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ABSTRACT

An economy is presented that has a unique equilibrium under a money supply rule that equates real growth rates with money growth rates. Two schemes for pegging the nominal interest rate are then considered. When the government stands ready to exchange interest bearing assets for currency on demand, a unique equilibrium also results. However, depending on the choice of interest rate targets, it may be possible to achieve a Pareto improvement by using taxes and transfers to accomplish endogenous adjustments in the money supply (when the focus is on steady states). The latter method of pegging the interest rate may allow both stationary sunspot equilibria and a continuum of perfect foresight equilibria to exist, however. Thus, even restricting attention to interest rate pegs, a tension is observed between efficiency and determinacy of equilibrium. Interestingly, in the equilibria considered, the price level is always determinate under an interest rate peg. However, the time path for the money supply and all real variables may be indeterminate.

This paper relates to three topics that have each generated a substantial literature. The first topic concerns alternative choices of an operating procedure for monetary policy, for instance, whether an "interest rate" or a "money supply" rule should be followed. The second concerns situations under which stationary sunspot equilibria (evolving according to finite state Markov chains) exist. And the third topic is the apparent tension between policies that support efficient allocations as equilibria, but admit multiple equilibria, and policies that admit only one equilibrium, but require that that equilibrium be inefficient.

With respect to the first topic, there is clearly a large literature on the properties of "interest rate" versus "money supply" rules. Most of this literature proceeds using variants of an IS–LM model.[1] Yet, despite some underlying similarities of approach, very different conclusions are reached regarding the desirability of interest rate rules. For instance, Barro (1989, pp. 4, 28) argues that interest rate targeting appears to be a reasonable objective of monetary policy. Parkin (1978) reaches a similar conclusion for a small open economy following certain exchange rate policies. Dotsey and King (1983), on the other hand, argue that interest rate pegs are inferior to other feedback policies because interest rate pegs destroy any information conveyed by interest rates that are free to vary. This literature is also inconclusive on the issue of indeterminacy of equilibrium under interest rate pegging. McCallum (1986, p. 149) argues that "pure pegging – a standing offer to buy and sell securities at a specified interest rate – does not constitute a well–formulated monetary policy" because it allows multiple stochastic processes for equilibrium quantities. On the other hand, Woodford (1988) produces a cash–in–advance model in which such a "pure peg" is consistent with a unique equilibrium.

This paper produces a general equilibrium model sharing several features of some of the models just discussed. In particular, as in Barro (1989) and McCallum (1981, 1986) non–investment income is exogenous, and as in Barro (1989) the (expected) real interest rate also cannot be affected by the choice of policy. The model is formulated in such a way that under fixed time paths for the money supply (that do not involve growth rates that are "too far" from the real growth rate) there is a unique equilibrium. An analysis is then undertaken of "interest rate pegs." As pointed out by McCallum (1986), however, the statement that the interest rate will be pegged and the money supply will be endogenous is not an adequate description of policy. In particular, one must specify how money supply adjustments will occur. Under one natural specification – the "pure peg" described above – a unique equilibrium results whenever the nominal interest rate is pegged at a non–zero level. Thus the analysis produces a result similar to that in Woodford (1988) and contrary to that in McCallum (1986). However, unless the interest rate is pegged at a level implying exactly zero inflation, this method of fixing the interest rate is necessarily inefficient in a sense to be made precise.

An alternative method of pegging the interest rate is to allow money supply adjustments to occur through lump–sum taxes and transfers. When only steady state equilibria are considered, such a method may be Pareto superior to a "pure–peg" accomplished by having the government stand ready to exchange money for other assets. However, this system for pegging interest rates can allow indeterminacies to arise; both in the form of a continuum of non–stationary perfect foresight equilibria and in the form of stationary sunspot equilibria. Interestingly, these equilibria all have determinate sequences for the price level: only variation in the money stock and real allocations is observed.

The second topic to be addressed concerns situations where stationary sunspot equilibria can arise. In contexts where a fixed (constant) time path for the money supply is assumed, Azariadis (1981) shows that stationary sunspot equilibria exist [in two–period, overlapping generations models] only if savings functions are decreasing in the rate of interest in a neighborhood of the unique monetary steady state. In the same context, Azariadis and Guesnerie (1986) show that stationary sunspot equilibria exist iff an economy also displays equilibrium 2–cycles (assuming all goods are strictly normal). These conditions may (to some) make it appear that such sunspot equilibria are empirically implausible. However, it is typically argued [see, e.g., McCallum (1981)] that the actual conduct of monetary policy approximates an interest rate rather than a money supply rule. As will be shown, stationary sunspot equilibria can then easily arise despite the absence of two–period cycles, and even though savings functions are strictly increasing in the rate of interest.

Finally, some literature exists that explores an apparent "tension" between efficiency and determinacy of equilibrium.[2] An interesting example of this tension is produced by Woodford (1988), who gives conditions in a cash–in–advance model where relatively low money growth rates are (in terms of steady state welfare) more efficient than higher money growth rates. However, low rates of money growth are also more conducive to the existence of sunspot equilibria. Finally, Woodford's model has the feature that "pure interest rate pegs" give rise to a unique equilibrium. Then, since any steady state equilibrium that arises under a money supply rule can be replicated by an interest rate peg, this appears to argue for the latter as the best choice of policy.

In the model produced here, an interest rate peg accomplished by having the government stand ready to exchange money for other assets also yields a unique equilibrium. However, it is also often an inefficient policy. The interest rate can be pegged by other means though, for instance by injecting any necessary money through lump–sum taxes/transfers. However, this method of pegging the interest rate will, for many economies, allow sunspot (and other) equilibria to exist. Thus this paper illustrates a sense in which there is a tension between determinacy and efficiency, even within the class of policies consistent with a given pegged level of the interest rate.

The results just described are obtained in the context of a two–period lived, overlapping generations model. (This facilitates a comparison of the results on stationary sunspot equilibria with those of Azariadis (1981) or Azariadis–Guesnerie (1986)). In this model there is an exogeneously given endowment of a single consumption good in each period, and in addition there is a non–stochastic linear storage technology. (The latter fixes the real interest rate faced by agents in the model.) Young agents in each period decide how much to save (store) given the return on storage (which exceeds the rate of population growth by assumption), subject to a reserve requirement which forces money to be held. Finally, the utility functions of agents are restricted in a way that implies savings functions which are non–decreasing in the real rate of return on savings.

