Rochester Center for
Economic Research

Credible Borrowing Constraints with Renegotiable Debt

Kahn, James A.

Working Paper No. 175
February 1989

University of
Rochester

CREDIBLE BORROWING CONSTRAINTS WITH RENEGOTIABLE DEBT

by

James A. Kahn
Department of Economics
University of Rochester
Rochester, NY 14627

Working Paper No. 175

February 1989

This is a revision of an earlier paper entitled "Credit Rationing and Credible Borrowing Constraints with Renegotiable Debt."

Abstract

This paper presents a two-period model of sovereign debt in which reputation helps to sustain a higher level of lending than would take place with complete information. Sanctions against defaulters are renegotiation-proof in the sense that second-period lending is optimal given the information revealed by the borrower's actions. Default is a signal of poor future investment opportunities, thereby making credible a lender's threat to restrict credit to a delinquent borrower.

Models of sovereign debt that rely on reputation as the basis of the market's existence have not been very successful. It is difficult to come up with reasonable models that sustain positive levels of indebtedness without the ability of lenders to recover assets or otherwise directly penalize borrowers in the event of a default. One difficulty is making the subsequent refusal to lend to a defaultor both costly enough to the borrower and a credible threat on the part of the lender so as to deter dishonest default. Lenders may threaten to cut off a borrower who defaults, but such a threat is not necessarily credible if in equilibrium there are gains to trade in the next period. This difficulty in committing to a punishment may be particularly severe if there are a large number of competing lenders, since one lender may be able to bind himself to punish a borrower who dishonestly defaults, but it is less plausible to think that he could bind other lenders as well.[1] The basic point is that as soon as a loan is made, repayment is something that in principle can be subject to negotiation.

At the same time, the alternative view that borrowers have enough assets at risk of seizure by lenders to sustain the market has to cope with the fact that measures of the assets at risk are frequently only a small fraction of borrowing countries' debts.[2] For example, according to Bulow and Rogoff (1989c), a number of highly indebted countries have debt/GNP ratios that exceed one while their debt repayments amount to less than 5 percent of GNP. Of course these are bad realizations: The low

---

[1]Lenders under one jurisdiction may be able to do so if such arrangements are legally enforceable within their jurisdiction, but they would be unable to coerce others into joining them. International loans are frequently made by consortiums of lenders that can bind members to certain rules. The credibility problem still affects the consortium as much as it would an individual lender.

[2]The discretionary aspect of bankruptcy or default is not limited to markets with sovereign agents. Borrowers are generally in a much better position to evaluate their own assets than are their creditors or other outsiders. Restrictions such as laws against slavery may also prevent creditors from fully recovering the assets of bankrupt borrowers. Even if in practice there is a legal system or other mechanism to enforce some degree of honesty, the model in this paper can be viewed as referring to the residual of unenforceability that remains.

repayments may represent the realization of an implicit contract that reduces repayments in response to bad realizations of investments, or they may be low because the value of the assets at risk declined along with GNP. Nonetheless, there is little hard evidence that indebted countries' assets at risk are typically great enough to account for the size of their debts.

Bulow and Rogoff (1989b) argue that for reputation to sustain the market some combination of incomplete information and the inability of the borrowing country to enter into alternative contracts that dominate debt must be present, and even then it is not clear that it is possible. Regarding credibility, it seems absolutely necessary that a default (in equilibrium) reveal information about the defaultor if lenders are to alter their lending decisions in response. If lenders' information about a borrower is unaffected by a default (other than the direct effect on the borrower's assets), then the subsequent indebtedness of the borrower (given that it was previously thought to be optimal) cannot be affected by the default.[3] Thus the term "reputation" as used in this paper refers specifically to private information that is revealed by an agent's public actions (see Kreps and Wilson (1982)).

This paper explores a model that takes elements of both the reputation and the Bulow–Rogoff direct retribution approach. It shows that reputation may play a role in a country's willingness to repay its debts provided that with some probability the country will develop to a point where its stake in world markets is sufficient to back its debts. The idea is that the enforcement problem could be more serious for poor

[3]According to this view of credibility, the "review" strategies analagous to those of, for example, Radner (1985), Rubinstein (1979), Rubinstein and Yaari (1983) do not represent credible threats if the punishment takes the form of subsequent refusal to lend. Stiglitz and Weiss (1983) and Haubrich (1986) have multiperiod models of borrowing and lending in which banks punish defaultors. Eaton and Gersovitz (1986) and Grossman and Van Huyck (1988) consider reputation–based sovereign debt. None of these authors addresses the credibility issue, however. Kletzer's (1988) approach is more closely related.

underdeveloped countries than for developed or rapidly developing countries. Thus creditors' ability to impose penalties on defaultors may depend on the borrower's wealth or "state of development". This is based on the (admittedly casual) observation that debt repudiation by a major developed country such as the U.S. or Japan seems to be (for some reason) regarded as an absurdity, but is not regarded as such for poorer countries.

The model of Bulow and Rogoff (1989a) implies that the probability of default depends inversely on the country's stake in international trade, seizable assets abroad, and so forth. In earlier stages of development a country may not have very much to lose from the sanctions imposed as a consequence of default, even relative to its small GNP, yet as it grows its susceptibility to such penalties may grow by more than in proportion. For example, successful growth might lead to an increase in the export/GNP ratio, which would ultimately back the country's debt. Table 1 gives some examples of export/GNP ratios over time for successfully developing countries.

Table 1: Exports and GDP of Successfully Developing Countries*

Country	Exports**		GDP**		100xExports/GDP	
	1960	1975	1960	1975	1960	1975
Taiwan	100	1387	985	3399	10.2	40.8
S. Korea	77	2748	2818	9803	2.7	28.0
Thailand	91	278	683	2041	13.3	13.6
Indonesia	1241	3048	6298	14387	19.7	21.2

*Source: *World Tables* (published for the World Bank)
**In real terms, measured in the domestic currency.

Not surprisingly, exports grow both in absolute and relative terms. But the point of the paper is that the mere possibility of assets that will appear in the future might not suffice to sustain large quantities of lending in the present without some additional reputational mechanism such as is described below.

