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Abstract

Contrary to the belief that capital overaccumulation is impossible in an economy with land—i.e., a non-reproducible factor of production, this note shows that the possibility of dynamic inefficiency depends on the income share of land in steady states and the elasticity of substitution between land and other factors of production in CES production functions, and tries to find empirical evidence of the substitutability.

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I. Introduction.

In an overlapping generation model, Diamond(1965) showed that an economy can reach a steady state with capital overaccumulation. An economy with capital overaccumulation is said to be dynamically inefficient since a Pareto improvement can be achieved by allowing the current generation to consume the existing capital stock and leaving the consumption of all future generations intact.

Whether an economy is dynamically efficient is an important issue in positive as well as normative analysis. Tirole (1985) proved that dynamic inefficiency is a necessary condition for the existence of speculative bubble. Weil (1987) showed that dynamic efficiency is necessary for an operative bequest motive and thus, for the Ricardian Equivalence theorem. As a policy issue, the presumption that an economy is dynamically efficient underlies the arguments for increased national savings. Empirically, Abel, Mankiw, Summers and Zeckhauser (1989) conducted a study to test whether actual economies are dynamically inefficient.

However, some economists argue that capital overaccumulation is impossible *a priori* if an economy includes an asset that is productive and non-reproducible --i.e., if an economy is one with *land.* (See McCallum (1987).) Intuitively speaking, with a finite amount of land and growing population, land becomes progressively scarce so that the price of land will explode. Most of private savings will be absorbed in paying a high price of land, instead of accumulating capital.[1] Since no one would dare to deny the

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1 Section II shows the fallacy of this handy reasoning: the possibility of

existence of land, it is often argued that the possibility of dynamic inefficiency should not be regarded as a matter of concern.

This note shows that the above argument is based on the particular assumption that land is an *essential*[2] factor of production and the production function is Cobb-Douglas. When land is *essential*, only the trivial steady state of zero output and capital per capita is possible as land per capita vanishes. To avoid this triviality, *a balanced growth path* is analyzed where output per capita grows at a constant rate. Furthermore, to require a constant output capital ratio along a balanced growth path, the Cobb-Douglas production function is adopted among production functions where land is essential.[3] However, as we will see in section II, the requirement that factor shares be constant (due to the Cobb-Douglas production function) together with the

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dynamic inefficiency is not exactly a question of whether land becomes scarce eventually. In case of the Cobb-Douglas production function with technical progress, land does not become scarce and the price of land will not grow enough to absorb most of savings, but still, dynamic inefficiency is not possible.

2 Dasgupta and Heal (1974) and Solow (1974) call a factor L *essential* in the production function, Q = F(K,N,L), (1) if L=0 entails Q=0 and (2) the average product of L has no upper bound. Only the Cobb-Douglas production function is *essential* among CES production functions.

3 An example of a non-Cobb-Douglas production function with essential land is $Y_t = (K_t + N_t)^h L_t^{1-h}$, $0<h<1$, where K, N, L are capital, labor, and land, respectively.

introduction of a balanced growth path is sufficient to insure the dynamic efficiency. In this sense, the impossibility is not derived from the mere existence of *essential* land, but rather from this particular characteristic of the Cobb-Douglas production function.

In section II, a counter example is provided with a CES production function where the elasticity of substitution between land and other factors of production is greater than one. When a producer can easily substitute other factors of production for land as land becomes expensive, the price of land will be bounded and cannot absorb most of savings, leaving the possibility of capital overaccumulation intact. In proving the possibility of dynamic inefficiency, our model generalizes Tirole's (1985) analysis of deterministic bubbles on assets yielding constant rents: In our model, since rents (marginal product of land) change with the interest rates, additional restrictions should be made for the uniqueness of a non-steady state equilibrium path.

However, this note shows that the high elasticity of substitution is not sufficient to preclude dynamic inefficiency unless we confine our interest to CES production functions. With more general production functions, dynamic inefficiency is impossible if the income share of land in steady state does not vanish.

Since the income share of land and the substitutability of land in CES production functions are crucial to the understanding of dynamic inefficiency, it would be interesting to see empirical evidence. Section III examines the historical tendencies of the share of land in national income.

