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# THE INDIRECT AND DIRECT SUBSTITUTION EFFECTS

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## *Abstract*

The classification of two goods as substitutes or complements by the sign of the substitution term defined by Hicks (1939) intimately involves the relation of each good to the other goods. Hence Hicks's definition may lead to a counter-intuitive classification when a third good possesses a strong influence. The purpose of this note is not to propose an alternative classification but to characterize the effect on the substitution term of a specified third good. The effect is characterized by a further decomposition of the substitution term: Given a third good, the substitution term will be decomposed into what we call the direct and the indirect substitution terms, the former being free from the effect of the third good and the latter characterizing the effect. This characterization enables one to verify whether a given third good causes two goods to be substitutes (complements) when the two goods would be complements (substitutes) if the effect of the third good were eliminated. Thus when an strange substitution-complement relationship of two goods is found in empirical or theoretical work, it is possible to check if any third good causes the counter-intuitive result.

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*Key Words:* Substitution Effect; Substitute; Complement.



## *I. Introduction*

The classification of two goods as substitutes or complements by the sign of the substitution term defined by Hicks (1939) intimately involves the relation of each good to the other goods, as was emphasized by Samuelson (1974) and Sono (1961) among others. Hence Hicks's definition may lead to a counter-intuitive classification when a third good possesses a strong influence. This defect of Hicks's definition lead Samuelson (1974) to propose an alternative definition for substitutes and complements. The purpose of this note is not to propose an alternative classification but to characterize the effect on the substitution term of a specified third good. This task is important because Hicks's definition is most frequently used in spite of the deficiency.

As discussed below, it is possible to give an intuitive argument how the classification of two goods are affected by a third good. The main goal for our characterization of the effect of a third good is to provide the intuitive argument with precise and quantitative content. The effect is characterized by a further decomposition of the substitution term: Given a third good, the substitution term will be decomposed into what we call the direct and the indirect substitution terms, the former being free from the effect of the third good and the latter characterizing the effect. This characterization enables one to verify whether a given third good causes two goods to be substitutes (complements) when the two goods would be complements (substitutes) if the effect of the third good were eliminated. Thus when an strange substitution-complement relationship of two goods is found in empirical or theoretical work, it is possible to check if any third good causes the counter-intuitive result. Ogaki (1988) applied the concept of the indirect and direct substitution effects to theoretical work in international financial economics.

*An Intuitive Argument*

Samuelson (1974) offered an illuminating example:

...sometimes I like tea and cream....I also sometimes take cream with ny coffee. Before you agree that cream is therefore a complement to both tea and coffee, I should mention that I take much less cream in my cup of coffee than I do in my cup of tea. Therefore, a reduction in the price of coffee may reduce my demand for cream, which is an odd thing to happen between so called complements.

Though Samuelson treats the uncompensated price change here, it is obvious that this example is also applicable to the compensated price change. Thus coffee and cream may be classified as substitutes rather than complements in Hicks's definition.

This example can be explained as follows. Suppose that a compensated price reduction in the price of coffee is experienced by a consumer. Then there exist two kinds of effects which work in opposite directions on the demand for cream. One kind of effect works directly: since the consumer tends to consume coffee and cream together, the demand for cream is increased. The other kind of effect works indirectly via the demand for tea: he now demands less tea since coffee and tea are substitutes, and less consumption of tea leads to less demand for cream. We may call the former the direct substitution effect, and the latter the indirect substitution effect. Though coffee and cream are direct complements, coffee and cream are indirect substitutes with respect to tea as the argument above shows. If the indirect substitution effect is greater than the direct substitution effect in absolute value, coffee and cream are substitutes in Hicks's sense.

In the example, coffee and cream are indirect substitutes because cream is a complement of a substitute of coffee, namely tea. Similar intuitive argument can be employed to show that a substitute of a substitute is an indirect complement that a complement of a complement is an indirect

complement.

