Full Insurance in the Presence of Aggregate Uncertainty

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1. Introduction

In this paper, I analyze and test the implications of full consumption insurance in the presence of aggregate uncertainty. The object is to determine how much mileage can be obtained from a model with complete markets, omitting such features as private information or liquidity constraints. The goal of this research is not to provide evidence that all markets are perfect, but rather to determine if market imperfections or lack of completeness are essential features in explaining consumption allocations. Hence a complete markets model provides a useful benchmark without requiring researchers to literally accept market perfection.

An insurance scheme may be approximated in the actual economy by various risk–sharing opportunities and markets. For example, some possible sources of insurance include stocks and securities markets, unemployment insurance, contracts between employer and employee, crop insurance for farmers, and insurance among family members or close communities.

The principal implication of risk sharing is that individual consumption varies positively with aggregate consumption. This positive relationship, together with fluctuations in aggregate consumption, determines the volatility of individual consumption. Hence the key aspect of full insurance is that individual consumption responds to aggregate risk but not to idiosyncratic risk. In the current setting, aggregate risk is represented by aggregate consumption and idiosyncratic risk includes changes in individual income and employment status. The implications of risk sharing are emphasized in the early work by Wilson (1968) and in more recent work by Constantinides (1982), Scheinkman (1984), Eichenbaum, Hansen, and Richard (1987), and Townsend (1987). Empirical work related to the current approach includes Leme (1984), Mace (1988), Cochrane (1988), and Townsend (1989).

The current research is closely related to the literature on the permanent income hypothesis (PIH). Unlike the PIH, all changes in idiosyncratic income, both permanent and transitory, are insured in the current risk–sharing model. Hence the current model can be viewed as an extreme version of the PIH. Understanding the benchmark case of full insurance may help us to sort out the large volume of mixed evidence on various forms of the PIH.

The paper is organized as follows. A model of risk sharing is analyzed in section 2. For homothetic preferences, the principal implication of risk sharing is that individual consumption varies positively with aggregate consumption. While an individual's share of aggregate consumption is determined by the individual's wealth relative to aggregate wealth, changes in individual consumption are determined by changes in aggregate consumption.

The testable implications of risk sharing are developed in section 3: in particular, changes in individual consumption are determined by changes in aggregate consumption rather than by changes in individual income or employment status. The data source is described in section 4. I use observations on consumption and income for 10,695 households from the Consumer Expenditure Survey (CES) for 1980–1983. In section 5, the results are reported and various econometric issues are addressed. Many of the results are fully consistent with the risk sharing implications. Once the change in aggregate consumption is accounted for, the change in household income does not help to explain the change in household consumption.

The summary and conclusions are provided in the final section. Overall, many of the results are consistent with full consumption insurance.

2. Theory of Risk Sharing

The major implication of risk sharing is that individual consumption varies positively with aggregate consumption. This positive relationship between individual and aggregate consumption is not a new result in the literature on risk sharing and complete markets. The model analyzed in this section draws heavily from this literature. Of the early risk–sharing literature, studies by Wilson (1968) and Kihlstrom and Pauly (1971) emphasize particular preference specifications and the associated sharing rules. The implications of risk sharing are also emphasized in more recent work by Constantinides (1982), Eichenbaum, Hansen, and Richard (1987), and Townsend (1987).

The current approach to risk sharing is also closely related to the literature pertaining to aggregation properties of preferences. The characterization of risk–sharing outcomes for a class of homothetic preferences stems from the early work on preferences by Gorman (1953). This includes the study of syndicate behavior by Wilson (1968) and of Rubinstein's (1974) composite individual. The aggregation property of preferences in the presence of uncertainty is examined in a static risk–sharing model by Brennan and Kraus (1978) and in a dynamic risk–sharing model by Eichenbaum, Hansen, and Richard (1987).

The risk–sharing problem is cast in the setting of a social planner.[1] The planner maximizes the weighted sum of the expected utilities of individuals subject to an aggregate resource constraint. Optimal resource allocation entails a distribution of the aggregate endowment which equalizes weighted marginal utilities across individuals. The solutions are specialized for two specifications of homothetic preferences: exponential and power utility. For exponential utility, the first differences of consumption, net of preference shocks, are equalized across individuals. The growth rates of consumption are equalized across individuals in the case of power utility.

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[1]The risk–sharing implications developed in this paper also apply to a decentralized economy (e.g., Mace (1988)).

Optimal Resource Allocation

Described below are the general characteristics of the economy, including the information structure, preferences, and endowments. The analysis is limited to the case of a single nondurable good. The risk–sharing implications continue to hold for more general homothetic preferences and for various assumptions on the number, separability, and durability of goods. These results are summarized in the appendix.

(i) Information

Individuals' common information at time t is represented by events $s_{\tau t}$, $\tau = 1, 2, ..., S$, where each event is a collection of states of the world. The number of events S is finite but the number of states may be infinite due to the accumulation of information over time. $\pi(s_{\tau t}) \in [0, 1]$ denotes the probability that event τ occurs at time t with $\sum_{\tau=1}^{S} \pi(s_{\tau t}) = 1$, for all t.

(ii) Preferences

There are J infinitely–lived consumers. Consumer j has preferences for the consumption good, thus expected lifetime utility is expressed

$$\sum_{t=0}^{\infty} \beta^t \sum_{\tau=1}^{S} \pi(s_{\tau t}) \; U[C_t^j(s_{\tau t}), b_t^j(s_{\tau t})] \qquad (2.1)$$

where $C_t^j(s_{\tau t})$ denotes consumption for individual j in event τ at time t, $b_t^j(s_{\tau t})$ a preference shock, and $0<\beta<1$ the discount factor.

(iii) Endowments

Each individual j receives an exogenous endowment of the consumption good.

(2.2) $$y_t^j(s_{\tau t}) = \bar{y}_t^j + \eta_t^j(s_{\tau t}) + \epsilon_t^j(s_{\tau t})$$

where $\bar{y}_t^j$ denotes a deterministic component of output, $\eta_t^j(s_{\tau t})$ represents the aggregate shock to individual j's endowment, and $\epsilon_t^j(s_{\tau t})$ denotes the idiosyncratic shock. Although $\eta_t^j(s_{\tau t})$ is labeled an aggregate shock, its magnitude may differ across individuals. For example, some endowments are more cyclical than others.

Aggregating equation (2.2) over J individuals, the aggregate endowment is given by

(2.3) $$y_t^a(s_{\tau t}) = \bar{y}_t^a + \eta_t^a(s_{\tau t}) .$$

The aggregate variables are simply averages of the individuals and tend to the economy averages as the number of individuals J becomes large. Aggregate uncertainty is represented by $\eta_t^a(s_{\tau t}) \neq 0$ for at least one event, for all t. The idiosyncratic shock is defined such that $\epsilon_t^a(s_{\tau t}) = 0$ for all events and for all t.[2] This property is also an approximation: $\epsilon_t^a(s_{\tau t})$ approaches zero as J tends to infinity.

This particular form of endowments is employed in order to stress the feature of aggregate uncertainty. It is not crucial for the principal implications of risk sharing: more general technologies render the same implications.

