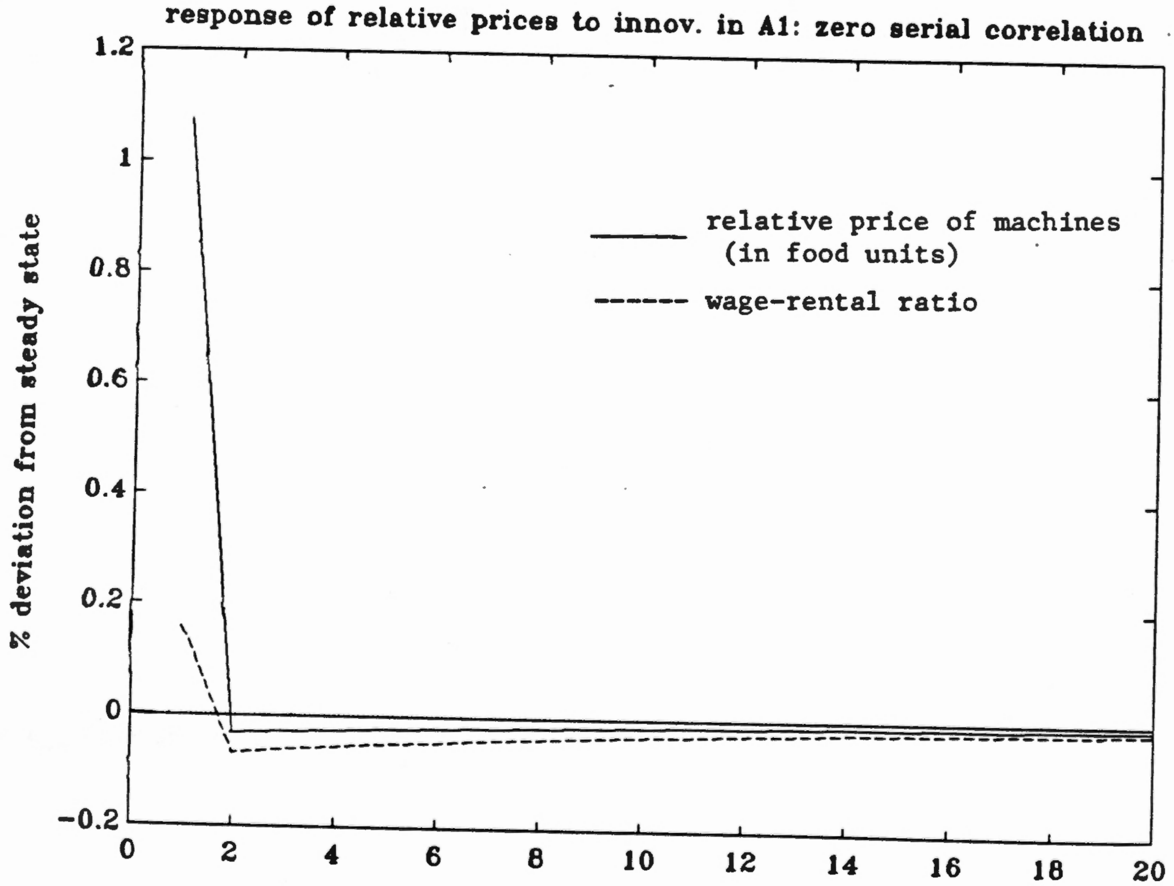


Figure 8

