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1. INTRODUCTION

The causes and the consequences of credit rationing have attracted a great deal of attention among economists. Of particular interest have been the consequences of credit rationing for a broad array of government policy actions, ranging from general monetary and fiscal policy actions to very specific targeted credit or credit subsidy policies. Credit rationing is often argued to be a channel through which otherwise neutral monetary policy may have real effects [Tobin (1980)]. In addition, even in the U.S., but much more so in developing countries, government loan and credit subsidy programs targeted at the "victims" of credit rationing consume considerable resources and, one expects, put upward pressure on interest rates.

Despite the interest in credit rationing and its implications for policy, most research related to credit rationing and its policy consequences to date has proceeded in partial equilibrium contexts. [A short list of examples includes Stiglitz and Weiss (1981), Williamson (1987), Besanko and Thakor (1987), Smith and Stutzer (1988), and Gale (1989).] More specifically, it is common in models of credit rationing to ignore potential effects of rationing on market interest rates in general equilibrium. Yet such effects must surely exist if credit rationing provides a distinct channel through which macroeconomic policies operate. Moreover, the kinds of informational frictions that frequently underlie credit rationing have been argued [see, for instance, Friedman (1960), Brunner and Meltzer (1971), or Smith (1986)] to create a role for the provision of government issued fiat currency. Thus there must be at least two motives for studying the general equilibrium consequences of adverse selection and moral hazard.

Furthermore, if the existence of credit rationing creates a presumption in favor of government intervention, then public policy in the credit market must have its own general equilibrium consequences. As far as we know, these consequences have not been considered.[1] What are the implications of adverse selection for equilibrium returns on assets, including currency? Will economies with serious problems of private information tend to exhibit systematically different yields than do the corresponding Arrow–Debreu economies? Are they subject to similar inflationary pressures?

Secondly, if indeed the existence of a set of rationed borrowers does raise aggregate savings at any interest rate, and hence put downward pressure on equilibrium interest rates, what welfare consequences follow from this? For example, is it possible that reductions in interest rates caused by rationing are large enough so that credit rationing has a positive impact on the utility of unrationed (or even rationed) borrowers?

Finally, we ask what the implications of credit rationing are for the set of equilibria that can arise in models with both outside and inside "money," i.e. with both public and private debt. In particular, can the informational frictions that give rise to credit rationing substantially shrink or expand the set of equilibria? What do they imply for the dynamic (e.g., local stability) properties of equilibria of interest? The large recent literature on general equilibrium with incomplete markets suggests that private information should enlarge the set of equilibrium allocations.

This paper embeds a model of credit rationing into a pure exchange economy with or without national debt. More specifically, we look at an overlapping generations economy in which heterogeneous young agents engage in intragenerational borrowing and lending, and possibly in intergenerational exchange as well. However, intragenerational lending is encumbered by an adverse selection problem, which produces credit rationing in equilibrium. In particular, borrowers who have random old age endowments may default in unfavorable states of nature. Each young borrower knows his own default probability. If this information is private, lenders will seek to elicit it by using credit rationing as a self–selection mechanism. The sorting device actually employed, familiar from Rothschild and Stiglitz (1976), restricts the credit received by agents claiming to have low default probabilities.

Embedding private information in a pure exchange overlapping generations model allows us to examine some of the general equilibrium consequences of credit rationing. First, a version of the model with no outside "money" is considered. Here informational frictions can affect the set of equilibria in a smooth manner, if (pure strategy) equilibria exist under private information. However, the general equilibrium framework has substantial implications for the welfare consequences of credit rationing. For instance, in the absence of outside assets, credit rationing tends to result in reductions in equilibrium interest rates relative to the situation under full information. Then the informational friction has adverse consequences for savers. Interestingly, it is possible that credit rationing forces interest rates down enough so that rationed borrowers benefit from the informational friction that produces rationing in the first place. This, of course, would tend to attenuate arguments in favor of policies that benefit rationed borrowers.

We introduce next a constant stock of government debt. This permits two issues to be addressed. One of them is the effect of credit rationing on the set of dynamical equilibria. One result that emerges (which is to be expected given that this rationing tends in general to promote savings) is that some economies that are "classical" (in Gale's (1973) usage) under full information are "Samuelsonian" (again Gale's usage) under private information. In other words, economies that admit no equilibria with valued fiat money under full information will do so under private information. This finding provides support for arguments, like those given by Friedman (1960, p. 6–8), that private information provides a role for fiat currency that would not be present if "promises to pay were always fulfilled."

Private information (and the credit rationing that results from it) may turn out to have dramatic consequences for the structure of equilibria in economies with valued national debt. Under perfect information these economies generally have at least two stationary equilibria and a continuum of dynamical ones. However, if adverse selection becomes serious enough, then the only pure strategy equilibrium is the unstable steady state. This suggests that informational frictions may shrink the set of competitive equilibria, i.e., produce determinacy in dynamical economies that would feature indeterminate equilibria under full information and perfect foresight.

The last situation considered is one of a government faced with monetizing a fixed deficit of given size. Here two results are obtained. First, adverse selection enables the central bank to monetize larger deficits for any given economy than would be possible under public information. This is simply because credit rationing tends to increase aggregate savings, and hence enhance the inflation–tax base at any interest rate. Second, informational frictions that give rise to credit rationing can be either inflationary or deflationary, depending on which side of the Laffer curve an economy finds itself. If these forces are deflationary then, interestingly, lenders benefit from facing an adverse selection problem in loan markets. This again emphasizes that the welfare repercussions of credit rationing depend heavily on its general equilibrium implications for rates of return.

Before giving the details of the analysis, we should stress that these results are obtained in the context of a pure exchange economy where all loans are for consumption purposes. The consequences of credit rationing in production economies where investment is financed via private lending are the topic of future research.

2. THE MODEL

A. Physical Environment

The economy considered is one of pure exchange: it consists of a sequence of two–period lived overlapping generations, as well as an initial old generation. All young generations are identical in size and composition, and consist of three groups of agents, to be described below. Time is discrete and indexed by $t = 0,1,...$ A single non–storable good is consumed at each date.

The first group of young agents will, somewhat loosely, be termed "savers." This group can actually be arbitrarily heterogeneous, containing agents with different preferences whose endowment is positive in youth and zero in old age. A fraction μ of the population belongs to this group. The economic behavior of the group as a whole is represented by an aggregate savings function, $s(r_t)$, where r_t denotes the (gross) rate of interest paid to members of this group at t. We assume that s is continuously differentiable.

The second group of agents will be termed "borrowers." There is a large number of these individuals (technically a continuum). All borrowers have the same additive utility function, $u(c_1) + v(c_2)$, where c_j denotes age j consumption, and u and v are standard utility functions defined on the consumption set R_+ such that $v(0) = 0$. For simplicity, borrowers are assumed to have no endowment when young.

The old age endowment of borrowers is a binary random variable $\tilde{w}$ whose realization is drawn from the set $\{0,w\}$. Borrowers are divided into two types, indexed by $i \in \{H,L\}$: type i has probability p_i of receiving a positive second period endowment. We suppose $1 \geq p_L > p_H > 0$, so type L borrowers have a lower probability of zero endowment. Realizations of $\tilde{w}$ are independent over borrowers, who comprise a fraction $1-\mu$ of each young generation. A fraction $\lambda \in (0,1)$ of borrowers is of type H, and $1 - \lambda$ are of type L.

The third group of agents is termed "intermediaries". Any saver can establish an intermediary, which accepts deposits, makes loans, and earns zero profits. Intermediary behavior is described in detail in the next section.

Finally, we introduce a government. At first, the government issues a constant stock of interest–bearing nominal debt (which may be negative) at t=0, and is otherwise inactive. Section 6 considers the case of a government faced with a deficit to be monetized.

B. Individual Behavior

Intermediaries are the key element of our economy. On the deposit side of the market, intermediaries are competitive taking the (gross) deposit rate r_t at t as given. Then each intermediary that operates at t pays r_t on deposits, and accepts all deposits offered. Aggregate saving is simply $s(r_t)$.

On the loan side intermediaries are Nash competitors. They offer loan contracts to borrowers, which consist of pairs (R_{it}, ℓ_{it}), where R_{it} is a gross interest rate on loans offered to type i borrowers at t, and ℓ_{it} is a loan quantity offered to type i borrowers at t. Throughout we restrict attention to pure strategies for intermediaries.

Intermediaries, then, choose contract terms (R_{it}, ℓ_{it}); $i \in \{H,L\}$, taking the contracts offered by other intermediaries as given. In offering these contracts an intermediary may attempt to induce self–selection of borrowers. If a particular intermediary attempts to induce self–selection, its contracts must be incentive compatible. It will be assumed that any borrower can borrow from only one intermediary, so that a borrower taking a type i contract has $c_1 = \ell_{it}$, $c_2 = w - R_{it}\ell_{it}$ if $\tilde{w} = w$, and $c_2 = 0$ otherwise. Then self–selection requires that, $\forall t$,

$$(1) \qquad u(\ell_{Ht}) + p_H v(w - R_{Ht}\ell_{Ht}) \geq u(\ell_{Lt}) + p_H v(w - R_{Lt}\ell_{Lt})$$

$$(2) \qquad u(\ell_{Lt}) + p_L v(w - R_{Lt}\ell_{Lt}) \geq u(\ell_{Ht}) + p_L v(w - R_{Ht}\ell_{Ht}).$$

Notice that $c_2 = 0$ if $\tilde{w} = 0$, so that state contingent claims trading with respect to second period endowment realizations has been ruled out for simplicity. Finally, if an intermediary chooses not to induce self–selection, its contract offers satisfy $(R_{Ht}, \ell_{Ht}) = (R_{Lt}, \ell_{Lt})$, which trivially satisfy (1) and (2).