The credit restrictions that occur in the model take the form of a default penalty that restricts subsequent borrowing. This penalty discourages dishonest defaults, and is shown to be a credible threat. In other words, once a default has occurred, competing lenders all find it in their own selfish interest to carry out the threat. The credibility is perhaps enhanced by the fact that punishments actually do occur in equilibrium; that is, the threats are *not* of the sort that once made never have to be carried out.

The equilibrium features credit rationing as an integral part of the solution, and as a determinant of the growth rates of output and consumption in the developing country. The credit market may also fail to get started in the initial period if the initial wealth of the borrowers is sufficiently small. This suggests that such countries would have to rely on internal finance for growth until they reached the point where they could begin to obtain access to credit markets. Then growth would accelerate, and then finally level off upon obtaining complete access to credit. These arguments are, however, only implicit in the present version of the paper, because there is no asset accumulation in the model.

The focus of the paper is not game–theoretic. The model presented below will be a simple two–period model in which, because of the nature of the enforcement mechanism, at least some borrowers behave "honestly" in the final period. Thus the enforcement problem only arises prior to the last period. Extensions to longer (or infinite) time horizons are left to the imagination of the reader.

1. The Model

This section develops a model of a multi–period loan market with asymmetric information. The model has several distinguishing features. First, borrowers in early periods have the option of simply reneging on their debt obligation, with the only possible retribution by lenders being subsequent refusal to make additional loans.

Second, the notion of equilibrium requires that any such retribution by lenders be renegotiation–proof (in a sense to be made explicit below).

The credit market consists of two types of agents: Competitive, risk–neutral potential lenders with alternative investments that have an expected return of R, and risk–averse or risk–neutral potential borrowers with an array of mutually exclusive investment projects each of which has a marginal return (at zero investment) that exceeds R. Risk–neutrality of the lenders is just a simplifying assumption, justified by the notion that the lender is much wealthier than the borrower, so that the risky debt represents a negligible share of its assets.

There is asymmetric information regarding the choice and quality of the project, and regarding the outcome of the investment. The discretion over the choice of investment projects creates a problem of moral hazard, and induces lenders to reduce the size of their loans to make them incentive–compatible. The asymmetric information with respect to the outcome of the investment is not crucial except to motivate the use of debt rather than equity. The paper does not, however, have a proof of the optimality of the debt contract relative to all possible contracts. Future versions of this paper may address this question.

The presence of moral hazard in the choice of investment project is not strictly necessary for most of the results. There are several reasons for the desirability of beginning with a second–best credit market: First, with risk–averse borrowers the first–best loan size is unrealistically large (larger than the size of the investment) so as to permit complete risk–sharing, while with risk–neutral borrowers the loan size is indeterminate and can be arbitrarily large. Second, in the risk–neutral case the indeterminacy of the loan size allows achievement of the first–best in all periods by the implausible device of making the last period loan sufficiently large (again larger than would be needed to finance the first–best level of investment). Finally, it turns out that the results with second–best lending are more robust to generalizations of the model such as allowing for a continuum of borrower types. The main point is that the results are more realistic with an underlying model of the loan market that is more realistic.[4]

Bulow and Rogoff (1989b) argue that the extent of private information is unlikely to be very significant in practice. There are two justifications for basing a model on the presence of information asymmetries. First, one need not take literally the interpretation of the privately known quantities as productivity or investment quality parameters. There may be private information about future government policies that affect productivity, for example. Second, there is evidence that international capital flows do not serve primarily to share consumption risk across countries, and to the extent they do it is very incomplete. Capital inflows are often associated with high realizations of output, rather than the reverse. Consumption is correlated with output, and investment tends to be correlated with savings.[5] Thus there appears to be substantial variability of investment opportunities across countries, which at least is consistent with the possibility of asymmetric information, especially with the broader view of the term "investment opportunities". The model about to be presented is consistent with these facts in that there is incomplete risk–sharing, and an important determinant of capital flows is the variability of productivity rather than the desire to smooth consumption in the face of variable output realizations. High output realizations may be associated with capital inflows to the extent they are correlated with good investment opportunities.

There are two time periods denoted t=1,2. At the beginning of t=1 borrowers and lenders agree to loans of a particular size, and investments are undertaken. At the end of the period the outcome of the investments are realized, and borrowers acquire private information regarding their opportunities for the next time period.

---

[4]The model of lending with moral hazard is taken from Kahn (1988a).
[5]See Atkeson (1987), Tesar (1988), among others.

Given this, they decide how much (if any) of their loan they wish to repay, they consume what remains, and then come to terms with lenders for any new loans. To keep the model as simple as possible, it is assumed for now that there are just two types of borrowers, denoted g and b (or "good" and "bad"), and that each borrower has a choice of two projects, denoted r and s (risky and safe). For any individual type i borrower the project payoffs y_{ij} (i=g,b, j=r,s) are:

$$(1) \qquad y_{ij} = \begin{cases} \theta_{ij} f(x)/p_j & \text{prob. } p_j \\ 0 & \text{prob. } 1-p_j \end{cases}$$

where x is the amount invested in the project, θ_{ij} is the private information of the borrower, and where $f' > 0$, $f'' < 0$. Thus projects of a given type j have the same risk (as measured by p_j), but possibly different expected returns depending on the type of the borrower.

Borrowers can also invest in a risk-free project (or asset)—either exclusively or along with one of the risky projects—that yields a rate of return R, the same rate that is the opportunity cost of the lenders. The following assumptions make the incentive problems interesting:

$$(A1) \qquad \theta_{bj} < \theta_{gj}, \ \theta_{ir} < \theta_{is} \quad (i=g,b;\ j=r,s),$$

$$(A2) \qquad \theta_{ir}/p_r > \theta_{is}/p_s \quad (i=g,b).$$

Assumption A1 says simply that type g borrowers' projects have a higher expected return than those of type b borrowers, and a type g borrower's type s project has a higher return than his type r project. The first is just definitional, the second leads to the moral hazard problem, i.e. the conflict between borrowers' and lenders' interests. Assumption A2 guarantees that the moral hazard problem will require credit rationing (at least under risk–aversion). It ensures that at the first–best level of lending the borrower will be tempted to substitute the type r project for the type s project.

