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by

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ABSTRACT

This paper is an attempt to contribute to the integration of business cycle analysis with long-term growth. A real business cycle model with endogenous growth is developed and estimated with U.S. data. It predicts that output and the real wage follow integrated processes, while the process describing hours of work is stationary. In the present framework wage movements do not have to be transitory to generate fluctuations in labor effort.

The reduced form is a constrained bivariate output/hours (or real wage/hours) VAR process. The bivariate setup provides a useful framework to analyze the persistence of output fluctuations, given that the theory implies that hours of work contain information about future output movements.

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1. Introduction

The analysis of business cycles requires that the researcher come to grips with the presence of long-term growth in macroeconomic time series. The traditional approach has been to assume that growth occurs as the result of exogenous technological progress, which can be captured by a deterministic function of time. A measurement of the business cycle is then the residuals obtained from fitting a deterministic trend to the series.

An alternative approach is to treat long-term growth as endogenous, as in King and Rebelo (1986) and King, Plosser, Stock and Watson (1986). These models follow Lucas (1985) and Romer (1986), in the incorporation of mechanisms generating sustained growth without exogenous technological change. A major implication of this framework is that temporary shocks have permanent effects. Hence, the decomposition of a series into a deterministic trend and a stationary cycle is inconsistent with this theory.

The present paper pursues this line of research, which integrates the analysis of business fluctuations with long-term growth considerations. A real business cycle model with endogenous growth is developed. It predicts that both output and the real wage (in logarithms) are generated by processes with unit roots, while hours of work follow a stationary process. In the present model fluctuations in hours of work reflect the changing relative labor productivity in home and market activities. A consequence of this setup is that real wage fluctuations do not have to be temporary in order to generate fluctuations in labor effort. A related implication is that the secular increase in the real wage is consistent with stationary hours of work, even though there is no income effect at work.

The empirical analysis focuses on two alternative bivariate systems derived from the model: output/hours, and the real wage/hours. The model predicts that both systems follow constrained bivariate vector autoregressions. These processes are estimated with post-World War II U.S. data, and the restrictions are tested. Structural parameters related to labor supply and capital accumulation, as well as the parameters of the joint process of two exogenous shocks affecting this economy, are estimated. One disturbance is a standard shock to the production function, as is usually found in the real business cycle models, and the other is a disturbance to capital accumulation.

The bivariate output/hours setup provides a useful framework to analyze the persistence of output fluctuations, given that the theory implies that hours contain information about future output growth. In this sense the present paper extends the work by Nelson and Plosser (1982), Campbell and Mankiw (1987), Watson (1986), Cochrane (1988) and others, who used a univariate framework. The question of the permanence of output fluctuations can also be posed in terms of how the movements in actual output relate to those in its long-run stochastic trend. The procedure adopted here to obtain a stochastic trend is a multivariate version of the method suggested by Beveridge and Nelson (1981), as discussed in King, Plosser, Stock and Watson (1986).

The paper proceeds as follows. In Section 2 the setup of the model is described along with a discussion of the special features of tastes and technology. The solution to the model is presented in Section 3 and its implications for the co-movement of real wages and labor effort are discussed in Section 4. The estimation is reported in Section 5, and Section 6 addresses the calculation of the stochastic trend in output and the persistence of fluctuations. Section 7 contains concluding remarks.

2. The Setup of the Model

The framework is a stochastic growth model of the type used in Kydland and Prescott (1982), Long and Plosser (1983) and King, Plosser, Stock and Watson (1986). As in the latter two papers the present model has a log-linear structure, so that a closed-form solution can be obtained. Here, however, hours of work are not constant in equilibrium and capital does not fully depreciate in one production period.

The economy is composed of a large and constant number of identical households and identical firms interacting in a competitive environment. The technology and preferences are specified and discussed next. Special attention is given to the aspects in which the present specification diverges from that usually used in real business cycle models.

The representative firm produces output according to the technology

$$(1)\quad Y_t = F(K_t, H_t L_t, z_{1t}) = A_o K_t^{\alpha}(H_t L_t)^{1-\alpha}\exp(z_{1t}), \quad 0 < \alpha < 1,$$

where z_{1t} is a productivity shock, K_t is physical capital in productivity units, H_t is an index of knowledge and L_t is labor input in time units. Hence, the accumulation of human capital has the effect of Harrod-neutral technological progress. The shock z_{1t} follows a stationary process to be specified below.

The capital stock evolves as

$$K_{t+1} = K_t G(I_t/K_t)\exp(z_{2t+1}), \quad G' > 0, \quad G'' < 0,$$

where I_t is the amount of resources devoted both to investment that increases the quantity of capital and to research that improves its quality. Capital accumulation is subject to the stationary disturbance z_{2t}. This is the type of capital evolution equation used by Lucas and Prescott (1971). Unlike the standard linear form (i.e., $K_{t+1} = K_t(1-d) + I_t$) it exhibits decreasing returns, which can be interpreted as reflecting adjustment costs in increasing the volume of capital or diminishing returns in research activities. The specific form adopted for the function G is $(I_t/K_t)^{1-\delta}$, $0 < \delta < 1$, so that the capital evolution equation becomes[1]

$$(2) \quad K_{t+1} = A_1 K_t (I_t/K_t)^{1-\delta} \exp(z_{2t+1}).$$

An alternative interpretation can be given to (2) by rewriting it as $K_{t+1} = A_1 K_t^{\delta} I_t^{1-\delta} \exp(z_{2t+1})$. The parameter δ can be associated then with the relative quality of old capital relative to new investment goods.

The shocks z_{1t} and z_{2t} are assumed to follow the vector autoregressive process $\phi(B)z_t = a_t$ where $z_t = (z_{1t}, z_{2t})'$, a_t is the vector white noise $(a_{1t}, a_{2t})'$ and $\phi(B)$ is a 2×2 matrix polynomial of order p in the backshift operator B.

As in Arrow (1962) and Romer (1986), knowledge is assumed to grow proportionally to and as a by-product of the accumulated investment and research activities in the economy:

$$(3) \quad H_t = \bar{K}_t,$$

where $\bar{K}_t$ is the average capital stock across firms.[2] Thus, the production function of the representative firm--equation (1)--displays increasing returns at the social level, but the model is consistent with competitive equilibrium since each firm takes H_t as given.

The representative household maximizes

$$E_t \sum_{j=0}^{\infty} \beta^j U[C_{t+j}, H_{t+j}(L - G(L_{t+j}))], \qquad 0 < \beta < 1,$$

where C_t is the flow of consumption goods per-household, L is the flow endowment of time, and $H_t(L-G(L_t))$ measures effective leisure time as input in home activities, which in turn produce utility. The function $G(\cdot)$, with $G'(\cdot) > 0$, $G''(\cdot) > 0$, represents a fatigue effect that reduces effective leisure time. Following Ghez and Becker (1975) and Heckman (1976), we incorporate the notion that knowledge increases the productivity of the input of time in home activities. Hence, H_t not only increases market labor productivity, but also the marginal disutility from labor supply. This effect is captured by

$$(4) \quad U = \ln[C_t - H_t L_t^{1+\omega}], \quad \omega > 0.$$

This form implies that the direct positive effect of H_t (which is not a choice variable) on utility is neglected. The corresponding formulation including this effect would have been

$$(5) \quad U = \ln[C_t + H_t(L - L_t^{1+\omega})].$$

The form in (4) was chosen instead of (5) because it allows for a closed-form solution to the model. Given the important role that this type of criterion function plays in the paper, it is in order to elaborate first on the economic implications of (4)-(5), and then to assess how the approximation taken in (4) is likely to affect the results.

An important property of both (4) and (5) is that the marginal rate of substitution between consumption and labor supply is independent of the consumption level. Hence, L_t can be solved independently of the intertemporal optimization over consumption and saving. This, however, would hold not only for the logarithmic function but also for any function applied to the composite $C_t - H_tL_t^{1+\omega}$ or to $C_t + H_t(L - L_t^{1+\omega})$. Defining W_t as the real wage at time t, the condition for optimal labor supply for both (4) and (5) is

$$(6) \quad -\frac{U_\ell(t)}{U_c(t)} = (1+\omega)H_tL_t^\omega = W_t.$$

Labor supply is a stationary variable if H_t and W_t have the same stochastic trend. This holds in the model. The accumulation of human capital will affect not only the real wage, but also the opportunity cost of supplying labor. The important characteristic of (4)-(5) is that there is no income effect at work. Labor supply remains stationary as the real wage increases over time, because of parallel increasing productivity in home activities.