As will be apparent, the behavior of intermediaries here is exactly that analyzed by Rothschild and Stiglitz (1976). Following them, we require that all contract offers must at least break even individually (in expected terms). Then any separating contracts $(R_{Ht}, \ell_{Ht}) \neq (R_{Lt}, \ell_{Lt})$ must satisfy $p_i R_{it} \geq r_t$. Similarly, any pooling contract $(R_{Ht}, \ell_{Ht}) = (R_{Lt}, \ell_{Lt})$ satisfies $[\lambda p_H + (1-\lambda)p_L]R_t \geq r_t$.

Assuming free entry into the intermediation industry, we focus on equilibria where each intermediary serves infinite numbers of lenders and depositors. Thus intermediary profits on type i contracts are $(p_i R_{it} - r_t)\ell_{it}$ per borrower at t. To complete the description of agents' behavior, borrowers simply accept their most preferred loan contract from among the set of offered contracts.

To summarize, savers save $s(r_t)$ in the aggregate. Some or all of this amount is deposited with intermediaries, which offer the loan contracts (R_{it}, ℓ_{it}) to type i; $i \in \{H,L\}$. Equilibrium loan contracts are described next.

3. EQUILIBRIUM LOAN CONTRACTS

A. Full Information

As a benchmark, we describe briefly Nash equilibrium loan contracts under full information. Clearly, free entry implies that zero profits must obtain in equilibrium. Since intermediaries take r_t as given, loan rates satisfy

$$(3) \qquad R_{it} = r_t/p_i;\ i = H,L.$$

Further, under full information, the self–selection constraints (1) and (2) are irrelevant. Thus competition among banks for borrowers implies that ℓ_{it} must be maximal for type i borrowers at t, given that R_{it} satisfies (3). Then ℓ_{it} is defined from

$$\ell_{it} = \underset{0 \le \ell \le w/R_{it}}{\operatorname{argmax}} \; [u(\ell) + p_i v(w - R_{it}\ell)]$$

Denote the maximizing value of ℓ_{it} by $\ell(r_t, p_i)$, where $R_{it} = r_t/p_i$. Under full information, equilibrium contracts are just $(r_t/p_i, \ell(r_t, p_i))$. We conclude this section by observing that these contracts will not satisfy the self–selection constraint (1) if $\ell(r_t, p_i) > 0 \ \forall$ i, and borrowers enjoy a positive gain from trade.

Proposition 1. If $\ell(r_t,p_i) > 0\ \forall i$, and if $u[\ell(r_t,p_i)] + p_i v[w - (r_t/p_i)\ \ell(r_t,p_i)]$

$> u(0) + p_i v(w)\ \forall$ i, then

$$u[\ell(r_t,p_L)] + p_H v[w - (r_t/p_L)\ \ell(r_t, p_L)] > u[\ell(r_t,p_H)] + p_H v[w - (r_t/p_H)\ \ell(r_t,p_H)].$$

The proof is given in the appendix. The proposition asserts that, under private information, the incentive constraint (1) must hold with equality at any separating equilibrium.

B. Private Information

Having shown that the self–selection constraint (1) binds on any contract announcements that cause borrowers to self–select, it is now straightforward to describe Nash equilibrium contracts (in pure strategies). In particular, identical arguments to those given by Rothschild and Stiglitz (1976) establish that (i) any equilibrium contracts must induce self–selection (so $(R_{Ht}, \ell_{Ht}) \neq (R_{Lt}, \ell_{Lt})$); (ii) any equilibrium contracts must earn zero profits, and (iii) type i contracts must be maximal for type i borrowers among the set of contracts that satisfy (1)–(3), given other available contracts. Notice that (i) and (ii) imply that the zero–profit condition (3) continues to obtain. Also, it is the case that no pure strategy Nash equilibrium loan contracts need exist. Since conditions that imply the existence of equilibrium loan contracts in pure strategies play an important role in determining what the overall set of equilibria is, these conditions will be discussed in detail below.

Before giving the details of any candidate equilibrium loan contracts, it will be useful to display equilibrium contracts diagramatically for any given value of r_t. Consider the situation in Figure 1. The gross loan rate of interest R appears on the vertical axis, and the loan quantity ℓ appears on the horizontal axis. Figure 1 depicts a representative indifference curve for type H borrowers, and three indifference curves for type L borrowers. Of course, each indifference curve in the figure is described by a locus of the form $u(\ell) + p_i v(w-R\ell) = k$, where k is an arbitrary constant. Then the slope of a type i indifference curve through any (ℓ, R) pair is given by

$$(4) \qquad \left.\frac{dR}{d\ell}\right|_{dU_i=0} = \frac{u'(\ell)}{p_i v'(w-R\ell)\ell} - \frac{R}{\ell}.$$

It is then immediate that the type H indifference curve through any point has an algebraically larger slope than the type L indifference curve through the same point. Thus a standard "single–crossing property" for preference maps obtains.

As shown in Figure 1, equilibrium loan rates R_{it} must satisfy $R_{it} = r_t/p_i$. Then competition among intermediaries for type H borrowers must imply that type H borrowers receive their most preferred contract (among those earning zero profits). Therefore, type H contracts occur at the tangency between a type H indifference curve and the type H zero profit locus ($R_{Ht} = r_t/p_H$).

Similarly, type L contracts must be maximal for type L agents among the set of incentive–compatible contracts earning non–negative profits when taken by type L agents ($R_{Lt} \geq r_t/p_L$). As shown in that figure, the self–selection constraint binds on the determination of the type L contract, which must lie on or above the type H indifference curve through H. It is apparent that the most preferred such contract for type L agents occurs at the point L_1, where the type H indifference curve through H intersects the type L zero profit locus. It is also clear from Figure 1 that the intersection implying the smallest value of ℓ_{Lt} is the relevant one. And finally, at the interest factor r_t/p_L, type L agents would notionally like to borrow the amount associated with the point L*. Thus type L agents face credit rationing in a separating equilibrium.[2]

It is now straightforward to describe candidate equilibrium loan contracts fully. As shown in Figure 1, type H contracts are unencumbered by any considerations of self–selection. Then $R_{Ht} = r_t/p_H$, and ℓ_{Ht} maximizes $u(\ell) + p_H v[w - (r_t/p_H)\ell]$. Then, $\ell_{Ht} = \ell(r_t, p_H)$.

Focusing for the present on interior solutions, ℓ_{Ht} satisfies the first–order condition

$$(5) \qquad u'(\ell_{Ht}) = r_t v'[w - (r_t/p_H)\ell_{Ht}]$$

This implies that ℓ is a decreasing function of r_t whenever $\ell > 0$.

To describe candidate equilibrium contracts for type L agents, recall that $R_{Lt} = r_t/p_L$ and that ℓ_{Lt} occurs at the (smallest) intersection of the type H indifference curve through H and the type L zero profit locus. Then, letting $\ell_{Lt} \equiv f(r_t)$, $f(r_t)$ satisfies

$$(6) \qquad u[f(r_t)] + p_H v[w - (r_t/p_L)f(r_t)] = u[\ell(r_t,p_H)] + p_H v[w - (r_t/p_H)\ell(r_t,p_H)].$$

Then, as Figure 1 suggests, $f(r_t) < \ell(r_t,p_L)$. Moreover, as will again be apparent from the figure, $u'[f(r_t)] > r_t v'[w - (r_t/p_L)f(r_t)]$. Then it is straightforward to see that f too is a decreasing function of r_t.

C. Existence of Pure Strategy Nash Equilibria

By construction, no contract exists that earns non–negative profits and that type H borrowers prefer to the contract $(R_{Ht}, \ell_{Ht}) = [r_t/p_H, \ell(r_t,p_H)]$. Similarly, there is no profitable contract that type L borrowers prefer to $(R_{Lt}, \ell_{Lt}) = [r_t/p_L, f(r_t)]$ and that does not attract type H borrowers. Hence if the contracts specified are not Nash equilibrium contracts, some intermediary will have an incentive to offer a pooling contract, which will then attract agents in their population proportions. When a pooling contract exists that (a) earns non–negative profits when it attracts borrowers in their population proportions, and (b) is preferred by all agents to the separating contracts discussed above, those separating contracts do not constitute a Nash equilibrium. Moreover, for the same reasons given by Rothschild and Stiglitz (1976), no pooling contracts will be observed in equilibrium. Then there is no equilibrium in pure strategies when conditions (a) and (b) above are satisfied.

Letting $p(\lambda) \equiv \lambda p_H + (1-\lambda)p_L$, the most preferred pooling contract for type L agents has $R_t = r_t/p(\lambda)$, and sets ℓ_t to maximize $u(\ell) + p_L v\{w - [r_t/p(\lambda)]\ell\}$. The solution sets $\ell_t = \ell[p_L r_t/p(\lambda), p_L]$. Then, a pooling contract satisfying (a) and (b) fails to exist iff

$$(7) \qquad u[f(r_t)] + p_L v[w - (r_t/p_L)f(r_t)] \geq u\{\ell[p_L r_t/p(\lambda), p_L]\} + p_L v\{w - [r_t/p(\lambda)]\ell[p_L r_t/p(\lambda), p_L]\}.$$

It should be immediately apparent that (7) fails for $\lambda=0$. It is also straightforward to show that (7) holds with strict inequality for $\lambda=1$. Then by continuity there exists a value $\bar{\lambda}(r_t) \in (0,1)$ such that (7) holds with equality. For completeness, $\bar{\lambda}(r_t)$ is uniquely defined by

$$(8) \qquad u[f(r_t)] + p_L v[w - (r_t/p_L)f(r_t)] = u\{\ell[p_L r_t/p[\bar{\lambda}(r_t)], p_L]\} + p_L v\{w - [r_t/p[\bar{\lambda}(r_t)]]\ell[p_L r_t/p[\bar{\lambda}(r_t)], p_L]\}.$$

Clearly, then, a Nash equilibrium in pure strategies exists iff $\lambda \geq \bar{\lambda}(r_t)$.