Borrowers have identical initial endowments w. To ensure that there will be some demand for borrowing, we assume

(A3) $$\theta_{gs}f'(w) > R.$$

Lenders are assumed to be competitive in the sense that they will gladly make a loan of any size that gives them an expected rate of return of R, and they will undercut any other lender's agreement that involves an expected return greater than R. They will, however, take account of borrowers incentives in assessing their expected return on a loan. Loan market transactions are public information (so that the total indebtedness of any borrower is known), but actions by borrowers in making use of the loans (such as the quality and size of the investment) are not.

The following additional assumptions will be maintained unless stated otherwise:

(A4) Each borrower begins each time period with an identical exogenous endowment w. All output in each period is either consumed or paid to lenders.

(A5) Two–period contracts are not feasible.

Assumption A4 is not essential except for analytical tractability. It rules out asset accumulation over time, a phenomenon that is important in some models but not in this one. A5 rules out a kind of "pooling" arrangement whereby all borrowers are given identical two–period loans at the outset. Such an arrangement would not necessarily be preferred to the single–period contracts because of its failure to make use of interim information. One should think of the periods in the model as ten or twenty years, so that one–period loans are already fairly long–term, at least long enough that there might be substantial information in the intervening time. Even if two–period loans were preferable, they would amount to a precommitment to lend again in the second period. If that kind of precommitment were feasible then it would be hard to argue that a commitment *not* to lend again in the event of default would not be feasible.

1.1 Optimal Loan Size with Complete Enforcement

To solve for the equilibrium of the model, it is necessary to start in the last period. As stated in the introduction, there is an enforcement mechanism that induces some borrowers to repay their loans if they are successful in the last period. The idea is that good (and successful) borrowers have more to lose from punitive actions such as interference with trade, seizure of assets abroad, and so forth. Specifically, the assumption is

(A6) By the end of period 2, successful borrowers have assets at risk that just cover the second period loan.

This assumption is made just for the sake of concreteness, and can be weakened substantially without affecting the results in any important way. All that is necessary is that some lending to type g borrowers be possible in the last period. For the moment we will assume *complete* enforcement in the last period, meaning that all successful borrowers, both type g and type b, will repay their loans. This will be relaxed in a subsequent section.

The reason for focusing on the last period in this way is that it turns out that the equilibrium loan in the first period will depend crucially on the value of a "good" loan in the second period to a bad borrower. Such a loan will not in fact take place in equilibrium, but its value determines how much a good borrower can be induced to repay at the end of the first period. This is because the good borrower need only repay enough to signal the quality of his opportunities, and that amount is precisely (or perhaps ϵ more than) the maximum a bad borrower would be willing to repay to obtain the same loan on the same terms. This quantity then feeds into the lenders' decisions regarding the size of first period loans in the obvious way.

We can begin with analysis of the optimal second period loan under the assumption that borrowers' types are public information. This is done with the presumption that borrowers will by then have signaled their types by their prior actions, and we now want to impose *ex post* optimality on subsequent decisions. That is, given that borrowers' types have been signaled, their past behavior—however immoral or dishonest—cannot affect current lending decisions.

Consider a type i borrower. Given A1, the borrower would invest some of the proceeds of a small loan at the "fair" rate $R_L = R/p_s$ in the safe project. This is because the safe project stochastically dominates the risky project, so the borrower would strictly prefer it at a zero and hence at a loan of small size as well. Letting V_{ij} denote expected utility for a type i borrower who obtains a loan of size ℓ and invests in a type j project (j=r or s), we have

$$(4) \qquad V_{ij}(\ell;R_L) = \max_{x \leq w+\ell} p_j u\left[\theta_{ij}f(x)/p_j + (w+\ell-x)R - R_L\ell\right] + (1-p_j)u\left[(w+\ell-x)R\right],$$

where competition among leaders implies $R_L = R/p_s$. Let $\Phi_i(\ell) = V_{is}(\ell) - V_{ir}(\ell)$. It is easy to see that Φ_i is continuous (in fact it is differentiable), and that $\Phi_i(0) > 0$.

Let ℓ^* denote the value of ℓ that maximizes $V_i(\ell)$, and let $\bar{\ell}$ be the solution to the problem

$$(5) \qquad \max_{\ell \geq 0} V_{is}(\ell)$$

subject to

(6) $$\Phi_i(\ell) \geq 0.$$

We will say that "credit rationing" may occur if $\bar{l} < \ell^*$.[6] Condition (6) is an incentive compatibility constraint that requires that the borrower prefer to invest in the good project. Such rationing need not occur, though. The solution with rationing must be compared with the solution to

(7) $$\max_{\ell \geq 0} V_{ir}(\ell),$$

with $R_L = R/p_r$, since with risk–aversion and (6) binding it is possible that investing in the bad project without credit rationing is preferred to investing in the good project with rationing. It is easily verified that the rationed equilibrium will be preferred for θ_{is} sufficiently larger than θ_{ir} and/or if borrowers are not too risk–averse. This will be imposed in what follows.

We have already argued that Φ is continuous and that $\Phi_i(0) > 0$. Hence to derive credit rationing as an equilibrium we must derive conditions under which $\Phi_i(\ell^*) < 0$, and under which the solution to (5)–(6) is preferred to the solution to (7). The assumption of binomial distributions for project outputs simplifies the derivation of ℓ^* under risk–aversion because it allows complete insurance to occur. The optimal investment in the safe project is

(8) $$x^*_{is} = f'^{-1}(R/\theta_{is}).$$

If borrowers are risk–neutral, the first–best loan size is not pinned down, except that it must be at least $x^*_{is} - w$. If they are risk–averse, complete insurance implies that the first–best loan size would be

[6] This type of moral hazard problem with debt is well known. Stiglitz and Weiss (1981) suggest a type of credit rationing as a response, but there analysis only appled to the case of a monopolistic lender and discrete loan sizes.

$$(9) \qquad \ell_i^* = \theta_{is} f(x_{is}^*)/R,$$

which implies that $\ell_i^* > x_{is}^*$ regardless of w. The lender essentially invests in the project and bears all of the risk, while providing the borrower with consumption equal to wR plus a constant related to the expected value of the project's output. While the binomial assumption is crucial for the simplicity of this result, it is not essential for the main qualitative results of the paper.