In this environment, a fixed and constant time path for the money supply implies the existence of a unique equilibrium. (This will also be the case for constant money growth rates sufficiently near zero.) Interest rate pegs are then analyzed. One method of fixing the interest rate is to allow the money supply to adjust endogenously to money demand, with any money supply changes accomplished through lump–sum taxes/transfers. Under appropriate pegged levels for the nominal interest rate, the model produces a steady state equilibrium identical to the unique equilibrium that obtains under a money supply rule. However, the dynamic (e.g., stability) properties of the steady state equilibrium under this means of pegging the interest rate can differ substantially from those of the steady state that obtains under a money supply rule. If the steady state equilibrium is asymptotically stable under an interest rate peg, a continuum of perfect foresight equilibria as well as an open set of stationary sunspot equilibria exist. All such equilibria have determinate price level sequences, but the nominal money supply, storage (capital accumulation) and real allocations are indeterminate. Thus certain money supply rules imply determinacy in this model while this interest rate peg does not. However, in contrast to the usual situation, no indeterminacy with respect to the price level is observed.

Finally, interest rate pegs achieved by having the government stand ready to exchange money for other assets are considered. If the nominal interest rate is pegged at a feasible level, the result is a unique equilibrium. However, maintenance of this equilibrium requires the government to accumulate claims to consumption. Since such claims held by the government (by assumption) do not contribute to the utility of individuals, this situation is an inefficient one. Thus an interest rate peg involves some tension between the efficiency and the determinacy of equilibrium.

The remainder of the paper proceeds as follows. Section I describes the physical environment and trading structure of the model. Section II describes equilibria under a constant time path for the money supply. Section III considers interest rate pegs accomplished via lump–sum taxes or transfers, while section IV considers pegs accomplished through open market operations. Section V concludes and offers some comments on the nature of the model specification.

I. THE MODEL

A. Environment

The economy consists of an infinite sequence of two–period lived overlapping generations, plus a set of one–period lived initial old agents. Let time be indexed by $t = 1,2,\ldots$. Then, at $t = 1$, the initial old are endowed with the initial per capita money supply, $M_0 > 0$, and nothing else. Also, at each date $t \geq 1$ a new young generation is born. All young agents in each generation are identical. Moreover, all generations are of the same size, having N members. (All per capita quantities are expressed in terms of the size of a generation.)

There is a single consumption good at each date. Young agents receive a constant endowment of this good $w > 0$ when young, and have no (gross of tax) endowment of the good when old. The good can be stored, however. If a representative young agent at t stores k_t units of the good, xk_t units are received at $t + 1$. It is assumed that $x > 1$ holds. Note that x is the real rate of interest on capital (storage), which is fixed by technology.[4]

Finally, let c_{it} denote consumption in period i of a representative member of generation t. Then all young members of this generation have the additive utility function $u(c_{1t}) + v(c_{2t})$, with u and v defined on R_+. It is assumed that $u', v' > 0$, $u'' \leq 0$, $v'' < 0$, and that

$$(1) \qquad cv''(c)/v'(c) \geq -1 \qquad \forall c.$$

B. Trading

At each date some quantity of money is outstanding. The per capita quantity of money at t is denoted M_t, and p_t is the time t price level.

Young agents at each date choose a quantity of the good to store, k_t, and a quantity of real money balances to carry between periods, denoted z_t. In addition, the government distributes lump–sum transfers (or if these are negative, assesses taxes). Let τ_{1t} denote lump–sum transfers to young agents at t, and let τ_{2t} denote similar transfers to old agents at t. Then young agents at date t face the budget constraints

(2) $$c_{1t} = w + \tau_{1t} - k_t - z_t$$

(3) $$c_{2t} = xk_t + (p_t/p_{t+1})z_t + \tau_{2t+1}.$$

It is assumed that $k_t \geq 0$.

In addition to (2) and (3), the young at t face a reserve requirement:[5]

(4) $$z_t \geq \lambda k_t\,; \quad \lambda > 0.$$

This requirement merits some comment, since it appears to require that agents hold reserves against assets rather than liabilities. However, the following reinterpretation is possible. Suppose that agents cannot store their own good (as they cannot consume the proceeds of their own production in cash–in–advance models). Then let each agent take resources equal to $k_t + z_t$ to a "bank," which intermediates goods storage at t. Then the bank has a per capita liability of $k_t + z_t$ at t, against which it is required to hold a cash reserve with a real value of z_t. If the bank earns zero profits, then each depositor receives a pro rata share of the bank's proceeds at t+1, which will be $xk_t + (p_t/p_{t+1})z_t$. Therefore, (2)–(4) simply "consolidate" the balance sheet of banks and their depositors.

The (gross) nominal interest rate in this economy is xp_{t+1}/p_t. Throughout the focus will be on situations where the nominal interest rate is positive, so $p_t/p_{t+1} < x$.

Young agents at t, then, choose c_{1t}, c_{2t}, $k_t \geq 0$ and z_t to maximize $u(c_{1t}) + v(c_{2t})$

subject to (2)–(4). Since $p_t/p_{t+1} < x$, the reserve requirement binds, and the problem of a young agent at t can be written as follows. Let $Q_t = k_t + z_t$ denote total saving, and let $\phi \equiv (1 + \lambda)^{-1}$. Then $k_t = \phi Q_t$ while $z_t = \lambda k_t = (1 - \phi)Q_t$. In addition, define

$$R_t = \phi x + (1 - \phi)(p_t/p_{t+1}),$$

so that R_t is the weighted return on agents' portfolios. Then young agents can be viewed as choosing $Q_t \in [0,w]$ to maximize $u(w + \tau_{1t} - Q_t) + v(R_tQ_t + \tau_{2t+1})$. Throughout attention will be restricted to situations where this problem has an interior solution. Therefore, Q_t will satisfy

$$u'(w + \tau_{1t} - Q_t) = R_t v'(R_tQ_t + \tau_{2t+1}) \tag{5}$$

Note that (5) implicitly defines a savings function $Q_t = f(R_t, \tau_{1t}, \tau_{2t+1})$. Letting f_j denote the partial derivatives of f,

$$f_1(R_t, \tau_{1t}, \tau_{2t+1}) = \frac{v'\left[1 + (R_tQ_tv''/v')\right]}{-(u'' + R_t^2v'')}$$

Notice that, since $c_{2t} = R_tQ_t + \tau_{2t+1}$, $f_1(R_t, \tau_{1t}, 0) \geq 0$, by the assumption of equation (1). Moreover, if $f_1(R_t, \tau_{1t}, 0) > 0$ and if f has continuous partial derivatives, $f_1 > 0$ will hold if τ_{2t+1} is sufficiently small. Finally, for future reference,

$$f_3(R_t, \tau_{1t}, \tau_{2t+1}) = -R_tv''/(u'' + R_t^2v'') < 0$$

It remains to describe the choices open to the government. The government is free to store the good, and is not subject to the reserve requirement.[6] Then, letting k_{gt} denote government storage at t in per capita terms, the government faces the budget constraint

$$\tau_{1t} + \tau_{2t} = (M_t - M_{t-1})/p_t - k_{gt} + xk_{gt-1}\ ;\ t \geq 1, \tag{6}$$

with M_0 and $k_{g0} = 0$ given.