The properties of the set of equilibria with outside assets depend critically on the properties of $\bar{\lambda}(r_t)$, and in particular, on whether $\bar{\lambda}(r_t)$ is increasing or decreasing in r_t. In general the sign of $\bar{\lambda}'(r_t)$ is ambiguous. Some examples illustrating this are as follows.

Example 1. Let $u(c_1) = c_1$ and $v(c_2) = \beta c_2$. Then for all $r_t < 1/\beta$, $\ell(r_t,p_i) = p_i w/r_t$. In addition, direct substitution into (1) at equality establishes that

$$f(r_t) = \frac{(p_H w/r_t) - p_H \beta w}{1 - (p_H/p_L)\beta r_t}.$$

Then (8) implies that $\bar{\lambda}(r_t)$ is given by

$$\bar{\lambda}(r_t) = \frac{1 - \beta r_t}{1 - (p_H/p_L)\beta r_t}$$

for $\beta r_t < 1$. Then $\bar{\lambda}$ is clearly decreasing.

Example 2. Let $u(c_1) = -c_1^{-\gamma}/\gamma$; $\gamma > -1$, and let $v(c_2) = \beta c_2$. Then it is straightforward but tedious to show that $\bar{\lambda}$ is independent of r_t for all r_t satisfying $(r_t/p_H)\ell(r_t,p_H) < w$.[3]

4. GENERAL EQUILIBRIUM WITH INSIDE MONEY

The loan market model of the previous section is now embedded in a complete general equilibrium setting. In this section the situation with no outside assets is considered. Outside assets are introduced in section 5. We discuss first the benchmark case of full information.

A. Full Information

Under full information an equilibrium is a sequence of contracts $\{(R_{it},\ell_{it})\}_{t=0}^{\infty}$; i = H,L, and a sequence of deposit rates $\{r_t\}_{t=0}^{\infty}$ such that (a) no intermediary can earn positive expected profit by offering an alternative set of contracts at the same r_t; and (b) the asset market clears. Asset market clearing with no currency or national debt requires that

$$\mu s(r_t) = (1 - \mu)[\lambda\ell(r_t,p_H) + (1-\lambda)\ell(r_t,p_L)] \tag{9}$$

Defining the economy–wide savings function $h(r_t)$ by $h(r_t) = \mu s(r_t) - (1-\mu)[\lambda\ell(r_t,p_H) + (1-\lambda)\ell(r_t,p_L)]$, we can rewrite equation (9) as $h(r_t) = 0$.

This model of full information is simply the standard overlapping generations model with heterogeneous agents. Since "savers" will not save if r = 0 and "borrowers" will not borrow at $r = \infty$, at least one positive, finite r_t will exist that satisfies $h(r_t) = 0$. In general there is an odd number of such interest rates.

B. Private Information

Under private information, an equilibrium is a sequence of contracts $\{(R_{it},\ell_{it})\}_{t=0}^{\infty}$; $i = H,L$, and a sequence $\{r_t\}_{t=0}^{\infty}$ such that (i) (R_{it},ℓ_{it}) for $i = H,L$, satisfies relations (1)–(3); (ii) given r_t, no intermediary has an incentive to offer a set of contracts satisfying (1)–(3) other than (R_{it},ℓ_{it}); and (iii) the asset market clears.

Asset market clearing here requires that

$$\text{(10)} \quad \mu s(r_t) = (1-\mu)[\lambda\ell(r_t,p_H) + (1-\lambda)f(r_t)]$$

since now type L borrowers are rationed, receiving loans of size $f(r_t)$ at t. Finally, in light of the discussion of section 3C, the contracts described there satisfy condition (ii) iff $\lambda \geq \bar{\lambda}(r_t)$. If this condition fails, there is no pure strategy equilibrium for intermediaries.

To express (10) more conveniently, define the aggregate savings function $g(r_t)$ by

$$g(r_t) \equiv \mu s(r_t) - (1-\mu)[\lambda\ell(r_t,p_H) + (1-\lambda)f(r_t)].$$

Then any equilibrium satisfies $g(r_t) = 0$, $R_{it} = r_t/p_i$, $\ell_{Ht} = \ell(r_t,p_H)$, $\ell_{Lt} = f(r_t)$, and $\bar{\lambda}(r_t) \leq \lambda \ \forall$ t.

The effects of credit rationing on equilibrium interest rates r_t can be seen by comparing the functions h and g. Clearly,

$$g(r_t) - h(r_t) \equiv (1-\mu)(1-\lambda)[\ell(r_t,p_L) - f(r_t)] \geq 0.$$

Here the inequality follows from the fact that $\ell(r_t,p_L) \geq f(r_t)$, and it is strict if $\ell(r_t,p_L) > 0$. Therefore, the general consequences of private information depend on the properties of the functions s and $\bar{\lambda}$.

Suppose first that s is an increasing function of r, as depicted in Figure 2a. Then, since the first partial derivative $\ell_r(r_t,p_i) \leq 0$, there is a unique equilibrium under full information. For similar reasons, there is at most one equilibrium under private information. If a pure strategy equilibrium exists under private information, it must occur at a lower interest rate than that which obtains under full information (i.e., $g^{-1}(0) < h^{-1}(0)$). All of this is apparent from Figure 2a.

If s is locally decreasing in r, private information can impact on the set of equilibrium interest rates in more subtle ways. Two of these are depicted in Figures 2b and 2c. In Figure 2b private information enlarges the set of equilibrium interest rates. (Of course, some or all of these rates may fail to satisfy $\bar{\lambda}(r) \leq \lambda$. If $\bar{\lambda}'(r_t) = 0$, for instance, either all or none of them are admissible equilibria.) All equilibrium rates under private information lie below the unique equilibrium interest rate under full information. And in Figure 2c private information causes the set of equilibrium interest rates to shrink.

While private information affects the set of "inside money" equilibria when the income effect of an interest–rate change dominates the corresponding substitution effect for some values of r_t, there are no large qualitative effects, and little can happen that is qualitatively much different from the case of full information. This will not be the case when we introduce currency.

C. Welfare Consequences of Credit Rationing

When the savings function s is monotone, private information introduces two potential sources of inefficiency relative to the situation under full information. First, it causes the standard welfare losses associated with binding incentive constraints, and second, it puts downward pressure on the rate of interest. Who bears the adverse welfare consequences of the informational frictions? In partial equilibrium, the answer would be as follows: Since r_t is fixed, savers and type H borrowers suffer no adverse consequences from adverse selection; all such consequences are born by rationed type L borrowers. Once r_t is allowed to change, our answer must change as well. First, clearly anyone saving in positive amounts suffers when interest rates are reduced. Thus, if s is monotone, private information must reduce interest rates (if an equilibrium in pure strategies exists) and make savers worse off. For the same reason, type H borrowers benefit from the adverse selection problem they cause. The welfare consequences of credit rationing for type L borrowers are no longer obvious; they cannot borrow as much as they would like at the going interest rate, but potentially benefit from declining interest rates. It turns out that the latter effect can dominate. This is now illustrated by example.

Example 3. Let $s(r_t) \equiv 1$, let $u(c_1) = c_1$, let $v(c_2) = \beta c_2$, let $\beta = 1.452$, and let $p_H = .3$, $p_L = .51$, $\mu = .2$, $w = .5$, and $\lambda = .8$. Then the unique equilibrium under full information has $r_t = .684 \; \forall \; t$. The utility of type L borrowers in this equilibrium is $p_L w/r = .3728$.

Under private information the unique equilibrium value of $r_t = .53$. Using the expression for $f(r_t)$ given in example 1, $f(.53) = .1569$. Then type L utility under private information is $\beta p_L w + (1-\beta r)f(r) = .4065$.[4] Thus in this example all the adverse welfare consequences of credit rationing are born by savers. The "victims" of credit rationing (type L borrowers) benefit by it.[5]

5. NATIONAL DEBT

A government playing some role in the economy is now introduced. The government has constant per capita expenditures of $G \geq 0$, all of which are financed by issuing government debt.[6] Let z_t denote the real value of government debt issued at t, which may be positive, negative, or zero. Satisfaction of the government budget constraint at each date requires that

(11) $G = z_t - r_{t-1}z_{t-1}$,

where government debt bears the rate of return r_t, and is default free. For the remainder of this section we assume G=0. Positive deficits are examined in section 6. For simplicity, it is henceforth assumed that $s'(r_t) \geq 0$. That implies $g'(r_t) \geq 0$ and $h'(r_t) \geq 0 \ \forall \ r_t$ as well.

A. <u>Full Information</u>

Again the situation under full information is considered as a benchmark. Here an equilibrium is a sequence of contracts $\{(R_{it}, \ell_{it})\}_{t=0}^{\infty}$ for i = H,L, a sequence of deposit rates $\{r_t\}_{t=0}^{\infty}$, and a sequence $\{z_t\}_{t=0}^{\infty}$ (z_0 given) such that (a) given r_t, no intermediary can earn a positive expected profit by announcing a contract other than (R_{it}, ℓ_{it}); (b) the asset market clears; and (c) the government budget constraint (11) is satisfied with G=0.

Asset market clearing now requires that

(12) $h(r_t) = z_t$.

What equilibria exist for this economy depends on the unique, generationally autarkic, interest rate $r^* = h^{-1}(0)$. Following Gale (1973), the case where $h^{-1}(0) \geq 1$ is termed the "classical case," while the case with $h^{-1}(0) < 1$ is termed the "Samuelson case." We now briefly review the set of equilibria in each case.