We can now demonstrate that the loan size given by (9) is not incentive-compatible:

Proposition 1: $\Phi_i(\theta_{is} f(x_{is}^*)/R) < 0$.

Proof: The proof of this proposition works as follows: We can show that it is possible for an agent receiving a loan of ℓ_i^* to invest in the r project and receive a higher payoff regardless of the outcome of the project. Given the solutions in (8) and (9), we get

$$(10) \qquad \Phi_i(\ell_i^*) = u((w+\ell^*-x_{is}^*)R) - p_r u(\theta_{ir} f(\hat{x}_{ir})/p_r - R\ell_i^*/p_s + (w+\ell_i^*-\hat{x}_{ir})R) - (1-p_r)u((w+\ell_i^*-\hat{x}_{ir})R).$$

where $\hat{x}_{ir}$ solves the problem (4) with $\ell = \ell_i^*$, j=r. It suffices, however, to show that there exists a value of $x_{ir} \leq x_{is}^*$ such that

$$(11) \qquad \theta_{ir} f(x_{ir})/p_r - R\ell_i^*/p_s + (w+\ell_i^*-x_{ir})R > (w+\ell^*-x_{is}^*)R.$$

This would guarantee that the borrower would prefer to invest in the r project, since the payoff would be higher than that from investing in the s project regardless of whether the r project succeeded or failed. Using (9) to substitute for ℓ_i^* in (11), we get

$$(12) \qquad \theta_{ir}f(x_{ir})/p_r - \theta_{is}f(x^*_{is})/p_s + (x^*_{is}-x_{ir})R \;>\; 0.$$

Assumption A2 clearly implies that this condition is satisfied for $x_{ir} = x^*_{is}$, which proves that $\Phi_i(\theta_{is}f(x^*_{is})/R)$ is negative. □

This result does not extend to risk–neutrality for all possible first–best loans. Consider the smallest possible first–best loan $\ell^*_i = x^*_{is} - w$. Then we have

$$(13) \qquad \Phi_i(\ell^*_i) \;=\; \theta_{is}f(x^*_{is}) - R(x^*_{is} - w) - \\ [\theta_{ir}f(x^*_{ir}) - p_rR(x^*_{is}-w)/p_s + (x^*_{is}-x^*_{ir})R]$$

Assumption A2 is not sufficient to guarantee that this expression is negative. It can be negative, however, depending on the parameter values. Moreover, the important implication of the proposition is that incentive–compatibility puts an upper bound on the loan size in the second period.

Thus we have shown that for either type of borrower, $\Phi_i(\ell) > 0$ for sufficiently small ℓ and $\Phi_i(\ell) < 0$ for sufficiently large ℓ. It is not hard to show that under weak regularity assumptions $d\Phi_i/d\ell < 0$ for $\ell \in [0,\ell^*]$. Hence the equilibrium loan size $\bar{l}_i$ is then determined from the solution to the equation $\Phi(\bar{l}_i) = 0$.

1.2. The First Period Repayment Decision

We next consider the repayment decision at the end of the previous period. In doing so we do not impose a non–negativity constraint on consumption in the first period. It would be easy and not particularly interesting to reformulate the model to be consistent with such a constraint by having a nonzero payoff in the event the projects fail. This is not done for the sake of reducing the already burdensome quantity of notation.

Borrowers have just learned their types for the next period, either g or b. Define the functional Γ as follows:

$$(14) \qquad \Gamma(i,i') = \max\ \{V_{is}(\ell_{i'}),V_{ir}(\ell_{i'})\} - V_{is}(\ell_i)\ .$$

$\Gamma(i,i')$ is the gain to a borrower of type i from "declaring himself" to be type i'. The first term in (14) takes account of the possibility that in declaring himself to be type i', the type i borrower may be better off investing in the r project than the s project. The incentive-compatibility constraint (6) assures that he will choose the s project if he declares his type honestly.

We start with a conjecture that the equilibrium is characterized by type g borrowers repaying some quantity z, and type b borrowers defaulting. Let $\overline{W}_b$ denote the maximum assets (prior to repayment) that a type b borrower could have at the end of the first period, and let $\underline{W}_g$ denote the minimum that a type g borrower could have. Incentive-compatibility requires the conditions

$$(15) \qquad u(\overline{W}_b) - u(\overline{W}_b - z) \ \geq\ \beta\Gamma(b,g),$$

and

$$(16) \qquad u(\underline{W}_g) - u(\underline{W}_g - z) \ \leq\ -\beta\Gamma(g,b).$$

where β is a discount factor. Condition (15) says that a type b borrower will be willing to pay no more than z in order to get treated like a type g, while (16) says that a type g will be willing to pay at least z to avoid being treated like a type b. $\Gamma(b,g)$ is generally positive because a type b borrower would value getting a larger loan at the same or lower interest rate. Similarly, $\Gamma(g,b)$ is generally negative, i.e. a type g borrower will generally pay something to avoid being treated like a type b.

In fact it should be clear that equation (15) must be binding in the conjectured equilibrium because of the assumption that borrowers have complete discretion over how much to repay. If (15) were not binding, a type g borrower could repay less than z and still distinguish himself from a type b. Therefore the repayment by type g borrowers is determined by condition (15) holding with equality.[7] If this violates condition (16) then the separating mechanism breaks down, and no lending of this sort can take place. Before analyzing these conditions in more detail it will be helpful to consider the initial lending decision conditional on z.

1.3 First Period Lending

The question of whether there can be positive lending in the first period hinges on whether $z > 0$. That this is possible is clear from considering the case in which borrowers are risk–neutral. Then z would certainly be positive becaause a risk–neutral type b borrower would always be willing to pay some positive amount for a loan at a lower than "actuarially fair" rate. This suggests that so long as agents are not too risk–averse or w is sufficiently large there can be positive borrowing.