The criterion functions (4)-(5) also have important implications regarding the effect of the real wage on labor effort. A standard utility function, like the Cobb-Douglas, has the important characteristic that only *temporary* movements in the real wage affect labor supply. Permanent movements do not alter labor effort because it generates offsetting substitution and income effects. It can be observed in (6) that, under the present utility function, real wage movements have a positive effect on labor supply regardless of whether they are permanent or transitory. What matters for this response is whether the increased labor productivity driving the real wage affects home productivity (H_t) contemporaneously or with a lag. Labor supplied in the market is, in the present setup, a result of shifting relative productivities at work and at home.

We turn now to the implications of using the form in (4) rather than that in (5). As mentioned above, (4) neglects the direct effect of the externally evolving human capital on utility, reflected in H_tL. As seen above, since this term does not affect the marginal rate of substitution between consumption and leisure, the conditions for optimal labor supply following from (4) and (5) are identical.

The marginal rate of substitution between current and future consumption determines the investment decision. The term H_tL, which is not a choice variable, reduces the marginal utility from current consumption but, at the same time, it also reduces that of future consumption. Hence, the exclusion of H_tL does not systematically bias the investment decision. As a consequence, in a deterministic version of the model the balanced growth path is not affected at all by the exclusion of H_tL. This is shown in Appendix A. The question now is how the investment decision following from (4) approximates the investment decision from (5) in the stochastic framework.

Appendix A describes the numerical simulation of the exact choice under two extreme cases regarding the stochastic nature of the shocks. Our conclusion from those exercises is that the adoption of (4) instead of (5) should have little impact on the investment decision. Hence, the behavior of the variables in the present model seems to approximate closely to that following from a model with the criterion function (5).

Finally, the choices in the economy are constrained by:

(7) $Y_t = C_t + I_t$.

3. Solution of the Model

The interaction of optimizing households and firms in competitive equilibrium is analyzed by solving the following representative agent's dynamic problem

$$V(K_t,\bar{K}_t,\underline{z}_t) = \max_{\{C_t,I_t,L_t\}} \{\ln(C_t - \bar{K}_t L_t^{1+\omega}) + \beta E_t V(K_{t+1},\bar{K}_{t+1},\underline{z}_{t+1})\},$$

where $\underline{z}_t = (z_t, z_{t-1}, \ldots, z_{t-p+1})$,

subject to:

$$C_t = Y_t - I_t,$$

$$Y_t = A_o K_t^{\alpha} (\bar{K}_t)^{1-\alpha} L_t^{1-\alpha} \exp(z_{1t}), \text{ and}$$

$$K_{t+1} = A_1 K_t^{\delta} I_t^{1-\delta} \exp(z_{2t+1}).$$

Additionally, the individual agent takes the evolution of the average level of capital in the economy as given by the process:

$$(2') \quad \bar{K}_{t+1} = A_2 (\bar{K}_t)^{\tau_1} \exp[\textstyle\sum_{j=0}^{p} \tau_{2j} z_{1,t+1-j} + \tau_{3j} z_{2,t+1-j}], \qquad \tau_1 < 1/\beta.$$

Since this process is taken as given, the individual agent does not recognize the effect of his own decisions on $\bar{K}_{t+1}$.

For any arbitrary set of coefficients in (2′) (satisfying the condition $\tau_1 < 1/\beta$ for finiteness of utility) the model can be solved by conjecturing that the value function is of the form

$$V(K_t, \bar{K}_t, \underline{z}_t) = D_o + D_1 \ln K_t + D_2 \ln \bar{K}_t + \textstyle\sum_{j=0}^{p-1} D_j^* z_{t-j},$$

where D_j^* is a 1×2 vector.

The solution is:

$$(8) \quad L_t = A_\ell K_t^{\frac{\alpha}{\alpha+\omega}} (\bar{K}_t)^{\frac{-\alpha}{\alpha+\omega}} \exp(z_{1t})^{\frac{1}{\alpha+\omega}}, \qquad A_\ell = \left[A_o\left(\frac{1-\alpha}{1+\omega}\right)\right]^{\frac{1}{\alpha+\omega}}$$

$$(9)\quad Y_t = A_y K_t^{\frac{\alpha(1+\omega)}{\alpha+\omega}} (\bar{K}_t)^{\frac{\omega(1-\alpha)}{\alpha+\omega}} \exp(z_{1t})^{\frac{1+\omega}{\alpha+\omega}}, \quad A_y = \left[A_o^{1+\omega}(\frac{1-\alpha}{1+\omega})^{1-\alpha}\right]^{\frac{1}{\alpha+\omega}},$$

$$(10)\quad I_t = bY_t, \quad b = \frac{\beta D_1(1-\delta)}{1+\beta D_1(1-\delta)} (\frac{\alpha+\omega}{1+\omega})$$

$$(11)\quad C_t = (1-b)Y_t,$$

$$(12)\quad K_{t+1} = A_k K_t^{\delta+\frac{\alpha(1+\omega)}{\alpha+\omega}(1-\delta)} (\bar{K}_t)^{\frac{\omega(1-\alpha)(1-\delta)}{\alpha+\omega}} \exp(z_{1t})^{\frac{(1+\omega)(1-\delta)}{\alpha+\omega}} \exp(z_{2t+1}),$$

$$A_k = A_1(bA_y)^{1-\delta}.$$

The coefficients D_1 and D_2 are

$$D_1 = \frac{\alpha(1+\omega)/(\alpha+\omega)}{1-\beta[\frac{\alpha+\omega(\delta+\alpha(1-\delta))}{\alpha+\omega}]} > 0.$$

$$D_2 = \frac{\omega(1-\alpha)[1+\beta D_1(1-\delta)]/(\alpha+\omega)}{1-\beta\tau_1} > 0.$$

Since D_1 is positive (and $0 < \alpha < 1$), this implies that $0 < b < 1$. The other coefficients of the value function, which also can be calculated, are not relevant for our purposes. Substituting D_1 into eq. (9) yields

$$(10')\quad I_t = \frac{\alpha\beta(1-\delta)}{1-\delta\beta} Y_t.$$ [3]

As in the similar contexts analyzed by Romer (1986), King and Rebelo (1986) and King, Plosser, Stock and Watson (1986), the model generates an endogenous "engine of growth". This role is taken in the present model by the capital stock process in equation (12). Consider first the case where the externality does not exist and hence $\bar{K}_t$ drops from equation (12). Since the exponent of K_t is less than one, this would imply that the stochastic difference equation in (12) is a stationary process. Therefore, no long-term growth would occur in this economy.

The presence of the externality implies that K_{t+1} depends not only on K_t but also on $\overline{K}_t$. Now, since all firms are identical the equality $\overline{K}_t = K_t$ holds. Then, equation (12) becomes

$$(12') \quad K_{t+1} = A_k K_t \exp(z_{1t})^{\frac{(1+\omega)(1-\delta)}{\alpha+\omega}} \exp(z_{2t+1}).$$

Hence, the disturbances z_{1t} and z_{2t+1} have permanent effects on the capital stock, which accumulates at a stochastic but stationary rate.

Since $\overline{K}_t = K_t$, the process for the average capital stock in (2′) coincides with the process for K_t in (12′). Hence, the arbitrary parameters of (2′) should be identical to the corresponding coefficients in (12′). In particular $\tau_1 = 1$, satisfying the condition $\tau_1 < 1/\beta$.[4]

We turn now to consider the solution to the two variables of main interest: output and hours of work. Since output depends on the capital stock, the nonstationarity of the latter is absorbed by output as well. With $\overline{K}_t = K_t$, equation (9) becomes

$$(9') \quad Y_t = A_y K_t \exp(z_{1t})^{\frac{1+\omega}{\alpha+\omega}},$$

where the "trend" in output is generated by the accumulation of K_t.

When $K_t = \overline{K}_t$ the process for hours of employment in equation (8) becomes:

$$(8') \quad L_t = A_\ell \exp(z_{1t})^{\frac{1}{\alpha+\omega}}.$$

The capital stock drops from the labor equation because of the two opposite forces that it exerts on labor supply. Since knowledge affects proportionally both the market and the non-market marginal productivities of labor, productivity at home rises as much as in the market place.

To establish the link between output and employment fluctuations in an empirically useful form, the solution for output is expressed in terms of its rate of change. From equations (8′), (9′) and (12′) and using lower-case letters for natural logs, it follows that

$$(13)\quad \Delta y_t = \mu + \left(\frac{1+\omega}{\alpha+\omega}\right)(1-\delta B)z_{1t} + z_{2t}, \qquad \mu = \ln A_k$$

$$(14)\quad \ell_t = \ln(A_\ell) + \frac{1}{\alpha+\omega}\, z_{1t}.$$

These two equations form the main structural empirical model to be estimated.