II. EQUILIBRIA UNDER A MONEY SUPPLY RULE

A. Perfect Foresight Equilibria

The set of perfect foresight equilibria under a fixed (constant) time path for the money supply is now described. The possibility of stationary sunspot equilibria is discussed in the next section.

For notational simplicity, it is assumed that the time path of the money supply is set so that $M_t = M_0 = 1\ \forall\ t.$[7] In addition, since no lump–sum transfers or asset exchanges are required to accomplish any changes in the money supply, $k_{gt} = \tau_{1t} = \tau_{2t} = 0\ \forall\ t$. Then a perfect foresight equilibrium under these policy settings is a time path for $\{k_t, z_t, Q_t, p_t\}$ satisfying the money market clearing condition

$$1/p_t = z_t = (1-\phi)Q_t \tag{7}$$

the first order condition (recall that $\tau_{1t} = \tau_{2t+1} = 0$)

$$u'(w - Q_t) = R_t\, v'(R_t Q_t) \tag{5'}$$

and $k_t = \phi Q_t\ \forall\ t \geq 1$.

Recall that $R_t = \phi x + (1-\phi)(p_t/p_{t+1})$, and note from (7) that $p_t/p_{t+1} = Q_{t+1}/Q_t$. Using these relations in (5′) yields the equilibrium law of motion for Q_t:

$$u'(w - Q_t) = [\phi x + (1-\phi)(Q_{t+1}/Q_t)]\ v'\ [\phi x Q_t + (1-\phi)Q_{t+1}] \tag{8}$$

There is a unique steady state equilibrium value for Q_t, denoted $\tilde{Q}$, which satisfies[8]

$$(9) \qquad u'(w-\tilde{Q}) = (\phi x + 1 - \phi)v'[(\phi x + 1 - \phi)\tilde{Q}].$$

Moreover, from (8),

$$(10) \qquad \left.\frac{dQ_{t+1}}{dQ_t}\right|_{\tilde{Q}} = \frac{u'-\phi xv' - Q_t u'' - [\phi x Q_t + (1-\phi)Q_{t+1}]\phi x v''}{(1-\phi)v'\{1 + [\phi x Q_t + (1-\phi)Q_{t+1}](v''/v')\}} > 1$$

The inequality follows from the assumption of equation (1), and the fact that (5) implies $u' - \phi xv' = (1-\phi)v'$, where these functions are evaluated at $\tilde{Q}$.

Equation (10), of course, asserts the instability of the unique steady state equilibrium. Then the law of motion for Q_t given by (8) looks as shown in Figure 1. As is apparent, since $Q_t \geq 0$ must hold, the only equilibrium time path for Q_t has $Q_t = \tilde{Q}\ \forall\ t$.

B. Stationary Sunspot Equilibria

The possibility of stationary sunspot equilibria (s.s.e.'s) evolving according to a two–state Markov chain is now investigated. For notational purposes, let $e \in \{1,2\}$ denote the current period state, while e' denotes "next period's" state. Further, let $\pi(e) = \text{prob}(e' = 1{:}e)$, so $\pi(e)$ is the probability that $e' = 1$ conditional on the current period state being e. Finally, let Q(e), k(e), z(e), and p(e) denote total savings, storage, real balances, and the price level, respectively, if the current period state is e. Finally, as above, for the remainder of the section $k_{gt} = \tau_{1t} = \tau_{2t} = 0$ and $M_t = 1\ \forall\ t$.

It is assumed that all trading occurs after the realization of the current period state at each date. Trading is exactly as described previously,[9] so that a young agent born in state e chooses k(e), z(e), $c_{1t}(e)$, $c_{2t}(e,e')$ to maximize $u[c_{1t}(e)] + \pi(e)v[c_{2t}(e,1)] + [1-\pi(e)]v[c_{2t}(e,2)]$ subject to

$$c_{1t}(e) = w - k(e) - z(e)$$

$$c_{2t}(e,e') = xk(e) + [p(e)/p(e')]\, z(e)$$

and $z(e) \geq \lambda\, k(e)$.

As above, the focus is on situations in which the reserve requirement is binding.[10] Then the problem of a young agent can be reformulated as choosing $(1 + \lambda)k(e) \equiv Q(e) \in [0,w]$ to maximize

$$u[w-Q(e)] + \pi(e)v\{[\phi x + (1-\phi)p(e)/p(1)]Q(e)\} + [1-\pi(e)]v\{[\phi x + (1-\phi)p(e)/p(2)]Q(e)\}.$$

The first order condition for this problem is

$$\text{(11)} \qquad u'[w-Q(e)] = \pi(e)[\phi x + (1-\phi)p(e)/p(1)]v'\{[\phi x + (1-\phi)p(e)/p(1)]Q(e)\}$$
$$+ [1-\pi(e)]\,[\phi x + (1-\phi)p(e)/p(2)]v'\{[\phi x + (1-\phi)p(e)/p(2)]Q(e)\}; \quad e = 1,2.$$

As in Azariadis (1981), an s.s.e. is a set of values $Q(e)$, $k(e)$, $z(e)$, $p(e)$ and $\pi(e)$, $e \in \{1,2\}$, satisfying the asset market clearing condition

$$\text{(12)} \qquad 1/p(e) = z(e) = (1-\phi)Q(e); \quad e = 1,2,$$

$k(e) = \phi Q(e)$, $0 \leq \pi(e) \leq 1$, and (11). It is then easy to establish that this economy has no non–trivial s.s.e. (in which the reserve requirement binds). To see this, note that from (12), $p(e)/p(e') = Q(e')/Q(e)$. Substituting this relation into (11) for $e = 1,2$, yields

$$\text{(13)} \qquad u'[w-Q(1)] = \pi(1)(\phi x + 1-\phi)v'[(\phi x + 1-\phi)Q(1)]$$
$$+ [1-\pi(1)][\phi x + (1-\phi)Q(2)/Q(1)]v'[\phi xQ(1) + (1-\phi)Q(2)]$$

$$(14) \qquad u'[w-Q(2)] = \pi(2)[\phi x + (1-\phi)Q(1)/Q(2)]v'[\phi xQ(2) + (1-\phi)Q(1)] + [1-\pi(2)](\phi x + 1-\phi)v'[(\phi x + 1-\phi)Q(2)].$$