The social planner maximizes the weighted sum of the expected utilities of the J individuals, given by the objective function (2.4), by choosing an allocation of consumption across individuals subject to the aggregate resource constraint in equation

[2]A shock in this economy is any output movement away from the deterministic component $\bar{y}_t^j$. The idiosyncratic shock, $\epsilon_t^j(s_{\tau t})$, does not have to be unanticipated. It should not be interpreted as transitory income. The only restriction is that $\epsilon_t^a(s_{\tau t}) = 0$. Depending on the generating processes, there may be components of both $\eta_t^j(s_{\tau t})$ and $\epsilon_t^j(s_{\tau t})$ which are fully anticipated.

(2.5) for each date and event.

$$\sum_{j=1}^{J} \omega^j \sum_{t=0}^{\infty} \beta^t \sum_{\tau=1}^{S} \pi(s_{\tau t}) \; U[C_t^j(s_{\tau t}), b_t^j(s_{\tau t})] \tag{2.4}$$

$$\sum_{j=1}^{J} C_t^j(s_{\tau t}) = \sum_{j=1}^{J} y_t^j(s_{\tau t}) \tag{2.5}$$

with the planner's weights ω^j satisfying $0 < \omega^j < 1$ and $\sum_{j=1}^{J} \omega^j = 1$. (See Negishi (1960).)

The first-order conditions for individual j include

$$\omega^j \beta^t \; \pi(s_{\tau t}) \; U'[C_t^j(s_{\tau t}), b_t^j(s_{\tau t})] = \mu_t \tag{2.6}$$

where μ_t is the Lagrange multiplier associated with the resource constraint (2.5) at time t. With the state notation suppressed for expositional convenience, for any two individuals j and k, the first-order conditions imply that

$$\omega^j \; U'[C_t^j, b_t^j] = \omega^k \; U'[C_t^k, b_t^k] \; . \tag{2.7}$$

The aggregate endowment is distributed across individuals such that weighted marginal utilities are equated across individuals; more precisely, individual consumption varies positively with the aggregate endowment (e.g., Townsend (1987)).

<u>Preference Specifications</u>

Preferences are specialized to a class of homothetic functions. The first specification is exponential utility. The preferences are both time and state separable.

$$U[C_t^j, b_t^j] = -\frac{1}{\sigma}\exp\{-\sigma[C_t^j - b_t^j]\} \tag{2.8}$$

where $\sigma > 0$. Individuals have the same coefficient of constant absolute risk aversion σ.

The first–order conditions in (2.6) are rewritten as

$$\hat{\mu}_t = \omega^j \exp\{-\sigma[C_t^j - b_t^j]\} . \tag{2.9}$$

where $\hat{\mu}_t = \dfrac{\mu_t}{\beta^t \pi_t}$.

Taking the logarithm of equation (2.9) implies

$$\log \hat{\mu}_t = \log \omega^j - \sigma(C_t^j - b_t^j) . \tag{2.10}$$

Aggregating equation (2.10) over J individuals yields

$$\log \hat{\mu}_t = \omega^a - \sigma(C_t^a - b_t^a) \tag{2.11}$$

where $\omega^a = \frac{1}{J}\sum_{j=1}^{J} \log \omega^j$, $C_t^a = \frac{1}{J}\sum_{j=1}^{J} C_t^j$, and $b_t^a = \frac{1}{J}\sum_{j=1}^{J} b_t^j$.

Combining equations (2.10) and (2.11) implies that consumption for individual j is

$$C_t^j = C_t^a + \frac{1}{\sigma}[\log \omega^j - \omega^a] + [b_t^j - b_t^a] \tag{2.12}$$

The major implication of risk–sharing is reflected in equation (2.12): individual consumption varies positively with aggregate consumption which varies by state and over time.

Consumption for individual j is above (below) the economy–wide average of consumption if the sign of [log $\omega^j - \omega^a$] is positive (negative). This term is positive for individuals with above average wealth. Hence there is a positive relationship between the share of aggregate consumption allocated to individual j and the magnitude of individual j's wealth relative to aggregate wealth. Also note that the planner's weight (ω^j) does not depend on time. Therefore, an individual's share of aggregate consumption does not vary over time. However, the level of individual consumption fluctuates over time and across states due to fluctuations in aggregate consumption.

The risk–sharing implication is expressed in terms of changes by taking the first difference of equation (2.12).

$$(2.13) \qquad C^j_{t+1} - C^j_t = C^a_{t+1} - C^a_t + \{[b^j_{t+1} - b^j_t] - [b^a_{t+1} - b^a_t]\}.$$

Consequently, the changes in consumption, net of preference shocks, are equalized across individuals. The individual fixed effect in equation (2.12), [log $\omega^j - \omega^a$], is removed with the first difference.

The risk–sharing implication that individual consumption varies positively with aggregate consumption also holds for an additional preference specification: power utility with multiplicative preference shocks. Again, this is illustrated for the case of a single nondurable good.

$$(2.14) \qquad U[C^j_t, b^j_t] = \exp\{\sigma\, b^j_t\} \frac{1}{\sigma} [C^j_t]^{\sigma} .$$

Strict concavity requires $\sigma < 1$. Individuals have the same coefficient of constant relative risk aversion $(1 - \sigma)$.[3]

The logarithm of consumption for individual j is

$$(2.15) \qquad \log C_t^j = \log C_t^a + \frac{1}{1-\sigma} [\log \omega^j - \omega^a] + \frac{\sigma}{1-\sigma} [b_t^j - b_t^a]$$

where $\omega^a = \frac{1}{J} \sum_{j=1}^{J} \log \omega^j$, $C_t^a = \exp \{\frac{1}{J} \sum_{j=1}^{J} \log C_t^j\}$, and $b_t^a = \frac{1}{J} \sum_{j=1}^{J} b_t^j$.

Taking the first–difference of equation (2.15) yields

$$(2.16) \qquad \log C_{t+1}^j - \log C_t^j = \log C_{t+1}^a - \log C_t^a + \frac{\sigma}{1-\sigma} \{[b_{t+1}^j - b_t^j] - [b_{t+1}^a - b_t^a]\}.$$

where $\log C_{t+1}^a - \log C_t^a = \frac{1}{J} \sum_{j=1}^{J} [\log C_{t+1}^j - \log C_t^j]$.

Hence for power utility there is a positive and linear relationship between the growth rates of individual consumption and the growth rate of aggregate consumption.

In summary, a major implication of risk sharing is that individual consumption varies positively with aggregate consumption. This relationship is further specialized for two classes of homothetic preferences. For exponential preferences, the changes in consumption, net of preference shocks, are equalized across individuals. For a class of power utility functions with multiplicative preference shocks, the growth rates of consumption, net of preference shocks, are equalized across individuals.

3. Empirical Implications

The risk–sharing implications are recast for empirical implementation. The major

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[3]Some special cases of power utility include logarithmic and Cobb–Douglas. See the appendix for more details.

implication is that changes in individual consumption are determined by changes in aggregate consumption. Empirical specifications for individual observations are developed for both exponential and power utility.

