

Real Business Cycles and the Test of the Adelmans

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### ABSTRACT

This paper conducts a modern variant of the test proposed and carried out by Adelman and Adelman [1959]. Using the methods developed by Burns and Mitchell [1946], we see if we can distinguish between the economic series generated by an actual economy and those analogous artificial series generated by a stochastically perturbed economic model. In the case of the Adelmans, the model corresponded to the Klein-Goldberger equations. In our case, the model corresponds to a simple real business cycle model. The results indicate a fairly high degree of coincidence in key economic aggregates between the business cycle characteristics identified in actual data and those found in our simulated economy.

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## 1 Introduction

The most prominent and detailed description of business cycle regularities is due to a generation of researchers assembled by Wesley Clair Mitchell at the National Bureau of Economic Research (NBER) in New York in the 1940s and 1950s. These researchers pursued and expanded the research program initiated by Mitchell [1927] and Burns and Mitchell [1946]. They analyzed hundreds of economic time series producing a wealth of information about the pace and pattern of economic activity in a variety of industrialized countries. This massive body of empirical work now provides the dominant background to most academic and policy discussion of macroeconomic developments.

It would be difficult to overstate the cumulative impact of this body of research. When we talk about business cycles, most of us have in mind one variant or another of the visual summary statistics produced by Burns, Mitchell and their co-workers. To take one common example, when we think about the evolution of an industry's production over time, we frequently think about a plot of the time series with recession components highlighted by shaded areas. For another, our sense of how long expansion and recession episodes last is based on measurements that continue to be made by the NBER to this day, following the path of Burns and Mitchell. To cite yet a third example, our view of the extent to which particular economic variables move with the cycle in aggregate economic activity--for example, investment tightly related and real wages not so tightly linked--is heavily influenced by business cycle plots and conformity statistics of the type developed by Burns and Mitchell. In fact, these results are so much a part of our thinking that it is relatively common to evaluate models in terms of whether they *replicate* the stylized facts of business cycles as isolated by Burns and Mitchell.

Yet, from a modern perspective, the Burns and Mitchell procedures produced summary statistics with unknown distributional properties. This vacuum was primarily due to the complexity of the computations and macroeconomists' lack of detailed knowledge about the dynamic

economic model generating the time series (Koopmans [1947]). This paper is the first product of a research project that is designed to shed light on the character of the Burns and Mitchell procedures. In the current context, we are mainly interested in following the path of Adelman and Adelman [1959], using the Burns and Mitchell procedures to *evaluate* a specific economic model. Our intention, however, is to build on skills gained in execution of this project to ultimately provide a broader based evaluation of the procedures developed by Burns and Mitchell so that we may better understand the facts and figures that underlie our description of business cycles.

Although the NBER business cycle research was not designed to evaluate specific economic theories, it has of course been used--beginning with Mitchell [1927]--to guide the development of economic models because it represented a group of "stylized facts" about economic fluctuations to which theories could be compared. This approach to model evaluation was made explicit by Adelman and Adelman [1959], who developed summary statistics based on the Burns and Mitchell methodology for the time series generated by a stochastically perturbed variant of the Klein-Goldberger [1959] model. Then, the Adelmans compared these statistics to those for the U.S. economy as reported in Mitchell [1951]. As stated by Lucas [1980];

"the Adelmans posed, in a precise way, the question of whether an observer armed with the methods of Burns and Mithcell [1946] could distinguish between a collection of economic series generated artificially by a computer programmed to follow the Klein-Goldberger equations and the analogous series generated by an actual economy. The answer, to the evident surprise of the Adelmans (and, one suspects, of Klein and Goldberger, who has in no way directed their efforts to meeting this criterion) was *no*. This achievement signaled a new standard for what it means to understand business cycles. One exhibits understanding of business cycles by constructing a *model* in the most literal sense: a fully articulated economy which behaves through time so as to imitate closely the time series behavior of actual economies."

More specifically, the Adelmans showed that only *one* of three types of displacements was capable of generating business cycle movements that resembled those isolated by the NBER



researchers. Neither transitional responses from initial conditions nor sample variation in exogenous variables was capable of reproducing the stylized facts isolated by the NBER researchers. Rather, it was necessary to add random shocks to the structural equations of the Klein-Goldberger model. Then, the model's internal dynamic structure led to time series that resembled those of the U.S. economy from the perspective of an NBER researcher.

The objective of this paper is to conduct a modern variant of the Adelman's project, taking advantage of the development in economic theory, econometric method and computing power that have occurred since the 1950s. First, we use a representative "real business cycle model" that is driven by a stochastic process for total factor productivity, as a basic example of a fully articulated model economy. Second, since the reduced form of this model is a system of log linear system of difference equations, its time and frequency domain implications can be determined with precision. Third, drawing on work by Bry and Boschan [1971], we use computer algorithms for business cycle analysis that avoid some of the difficulties of replication that are evident in Burns and Mitchell's early work. Finally, given developments in computing power since 1959, it is possible for us to be more explicit about the *distribution* of the Burns and Mitchell measures than it was for the Adelmans. We generate time series from our real business cycle model and compute various summary measures, with an eye toward appraising the capacity of this artificial economy to mimic the time series of the U.S. economy.

In addition, a virtue of our approach is that it enables us to shed light on the mapping between the stochastic properties of an underlying economic model and the summary statistics of the Burns and Mitchell procedures. Articulation of this mapping is important because the Burns and Mitchell methods of business cycle analysis have not been previously subjected to this type of scrutiny. However, we defer until later a detailed and more complete evaluation of these business cycle methods.

Our discussion is organized as follows. In section 2, we provide a brief overview of business cycle research as described in Burns and Mitchell [1946] and codified by Bry and Boschan [1971]. We discuss our variant of this approach--which can be mechanically applied to time series generated by an artificial economy. Then, we present results of applying this method to the post World War II behavior of some major macroeconomic time series including the five series (measures of consumption, investment, output, labor input and the real wage) we studied in a pair of recent papers on real business cycles (King, Plosser and Rebelo [1988a,b]). In section 3, we provide an overview of the basic real business cycle model that we employ in our analysis and summarize the model's stochastic properties. In section 4, we discuss the results of simulating this model and compare the business cycle characteristics of U.S. and model generated time series using the methods developed by Burns and Mitchell. In section 5 we make a crude comparison between the summary measure proposed by Burns and Mitchell and the more common statistical measures of standard deviation and correlation.. The final section contains a summary and discussion of research plans.

## 2 Implementation of the Burns and Mitchell Methodology

Burns and Mitchell's 1946 treatise, *Measuring Business Cycles* provides the most complete description of their approach to processing information and summarizing the central characteristics of business cycles. In this section we briefly describe key aspects of the Burns and Mitchell method and our implementation of their procedures.

### 2.1 Reference Cycle Determination

In order to investigate and measure the characteristics of different series over the business cycle, it is necessary to define the business cycle and to determine when it occurs. Burns and Mitchell adopted the working definition proposed earlier by Mitchell:

Business cycles are a type of fluctuations found in the aggregate economic activity of nations that organize their work mainly in business enterprises: a cycle consists of expansions occurring at about the same time in many economic activities, followed by similarly general recessions, contractions, and revivals which merge into the expansion phase of the next cycle; this sequence of changes is recurrent but not periodic; in duration business cycles vary from more than one year to ten or twelve years; they are not divisible into shorter cycles of similar character with amplitudes approximating their own. [Mitchell (1927, p. 468)]

While vague, the definition does place restrictions on both duration and amplitudes that must be satisfied to qualify an episode as a business cycle. The first step, therefore, is to determine the periods of expansion and contraction in "aggregate economic activity" or, in the terminology of Burns and Mitchell, to determine the dates or turning points for the reference cycle. Burns and Mitchell, however, were not satisfied with any of the measures of aggregate economic activity available to them so they did not wish to determine the reference cycle dates using only aggregate measures. Instead they relied primarily on two sorts of information: i) descriptive evidence from business publications and general business conditions indexes and ii) the "specific cycles" found in many individual series and the tendency for the turning points from expansion to contraction (and vice versa) to sometimes cluster at certain dates. Based on this information a set of

reference cycle dates were selected that specified the turning points (peaks and troughs) in "aggregate economic activity." This process was time-consuming and involved considerable researcher specific judgement.

Our implementation of the Burns and Mitchell procedures, to the data of the post-war U.S. economy and to our simulated artificial real business cycle economy, differs only in the manner in which the reference cycle dates are selected. We depart from Burns and Mitchell in two ways. First, we determine the turning points in the reference cycle using a measure of aggregate economy activity such as real per capita GNP. Second, the actual turning points are selected using an algorithm developed by Gerhard Bry and Charlotte Boschan [1971]. The advantage of the Bry and Boschan method is that it is quickly and easily implemented and can be readily replicated. The disadvantage, of course, is that it removes from the process the detailed knowledge and judgement of an experienced observer of business cycles.

The Bry and Boschan procedure for selecting turning points in any series is summarized in Table 1. The general procedure, followed also by Burns and Mitchell, is to look for turning points in some smoothed version of a seasonally adjusted series so as not to be misled by "erratic" movements. Bry and Boschan implement this by beginning with a highly smoothed series (a 12-month moving average) to find initial estimates of the turning points. Using these initial estimates, a somewhat less smooth curve is investigated (a Spencer curve)<sup>1</sup> to refine the dates of the turning points. This process is repeated using a short-term (3 month) moving average. The final turning points are determined using the unsmoothed series and verifying that the turns sat-

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<sup>1</sup>The Spencer Curve is a 15-month centered moving average with the terms near the center receiving the largest weight and the extreme terms receiving a negative weight. The actual weights on terms  $t-7$  to  $t+7$  are [-.0094, -.0188, -.0156, .0094, .0656, .1438, .2094, .2313, .2094, .1438, .0656, .0094, -.0156, -.0188, -.0094].

isfy a set of restrictions given in part V of Table 1. Of particular interest is that business cycles must be no less than 15 months long and all phases (expansions and contractions) must be at least 5 months in duration.