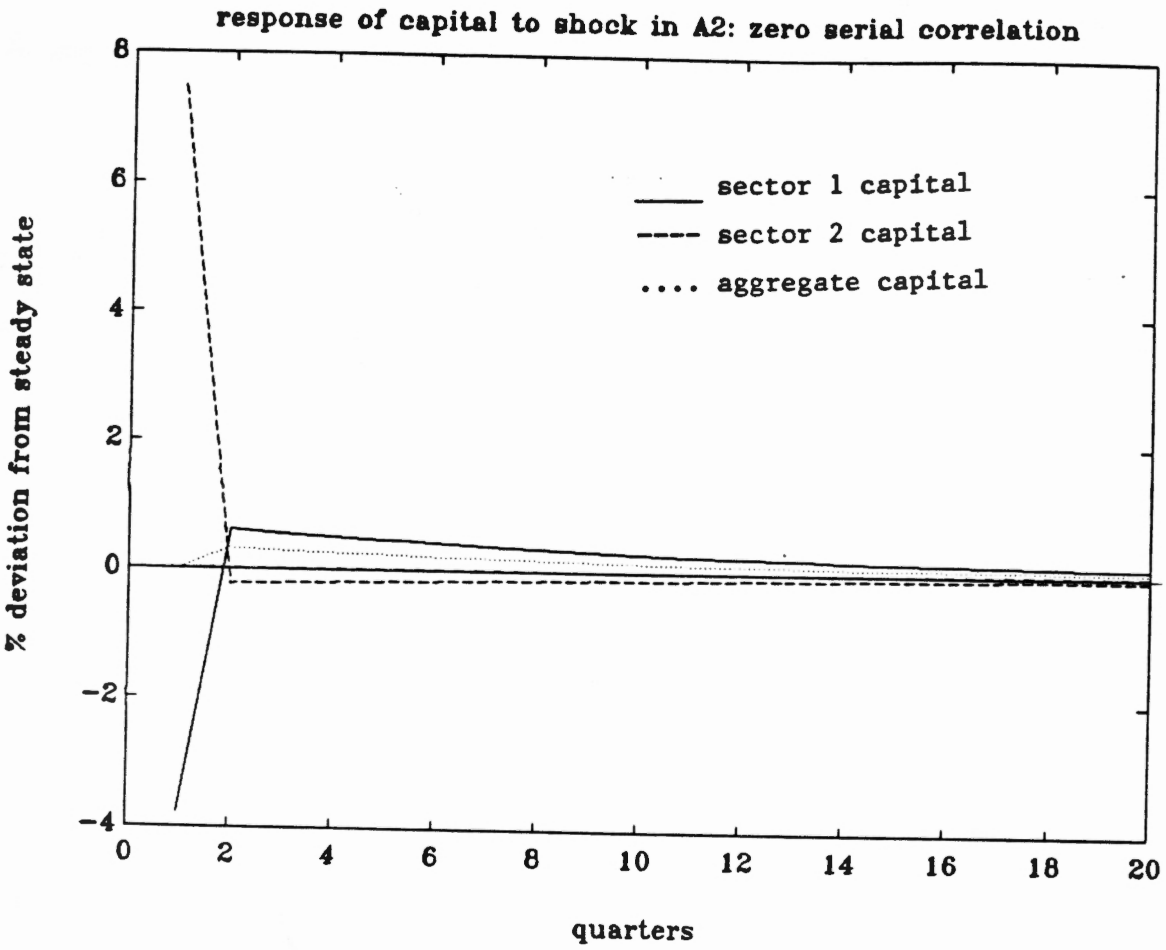


Figure 9