II. Counter examples

The economy consists of overlapping generations of identical, two period lived individuals. The generation born at time t equals $N_t = (1+n)^t N_0$. A consumer lives for two periods, but works only during the first. He supplies labor inelastically. The stock of land L_0 is fixed so that land per capita l_t is decreasing at rate n. Output is produced by competitive firms through a CES production function: $Y_t=(K_t^\rho + N_t^\rho + L_t^\rho)^{1/\rho}$, where Y_t is total output and K_t is the capital stock. *We assume that the elasticity of substitution between land and capital is greater than one*-- i.e., $0<\rho<1$. In intensive form, $y_t = f(k_t, l_t) = (1 + k_t^\rho + l_t^\rho)^{1/\rho}$.

Competitive profit maximization requires that the rental price of capital (r_t) and land (r_t^*), and the wage rate (w_t) be equal to the marginal productivity of capital, land, and labor respectively:

(1) $$r_t = (1 + k_t^\rho + l_t^\rho)^{(1-\rho)/\rho} k_t^{\rho-1} ,$$

(2) $$r_t^* = (1 + k_t^\rho + l_t^\rho)^{(1-\rho)/\rho} l_t^{\rho-1} ,$$

(3) $$w_t = (1 + k_t^\rho + l_t^\rho)^{(1-\rho)/\rho} .$$

The income of a consumer born at time t is his wage, w_t. Out of his income he consumes C_{1t} and saves the rest. His saving is channeled to buying either capital or land. The price of land is q_t. Capital and land bear interest r_{t+1} and r_{t+1}^* in the next period, and land is sold at price q_{t+1}. All the proceeds are used for C_{2t}, the second period per capita consumption of the generation t. A consumer hence faces the following budget constraints:

(4) $$C_{1t} + (1+n)k_{t+1} + (1+n)q_t l_{t+1} = w_t ,$$

(5) $$C_{2t} = (1 + r_{t+1})(1+n)k_{t+1} + (q_{t+1} + r_{t+1}^*)(1+n)l_{t+1} .$$

A no arbitrage condition requires that the return from holding each asset should be the same;

(6) $$1 + r_{t+1} = \frac{q_{t+1} + r^*_{t+1}}{q_t}, \quad \forall\ t.$$

A time separable log utility function is assumed in this example:

(7) $$U(\ C_{1t},\ C_{2t}\ ) = \ln C_{1t} + \alpha \ln C_{2t}, \qquad 0<\alpha<1.$$

Before we analyze the dynamics of equilibrium paths, let us decompose q_t into two parts: fundamental and bubble. Since land yields the interest rate r^*_t, its fundamental price f_t is:

(8) $$f_t \equiv \sum_{s=t+1}^{\infty} \frac{r^*_s}{(1+r_{t+1})\cdots(1+r_s)} .$$

The other component of q_t is bubble, β_t, which pertains to the ownership of land. Under perfect foresight, bubble must bear the same yield as capital and must be positive; $\beta_{t+1} = (1+r_{t+1})\beta_t$, $\beta_t>0\ \forall\ t$. By definition, $q_t = f_t + \beta_t$. Then, from the market equilibrium and arbitrage conditions, we can characterize the dynamics of equilibrium paths by four difference equations:

(9) $$(1+n)k_{t+1} + f_t l_t + \beta_t l_t = \frac{\alpha}{1+\alpha}(1 + k^\rho_t + l^\rho_t)^{(1-\rho)/\rho} ,$$

(10) $$f_{t+1} l_{t+1} = \frac{1+(1+k^\rho_{t+1}+l^\rho_{t+1})^{(1-\rho)/\rho} k^{\rho-1}_{t+1}}{1 + n} f_t l_t - (1+k^\rho_{t+1}+l^\rho_{t+1})^{(1-\rho)/\rho} l^\rho_{t+1} ,$$

(11) $$\beta_{t+1} l_{t+1} = \frac{1+(1+k^\rho_{t+1}+l^\rho_{t+1})^{(1-\rho)/\rho} k^{\rho-1}_{t+1}}{1 + n} \beta_t l_t, \qquad \beta_t \geq 0 ,$$

(12) $$l_{t+1} = \frac{1}{1+n} l_t, \quad l_0 \text{ and } k_0 \text{ are given.}$$

Definition: Following Tirole (1985), an equilibrium is called bubbly if $\beta_t l_t > 0 \ \forall \ t$, and bubbleless if $\beta_t l_t = 0 \ \forall \ t$. An equilibrium is asymptotically bubbleless if $\beta_t l_t$ converges to zero. Also an equilibrium is asymptotically rentless if $f_t l_t$ converges to zero-- that is, the value of land per capita becomes negligible in the long run.