For exponential preferences, there is a direct relationship between the change in individual j's consumption and the change in aggregate consumption. (Refer to equation (2.13).)

$$\Delta C_t^j = \Delta C_t^a + [\Delta b_t^j - \Delta b_t^a] \tag{3.1}$$

where $\Delta C_t^j = C_t^j - C_{t-1}^j$, $\Delta C_t^a = \frac{1}{J}\sum_{j=1}^{J} \Delta C_t^j$, and $\Delta b_t^a = \frac{1}{J}\sum_{j=1}^{J} \Delta b_t^j$.

Equation (3.1) presents a nontrivial approach for testing the full insurance implications: regress the changes in individual consumption onto the change in aggregate consumption and other right–hand side variables such as changes in individual income and changes in employment status. All variables other than the aggregate consumption variable are predicted to enter insignificantly. This reflects the key feature of risk sharing: individual consumption responds to aggregate risk but not to idiosyncratic risk. Formally, the empirical specification is

$$\Delta C_t^j = \beta_1 \Delta C_t^a + \beta_2 \Delta y_t^j + u_t^j \tag{3.2}$$

where Δy_t^j is the change in individual j's income. The disturbance term u_t^j includes the time–varying component of both individual and aggregate preference shocks and might also include measurement errors from the consumption and income data. The predictions of the risk–sharing model are $\beta_1 = 1$ and $\beta_2 = 0$. The model also predicts a zero coefficient for other right–hand side variables such as change in employment status.

For econometric reasons, the implication for differenced consumption of equation (2.13) is exploited in the empirical work rather than the implication for level consumption from equation (2.12). Remember that equation (2.13) is the first difference of equation (2.12). Individual j's additive fixed effect, $[\log \omega^j - \omega^a]$ in equation (2.12), is removed by first differencing. Using the first–difference specification avoids problems from an omitted–variables bias when the fixed effect is not observed by the econometrician.

In the previous section, risk–sharing implications are also derived for a class of power utility functions. For these preferences, there is a direct relationship between the growth rates of individual consumption and the growth rate of aggregate consumption. (Refer to equation (2.16).)

(3.3) $$\Delta\log C_t^j = \Delta\log C_t^a + \frac{\sigma}{1-\sigma}[\Delta b_t^j - \Delta b_t^a] \quad .$$

where $$\Delta\log C_t^j = \log C_t^j - \log C_{t-1}^j, \ \Delta\log C_t^a = \frac{1}{J}\sum_{j=1}^{J}\Delta\log C_t^j ,$$

and $$\Delta b_t^a = \frac{1}{J}\sum_{j=1}^{J}\Delta b_t^j.$$

The empirical specification for power utility is

(3.4) $$\Delta\log C_t^j = \beta_1\Delta\log C_t^a + \beta_2\Delta\log y_t^j + v_t^j$$

where $\Delta\log y_t^j$ is the growth rate of individual j's income. As before, the disturbance term v_t^j includes the time–varying components of individual and aggregate preference shocks and might also include measurement errors from the data. The risk–sharing model predicts that $\beta_1 = 1$ and $\beta_2 = 0$. The consumption growth rate implication of equation (2.16) is implemented rather than the implication for logarithmic consumption from equation (2.15). As with the previous specification for exponential preferences, the individual fixed effects are removed when the observations are first–differenced.

In summary, the implications of risk sharing are recast for empirical implementation. Fluctuations in individual consumption are determined by fluctuations in aggregate consumption rather than by flucutations in own income. Two empirical specifications are presented. For exponential preferences, the change in individual consumption is determined by the change in aggregate consumption rather than by the change in individual income. For a class of power utility functions, the growth rate of individual consumption is determined by the economy–wide average of consumption growth rates rather than by the growth rate of individual income.

4. Data

Equations (3.2) and (3.4) are the specifications tested in section 5: one involves growth rates of consumption and income and the other involves first differences of the levels. The data requirements for these tests are observations on consumption and income at the individual or household level. Also required is a minimum of two observations for each individual or household for computing first differences and growth rates. Hence cross–sectional data are not suitable.

The implications are tested using data from the Consumer Expenditure Survey (CES) for 1980–1983. The CES data satisfy the two criteria of consumption and income observations at the household level and there are at least two data points for each household. The surveys are sponsored by the U.S. Bureau of Labor Statistics (BLS) and administered by the U.S. Bureau of the Census. The survey provides unusually rich consumption data, as well as data on income, assets and liabilities, employment, and numerous demographic characteristics.

The CES data contain a panel aspect. The data are actually a collection of overlapping panels of one year duration with a quarterly sampling frequency. Aside from attrition, data are available for each household from four consecutive quarterly interviews. The one year panels overlap due to the rotating nature of the sample: 20 percent of the sample is replaced by new households each quarter. The annual sample size varies between 4,800 and 5,000 households for 1980–1983.

The panel aspect of the CES data is suitable for testing the implications in differenced form in equations (3.2) and (3.4). Pooling the differenced household observations exploits both the cross–sectional and time series aspects of these specifications. Note that the aggregate consumption variable on the right–hand side does not vary across individuals at each point in time. However, there is individual variation in the other right–hand side variable, individual income.

Household expenditures are reported on a monthly basis in all interviews. Approximately 500 expenditure categories are available on each quarterly questionnaire. An enormous amount of aggregation is required to reduce expenditure categories to a manageable number. For each household, I group expenditures into three broad categories of services, nondurables, and durables and into thirteen more narrowly defined categories.

Income is reported for the year ending at the time of the interview. Disposable income is the income measure used in the empirical analysis. Disposable income is defined as before–tax income minus income taxes, deductions for social security and other pension plans, and occupational expenses such as union dues. Household income before taxes is the sum of regular income and other income such as lump sum receipts. Regular income includes such items as wages, salaries, pension income, and interest income.[4]

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[4]See Appendix C of Mace (1988) for details on the surveys, exact definitions of the expenditure groups and income, sample exclusions, and sample size.

(i) Full Sample

The full sample consists of 227,949 monthly observations for the 1980Q1–1984Q1 period. This represents approximately 20,000 households with each household contributing a maximum of 12 monthly observations. Summary statistics for income and the broad expenditure groups of total consumption, services, nondurables, and durables are displayed in Table 1. Due to the rotating nature of the sample, the statistics are computed on a monthly basis for each quarter.

Although mean before–tax income exceeds mean consumption in all quarters, mean consumption exceeds mean disposable income. This is not a surprising outcome due to the different survey collection procedures used for expenditures and income. At each interview, expenditures are reported for the three previous months while income is reported for the twelve previous months. With positive growth of consumption and income over time, this result could follow. Also, the income data includes losses from farm and nonfarm businesses. In addition, disposable income is not only net of income taxes but also retirement deductions and occupational expenses.

(ii) Household Sample

The household sample includes 10,695 households, for which there is one differenced observation per household. The two observations used for differencing are from the first and last interviews. There are nine months between these two interviews. Expenditures are from the last month reported in both interviews.