*Definition and Properties of the Indirect  
and Direct Substitution Effects*

In order to give the intuitive argument a precise content, we propose a definition of the direct and indirect substitution terms. The definition is given by a further decomposition of the substitution term. Suppose there is a compensated price change in coffee, and consider the change in demand for cream. In order to remove the effect of a third good, say tea, let us consider the change in demand of cream when the consumption of tea is kept constant. We may call the change as the direct substitution effect between coffee and cream with respect to tea.<sup>1</sup> The difference between Hicks's substitution effect and the direct substitution effect may be called the indirect substitution effect. The indirect substitution effect characterizes the effect of the specified third good on the substitution effect.

The usefulness of the decomposition depends on the two properties summarized as follows. Since the direct substitution effect is nothing but the substitution effect under "straight" or "specific commodity" rationing, almost all the properties of both the direct and the indirect substitution effects can be derived from results in the literature of rationing.

First, it is easily seen that the matrices of both the direct and the indirect substitution terms are symmetric and negative semi-definite. That the matrix of the indirect substitution effects is negative semi-definite is a special case of a result proved by Neary and Roberts (1980). Because of the symmetry property, the following terminology is legitimate. If the direct (indirect) substitution effect between two goods are positive, the

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<sup>1</sup>The direct and indirect substitution effects are defined with respect to a specified third good. However, we omit the phrase "with respect to ..." when the third good is clear by context.



goods are called direct (indirect) substitutes; if negative, direct (indirect) complements.

Second, some results of Tobin and Houthakker (1951) can be utilized to show that the indirect substitution term between coffee and cream with respect to tea equals the substitution term between coffee and tea multiplied by the substitution term between tea and cream divided by the own substitution term of tea. This property may be called the sign property of the indirect substitution effect: Since the own substitution term of tea is always negative, the sign of the indirect substitution term between tea and coffee is determined by those of the substitution term between coffee and tea and the substitution term between tea and cream.

It follows from the sign property that a complement of a substitute is an indirect substitute, that a substitute of a substitute is an indirect complement, and that a complement of a complement is an indirect complement. Thus our definition formalizes the intuitive argument given above. The sign property also enables us to calculate the direct and the indirect substitution effects only from the knowledge of substitution terms. Thus whenever an empirical researcher obtains a strange result about substitute-complement relationship of two goods, he can check if some third good or composite good exists which renders the two goods substitutes or complements through the indirect substitution effect. For example, if a researcher finds that coffee and cream are substitutes, he can check whether or not they are direct complements with respect to tea if he knows the relevant elasticities of substitutions.

In the next section, the direct and the indirect substitution terms are defined formally, and simple proofs are given to the two properties, using duality approach to analyze consumer behavior under rationing as in Gorman

(1976) Neary and Roberts (1980) and Deaton (1981). An empirical illustration is given in the last section.

## *II. Definitions and Properties of the Indirect and Direct Substitution Terms*

Let  $x_i^c(p, p_n, u)$  be Hicksian demand function for good  $i$  ( $i=1, \dots, n$ ), and  $\tilde{x}_i^c(\bar{x}_n, p, p_n, u)$  be compensated constrained demand function for good  $i$  ( $i=1, \dots, n-1$ ), when the consumer is forced to consume an amount  $\bar{x}_n$  of  $x_n$ . Here  $p=(p_1, \dots, p_{n-1})'$ .

Let

$$S_{ij}(p, p_n, u) = \partial x_i^c / \partial p_j(p, p_n, u), \quad i=1, \dots, n, \quad j=1, \dots, n,$$

be the substitution term.<sup>2</sup> We define the direct substitution term with respect to the  $n$ th good

$$S_{ij}^d(p, p_n, u) = \partial \tilde{x}_i^c / \partial p_j(x_n^c(p, p_n, u), p, p_n, u), \quad i=1, \dots, n-1, \quad j=1, \dots, n-1,$$