**Table 1**  
**PROCEDURE FOR PROGRAMMED DETERMINATION**  
**OF TURNING POINTS**

- 
- 
- I. Determination of extremes and substitution of values.
  - II. Determination of cycles in 12-month moving average (extremes replaced).
    - A. Identification of points higher (or lower) than 5 months on either side.
    - B. Enforcement of alternation of turns by selecting highest of multiple peaks (or lowest of multiple troughs).
  - III. Determination of corresponding turns in Spencer curve (extremes replaced).
    - A. Identification of highest (or lowest) value within  $\pm 5$  months of selected turn in 12-month moving average.
    - B. Enforcement of minimum cycle duration of 15 months by eliminating lower peaks and higher troughs of shorter cycles.
  - IV. Determination of corresponding turns in short-term moving average of 3 to 6 months, depending on MCD (months of cyclical dominance).
    - A. Identification of highest (or lowest) values within  $\pm 5$  months of selected turn in Spencer curve.
  - V. Determination of turning points in unsmoothed series.
    - A. Identification of highest (or lowest) value within  $\pm 4$  months, or MCD term, whichever is larger, of selected turn in short-term moving average.
    - B. Elimination of turns within 6 months of beginning and end of series.
    - C. Elimination of peaks (or troughs) at both ends of series which are lower (or higher) than values closer to end.
    - D. Elimination of cycles whose duration is less than 15 months.
    - E. Elimination of phases whose duration is less than 5 months.
  - VI. Statement of final turning points.
- 
- 

Note: This table is taken from Bry and Boschan [1971, pg. 21]

Bry and Boschan present two types of evidence that are useful in evaluating their procedures. First, they study a series that was extensively studied by Burns and Mitchell; monthly bituminous coal production from 1914 to 1935. The thirteen turning points selected by Burns and Mitchell correspond exactly to ones selected by the program in all cases but one and this was

a dating difference of three months. The only other discrepancy was that the programmed procedures isolated a minor cycle that was ignored by Burns and Mitchell. The second type of evaluation conducted by Bry and Boschan was to compare the turning points selected by their program in over fifty series covering the period 1947-1966 to the turning points selected by the National Bureau staff. Of the turns identified by the staff (435 in total) almost 95% of them were identified by the program and 90% had identical dates. The only systematic discrepancy arose because the program procedures tended to find about 15% more turns than the staff.

To illustrate these procedures we have computed the turning points for real GNP and per capita real GNP. One issue raised by these series is that they are quarterly rather than monthly series. Bry and Boschan handle quarterly data by simply setting each month of the quarter equal to the quarterly value and proceeding to treat the series as monthly. This interpolation procedure is similar, but not identical, to that employed by Burns and Mitchell. For subsequent analysis the turning point is arbitrarily assigned to the middle month of the quarter. The turning points are presented in Fig. 1a and 1b. For comparison purposes we have labeled the turning points determined by the NBER business cycle dating committee. The NBER dates are not simply based on the behavior of GNP but on many series and thus there is no *a priori* reason the procedures should yield the same results.

There are two important observations to be made about Fig. 1a. First, working solely from real GNP the program procedures do not recognize the NBER recession that began in April 1960 and ended in February 1961. The smoothing process explicit in the programmed procedures effectively eliminates any evidence of a recession during this period. Second, the program procedure does not recognize the NBER recession that began in January 1980 and ended in July

Fig. 1a  
TURNING POINTS IN  
REAL GNP

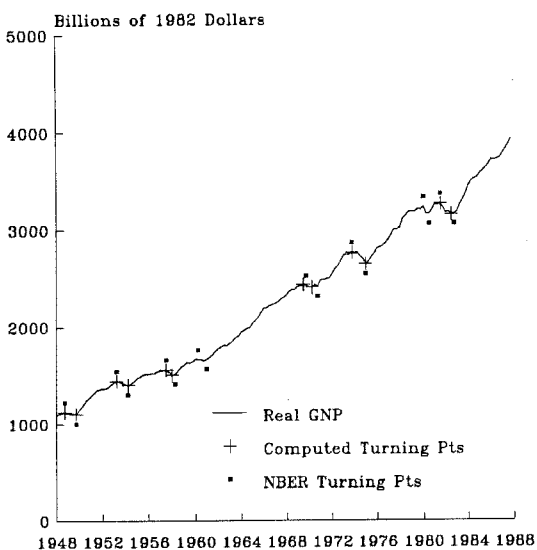
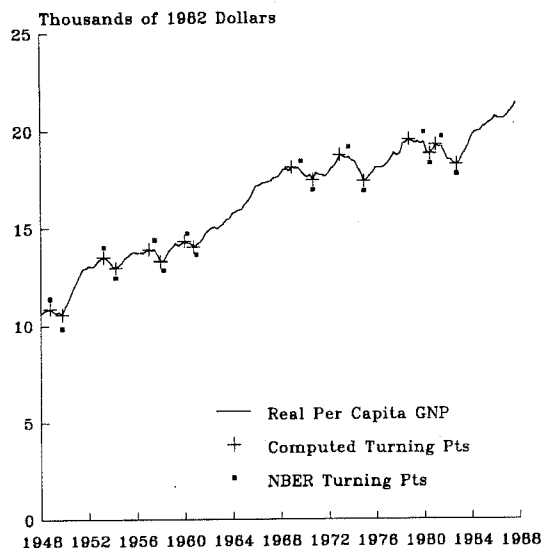


Fig. 1b  
TURNING POINTS IN  
REAL PER CAPITA GNP



1980. The reason arises from the phase duration restriction. The program recognized a peak in March 1980 and a trough in June 1980, but since it lasted less than five months it was rejected as a business cycle.<sup>2</sup>

Fig. 1b presents the turning points in real per capita GNP. While Burns and Mitchell did not consider per capita measures in their analysis, we do for the simple reason that the sorts of representative agent models we are working with in Section 3 are more naturally interpreted as describing per capita quantities. It is apparent, however, that real per capita GNP displays somewhat different cyclical patterns than real GNP. The reasons are easy to understand. By deflating by population, the peaks will tend to occur earlier than the peaks in total GNP. This holds true for six of the eight peaks occurring in the post-war period. Working with the per capita measure also results in the return of the 1960 recession because the decline is more pronounced and the

<sup>2</sup>Consistent with the July 1980 selection, the NBER dates the trough as 1980 III even though real GNP was actually lower in 1980 II than in 1980 III.

1980 recession because the peak is identified so much earlier. Trough dates tend to correspond more closely because the typical rate of decline preceding a trough and the rate of increase following a trough are larger than the rate of increase preceding a peak. Thus deflating by population has little impact on the location of the trough.

Table 2 summarizes the turning points selected by the algorithm of Bry and Boschan and the business cycle peaks and troughs selected by the NBER. As can be seen from the table, the dates of the troughs coincide almost exactly, but the peaks in per capita GNP frequently lead the NBER selected peaks. The table also summarized the duration (in months) of the business cycles identified from real per capita GNP.

**Table 2**  
**COMPARISON OF TURNING POINTS**  
**IN THE POST-WAR PERIOD**  
**1948-1987**

Turning Points Based on Real per Capita GNP		Reference Cycle Dates Selected by the NBER		Timing Differences (in Quarters)		Duration (Months)		
Trough	Peak	Trough	Peak	Trough	Peak	Expansion	Contraction	Cycle <sup>a</sup>
-	1948 IV	-	1948 IV	-	0	-	12	-
1949 IV	1953 II	1949 IV	1953 II	0	0	42	12	54
1954 II	1957 I	1954 II	1957 III	0	2	35	12	47
1958 I	1960 I	1958 II	1960 II	1	1	24	9	33
1960 IV	1969 I	1961 I	1969 IV	1	3	99	21	120
1970 IV	1973 I	1970 IV	1973 IV	0	3	27	24	51
1975 I	1978 IV	1975 I	1980 I	0	5	45	21	66
1980 III	1981 I	1980 III	1981 III	0	2	6	21	27
1982 IV	-	1982 IV	-	0	-	-	-	-
					Avg.	39.5	17.1 <sup>b</sup>	56.6

<sup>a</sup>Cycle durations are computed on a trough-peak-trough (T-P-T) basis beginning with the trough in 1949 IV.

<sup>b</sup>The average contraction length is computed by ignoring the initial contraction from 1948 IV to 1949 IV to be comparable with the full cycle calculation. The average contraction length including the initial contraction is 15.9.



The average expansion lasted almost 40 months, the average contraction lasted just over 17 months and the cycle averaged almost 57 months. As expected from the timing differences, these averages differ slightly from the NBER dates where the average expansion is 45 months, average contraction is just over 11 months and the cycle averaged 56 months.

## 2.2 Summary Measures of Cyclical Behavior

With the reference cycle defined by the turning points determined above, the basic measures of cyclical behavior, as developed by Burns and Mitchell, are easily computed. In this development we follow Burns and Mitchell as closely as possible. Our task has been simplified by Bry and Boschan, who programmed the summary statistics described below.