"<u>Classical Case</u>." If $h^{-1}(0) \geq 1$, then there are two steady state equilibria. One has $z_t = 0$ and $r_t = h^{-1}(0)$, and the other has $r_t = 1$ and $z_t = h(1) < 0$.[7] In addition, the "golden rule" steady state is asymptotically stable, so there exist a continuum of non–stationary equilibria converging monotonically to the golden rule steady state. From (11) and (12) it is obvious that these equilibria satisfy $z_t = h(z_{t+1}/z_t)$. The set of equilibria is depicted in Figure 3a; see also Cass et al. (1979).

"**Samuelson Case**." If $h^{-1}(0) < 1$, then there are again two steady state equilibria: one with $r_t = h^{-1}(0)$ and $z_t = 0$; and the "golden rule" steady state with $r_t = 1$ and $z_t = h(1) > 0$. In this case the golden rule steady state is unstable, and there is a continuum of non–stationary equilibria evolving according to $z_t = h(z_{t+1}/z_t)$ and converging to $z = 0$. The set of equilibria is depicted in Figure 3b.

B. Private Information

Some authors (e.g., Gale (1973)) have been uncomfortable with the overlapping generations model as a model of money because they accept Irving Fisher's reasons for interest rates in excess of natural growth rates ($h^{-1}(0) > 1$). They thus regard the "classical case" as the "natural" situation under full information.

At the same time, other authors (e.g., Friedman (1960)) have argued that informational frictions create a role for government provision of currency and similar assets. The possibility, illustrated in section 4, that private information reduces the "inside money" equilibrium interest rate, also raises the possibility that private information can convert a "classical case" economy under full information ($h^{-1}(0) \geq 1$) into a "Samuelson case" economy ($g^{-1}(0) < 1$). This possibility would provide a validation of Friedman's argument. This issue, and other implications of private information for the set of equilibria are now investigated.

Under private information, an equilibrium is a sequence of contracts $\{(R_{it},\ell_{it})\}_{t=0}^{\infty}$, a sequence $\{r_t\}_{t=0}^{\infty}$, and a sequence $\{z_t\}_{t=0}^{\infty}$ such that (i) contract offers satisfy (1) – (3) $\forall$ t, (ii) given r_t, no intermediary has an incentive to offer a set of contracts other than (R_{it},ℓ_{it}), (iii) the asset market clears, and (iv) the government budget constraint is satisfied. As before, conditions (i) and (ii) are satisfied by the contracts described in section 3 iff $\lambda \geq \bar{\lambda}(r_t)$ holds $\forall$ t. With outside assets, (iii) now requires that

$$(13) \quad g(r_t) = z_t$$

Finally, we will say that an economy is "classical" ("Samuelson") under full information if $h^{-1}(0) \geq (<) 1$. An economy is "classical" ("Samuelson") under private information if $g^{-1}(0) \geq (<) 1$. Equilibria of each type under private information are now described.

B.a. "Classical" Economies

Suppose $g^{-1}(0) \geq 1$. Then the set of candidate equilibria consists of two potential steady states.[8] One has $r_t = g^{-1}(0)$ and $z_t = 0$, and one ("the golden rule") has $r_t = 1$ and $z_t = g(1) < 0$. In addition, there are potentially a continuum of non–stationary equilibria with $r_t = z_{t+1}/z_t$ and z_t evolving according to $z_t = g(z_{t+1}/z_t)$.

These candidate equilibria may or may not satisfy $\lambda \geq \bar{\lambda}(r_t)$, however. Three possibilities are of interest in this respect.

Case 1. $\bar{\lambda}'(r_t) = 0$. In this case, either all of the candidate equilibria satisfy $\lambda \geq \bar{\lambda}(r_t)$, or none of them do. Then the set of equilibria (where only pure strategy equilibria are considered with respect to contracts) is either empty, in which case only mixed strategy equilibria with respect to contracts would be observed, or the set of equilibria is essentially the same as in the classical case under full information.

Case 2. $\bar{\lambda}'(r_t) < 0$, but $\lambda \geq \bar{\lambda}(1)$. Then again, all candidate equilibria satisfy $\lambda \geq \bar{\lambda}(r_t)$, subject to the additional requirement that $\bar{\lambda}(z_2/z_1) \leq \lambda$ hold. Any equilibrium paths with $z_t \geq g(1)\ \forall\ t$ satisfy this requirement.

Case 3. $\bar{\lambda}'(r_t) < 0$, and $\bar{\lambda}(1) > \lambda \geq \bar{\lambda}[g^{-1}(0)]$. In this case, the steady state equilibrium with $r_t = g^{-1}(0)\ \forall\ t$ satisfies $\lambda \geq \bar{\lambda}(r_t)$. All other candidate equilibrium paths for this economy (abstracting from equilibria associated with mixed strategies) have $r_t \to 1$, and hence eventually violate $\lambda \geq \bar{\lambda}(r_t)$. Of course, if $\bar{\lambda}[g^{-1}(0)] > \lambda$, this economy has no (pure strategy) equilibria under private information.

In case 3, then, the presence of private information has dramatic consequences for the set of equilibria, forcing the economy to a unique equilibrium path that is associated with the use of pure strategies by intermediaries.

B.b. <u>"Samuelson" Economies</u>

When $g^{-1}(0) < 1$, the set of potential equilibria that do not require intermediaries to employ mixed strategies are as follows. There are two candidate steady states, one having $r_t = g^{-1}(0)$, and another having $r_t = 1$, and $z_t = g(1) > 0$. In addition, the golden rule steady state is unstable, while the steady state equilibrium with $z_t = 0$ is asymptotically stable. Then potentially there are a set of non–stationary candidate equilibria with $r_t = z_{t+1}/z_t$, and $z_t = g(z_{t+1}/z_t)$ that converge to the steady state equilibrium with $z_t = 0$.

As in the classical case, some or all of these candidate equilibria may fail to satisfy $\lambda \geq \bar{\lambda}(r_t)\ \forall\ t$. As before, the following three cases illustrate possibilities of interest.

<u>Case 1</u>. $\bar{\lambda}'(r_t) = 0\ \forall\ r_t$. As in the classical case, the set of equilibria consistent with pure strategies contains all of the candidate equilibria described, or none of them.

<u>Case 2</u>. $\bar{\lambda}'(r_t) < 0$ and $\lambda \geq \bar{\lambda}[g^{-1}(0)]$. Then all candidate equilibria with $z_t \geq 0\ \forall\ t$ satisfy $\lambda \geq \bar{\lambda}(r_t)$.

<u>Case 3</u>. $\bar{\lambda}'(r_t) < 0$, and $\bar{\lambda}[g^{-1}(0)] > \lambda \geq \bar{\lambda}(1)$. In this case, the only candidate equilibrium path that is consistent with the existence of a pure strategy equilibrium for loan contracts is the monetary steady state with $r_t = 1\ \forall\ t$. Again private information dramatically reduces the set of equilibria not requiring mixed strategies.

It is straightforward to produce examples illustrating this result. One is as follows.

<u>Example 4</u>. Let $s(r_t) = 1 \; \forall \; r_t$, and let $u(c_1) = c_1$, $v(c_2) = \beta c_2$. Further, let $\beta = .4$, $p_H = .3$, $p_L = .6$, $\lambda = .8$, $\mu = .5$, and $w = 1$. Then, under full information, there is a steady state with $r_t = h^{-1}(0) = .36$, a golden rule steady state with $r_t = 1$, and a continuum of non–stationary equilibria. However, as in example 1, $\bar{\lambda}'(r_t) < 0 \; \forall \; r_t < 1/\beta$. Moreover $\bar{\lambda}^{-1}(.8) = (1 - \lambda)/\beta[1 - \lambda(p_H/p_L)] = .833$. Thus no equilibrium path under private information is consistent with the existence of a pure strategy equilibrium if it has $r_t < \bar{\lambda}^{-1}(\lambda) = .833$ for any t.

Under private information, $g^{-1}(0) = .2962$, and all candidate equilibria other than the monetary steady state have $r_t \to g^{-1}(0)$. Thus the only equilibrium path with $r_t \geq \bar{\lambda}^{-1}(\lambda) \; \forall$ t is the monetary steady state!

C. <u>Private Information and the "Role" for Money</u>

As alluded to previously, Fisher has argued that the "natural" situation under full information is what we now call the "classical case". Others have argued that informational frictions create a role for outside money which would not be present if all information were freely available. These arguments suggest a sense in which private information "converts" classical case full–information economies into Samuelson case economies. This in fact happens here whenever $h^{-1}(0) \geq 1 > g^{-1}(0)$. It is easy to characterize when this can occur.

<u>Proposition 2</u>. Suppose $h(1) \leq 0$. Then $g(1) > 0$ iff

$$(14) \quad \ell(1,p_L) - f(1) > -h(1)/(1 - \mu)(1 - \lambda) \geq 0.$$

The proposition follows from observing that $g(r_t) = h(r_t) + (1 - \mu)(1 - \lambda)[\ell(r_t,p_L) - f(r_t)]$.

Proposition 2 asserts that private information converts a classical case economy ($h(1) \leq 0$) into a Samuelson case economy ($g(1) > 0$) <u>iff credit rationing is sufficiently severe at the golden rule steady state</u>, in the sense defined by (14). Moreover, for any savings function $s(r_t)$ satisfying $s(1) > 0$, and for any specification of borrower preferences, endowments, and default probabilities (which imply $\ell(r_t,p_i)$ and $f(r_t)$), the parameters μ and λ can be chosen so that (14) holds. In fact, they can be chosen so that $\lambda \geq \bar{\lambda}(1)$ holds as well, since $\bar{\lambda}(1) < 1$ and $h(1)$ can be made arbitrarily small by choice of μ. Thus this model validates arguments that preference and income patterns can make the classical case the natural situation, while informational frictions create a role for government issued fiat currency.