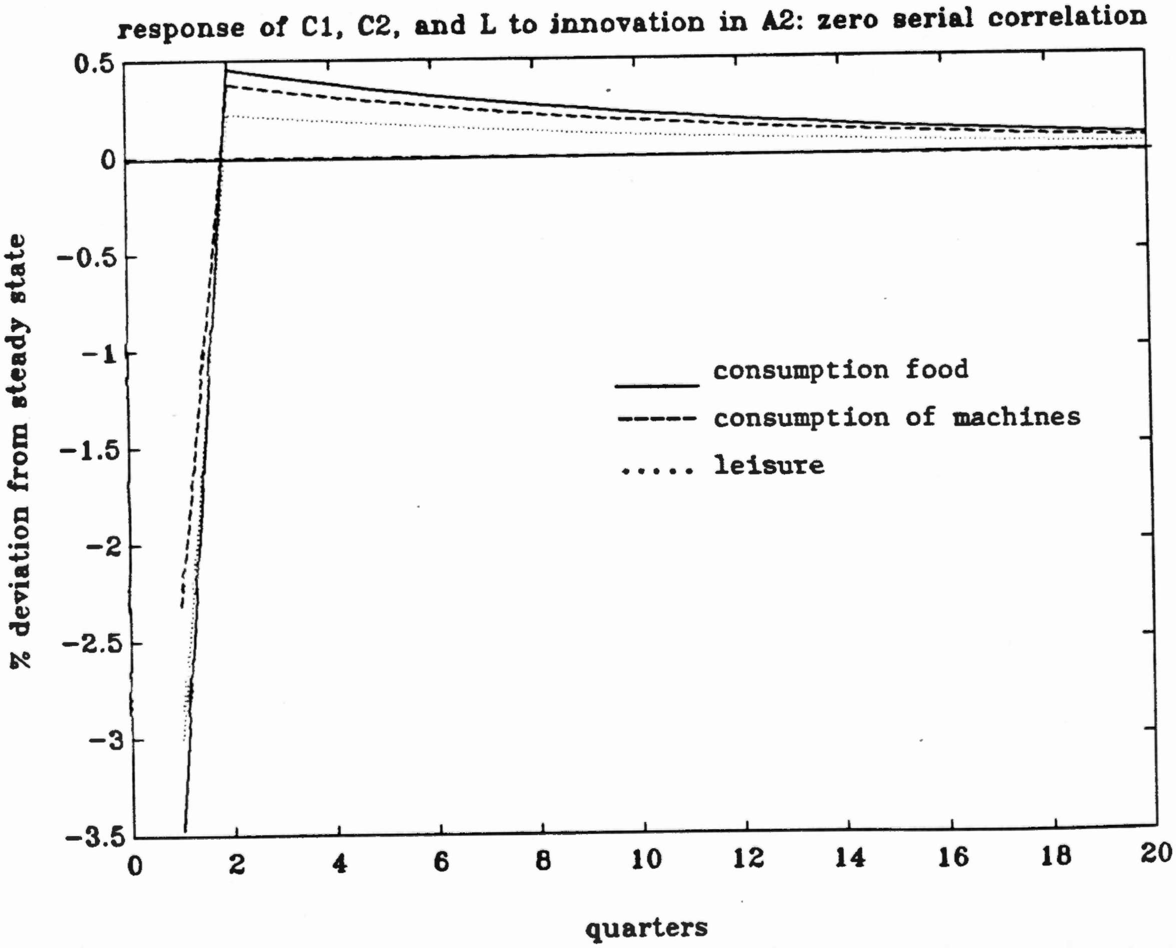


Figure 10