*Reference Cycle Patterns and Cycle Relatives.* Each reference cycle is divided into nine stages. The initial stage (I) includes the three months centered on the initial trough, stage V is the three months centered on the reference cycle peak, and stage IX is the three months centered on the terminal trough. The expansion phase (stage I to V) is divided into three substages (II, III, and IV) of equal length (excluding those months included in stages I and V). The contraction phase is similarly divided to obtain stages VI, VII and VIII. Since expansions and contraction are generally not of equal length, the number of months in the expansion stages (II, III and IV) will not be the same as in the contraction stages (VI, VII and VIII). Moreover, since reference cycle expansions (contractions) will be of differing lengths, stages II, III and IV (VI, VII and VIII) will not contain the same number of months across reference cycles. Breaking cycles into stages results in each reference cycle having associated with it two units of measurement. One is measured in calendar time and the other is measured in business cycle time.

In order to compare the behavior of different series over the cycle and across cycles, the series are typically converted to *cycle relatives*. This conversion is accomplished by computing the average value of the series over a particular reference cycle (the cycle base) and expressing

each observation as a percentage of this average. Thus a cycle relative value of 100 is equivalent to the average value of the series over the cycle. The conversion to cycle relatives makes it possible to compare cycles at different points in time.<sup>3</sup> To do so, the Burns and Mitchell procedure eliminates an inter-cycle trend. But, the potential for an intra-cycle trend remain. Thus, if a series exhibits substantial growth over a cycle, the cycle relatives will have an upward drift.

A controversial step in the Burns and Mitchell procedures involves averaging cycle relatives at each stage across reference cycles. This results in an average cycle relative at each stage of the business cycle. Plots of these average reference cycle patterns are intended to show how different series behave, on average, over the business cycle. These *business cycle plots* provide an important graphical summary of the information contained in the Burns and Mitchell approach.

*Amplitudes.* One important summary measure that is easily discernible from the business cycle plots is the amplitude of a specific series over the reference cycle. Amplitudes are measured for both expansions and contractions as well as the full cycle. The amplitude during expansion is simply the cycle relative at the peak (stage V) minus the cycle relative at the initial trough (stage I). For contractions, the amplitude is the cycle relative at the terminal trough (stage IX) minus the cycle relative at the preceding peak (stage V). Note that for a series that typically rises during expansions and falls during contractions the amplitude will be positive for expansions and negative for contractions. The full cycle amplitude is simply the expansion amplitude minus the contraction amplitude.

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<sup>3</sup>Cycle relatives are generally not appropriate for series that contain significant numbers of negative values. In such instances the analysis is usually carried out in the original units. This is the case, for example, for the yield spread and the inflation rate discussed in Section 2.3 below.

*Conformity.* While the business cycle plots provide a useful visual impression of the degree to which a series moves with the business cycle, Burns and Mitchell developed a quantitative device for assessing this trait. This summary measure is called conformity and can be associated with expansions, contractions and the full cycle. To compute the conformity of a series during reference cycle expansions, a value of one is assigned to each expansion for which the average per month change in the cycle relative from stage I to stage V (trough to peak) is positive. For those expansions where the average per month change is negative (that is the series falls during an expansion), a value of minus one is assigned. The average of this series of ones and minus ones (multiplied by 100) is the index of conformity. A conformity of +100 corresponds to a series that on average rises during each reference cycle expansion and a conformity of -100 corresponds to a series that on average falls during each reference cycle expansion. Conformity during contractions is measured in analogous fashion except that a value of plus one is assigned if the average per month change in the cycle relative from stage V to stage IX is negative. Conformity of +100 during contractions is associated with a series that on average declines during every reference cycle contraction.

Conformity for the full cycle is determined not by the average conformity during expansions and contractions, but by the relative behavior of the series during the two phases. A value of one is assigned to each cycle that the average per month change during the expansion phase exceeds the average per month change during the contraction phase. Thus a series might rise during every expansion and contraction so that conformity is +100 during expansions and -100 during contractions, but if the rate of monthly increase in expansions exceeds that in contractions the full cycle conformity would be +100.

### *2.3 Summary Measures for the United States 1948-1987*

In this section we employ the methods of the Burns and Mitchell outlined above to characterize the business cycle in the post-war U.S. economy. Our primary focus is on five commonly discussed macroeconomic aggregates: per capita values of GNP, consumption of nondurables and services, gross fixed investment, average weekly hours and a measure of the real wage.<sup>4</sup> These series also most closely correspond to the conceptual series generated by the real business cycle model discussed in Section 3. Following Burns and Mitchell, no prior filtering or detrending of the data has been undertaken except for the implicit filtering implied by the use of seasonally adjusted data and per capita quantities.

Reference cycle dates are computed based on the methods of Bry and Boschan summarized in Table 1 using aggregate real per capita GNP. In Fig. 2a-2e, we plot the quarterly time series for our five macroeconomic quantities and indicate the reference cycle peaks and troughs based on real per capita GNP and those selected by the NBER business cycle dating committee. These pictures highlight the major features of the data familiar to macroeconomists: expansion phases are longer than contraction phases; contractions are accompanied by sharp changes in labor input and investment; and consumption responds far less than output to recession. In Fig. 2f we have plotted the GNP deflator over the post-war period. It displays no special cyclical tendencies, instead rising during both expansions and contractions.

Next, we present a compendium of business cycle plots of the form employed extensively by Burns and Mitchell. These plots, presented in Fig. 3a-3f, display the average behavior, in cycle relatives, over the nine stages of the business cycle for each of the six macroeconomic time series. Each of these plots are constructed in the same manner and drawn to the same scale. They plot the average cycle relative for each stage where the average is taken over the seven

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<sup>4</sup>See the appendix for a complete description of the data series.

Fig. 2a  
REAL PER CAPITA GNP

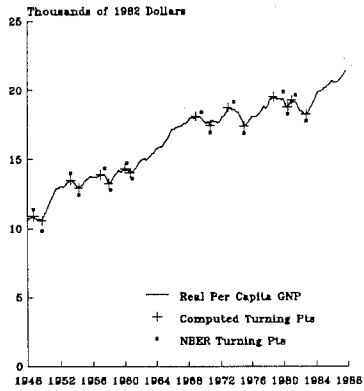


Fig. 2b  
REAL PER CAPITA CONSUMPTION

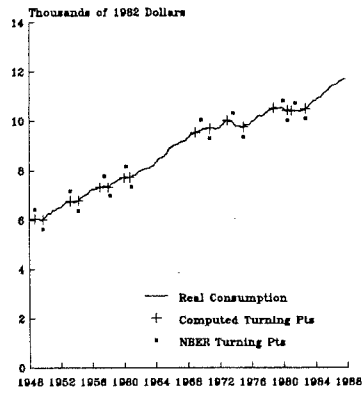


Fig. 2c  
REAL PER CAPITA FIXED INVESTMENT

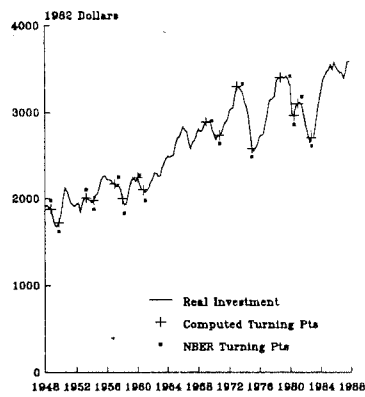


Fig. 2d  
PER CAPITA WEEKLY HOURS WORKED

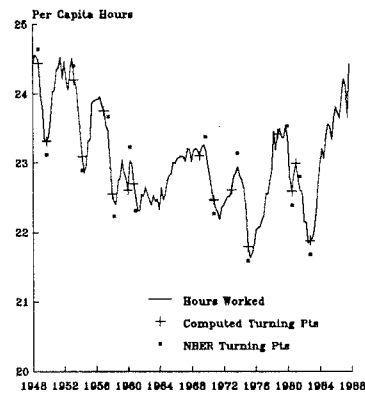


Fig. 2e  
REAL HOURLY WAGES

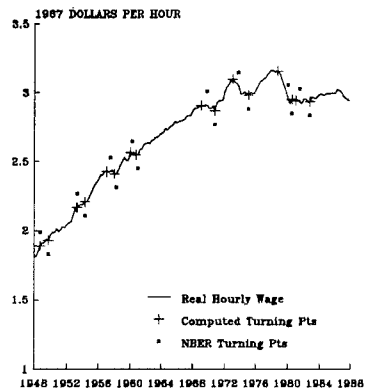
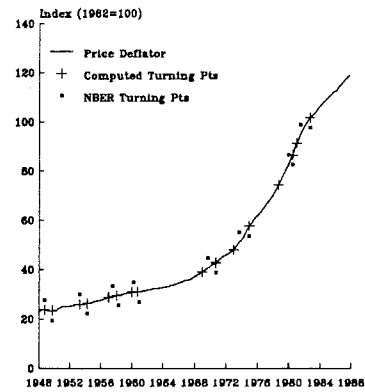


Fig. 2f  
IMPLICIT PRICE DEFLATOR



complete trough-peak-trough (T-P-T) cycles beginning with the trough in 1949 IV and ending with the trough in 1982 IV. The horizontal axis (T to T) measures the average duration of the business cycle in months (~57 months). The horizontal distance from T to P is the average length of expansion phases (~40 months) and from P to T is the average length of contraction phases (~17 months). The vertical tick marks indicate the midpoints for each of the nine stages. Stage I coincides with the initial trough, stage V with the peak and stage IX corresponds to the terminal trough. The average cycle relatives for each stage are plotted at these midpoints. The height of these tick marks is a measure of dispersion across cycles. More precisely they represent the mean absolute deviation across the seven cycles at each stage and their magnitudes are read off the right-hand side vertical axis. Note that both the right and left vertical axis in each figure have the same units and scale.