In this example we can easily see two steady state equilibria. One equilibrium is Diamond's bubbleless and rentless equilibrium, ($f_t l_t \rightarrow \beta_t l_t \rightarrow 0$, $l_t \rightarrow 0$, $k_t = k_D$), where $(1+n)k_D = \frac{\alpha}{1+\alpha} (1+k_D^{\rho})^{(1-\rho)/\rho}$. Note that Diamond's equilibrium can be either dynamically efficient or inefficient, irrespective of the existence of land. We define r_D as the interest rate at Diamond's equilibrium. The other steady state is a bubbly equilibrium. Define $\hat{b}$ and k_b such that:

$$r = (1+k_b^{\rho})^{(1-\rho)/\rho} k_b^{\rho-1} = n, \text{ and } (1+n)k_b - \hat{b} = \frac{\alpha}{1+\alpha} (1+k_b^{\rho})^{(1-\rho)/\rho} .$$

Then ($f_t l_t \rightarrow 0$, $\beta_t l_t \rightarrow \hat{b}$, $l_t \rightarrow 0$, $k_t = k_b$) is a bubbly equilibrium. Note that this bubbly equilibrium can happen only when Diamond's equilibrium is inefficient-- i.e., $r_D < n$.

So far the model with land is not different from the model without land. However, unlike Tirole's model, non-steady state equilibrium paths are not stable due to time varying rents. Assumption A is introduced as a sufficient condition for the uniqueness of an asymptotic bubbleless equilibrium. Proposition 1 and 2 summarizes the dynamics of the model.

Assumption A: As land per capita diminishes on an equilibrium path, the marginal productivity of land does not grow as fast as, or faster than, the population growth rate n.

The violation of Assumption A implies that, as land per capita decreases, land becomes proportionally more productive so that output per capita grows to infinity. Assumption A disposes of these divergent paths and makes $f_t l_t$ converge to zero, which is necessary for the proof of proposition 1.

Proposition 1: Under Assumption A,

(a) If $r_D > n$, there exists a unique bubbleless equilibrium and the interest rate converges to r_D.

(b) If $0 < r_D < n$, there exists a maximum feasible bubble $\hat{\beta}_0 l_0 = \hat{b}_0$, such that

(i) for any $\beta_0 l_0 = b_0$ in $[0, \hat{b}_0)$, there exists a unique asymptotically bubbleless equilibrium with initial bubble b_0 and the interest rate converges to r_D.

(ii) there exists a unique bubbly equilibrium with initial bubble $\hat{b}_0$. The bubble per capita converges to $\hat{b}$ and the interest rate converges to n.

Proof: The proof is basically equivalent to the proof of proposition 1 in Tirole (1985). Assumption A is needed for the proof of Lemma 1 in his appendix 1. (available by request.)

Proposition 2: If $r_D < n$, the asymptotically bubbleless equilibria are inefficient and the asymptotically bubbly equilibrium is efficient.

Proof: Equivalent to the proof of proposition 2 in Tirole (1985).

In summary, this example shows that an economy can be dynamically inefficient irrespective of the existence of land. We now turn to the question how the Cobb-Douglas production function can exclude dynamic inefficiency.

First, with the Cobb-Douglas function, a balanced growth path is analyzed in order to avoid the trivial steady state where output and capital per capita are zero. One can easily forget that the Golden rule criterion of dynamic inefficiency is based on a steady state where the growth rate of output per capita is zero. In a balanced growth path where output per capita grows at a non-zero constant rate g, an interest rate smaller than n can be dynamically efficient. For example, think of the Cobb-Douglas function with factor exponents α_1, α_2 and α_3 (for labor, capital, and land, respectively). The growth rate g satisfies the condition $(1+g)^t = (1+n)^{(\alpha-1)t}$, where $\alpha = \alpha_1/(\alpha_1+\alpha_3) < 1$. Then a sufficient condition for dynamic efficiency in a balanced growth path is[4];

$$(13) \quad \lim_{t\to\infty} \frac{(1+n)^t}{(1+r)^t}(1+g)^t = \lim_{t\to\infty} \frac{(1+n)^t}{(1+r)^t}(1+n)^{(\alpha-1)t} = 0, \text{ i.e.,} \quad 1+r > (1+n)^{\alpha}.$$