Proposition 2 also has the following, closely related corollary. Recalling that $h'(r_t) \geq 0$ and $g'(r_t) \geq 0 \; \forall \; r_t$ by assumption, we have

Corollary. If $h(1) \leq 0$ and if (14) holds, then the golden rule steady state is asymptotically stable under full information, and unstable under private information.

Thus, if the conditions of proposition 2 hold, and in particular, if credit rationing is sufficiently severe, this rationing introduces two sources of inefficiency. One is the standard static inefficiency associated with binding incentive constraints. The other is that credit rationing causes locally stable, low interest rate ($r_t < 1$) equilibria to exist, which are dynamically inefficient as well.[9] This result is illustrated in Figure 4. It is also illustrated by the following example.

Example 5. As in the previous examples, $u(c_1) = c_1$ and $v(c_2) = \beta c_2$. $s(r_t) = .55 \; \forall \; r_t$ holds, and $\beta = .95$, $p_H = .3$, $p_L = .6$, $\mu = .5$, $\lambda = .8$, and $w = 2$. Then under full information $h^{-1}(0) = 1.3091$, while under private information $g^{-1}(0) = .9212$. It is straightforward to check that $\bar{\lambda}[g^{-1}(0)] < .8$.

D. Welfare Consequences of Credit Rationing

Assuming that a particular economy has a golden rule steady state equilibrium under private information, it is apparent that the welfare consequences of private information associated with this equilibrium are exactly the same as one finds in partial equilibrium contexts. In particular, in this equilibrium $r_t = 1$ independently of whether information is symmetric or not. Thus all welfare consequences of private information fall on type L borrowers.

6. CREDIT RATIONING AND DEFICIT FINANCE

We now consider the situation of a government faced with monetizing a constant deficit ($G > 0$). It will be convenient to add the following notation: M_t is the time t per capita money supply (M_{-1} given), and p_t is the time t price level. Then $z_t = M_t/p_t$, and $z_{-1} = M_{-1}/p_0$.

The definition of an equilibrium is the same as in the previous section under both full and private information. For the remainder of the section we focus on steady state equilibria. Also, since no new issues arise with respect to the existence of equilibria here, it is assumed that $\bar{\lambda}'(r_t) = 0 \ \forall \ r_t$. Then either all candidate equilibria examined satisfy $\lambda \geq \bar{\lambda}(r_t)$, or none do. Finally, in addition to assuming that $h'(r_t) \geq 0$ and $g'(r_t) \geq 0$, we assume that $h''(r_t) \leq 0$ and that $g''(r_t) \leq 0$ as well.[10]

A. Full Information

Beginning again with the full information case, the market clearing condition (12) can be substituted into the government budget constraint (11) to obtain

$$(15) \qquad G = (1 - r)\, h(r),$$

where time subscripts have been omitted. Determination of r is depicted in Figure 5. Assuming G is sufficiently small ($G_0 < \max\,(1 - r)h(r)$), there will be two steady state values of r, denoted $\underline{r}$ and $\bar{r}$. Associated with each of these values is an initial price level p_0, satisfying

$$G = h(r) - M_{-1}/p_0.$$

B. <u>Private Information</u>

The definition of an equilibrium under private information is the same as in section 5. Then, substituting the market clearing condition (13) into the government budget constraint (11) yields the (steady state) equilibrium condition

$$(16) \quad G = (1 - r)g(r)$$

Since $g(r) = h(r) + (1 - \mu)(1 - \lambda)[\ell(r,p_L) - f(r)] > h(r)$, the right–hand side of (16) exceeds the right–hand side of (15) $\forall\, r \in [h^{-1}(0),1)$. The right–hand side of (16) is depicted in Figure 5. We now make several observations about the consequences of credit rationing.

First, private information allows the government to monetize larger deficits in the steady state than is possible under full information. As shown in figure 5, the deficit G_1 can be financed under private information, but not under full information. This is simply a reflection of the fact that credit rationing has the effect of increasing aggregate savings at each interest rate.

Secondly, the consequences of private information for inflation depend on which side of the "Laffer curve" the economy finds itself. On the "good side" of the Laffer curve (the downward–sloped branch) credit rationing is deflationary ($r = p_t/p_{t+1}$), as shown in Figure 5, while on the "bad side" of the Laffer curve (the upward–sloped branch) credit rationing reduces interest rates, and is therefore inflationary. Of some interest is the possibility that credit rationing will raise interest rates (relative to the equilibrium under full information). Moreover, under private information, the initial price level p_0 satisfies

$$G = g(r^*) - M_{-1}/p_0.$$

Since $g(r^{*}) > g(\bar{r}) \geq h(\bar{r})$, an economy on the "good" side of the Laffer curve will find that credit rationing acts to reduce the price level as well. Of course the opposite is true on the "bad" side of the Laffer curve.

Third, the welfare consequences of credit rationing are now very ambiguous, depending heavily on the economy's position on the Laffer curve. On the "good" side of the curve private information tends to raise interest rates. Hence savers actually benefit from facing an adverse selection problem in loan markets. Interestingly, higher interest rates associated with credit rationing in this case act to the detriment of type H borrowers, who are now harmed by the adverse selection problem that their presence creates. And clearly type L borrowers are injured both by rationing and by higher interest rates. On the "bad" side of the Laffer curve these results are reversed for savers and type H borrowers, while the welfare consequences of credit rationing for type L borrowers are ambiguous. To summarize, in the presence of deficits the welfare consequences of credit rationing for any particular group are particularly difficult to assess, and this assessment requires knowledge of where on the Laffer curve the economy lies. This comment suggests that it is dangerous to evaluate policy proposals intended to alleviate credit rationing without taking account of their general equilibrium effects.

7. CONCLUSIONS

This paper examines the impact of adverse selection on the set of competitive equilibria in a dynamic pure–exchange economy with overlapping generations of heterogeneous, two–period lived individuals. We measure the implications of adverse selection by how much the resulting credit–rationed equilibria (in pure strategies) differ from the corresponding benchmark of full–information perfect–foresight competitive allocations.

Suppose first that the only asset available to agents is "inside money", that is, private loans. Then adverse selection may shrink, leave unchanged or enlarge the set of competitive equilibria; broadly speaking it tends to reduce interest rates because credit rationing discourages borrowing.

Next we allow both "inside money" and "outside money" (that is, we introduce a non–zero stock of public debt) while keeping the flow of government deficits constant at zero. In this case we find, given some mild restrictions on savings functions, that there exists an open set of economies with the following property: <u>all of them are classical economies with a continuum of equilibria under full information, but become Samuelson economies with a unique monetary equilibrium under adverse selection</u>. This open set contains all economies in which low–risk borrowers suffer from sufficiently tight credit rationing at the "golden rule" rate of interest, that is, whenever the yield on safe bank deposits equals the natural rate of growth.

Adverse selection enables governments to finance a larger flow of deficits in the steady state than would be possible in a regime of public information. This result confirms what we know from the quantity rationing literature referred to earlier. We find in addition, that, along the stable ("bad") branch of the Laffer curve, adverse selection reduces the steady–state rate of interest, raises the corresponding inflation rate and lowers the steady–state welfare of savers. All these comparative statics results are reversed on the unstable ("good") branch of the Laffer curve.

Finally, we note a very special feature of credit rationing in pure–exchange economies: <u>it changes prices, not endowments</u>. This distinction is not valid in production economies in which adverse selection has potentially powerful <u>income</u> effects.

APPENDIX

Proposition 1. If $\ell(r_t,p_i) > 0\ \forall$ i, and if $u[\ell(r_t,p_i)] + p_i v[w - (r_t/p_i)\ell(r_t,p_i)] >$ $u(0) + p_i v(w)\ \forall$ i, then

(A.1) $$u[\ell(r_t,p_L)] + p_H v[w - (\frac{r_t}{p_L})\ell(r_t,p_L)] > u[\ell(r_t,p_H)] + p_H v[w - (\frac{r_t}{p_H})\ell(r_t,p_H)].$$

Proof. There are several possible cases to be considered. In turn, these are as follows. (a) Since $c_2 \geq 0$ must hold, $\ell(r_t,p_i) \leq p_i w/r_t$. One possibility, then, is $\ell(r_t,p_i) = p_i w/r_t\ \forall$i. (b) $0 < \ell(r_t,p_i) < p_i w/r_t\ \forall$ i. (c) $0 < \ell(r_t,p_i) < p_i w/r_t$ for one i, while $\ell(r_t,p_i) = p_i w/r_t$ for the other type. Each case is now considered.

(a) If $\ell(r_t,p_i) = p_i w/r_t\ \forall$i, (A.1) clearly holds. Parenthetically this must be the case if utility is linear and $u[\ell(r_t,p_i)] + p_i v[w - (\frac{r_t}{p_i})\ell(r_t,p_i)] > u(0) + p_i v(w)$. Therefore cases (b) and (c) will be relevant only if $u'' < 0$ or $v'' < 0$ (or both).

(b) By hypothesis, $\ell(r_t,p_i)$ satisfies

(A.2) $$u'[\ell(r_t,p_i)] = r_t v'[w - (r_t/p_i)\ell(r_t,p_i)]$$

and either $u'' < 0$ or $v'' < 0$. Then we claim that

(A.3) $$(p_L/p_H)\ell(r_t,p_H) > \ell(r_t,p_L) \geq \ell(r_t,p_H).$$

(A.3) establishes the proposition, as is apparent from (A.1).