Now suppose that $Q(1) > Q(2)$. Then equation (1), which implies that $cv'(c)$ is non–decreasing in c, also implies that

$$(15) \qquad (\phi x + 1-\phi)v'[(\phi x + 1-\phi)Q(1)] \geq [\phi x + (1-\phi)Q(2)/Q(1)]v'[\phi xQ(1) + (1-\phi)Q(2)]$$

Moreover, (13) and (15) imply that

$$(16) \qquad (\phi x + 1-\phi)v'[(\phi x + 1-\phi)Q(1)] \geq u'[w-Q(1)]$$

Recalling that the steady state value of Q_t, $\tilde{Q}$, is defined by (9), (16) implies that $\tilde{Q} \geq Q(1)$. Similarly, since $cv'(c)$ is non–decreasing,

$$(17) \qquad [\phi x + (1-\phi)Q(1)/Q(2)]v'[\phi xQ(2) + (1-\phi)Q(1)] \geq (\phi x + 1-\phi)v'[(\phi x + 1-\phi)Q(2)]$$

Moreover, (17) and (14) imply that

$$(18) \qquad u'[w-Q(2)] \geq (\phi x + 1-\phi)v'[(\phi x + 1-\phi)Q(2)]$$

or that $Q(2) \geq \tilde{Q}$. But then $Q(2) \geq Q(1)$, contradicting the original supposition. A similar contradiction arises if it is assumed that $Q(2) > Q(1)$. Thus this economy has no non–trivial s.s.e.

Under a constant time path for the money supply, this economy does not differ substantially from that considered by Azariadis (1981). Hence, equation (1), which implies that $f(R,0,0)$ is monotone in R, rules out the possibility of expectations driven cycles. As will be seen, this is not the case under certain schemes for pegging the interest rate.

III. INTEREST RATE PEGS ACCOMPLISHED THROUGH LUMP–SUM TAXATION

For the remainder of the paper, it is assumed that the government pegs the nominal interest rate. As noted above, the (gross) nominal interest rate for this economy is xp_{t+1}/p_t, so pegging the nominal interest rate is equivalent to pegging the inflation rate p_{t+1}/p_t. Thus the government can be viewed as fixing a target sequence for the price level, denoted $\{p_t^*\}$.[11] For simplicity, it is assumed that $p_t^*/p_{t+1}^* = \rho \; \forall \; t \geq 1$. It is further assumed that $0 < \rho < x$, so that the nominal interest rate is pegged at a positive level.

This section focuses on an interest rate peg in which any necessary movements in the money supply are accomplished through lump–sum taxes/transfers. Then, since the government makes no asset exchanges, $k_{gt} = 0 \; \forall \; t$ is assumed to hold. It is also assumed that $\tau_{1t} = 0 \; \forall \; t$, so that all transfers are made to old agents. It is easily checked that only minor modifications are required if lump–sum transfers are made to young rather than old agents.

A. Perfect Foresight Equilibria

The perfect foresight equilibria of this economy are now characterized; s.s.e.'s are considered in the next section.

To begin, recall that young agents choose c_{1t}, c_{2t}, k_t, and z_t to maximize $u(c_{1t})$ + $v(c_{2t})$ subject to $c_{1t} = w - k_t - z_t$; $c_{2t} = \tau_{2t+1} + xk_t + (p_t^*/p_{t+1}^*)z_t$; and $z_t \geq \lambda k_t$. Since $p_t^*/p_{t+1}^* = \rho < x$ the reserve requirement binds. Then, again letting $Q_t = (1 + \lambda)k_t$, the problem of a representative young agent at t is to choose $Q_t \in [0,w]$ to maximize $u(w-Q_t)$ + $v(R^*Q_t + \tau_{2t+1})$, where $R^* \equiv \phi x + (1-\phi)(p_t^*/p_{t+1}^*) = \phi x + (1-\phi)\rho$. The first order condition associated with this problem is

$$u'(w-Q_t) = R^* v'(R^* Q_t + \tau_{2t+1}). \tag{19}$$

Finally, from the government budget constraint (6),

$$(20) \qquad \tau_{2t} = (M_t - M_{t-1})/p_t^*$$

since $k_{gt} = \tau_{1t} = 0 \; \forall \; t$.

A perfect foresight equilibrium for this economy is a sequence $\{Q_t, k_t, z_t, M_t, \tau_{2t}\}$ satisfying (19), (20), $k_t = \phi Q_t$, $z_t = (1-\phi)Q_t$, and the money market clearing condition

$$(21) \qquad M_t/p_t^* = (1-\phi)\, Q_t.$$

(Recall that $\phi = (1+\lambda)^{-1}$.)

It is now straightforward to characterize the set of perfect foresight equilibria. From (20) and (21), $\tau_{2t+1} = (1-\phi)Q_{t+1} - \rho(1-\phi)Q_t$. Substituting this into (19), and using $R^* = \phi x + (1-\phi)\rho$, yields the expression

$$(22) \qquad u'(w-Q_t) = R^* v'[R^* Q_t + (1-\phi)Q_{t+1} - \rho(1-\phi)Q_t] = R^* v'[\phi x Q_t + (1-\phi)Q_{t+1}],$$

which gives the equilibrium law of motion for Q_t.

This law of motion is depicted in Figure 2. From (22),

$$(23) \qquad \frac{dQ_{t+1}}{dQ_t} = \frac{-u'' - \phi x R^* v''}{(1-\phi)R^* v''} < 0.$$

In addition, there is a unique steady state equilibrium value, denoted Q^*, satisfying

$$(24) \qquad u'(w-Q^*) = R^* v'[(\phi x + 1-\phi)Q^*].$$

From (23), this steady state equilibrium is asymptotically stable if

$$(25) \qquad \left.\frac{dQ_{t+1}}{dQ_t}\right|_{Q^*} = \frac{-u'' - \phi x R^* v''}{(1-\phi)R^* v''} > -1$$

where u'' and v'' are evaluated at Q^*. Then, if (25) holds, there is a continuum of non–stationary equilibria with $\{Q_t\}$ converging to Q^*. A representative time path for Q_t is depicted in Figure 2.

Assuming that (25) holds, then, the policy of pegging an interest rate in the manner described leads to an indeterminacy in the form of a continuum of perfect foresight equilibria. This is despite the fact that there is a unique equilibrium under a constant time path for the money supply. Moreover, (25) can easily hold. In fact (25) is equivalent to the condition

$$(25') \qquad 1-\phi(1 + x) > u''/R^* v''.$$

If, for instance, $u'' \equiv 0$, (25′) holds whenever $\phi < (1+x)^{-1}$.