It is to be noted that the partial derivative is evaluated at  $\bar{x}_n = x_n^c(p, p_n, u)$ . Otherwise, it would not be possible to interpret  $S_{ij}^d$  and  $S_{ij}^i$  defined below as a decomposition of  $S_{ij}$ . The indirect substitution term with respect to the  $n$ th good is defined by

$$S_{ij}^i(p, p_n, u) = S_{ij}(p, p_n, u) - S_{ij}^d(p, p_n, u), \quad i=1, \dots, n-1, \quad j=1, \dots, n-1.$$

Let  $S=[S_{ij}]$  be the matrix of substitution terms for goods  $i, j=1, \dots, n-1$ . Similarly let  $S^d=[S_{ij}^d]$  and  $S^i=[S_{ij}^i]$  be the matrix of direct and the indirect substitution terms respectively ( $i, j=1, \dots, n-1$ ).

Let us examine the properties of the direct and indirect substitution

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<sup>2</sup>We will introduce conditions for the partial derivative to exist later. For convenience  $S_{ij}$  is defined as function of  $(p, p_n, u)$ . On substituting the indirect utility function to  $u$ , we can regard  $S_{ij}$  as a function of the prices and income.

terms. For this purpose it is convenient to introduce expenditure functions. When the consumer is unconstrained the expenditure function takes the form

$$m(p, p_n, u) = \inf_{x, x_n} [p'x + p_n x_n; v(x, x_n) \geq u],$$

where  $x = (x_1, \dots, x_{n-1})$ , and  $v(x, x_n)$  is the utility function. When the consumer is constrained an expenditure function may be defined as

$$\tilde{m}(\bar{x}_n, p, p_n, u) = \inf_x [p'x + p_n x_n; v(x, x_n) \geq u].$$

We assume that  $\tilde{m}(\bar{x}_n, p, p_n, u)$  evaluated at  $\bar{x}_n = x_n^c(p, p_n, u)$  is twice continuously differentiable with respect to  $(\bar{x}_n, p, p_n)$  in a nonempty open set  $T$  of  $R^{n+1}$ , and that  $m(p, p_n, u)$  is twice continuously differentiable in  $T$ . This assumption implies, among the other things, that the infima for  $m(p, p_n, u)$  and  $\tilde{m}(x_n^c(p, p_n, u), p, p_n, u)$  are uniquely attained by  $x_i^c(p, p_n, u) = \partial m / \partial p_i$ ,  $i=1, 2, \dots, n$ , and  $\tilde{x}_i^c(p, p_n, u) = \partial \tilde{m} / \partial p_i$ ,  $i=1, 2, \dots, n-1$ , respectively in  $T$ .

*Property 1:* For any  $(p^*, p_n^*, u^*) \in T$ , the direct and indirect substitution matrices,  $S^d$  and  $S^i$ , are symmetric and negative semi-definite.

*Proof:* It is easily seen that  $S^d$  is negative semi-definite because  $S_{ij}^d = \partial^2 \tilde{m} / \partial p_i \partial p_j$ . In order to prove that  $S^i$  is symmetric and negative semi-definite, fix  $(p^*, p_n^*, u^*)$  in  $T$  and define  $g(p) = m(p, p_n^*, u^*) - \tilde{m}(x_n^c(p^*, p_n^*, u^*), p, p_n^*, u^*)$ . Since the minimization subject to more constraints never leads to smaller values of the objective function,  $g(p) \leq 0$  for any  $p$ . On the other hand, by the definition of  $x_n^c$ ,  $g(p^*) = 0$ . Hence  $g(p)$  attains maximum at  $p^*$ . Since  $g(p)$  is twice continuously differentiable at  $p^*$ ,  $d^2 g(p^*) / (dp)^2$  is symmetric and negative semi-definite by the second order necessary condition. Finally we note that  $d^2 m / (dp)^2 (p^*, p_n^*, u^*) - d^2 \tilde{m} / (dp)^2 (x_n^c(p^*, p_n^*, u^*), p^*, p_n^*, u^*) = S^i(p^*, p_n^*, u^*)$ . Q.E.D.