Second, irrespective of the choice of production function, the budget constraint implies that $\frac{f_t l_t}{y_t}$ should be less than one. From the definition (8),

$$(14) \quad \frac{f_t l_t}{y_t} = \left[\sum_{s=t+1}^{\infty} \frac{r^*_s}{(1+r_{t+1})\cdots(1+r_s)} \right] \frac{l_t}{y_t} \quad .$$

In a balanced growth path, one necessary condition for (14) to be finite is;

$$(15) \quad \lim_{t\to\infty} \left[\frac{r^*_t l_t}{y_t} \right] \left[\frac{(1+n)^t (1+g)^t}{(1+r)^t} \right] = 0 \ .$$

Since the Cobb-Douglas function requires that factor shares be constant, the first limit in (15) is a non-zero constant. Therefore, the second limit should go to zero, which is exactly the efficiency condition (13). With a non-Cobb-Douglas production function, however, even when land is essential,

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[4] For a proof, see Cass (1972) and McCallum (1987).

the income share of land can vanish in a balanced growth path and the second limit does not necessarily go to zero: Essential land does not generally preclude the possibility of dynamic inefficiency.[5]

Also, note that land being substitutable is not sufficient for the possibility of dynamic inefficiency. For example, when there is land-augmenting technical progress of the form $(1+n)^t$, dynamic inefficiency is not possible even when the elasticity of substitution between land and other factors of production is greater than one[6]. However, if we confine our interest to CES production functions without technical progress, high elasticity of substitution between land and other factors is sufficient for the possibility of dynamic inefficiency.

III. How Important Is Land?

Since the possibility of dynamic inefficiency depends on the income share of land in steady state, it would be interesting to examine the historical tendencies of the income share of land. Also, if land is not an essential factor and easily substitutable, a rough guess would be that the value of land

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5 The non Cobb-Douglas example in the footnote 3 has a positive income share of land and dynamic inefficiency can be precluded if the economy has a balanced growth path. However, it is not clear how to define a balanced growth path with this function; the capital-output ratio cannot be constant in a balanced growth path in this case.

6 This can be proved easily by applying (15), since the income share of land does not vanish.

relative to other tangible assets should have declined.

Table 1 and 2 record the time series of the percentage distribution of national income and net national product among factors of production. Table 3 examines the historical tendencies of the proportion of land value to the reproducible tangible assets. Table 1 and 2 adopt the different methods of imputation for the labor share of income. Table 1 is based on the assumption that the percentage of income allocatable to labor is the same in proprietorship and partnership as in corporations. Table 2 uses a different assumption that the after-tax rate of return of the non-corporate sector is the same as that of the corporate sector. As the non-corporate sector is believed to have a lower rate of return, the latter assumption tends to allocate more to the capital.

The share of land declined from about 9 per cent to 3 per cent from 1900 to 1950. Citing Dennison's (1962) table, Nordhaus and Tobin (1972) concluded that this trend is compatible with the assumption that the elasticity of substitution of land for other factors is greater than one. With the pre 1970 data, they estimated a three factor CES production function whose elasticity of substitution between land and the other joint factors is about 2. However, after 1950, Table 1 indicates that the decline in the share of land considerably slowed, making Nordhaus and Tobin's estimation implausible.

The same pattern is observed in Table 2 and 3. From 1900 to 1945, the proportional value of land in tangible assets dropped almost in half from 59 percent to 29 percent. However, there seems to be no declining tendency in postwar data. The last column of Table 3 shows an enormously high value of land in Japan. If we regard Japan as the limit case of declining land per capita, it indicates that the relative land value will grow eventually as land becomes scarce.[7]

It is likely that the declining tendency of the land income share and value before the fifties reflected primarily the diminishing importance of agriculture within the economy. As the agricultural sector reached its lower bound, land seems to have become less substitutable, and the Cobb-Douglas production function seems to be a reasonable approximation for the postwar data. Dynamic inefficiency may be precluded based on the empirical evidence of the postwar period.

IV. Conclusion.

This note shows that capital overaccumulation is possible *a priori* in an economy with land. The income share of land in the steady state and the substitutability between land and other factors of production in CES production functions are crucial to understand the possibility of dynamic inefficiency.

Another interesting case that can allow for the possibility of dynamic inefficiency in an economy with land is the differential capital gains tax: when capital gains from landholding are taxed more heavily than other capital gains, the price of land may not grow high enough to prevent capital overaccumulation even with the Cobb-Douglas production function. Likewise, the uncertainty of the value of underground materials reduces the price of land,

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[7] This argument does not take into account of the effect of different technical progress rate and macroeconomic policies on the price of land. For the discussion of the influence of macroeconomic policies on the land price, see Peter Boone (1989).

and its effect may be the same as the capital gains tax on land.