The indeterminacy that obtains here under a pegged interest rate is substantially different than that discussed by Sargent and Wallace (1975) or McCallum (1981, 1986), however. In particular, the price level sequence is determined by the initial target value p_1^* and the target deflation rate ρ. The indeterminacy here relates to the nominal money supply $\{M_t\}$ and all real allocations. Moreover, the indeterminacy arises even if $\rho = 1$, so that the interest rate peg implies the same equilibrium inflation rate as a constant money supply. This is now illustrated by example.

<u>Example</u>. Let $u(c_1) = \ln c_1$ and $v(c_2) = \beta \ln c_2$; $\beta < 1$. Then under a money supply rule specifying $M_t = 1 \ \forall\, t$, the unique equilibrium has $Q_t = \beta w/(1 + \beta)$. Under an interest rate peg, (22) reduces to the condition.

$$Q_{t+1} = \beta R^* w/(1-\phi) - \{[R^*(1+\beta)/(1-\phi)] - \rho\}Q_t, \tag{26}$$

while (25) becomes $(1+\beta)R^*/(1-\phi) < 1+\rho$. If $\rho = 1$, then this condition can be manipulated to obtain $1-\beta > \phi[x+1+\beta(x-1)]$, which will hold whenever ϕ is sufficiently small. Thus a continuum of perfect foresight equilibria exists for small ϕ, even if $\rho = 1$.

This example also illustrates an additional point. If $1+\rho > (1+\beta)R^*/(1-\phi)$, it is easily verified that this economy has no equilibrium two–cycle. The next section demonstrates that the same condition implies the existence of non–trivial s.s.e.'s. Thus s.s.e.'s can exist even if all goods are strictly normal, and an economy has no equilibrium two–cycle.

B. Stationary Sunspot Equilibria

This section retains the notation of section II.B. The additional notation required is that $\tau(e,e')$ denotes the lump–sum transfer made to old agents who experienced state e when young, and e' when old, and M(e) denotes the per capita money supply in state e. From the government budget constraint,

$$\tau(e,e') = [M(e') - M(e)]/p^*_{t+1}. \tag{27}$$

Since $\{p^*_t\}$ continues to be the price level sequence, and since $p^*_t/p^*_{t+1} = \rho < x$, the reserve requirement binds, so that $z(e) = \lambda k(e)$. Then, again letting $Q(e) = (1+\lambda)k(e)$, a young agent born in state e chooses $Q(e) \in [0,w]$ to maximize

$$u[w-Q(e)] + \pi(e)v[R^*Q(e) + \tau(e,1)] + [1-\pi(e)]v[R^*Q(e) + \tau(e,2)].$$

The first order condition associated with this problem is

$$u'[w-Q(e)] = \pi(e)R^*v'[R^*Q(e)+\tau(e,1)] + [1-\pi(e)]R^*v'[R^*Q(e)+\tau(e,2)];\ e=1,2. \tag{28}$$

It is now possible to define an s.s.e. for this economy as a set of values Q(e), k(e), z(e), M(e), $\tau(e,e')$, and $\pi(e)$ satisfying (27) and (28), $k(e) = \phi Q(e)$, $z(e) = (1-\phi)Q(e)$, $0 \leq \pi(e) \leq 1$, and the money market clearing condition

$$M(e)/p_t^* = (1-\phi)Q(e). \tag{29}$$

It is straightforward to construct non–trivial s.s.e.'s if the steady state of the previous section is locally stable, i.e., if (25) holds. The construction is quite standard [following that of Azariadis (1981)], so that its steps are only sketched here. To begin, (29) and (27) imply that $\tau(e,e')$ is given by

$$\tau(e,e') = (1-\phi)Q(e') - \rho(1-\phi)Q(e).$$

Substituting this expression into (28) yields the conditions

$$u'[w-Q(1)] = \pi(1)R^*v'[(\phi x + 1-\phi)Q(1)] + [1-\pi(1)]R^*v'[\phi xQ(1) + (1-\phi)Q(2)] \tag{30}$$

$$u'[w-Q(2)] = \pi(2)R^*v'[\phi xQ(2) + (1-\phi)Q(1)] + [1-\pi(2)]R^*v'[(\phi x + 1-\phi)Q(2)], \tag{31}$$

where $R^* = \phi x + (1-\phi)\rho$ has been used to obtain (30) and (31). Then, solving (30) and (31) for $\pi(1)$ and $\pi(2)$ yields

$$\pi(1) = \frac{R^*v'[\phi xQ(1) + (1-\phi)Q(2)] - u'[w-Q(1)]}{R^*v'[\phi xQ(1) + (1-\phi)Q(2)] - R^*v'[(\phi x+1-\phi)Q(1)]} \tag{30'}$$

$$\pi(2) = \frac{R^*v'[(\phi x + 1-\phi)Q(2)] - u'[w-Q(2)]}{R^*v'[(\phi x + 1-\phi)Q(2)] - R^*v'[\phi xQ(2) + (1-\phi)Q(1)]} \tag{31'}$$

It is now shown that $Q(1)$ and $Q(2)$ can be chosen so that the values of $\pi(1)$ and $\pi(2)$ implied by (30′) and (31′) are in the interior of the unit interval. In particular, choose $w > Q(1) > Q^* > Q(2) > 0$. Then the denominators on the right–hand sides of (30′) and (31′) are both positive. Moreover, $u'[w-Q(1)] > R^* v'[(\phi x + 1-\phi)Q(1)]$ and $u'[w-Q(2)] < R^* v'[(\phi x + 1-\phi)Q(2)]$ both hold. Therefore, the values of $\pi(1)$ and $\pi(2)$ given by (30′) and (31′) are both between zero and one if $Q(1)$ and $Q(2)$ can be selected to satisfy

$$R^* v'[\phi x Q(1) + (1-\phi)Q(2)] \geq u'[w-Q(1)] \tag{32}$$

$$u'[w-Q(2)] \geq R^* v'[\phi x Q(2) + (1-\phi)Q(1)] \tag{33}$$

as well as $w > Q(1) > Q^* > Q(2) > 0$. However, the fact that this is possible follows in a standard way from the asymptotic stability of the steady state equilibrium. Thus (25) implies the existence of a set of non–trivial s.s.e.'s.