To prove (A.3), note first that either $\ell(r_t,p_L) \geq \ell(r_t,p_H)$ or $(p_L/p_H)\ell(r_t,p_H) \geq \ell(r_t,p_L)$ (or both). For if not, then $\ell(r_t,p_H) > \ell(r_t,p_L) \geq (p_L/p_H)\ell(r_t,p_H)$, contradicting $p_L/p_H > 1$. Now suppose, for the purpose of deriving a contradiction, that $\ell(r_t,p_H) > \ell(r_t,p_L)$. From (A.2),

$$\text{(A.4)} \qquad u'[\ell(r_t,p_L)] = r_t v'[w - (\frac{r_t}{p_L})\ell(r_t,p_L)] \leq r_t v'[w - (\frac{r_t}{p_H})\ell(r_t,p_H)] = u'[\ell(r_t,p_H)]$$

must hold, and the inequality must be strict if $v'' < 0$. But then $\ell(r_t,p_H) \leq \ell(r_t,p_L)$ if $u'' < 0$, contradicting the supposition. And, since either $u'' < 0$ or $v'' < 0$ (or both), (A.4) cannot hold if $u'' = 0$. Thus $\ell(r_t,p_H) > \ell(r_t,p_L) > 0$ is not possible.

Then, if (A.3) is false, $\ell(r_t,p_L) \geq (p_L/p_H)\ell(r_t,p_H)$. From (A.2), this implies that

$$\text{(A.5)} \qquad u'[\ell(r_t,p_L)] = r_t v'[w - (\frac{r_t}{p_L})\ell(r_t,p_L)] \geq r_t v'[w - (\frac{r_t}{p_H})\ell(r_t,p_H)] = u'[\ell(r_t,p_H)]$$

where the inequality is strict if $v'' < 0$. Since either $u'' < 0$ or $v'' < 0$, (A.5) is a contradiction if $u'' = 0$. But if $u'' < 0$, (A.5) implies that $\ell(r_t,p_H) \geq \ell(r_t,p_L)$, contradicting the supposition. Then $\ell(r_t,p_L) \geq (p_L/p_H)\ell(r_t,p_H)$ is impossible, establishing the claim for case (b).

(c) Suppose first that $\ell(r_t,p_H) = p_H w/r_t$ while $\ell(r_t,p_L) < p_L w/r_t$. Then, if (A.1) is not true, $\ell(r_t,p_H) > \ell(r_t,p_L)$ must hold. But then, by hypothesis,

$$\text{(A.6)} \qquad r_t v'(0) \leq u'[\ell(r_t,p_H)] \leq u'[\ell(r_t,p_L)] = r_t v'[w - (\frac{r_t}{p_L})\ell(r_t,p_L)]$$

Moreover, since either $u'' < 0$ or $v'' < 0$, this implies that $w \leq (r_t/p_L)\ell(r_t,p_L)$, contrary to assumption.

Then, if this case is to obtain without satisfying (A.1), $\ell(r_t,p_L) = p_L w/r_t$ must hold, while $\ell(r_t,p_H) < p_H w/r_t$. Then clearly $\ell(r_t,p_L) > \ell(r_t,p_H)$, and by hypothesis

$$\text{(A.7)} \qquad r_t v'(0) \leq u'[\ell(r_t,p_L)] \leq u'[\ell(r_t,p_H)] \leq r_t v'[w - (\frac{r_t}{p_H})\ell(r_t,p_H)]$$

But (A.7) and $u'' < 0$ or $v'' < 0$ contradicts $p_H w/r_t > \ell(r_t,p_H)$. This establishes the proposition.

NOTES

1. Exceptions appear in the quantity–rationing literature, e.g., Puhakka (1985), Bertocchi (1989).

2. It may seem troublesome to have borrowers with low default probabilities be the rationed group. However, for most economies this appears to be the relevant situation. Claims that some "good" borrowers are unfunded or underfunded while "poor" borrowers are simultaneously fully funded (or perhaps overfunded) abound in the development literature. On this point see, for instance, McKinnon (1973, p. 8).

3. As yet we have no examples with $\bar{\lambda}'(r_t) > 0$. However, we have also not been able to rule this case out.

4. Using the expression for $\bar{\lambda}(r_t)$ given in example 1, $\bar{\lambda}(.53) = .421 < .8$.

5. It is not the case that this result depends in any way on having $r < 1$. An alternative example retains all the features of this one, but sets $p_H = .6$, $p_L = .71$, and $\beta = .7939$. Then under full information $r_t = 1.244 \; \forall \; t$. Under private information $r_t = 1.0831 \; \forall \; t$. It is easy to check that type L borrowers are better off under credit rationing than under full information, and that $\bar{\lambda}\,(1.0831) = .513 < .8$.

6. Government taxes, if they are levied on borrowers, can have incentive effects depending on how and when they are assessed. So, for simplicity, taxation is suppressed. Of course a fixed set of taxes on savers causes no potential difficulties, and then G can be reinterpreted as the per capita deficit.

7. Assuming, of course, that $h(1) \neq 0$.

8. Again, assuming that $g(1) \neq 0$.

9. Of course this statement presupposes that $\lambda \geq \bar{\lambda}[g^{-1}(0)]$.

10. Second–derivative assumptions are used for illustrative purposes to produce single–peaked Laffer curves. In general, the steady–state deficit may have several "humps" when expressed as a function of the interest rate. Still, it is not difficult to produce examples satisfying these curvature conditions. For instance, if $u(c_1) = \ln c_1$ and $v(c_2) = \beta c_2$, $\bar{\lambda}'(r_t) = 0$. $h' > 0$ and $g' > 0$ hold if $s'(r_t) \geq 0$, and $g'' < 0$ holds if $h'' \leq 0$. Also, $h'' < 0$ is implied in this instance by $s''(r_t) \leq 0$.

REFERENCES

Bertocchi, Graziella

Besanko, David and Anjan Thakor, "Collateral and Rationing: Sorting Equilibria in Monopolistic and Competitive Credit Markets," *International Economic Review*, 28 (3), 1987, p. 671–89.

Brunner, Karl and Alan Meltzer, "The Uses of Money: Money in the Theory of an Exchange Economy," *American Economic Review*, 61, 1971, p. 784–805.

Cass, David, Masahiro Okuno and Itzhak Zilcha, "The Role of Money in Supporting the Pareto Optimality of Competitive Equilibria," *Journal of Economic Theory*, 20, 1979, p. 41–80.

Friedman, Milton, *A Program for Monetary Stability*, Fordham University Press, New York, 1960.

Gale, David, "Pure Exchange Equilibrium of Dynamic Economic Models," *Journal of Economic Theory*, 6, 1973, p. 12–36.

Gale, William G., "Information, Collateral, and Government Intervention in Credit Markets," manuscript, 1989.

McKinnon, Ronald I., *Money and Capital in Economic Development*, Brookings Institute, Washington, 1973.

Puhakka, Mikko, "Credit Rationing in a Model of National Debt," Ch. 4 of an unpublished doctoral dissertation, University of Pennsylvania, 1985.

Rothschild, Michael and Joseph Stiglitz, "Equilibrium in Competitive Insurance Markets: the Economics of Incomplete Information," *Quarterly Journal of Economics*, 91, 1976, p. 629–50.

Smith, Bruce D., "Limited Information, Money and Competitive Equilibrium," *Canadian Journal of Economics*, 19, 1986, p. 780–97.

Smith, Bruce D. and Michael Stutzer, "Credit Rationing and Government Loan Programs: a Welfare Analysis," manuscript, 1988.

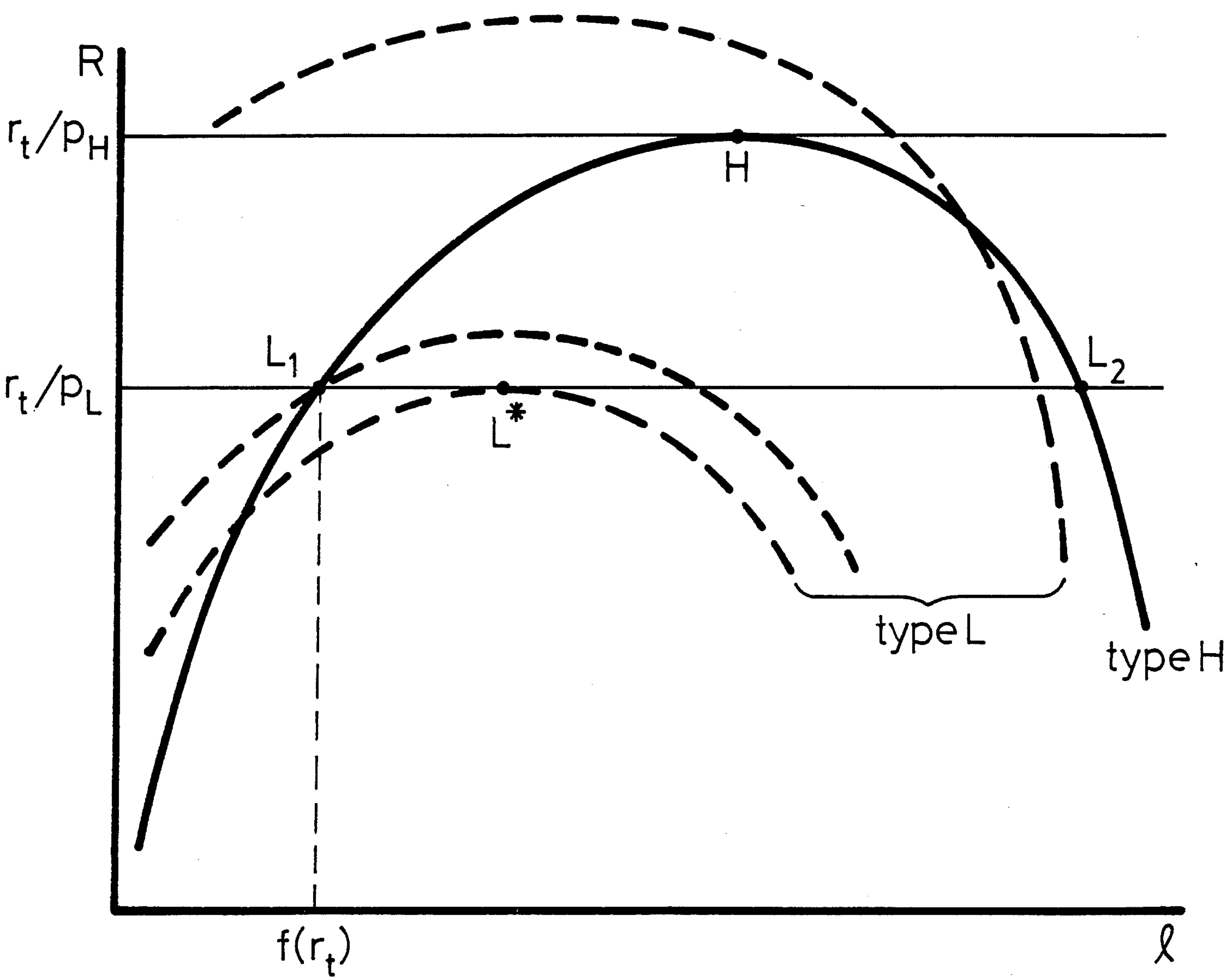
Stiglitz, Joseph E. and Andrew Weiss, "Credit Rationing in Markets with Imperfect Information," *American Economic Review*, 71, 1981, p. 393–410.