The choice of the first and last interviews is based on the survey's collection procedures for income. Data on expenditures are collected in all interviews while the primary income data are collected in only the first and last interviews. Changes in employment status are collected in all interviews and this information is used by the BLS to adjust the income data for the second and third interviews. Income changes

TABLE 1

MEANS AND STANDARD DEVIATIONS OF CONSUMPTION AND INCOME[a]

	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1
	1980				1981				
Total Consumption	1,378.2 (1,547.9)	1,396.8 (1,829.0)	1,430.1 (1,568.0)	1,431.9 (1,463.1)	1,315.0 (1,361.8)	1,349.1 (1,414.5)	1,380.0 (1,424.3)	1,391.8 (1,473.9)	
Services	631.3 (706.1)	591.7 (594.5)	648.6 (738.5)	639.8 (670.6)	616.3 (589.5)	593.7 (596.1)	628.1 (666.2)	633.4 (703.8)	
Nondurables	522.6 (383.4)	523.1 (372.5)	529.4 (409.5)	542.9 (399.3)	492.3 (333.5)	506.7 (344.6)	520.2 (387.4)	501.0 (383.3)	
Durables	225.4 (1,138.6)	279.5 (1,525.2)	252.8 (1,091.8)	249.7 (983.8)	206.6 (1,042.1)	246.6 (1,054.4)	230.7 (979.2)	257.0 (1,039.5)	
Disposable Income	1,271.2 (1,119.0)	1,274.2 (1,096.0)	1,240.8 (1,097.4)	1,227.6 (1,067.9)	1,223.4 (1,059.6)	1,247.9 (1,064.3)	1,253.5 (1,073.5)	1,257.2 (1,090.6)	
	1982				1983				1984
Total Consumption	1,289.4 (1,298.3)	1,346.1 (1,382.0)	1,359.3 (1,448.9)	1,413.7 (1,444.7)	1,336.7 (1,399.5)	1,405.4 (1,585.4)	1,450.5 (1,728.4)	1,443.6 (1,614.6)	1,407.6 (1,767.4)
Services	639.7 (612.1)	608.0 (570.9)	639.8 (651.6)	656.6 (729.3)	662.0 (651.2)	642.7 (624.7)	697.5 (939.6)	654.6 (763.8)	681.6 (697.5)
Nondurables	454.1 (321.2)	489.1 (353.7)	495.3 (378.2)	509.9 (390.4)	467.4 (331.6)	495.3 (364.0)	503.2 (405.7)	514.8 (399.6)	463.0 (391.8)
Durables	194.6 (910.2)	248.8 (1,040.7)	224.0 (1,030.2)	247.7 (943.9)	208.2 (1,013.1)	268.2 (1,230.8)	250.1 (1,139.6)	272.7 (1,101.5)	261.4 (1,431.2)
Disposable Income	1,284.7 (1,140.5)	1,313.5 (1,176.3)	1,292.5 (1,201.3)	1,284.5 (1,221.3)	1,315.7 (1,237.6)	1,354.2 (1,276.3)	1,333.7 (1,254.3)	1,325.8 (1,262.0)	1,317.3 (1,260.7)

[a]Standard deviations are in parentheses. Means and standard deviations are expressed on a monthly basis in 1982 dollars. Each expenditure group is deflated by the relevant component of the nonseasonally adjusted monthly CPI index. The original, annual income data are deflated by annual averages of the monthly CPI for all items.

are reflected in the data for the middle interviews only if there is a change in employment status.

The aggregate consumption variable for the first–difference specification (equation (3.2)) is constructed from the entire sample of 10,695 households. Expressed as a range over 12 months, the numbers of households used to compute monthly values of aggregate consumption are as follows: 214–492 households in 1980, 424–520 in 1981, 518–577 in 1982, 262–579 in 1983. There are 282 households in both January and February of 1984.

The sample for the growth rate specification (equation (3.4) is a subset of the above sample. Because logarithms are employed, a household is included in the sample only if both the reported expenditure and income are positive for both interviews. Hence the numbers of observations differ by the measure of consumption. For example, the numbers of households used to compute the aggregate consumption variables in 1982 range from 120–165 for household furnishings to 440–510 for total nondurables.

5. Results and Econometric Issues

In this section, the empirical specifications are tested using data from the CES for 1980–1983. Results are reported for various measures of consumption: the broad categories of total consumption, services, nondurables, and durables and the more narrowly defined categories of food, housing, utilities, household furnishings, clothing, medical, transportation, and recreation.[5] The consumption data are expenditures rather than service flows. For the homothetic preferences analyzed here, the implications of

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[5]The percentages of total CES expenditures accounted for by services, nondurables, and durables are 44, 36, and 20, respectively. For the more narrow categories, food accounts for 22.3 percent of total expenditures, transportation for 20.5, housing for 15.2, and utilities for 8 percent. Recreation, clothing, and household furnishings each account for 5–6 percent and medical accounts for 4.7 percent. Results are not reported separately for domestic services, education, personal care, personal business, and contributions. Each accounts for less than 2.8 percent of total expenditures.

risk sharing are applicable to all measures of consumption, with one exception. The exception is durable goods for the case of power utility with multiplicative preference shocks. (See the appendix.)

Household Tests

Two specifications are tested at the household level: first differences for exponential preferences (equation (3.2)) and growth rates for power utility (equation (3.4)). The sample consists of two observations for each of 10,695 households. As previously discussed, the two observations represent the first and last interviews for each household. There are nine months between these two interviews. For the first difference specification, there is one differenced observation of consumption and income for each of the 10,695 households.

The sample for the growth rate specification is a subset of the above sample. Because logarithms are employed, a household is included in the sample only if both the reported expenditure and income are positive for both interviews. Hence the numbers of observations differ by the measure of consumption.

In addition to household income, employment status dummy variables are also used in the empirical tests. The survey collects data on the employment status of the reference person, spouse, and others in the household. There are eight possible categories for employment status: (1) reference person only, (2) reference person and spouse, (3) reference person, spouse, and others, (4) reference person and others, (5) spouse only, (6) spouse and others, (7) others only, and (8) no earners. There are 64 possible combinations of changes in employment status between the first and last interviews. Household income is replaced by the employment status dummy variables in half of the tests. The risk–sharing model predicts that the coefficients on these dummy variables are all zero.

Results are reported for twelve measures of consumption. The data are deflated by the relevant components of the CPI index. The data are neither seasonally adjusted nor detrended prior to estimation. The coefficients are estimated using ordinary least squares. Various econometric issues concerning measurement error and possible estimation biases are addressed later in the section.

Results are reported in Table 2 for the first–difference specification. Two regressions are reported for each measure of consumption. The dependent variable in all regressions is the first difference of household consumption. The first difference of aggregate consumption is included on the right–hand side in both regressions. The first regression includes the first difference of household income as a right–hand side variable while the last regression includes employment status dummy variables (ΔE_t^j).

A summary of the two regressions is given below.