Equation (13) shows that the disturbance to capital accumulation, z_{2t}, has a permanent effect on output, while the effect of the production function disturbance, z_{1t}, is partially reversed next period according to the parameter δ $(0 < \delta < 1)$. Note that the theory rules out the possibility of the existence of a deterministic trend in output. For this to be the case the variance of z_{2t} would have to be zero and, more importantly, the moving-average coefficient δ would have to be one--implying that investment does not contribute at all to future productive capacity.

Finally, equation (14) implies that the movements of hours of employment reflect the productivity shock z_{1t}. Hence, by looking at the ℓ_t series one can identify, in the Box-Jenkins sense, the process describing the production function shock in a univariate setting.

4. The Real Wage and Labor Effort

This model generates fluctuations in labor effort through a different channel than the intertemporal substitution mechanism stressed originally by Lucas and Rapping (1969), and incorporated in the real business cycle models. The latter mechanism has to do with the response of labor supply to the deviations of the real wage from the normal future level. Hence, for example, if the real wage follows a random walk this channel does not operate, and labor effort does not respond to real wage movements. This is not the case in the present model. To illustrate this, the solution of the model can be used to derive the real wage that would support, in a competitive environment, the process for L_t derived from the model.

Equalizing the real wage, W_t, to the marginal productivity of labor at the solution values yields

$$(15)\quad W_t = A_w K_t \exp(z_{1t})^{\frac{\omega}{\alpha+\omega}}, \qquad\qquad A_w = (1-\alpha)A_o A_\ell^{-\alpha}\ .$$

From the process of K_t in (12′), the first difference of the log of the real wage can be expressed as

$$(16)\quad \Delta w_t = \mu + \frac{1}{\alpha+\omega}[\omega + (1-\delta(1+\omega))B]z_{1t} + z_{2t}, \qquad\qquad \mu = \ln A_k.$$

To highlight the implications of the present specification assume for a moment that $\delta(1+\omega) = 1$, which is consistent with the theory, and that z_{1t} and z_{2t} are white noise. In this case the real wage follows a random walk with drift. However, labor effort still co-moves with productivity according to equation (14).

The different implications of the present mechanism, relative to the intertemporal substitution mechanism, is that what matters for labor supply here is the contemporaneous productivity differential in market and home activities, represented by the shock z_{1t}. As market productivity, and hence the real wage, changes with a given realization of z_{1t} labor supply reacts positively. Over time the transitory shock generates the accumulation of capital and knowledge, which cancel each other as to the labor supply decision. The increment in the real wage typically becomes permanent, given the accumulation of capital, but as the shock z_{1t} dissipates so does the labor fluctuation.

Another aspect of the relationship between real wages and hours worked was addressed recently by Christiano and Eichenbaum (1988). They stress the contrast between the positive correlation between real wages and hours, predicted by the existing real business cycle models, and the old observation that the actual correlation is very weak. Christiano and Eichenbaum argue that it is important to incorporate shocks shifting labor supply, in addition to the usual productivity shocks affecting labor demand, for a model to conform with the observed correlations.

Given the present mechanism, by which human capital accumulation can be seen as shifting labor supply, it is interesting to address the model's implications regarding the co-movement of real wages and hours of work. Consider first the innovations in these two variables. From equations (14) and (16) if follows that $(1/(\alpha+\omega))a_{1t}$ and $(\omega/(\alpha+\omega))a_{1t} + a_{2t}$ are the innovations in hours and wages respectively. If a_{1t} and a_{2t} are independent, the correlation between innovations in hours and real wages is positive since $\omega > 0$. This can be interpreted along standard lines: a_{1t} affects market productivity of labor. Hence, its effect can be visualized as a shift of the demand schedule, which has positive effects on hours and the real wage. The component a_{2t} appears only in the real wage innovation. This is so because it affects both home and market productivities in the same way, reducing labor supply and increasing labor demand. This results in higher wages at the same level of hours worked.

The observed negative correlation, however, does not refer to innovations but to detrended of first-differenced real wages and hours. In the context of this model, there is no independent trend, as is usually assumed. Hence, we will consider the theoretical correlation of the first-differenced variables. The relevant expressions are:

$$(16)\quad \Delta w_t = \mu + \frac{\omega}{\alpha+\omega} z_{1t} + \left(\frac{1-\delta(1+\omega)}{\alpha+\omega}\right) z_{1t-1} + z_{2t}$$

$$(14')\quad \ell_t = \frac{1}{\alpha+\omega} z_{1t} - \frac{1}{\alpha+\omega} z_{1t-1}$$

If $\delta(1+\omega) < 1$ holds, the coefficients of z_{1t-1} in the two equations have opposite signs. This implies that, at times, the real wage and hours may be moving in opposite directions. This would tend to reduce the positive correlation generated by the current z_{1t}.

What happens can be interpreted as movements of labor demand and supply schedules. Consider a positive productivity shock z_{1t}. Contemporaneously, labor demand increases, thus generating a positive co-movement of hours and the real wage. Abstracting from subsequent disturbances, as the shock dissipates the demand curve does not return to the initial position because of the accumulation of new capital. This accumulation is larger the smaller is δ (recall the condition $\delta(1+\omega)<1$ above). Additionally, the parallel increase in H_t shifts the supply schedule to the left. The smaller the contraction in labor demand and the larger the contraction in labor supply, the more likely it is that, as the shock dissipates, the co-movement of hours and real wages becomes negative.

5. Estimation

This section presents the derivation of reduced form representations for the model and the testable restrictions it imposes. Then, the empirical results are reported.

The structural form of the model, in (13) and (14), can be written as:

$$(17)\quad x_t = \Omega(B)z_t,$$

where $x_t = [\Delta y_t - \mu,\ \ell_t - \ln A_\ell]'$, $\quad \Omega(B) = \begin{bmatrix} (1+\omega)(1-\delta B) & 1 \\ 1 & 0 \end{bmatrix}$,

and the factor $1/(\alpha+\omega)$ has been absorbed into z_{1t} (since α turns out to be not identifiable).

As mentioned in Section 2, z_t is assumed to follow a vector autoregressive process of order p given by $\phi(B)z_t = a_t$, where $a_t \sim \text{iin}[0,\Sigma]$, $\Sigma = [\sigma_{ij}]$ for i,j=1,2, and $\phi(B)$ is a 2×2 matrix polynomial of order p with $\phi(0) = I$.

A vector autoregressive process is much easier to estimate than a vector ARMA(p,q) with $q > 0$, especially when restrictions are to be imposed. From (17) it appears that x_t follows a vector ARMA(p,1) process if z_t is a VAR(p) process. However, it turns out that x_t has a VAR(p+1) representation as well. This is so because $\det[\Omega(B)] = -1$, so that $\Omega(B)^{-1}$ is also a first order polynomial (rather than being of infinite order as is usually the case) given by:

$$\Omega(B)^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & -(1+\omega)(1-\delta B) \end{bmatrix},$$

so that

(18) $\phi(B)\Omega(B)^{-1}x_t = a_t,$

and hence x_t follows a VAR(p+1) process. This simplifies the estimation work considerably.

However, (18) is not the usual VAR representation since $\Omega(0)^{-1} \neq I$. The standard representation, with the autoregressive operator normalized to be the identity matrix when evaluated at zero, is given by:

$$\Omega(0)\phi(B)\Omega(0)^{-1}\Omega(0)\Omega(B)^{-1}x_t = \Omega(0)a_t,$$

or

(19) $\tilde{\phi}(B)\tilde{\Omega}(B)^{-1}x_t = \tilde{a}_t,$

where

(20) $\tilde{\phi}(B) = \Omega(0)\phi(B)\Omega(0)^{-1},$

$\tilde{\Omega}(B) = \Omega(B)\Omega(0)^{-1}$, and

$\tilde{a}_t \sim \text{iin}[0,\tilde{\Sigma}], \quad \tilde{\Sigma} = \Omega(0)\Sigma\,\Omega(0).$

The polynomial $\tilde{\phi}(B)$ is also of order p with $\tilde{\phi}(0) = I$, $\tilde{\Omega}(B)^{-1}$ is of order 1 with $\tilde{\Omega}(0)^{-1} = I$ and is given by

(21) $\tilde{\Omega}(B)^{-1} = \begin{bmatrix} 1 & \lambda B \\ 0 & 1 \end{bmatrix}, \qquad \lambda = \delta(1+\omega).$

The representation in (19) can be viewed as the reduced form with reduced form parameters: λ, $\tilde{\sum}$, and the 4xp coefficients in $\tilde{\phi}(B)$. The VAR(p+1) process is constrained by the model to be of the form $\tilde{\phi}(B)\tilde{\Omega}(B)^{-1}$ so that three testable restrictions are imposed on the VAR(p+1) operator. That is, if one estimates an unrestricted VAR(p+1) process there would be 4(p+1) free parameters while the restricted VAR(p+1) depends only on 4p+1 free parameters: λ and $\tilde{\phi}(B)$[5].

