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TRANSITIONAL DYNAMICS AND ECONOMIC GROWTH IN THE NEOCLASSICAL MODEL

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Abstract

An understanding of the *qualitative* nature of the transitional dynamics of the neoclassical model—the process of convergence from an initial capital stock to a steady state growth path—is a key part of the shared knowledge of most economists. It forms the basis, for example, of the widespread interest in hypotheses about convergence of levels of national economic activity. Based on several quantitative experiments undertaken in the 1960s with fixed savings rates versions of the neoclassical model, many economists further believe that the transition process can be lengthy, potentially rationalizing differences in growth rates across countries that are sustained for decades.

In this paper, we undertake a systematic *quantitative* investigation of transitional dynamics within the most widely employed versions of the neoclassical model with intertemporally optimizing households. Lengthy transitional episodes arise only if there is very low intertemporal substitution. But, more important, we find that the simplest neoclassical model inevitably generates a central implication that is traced to the production technology. Whenever we try to use it to explain major growth episodes, the model produces a rate of return that is counterfactually high in the early stages of development. For example, in seeking to account for U.S.-Japan differences in post war growth as a consequence of differences in end-of-war capital, we find that the immediate postwar rate of return in Japan would have had to exceed 500% per annum.

Frequently employed variants of the basic neoclassical model—those that introduce adjustment costs, separate production and consumption sectors, and international capital mobility—can potentially sweep this marginal product implication under the rug. However, such alterations necessarily cause major discrepancies to arise in other areas. With investment adjustment costs, for example, the implications resurface in counterfactual variations in Tobin's Q.

We interpret our results as illustrating two important principles. First, systematic quantitative investigation of familiar models can provide surprising new insights into their practical operation. Second, explanation of sustained cross country differences in growth rates will require departure from the familiar neoclassical environment.

The neoclassical model of capital accumulation developed by Solow [1956], Swan [1963], Cass [1965], and Koopmans [1965] is one of the major theoretical paradigms for dynamic economic analysis. It has been the impetus for much theoretical research into the behavior of dynamic systems, including elucidation of such key properties as the local and global turnpike theorems. In the hands of Solow [1957], Denison [1962] and their followers, the basic neoclassical model has further provided an empirical framework that has stimulated important research into the sources and nature of economic growth.

Virtually every professional economist trained in the last two decades is familiar with the central properties and the intuitive mechanics of the basic neoclassical growth model. The model is so familiar that the reader may be skeptical that there is anything new to learn about it. In this paper, by contrast, we take the view that its transition path dynamics are largely unexplored from a *quantitative* standpoint and that this exploration is essential to understanding whether the model can plausibly explain major differences in rates of economic growth over time and across countries. We examine the transitional dynamics of the most common versions of the neoclassical model for a wide range of parameter values. On this basis, we conclude that—while some features of the adjustment path toward the steady state are model and parameterization specific—there is a key, common counterfactual implication of all the models examined. An important role for transitional dynamics in explaining growth over long periods is inconsistent with observed variation in interest rates, asset prices and factor shares over time and across countries.

The organization of the paper is as follows. In Section II, we provide the specific discrete time version of the basic neoclassical model that we use throughout the paper, including some discussion of alternative modes of saving behavior along the lines of Solow [1956] and of the alternative put forward by Ramsey [1928], Cass [1965], and Koopmans [1965]. In section III, we review key quantitative analyses by R. Sato [1963] and Atkinson [1969] that have led macroeconomists to view transitional dynamics as potentially very protracted and, hence, as potentially capable of explaining sustained cross country differences in growth rates.

In section IV, we provide our basic experiments, computing the transition paths that the neoclassical model must follow if it is to explain seven fold growth in output over a century. This experiment was selected because seven luckily corresponds to key differences in U.S. history and in the international cross section. First, it is roughly the ratio of U.S. per capita real gross domestic product currently to that of a century ago. Second, in the international cross section of Summers and Heston [1984], it also corresponds to the gap between poor countries and the U.S. in 1950. Thus, we investigate the quantitative nature of transitional dynamics if capital is initially such that output is one seventh of its stationary value. We find that transitional dynamics are every rapid—unless the intertemporal elasticity of substitution is much smaller than the range generally considered by macroeconomists. Consequently, the conclusions of our investigation differ importantly from the traditional view that originates in Sato's [1963] experiments.

In computing growth paths under some alternative assumptions about saving behavior—corresponding to alternative values of the intertemporal substitution elasticity—we find a recurrent puzzle. Even if transitional dynamics are required to account for only one half of this growth, then the real rate of return is counterfactually high at the beginning of the century (about 40 percent per year). We identify this implication with a basic characteristic of the production technology in the neoclassical model: there are major variations in the marginal return to the reproducible factor, physical capital, if the level of the capital stock is varied over the ranges we consider. For this reason, we conduct a detailed investigation of alternative neoclassical production technologies—varying, for example, the elasticity of substitution between factors and the steady state factor shares—and find little change in the implications of our basic model. In section IV.2, we then explore the robustness of our result to some alterations in the basic model: (i) extensions to distinct technologies for production of consumption and capital goods; (ii) the introduction of adjustment costs; and (iii) consideration of a small open economy facing a given real interest rate. These modifications can permit us to overcome the real interest rate implications, but they do so only at the cost of producing some other, related counterfactual behavior. For example, with the introduction of adjustment costs, the link between marginal product of capital and the real interest rate is weakened. But the model then implies counterfactual variation in the relative price of installed capital and new investment goods, i.e., Tobin's [1969] "q".