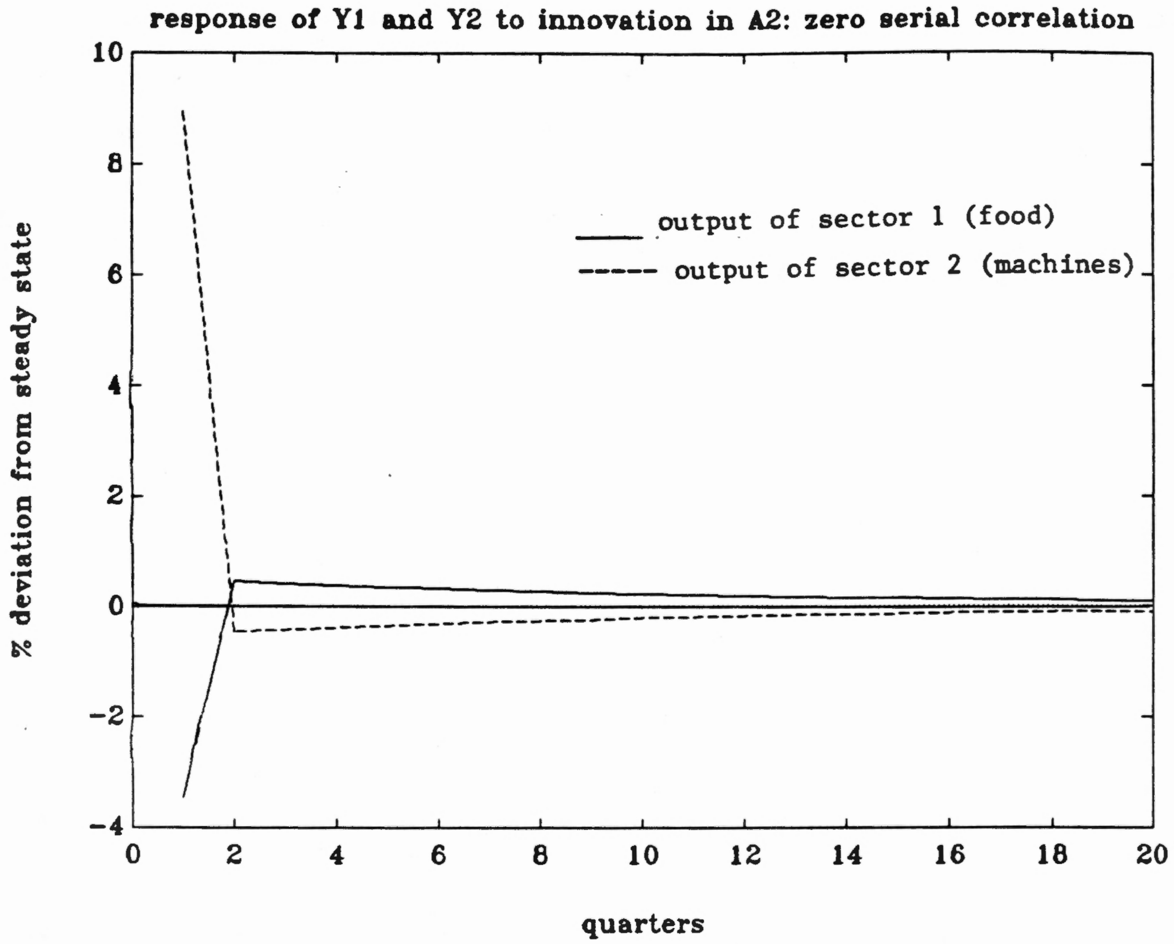


Figure 11