Tobin, James, *Asset Accumulation and Economic Activity*. Chicago: University of Chicago Press, 1980.

Williamson, Stephen D., "Costly Monitoring, Loan Contracts, and Equilibrium Credit Rationing," *Quarterly Journal of Economics*, 1987, p. 135–45.

Figure 1

Determination of Equilibrium Contracts



Utility increases toward the south.

Figure 2a

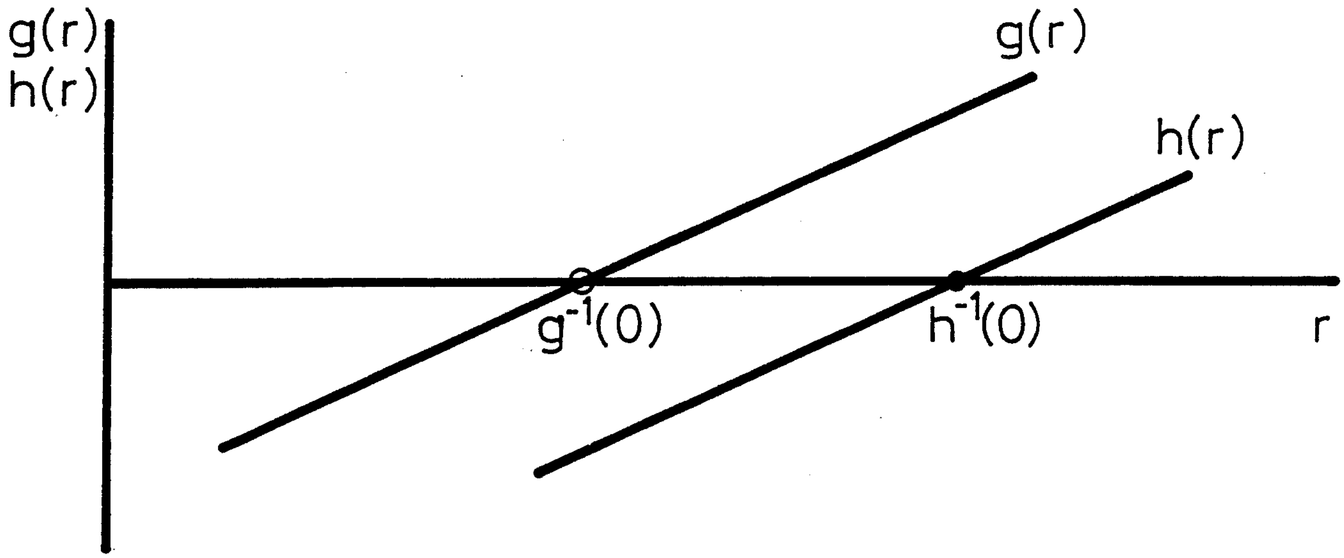


Figure 2b

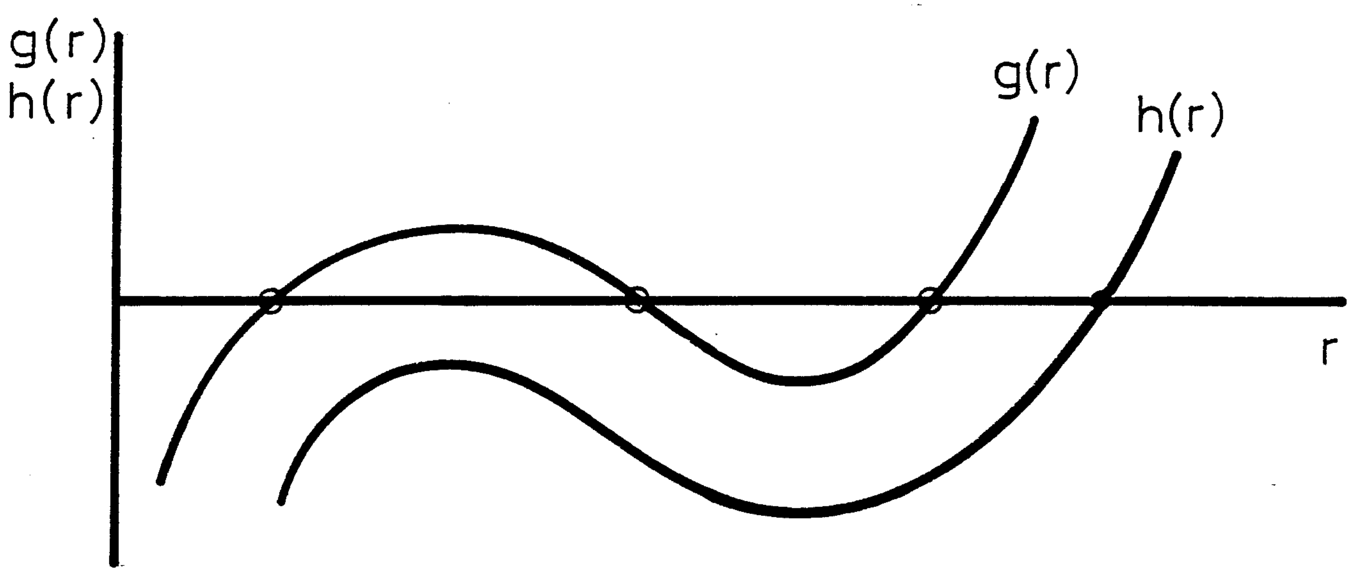


Figure 2c

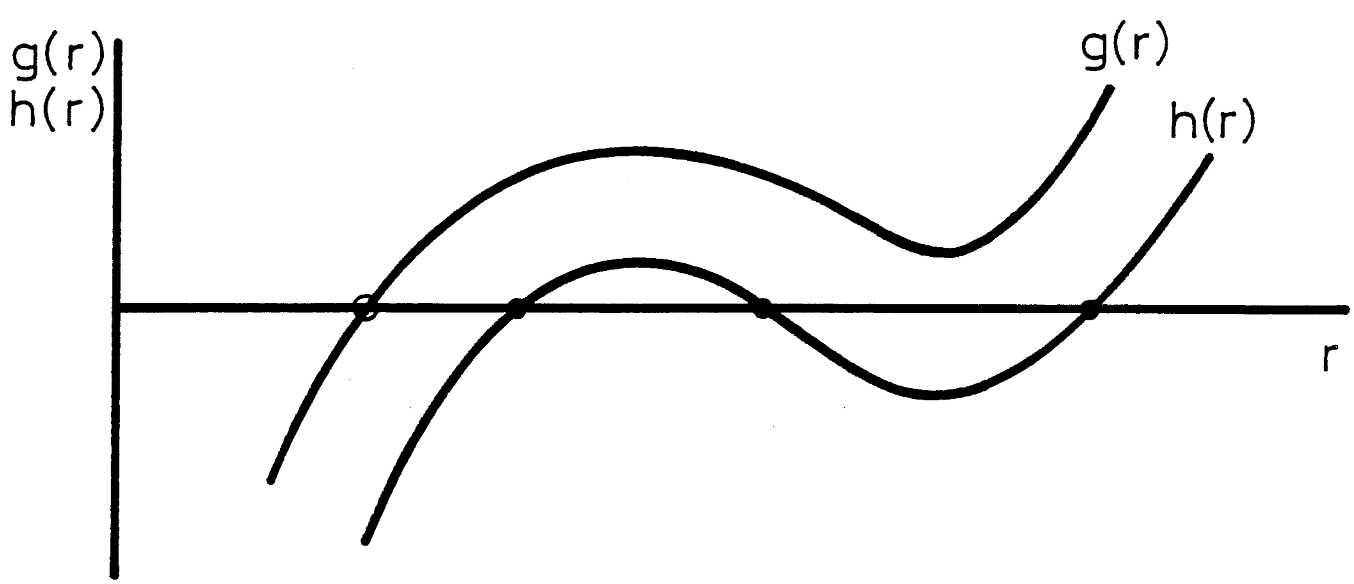


Figure 3a : "Classical Case"