The business cycle behavior of per capita real GNP, of course, coincides with the reference cycle. At its peak, it is 5% higher than its reference cycle average, while the initial trough is about 7.5% below the reference cycle average. Per capita real consumption displays little responsiveness to contraction phases although there is evidence of slowing of its average monthly growth rate which is reflected in Fig. 3b by the reduced slope during recessions. The cyclical behavior in investment is presented in Fig. 3c. Investment tends to rise more sharply during expansion phases and decline more sharply during contraction phases than does either output or consumption. Labor input, as measured by weekly hours, is much smoother, but also moves with the cycle. Notably the real wage, Fig. 3e, displays clear pro-cyclical tendencies while the price level, Fig. 3f, rises in during both expansion and contractions.

Table 3a presents the important summary measures of the business cycle characteristics of these six key macroeconomic series. The first group of statistics measures the conformity of each series to the reference cycle in GNP. Investment and hours display conformity of +100. The real wage is also pro-cyclical as seen from its business cycle plot, but does not conform

Fig. 3a  
REAL PER CAPITA GNP  
Average of 7 Cycles: 1949 IV - 1982 IV

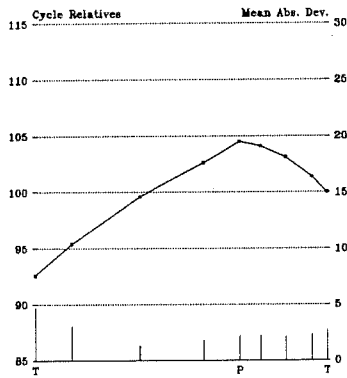


Fig. 3b  
REAL PER CAPITA CONSUMPTION  
Average of 7 Cycles: 1949 IV - 1982 IV

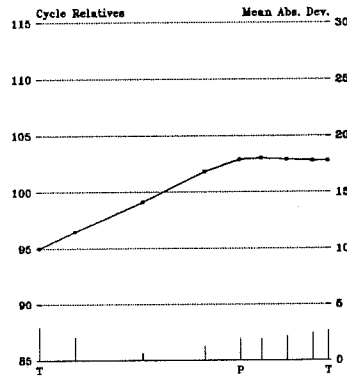


Fig. 3c  
REAL PER CAPITA FIXED INVESTMENT  
Average of 7 Cycles: 1949 IV - 1982 IV

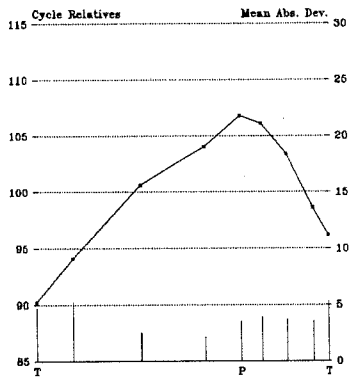


Fig. 3d  
WEEKLY HOURS WORKED  
Average of 7 Cycles: 1949 IV - 1982 IV

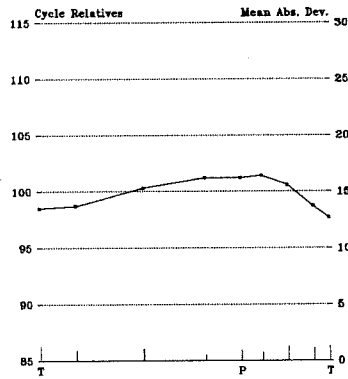


Fig. 3e  
REAL HOURLY WAGE RATE  
Average of 7 Cycles: 1949 IV - 1982 IV

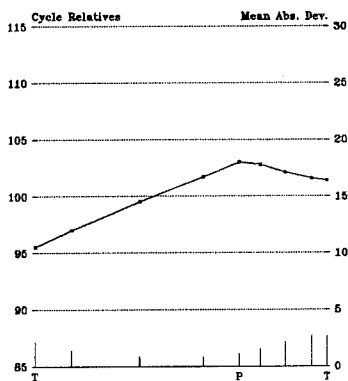
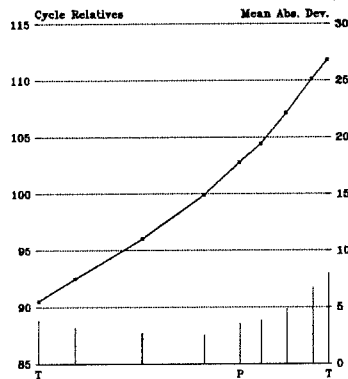


Fig. 3f  
IMPLICIT PRICE DEFLATOR FOR GNP  
Average of 7 Cycles: 1949 IV - 1982 IV



perfectly. As seen in the plots, consumption conforms well during expansion phases, but displays neither positive nor negative conformity during contractions. The price level actually conforms negatively during contractions and thus displays little overall cyclical conformity.

**Table 3a**  
**BURNS AND MITCHELL CYCLICAL MEASURES**  
**FOR KEY QUARTERLY AGGREGATES**  
 (Based on 7 cycles: 1948-1987)

Series	Index of Conformity <sup>a</sup>			Reference Cycle Amplitudes <sup>b</sup>		
	Expansions	Contractions	Cycle	Expansions	Contractions	Cycle
Quarterly Series:						
Real Per Capita GNP	+100	+100	+100	11.9/.32 (6.9)	-4.5/-.27 (1.1)	16.4/.30 (6.7)
Real Per Capita Consumption of Nondurables and Services	+71	0	+85	7.9/.17 (4.6)	-0.1/-.00 (1.0)	8.0/.13 (4.1)
Real Per Capita Fixed Investment	+100	+100	+100	16.6/.50 (7.6)	-10.5/-.59 (5.8)	27.1/.52 (11.2)
Average Weekly Hours Per Capita	+100	+100	+100	2.7/.10 (1.6)	-3.6/-.22 (1.2)	6.3/.13 (2.5)
Real Hourly Wage	+71	+57	+85	7.6/.19 (3.1)	-1.6/-.07 (2.1)	9.2/.16 (3.3)
Implicit Price Deflator for GNP	+100	-85	+35	12.3/.39 (6.3)	9.0/.44 (6.5)	3.3/.03 (6.1)
Inflation Rate <sup>c</sup>	+57	0	+28	1.4/.06 (1.4)	-.9/-.10 (2.7)	2.3/.07 (3.9)

<sup>a</sup>The method for calculating the conformity index is detailed in the text. In general a conformity index of +100 says that a series rises (falls) during each expansion (contraction) and rises at a faster rate during expansions than during contractions. An index of -100 says indicates the opposite (e.g. series that falls during every expansion would have a conformity of -100 for the expansion phase).

<sup>b</sup>The first amplitude measure indicates the total change in cycle relatives during each phase of the cycle. The number to the right converts that total change to a per month change. The number in parentheses below is the mean absolute deviation across seven cycles of the total change in cycle relatives for each phase.

<sup>c</sup>See footnote c Table 3b.



Table 3b  
**BURNS AND MITCHELL CYCLICAL MEASURES  
 FOR SOME ADDITIONAL SERIES OF INTEREST**  
 (Based on 7 cycles: 1948-1987)

Series	Index of Conformity <sup>a</sup>			Reference Cycle Amplitudes <sup>b</sup>		
	Expansions	Contractions	Cycle	Expansions	Contractions	Cycle
Quarterly Series						
Real Per Capita Consumption of Durables	+100	+85	+85	19.5/5.8 (12.0)	-9.0/4.8 (6.8)	28.5/48 (18.3)
Real Per Capita Government Expenditures	-14	+14	+14	9.6/17 (16.7)	-2.8/-17 (4.5)	11.3/17 (21.1)
Real Per Capita Exports	+100	-42	+14	16.1/48 (6.2)	2.0/06 (11.9)	14.1/34 (11.5)
Real Per Capita Imports	+100	+42	+85	23.5/69 (10.9)	-3.8/-24 (8.0)	27.3/53 (10.6)
Change in Real Per Capita Inventories <sup>c</sup>	+100	+100	+100	292./14.6 (70.5)	-315./-19.7 (112.)	608./13.8 (183.)
Monthly Series:						
Yields on Three Month Treasury Bills	+100	+71	+100	63.8/2.6 (27.7)	-41.2/-3.3 (31.2)	105.0/2.4 (49.2)
Yields on AAA Corporate Bonds	+71	+14	+28	17.5/64 (13.2)	5.3/09 (16.4)	12.2/33 (22.2)
Yield Spread <sup>c</sup>	-100	-85	-85	-1.8/-13 (.9)	2.1/12 (.9)	-3.9/-09 (1.7)
Expost Real Rate on Treasury Bills <sup>c</sup>	-42	-42	-14	.2/17 (3.1)	1.2/06 (.9)	-.9/01 (3.3)
Consumer Price Index	+100	-85	-14	10.3/33 (6.4)	10.5/52 (7.5)	-.3/-03 (5.8)
S & P 500 Stock Price Index	+100	+14	+57	29.4/83 (12.1)	-1.1/03 (16.5)	30.5/53 (26.0)
Unemployment Rate <sup>c</sup>	-100	-100	-100	-1.9/-05 (.87)	2.6/16 (.62)	-4.5/-09 (.99)
Employment Rate <sup>c</sup>	+100	+100	+100	1.5/05 (.85)	-1.3/-10 (.51)	2.8/11 (1.06)
Real Balances	+69	+23	+38	7.7/11 (7.7)	-3.2/-13 (5.0)	10.9/14 (12.7)

<sup>a</sup>See footnote a Table 3a.

<sup>b</sup>See footnote b Table 3a.

<sup>c</sup>These amplitude measures are stated in original units rather than cycle relatives. For example, the yield spread, ex post real rates and inflation rates are all stated in percent per annum. The unemployment rate and employment rate are stated in percentage points and the inventories measure is expressed in real per capita dollars.