Again, the existence of non–trivial s.s.e.'s implies that there is an indeterminacy associated with the policy of pegging an interest rate (in this way) that does not arise under a constant time path for the money supply. Such an indeterminacy can arise even if $\rho = 1$, as illustrated above. Finally, it bears reiterating that there is no indeterminacy regarding the time path for the price level, although any s.s.e. displays random fluctuations in the money stock, the capital stock, and real allocations.

IV. INTEREST RATE PEGS ACCOMPLISHED THROUGH OPEN MARKET OPERATIONS

This section considers the case of what McCallum (1986) terms a "pure" interest rate peg, in which the government stands ready to exchange interest bearing assets for currency on demand. As will be seen, this method of fixing the interest rate leads to a unique equilibrium. However, this equilibrium is generally inefficient relative to the steady state equilibrium of section III except for very particular choices of ρ and p_1^*.

In this section the assets are storage, so $k_{gt} \neq 0$ must be allowed for, and currency. The modifications necessary to permit bonds to be added, and the resulting changes in equilibrium outcomes, are discussed at the end of the section.

As in section III, the government may be viewed as selecting a target sequence for the price level, denoted $\{p_t^*\}$. As above, it is assumed that $p_t^*/p_{t+1}^* = \rho < x \ \forall \ t \geq 1$. Moreover, under a "pure" interest rate peg, $\tau_{1t} = \tau_{2t} = 0 \ \forall \ t$. Thus the government budget constraint becomes

$$(34) \qquad k_{gt} = xk_{gt-1} + (M_t - M_{t-1})/p_t^*$$

with $M_0 > 0$ and $k_{g0} = 0$ given.

Under this scheme for pegging interest rates, the total savings of young agents at t is given by $Q_t = f(R^*,0,0)$, where as above, $R^* = \phi x + (1-\phi)\rho$. Moreover, since $\rho < x$ the reserve requirement binds, so $z_t = (1-\phi)f(R^*,0,0)$. Thus money market clearing requires

$$(35) \qquad M_t/p_t^* = (1-\phi)\, f(R^*,0,0)$$

Finally, $c_{1t} = w - f(R^*,0,0)$ and $c_{2t} = R^* f(R^*,0,0)$. Then with $\{p_t^*\}$ given, all equilibrium quantities are determined and a unique equilibrium exists.

It remains to consider the behavior over time of the government's portfolio. Substituting (35) into (34) implies that government asset holdings, k_{gt}, evolve according to

$$(36) \qquad k_{gt} = xk_{gt-1} + M_t/p_t^* - \rho(M_{t-1}/p_{t-1}^*) = xk_{gt-1} + (1-\rho)(1-\phi)f(R^*,0,0)$$

for $t > 1$, where it will be recalled that $k_{gt} \geq 0$ must hold.[12] Also, at $t = 1$, (34) and (35) imply that

(37) $$k_{g1} = M_1/p_1^* - M_0/p_1^* = (1-\phi)f(R^*,0,0) - M_0/p_1^*.$$

Then, since $k_{g1} \geq 0$ must hold, p_1^* must (for feasibility) be set so that $M_0/p_1^* \leq (1-\phi)f(R^*,0,0)$.

The solution to (36) is given by

(38) $$k_{gt+1} = x^t k_{g1} + (x^t-1)(1-\rho)(1-\phi)f(R^*,0,0)/(x-1).$$

Substituting (37) into (38) and rearranging terms yields

(39) $$k_{gt+1} = x^t[(x-\rho)(1-\phi)f(R^*,0,0)/(x-1) - (M_0/p_1^*)] - (1-\rho)(1-\phi)f(R^*,0,0)/(x-1)$$

as the value for government storage at t + 1. Evidently, p_1^* and ρ must be set so that

(40) $$(x-\rho)(1-\phi)f(R^*,0,0)/(x-1) \geq M_0/p_1^*$$

since otherwise $k_{gt+1} < 0$ must eventually occur (contrary to feasibility). In addition, (40) can hold with equality only if $\rho \geq 1$, since otherwise (40) at equality would contradict $M_0/p_1^* \leq (1-\phi)f(R^*,0,0)$. Thus, unless (40) holds with equality and $\rho = 1$, government storage must necessarily be positive $\forall$ t. And, if (40) holds as an inequality, government storage grows without bound. In fact, in this case the discounted present value of future government storage is always positive.

If either (40) holds as a strict inequality, or $\rho > 1$ (or both), storage of goods occurs which results in no future consumption (only more future storage by the government). This situation is clearly not efficient. It is now established that it is generally inefficient relative to the steady state equilibrium that obtains in section III. This statement requires some justification, however, since the equilibrium of section III need not be Pareto optimal. By way of providing such a justification, it is assumed that target values ρ and p_1^* are given, and that

p_1^* is set so that $M_0/p_1^* < (1-\phi)Q^*$ (where Q^* is the steady state savings level of section III). To establish the inefficiency of the "pure" peg under these conditions, it is easiest to consider two cases.

<u>Case 1</u>. Suppose $\rho \leq 1$, so that $R^* \leq \phi x + 1-\phi$. Let c_{1t}^* and c_{2t}^* denote young and old consumption, respectively, under the steady state equilibrium of section III, while $\hat{c}_{1t}$ and $\hat{c}_{2t}$ denote the same equilibrium quantities under the "pure" peg. Then $c_{1t}^* = w-Q^*$ and $c_{2t}^* = \tau_{2t+1} + R^*Q^* = (\phi x + 1-\phi)Q^*$. Also, $\hat{c}_{1t} = w-f(R^*,0,0)$, while $\hat{c}_{2t} = R^*f(R^*,0,0)$. Then it is claimed that $c_{1t}^* \geq \hat{c}_{1t}$ and $c_{2t}^* \geq \hat{c}_{2t}$. The first of these inequalities is obvious, since Q^* satisfies (note that $\tau_{2t} = (1-\rho)(1-\phi)Q^*$)

$$Q^* = f[R^*,0,\ (1-\rho)(1-\phi)Q^*] \leq f(R^*,0,0),$$

where the inequality follows from $f_3 < 0$. Moreover, $(\phi x + 1-\phi)Q^* \geq R^*f(R^*,0,0)$ holds. To see this, note that $f(R^*,0,0)$ satisfies

$$u'[w-f(R^*,0,0)] = R^*v'[R^*f(R^*,0,0)], \tag{41}$$

while from (24), Q^* satisfies

$$u'(w-Q^*) = R^*v'[(\phi x + 1-\phi)Q^*]. \tag{42}$$

Then suppose $R^*f(R^*,0,0) > (\phi x + 1-\phi)Q^*$. From (41) and (42) this implies that

$$u'[w-f(R^*,0,0)] = R^*v'[R^*f(R^*,0,0)] < R^*v'[(\phi x + 1-\phi)Q^*] = u'(w-Q^*),$$

or that $Q^* > f(R^*,0,0)$. But this is a contradiction.