Unfortunately, the structural parameters in the model are not identifiable without one additional restriction. To see this one can match the structural parameters with their reduced form counterparts: $\tilde{\sum}$ and $\sum$, $\tilde{\phi}(B)$ and $\phi(B)$, but λ gets matched with δ and ω. Thus, one more restriction is required in order to identify all the structural parameters.

A natural, though arbitrary, identifying restriction is to set $\sigma_{12} = 0$ so that the innovations driving z_{1t}, the shocks to the production function, are independent of the innovations driving z_{2t}, the shocks to capital accumulation. Hence, any interaction between z_{1t} and z_{2t} occurs after a lag of at least one period. From (20), it follows that $1+\omega = \tilde{\sigma}_{12}/\tilde{\sigma}_{22} - \sigma_{12}/\sigma_{11}$ so that if $\sigma_{12} = 0$, ω can be calculated as $1+\omega = \tilde{\sigma}_{12}/\tilde{\sigma}_{22}$. Once ω is determined, the definition of λ in (21) implies that $\delta = \lambda/(1+\omega)$. Given ω and δ, one can calculate $\Omega(0)$ and then use (20) to obtain $\phi(B)$ and $\sum$.

The model can also be estimated using the alternative system real wage/hours expressed in equations (14) and (16). This pair of equations can be written as

(22) $s_t = \Psi(B)z_t$

where $s_t = [\Delta w_t - \mu,\ \ell_t - \ln A_\ell]'$,

$$(23)\quad \Psi(B) = \begin{bmatrix} \omega+(1-\delta(1+\omega))B & 1 \\ 1 & 0 \end{bmatrix},$$

and, again, the factor $1/(\alpha+\omega)$ has been absorbed into z_{1t}. Just as before, $\det[\Psi(B)] = -1$ so that s_t has the constrained VAR(p+1) representation:

$$(24)\quad \bar{\phi}(B)\bar{\Psi}(B)^{-1}s_t = \bar{a}_t,$$

where

$$(25)\quad \bar{\phi}(B) = \Psi(0)\phi(B)\Psi(0)^{-1},$$

$$\bar{\Psi}(B) = \Psi(B)\Psi(0)^{-1},$$

$$\bar{a}_t \sim \text{iin}[0,\bar{\Sigma}],\quad \bar{\Sigma} = \Psi(0)\Sigma\ \Psi(0),$$

and

$$\bar{\Psi}(B)^{-1} = \begin{bmatrix} 1 & -\gamma B \\ 0 & 1 \end{bmatrix},\qquad \gamma = 1-\delta(1+\omega).$$

Since the reduced forms for both the output/hours and the real wage/hours models are of the same form, the same computer program can be used to estimate both models. However, the calculation of the structural parameters, assuming again that $\sigma_{12} = 0$, is now $\omega = \bar{\sigma}_{12}/\bar{\sigma}_{22}$, and $\delta = (1+\gamma)/(1+\omega)$. Once ω and δ are calculated, one can obtain $\Psi(0)$ from (23) and then use (25) to calculate $\phi(B)$ and Σ .

The theory does not provide guidance regarding the length of the basic time unit, and hence about the frequency of the observations to use in the estimation. We opted for using alternatively annual and quarterly data as following from two alternative ways of interpreting the model.

The results for the output/hours system are reported first, and then the real wage/hours system is briefly addressed. The use of wage data proxying for the theoretical spot return on labor effort is admittedly problematic. In spite of this, we reestimated the model with real wages as a way to provide additional evidence about the model.

The estimation was carried out with U.S. data. The variable Y_t is measured by real GNP and L_t is total employment times average weekly hours. Since the model is based on a representative agent and constant population, both output and total hours were divided by the working age population (between 16 and 64 years of age). The labor data are from the Current Population Survey, which is a survey of households. The real wage is measured by nominal average hourly earnings in total private nonagricultural establishments divided by the GNP deflator. The quarterly data are seasonally adjusted.[6]

The annual results for the output growth/hours system in equation (19) are reported first. The sample period is 1954-1987. The estimation requires the specification of the order of the polynomial $\tilde{\phi}(B)$, which is equal to the order of $\phi(B)$. We picked p=2 given that, using the Box-Jenkins identification procedure, the univariate annual hours series that is theoretically proportional to z_{1t} appears to be described very well as an AR(2). Hence, to allow for the possibility that z_{1t} and z_{2t} do not interact, at least a second order for $\phi(B)$ is required.

The maximum likelihood estimates are presented in Table 1. The restrictions implied by the model on the reduced form, tested against the alternative of an unrestricted VAR of order 3, produce the likelihood-ratio test statistic of 0.80. The 5% critical value is $\chi^2_{.05}(3) = 7.8$, so that the restrictions can be easily accepted. Previewing similar tests performed with the three other data sets, in none of these cases was it possible to reject the restrictions either. These results are not interpreted as evidence that the model is correct, but, rather, that the restrictions are weak. We prefer to see the model as a tool with which to organize and theoretically interpret the data. Another way of "testing" the model is then to judge whether the interpretation is sensible.

Using the identifying assumption $\sigma_{12} = 0$, the structural parameter values are $\omega = 0.28$ and $\delta = 0.66$. The estimate of ω indicates a high elasticity of labor supply with respect to the real wage. This elasticity corresponds to $1/\omega \simeq 3.6$. In micro-data studies (see for example MaCurdy (1981) and Ashenfelter (1984)) the estimates of a similar elasticity are much smaller. However, as Heckman (1984) points out, most of the variation in total hours comes from movements in the number of workers rather than in hours per worker. Since the macro-data used here captures the total variation in hours, one would expect a higher estimate for that elasticity. See below, however, about the assumption $\sigma_{12} = 0$ underlying the identification of ω. The estimate of δ does not have, to our knowledge, comparable values in the literature. Recall that the capital accumulation equation is $K_{t+1} = A_1 K_t^{\delta} I_t^{1-\delta}$, and hence $1-\delta$ represents the elasticity of the next period capital stock with respect to current investment. A rough comparison, though, can be made with the corresponding elasticity in the standard linear capital evolution equation. In the latter case, the elasticity equals the ratio of gross investment to the usual measure of the capital stock. This ratio is in the order of magnitude of 0.1, much smaller than the $1-\delta$ value of 0.37 obtained here. Hence, the present estimate indicates higher relative importance of current investment for future productive capacity than under the usual capital accounting.

The estimation with quarterly data requires the determination of the order of $\tilde{\phi}(B)$ for this case. Given the thinner choice of p to be made now, we used here the criterion suggested by Hannan (1980) for the determination of the order of an ARMA process.[7] Surprisingly, the best choice turns out to be 3, implying again p=2. Hence, it will be harder for the estimated quarterly system to capture the long swings in hours picked by the annual data.

The results from the quarterly estimation are reported in Table 2. The likelihood-ratio test produces the statistic 2.6 (compared with the 5% 7.8 critical value). However, a problem for the model with these results is that the estimates of ω and δ are -0.68 and 2.5, which sharply contradict the theory. The parameter ω is theoretically constrained to be positive, and δ should be between zero and one. The assumption responsible for those estimates is $\sigma_{12} = 0$. This assumption implies that $\tilde{\sigma}_{11} = (1+\omega)\sigma_{11} + \sigma_{22}$ and $\tilde{\sigma}_{22} = \sigma_{11}$. Note, however, that in Table 2: $\tilde{\sigma}_{11} < \tilde{\sigma}_{22}$. Hence, if $\sigma_{12} = 0$, ω has to be negative. Then, given $\hat{\lambda}$, the estimate $\hat{\delta}$ results in a value higher than one.

The empirical results can be reconciled with theoretically acceptable values for ω and δ if σ_{12} is negative rather than zero. This would allow for ω to be positive, which would result in a value for δ less than 0.80 $(\hat{\lambda})$. We return to this point below.

Results from the estimation of the real wage/hours system in equation (24) are reported in Table 3. The quarterly series on wages is available only from 1964. Again, the restrictions cannot be rejected at the 5% level, although with annual data the statistic is 7.3, close to the critical value of 7.8. The table also reports the estimated structural parameters under $\sigma_{12} = 0$. With quarterly data the estimates are $\hat{\omega} = 0.02$ and $\hat{\delta} = 0.97$, which are barely but within the theoretically acceptable region. The corresponding annual estimates are $\hat{\omega} = 0.15$ and $\hat{\delta} = 1.31$. Here $0 < \hat{\delta} < 1$ is violated. It is interesting to note that if σ_{12} is negative rather than zero, the values for ω will be larger and those for δ smaller. That will shift the quarterly estimates further into the theoretical region, and move the annual estimates into that region.