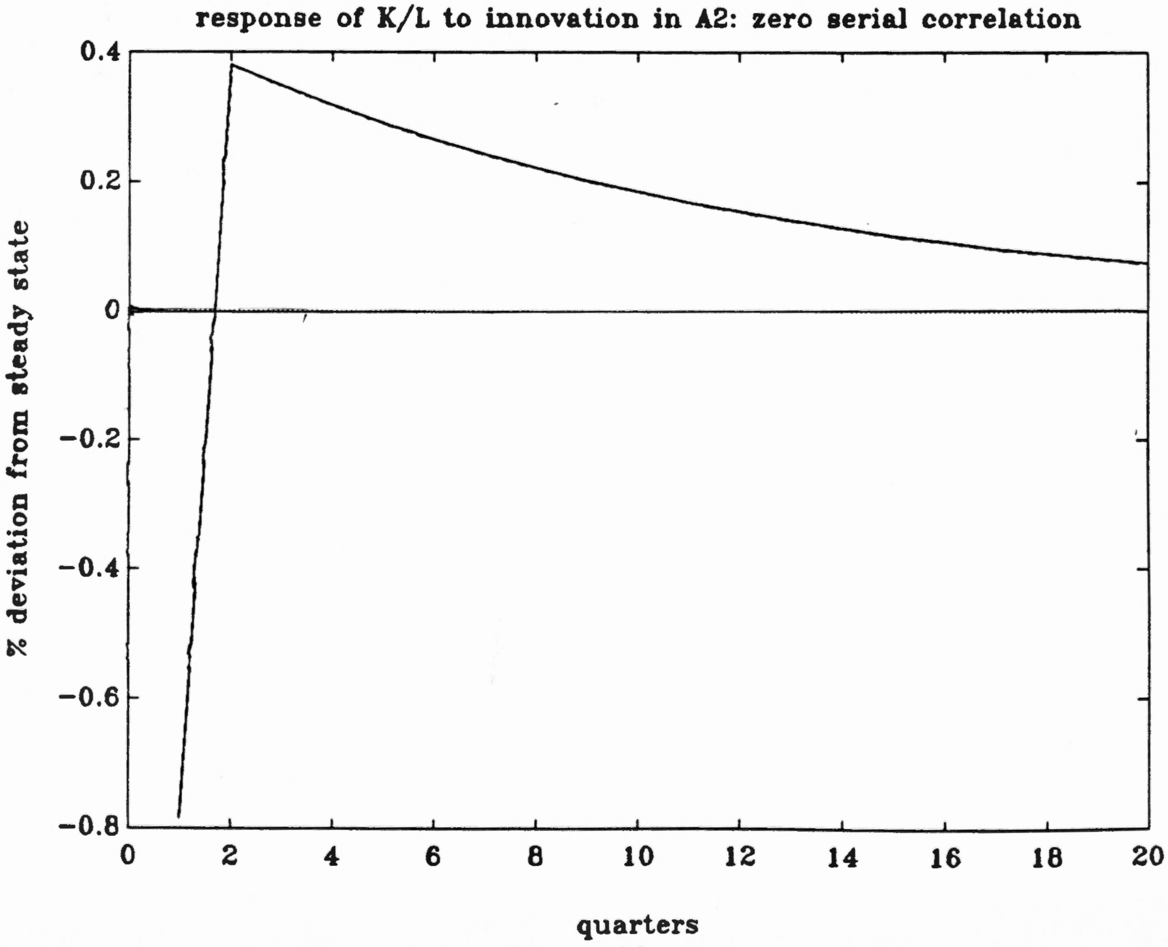
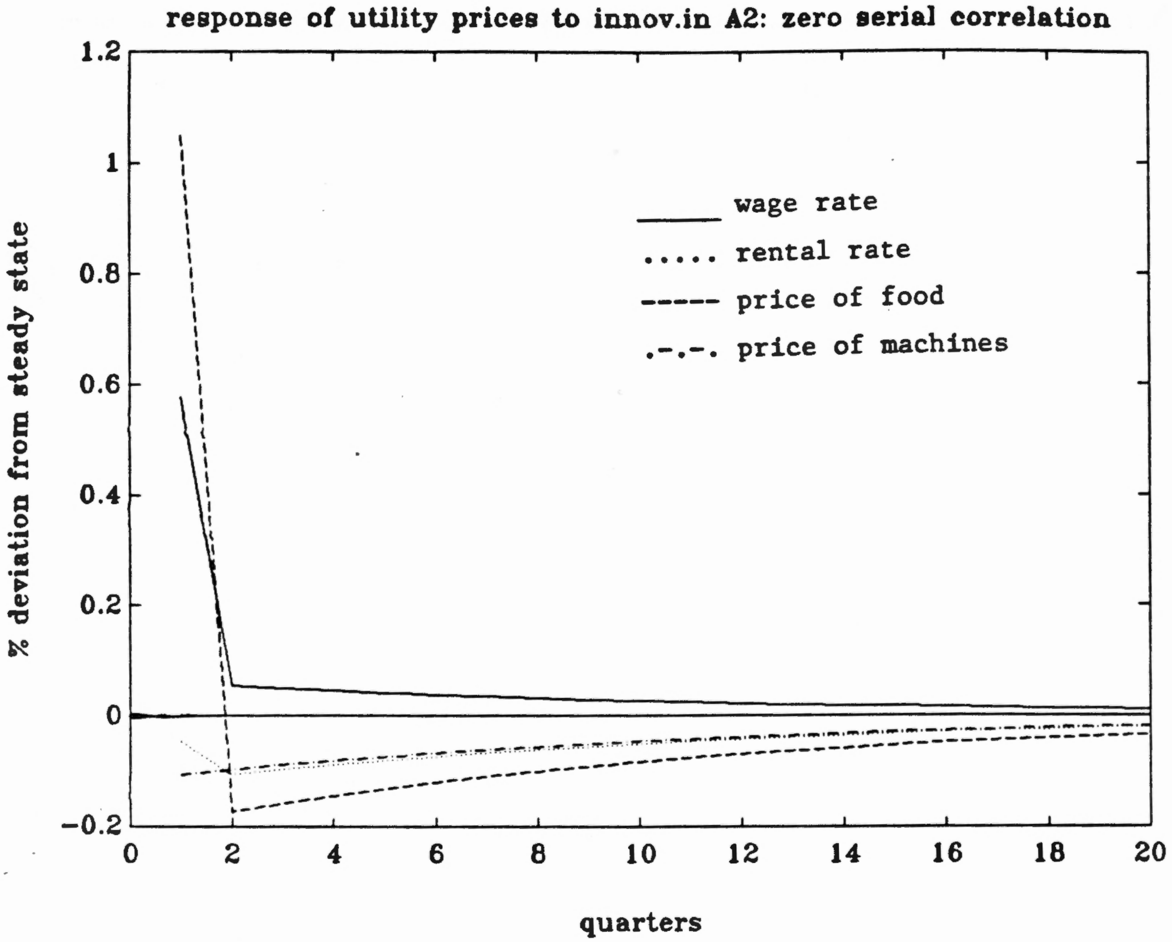