That  $S^1$  is negative semi-definite is just a general Le Chatelier-type result, and is a special case of the result proved by Neary and Roberts (1980, p.34).

Property 2: For any  $(p, p_n, u) \in T$ ,  $S_{ij}^1 = S_{in} S_{nj} / S_{nn}$  for  $1 \leq i \leq n-1$ ,  $1 \leq j \leq n-1$ .<sup>3</sup>

proof: We first prove that

$$(1) \quad \partial \tilde{m} / \partial \bar{x}_n (x_n^c(p, p_n, u), p, p_n, u) = 0$$

for any  $(p, p_n, u) \in T$ . Define  $f(\bar{x}_n) = \tilde{m}(\bar{x}_n, p, p_n, u)$ . By the same logic as in the last proof,  $f(\bar{x}_n)$  attains the minimum at  $\bar{x}_n = x_n^c(p, p_n, u)$ . Hence by the first order necessary condition,  $df/d\bar{x}_n(x_n^c(p, p_n, u)) = \partial \tilde{m} / \partial \bar{x}_n(x_n^c(p, p_n, u), p, p_n, u) = 0$ .

Differentiating (1) with respect to  $p_j$  yields

$$(2) \quad \frac{\partial^2 \tilde{m}}{\partial p_j \partial \bar{x}_n} + \frac{\partial^2 \tilde{m}}{(\partial \bar{x}_n)^2} \frac{\partial x_n^c}{\partial p_j} = 0, \quad j=1, \dots, n.$$

Differentiating  $\partial \tilde{m} / \partial p_n = \bar{x}_n$  with respect to  $\bar{x}_n$ , we obtain  $\partial^2 \tilde{m} / (\partial p_n \partial \bar{x}_n) = 1$ . Substituting this into equation (2) for  $j=n$ , we can conclude that  $\partial x_n^c / \partial p_n = S_{nn}$  is different from zero, and that

$$(3) \quad - \frac{\partial^2 \tilde{m}}{(\partial \bar{x}_n)^2} = \frac{1}{\partial x_n^c / \partial p_n}$$

Substituting (3) into (2) yields

$$(4) \quad \frac{\partial^2 \tilde{m}}{\partial p_j \partial \bar{x}_n} = \frac{\partial x_n^c / \partial p_j}{\partial x_n^c / \partial p_n}.$$

On the other hand,  $m(p, p_n, u) = \tilde{m}(x(p, p_n, u), p, p_n, u)$  by definition.

Differentiating this identity with respect to  $p_j$  and using (1) give

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<sup>3</sup>It will be shown that  $S_{nn}$  is different from zero for any  $(p, p_n, u) \in T$  in the course of the proof.

$$(5) \quad \frac{\partial m}{\partial p_j} = \frac{\partial \tilde{m}}{\partial p_j} + \frac{\partial \tilde{m}}{\partial \bar{x}_n} \frac{\partial x_n^c}{\partial p_j} = \frac{\partial \tilde{m}}{\partial p_j} (x_n^c(p, p_n, u), p, p_n, u), \quad j=1, \dots, n.$$

Differentiating (5) with respect to  $p_j$ , we obtain

$$(6) \quad \frac{\partial^2 m}{\partial p_j \partial p_j} = \frac{\partial^2 \tilde{m}}{\partial p_i \partial p_j} + \frac{\partial^2 \tilde{m}}{\partial p_j \partial x_n} \frac{\partial x_n^c}{\partial p_i}, \quad i=1, \dots, n, \quad j=1, \dots, n.$$

Substituting from (4) into (6) yields

$$(7) \quad \frac{\partial^2 m}{\partial p_j \partial p_j} - \frac{\partial^2 \tilde{m}}{\partial p_i \partial p_j} = \frac{1}{\partial x_n / \partial p_n} \frac{\partial x_n^c}{\partial p_i} \frac{\partial x_n^c}{\partial p_j} \quad i=1, \dots, n, \quad j=1, \dots, n.$$

In particular, when  $1 \leq i \leq n-1$ ,  $1 \leq j \leq n-1$ , (7) reads  $S_{ij}^i = S_{ij}^d - S_{in}^d S_{nj} / S_{nn}$ .  
Q.E.D.