The results above imply that for the present model to be a reasonable description of all the four data sets used the innovations a_{1t} and a_{2t} should be negatively correlated rather than be independent. However, this would imply that the annual output/hours estimate $\hat{\omega} = 0.28$, which seems low but still theoretically reasonable under $\sigma_{12} = 0$, should be taken only as a lower bound. Hence, the elasticity of labor supply to the real wage emerging from that data set should be lower than the corresponding value of 3.6. Also, $\hat{\delta} = 0.66$ would be considered in this case only as an upper bound.

6. Stochastic Detrending and Long-Term Impulse Responses

The present model provides a new angle to analyze the question of how persistent output fluctuations are, which can also be posed in terms of how the movements in actual output relate to those in its long-run trend. The existing evidence on this issue varies. Nelson and Plosser (1982) and Campbell and Mankiw (1987) found little or no evidence of temporary output fluctuations, while results in Watson (1986) and Cochrane (1988) do indicate that a large part of output movements are reversed later on.

The theoretical structure of this model, within the bivariate setup, can be used to obtain another insight into the decomposition of output into permanent and transitory movements. To see this it is convenient to rewrite equations:

(13) $\Delta y_t = \mu + \frac{1+\omega}{\alpha+\omega}(1 - \delta B)z_{1t} + z_{2t}$, and

(14) $\ell_t = \ln(A_\ell) + \frac{1}{\alpha+\omega} z_{1t}$.

Equation (13) says that output movements are generated by two types of shocks, z_{1t} and z_{2t}. Abstracting from the exogenous stochastic properties of these shocks, the model implies that an output movement generated by z_{1t} is partly temporary, given that $0 < \delta < 1$, and that when it is generated by z_{2t} it is permanent. The theoretical interpretation of these different impacts is that z_{1t} affects the market efficiency of labor, and hence it induces a fluctuation due to temporary labor effort. The reason why only a part of this fluctuation is reversed--z_{1t-1} is multiplied by δ which is less than 1--is because capital accumulates following the shock. By contrast, z_{2t} augments permanently the productivity of labor in both market and home activities, without generating a movement in labor effort. Hence, this type of output movement is permanent.

Given the different implications of the two shocks, the bivariate setup (13)-(14) is particularly useful. Since in (14) hours move proportionally to z_{1t} the hours series makes it possible to identify the output movements which have a temporary component.

To measure the degree of persistence of output movements, the bivariate system was used to compute the long-run impulse responses to the structural innovations. Then, the procedure was followed up by computing a stochastic trend in output. The reported results correspond to the annual data and estimates. It is important to note that these calculations are based on the reduced form (19), which does not depend on the identification of ω and δ.

The long-term impulse response of a unit size innovation corresponds to the sum of coefficients in the possibly infinite moving average representation of the growth rates. To obtain such a representation one can rewrite equation (19) as

(26) $x_t = \psi(B)\tilde{a}_t$,

where $\psi(B) = \tilde{\Omega}(B)\tilde{\phi}(B)^{-1}$ and $\psi(0) = I$. Let $\psi_{ij}(B) = \sum_{k=0}^{\infty}\psi_{ij}^{k}B^{k}$ be the $(i,j)^{th}$ component of $\psi(B)$. Then, for output we have

(27) $\Delta y_t = \psi_{11}(B)\tilde{a}_{1t} + \psi_{12}(B)\tilde{a}_{2t}$,

with $\tilde{a}_{1t} = (1+\omega)a_{1t} + a_{2t}$ and $\tilde{a}_{2t} = a_{1t}$ (from equation (19)). Hence, (27) can be expressed as

$$\Delta y_t = \left[\psi_{11}(B) + \frac{\psi_{12}(B)}{1+\omega}\right](1+\omega)a_{1t} + \psi_{11}(B)a_{2t}.$$

Since $\psi_{11}(0) = 1$ and $\psi_{12}(0) = 0$ (from $\psi(0) = I$), an innovation in output of, say, one percent corresponds either to $(1+\omega)a_{1t}$ or a_{2t} of the same magnitude (or a combination of both). The calculation of the long-term effect of each one of these shocks requires the values of $\psi_{1j}(1) = \sum_{k=0}^{\infty}\psi_{1j}^{k}$, j=1,2. These values--which take into account the total impact of a current innovation on growth rates over the entire infinite future--are calculated using the estimated parameters as $\psi(1) = \tilde{\Omega}(1)\tilde{\phi}(1)^{-1}$.

The long-term effect of the innovation $(1+\omega)a_{1t}$ turns out to be 0.53. For comparison, with a process following a deterministic trend plus stationary fluctuations, the long-term response is zero. In the random walk case the corresponding value is one, since with the long-run level of the variable shifts with the actual level. The value of 0.53 indicates an intermediate case where about half of the output innovation is "undone" over the future. Hence, production function shocks generate output movements that are largely temporary.

The situation is very different with respect to the capital innovation a_{2t}. For this shock the long-term response is 2.4. Hence, the effect of a given innovation on output due to this source builds up over time to more than twice the size of its initial impact.

The procedure followed to compute the detrended output series, which is closely related to the calculation of the long-term impulse responses, is based on a bivariate counterpart of that suggested by Beveridge and Nelson (1981) as discussed in King, Plosser, Stock and Watson (1986).

Detrending is achieved by the decomposition of the level of y_t into a permanent and a cyclical component:

$$y_t = y_t^p + y_t^c,$$

where y_t^p is a random walk (plus drift) representing the stochastic trend and y_t^c is a stationary component. The decomposition is obtained by rewriting equation (27) as

$$(28) \quad y_t = \left[\psi_{11}(1)\frac{\tilde{a}_{1t}}{1-B} + \psi_{12}(1)\frac{\tilde{a}_{2t}}{1-B}\right] + \left[\frac{(\psi_{11}(B) - \psi_{11}(1))}{1-B}\tilde{a}_{1t} + \frac{(\psi_{12}(B) - \psi_{12}(1))}{1-B}\tilde{a}_{2t}\right].$$

The expression in the first brackets is a random walk and is thus defined as the permanent component y_t^p. It is related to the long-run impulse responses because it represents the long-run level of output (except for the deterministic growth).

The expression in the second brackets of (28) is defined as y_t^c. Using the fact that the residuals $\tilde{a}_{1t}$ and $\tilde{a}_{2t}$ can be written in terms of the observable variables, it is shown in Appendix B that y_t^c can be expressed as

$$(29) \quad y_t^c = \Gamma_1(B)\Delta y_t + \Gamma_2(B)\ell_t,$$

where $\Gamma_1(B)$ and $\Gamma_2(B)$ are finite order polynomials whose coefficients can be calculated from the estimated parameters. Since Δy_t and ℓ_t are stationary, equation (29) implies that y_t^c is also stationary. Once y_t^c is calculated from (29), the stochastic trend can be obtained as $y_t^p = y_t - y_t^c$.

This method of calculating the permanent and transitory components differs from the procedure suggested by Beveridge and Nelson. That procedure is to calculate y_t^p by recursively producing forecasts of the future levels of the time series, which approach the trend value as the forecast period increases. Here, given the vector autoregressive form, we can obtain and calculate closed-form expressions for the trend and the transitory component.

Figure 1 depicts the stochastic trend along with the actual values of output. The resulting values for y_t^c and the cyclical movements in total hours, which is ℓ_t itself minus a constant, are plotted in Figures 2 and 3. The cycles in hours exhibit troughs in the years 1958, 1961, 1964, 1971, 1975 and 1982, most of them matching the conventional chronology of business cycles. These fluctuations tend also to be reflected, to some extent, in the cycles of output depicted in Figure 2.

To illustrate the model's interpretation of the detrending and impulse response calculations it is of particular interest to consider the markedly different behavior of detrended output and hours in the 1974-1975 episode. Figure 3 shows that hours turn downwards slightly in 1974, and then they decline sharply in 1975. In contrast, detrended output surprisingly turns upwards in 1974 and only in 1975 it declines, but not nearly as sharply as hours (Figure 2).

The source of the difference lies in the sharp negative innovation to capital accumulation in 1974. Note in Figure 5 that the value of a_{2t} is below -2%, the lowest value in the sample.[8] Given the long-run impulse response of 2.4 to capital accumulation innovations (see above) the long-run value of output--or trend--declines by about 5.5% in 1974 (this is the calculated permanent loss of annual output due to that particular negative shock). Now, for the measurement of the temporary component, what matters is that the trend declines by more than actual output (see Figure 1), producing the recorded increase in detrended output in 1974. Then, the observation of 1975 is dominated by the productivity innovation of -3.8%, also the largest in the sample (see Figure 4) which generates a decline in detrended output. Recall that for this type of disturbance the long-run effect is 0.52, so that the trend responds by less than actual output.