Suppose for the moment that all agents are type g in the first period, and that the realization of the first–period projects are perfectly correlated with type in the second period: Borrowers who are successful in the first period are type g in the second, while those who are unsuccessful in the first are type b in the second. Let ℓ^1 denote the equilibrium first–period loan, and x^1 the first–period investment. Then we have

$$(17) \qquad \underline{W}_g = \frac{\theta_{gs} f(x^1)}{p_s} + (w+\ell^1 - x^1)R,$$

$$(18) \qquad W_b = (w+\ell^1-x^1)R,$$

where the subscript 1 indicates first period quantities.

The amount that a type b would repay at the end of the first period can be found by substituting (18) into (15) and solving for z. Call this amount z*. If there

---

[7] It is possible that (6) could bind first. This is discussed below.

were slack in the constraint in the second period that assets at risk cover total indebtedness, then z* could represent partial repayment of the first period loan, while the remainder is rolled over and paid in the second period. A6 rules this out by assuming that there are not sufficient assets at risk in the second period to cover more than the second period loan. Consequently the entire loan must be repaid in the first period, so the equilibrium first period loan ℓ^1 is $z^* p_s/R$, the amount that would give the lender an expected return of R. Since this quantity is normally smaller than the quantity ℓ^*_g, incentive–compatibility with regard to project choice should be satisfied.

The following numerical example shows that first period loans can be very small relative to the first–best level, and that it is easy to find examples where the market can not get started.

Example 1: Suppose $u(c) = \frac{c^{1-\alpha}-1}{1-\alpha}$, $f(x) = x^{\gamma}$ (where $\alpha \geq 0$, $0 < \gamma < 1$), $R = 1.01$, $\beta = 0.99$, $\gamma = 0.5$, $a = 0.5$. Table 1 gives some results for various values of w and α, using the parameter values $\theta_{gs} = 3$, $\theta_{gr} = 2$, $\theta_{bs} = 1.5$, $\theta_{br} = 1$, $p_s = 0.9$, $p_r = 0.5$. For these parameters the first–best loan would be 4.411, and the first–best level of expected production would be 4.455.

Table 2: Examples with Complete Enforcement

α	w	ℓ^1	$\bar{\ell}_g$	Ey^1_g	Ey^2_g	Ec^1_g-wR	Ec^2_g-wR
0.25	5	0.681	2.751	4.364	4.423	2.227	2.227
	3	0.584	2.748	4.291	4.408	2.225	2.227
	2	0.408	2.745	4.147	4.394	2.217	2.227
0.5	5	0.573	2.744	4.259	4.391	2.223	2.227
	3	0.440	2.737	4.094	4.360	2.213	2.227
	2	0.302	2.731	3.836	4.332	2.184	2.226

The first–best loan size, output level, and consumption level (less wR) would all be the same in periods 1 and 2, and would be independent of w and α. Thus the market imperfections have noticeable effects on first–period output and consumption, the more so the lower the initial wealth w and/or the higher the risk–aversion coefficient. There is also a considerable reduction in risk–sharing, especially in the first period.

At lower levels of wealth (e.g. w=1) the value of z that satisfies (15) with equality violates (16). Consequently the mechanism described in this paper breaks down and no lending takes place. Whether some other mechanism can arise to take its place depends on whether long–term loans of the sort described in the introduction can be made. While this seems at least possible under the present assumption that all successful borrowers repay their loans in period 2, under the more reasonable assumptions described in the next section it would be difficult to sustain much lending of this sort.

The situation is a little more complicated if the outcome in period 1 is not perfectly correlated with type in period 2, but the model works in basically the same way, with $\overline{W}_b$ and $\underline{W}_g$ switched in equations (17) and (18). The only caveat is that there could be trouble if (16) binds rather than (15) for some borrowers and not others (which is a more likely possibility with $\overline{W}_b > \underline{W}_g$). Then either only successful type g borrowers would repay, or some combination of successful type gs, successful type bs, and unsuccessful type gs would repay, with new "good" loans taking place at rates that take account of the pooling of types. This adds nothing particularly interesting, so we will assume that (15) binds, in which case the results do not change very much from those in the example.

To summarize the results thus far: The level of first period lending is determined completely by the incentive–compatibility constraint (15) on borrowers that will be type b in period 2. This constraint determines the amount that a type b would pay at the end of period 1 for the option of a loan in period 2, and hence is infimum of what a type g would have to pay to distinguish himself from type b. The separating mechanism breaks down if a type g is not willing to pay that much.

1.3.1 Partial Enforcement

A more reasonable assumption about second period repayments is that only good borrowers have an incentive to repay their loans. This is more consistent with the underlying story that successful borrowers reach a point where their assets at risk of seizure approach the level of their indebtedness. This section sketches the modification to the results from Section 1.1 required by the alternative assumption. Somewhat ironically, the effect of reducing the enforceability in the second period could be to *increase* the equilibrium first period loan. This occurs because the default penalty is now no loan at all rather than just a reduced loan.

Suppose borrowers who reveal themselves to be type i at the end of the first period have a probability φ_i of being type g at the end of the second period and repaying their loans. The implicit assumption in the previous section was that $\varphi_i = 1$ for both i=g and i=b. If instead $\varphi_i < 1$ then lending will generally have to be reduced. The reason for this is a little subtler than one might have thought: The loan rate must be higher to allow for the higher default rate, and this in turn makes loans at the previously derived levels not incentive–compatible. Since the borrower with $\varphi_i = 1$ was just indifferent between the r and s projects, the higher borrowing rate will break the tie and induce the borrower to choose the r project. This means that incentive–compatibility with $\varphi_i < 1$ requires a smaller loan.

To take one extreme case, suppose $\varphi_g = 1$ and $\varphi_b = 0$. Then second period loans to good borrowers will be exactly as derived in Section 1.1, while loans to bad borrowers will be zero. This in turn increases the temptation of a bad borrower to mimic a good borrower by repaying his loan, thereby raising z* and the equilibrium first–period loan. The following example gives results under this set of assumptions.