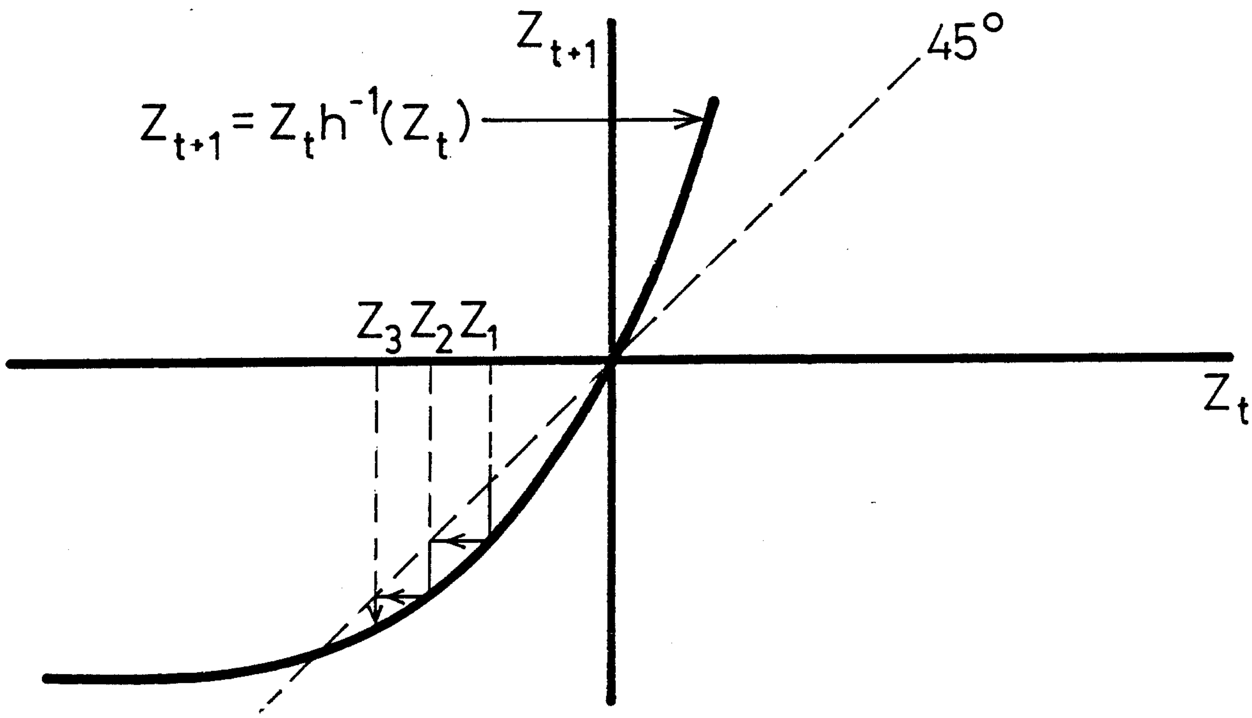


Figure 3b : "Samuelson Case"

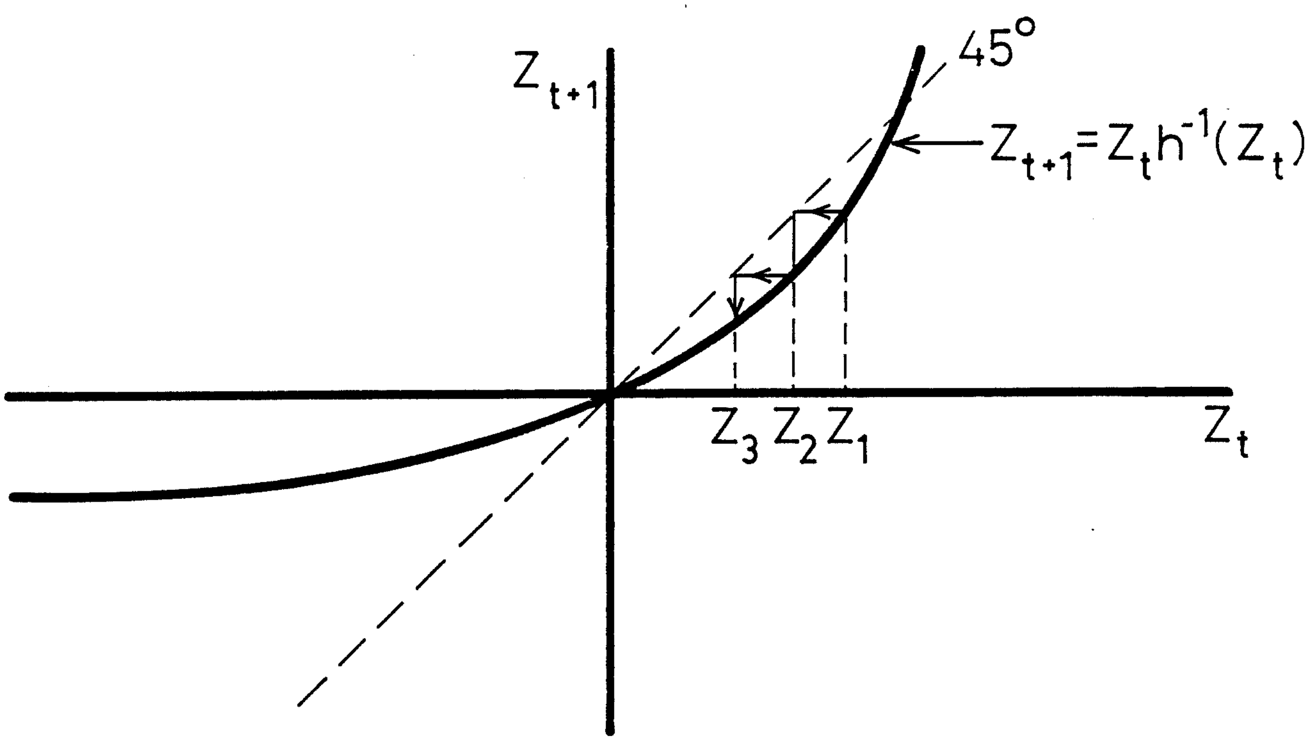
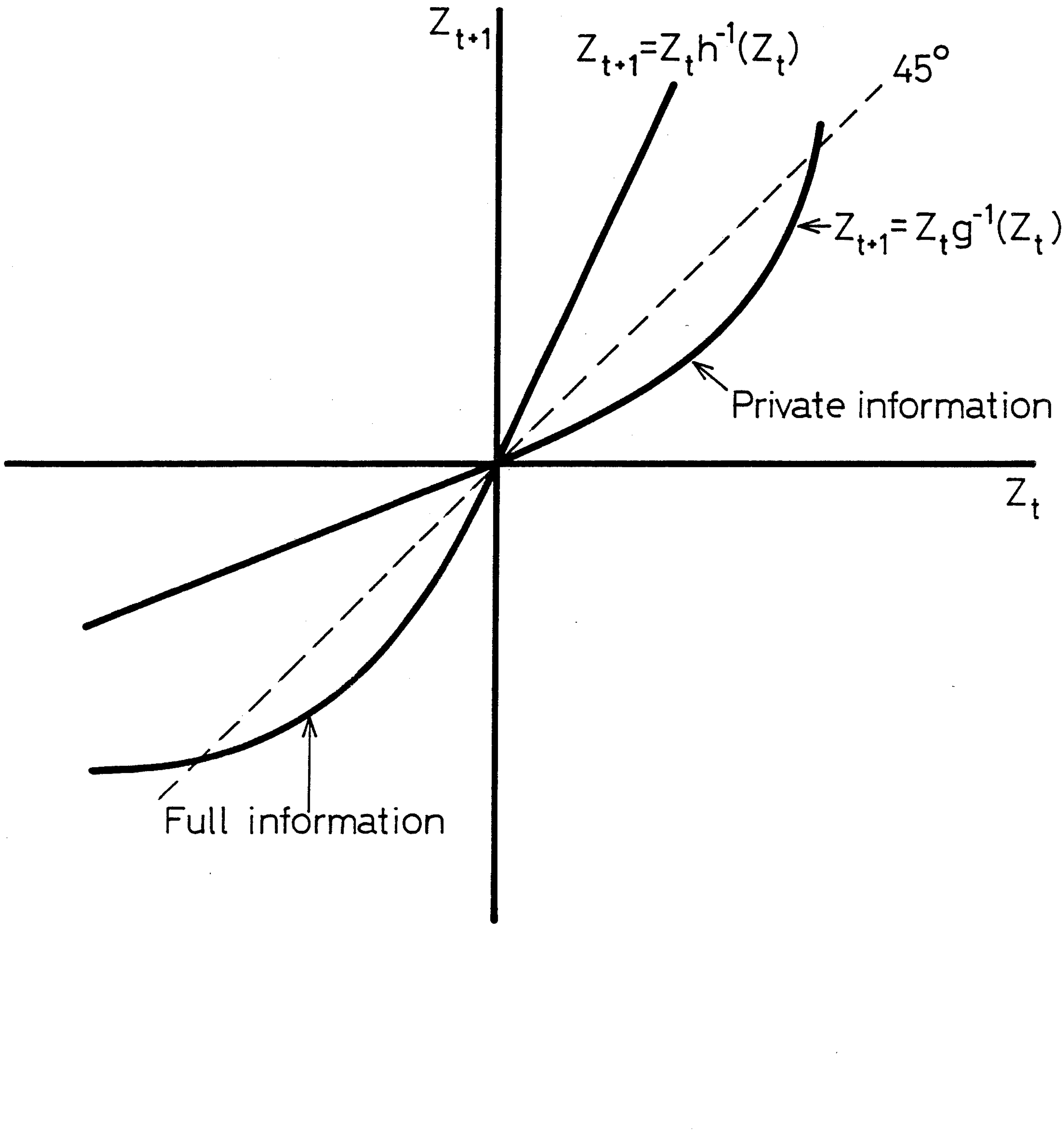


Figure 4



Z_{t+1}
$Z_{t+1}=Z_t h^{-1}(Z_t)$
45°
$Z_{t+1}=Z_t g^{-1}(Z_t)$
Private information
Z_t
Full information

Figure 5

Deficit Monetization