Also summarized in Table 3a are measures of each series' amplitude over the business cycle. As indicated previously, the phase amplitudes are measured (with exceptions discussed below) in the differences between cycle relatives at peaks and troughs and averaged across cycles. Thus real per capita GNP on average rises 11.9% of the cycle base during expansions and falls 4.5% of the cycle base during contractions. The full cycle amplitude is thus 16.4. The average absolute deviation across the seven cycles is given in parentheses. Because these measures are in business cycle time, expansions that are very long and very short in calendar time are weighted equally. Thus the 11.9% is an average across expansions that range from 6 to 99 months. The number to the right of the / expresses the average amplitude in cycle relatives per month. Thus the GNP measure rises on average 0.32% of the cycle base per month during expansions. Looking at the full cycle amplitudes of our key series further highlights the relative volatilities of different series. The amplitude of consumption is approximately one-half that of output and investment's amplitude is about one and three-quarters that of output. Both hours and the real wage display less volatility than output. The inflation rate, which is very volatile in percentage terms, shows substantial variation across cycle as indicated by the large average absolute deviations.

In order to provide a somewhat broader perspective on post-war U.S. business cycles, we have constructed business cycle plots for several additional series focusing particularly several financial variables. These plots are presented in Fig. 4a-4f and summary statistics for these variables are included in Table 3b along with several additional series. All of the plotted series are monthly using the same scale (except Fig. 4e and 4d) albeit a slightly different scale from the quarterly series in Fig. 2.

The business cycle features of the yield on three month treasury bills are displayed in Fig. 4a. It is a very pro-cyclical series displaying very large percentage changes (movements in cycle relatives). Note also the large average absolute deviations indicate variations in cyclical

Fig. 4a  
**THREE MONTH TREASURY BILL YIELDS**  
 Average of 7 Cycles: 1949 IV - 1982 IV

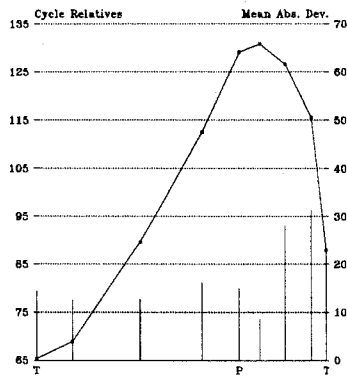


Fig. 4b  
**AAA CORPORATE BOND YIELD**  
 Average of 7 Cycles: 1949 IV - 1982 IV

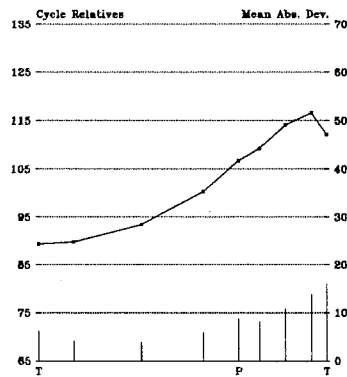


Fig. 4c  
**EX POST REAL YIELDS ON TBILLS**  
 Average of 7 Cycles: 1949 IV - 1982 IV

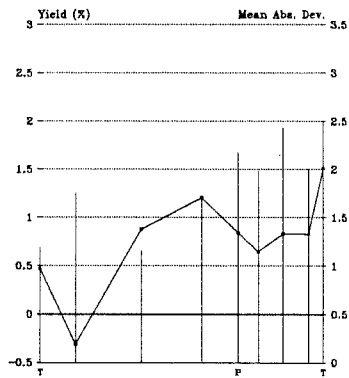


Fig. 4d  
**YIELD SPREAD: AAA - T-BILL**  
 Average of 7 Cycles: 1949 IV - 1982 IV

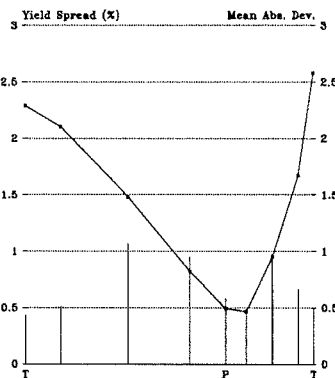


Fig. 4e  
**CONSUMER PRICE INDEX**  
 Average of 7 Cycles: 1949 IV - 1982 IV

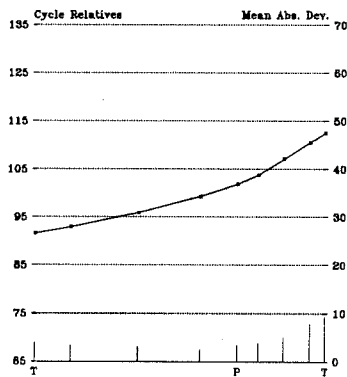
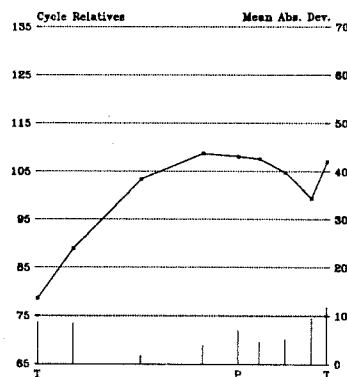


Fig. 4f  
**S & P 500 STOCK PRICE INDEX**  
 Average of 7 Cycles: 1949 IV - 1982 IV



behavior. The peak in this short-term interest appears to occur after the peak in the reference cycle. Long-term rates as measured by the yield on AAA corporate bonds displays much less conformity with the business as well as much less amplitude.

We have also constructed a measure of the ex post real rate of interest on the three month treasury bill and its business cycle characteristics are displayed in Fig. 4c. Note that the scale is no longer plotted in cycle relatives, but in units of the series itself. The reason for this is that this series exhibits numerous negative and zero values that make it unsuitable for converting to cycle relatives. As can be seen in the plot and in the table, ex post real rates display little reliable conformity with the cycle. The conformity measures suggest that it is more frequently counter-cyclical than not, but the wide variety of cyclical behavior indicated by the large average absolute deviations makes such a conclusion very dubious. We have also constructed a measure of the yield spread as the difference between the AAA corporate bond rate and the treasury bill rate. This series has a distinct counter-cyclical pattern, falling during expansions and rising during contractions. Fig. 4d is also expressed in units of the spread for the same reasons discussed above for the ex post real rate.

In Fig. 4e we present the business cycle plot for the consumer price index as a check on whether the cyclical behavior of prices as exhibited by the GNP deflator was robust to the choice of price index. As is apparent from the plot, it is. In fact, there is even less evidence of pro-cyclical prices here than there is with the GNP deflator.<sup>5</sup> Fig. 4f graphically presents the business cycle characteristic of the stock prices. Two things are worth noting. First, stock prices are roughly pro-cyclical. Second, their timing, however, differs somewhat from the reference cycle.

---

<sup>5</sup>The business cycle plot of the producer price index is virtually identical to that of the consumer price index.

Stock prices appear to peak one stage prior to the reference cycle peak and they tend to bottom out one stage before the trough in the reference cycle. Thus they exhibit the sorts of leading indicator qualities frequently attributed to them.

Table 3b summarizes the behavior of the series in Fig. 4a-4f along with a number of other series that are of general interest. First in Table 3a is the quarterly inflation rate which shows little conformity in the post-war period; rising in as many contractions as it falls, hence conformity during contractions is zero. Other quarterly series reported in Table 3b include per capita consumption of durables which has amplitudes that resemble investment more than consumption. Government expenditures, which exhibit little conformity. Inventories are expressed in original units and to no surprise display large amplitudes and high conformity. The unemployment rate and employment rate also behave as expected. Real balance show only mild conformity with the reference cycle.

In this section we have investigated nineteen series for the post-war U.S. economy using the techniques developed by Burns and Mitchell. We will use these findings as the benchmarks to compare the business cycle behavior of an artificial economy in the spirit of the investigation of the Adelman's nearly 30 years ago.

### 3 A Basic Real Business Cycle Model

Our objective is to follow the Adelman's path by studying the outcomes of an artificial economy with our variant of the Burns and Mitchell methods. The specific economy that we study is one that we have explored in detail elsewhere (King, Plosser, and Rebelo [1988a]), so that our presentation is deliberately brief. For reference purposes, the model is close to the "divisible labor" structure of Hansen [1985] or that explicated by Prescott [1986] or Plosser [1989], differing only slightly in the selection of parameter values. In contrast to these other investigations, however, the specification studied here includes trend growth.

#### 3.1 The Basic Neoclassical Model

The preference, technology and resource constraints--the deep structure of the model economy--are specified as follows:

*Preferences:* The representative agent values sequences of consumption ( $C_t$ ) and leisure ( $L_t$ ) according to

$$\sum_{t=0}^{\infty} \beta^t u(C_t, L_t),$$

where for simplicity we work with the momentary utility function  $u(C_t, L_t) = \log(C_t) + \theta \log(L_t)$ .

*Technology:* The production and accumulation technologies are

$$Y_t = A_t F(K_t, N_t X_t) \quad \text{and} \quad K_{t+1} = (1 - \delta)K_t + I_t,$$

where  $Y_t$  is output,  $N_t$  is labor output,  $K_t$  is capital, and  $I_t$  is investment. The function  $F$  is a constant returns-to-scale production function, chosen to be Cobb-Douglas,

$F(K_t, N_t X_t) = [K_t^{1-\alpha} (N_t X_t)^\alpha]$ , with  $0 < \alpha < 1$ . A trend in the series is induced by  $X_t$ , which is a labor augmenting technological shift that satisfies  $X_{t+1}/X_t = \gamma_x$ . Fluctuations are induced by  $A_t$ , which is a stationary total factor productivity shift that satisfies  $\log(A_t/\underline{A}) = \rho \log(A_t/\underline{A}) + \varepsilon_{A_t}$  with  $\underline{A} > 0, \rho > 0, E(\varepsilon_{A_t}) = 0$ , and  $E(\varepsilon_{A_t})^2 = \sigma_\varepsilon^2$ .