Example 2: Using the same parameters as in Example 1, we get the following results:

Table 3: Examples with Partial Enforcement

α	w	ℓ	l_g	Ey_g^1	Ey_g^2	Ec_g^1–wR	Ec_g^2–wR
0.25	5	0.681	2.751	4.364	4.423	2.227	2.227
	3	0.584	2.748	4.291	4.408	2.225	2.227
	2	0.408	2.745	4.147	4.394	2.217	2.227
0.5	5	0.573	2.744	4.259	4.391	2.223	2.227
	3	0.440	2.737	4.094	4.360	2.213	2.227
	2	0.302	2.731	3.836	4.332	2.184	2.226

The qualitative results are not very different from those in the previous example.

2. The Pooling (No Reputation) Alternative

In circumstances where first period loans are very small, or where the equilibrium in which type g borrowers repay and type b default breaks down (because (16) is violated), it seems reasonable to think that some sort of pooled equilibrium would work as an alternative. The idea would be that loans could be structured so that both bad and good borrowers would repay their loans. The disadvantages of such an arrangement are twofold: First, information from the repayment decision is ignored, leading to a possible misallocation of resources. Second the loans that are made will generally be smaller and on poorer terms (for the type g borrower) than under the signaling mechanism of Section 1. But where the latter is infeasible, or where the gains from the additional information are not that great, some lending of this type may be preferred.

The problem is that there is no incentive to repay first period loans because by assumption a default conveys no information; if it did we would be back in the equilibrium described in the previous sections. Consequently to consider the pooled solution we must drop assumption A5 and allow two–period contracts. Loans are made prior to t=1, and repaid after t=2. In lieu of A5 we assume

(A7) Success in period t is perfectly correlated with type in period t+1 (or at the end of period t).

(A8) Only those borrowers who are type g at the end of the second period repay their loans.

In the notation of the previous section, A8 says that $\varphi_g = 1$ and $\varphi_b = 0$.

Lenders' two–period opportunity cost is R^2. As mentioned earlier, a two–period loan in this model is like a commitment to lend again in the second period with no repayment in the first. The probability that the two–period loan will be repaid is p_s^2 provided the loan is designed so that type g borrowers choose type s projects. The interest rate on the loan commitment must take account of the larger proportion of defaults that will take place. If the commitment is to lend (ℓ_1,ℓ_2) in the two periods, the required repayment z must satisfy

$$p_s^2 z = R^2\ell_1 + R\ell_2.$$

The repayment is also constrained by the country's assets at risk. Thus if, as in assumption A6, the assets are only on the order of $R\bar{\ell}_g/p_s$ (where $\bar{\ell}_g$ comes from the previous section' analysis), i.e. enough to back second period loans in the one–period setup from the previous section, second period loans could not be as large in this arrangement. Moreover, even if the assets are larger, the much higher implicit interest rate would also tend to reduce the size of second period loans because the incentive–compatibility constraint on project choice would bind more severely.[8] Some numerical calculations (not reproduced here) show that under reasonable parameter assumptions lending of the type considered in previous sections will be preferred if it is possible.

These arguments suggest that reputation can indeed facilitate lending on a scale that would otherwise not be possible. If all loans had to be backed by assets at risk, lending could be constrained relative to the case where reputation is allowed for. The reduced lending in the no–reputation scenario arises for two reasons: First, the quantity of future assets at risk constrain the total accumulated indebtedness, and second, the higher interest rate in the second period reduces the amount of lending that is consistent with incentive–compatibility.

3. Conclusions

This paper has presented a model of a loan market in which repayment is non–enforceable in the short run, but where reputation sustains a higher level of lending than could take place solely on the basis of future enforceability. In contrast to other models of multi–period borrowing and lending, the cutoff of defaultors is renegotiation–proof. A borrower's choice to default signals to the market a poor investment opportunity, so there is no incentive to deviate from the "punishment".

Bulow and Rogoff (1989b) have argued that the existence of sovereign debt depends on the ability of lenders to recover some kinds of damages from (or to inflict losses on) borrowers that do not repay. This results in this paper do not contradict their findings, even though the model has all sorts of features such as private information not allowed for in their analysis. What the model does do is reconcile the

[8]Thus in a richer setting that actually modelled the growth of assets at risk, the smaller loans might slow the growth of the assets, thereby further reinforcing the tendency toward smaller loans.

large levels of sovereign debt that are observed to the apparently small levels of damages that could be recovered by lenders in the event of default. It is not clear whether reputation of the sort considered in this paper can by itself sustain sovereign lending permanently, but the paper shows that it can do so temporarily, thereby perhaps accelerating the growth of borrowers' wealth levels to the point that direct recovery of damages can sustain the market.

This paper is related to the literature on reputation with incomplete information (e.g. Kreps and Wilson, 1982a), insofar as the notion of "reputation" is based on revealed private information. But whereas they have incomplete information about the punish*er*, so that a reputation for "toughness" (or irrationality) can be cultivated, this paper has incomplete information regarding the punish*ees*, i.e. the borrowers. Moreover, it is not irrationality (the willingness of some borrowers to behave contrary to their own best interest) that drives the results, but rational decisions about whether to repay a loan given the borrower's private information about his investment opportunities.

Ultimately whether reputation is quantitatively important in the sovereign debt market is an empirical question. Indirect evidence could be found by getting a better sense of debtor countries' assets at risk, including the gains from whatever international trade could be interfered with in the event of a default. Examining extraordinary events such as debt repudiations by revolutionary governments (events that are as close as we can come to "out of equilibrium") might shed some light on this question.

References

Atkeson, A., "International Lending with Moral Hazard and Risk of Repudiation", mimeo, Stanford University, 1987.

Bulow, Jeremy, and Kenneth Rogoff, "A Constant Recontracting Model of Sovereign Debt," *Journal of Political Economy* (forthcoming, 1989a).

Bulow, Jeremy, and Kenneth Rogoff, "Sovereign Debt: Is to Forgive to Forget?" *American Economic Review* (forthcoming, 1989b).

Eaton, J. and Gersovitz, "Debt with Potential Repudiation," *Review of Economic Studies 48* (April 1981), 289–309.

Eaton, J., Gersovitz, and Stiglitz, "The Pure Theory of Country Risk," European Economic Review 30 (June 1986), 481–513.

Grossman, H. and J. Van Huyck, "Sovereign Debt as a Contingent Claim: Excusable Default, Repudiation, and Reputation," *American Economic Review* 78 (1988), 1088–1097.