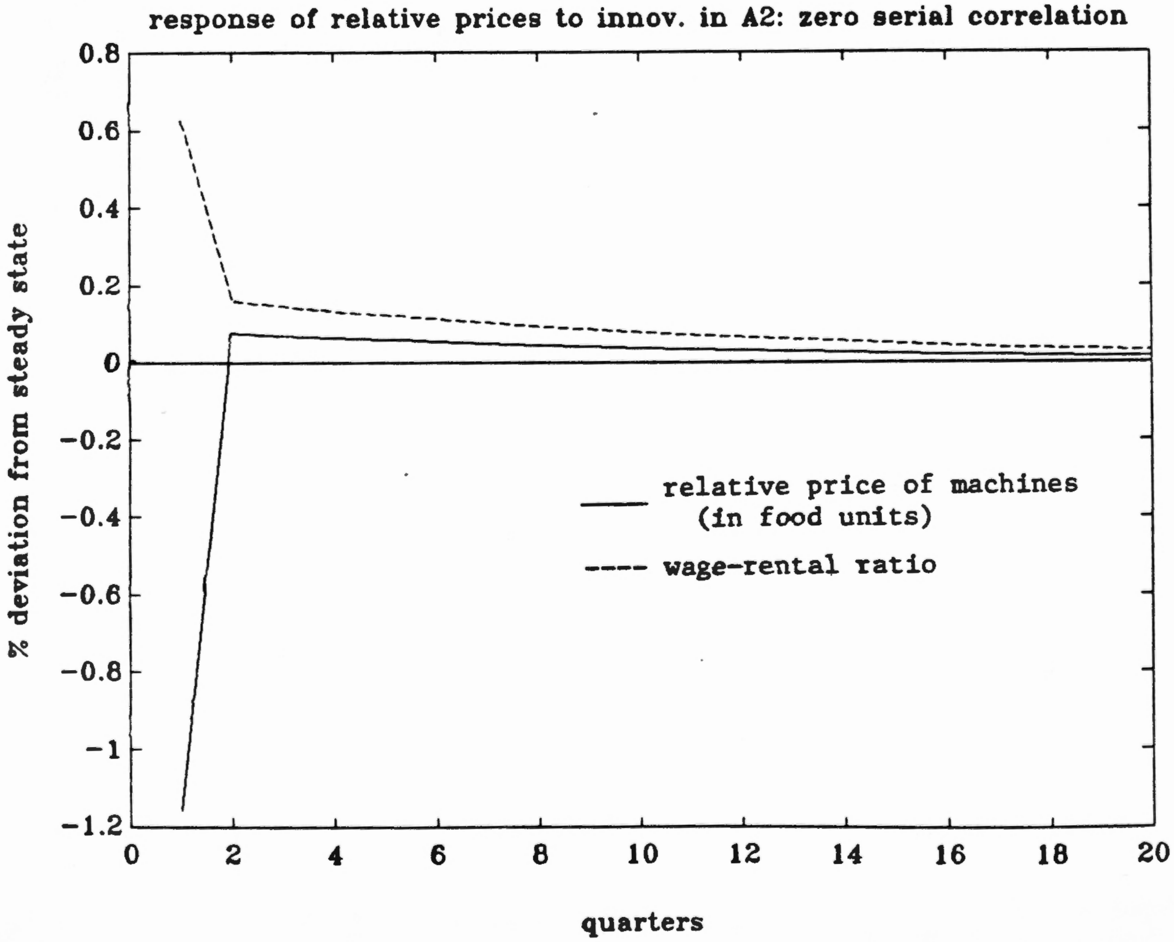