Then young agents cannot be better off under a "pure" peg, and apparently will be worse off if $\rho \neq 1$. Now consider the initial old agents. Under the "pure" peg their consumption is M_0/p_1^*. Under the section III steady state equilibrium their consumption is $M_0/p_1^* + \tau_{21} = (1-\phi)Q^*$, from equation (20). Since $(1-\phi)Q^* > M_0/p_1^*$ by assumption, the initial old are necessarily worse off under the "pure" peg.

Case 2. Now let $\rho > 1$, so that $R^* > \phi x + 1-\phi$. In this case, the same arguments as above imply that $Q^* = f[R^*,0,(1-\rho)(1-\phi)Q^*] > f(R^*,0,0)$, and $R^* f(R^*,0,0) > (\phi x + 1-\phi)Q^*$. Thus all young generations actually benefit from the "pure" peg. However, as above, under the pure peg the initial old consume the per capita amount M_0/p_1^*, while under the section III steady state equilibrium, they consume $(1-\phi)Q^* > M_0/p_1^*$. Moreover, in this case $M_0/p_1^* \leq (1-\phi)f(R^*,0,0)$ must hold for feasibility, and $Q^* > f(R^*,0,0)$, so the government must set $M_0/p_1^* < (1-\phi)Q^*$ (i.e., this is not just an assumption). Then a "pure" peg represents an inefficient transfer from the initial old to young agents if $\rho > 1$.

While the "pure" peg results in a unique equilibrium, then, it generally represents an inefficient situation relative to the steady state equilibrium that obtains under a tax/transfer scheme. However, if (25) holds, a tax/transfer scheme will allow for the existence of multiple equilibria. In short, a sort of tension between determinacy and efficiency is generally observed.

A natural question concerns how these results would be affected by the introduction of interest bearing government bonds. Suppose the government issues a quantity B_t of real one period bonds at t ($B_0 = 0$), which may be positive or negative. Suppose further that bond holdings are not subject to the reserve requirement. Then if bonds and other assets are to coexist, bonds must bear the (fixed) real return R^* at each date.

Suppose further that the government stands ready to exchange either bonds or storage for currency on demand. (The argument will not change if only bonds are exchanged for currency.) Then there is again a unique equilibrium. For brevity, its features will be sketched without any derivation. The equilibrium quantity of government storage is zero at each date, while $B_t = f(R^*,0,0) - (1 + \lambda)k_t$ holds $\forall$ t. Young agents borrow from the government in order to store the good. Their first period consumption is fixed at $w-f(R^*,0,0)$ and their old age consumption is $R^*f(R^*,0,0)$. However, private per capita storage at t evolves according to

$$k_{t+1} = x^t[(x-R^*)f(R^*,0,0)/(x-1) - (M_0/p_1^*)] + (R^*-1)f(R^*,0,0)/(x-1)$$

which grows without bound unless $M_0/p_1^* = (x-R^*)f(R^*,0,0)/(x-1)$. Thus again there is typically (increasingly larger) unconsumed storage at each date. In effect, relative to the situation without bonds, private agents store the good on behalf of the government, with the government financing this storage via loans. (Of course equilibrium storage is not the same here as it was previously, since government storage of the good is not subject to the reserve requirement.) In short, introducing government bonds will not affect the basic results.

V. CONCLUSIONS

An economy has been presented which has a unique equilibrium under a money supply rule that equates the rate of money growth and the rate of real growth. In addition, the behavior of the economy was considered under two schemes for pegging the nominal interest rate. Under a "pure" peg, a unique equilibrium results. However, under certain choices of targets, it is possible to achieve a Pareto improvement (when steady states are considered) by using a tax/transfer scheme to support the pegged level of the interest rate. The latter method of pegging the interest rate allows indeterminacies to arise under conditions that can easily be satisfied, however. Thus even when attention is restricted to interest rate pegs, some tension appears to exist between the determinacy and the efficiency of equilibria.

By way of conclusion, two objections are anticipated to the results just described. First, the analysis here is similar in spirit to that of Sargent and Wallace (1982), who analyze policies meant to peg the nominal interest rate at zero in an overlapping generations economy with a legal restriction. One criticism of Sargent–Wallace (1982), which might appear to apply here, is presented by McCallum (1986), who argues that money has no medium of exchange role in the kind of economies being considered. In McCallum's view, this argument suggests that the issues of efficiency or determinacy in a monetary economy addressed above (and addressed by Sargent–Wallace) are not well captured by an overlapping generations model.

In the context of this analysis, such an argument would appear to be misplaced. In particular, agents in the model here hold (base) money neither as a medium of exchange nor a store of value. Rather they are forced to do so by a reserve requirement. For many economies, particularly developing ones, this is a relevant situation. For instance, Taylor (1980) assumes that all base money in LDCs is held as required reserves, and offers a defense of the realism of this assumption. If Taylor's view is accepted, the assumptions made here about money holding appear relevant to a number of economies. Thus McCallum's argument can be telling in this context only if one believes that it is of no interest to discuss the issues considered here in the context of LDCs.

Second, the model used in sections II – IV included assumptions which rule out s.s.e.'s under a constant time path for the money supply. It was then used to demonstrate that, even in the presence of savings functions that are strictly increasing in the rate of return, and even though two period cycles need not exist, non–trivial s.s.e.'s can arise under certain methods for pegging the interest rate. Since it seems to be widely accepted that monetary policy actually operates by pegging interest rates, this would suggest that s.s.e.'s are more plausible empirically than would be indicated by earlier analyses. Here, however, it might be argued that the "realistic" assumption is that the monetary authority adopts a "pure" peg, or in other words, uses the policy analyzed in section IV. In particular, it seems common to view the monetary authority as pegging interest rates by standing ready to exchange interest bearing assets for currency, with real taxation fixed. In such a situation, s.s.e.'s will not exist.

In fact, however, such an argument would imply that changes over time in the money supply occur without any changes in government expenditure or real taxation, or in other words, that there is no monetization of deficits (or withdrawal of money as governments run surpluses). It would appear difficult to identify many periods or places where this is a realistic characterization of events. Thus it seems plausible to entertain the analysis of section III as a reasonable formulation of an interest rate peg. In this instance the analysis does suggest that s.s.e.'s are quite likely to be observed.