Another interesting episode is 1986-1987. In both years output growth and hours are high. The model translates these movements into large values (2.4% and 3.2%) for the innovation a_{1t}, which has temporary effects.

Informally, the bivariate output/hours detrending procedure amounts to the following. In a given period, if hours move in the same direction and by a similar percentage as output the model takes this as an underlying a_{1t} innovation. Hence, the output change is predicted to be largely transitory. If hours move significantly less than output, as in the period 1962-1964 and 1974, the model interprets it as an underlying a_{2t} innovations which have strong permanent effects. In these cases the trend shifts more than actual output.

7. Concluding Remarks

This paper studied the determination of output, the real wage and hours of work in a real business cycle model with endogenous growth. As in Arrow (1962) and Romer (1986), a characteristic of the model is a positive externality associated with investment activity. Investment is seen as producing the accumulation of knowledge, which in turn increases productivity at the social level. This form of technological progress is thought of as also affecting the productivity in home activities, which in turn produce utility.

The model is estimated in its output/hours form and in its real wage/hours form. Annual and quarterly data were used alternatively. The restrictions imposed by the model were not rejected in anyone of the data sets. These results are not taken as evidence that the model is appropriate, but that the restrictions are weak. We prefer to think of the model as a tool with which to interpret the data. Whether the interpretation is sensible or not is then a way of evaluating the model.

The way the model organizes the data can be summarized as follows. Hours of work depend only on the productivity in market activities relative to that in home activities. This feature has two main consequences. One is that the stationarity of the process of hours per-capita is not due to an income effect, but to parallel long-run technological progress in market and home activities. The other consequence is that real wage movements do not have to be temporary to generate labor fluctuations. The processes of output and the real wage, however, contain unit roots because of the endogenous growth mechanism. Shocks have permanent effects both on output and the real wage through their effects on capital and knowledge.

The model provides a new framework to perform and interpret a bivariate calculation of the persistence of output fluctuations. According to the model, output fluctuations are caused by two types of shocks. One is a disturbance to the production function--which has transitory effects since it alters the relative productivity of labor in market and home activities--and the other is a shock to capital accumulation, which has only permanent effects. Hours of work fluctuate with the first shock and hence it is possible to identify those output movements which contain a large transitory component. Hence, the present bivariate output/hours setup is a useful framework to carry out the decomposition of output into permanent and transitory components.

APPENDIX A

Consider a model identical to that in the text except for having the criterion function in (5)

$$U = \ln[C_t + H_t(L-L_t^{1+\omega})].$$

Given that this model does not have a closed-form solution, as that in the text, a different strategy should be adopted to solve for the equilibrium decision rules.

Define $\tilde{X}_t = X_t/H_t$, so that the model can be transformed into one which possess a deterministic steady state (see e.g. King, Plosser and Rebelo (1988) and Christiano and Eichenbaum (1988)).

In the transformed economy the planner's problem is to maximize

$$\text{(A.1)} \qquad E_t \sum_{j=0}^{\infty} \beta^j \ln(\tilde{C}_{t+j} + L - L_{t+j}^{1+\omega}) + \ln H_{t+j},$$

subject to

$$\text{(A.2)} \qquad \tilde{Y}_t = A_o \tilde{K}_t^{\alpha} L_t^{1-\alpha} \exp(z_{1t})$$

$$\text{(A.3)} \qquad \tilde{K}_{t+1} = A_1 \frac{H_t}{H_{t+1}} \tilde{K}_t^{\delta} \tilde{I}_t^{1-\delta} \exp(z_{2t+1})$$

$$\text{(A.4)} \qquad \tilde{Y}_t = \tilde{C}_t + \tilde{I}_t.$$

The first-order optimality conditions are

$$\text{(A.5)} \qquad (1-\alpha)A_o \tilde{K}_t^{\alpha} \tilde{L}_t^{-\alpha} \exp(z_{1t}) = (1+\omega)L_t^{\omega}$$

$$\text{(A.6)} \qquad \frac{1}{\tilde{Y}_t - \tilde{I}_t + L - L_t^{1+\omega}} = \beta E_t \frac{\alpha \tilde{Y}_{t+1}/\tilde{K}_{t+1} + (\frac{\delta}{1-\delta})\tilde{I}_{t+1}/\tilde{K}_{t+1}}{\tilde{Y}_{t+1} - \tilde{I}_{t+1} + L - L_{t+1}^{1+\omega}} \cdot (1-\delta) \frac{\tilde{K}_{t+1}}{\tilde{I}_t}.$$

From (A.5) and the production function it follows that

$$\text{(A.7)} \qquad L_t = A_\ell \tilde{K}_t^{\frac{\alpha}{\alpha+\omega}} \exp(z_{1t})^{\frac{1}{\alpha+\omega}},$$

$$\text{(A.8)} \qquad \tilde{Y}_t = A_y \tilde{K}_t^{\frac{\alpha(1+\omega)}{\alpha+\omega}} \exp(z_{1t})^{\frac{1+\omega}{\alpha+\omega}},$$

where A_ℓ and A_y are defined in equations (8) and (9).

Since in equilibrium $\bar{K}_t = K_t$, or $\tilde{K}_t = 1$ for all t, we have

$$\text{(A.9)} \qquad \frac{H_{t+1}}{H_t} = \frac{K_{t+1}}{K_t} = A_1 K_t^{\delta-1} I_t^{1-\delta} \exp(z_{2t+1}) = A_1 \tilde{I}_t \exp(z_{2t+1}),$$

$$\text{(A.10)} \qquad L_t = A_\ell \exp(z_{1t})^{\frac{1}{\alpha+\omega}} \text{ (identical to (8') in the text), and}$$

$$\text{(A.11)} \qquad \tilde{Y}_t = A_y \exp(z_{1t})^{\frac{1+\omega}{\alpha+\omega}}, \text{ or } Y_t = A_y K_t \exp(z_{1t})^{\frac{1+\omega}{\alpha+\omega}},$$

which is identical to (9') in this text. Substituting $\tilde{K}_t = \tilde{K}_{t+1} = \tilde{K}_{t+2} = 1$, and (A.9), (A.10), (A.11) in the Euler equation (A.6) yields:

$$\text{(A.12)} \qquad \frac{(\frac{1}{1-\delta})\tilde{I}_t}{(\frac{A_y - A_\ell^{1+\omega}}{A_y})\tilde{Y}_t - \tilde{I}_t + L} = \beta E_t \frac{\alpha \tilde{Y}_{t+1} + (\frac{\delta}{1-\delta})\tilde{I}_{t+1}}{(\frac{A_y - A_\ell^{1+\omega}}{A_y})\tilde{Y}_{t+1} - \tilde{I}_{t+1} + L},$$

where $A_y - A_\ell^{1+\omega} > 0$ (from the definitions of A_y and A_ℓ in (8)-(9)).

If L=0 the model is identical to that in the text, where the solution in (10) was that investment is proportional to output: $\tilde{I}_t = b\tilde{Y}_t$. Postulating this solution here, it can be easily checked that (A.12) holds, implying that $b = \alpha\beta(1-\delta)/(1-\delta\beta)$, as in (10') in the text.

Consider now the deterministic balanced growth case, where $z_{1t} = 0$. Equation (A.12) implies that

$$(\frac{1}{1-\delta})\tilde{I} = \beta[\alpha\, \tilde{Y} + (\frac{\delta}{1-\delta})\tilde{I}],$$

which, again, reduces to

$$\tilde{I} = \frac{\alpha\beta(1-\delta)}{1-\delta\beta}\, \tilde{Y}, \text{ or } I_t = \frac{\alpha\beta(1-\delta)}{1-\delta\beta}\, Y_t.$$

The solution of the original variables Y_t and K_t can now be calculated. From (A.11) it follows that along the deterministic balanced growth path

$Y_t = A_y K_t$

$I_t = bA_y K_t$, and, from the capital solution equation,

$K_{t+1} = A_k K_t$, where A_k is defined in equation (10).

These paths correspond to those in the text (equations (9′), (10′) and (12′)) with the shocks set to zero). Hence, the use of the criterion function (4) instead of (5) does not alter the deterministic growth version of the model.

The assessment of the stochastic behavior requires the numerical simulation of the investment decision using (A.12). In this equation the current $\tilde{I}_t$ decision depends on the state variable z_{1t} (through $\tilde{Y}_t$ in (A.11)) and on other variables only through their predictive power over z_{1t+1}. Hence, the decision about $\tilde{I}_t$ depends on z_{1t} and on the process driving this shock.