*Resource Constraints:* The resource constraints for goods and leisure are

$$C_t + I_t = Y_t \quad \text{and} \quad N_t + L_t = 1.$$

Values for technology and preference parameters are given in Table 4.

**Table 4**  
**ECONOMIC PARAMETER VALUES**

Depreciation Rate:	$\delta = .025$
Labor's Share:	$\alpha = .58$
Growth Rate:	$\gamma_x = 1.004$
Discount Factor:	$\beta = 1/(1 + .016) = .988$
Steady State Fraction of Time Spent Working:	$N = .20$
Std. Deviation of Technological Shifts:	$\sigma_\varepsilon = .0075$
Persistence Parameter:	$\rho = .95$

### 3.2 Approximate Dynamics

The equilibrium quantities for consumption, investment, output, and capital along with real wages will fluctuate stochastically around a common deterministic trend induced by  $X_t$ . On the other hand, hours are stationary random variables in this stochastic steady state. Approximating

this system,<sup>6</sup> we can develop a state space system for the logarithms of variables so that each variable can be written in the form  $\log(Y_t) = \log(Y) + \log(X_t) + \hat{y}_t$ , where  $\hat{y}_t$  is interpretable as the deviation from trend. The state space system which then describes the vector  $z_t = [\hat{y}_t, \hat{c}_t, \hat{N}_t, \hat{i}_t, \hat{w}_t]'$  is:

$$z_t = \Pi s_t$$

with state evolution

$$s_{t+1} = M s_t + \varepsilon_{t+1} \quad M = \begin{bmatrix} \mu_1 & \pi_{kA} \\ 0 & \rho \end{bmatrix}$$

where  $\hat{w}_t$  is deviations from trend of the log of the real wage rate,  $s_t = [\hat{k}_t, \hat{A}_t]'$  and

$\varepsilon_{t+1} = [0 \quad \varepsilon_{A,t+1}]$ . Coefficients in the matrices  $\Pi$  and  $M$ --implied by the parameters in Table 4--are given in Table 5. Stability of transitional dynamics ( $\mu_1 < 1$ ) is assured by diminishing returns to capital (holding fixed labor input). Thus,  $s_t$  is stationary so long as  $A_t$  is stationary ( $\rho < 1$ ).

---

<sup>6</sup>Our approximation method involves log linear approximation in a way made precise in the technical appendix to King, Plosser and Rebelo [1988a,b]. The method is closely related to that of Christiano [1988], but there are slight differences in the strategies used to solve the log-linear system, which should be of minor importance in the current context (since no distortions are present). Either approach is capable of exactly replicating the closed form solution of Long and Plosser [1983].



Table 5  
PARAMETERS OF THE LOG-LINEAR SYSTEM

	State Variable	
	Capital	Technology Shock
Consumption	$\pi_{ck} = .617$	$\pi_{cA} = .298$
Labor Input	$\pi_{Nk} = -.294$	$\pi_{NA} = 1.048$
Investment	$\pi_{ik} = -.629$	$\pi_{iA} = 4.733$
Output	$\pi_{yk} = .249$	$\pi_{yA} = 1.608$
Real Wage	$\pi_{wk} = .544$	$\pi_{wA} = .560$
Capital Stock (Next Period)	$\mu_1 = .953$	$\pi_{kA} = .137$

It is relatively easy to compute the population moments of the economic variables,  $z_t$  and  $s_t$ , by the two step procedure common in state space systems. First, one computes the moments of the states and then one exploits the simple relations that are readily shown to exist for moments of the  $z$  variables. For example, if  $V = E(s_t s_t')$  is the variance-covariance matrix of the states, then  $E(z_t z_t') = \Pi V \Pi'$  is the covariance matrix of the  $z$  variables. Results reported in Table 6 involve application of these ideas in a straightforward manner. Constructed in the manner detailed in King, Plosser and Rebelo [1988a], this table provides population moments for the baseline neo-classical model when a common deterministic trend is extracted from its trending series. Key pieces of information from this table are as follows: investment is almost twice as volatile as output; consumption and the real wage are about three-fourths as volatile; and labor input is

about one third as volatile. Generally, all series have high contemporaneous correlations with output, with consumption (.91) and labor input (.70) less strongly correlated than investment (.92) and the real wage (.95).

### *3.3 Simulating the System*

To simulate the system, we assume that the innovations to technology are drawn from a normal distribution with zero mean and standard deviation  $\sigma_e = .0075$ , which is taken from Prescott [1986]. The autoregressive parameter governing technology is  $\rho = .95$ . As discussed above, this implies a variance-covariance matrix of the stationary distribution for the state vector. We assume that  $s_0$  --the initial value of the state vector--is a random draw from its stationary distribution. Then, we draw a sequence of normal random variables for 160 quarters--the length of the postwar time period investigated in the preceding section--as innovations to the state vector. The results of one such simulation--are given in Fig. 5a-5e, which are comparable to the first set of plots explored in the prior section. We have indicated on these plots periods of recession which are determined by the turning point selected process of Bry and Boschan summarized in Table 1 applied to the output measure of the simulated economy. The notable characteristics of this simulation include many that we saw for the postwar United States: (i) the tendency for business cycle expansions to be longer and visually more gradual than business cycle contractions; (ii) the sharp movements in investment during contraction episodes; (iv) the tendency for consumption to flatten out rather than to decline during contractions, etc.

Table 6  
POPULATION MOMENTS: BASELINE MODEL

Variable	Std. Dev.	Std. Dev. Relative to $\hat{y}$	Autocorrelations			Cross-correlations with $\hat{y}_{t-j}$											
			1	2	3	12	8	4	2	1	0	-1	-2	-4	-8	-12	
$\hat{y}$	4.45	1.0	0.97	0.93	0.87	0.66	0.76	0.87	0.93	0.97	1.0	0.97	0.93	0.87	0.76	0.66	
$\hat{\epsilon}$	3.38	0.76	0.99	0.98	0.98	0.87	0.91	0.92	0.92	0.91	0.91	0.87	0.84	0.79	0.68	0.58	
$\hat{i}$	8.49	1.91	0.93	0.86	0.80	0.34	0.48	0.67	0.78	0.84	0.92	0.89	0.86	0.80	0.70	0.61	
$\hat{N}$	1.60	0.36	0.90	0.82	0.74	-0.01	0.15	0.38	0.52	0.61	0.70	0.67	0.65	0.61	0.54	0.46	
$\hat{w}$	3.53	0.79	0.99	0.98	0.97	0.83	0.89	0.93	0.94	0.94	0.95	0.91	0.88	0.82	0.71	0.61	

$\rho = 0.95, \sigma(\hat{\lambda}) = 2.40$

#### 4 Burns and Mitchell Measures for Simulated Economies

Fig. 5a-5e show one realization of a forty year history of quarterly data from a simple real business cycle model. The simulated series correspond to per capita measures of output, consumption and investment along with a measure of hours worked and the real wage rate. While interesting, it does not convey detailed information about the nature of business cycle behavior that is capable of being generated by the model. To study the business cycle characteristics of the model more thoroughly we randomly generate 100 of these forty year histories and investigate the distribution of the key business cycle measures of Burns and Mitchell. This exercise makes it possible for us to ask in a more precise way, what would Burns and Mitchell likely have found using their methods if the economic data arose from a simple real business cycle model. This is, in fact, the same question that was posed by the Adelmans.

In the post-war U.S. there have been seven complete business cycles independent of whether one begins with the peak in 1948 IV and ends with the peak in 1981 III (peak to peak cycles) or begins with the trough in 1949 IV and ends with the trough in 1982 IV (trough to trough cycles). As a matter of convention, we will arbitrarily focus on the cycles defined from trough to trough (T-P-T cycles). These seven T-P-T cycles averaged just over 56 months. The first question we ask of our model is how many business cycles would a researcher find in a forty year period of our simulated economy if he was armed with the techniques of Burns and Mitchell, as adapted by Bry and Boschan. Fig. 6 presents the frequency distribution of the number of complete T-P-T cycles found in our 100 simulated histories. The mean number of cycles found is 7.49 with a standard deviation of 1.63. The modal number of occurrences is eight. Forty five percent of the simulations exhibited either seven or eight cycles and sixty-four percent experience between six, seven and eight cycles. The maximum number of cycles identified in any forty year history is twelve and the minimum is four.