Haubrich, Joseph, "Financial Intermediation: Delegated Monitoring and Long–Term Relationships," mimeo, University of Pennsylvania, 1986.

Kahn, James, "Financial Intermediaries, Credit Rationing, and the Consequences of Regulatory Forbearance", mimeo, 1988a.

Kahn, James, "Bank Loans, Credit Rationing, and Investment Growth", mimeo, 1988b.

Kletzer, K., "Sovereign Debt Renegotiation under Asymmetric Information," Yale University, Economic Growth Center Discussion Paper No. 555, 1988.

Kreps, David, and Robert Wilson, "Reputation and Imperfect Information," *Journal of Economic Theory 27* (1982), 253–279.

Radner, Roy, "Repeated Principal–Agent Games with Discounting," *Econometrica 53* (September 1985), 1173–1198.

Rubinstein, Ariel, "An Optimal Conviction Policy for Offenses that May have Been Committed by Accident," in *Applied Game Theory*, edited by Brams, et al., W. Germany: Physica–Verlag, 1979.

Rubinstein, Ariel, and Menahem Yaari, "Repeated Insurance Contracts and Moral Hazard," *Journal of Economic Theory 30* (1983), 74–97.

Stiglitz, Joseph E. and Andrew Weiss, "Credit Rationing in Markets with Imperfect Information," *American Economic Review 71* (1981), 393–410.

Stiglitz, Joseph E. and Andrew Weiss, "Incentive Effects of Terminations," *American Economic Review 73* (1983), 912–927.

Tesar, L. "Savings, Investment, and International Capital Flows", Rochester Center for Economic Research W.P. No. 154, 1988.

Rochester Center for Economic Research
University of Rochester
Department of Economics
Rochester, NY 14627

1988–1989 DISCUSSION PAPERS

WP#118 ADVERSE SELECTION, AGGREGATE UNCERTAINTY, AND THE ROLE FOR MUTUAL INSURANCE COMPANIES
by Bruce D. Smith and Michael J. Stutzer, February 1988

WP#119 INTEREST ON RESERVES AND SUNSPOT EQUILIBRIA: FRIEDMAN'S PROPOSAL RECONSIDERED
by Bruce D. Smith, February 1988

WP#120 INTERNATIONAL FINANCIAL INTERMEDIATION AND AGGREGATE FLUCTUATIONS UNDER ALTERNATIVE EXCHANGE RATE REGIMES
by Jeremy Greenwood and Stephen D. Williamson, February 1988

WP#121 FINANCIAL DEREGULATION, MONETARY POLICY, AND CENTRAL BANKING
by Marvin Goodfriend and Robert G. King, February 1988

WP#122 BANK RUNS IN OPEN ECONOMIES AND THE INTERNATIONAL TRANSMISSION OF PANICS
by Peter M. Garber and Vittorio U. Grilli, March 1988

WP#123 CAPITAL ACCUMULATION IN THE THEORY OF LONG RUN GROWTH
by Paul M. Romer, March 1988

WP#124 FINANCIAL INTERMEDIATION AND ENDOGENOUS GROWTH
by Valerie R. Bencivenga and Bruce D. Smith, March 1988

WP#125 UNEMPLOYMENT, THE VARIABILITY OF HOURS, AND THE PERSISTENCE OF "DISTURBANCES": A PRIVATE INFORMATION APPROACH
by Bruce D. Smith, March 1988

WP#126 WHAT CAN BE DONE WITH BAD SCHOOL PERFORMANCE DATA?
by Eric Hanushek and Lori Taylor, March 1988

WP#127 EQUILIBRIUM MARKETING STRATEGIES: IS THERE ADVERTISING, IN TRUTH?
by Ignatius Horstmann and Glenn MacDonald, revised, March 1988

WP#128 REAL EXCHANGE RATE VARIABILITY UNDER PEGGED AND FLOATING NOMINAL EXCHANGE RATE SYSTEMS: AN EQUILIBRIUM THEORY
by Alan C. Stockman, April 1988

WP#129 POST-SAMPLE PREDICTION TESTS FOR GENERALIZED METHOD OF MOMENT ESTIMATORS
by Dennis Hoffman and Adrian Pagan, April 1988

WP#130 GOVERNMENT SPENDING IN A SIMPLE MODEL OF ENDOGENOUS GROWTH
by Robert J. Barro, May 1988

WP#131 FINANCIAL DEVELOPMENT, GROWTH, AND THE DISTRIBUTION OF INCOME
by Jeremy Greenwood and Boyan Jovanovic, May 1988

WP#132 EMPLOYMENT AND HOURS OVER THE BUSINESS CYCLE
by Jang-Ok Cho and Thomas F. Cooley, May 1988

WP#133 A REFINEMENT AND EXTENSION OF THE NO-ENVY CONCEPT
by Dimitrios Diamantaras and William Thomson, May 1988

WP#134 NASH SOLUTION AND UNCERTAIN DISAGREEMENT POINTS
by Youngsub Chun and William Thomson, May 1988

WP#135 NON-PARAMETRIC ESTIMATION AND THE RISK PREMIUM
by Adrian Pagan and Y. Hong, May 1988

WP#136 CHARACTERIZING THE NASH BARGAINING SOLUTION WITHOUT PARETO-OPTIMALITY
by Terje Lensberg and William Thomson, May 1988

WP#137 SOME SIMULATION STUDIES OF NON-PARAMETRIC ESTIMATORS
by Y. Hong and A. Pagan, June 1988

WP#138 SELF-FULFILLING EXPECTATIONS, SPECULATIVE ATTACKS AND CAPITAL CONTROLS
by Harris Dellas and Alan C. Stockman, June 1988

WP#139 APPROXIMATING SUBOPTIMAL DYNAMIC EQUILIBRIA: AN EULER EQUATION APPROACH
by Marianne Baxter, June 1988

WP#140 BUSINESS CYCLES AND THE EXCHANGE RATE SYSTEM: SOME INTERNATIONAL EVIDENCE
by Marianne Baxter and Alan C. Stockman, June 1988