(5.1) $$\Delta C_t^j = \hat{\alpha} + \hat{\beta}_1 \Delta C_t^a + \hat{\beta}_2 \Delta y_t^j$$

(5.2) $$\Delta C_t^j = \hat{\alpha} + \hat{\beta}_1 \Delta C_t^a + \sum_{k=1}^{M} \hat{\gamma}_k \Delta E_{kt}^j$$

where

ΔC_t^j = first difference of household consumption,

ΔC_t^a = first difference of aggregate consumption,

Δy_t^j = first difference of household income, and

ΔE_{kt}^j = dummy variables for change in household employment status.

The intercept term is included as a specification test.

Results

The consumption measures are listed in column (1) of Table 2. The intercept is reported in column (2). The coefficient on aggregate consumption ($\hat{\beta}_1$) is in column (3) and the coefficient on household income ($\hat{\beta}_2$) is in column (4). The standard errors of the estimated coefficients are in parentheses. In column (5), an F ratio is reported for the joint test of $\hat{\beta}_1 = 1$ and $\hat{\beta}_2 = 0$. An F ratio is also reported in column (6) for the regression with employment status, in which case the joint test is $\hat{\beta}_1 = 1$ and all coefficients on the employment dummy variables ($\gamma_k = 0$) are jointly zero. The R^2 is in column (7).

For most of the measures of consumption, the results from the first regression in Table 2 are consistent with the implications of risk sharing. In explaining the change in household consumption, the change in aggregate consumption enters with a coefficient of one while the change in household income has a zero coefficient. Based on the F test in column (5), the risk–sharing implications cannot be rejected for ten of the twelve goods (total consumption, services, durables, food, housing, utilities, household furnishings, medical, transportation, and recreation).[6] However, the results for food are marginal due to $\hat{\beta}_2 > 0$. For the remaining goods (nondurables and clothing), the risk–sharing implications are decisively rejected. These two rejections are the result of a significant coefficient on income.

Most of the results from the second regression in Table 2 are also consistent with full insurance. This regression includes employment status dummy variables instead of household income. The risk–sharing implications cannot be rejected for eight of the twelve goods (total consumption, services, durables, housing, utilities, household furnishings, medical, and recreation). The implications are rejected for nondurables,

[6]All statements regarding the statistical significance or rejection of the risk–sharing implications are based on a 5 percent level of significance.

TABLE 2

HOUSEHOLD CONSUMPTION REGRESSIONS[a]

– First Differences –

Consumption Measure	Intercept	ΔC_t^a	Δy_t^j	F Ratios: $\beta_1=1$, $\beta_2=0$	F Ratios: $\beta_1=1$, $\gamma_k=0\ \forall k$	R^2
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Total Consumption	–77.87 (19.32)	1.06 (.11)	.03 (.02)	1.27	—	.008
	–69.07 (43.55)	1.06 (.11)	—	—	1.11	.014
Services	–30.47 (7.48)	1.01 (.10)	.01 (.007)	1.14	—	.009
	–30.97 (16.47)	1.01 (.10)	—	—	.76	.013
Nondurables	–13.97 (3.33)	.99 (.06)	.01 (.003)	7.71[b]	—	.023
	–6.03 (7.57)	.98 (.06)	—	—	1.84[b]	.032
Durables	–32.44 (16.87)	1.03 (.15)	.004 (.02)	.06	—	.004
	–32.00 (38.36)	1.03 (.15)	—	—	1.01	.010
Food	–7.46 (2.12)	1.01 (.08)	.005 (.002)	2.52	—	.016
	–5.01 (4.76)	1.00 (.08)	—	—	1.69[b]	.025
Housing	–13.80 (3.45)	.92 (.10)	.004 (.003)	1.10	—	.008
	–9.37 (7.61)	.92 (.10)	—	—	.96	.013

TABLE 2 — Continued

Consumption Measure	Intercept	ΔC_t^a	Δy_t^j	F Ratios $\beta_1=1,\ \beta_2=0$	F Ratios $\beta_1=0,\ \gamma_k=0\ \forall k$	R^2
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Utilities	.62 (1.01)	1.00 (.04)	.002 (.001)	1.60	—	.060
	−.90 (2.29)	1.00 (.04)	—	—	.93	.065
Household Furnishings	−17.01 (10.69)	.94 (.18)	−.02 (.01)	1.72	—	.003
	−9.83 (24.28)	.94 (.18)	—	—	.90	.008
Clothing	−6.83 (2.18)	1.00 (.04)	.006 (.002)	3.51[b]	—	.058
	−6.05 (4.92)	1.00 (.04)	—	—	2.78[b]	.072
Medical	1.92 (2.79)	.98 (.16)	−.002 (.003)	.18	—	.004
	1.30 (6.36)	.99 (.16)	—	—	.92	.009
Transportation	−15.40 (12.11)	1.01 (.14)	.02 (.01)	1.68	—	.005
	−21.22 (27.41)	1.02 (.14)	—	—	1.39[b]	.013
Recreation	−11.65 (5.70)	.93 (.13)	.01 (.006)	2.78	—	.005
	−13.58 (13.00)	.94 (.13)	—	—	.20	.006

TABLE 2.— Continued

[a] Standard errors are in parentheses. For each measure of consumption, there are 10,692 degrees of freedom in the first regression and 10,633 in the second regression. The degrees of freedom for the F ratio numerators are 2 in the first regression and 61 in the second regression.

[b] Denotes significance at the 5 percent level for the F ratios.

food, clothing, and transportation. The employment status variables are significantly different from zero for these four goods.

Comparing the results for the two regressions, the eight goods for which the implications cannot be rejected in both regressions are total consumption, services, durables, housing, utilities, household furnishings, medical, and recreation. The significance of household income and employment status differs across two of the goods. Income is not significant although employment status is for transportation. For recreation, employment status is not significant although income is. Income and employment status are both significant for nondurables, food, and clothing.

Results are reported in Table 3 for the growth rate specification.[7] Two regressions are reported for each of the twelve goods. The specifications are identical to those in Table 2, with the first differences of household consumption, aggregate consumption, and household income replaced by growth rates.

For the first regression with income, there are now fewer results consistent with full insurance. Based on the F ratio in column (5), the test of the risk–sharing implications of $\hat{\beta}_1 = 1$ and $\hat{\beta}_2 = 0$ cannot be rejected for six goods (durables, housing, utilities, household furnishings, clothing, and transportation). The implications are rejected for the remaining six (total consumption, services, nondurables, food, medical, and recreation). All of the rejections in the first regression are the result of $\hat{\beta}_2 > 0$, with the exception of medical for which $\hat{\beta}_1 < 1$.

The results for the second regression with employment status are consistent with risk sharing for six goods (durables, household furnishings, clothing, medical, transportation, and recreation). The implications are rejected for total consumption, services, nondurables, food, housing, and utilities.

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[7]As previously noted, the numbers of observations differ by the measure of consumption due to requiring positive values for logarithms. The numbers of observations are listed at the end of Table 3 in note a.