NOTES

1. Examples of this literature include Poole (1970), Sargent and Wallace (1975, 1982), Parkin (1978), McCallum (1981, 1986), Dotsey and King (1983), Canzoneri, Henderson, and Rogoff (1983), Woodford (1988), and Barro (1989). Only Sargent–Wallace (1982) and Woodford (1988) do not utilize an IS–LM framework. Relatedly, Black (1987) advocates a policy of targeting sequences for the price level. It will be apparent that all of the discussion below also applies to attempts to "peg" the time path of the price level.
2. Examples include Smith (1988, 1989) and Woodford (1988).
3. Barro (1989) and McCallum (1986) discuss the possibilities of multiple equilibria, but do not explore the set of equilibria that exist under various policies.
4. Recall that Barro (1989) assumes an exogenously fixed (expected) real rate of interest.
5. This formulation follows that of section VI of Wallace (1981), and of Smith (1989).
6. As in Sargent and Wallace (1982), it is assumed that the government imposes no legal restrictions on itself.
7. It is easy to verify that all of the results of this section continue to obtain if M_t evolves according to $M_t = \sigma M_{t-1}$, if money is injected (or removed) via lump–sum transfers, if σ is sufficiently near one, and if (1) holds with strict inequality.
8. Notice that, so long as $\phi x v'(0) > u'(w)$, $Q_t = 0 \ \forall\ t$ is not an equilibrium.
9. In particular, there are no markets in which state contingent claims are traded. Since the young at t are born after the realization of the time t state, of course state contingent claims trading between young and old agents cannot occur. Then, since all young agents are identical, in equilibrium state contingent claims trades must be zero, so this assumption is innocuous.
10. It is easy to show that there are no s.s.e.'s in which the reserve requirement is not binding, or in which it binds in only one state.

11. The fact that a target value is chosen for p_1^* resolves the usual indeterminacies that arise in rational expectations models with pegged interest rates. This is a fairly standard resolution of this problem: see, e.g., McCallum (1981, 1986), Dotsey and King (1983), Canzoneri et.al.(1983), or Sargent and Wallace (1975, 1982). Also, the fact that pegging a nominal interest rate (plus an initial condition) is equivalent to choosing a time path for the price level implies that everything said here applies to Black's (1987) suggestion for pegging a time path for the price level (and allowing endogenous adjustment of the money supply).

12. $k_{gt} < 0$ could not be interpreted as lending by the government, since this would imply [by (34)] that the government lends at the real rate x. Since the most agents can earn on their investments is $R^* < x$, no such loans would be taken. Nor could it be interpreted as government borrowing, since such borrowing would drive out other assets. Situations that allow borrowing and lending by the government are discussed below.

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Figure 1

Law of motion for Q_t under a money supply rule.

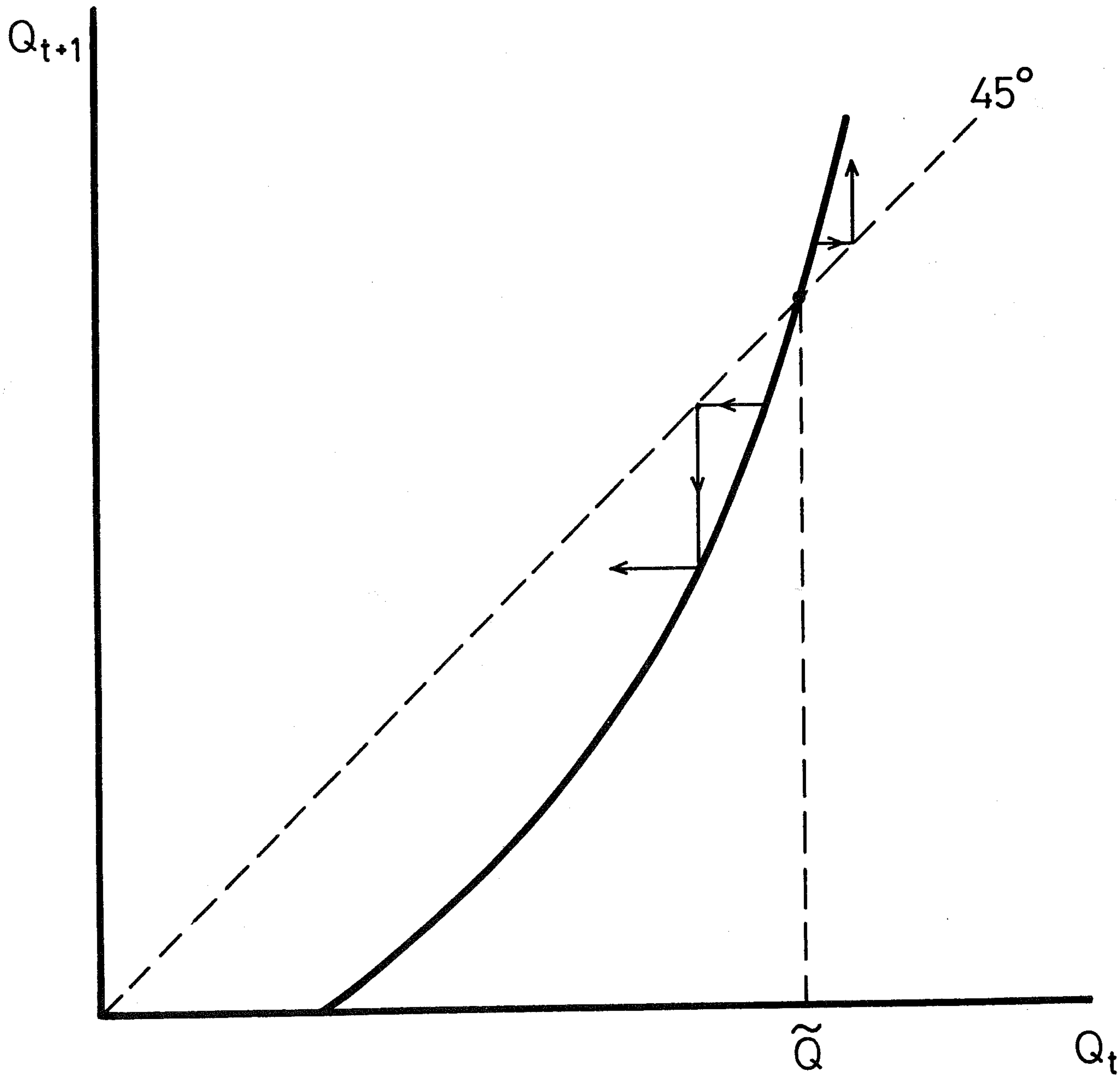

Figure 2

Law of motion for Q_t under an interest rate peg.