Fig. 5a  
SIMULATED OUTPUT

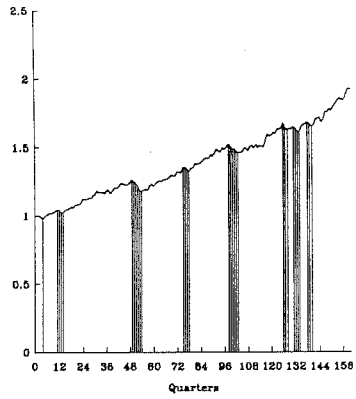


Fig. 5b  
SIMULATED CONSUMPTION

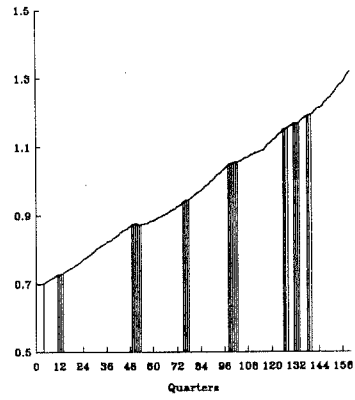


Fig. 5c  
SIMULATED INVESTMENT

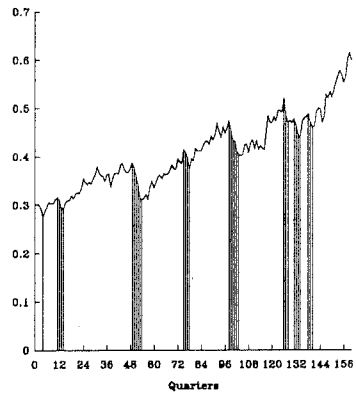


Fig. 5d  
SIMULATED HOURS WORKED

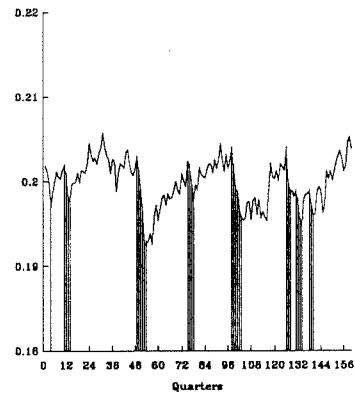


Fig. 5e  
SIMULATED WAGE RATE

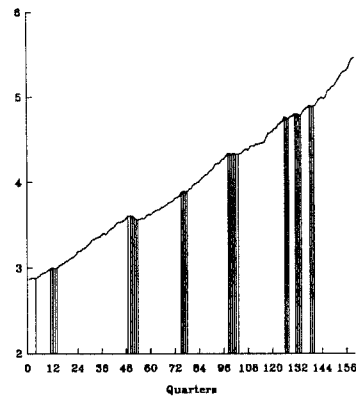
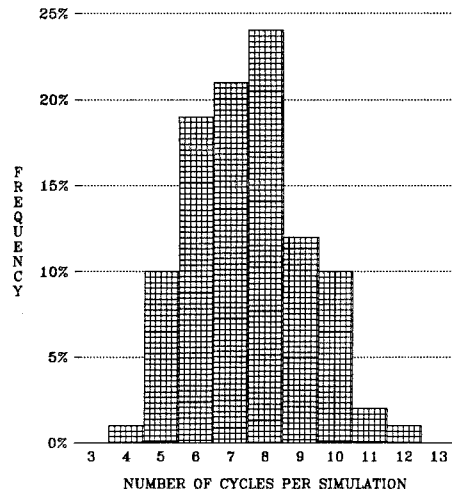


Fig. 6  
 T-P-T CYCLES IN 40 YEAR SAMPLE  
 Mean=7.49 Standard Deviation=1.63



The distribution of the average cycle duration for the 100 histories is presented in Fig. 7. The mean average cycle length in the simulations is 56 months, almost identical to the U.S. post-war experience. The standard deviation is 12.78 months. As expected this distribution is skewed somewhat to the right because of the restrictions on duration imposed by Burns and Mitchell. The minimum average duration is 36 months and the maximum is 116 months.

Fig. 8a and 8b break down the business cycle duration into the duration of the expansion and contraction phases. The duration of the average cycle expansion is 43.3 months with a standard deviation of 12.3 months. This corresponds to a value of 40 months for the post-war U.S. economy. The duration of the average cycle contraction is 12.78 months with a standard deviation of 2.07 months. This mean is close to the average post-war contraction of 11.4 months.

Fig. 7  
 DURATION OF CYCLES (T-P-T)  
 Mean=56.08 Standard Deviation=12.78

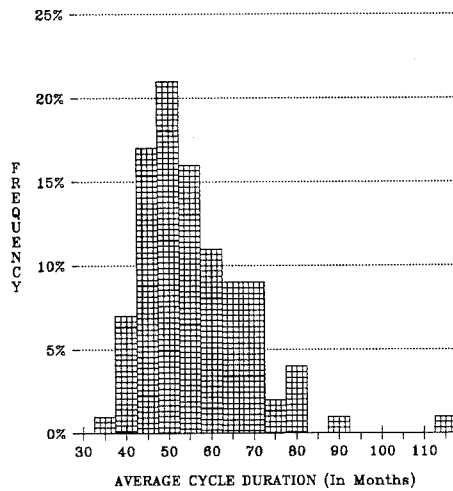


Fig. 8a  
 DURATION OF EXPANSIONS  
 Mean=43.30 Standard Deviation=12.31

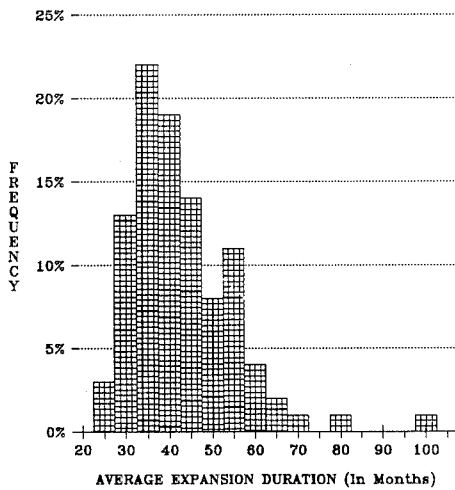
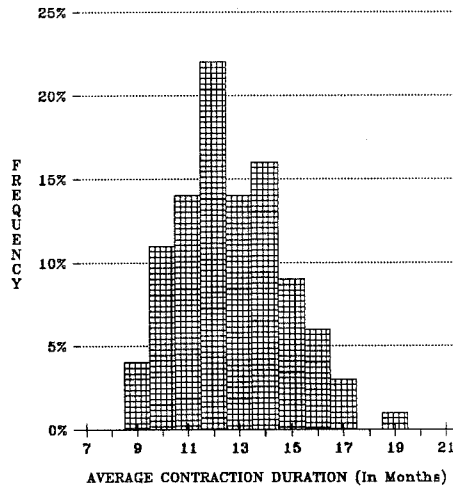


Fig. 8b  
 DURATION OF CONTRACTIONS  
 Mean=12.78 Standard Deviation=2.07



We have also computed the measures of amplitude for the different simulated series. Fig. 9a displays the frequency distribution of average output amplitudes for the 100 histories. The

Fig. 9a  
**OUTPUT CYCLE AMPLITUDES**  
 Mean=13.51 Standard Deviation=2.44

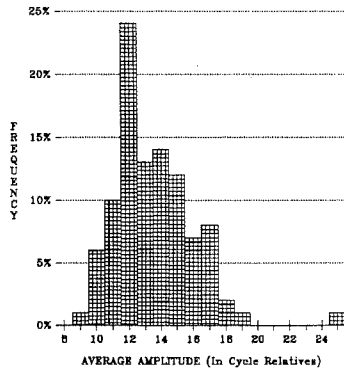


Fig. 9b  
**CONSUMPTION CYCLE AMPLITUDE**  
 Mean=5.82 Standard Deviation=1.78

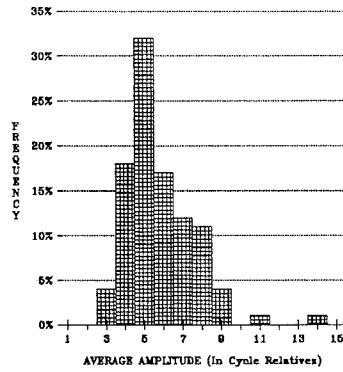


Fig. 9c  
**INVESTMENT CYCLE AMPLITUDES**  
 Mean=31.72 Standard Deviation=4.81

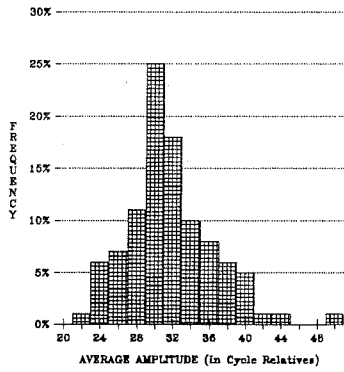


Fig. 9d  
**HOURS WORKED CYCLE AMPLITUDE**  
 Mean=6.09 Standard Deviation=0.90

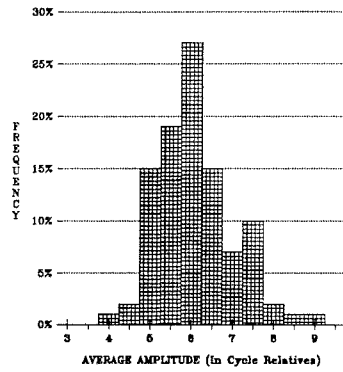
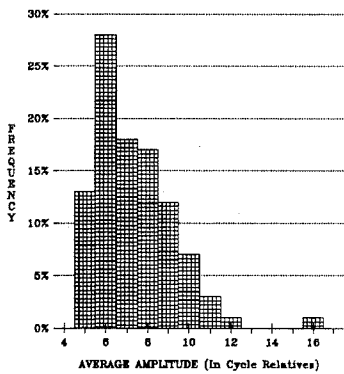


Fig. 9e  
**WAGE RATE CYCLE AMPLITUDES**  
 Mean=7.36 Standard Deviation=1.87





mean amplitude is 13.51 and the standard deviation is 2.44. This finding compares to a mean amplitude of 16.4 for per capita real GNP over the post-war period. Thus simulated output is slightly less variable than actual output. The mean consumption amplitude is 5.82 (Fig. 9b) and the mean investment amplitude is 31.72 (Fig. 9c). In the U.S. data these numbers compare to 8.0 and 27.1 respectively. This model yields slightly less variable consumption and slightly more variable investment but the relative variabilities for these aggregates is identical to that found in the data.

In the U.S. post-war period real wages have larger amplitude than hours worked (6.3 compared to 9.2). This characteristic also carries over to the simulated data, but with the model generating a little less variability in hours than appears in the data. The average amplitude for hours is 6.09 compared to 6.3 for the U.S. data and the average amplitude for real wages is 7.4 compared to 9.2 for the U.S. data.

It is interesting to recall that the Adelmans were able to generate significant differences in variability by subjecting each equation to a stochastic disturbance. However, the standard deviation of the stochastic disturbance shocking investment was about three times the standard deviation of the disturbance to consumption. Thus in the Adelman's experiment differential variability is largely generated exogenously. By contrast, the real business cycle model investigated here is driven by only one disturbance. Thus all differential amplitudes are determined by the model.