TABLE 3

HOUSEHOLD CONSUMPTION REGRESSIONS[a]

– Growth Rates –

Consumption Measure	Intercept	ΔC^a_t	Δy^j_t	F Ratios: $\beta_1=1$, $\beta_2=0$	F Ratios: $\beta_1=0$, $\gamma_k=0\ \forall k$	R^2
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Total Consumption	–.04 (.01)	1.06 (.08)	.04 (.007)	14.12[b]	—	.021
	–.01 (.01)	1.05 (.08)	—	—	1.85[b] (A)	.029
Services	–.02 (.01)	.93 (.10)	.04 (.01)	12.44[b]	—	.011
	.02 (.02)	.94 (.11)	—	—	1.34[b] (B)	.017
Nondurables	–.02 (.01)	.97 (.07)	.04 (.006)	22.69[b]	—	.027
	–.01 (.01)	.95 (.07)	—	—	1.76[b] (C)	.033
Durables	–.05 (.03)	1.00 (.06)	–.03 (.03)	.39	—	– .046
	–.14 (.09)	1.01 (.06)	—	—	1.20 (D)	0.57
Food	–.02 (.01)	.91 (.07)	.04 (.006)	18.67[b]	—	.020
	.01 (.01)	.89 (.07)	—	—	1.81[b] (E)	.027
Housing	–.05 (.01)	.79 (.12)	.01 (.01)	1.77	—	.006
	–.03 (.03)	.79 (.12)	—	—	1.47[b] (F)	.018

TABLE 3 — Continued

Consumption Measure (1)	Intercept (2)	ΔC_t^a (3)	Δy_t^j (4)	F Ratios: $\beta_1=1,\ \beta_2=0$ (5)	F Ratios: $\beta_1=0,\ \gamma_k=0\ \forall k$ (6)	R^2 (7)
Utilities	.02 (.01)	.98 (.05)	.002 (.01)	.10	—	.050
	-.01 (.02)	.98 (.05)	—	—	1.40[b] (G)	.058
Household Furnishings	-.02 (.04)	1.00 (.14)	-.004 (.04)	.01	—	.017
	.07 (.11)	.98 (.15)	—	—	.90 (H)	.032
Clothing	-.01 (.02)	1.01 (.04)	.04 (.02)	2.19	—	.078
	.03 (.05)	1.01 (.04)	—	—	.92 (I)	.085
Medical	.06 (.02)	.66 (.14)	.02 (.02)	3.85[b]	—	.004
	.12 (.04)	.65 (.14)	—	—	1.13 (J)	.013
Transportation	-.04 (.01)	.83 (.09)	.006 (.02)	1.72	—	.010
	-.07 (.04)	.84 (.09)	—	—	1.06 (K)	.017
Recreation	-.06 (.02)	.97 (.05)	.06 (.02)	7.44[b]	—	.042
	-.03 (.04)	.97 (.05)	—	—	.91 (L)	.046

TABLE 3. — Continued

[a] Standard errors are in parentheses. Degrees of freedom for the first regression are listed in order for the 12 measures of consumption: 9,230; 9,270; 9,295; 6,041; 9,281; 6,928; 9,009; 2,796; 6,799; 6,071; 8,386; 8,134.

For all measures of consumption, the degrees of freedom for the second regression are equal to the degrees of freedom for the F ratio denominator. The following lists the degrees of freedom for the numerator and denominator of the F ratios which correspond to the parentheses below the F ratios in the table.

A 58; 9,170	B 58; 9,210	C 58; 9,235	D 53; 5,986
E 58; 9,221	F 55; 6,871	G 58; 8,949	H 48; 2,746
I 57; 6,740	J 54; 6,015	K 58; 8,326	L 56; 8,076

[b] Denotes significance at the 5 percent level for the F ratios.

The common results for income and employment status in Table 3 are as follows: the implications cannot be rejected for durables, household furnishings, clothing, and transportation and are rejected for total consumption, services, nondurables, and food. The results for income and employment status differ for four goods. For housing and utilities, income is not significant although employment status is. For clothing and recreation, employment status is not significant although income is.

The growth rate implications for power utility, with multiplicative preference shocks, are not applicable in the case of durable goods. If the risk–sharing model is an accurate description of observed allocations and if power utility is the correct specification of preferences, then one expects to find the risk–sharing implications rejected only for the more durable goods. The results in Table 3 fail to support this scenario. Of the goods for which the risk–sharing implications cannot be rejected, three are possibly the most durable of the twelve: durables, household furnishings, and transportation. The implications are rejected for the least durable goods: services, nondurables, and food.

Comparing the results in Tables 2 and 3, the first–difference specification fares better than the growth rate specification. The results common to both specifications are rejections for nondurables in the first regression and for both nondurables and food in the second regression. The coefficients on income and employment status are significant for these goods. For both specifications, the implications cannot be rejected in the first regression for five goods (durables, housing, utilities, household furnishings, and transportation) and in the second regression for four goods (durables, household furnishings, medical, and recreation).

One possible explanation for the divergence in results between the first–difference and growth rate specifications is that lower income households are effectively given a larger weight than other households in the growth rate specification.[8] For example, the first–difference of income, $\Delta y^j_t = y^j_t - y^j_{t-1}$, is weighted by $1/y^j_{t-1}$ to compute the growth rate of income. This weight is larger the lower the household's income. If some households are not fully insured they would tend to be households in the lower income groups. Hence, in the growth rate specification, the group which would be more likely to cause rejections of the risk–sharing model are weighted most heavily.

Econometric Issues

Unbiased estimation of the coefficients requires a zero correlation between the disturbance term and the right–hand side variables of aggregate consumption and household income. The disturbance term may include both preference shocks and measurement errors from the data. For example, correlations between preference shocks to consumption and income might arise, such as an illness resulting in no employment and reduced consumption. Correlations between the disturbance and right–hand side variables might also arise due to measurement errors in the data. The following discussion is limited to the first–difference specification.

The current results do not correct for possible correlations between household income and the disturbance. However, a bias of the coefficient on aggregate consumption can be avoided by testing a modified version of the first–difference specification.

$$(5.3) \qquad \Delta C^j_t - \Delta C^a_t = \beta_2 \Delta y^j_t + u^j_t$$

The change in aggregate consumption is simply subtracted from the change in household consumption so that a unit coefficient on aggregate consumption is imposed.

[8]This was pointed out to me by Marjorie Flavin.

The only right–hand side variable is the change in household income or employment status. As before, the risk–sharing model predicts zero coefficients for these variables. The results (not reported here) are identical to the results in Table 2.

A related but more specific issue is whether the seasonal preference shocks to consumption and income are correlated. The CES consumption data are monthly observations and reveal large seasonal fluctuations. In contrast, the income data are reported as annual figures. In addition to household income, the risk–sharing model is also tested using quarterly changes in employment status. Hence seasonal preference shocks to consumption and employment status might be correlated.

To account for the common seasonal component of preference shocks, a simple procedure is to include seasonal dummy variables in the specifications in equations (5.1) and (5.2). For the two interview months used for each household, there are 12 possible combinations of months. For example, the first interview is in January and the last interview is in October, or the first interview is in February and the last interview is in November, etc. Hence eleven seasonal dummy variables and an intercept are incorporated.