The $\tilde{I}_t$ choice was simulated under two extreme cases regarding the z_{1t} process. One is when z_{1t} is i.i.d., and the other is when z_{1t} is a random walk. The model in the text requires stationarity for z_{1t}, so that the random walk should be interpreted only as a limiting case. To perform the simulations the following values were chosen for the parameters: $A_1 = 1$, $\alpha = 0.3$, and the annual data estimates $\hat{\delta} = 0.66$ and $\hat{\omega} = 0.28$. This implies that the mean of $\tilde{Y}$--A_y--is 0.49. The standard deviation of $\tilde{Y}$ was chosen as follows. The s.d. of $\ln L_t$ is $\frac{1}{\alpha+\omega}\sigma_{11}^{1/2}$, which is 0.025 in annual data. Correspondingly, the s.d. of $\ln\tilde{Y}_t$ is $(1+\omega)0.025 = 0.0325$. The value of L was picked as 0.34. The average of $L_t^{1+\omega}$ is $A_\ell^{1+\omega} = 0.26$. $L^{1+\omega}$ has the interpretation of total hours of work and rest.

Case (1): z_{1t} i.i.d.

A grid of 100 equally spaced points for $\tilde{Y}$ with equal probabilities was chosen so that the s.d. of $\tilde{Y}$ is 3.25%. An initial 100-vector guess (zeros) for $\tilde{I}$ was chosen and used on the right hand side of (A.12). With the resulting value on the RHS another choice vector $\tilde{I}$ was calculated from (A.12) on the LHS. This $\tilde{I}$ choice was used in a second iteration obtaining a new $\tilde{I}$ choice and so on, until one-by-one convergence of the elements in $\tilde{I}$.

The results: The average $\tilde{I}/\tilde{Y}$ is 0.26 with a standard deviation of 1.92%. When L=0 (the case in the text) the ratio $\tilde{I}/\tilde{Y}$ is 0.26 exactly.

Case (2): z_{1t} random walk

The same range of $\tilde{Y}$ was used, with equal $\frac{1}{3}$ probabilities of moving to the lower, higher, or staying in the same point. At the end points the probability $\frac{2}{3}$ was assigned to staying , and $\frac{1}{3}$ to the adjacent points. In this case for each point on the current grid for $\tilde{Y}$ there is an expected value on the RHS. Iterations on the $\tilde{I}$ choice were carried out using (A.12) until one-to-one convergence. Different grids sizes were used here: 10, 20, 50, and 100. In all cases the standard deviation of the $\tilde{I}/\tilde{Y}$ ratio around 0.26 is extremely small, less than 1/100 of one percent in the 10-grid case and even less as the grid size increases.

We view these results as follows. The true process for z_{1t} is more complicated than the two forms considered above, and jointly evolving with z_{2t}. However, given that z_{1t} is stationary but displays significant persistence, we conclude that the error in the investment choice caused by the approximate criterion (4) is far smaller that that in the white noise case.

APPENDIX B

In order to prove equation (29), we express equation (19) as

$$\text{(B.1)} \qquad \phi^*(B)x_t = \tilde{a}_t, \quad \phi^*(B) = \tilde{\phi}(B)\tilde{\Omega}(B)^{-1},$$

and rewrite for convenience equation (28):

$$\text{(B.2)} \qquad y_t = [\psi_{11}(1)\frac{\tilde{a}_{1t}}{1-B} + \psi_{12}(1)\frac{\tilde{a}_{2t}}{1-B}]$$

$$+ [\frac{\psi_{11}(B)-\psi_{11}(1)}{1-B}\tilde{a}_{1t} + \frac{\psi_{12}(B)-\psi_{12}(1)}{1-B}\tilde{a}_{2t}],$$

$$= y_t^p + y_t^c,$$

where $\psi_{ij}(B)$ is the $(i,j)^{th}$ component of $\psi(B) = \phi^*(B)^{-1}$. As shown below, ignoring the constant terms will not affect the generality of the argument.

Substituting the left-hand-side of (B.1) for $\tilde{a}_{1t}$ and $\tilde{a}_{2t}$ in (B.2), one obtains the y_t^p as a function of $(1-B)y_t$ and ℓ_t:

$$\text{(B.3)} \qquad y_t^p = \psi_{11}(1)\left[\frac{\phi_{11}^*(B)(1-B)y_t + \phi_{12}^*(B)\ell_t}{1-B}\right]$$

$$+ \psi_{12}(1)\left[\frac{\phi_{22}^*(B)\ell_t + \phi_{21}^*(B)(1-B)y_t}{1-B}\right]$$

$$= [\psi_{11}(1)\phi_{11}^*(B) + \psi_{12}(1)\phi_{21}^*(B)]y_t$$

$$+ \left[\frac{\psi_{12}(1)\phi_{22}^*(B) + \psi_{11}(1)\phi_{12}^*(B)}{1-B}\right]\ell_t = \Gamma_1^*(B)y_t + \frac{\Gamma_2^*(B)}{(1-B)}\ell_t,$$

where $\phi_{ij}^*(B)$ is the $(i,j)^{th}$ component of $\phi^*(B)$. Now, from the fact that $\psi(1)\cdot\phi^*(1) = I$ it follows that

$$\text{(B.4)} \qquad \Gamma_1^*(1) = 1 \qquad \Gamma_2^*(1) = 0,$$

so that $\Gamma_2^*(B) = \tilde{\Gamma}_2(B)(1-B)$, where $\tilde{\Gamma}_2(B)$ is a finite order polynomial. Hence:

$$\text{(B.5)} \qquad y_t^p = \Gamma_1^*(B)y_t + \tilde{\Gamma}_2(B)\ell_t.$$

Now, using the fact that $y_t^c = y_t - y_t^p$ it follows that

$$\text{(B.6)} \qquad y_t^c = (1 - \Gamma_1^*(B))y_t - \tilde{\Gamma}_2(B)\ell_t.$$

Since $\Gamma_1^*(1) = 1$ it follows that $(1-\Gamma_1^*(B)) = \Gamma_1(B)(1-B)$, where $\Gamma_1(B)$ is a finite order polynomial. We thus have:

$$\text{(B.7)} \qquad y_t^c = \Gamma_1(B) \cdot (1-B)y_t + \Gamma_2(B)\ell_t, \qquad \Gamma_2(B) = -\tilde{\Gamma}_2(B),$$

expressing the cyclical component as a finite linear combination of present and past values of $(1-B)y_t$ and ℓ_t. If $E[(1-B)y_t]=\mu\neq 0$, then one can obtain a zero mean y_t^c by substituting $(\Delta y_t - \mu) = \Delta y_t$ into (B.7). This proves (29).

FOOTNOTES

[1]A drawback of (2) is that if I_t is zero, K_{t+1} is zero as well. In any event, the present model will always generate positive investment.

[2]The assumption of proportionality in equation (3) is crucial for generating balanced growth in this model.

[3]Appendix A reports numerical simulations of the model using the exact utility function (5), under two extreme assumptions about z_{1t}: white noise and a random walk. The solutions for L_t and Y_t are identical to those in (8) and (9). The investment choice is not proportional to output, as in (10'). However, the deviations from proportionality seem reasonably small.

[4]Comparing (2') and (10'), it follows that $\tau_{2,1}=(1+\omega)(1-\delta)/(\alpha+\omega)$, $\tau_{3,0}=1$, $A_2=A_k$ and the rest of the coefficients in (2') are equal to zero.

[5]Strictly speaking we have to assume that $\tilde{\phi}(B)$ does not factor into $\phi^*(B)\tilde{\Omega}(B)$. If it did we would have $\tilde{\phi}(B)\tilde{\Omega}(B)^{-1}=\phi^*(B)$, where $\phi^*(B)$ could be any VAR operator, so that no restrictions on the reduced form would be implied.

[6]The only series unavailable in seasonally adjusted form is the average hours. It was adjusted by regressing the log of hours on four seasonal dummies. Adding to the residuals the average coefficient on the four dummies and taking the exponent produced the adjusted series.

[7]The procedure is to find the integer k=p+1 which minimizes $\ln[\det[\hat{\Sigma}_k]] + 8k\ln[\ln[N]]/N$, where N is the number of observations, and $\hat{\Sigma}_k$ is the estimated variance covariance matrix of the innovations from the unconstrained vector autoregressive of order k.

[8]One may think of this measured negative innovation in the capital stock as reflecting the drastic increase in oil prices, which rendered part of the capital stock economically obsolete.