However, this note does not claim that actual economies are dynamically inefficient. Rather, it attempts to show that dynamic inefficiency should not be considered a dead issue theoretically even in an economy with a fixed factor. The impossibility of the dynamic inefficiency should be grounded on the empirical evidence of the income share of land and its substitutability.

Table 1. Percentage Distribution of National Income among Factors of Production

period	labor	land	reproducible capital: total	reproducible capital: non-farm residential structure	reproducible capital: others	foreign
1909-13	69.5	8.9	21.2	3.3	17.9	0.4
1914-18	67.0	8.8	23.8	3.5	20.3	0.4
1919-23	69.5	7.0	22.7	3.4	19.3	0.8
1924-28	69.7	6.4	23.0	4.3	18.7	0.9
1929-33	69.2	6.2	23.6	4.5	19.1	1.0
1934-38	70.4	5.6	23.2	3.6	19.6	0.8
1939-43	72.1	4.9	22.4	2.8	19.6	0.6
1944-48	74.9	4.0	20.6	2.2	18.4	0.5
1949-53	74.5	3.4	21.6	2.5	19.1	0.5
1954-58	77.3	3.0	19.0	3.0	16.0	0.7

period	labor	land	reproducible capital: total	reproducible capital: non-farm residential structure	reproducible capital: others	foreign
1948-50	75.3	5.2	18.9	5.6	13.3	0.6
1951-55	76.5	4.7	18.1	5.6	12.5	0.7
1956-60	77.8	4.7	16.7	5.5	11.2	0.8
1961-65	76.8	5.1	17.2	6.1	11.1	0.9
1966-70	78.6	4.2	16.4	5.3	11.1	0.8
1971-75	81.4	3.2	14.1	4.7	9.4	1.3
1976-80	81.0	3.5	13.8	4.5	9.3	1.7
1981-84	81.5	3.9	12.7	4.8	7.9	1.9

***** The upper table is from Table 4 in Denison (1962). The lower table updates his table using the Balance Sheet for the U.S.Economy (1947-88) and NIPA data. The labor income is imputed under the assumption that the percentage of national income allocated to labor is the same in proprietorship and partnership as in corporations. To allocate the property income among land, capital and residential structure, three sectors are classified: agriculture, non-farm residential sector, and other private economy. Income in each sector is allocated among the relevant types of property in proportion to its relative current value in balance sheet.

Table 2. Percentage Distribution of NNP among Factors of Production.

period	labor	land	capital	foreign	sales tax
1946-50	62.2	6.4	26.0	0.4	5.0
1951-55	62.4	5.5	26.8	0.6	4.7
1956-60	64.4	5.2	24.8	0.6	5.0
1961-65	62.5	6.3	25.5	0.7	5.0
1966-70	63.8	5.8	25.0	0.7	4.7
1971-75	67.0	4.2	22.9	1.0	4.9
1976-80	66.3	4.8	22.8	1.5	4.6
1981-85	64.7	5.6	23.5	1.4	4.8

***** Table 2 is based on the data used by Jorgenson and Yun (1986) and the assumption that the after-tax rate of return of the non-corporate sector is the same as that of the corporate sector. This assumption and the exponential depreciation method allocate more to capital compared with Table 1.

Table 3. The Proportion of Land to Other Tangible Assets

period	U.S.1	U.S.2	Japan
1900	0.59	--	--
1912	0.60	--	--
1922	0.44	--	--
1929	0.41	--	--
1933	0.37	--	--
1939	0.32	--	--
1945	0.29	0.22	--
1946-50	0.26	0.19	--
1951-55	0.25	0.19	--
1956-60	0.28	0.20	--
1961-65	0.31	0.22	--
1966-70	0.28	0.21	1.61
1971-75	0.25	0.19	1.42
1976-80	0.29	0.22	1.17
1981-85	0.31	0.23	1.35

***** The figures represent the proportion of the market value of land to the replacement cost of other tangible assets. The data used for the first column are from Historical Statistics of the U.S.(Series F422-445) and the Balance Sheets for the U.S. Economy (1947-86), normalized for comparability. The second column uses the data by Jorgenson and Yun(1986). The first two data sets value the capital stock with straight-line depreciation whereas the third set values it with exponential depreciation. The Japanese data are from various issues of Annual Report on National Account.

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