The seasonal results (not reported here) are almost identical to the non–seasonal results in Table 2 and Table 3. The seasonal dummies are jointly insignificant in all cases. There are no changes in the significance of income and employment status when the seasonals are added. Overall, the results are not sensitive to the inclusion of seasonal dummies. However, these findings are not interpreted as an indication of no seasonals in preference shocks.

Lastly, a small sample issue arises because individual consumption, the dependent variable, is included in aggregate consumption. It appears that the montly samples from which aggregate consumption is constructed are sufficiently small for the individual to influence the aggregate. A solution is to create a different aggregate consumption variable for each individual by excluding that individual's consumption from the average. The aggregate consumption variable for individual j is denoted $C_t^{aj} = \frac{1}{J-1} \sum_{k \neq j}^{J} C_t^k$. An implication of risk sharing is that an individual's consumption moves together with the consumption of all other individuals.

The results (not reported here) using the modified aggregate consumption as a right-hand side variable show dramatic changes in $\hat{\beta}_1$ but no changes in $\hat{\beta}_2$. The coefficient on the modified aggregate consumption (β_1) is smaller for all goods. Hence, the monthly samples are "small enough" for the individual to influence the aggregate.

This small sample correction can also be combined with the modified test in equation (5.3). The modified aggregate consumption variable is simply subtracted from the change in household consumption.

$$(5.4) \qquad \Delta C_t^j - \Delta C_t^{aj} = \beta_2 \Delta y_t^j + u_t^j$$

As before, this imposes a unit coefficient on aggregate consumption. The results (not reported here) reveal no change in the coefficient on income.

To summarize, a biased coefficient on aggregate consumption is avoided through various modifications of the empirical specification. However, the current tests do not correct for possible biases of the coefficient on income. The current results are fairly consistent across income and employment status. This is encouraging if, as one would expect, there is less measurement error in employment status responses than in reported income.

6. Summary

The principal implication of risk sharing is that individual consumption varies positively with aggregate consumption. The solutions are specialized for two specifications of homothetic preferences: exponential and power utility. For exponential utility, the first differences of consumption, net of preference shocks, are equalized across individuals. The growth rates of consumption are equalized in the case of power utility.

In this paper, considerable success is registered for the benchmark case of full insurance. Once the change in aggregate consumption is accounted for, the change in household income does not help to explain the change in household consumption. This is for exponential preferences. Although the growth rate specification does not fare as well as the first–difference specification, many of the results for power utility are also consistent with full insurance.

The most striking results are the insignificant coefficients for the changes in household income and employment status. This is especially noteworthy considering the tremendous spectrum of households included in the sample. Those included are not limited to employed heads of households. Also included are single individuals as young as 18 years of age, the unemployed, the retired, and others not in the labor force such as welfare recipients.

Using a fresh data source adds further interest to the findings. The thoroughness of the CES consumption data is unmatched by any other source of household data for the U.S. Previous empirical work is primarily based on aggregate data or on food expenditures from the Panel Study of Income Dynamics.

The current evidence suggests that market imperfections or incomplete markets are not "the" essential features in explaining allocations. Although this might not hold perfectly for an entire life cycle, there appear to be ample sources of insurance in the economy. In addition to the traditional sources of stocks and securities markets, these include private and family sources of insurance and various institutional sources such as unemployment compensation, contracts between employers and employees, and crop insurance for farmers.

The current research does not identify the sources of the proposed insurance but it does suggest its presence. Additional research in the area of risk sharing is justified. The evidence in this paper strongly suggests that it is too soon to close the door on complete markets.

Appendix

The principal risk–sharing implications continue to hold for more general homothetic preferences. This includes the case of multiple goods, for both separable and nonseparable within period preferences, and for durable goods.

Multiple Nondurable Goods

I begin with a class of homothetic, power utility functions. (See Eichenbaum and Hansen (1987).) These preferences are both time and state separable, but are nonseparable across goods if $\sigma \neq \alpha$.

$$\text{(A.1)} \qquad U[C_t^j, b_t^j] = \frac{1}{\alpha}\{\sum_{i=1}^{m} \theta_i \; V[C_{it}^j, b_{it}^j]\}^{\frac{\alpha}{\sigma}} - \frac{1}{\alpha}$$

where i denotes good i, m is the number of goods, and $\sum_{i=1}^{m} \theta_i = 1$. For the case of a single good, m = 1, $\sigma = \alpha$, and $\theta_i = 1$. For power utility,

$$\text{(A.2)} \qquad V[C_{it}^j, b_{it}^j] = \exp\{\sigma b_{it}^j\} \; [C_{it}^j]^{\sigma} \; .$$

Strict concavity requires $\sigma < 1$. Individuals have the same coefficient of constant relative risk aversion $(1 - \sigma)$.

Some special cases of the utility function are noted. First, within period preferences are separable across goods if $\sigma = \alpha$.

$$\text{(A.3)} \qquad U[C_t^j, b_t^j] = \frac{1}{\sigma}\{\sum_{i=1}^{m} \theta_i \exp\{\sigma b_{it}^j\} \; [C_{it}^j]^{\sigma}\} - \frac{1}{\sigma} \; .$$

More specifically, preferences are logarithmically separable if $\sigma = \alpha = 0$.

$$\text{(A.4)} \qquad U[C_t^j, b_t^j] = \sum_{i=1}^{m} \theta_i \exp\{b_{it}^j\} \log C_{it}^j \ .$$

With multiplicative, rather than additive preference shocks, the utility function in equation (A.3) must be modified in order to obtain multiplicative preference shocks for the special case of logarithmic preferences. The current specification of $\exp\{\sigma b\}c^{\sigma} - 1$ in equation (A.3) must be replaced by $\exp\{b\}\ [c^{\sigma} - 1]$ so that preference shocks are included in the marginal utility of consumption.

The social planner's problem is now solved subject to m aggregate constraints, corresponding to the m goods. There are m marginal conditions for each individual j, as in equation (2.6). The marginal condition for each good is independent of the consumption and preference shocks of all other goods. Hence with separable preferences across goods, the risk–sharing implication of equation (2.12) holds in the case of multiple nondurable goods. With consumption indexed by good i,

$$\text{(A.5)} \qquad \log C_{it}^j = \log C_{it}^a + \tfrac{1}{1-\sigma} [\log \omega^j - \omega^a] + \tfrac{\sigma}{1-\sigma} [b_{it}^j - b_{it}^a],\ i = 1, ..., m \ .$$

The next extension involves preferences that are nonseparable across goods ($\sigma \neq \alpha$ in equation (A.1)). A special case is Cobb–Douglas. If $\alpha = 1$ and $\sigma = 0$,

$$\text{(A.6)} \qquad U[C_t^j, b_t^j] = \prod_{i=1}^{m} \exp\{\theta_i\ b_{it}^j\}\ [C_{it}^j]^{\theta_i} \ .$$

To illustrate, suppose preferences are Cobb–Douglas, there are two goods, and $\theta_1 + \theta_2 \neq 1$. Consumption of good 1 for individual j is

$$\text{(A.7)} \qquad \log C^j_{1t} = \log C^a_{1t} + \frac{1}{\phi}\,[\log \omega^j - \omega^a] + \frac{\theta_1}{\phi}\,[b^j_{1t} - b^a_{1t}] + \frac{\theta_2}{\phi}\,[b^j_{2t} - b^a_{2t}]$$

where $\phi = 1 - \theta_1 - \theta_2$. For Cobb–Douglas preferences, an individual's consumption of good 1 is positively related to the aggregate consumption of good 1. However, unlike in the separable case, consumption of good 1 depends on the preference shocks of both goods.