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Table 1

GNP/Hours Data Set*
Annual Observations: 1954-1987

Reduced Form Estimates

$$\hat{\lambda} = \underset{(0.25)}{0.84}$$

$$\hat{\tilde{\phi}}(B) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \underset{(0.28)}{0.37} & \underset{(0.45)}{0.28} \\ \underset{(0.19)}{0.24} & \underset{(0.24)}{0.86} \end{bmatrix} B - \begin{bmatrix} \underset{(0.28)}{0.18} & \underset{(0.33)}{-0.37} \\ \underset{(0.19)}{-0.28} & \underset{(0.20)}{-0.30} \end{bmatrix} B^2$$

$$\hat{\Sigma} = \begin{bmatrix} 0.00042 & \\ 0.00025 & 0.00019 \end{bmatrix}$$

Structural Parameter Estimates

$$\hat{\omega} = \underset{(0.12)}{0.28} \qquad \hat{\delta} = \underset{(0.26)}{0.66}$$

$$\hat{\phi}(B) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1.17 & 0.24 \\ -0.74 & 0.06 \end{bmatrix} B - \begin{bmatrix} -0.66 & -0.28 \\ 0.70 & 0.54 \end{bmatrix} B^2$$

$$\hat{\Sigma} = \begin{bmatrix} 0.00019 & \\ 0.0 & 0.00011 \end{bmatrix}$$

Likelihood-Ratio Test Statistic: 0.8

*Standard errors in parenthesis

Table 2

GNP/Hours Data Set*
Quarterly Observations: 1954:1 - 1987:4

Reduced Form Estimates

$$\hat{\lambda} = \underset{(0.32)}{0.80}$$

$$\hat{\tilde{\phi}}(B) = \begin{bmatrix} 1 & 0 \\ & \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \underset{(0.09)}{0.29} & \underset{(0.32)}{0.80} \\ & \\ \underset{(0.10)}{0.53} & \underset{(0.08)}{0.55} \end{bmatrix} B - \begin{bmatrix} \underset{(0.08)}{0.15} & \underset{(0.15)}{-0.42} \\ & \\ \underset{(0.10)}{0.28} & \underset{(0.25)}{-0.26} \end{bmatrix} B^2$$

$$\hat{\Sigma} = \begin{bmatrix} 0.00009 & \\ & \\ 0.00004 & 0.00012 \end{bmatrix}$$

Structural Parameter Estimates

$$\hat{\omega} = \underset{(0.14)}{-0.68} \qquad \hat{\delta} = \underset{(0.47)}{2.50}$$

$$\hat{\phi}(B) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.72 & 0.53 \\ 0.66 & 0.12 \end{bmatrix} B - \begin{bmatrix} -0.17 & 0.28 \\ -0.31 & 0.06 \end{bmatrix} B^2$$

$$\hat{\Sigma} = \begin{bmatrix} 0.0004 & \\ & \\ 0.0 & 0.0001 \end{bmatrix}$$

Likelihood-Ratio Test Statistic: 2.6

*Standard errors in parenthesis.

Table 3

Real Wage/Hours Data Set*

Annual Observations: 1954-1987

$\hat{\omega} = 0.15$ $\hat{\delta} = 1.31$
(0.12) (0.59)

Likelihood-Ratio Test Statistic: 7.3

Quarterly Observations: 1964:1 - 1987:4

$\hat{\omega} = 0.02$ $\hat{\delta} = 0.97$
(0.05) (0.19)

Likelihood-Ratio Test Statistic: 2.3

Figure 1

GNP PER-CAPITA AND TREND

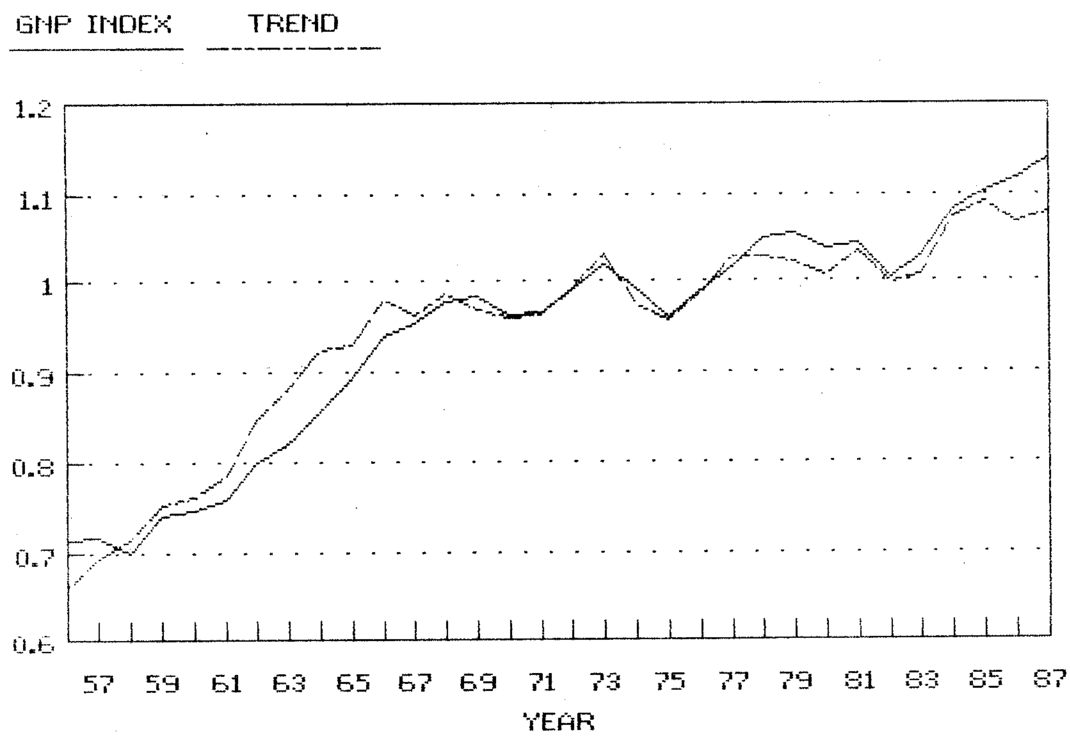

Figure 2

DETRENDED PER-CAPITA GNP

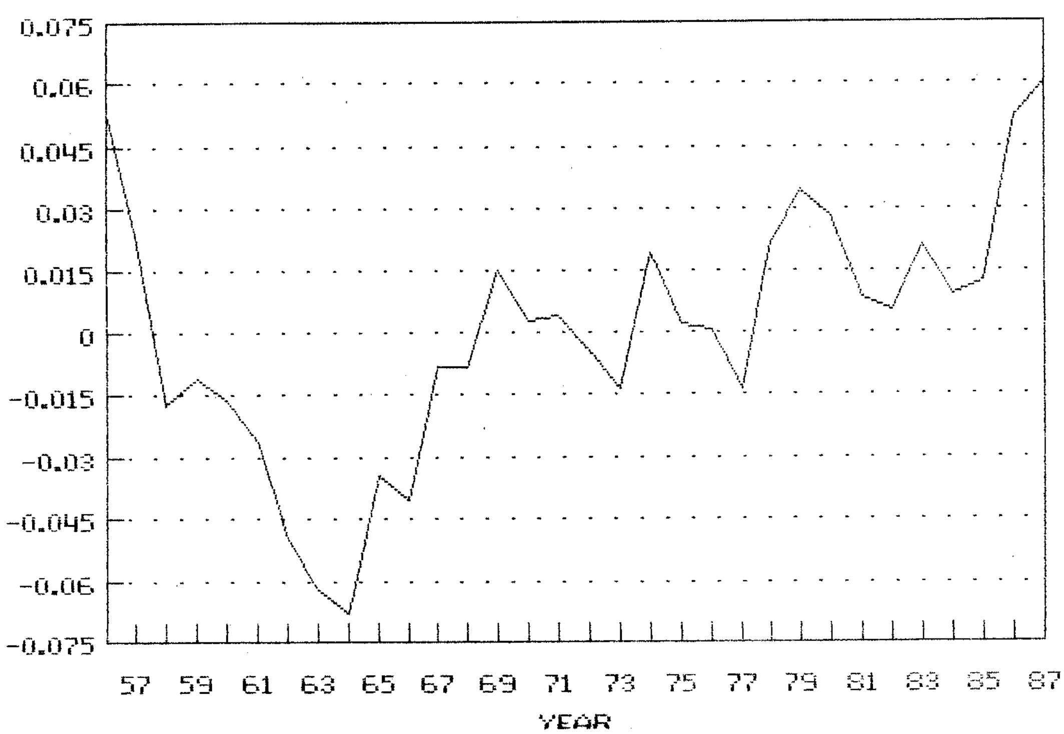

Figure 3

HOURS PER-CAPITA

(Percentage deviations from average)

Figure 4

INNOVATIONS