A visual impression of the business character characteristics generated by our simple real business cycle model is provided in Fig. 10a-10e. These business cycle plots have the same scale as those for the data. Comparing these pictures with Fig. 3a-3e, it would be difficult to distinguish the simulated data's characteristics from those of the actual data. This statement holds not only for the general shape of these plots, but also for the magnitudes of the average

Fig. 10a  
SIMULATED OUTPUT  
Average Based on 100 Histories

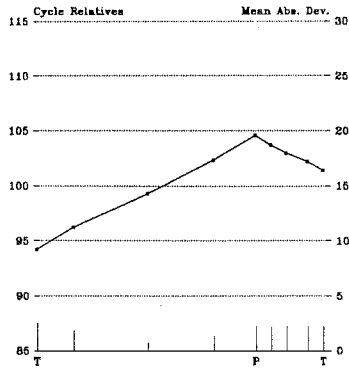


Fig. 10b  
SIMULATED CONSUMPTION  
Average Based on 100 Histories

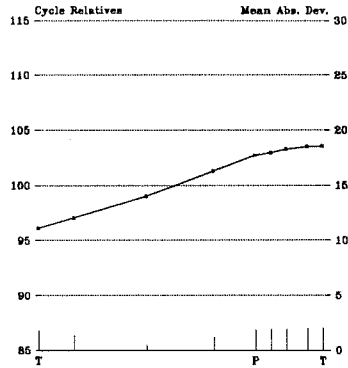


Fig. 10c  
SIMULATED INVESTMENT  
Average Based on 100 Histories

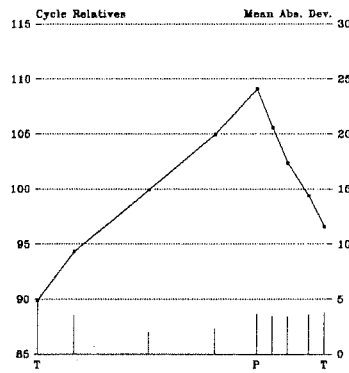


Fig. 10d  
SIMULATED HOURS WORKED  
Average Based on 100 Histories

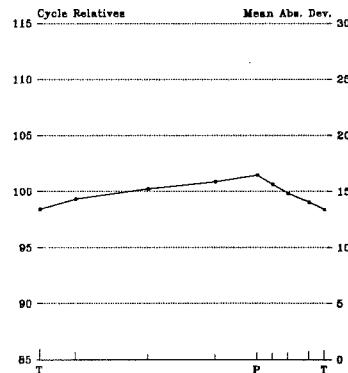
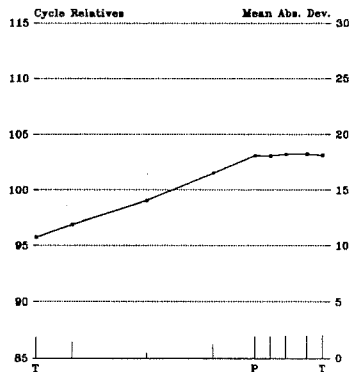


Fig. 10e  
SIMULATED WAGE RATE  
Average Based on 100 Histories



absolute deviations as indicated by the height of the bars on the horizontal axis. We could show the distribution of conformity measures produced by the simulated economy, but it would simply show that all of these aggregates exhibit +100 conformity in virtually all histories. Only consumption and the real wage deviate from this characterization and their average full cycle conformity is +99 across the 100 histories.

## 5 Comparing Alternative Business Cycle Measurements

The previous section illustrates that a simple real business cycle model is capable of replicating major business cycle features found in the data using the techniques of Burns and Mitchell. In this section we begin an investigation that shifts the focus from evaluating a model as in the Adelman's study to a more systematic investigation of the Burns and Mitchell procedures themselves and how they relate to more formal statistical procedures. We are currently pursuing this evaluation in detail in King and Plosser [1989], but at this stage it is useful to provide a benchmark for comparison.

Table 7 presents a crude comparison between the Burns and Mitchell characterizations as summarized by their measures of amplitude and conformity and common statistical measures such as standard deviation and correlation. The Burns and Mitchell measures are constructed as we have described in previous sections. One feature of their procedures is that they are computed without regard to whether the moments of the series exist or not.<sup>7</sup> This raises some important questions regarding how one might interpret the statistical properties of these measures. Nevertheless, we can compare these measures with statistical summary statistics that modern time series techniques might suggest. Measures such as standard deviation and correlation are useful but require some underlying assumptions if they are to be interpretable. For the simulated model, this is a straightforward exercise. The model generates series whose logged values are stationary fluctuations about a linear trend except for hours worked which is stationary. Thus standard deviations and correlations are computed based on these deviations from trend. Using the state-space formulation presented in Section 3 the population moments can be computed directly. The sample moments for the post-war U.S. data are computed as deviations from a common linear trend (in logs).

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<sup>7</sup>Burns and Mitchell consciously made this trade-off because they did not wish to be constrained or misled about business cycles based on purely statistic issues.

In part A of Table 7 we report the standard deviations of both the simulated series and the data along with the corresponding amplitude measure constructed using the methods of Burns and Mitchell. One easy way of comparing these measures is to look at how the two procedures rank the variability of the series. As is easily verified, the ranking of the key series are identical whether one uses standard deviation or amplitude.

**Table 7**  
**COMPARISON OF BUSINESS CYCLE SUMMARY MEASURES**

Series	U.S.	Simulated	U.S.	Simulated
<i>A. Cyclical Variability</i>				
	<u>Std. Deviations<sup>a</sup></u>		<u>Amplitude<sup>b</sup></u>	
Output	5.62	4.45	16.4	13.5
Consumption	3.86	3.38	8.0	5.8
Investment	7.61	8.49	27.1	31.7
Hours Worked	2.97	1.60	6.3	6.1
Real Wage	6.49	3.53	9.2	7.4
<i>B. Cyclical Conformity</i>				
	<u>Correlations with Output<sup>a</sup></u>		<u>Conformity<sup>b</sup></u>	
Output	1.00	1.00	+100	+100
Consumption	.85	.82	+71	+99
Investment	.60	.92	+100	+100
Hours Worked	.07	.79	+100	+100
Real Wage	.76	.90	+71	+99

<sup>a</sup>The sample and population measures of standard deviation and correlation with output are computed based on the percent deviations from trend rather than levels. The sample period is 1948 to 1987.

<sup>b</sup>The amplitude and conformity measures are computed using the Burns and Mitchell procedures outlined in section 2.

In part B of this Table we make a crude comparison between Burns and Mitchell's measure of conformity and the simple correlation coefficient of each series with output. Here, it is apparent that the conformity measure is a cruder metric, which does not necessarily imply a

poorer measure. Remember, conformity over the entire cycle may be +100 even though a series may rise during a contraction (thus negatively correlated with output during that period) as long as the per month rise during the expansion exceeds the per month rise during the contraction. Thus, while correlation and conformity clearly differ with respect to what they measure, it is unclear which method dominates the other. The answer clearly depends on the question at hand.

The correlation of hours worked with output in the post-war period (0.07) differs from what most macroeconomists take to be the "fact" (see the conformity measure) that hours is a strongly procyclical series. This observation is discussed in King, Plosser and Rebelo [1988a] and appears to be associated with the phenomenon that there are slow moving components to either output or hours that interact in ways to alter the correlation. For example, while the overall correlation is .07, dividing the sample in five year sub-periods produces correlation between .3 and .9 and averages almost .8. By looking at sub-periods we effectively permit the means of the two series to vary from one period to the next. This sensitivity of the correlation raises some important questions involving appropriate methods of detrending and/or inducing stationarity in order to compute meaningful sample moments.<sup>8</sup>

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<sup>8</sup>For example, see Nelson and Plosser [1982] and King and Rebelo [1989].

## 6 Summary and Future Plans

In this paper we have attempted to follow the path laid out by the Adelmans. Using the methods laid out by Burns and Mitchell, we see if we can distinguish between the economic series generated by an actual economy and those analogous artificial series generated by a stochastically perturbed economic model. In the case of the Adelmans, the model corresponded to the Klein-Goldberger equations. In our case, the model corresponds to a simple real business cycle model. The results indicate a fairly high degree of coincidence in key economic aggregates between the business cycles characteristics identified in actual data and those found in our simulated economy.

This, it seems to us, is a necessary hurdle that any business cycle model must clear. It is a particularly important hurdle given the importance of the Burns and Mitchell stylized facts and their influence on macroeconomists' thinking about business cycles. Nevertheless, our findings leave us uncomfortable. While no one has claimed that the Adelmans' test or the one we have conducted here represent particularly powerful tests of a model, it is somewhat troubling to us that two models as different as the Klein-Goldberger model and a neoclassical real business cycle model both are able to pass this "test."

There would appear to be two approaches to resolving this tension. First, it might well be the case that the findings of Burns and Mitchell are important stylized facts and that the implementation of the comparisons using their method have simply not been complete or thorough enough to help us distinguish among competing hypotheses. The second possibility is that the methods of Burns and Mitchell leads us to finding certain types of phenomenon regardless of the underlying model generating the data. If this latter possibility is the case then macroeconomists must face the possibility that tests of the sort we conduct here are not only not powerful, but much of what we think we know about economic fluctuations as organized by Burns and Mitchell may be an artifact of their procedures. In some ongoing research, King and Plosser [1989],

we are currently in the process of addressing the consequences of the Burns and Mitchell methods in a more thorough manner. Our goal is to obtain a better understanding of the link between their methods and their stylized facts.



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