The next example of nonseparable preferences involves the more general power utility in equation (A.2). While unable to obtain a direct analytical solution, conditions implied by the solutions for separable preferences are satisfied by the first order conditions in the nonseparable case. For example, in the two good case, the relationship between individual and aggregate consumption of the two goods is

$$\text{(A.8)} \qquad \log C^j_{1t} - \log C^j_{2t} - \frac{\sigma}{1-\sigma}[b^j_{1t} - b^j_{2t}] = \log C^a_{1t} - \log C^a_{2t} - \frac{\sigma}{1-\sigma}\,[b^a_{1t} - b^a_{2t}].$$

This condition is satisfied by the following expressions for consumption:

$$\text{(A.9)} \qquad \log C^j_{1t} = \log C^a_{1t} + \frac{1}{1-\sigma}\,[\log \omega^j - \omega^a] + \gamma_1[b^j_{1t} - b^a_{1t}] + \gamma_2[b^j_{2t} - b^a_{2t}]$$

$$\text{(A.10)} \qquad \log C^j_{2t} = \log C^a_{2t} + \frac{1}{1-\sigma}\,[\log \omega^j - \omega^a] + \gamma_3[b^j_{1t} - b^a_{1t}] + \gamma_4[b^j_{2t} - b^a_{2t}]\ .$$

If $\gamma_1 = \gamma_4 = \frac{\sigma}{1-\sigma}$ and $\gamma_2 = \gamma_3 = 0$, consumption in the nonseparable case is identical to that in the separable case. (See equation (2.15).) The Cobb–Douglas case is represented by $\gamma_1 = \gamma_3 = \frac{\theta_1}{\phi}$ and $\gamma_2 = \gamma_4 = \frac{\theta_2}{\phi}$.

Next, the risk–sharing implications continue to hold for various modifications of exponential preferences. For good i,

$$\text{(A.11)} \quad V[C^j_{it}, b^j_{it}] = -\exp\{-\sigma[C^j_{it} - b^j_{it}]\} \ .$$

For multiple goods and separable preferences ($\sigma = \alpha$), consumption of good i for individual j is

$$\text{(A.12)} \quad C^j_{it} = C^a_{it} + \frac{1}{\sigma}[\log \omega^j - \omega^a] + [b^j_{it} - b^a_{it}].$$

This is identical to the case of a single good.

For exponential preferences that are nonseparable ($\sigma \neq \alpha$), a direct analytical solution is not obtained. However, as with the previous example for two goods, it is possible to derive the relationship between individual and aggregate consumption of the two goods.

$$\text{(A.13)} \quad C^j_{1t} - C^j_{2t} - [b^j_{1t} - b^j_{2t}] = C^a_{1t} - C^a_{2t} - [b^a_{1t} - b^a_{2t}].$$

This condition is satisfied by the expression for consumption under separable preferences.

Durable Goods

The previous risk–sharing implications analyzed for nondurable goods also apply to the acquisition of durable goods in the case of exponential preferences. This does not hold for the class of power utility functions with multiplicative preference shocks.

A durable good is defined as a risk–free claim to services in current and future time periods. Preferences are defined over current services, where services refer to the previous nondurable goods. The number of services is equal to the number of durable goods.

The current treatment of durable goods concentrates on the measurement of durables for empirical implementation. Data are available on acquisitions of durable goods but not on the services derived from these goods. Although preferences are time separable for services they are not time separable for the durable good. The current analysis does not take into account this departure from time separability: allocations are in terms of services, which are simply proportional to the stocks of durable goods.

An additional assumption is that, in the initial trading period, the distribution of endowments of durable goods across individuals is identical to the distribution of the social planner's weights. This restriction on initial conditions, together with the former assumptions, is sufficient for the previous risk–sharing implications to carry over to the acquisition of durable goods.

Consider the case of a single durable good. For individual j, let D_t^j denote the time t acquisition of the durable good and let K_t^j denote the stock of the durable good. The stock depreciates at the rate $(1 - a)$, where $0 < a < 1$. Consequently, aK_{t-1}^j units of the stock from time $(t - 1)$ remain at time t. The current stock is equal to the remaining stock from time $(t - 1)$ plus the current acquisition of the durable good.

$$\text{(A.14)} \qquad K_t^j = a\, K_{t-1}^j + D_t^j \, .$$

Services are proportional to the stock of the durable good with w representing the factor of proportionality.

$$\text{(A.15)} \qquad C_t^j = wK_t^j \, .$$

The expression for service consumption is identical to that previously derived in equation (2.12). Substituting equation (A.15) into equation (2.12) yields

$$\text{(A.16)} \qquad K_t^j = K_t^a + \frac{1}{w\sigma}\, [\log \omega^j - \omega^a] + \frac{1}{w}\, [b_t^j - b_t^a],$$

where $K_t^a = \frac{1}{J} \sum_{j=1}^{J} K_t^j$.

Equation (A.14) is then substituted into equation (A.16) to yield

$$\text{(A.17)} \quad D_t^j = D_t^a + a[K_{t-1}^a - K_{t-1}^j] + \frac{1}{w\sigma} [\log \omega^j - \omega^a] + \frac{1}{w} [b_t^j - b_t^a],$$

where $D_t^a = \frac{1}{J} \sum_{j=1}^{J} D_t^j$.

Equation (A.17) is further reduced by substituting in the time (t – 1) expression for equation (A.16)

$$\text{(A.18)} \quad D_t^j = D_t^a + (1 - a) \frac{1}{w\sigma} [\log \omega^j - \omega^a] + \frac{1}{w} \{[b_t^j - ab_{t-1}^j] - [b_t^a - ab_{t-1}^a]\} .$$

Taking the first difference of equation (A.18) yields

$$\text{(A.19)} \quad \Delta D_t^j = \Delta D_t^a + \frac{1}{w} \{[\Delta b_t^j - a\Delta b_{t-1}^j] - \Delta b_t^a - a\Delta b_{t-1}^a]\}$$

where $\Delta D_t^j = D_t^j - D_{t-1}^j$ and $\Delta D_t^a = \frac{1}{J} \sum_{j=1}^{J} \Delta D_t^j$.

The risk–sharing implication for services carries over to the acquisition of durable goods. An individual's acquisition of durables varies positively with the aggregate acquisition of durables. As previously stated, this is important for empirical implementation since data are available on acquisitions of durable goods but not on the services derived from these goods.

The above result is sensitive to assumptions regarding initial conditions. If the initial distribution of endowments of durable goods differs from the distribution of planner weights then the risk–sharing implication for durables must be interpreted as an asymptotic one. Even then, the relationship between individual and aggregate acquisitions of durables is a useful approximation.

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