Since  $\partial^2 \tilde{m} / \partial p_j \partial \bar{x}_n = \partial \tilde{x}_j^c / \partial \bar{x}_n$ , (4) is nothing but equation (3.8) of Tobin and Houthakker (1951), and (6) is (5.2) of them. Thus these equations of them could be used to establish the property 2. However our proof is much simpler than that of Tobin and Houthakker which utilizes Jacobi's theorem.<sup>4</sup>

It is worth noting that the property 2 also holds in the elasticity form. Let  $e_{ij} = -S_{ij} p_j / x_i^c$  be the compensated cross-price elasticity of the  $j$ th price on the  $i$ th good. Define  $e_{ij}^d = -S_{ij}^d p_j / x_i^c$ ,  $e_{ij}^i = -S_{ij}^i p_j / x_i^c$  as direct and indirect cross-price elasticities, respectively. Then

$$(8) \quad e_{ij} = e_{ij}^d + e_{ij}^i,$$

and by the property 2, for any  $(p, p_n, u) \in T$ ,

$$(9) \quad e_{ij}^i = e_{in} e_{nj} / e_{nn}.$$

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<sup>4</sup>Pollak (1969, pp.72-73) provided another simple proof for Tobin and Houthakker's equation (3.8).

### III. An Empirical Illustration

In order to illustrate the use of our results, we apply them to point estimates of cross-price elasticities given in Deaton (1974). Deaton estimated  $c_{ij} = w_i e_{ij}$  using maximum likelihood methods for British data from 1900 to 1970, where  $w_i$  is the budget share of the  $i$ th good. We used the budget share  $w_i$  given in Deaton (1974, p.360) to convert estimates of  $c_{ij}$  reported in Table III for the symmetric version of the Rotterdam model into elasticities,  $e_{ij}$ . Once  $e_{ij}$ 's are obtained, indirect and direct cross-elasticities with respect to any third good can be calculated from equations (8) and (9).

Deaton's point estimates indicate that Footwear and clothing (hereafter Clothing for short) and Fuel are complements. This might be counter-intuitive given that one can decrease consumption of fuel by putting on more clothes in winter. In Table 1 indirect and direct cross-price elasticities for clothing and fuel are reported for the alternative choices of the third good. The direct cross-price elasticity with respect to Drink and tobacco (hereafter Drink for short) is positive. Thus Clothing and Fuel are direct substitutes and indirect complements with respect to Drink, and the indirect complementarity dominates the direct substitutability. This is because both Clothing and Drink and Drink and Fuel are good substitutes. Thus the counter-intuitive classification is due to the indirect effect from the third good, Drink.

Table 1 suggests that Clothing and Fuel are indirect complements with respect to some other third goods. The Clothing and Fuel are direct complements, however, when any other good than Drink is taken as the third good. Hence Drink is the only third good that causes Clothing and Fuel to

be complements. In this way, when an strange substitution-complement relationship of two goods is found in empirical work, it is possible to check which third good (if any) causes the counter-intuitive result.

TABLE 1

INDIRECT AND DIRECT CROSS-PRICE ELASTICITIES FOR  
CLOTHING AND FUEL WITH RESPECT TO  
DIFFERENT THIRD GOODS

third goods	Indirect cross-price elasticities	Direct cross-price elasticities
Food	-0.007	-0.028
Housing	0.002	-0.037
Drink and tobacco	-0.055	0.020
Travel and communication	-0.028	-0.007
Entertainment	-0.024	-0.011
Other goods	-0.019	-0.016
Other services	0.013	-0.048

NOTE: Point estimates reported in table III of Deaton (1974) were used to calculate indirect and direct cross-price elasticities for consumption of clothing when the price of fuel changes.

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