WP#141 RENT SHARING IN AN EQUILIBRIUM MODEL OF MATCHING AND TURNOVER
by Kenneth J. McLaughlin, June 1988

WP#142 CO-MOVEMENTS IN RELATIVE COMMODITY PRICES AND INTERNATIONAL CAPITAL FLOWS: A SIMPLE MODEL
by Ronald W. Jones, July 1988

WP#143 WAGE SENSITIVITY RANKINGS AND TEMPORAL CONVERGENCE
by Ronald W. Jones and Peter Neary, July 1988

WP#144 FOREIGN MONOPOLY AND OPTIMAL TARIFFS FOR THE SMALL OPEN ECONOMY
by Ronald W. Jones and Shumpei Takemori, July 1988

WP#145 THE ROLE OF SERVICES IN PRODUCTION AND INTERNATIONAL TRADE: A THEORETICAL FRAMEWORK
by Ronald W. Jones and Henryk Kierzkowski, July 1988

WP#146 APPRAISING THE OPTIONS FOR INTERNATIONAL TRADE IN SERVICES
by Ronald W. Jones and Frances Ruane, July 1988

WP#147 SIMPLE METHODS OF ESTIMATION AND INFERENCE FOR SYSTEMS CHARACTERIZED BY DETERMINISTIC CHAOS
by Mahmoud El-Gamal, August 1988

WP#148 THE RICARDIAN APPROACH TO BUDGET DEFICITS
by Robert J. Barro, August 1988

WP#149 A MODEL OF NOMINAL CONTRACTS
by Bruce D. Smith, August 1988

WP#150 A BUSINESS CYCLE MODEL WITH PRIVATE INFORMATION
by Bruce D. Smith, August 1988

WP#151 ASYMPTOTIC LIKELIHOOD BASED PREDICTION FUNCTIONS
by Thomas F. Cooley, August 1988

WP#152 MORAL HAZARD, IMPERFECT RISK-SHARING, AND THE BEHAVIOR OF ASSET RETURNS
by James A. Kahn, August 1988

WP#153 SPECIALIZATION, TRANSACTIONS TECHNOLOGIES, AND MONEY GROWTH
by Harold Cole and Alan C. Stockman, August 1988

WP#154 SAVINGS, INVESTMENT AND INTERNATIONAL CAPITAL FLOWS
by Linda L. Tesar, August 1988

WP#155 THE INFLATION TAX IN A REAL BUSINESS CYCLE MODEL
by Thomas F. Cooley and Gary D. Hansen, August 1988

WP#156 RAW MATERIALS, PROCESSING ACTIVITIES AND PROTECTIONISM
by Ronald W. Jones and Barbara J. Spencer, September 1988

WP#157 A TEST OF THE HARRIS ERGODICITY OF STATIONARY DYNAMICAL ECONOMIC MODELS
by Ian Domowitz and Mahmoud El-Gamal, September 1988

WP#158 SYMMETRIC STOCHASTIC GAMES OF RESOURCE EXTRACTION: THE EXISTENCE OF NON-RANDOMIZED STATIONARY EQUILIBRIUM
by Mukul Majumdar and Rangarajan Sundaram, September 1988

WP#159 SOME CONSEQUENCES OF CREDIT RATIONING IN AN ENDOGENOUS GROWTH MODEL
by Valerie Bencivenga and Bruce Smith, September 1988

WP#160 COMPETITIVE DIFFUSION
by Boyan Jovanovic and Glenn MacDonald, September 1988

WP#161 EXCHANGE RATES, THE CURRENT ACCOUNT, AND MONETARY POLICY
by Alan C. Stockman, October 1988

WP#162 DIAGNOSTIC TESTS FOR MODELS BASED ON INDIVIDUAL DATA: A SURVEY
by Adrian Pagan and Frank Vella, October 1988

WP#163 EFFICIENCY BOUND CALCULATIONS FOR A TIME SERIES MODEL WITH CONDITIONAL HETEROSKEDASTICITY
by John Heaton and Masao Ogaki, November 1988

WP#164 TAX ANALYSIS IN A DYNAMIC STOCHASTIC MODEL ON MEASURING HARBERGER TRIANGLES AND OKUN GAPS
by Jeremy Greenwood and Gregory W. Huffman, November 1988

WP#165 THE POSITIVE ECONOMICS OF METHODOLOGY
by James A. Kahn, Steven E. Landsburg, and Alan C. Stockman, November 1988

WP#166 MULTIPLIERS IN EQUILIBRIUM BUSINESS CYCLE MODELS
by Marianne Baxter and Robert G. King, November 1988

WP#167 DYNAMIC REAL TRADE MODELS: NEW DIRECTIONS FOR OPEN ECONOMY MACROECONOMICS
by Marianne Baxter, December 1988

WP#168 WHY IS PRODUCTION MORE VOLATILE THAN SALES? THEORY AND EVIDENCE ON THE STOCKOUT-AVOIDANCE MOTIVE FOR INVENTORY-HOLDING
by James A. Kahn, December 1988

WP#169 BUSINESS CYCLES, STYLIZED FACTS, AND THE EXCHANGE RATE REGIME: EVIDENCE FROM THE UNITED STATES
by Marianne Baxter, December 1988

WP#170 THE INDIRECT AND DIRECT SUBSTITUTION EFFECTS
by Masao Ogaki, December 1988

WP#171 RATIONAL RESPONSE TO UNPRECEDENTED POLICIES: THE 1979 CHANGE IN FEDERAL RESERVE OPERATING PROCEDURES
by Marianne Baxter, January 1989

WP#172 THE TRAGEDY OF THE COMMONS? A CHARACTERIZATION OF STATIONARY PERFECT EQUILIBRIUM IN DYNAMIC GAMES
by Prajit K. Dutta and Rangarajan K. Sundaram, January 1989

WP#173 EXTERNALITIES AND ASYMMETRIC INFORMATION
by Jeremy Greenwood and R. Preston McAfee, January 1989

WP#174 TESTING FOR CONTRACTING EFFECTS ON EMPLOYMENT
by Mark Bils, January 1989

WP#175 CREDIBLE BORROWING CONSTRAINTS WITH RENEGOTIABLE DEBT
by James A. Kahn, February 1989