Figure 12



response of utility prices to innov.in A2: zero serial correlation

Figure 13



response of relative prices to innov. in A2: zero serial correlation

Figure 14

World PPF for Two-Country Model:
Identical Technologies

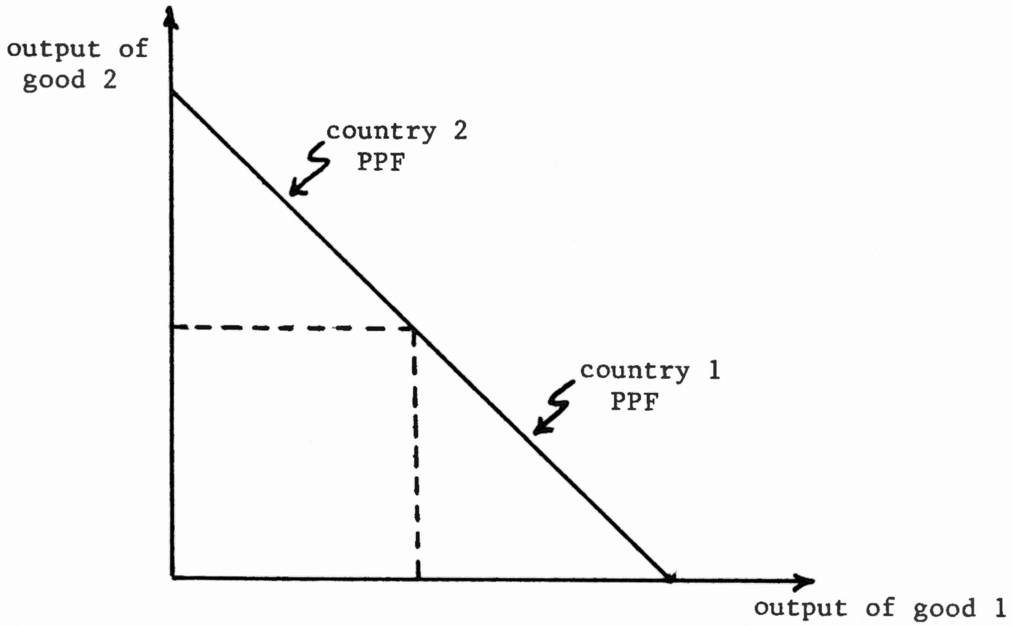


Figure 15

World PPF for Two-Country Model:
Different Technologies

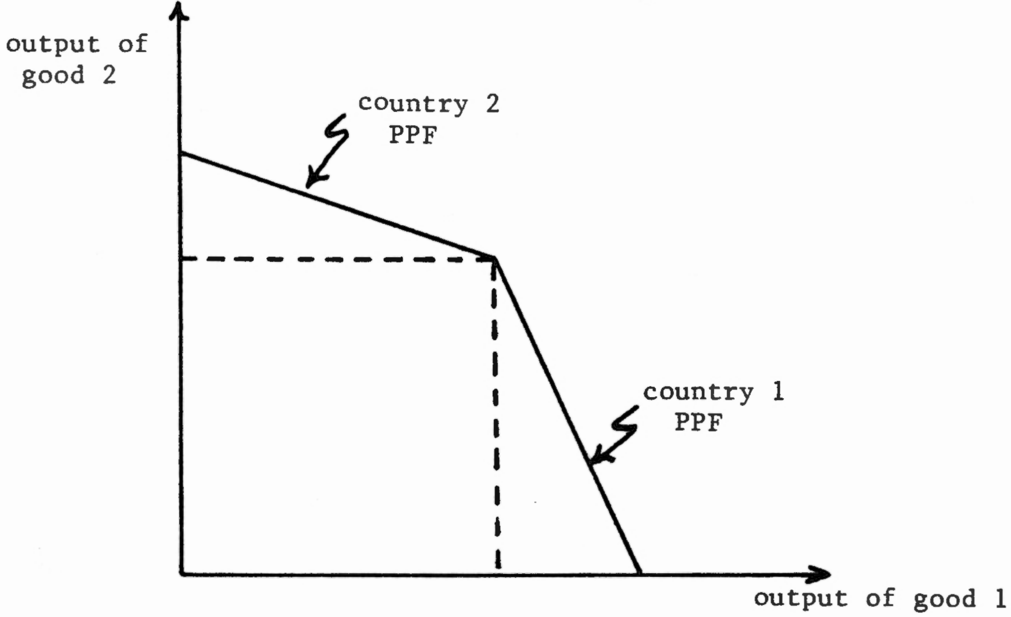


Figure 16

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