Overall, the results suggest that for realistic parameterizations of the production function there is a very minor role for neoclassical transitional dynamics in the explanation of observed growth rates. In our view, this pushes one to think about models of endogenous economic growth which, following Schultz [1961], Uzawa [1965], Romer [1986] and Lucas [1988], assign a larger role to other modes of accumulation, such as human capital formation or endogenous technical progress. But the strength of our negative results also gave us concern that our experiment was too extreme, i.e., that asking the neoclassical model to explain major portions of U.S. growth over the last century was just too much of a task. (Although, it is only fair to point out that, when we presented results on models of endogenous economic growth, many people suggested that the transitional dynamics of the neoclassical model were central to (i) explaining U.S. growth in this century or (ii) sustained cross country differences in growth rates.) For this reason, we decided to additionally consider a more restricted experiment suggested by Barro's [1987] discussion of the neoclassical model's content for understanding differential growth experiences for countries during the post World War II interval. This corresponds to the idea that for "losing" countries, the 1950 levels of output per capita can be used to identify the war induced decline in physical capital stocks. For example, during 1950–1980, Germany moved from a per capita output of 45% of the U.S. to 88% and Japan moved from 19% to 74%. In these growth experiences, in which we take the U.S. as defining the growth of the "technical frontier", we find that there continue to be major counterfactual implications of the basic neoclassical model: if all Japanese capital accumulation was to be financed by domestic saving, then its 1950 interest rate should have been nearly 500% in this alternative experiment. These extreme predictions for the real interest rate are also present under the assumption, implicit in descriptions of the convergence hypothesis such as Baumol [1986] and DeLong [1988], that technological progress is embodied. A final section provides some conclusions.

I. The Basic Neoclassical Model

In this section, we set out the basic neoclassical model of capital accumulation that will be used in our analysis. With minor modifications, the model is that of Solow [1956] translated to discrete time. At the heart of the model is a constant returns-to-scale aggregate production function,

$$(1) \quad Y_t = F(K_t, N_tX_t),$$

where Y_t is commodity output, K_t is physical capital, N_t is labor input (in man hours) and X_t is a measure of labor productivity. Holding fixed N_t and X_t, the production function has the familiar form displayed in Figure 1A, with positive and diminishing returns to the reproducible factor K_t. This implies that the marginal product of capital schedule, $D_1F(K_t,N_tX_t)$, has the familiar form displayed in Figure 1B.

In introducing technical change into the production function (1), we have expressed it in labor augmenting form so as to admit steady-state growth when technical change and labor input grow at constant rates. (See Swan [1963] and Phelps [1966]). This requirement is interpretable in two ways. First, if we have a general constant returns to scale production function, then we must literally require that only labor augmenting change is present. Second, if the production function is Cobb-Douglas $Y_t = A_t(X_{Kt}K_t)^{1-\alpha}(X_{Nt}N_t)^{\alpha}$, with $0 < \alpha < 1$, then we can always express all forms of technical change in a labor augmenting form by defining $X_t = X_{Nt}A_t^{(1/\alpha)}\ X_{Kt}^{(1-\alpha)/\alpha}$.

The additional equations of this familiar model are the resource constraint on consumption and investment,

$$(2) \quad C_t + I_t = Y_t,$$

the difference equation for the accumulation of capital,

$$(3) \quad K_{t+1} - K_t = I_t - \delta K_t,$$

and specifications of constant growth in labor input and labor productivity.

$$(4) \quad N_t = \gamma_N N_{t-1}$$

$$(5) \quad X_t = \gamma_X X_{t-1}$$

In expressions (4) and (5), γ_X and γ_N are "gross" growth rates, i.e., $\gamma_X - 1 = (X_t - X_{t-1})/X_{t-1}$.

In the basic neoclassical model, the common steady-state growth rate of many of the system's variables is $\gamma_X\gamma_N$. That is, denoting γ_Z as the gross growth rate of any variable Z, we have

$$(6) \quad \gamma_Y = \gamma_C = \gamma_K = \gamma_X\gamma_N.$$

Further, in a steady state many key ratios—such as consumption's share of output or labor's income share—are constant since numerator and denominator variables have equal growth rates.

Savings Behavior. We study the model under two alternative assumptions about savings behavior. The first is Solow's [1956] assumption that saving (net investment) is a fixed fraction of income,

$$(7) \qquad I_t = s\, Y_t + \delta K_t,$$

where s is the savings rate.

Our second specification is the Ramsey–Cass–Koopmans assumption that saving is an outcome of optimal consumption choices by an immortal family. Our specification of this family's preferences is

$$(8) \quad U_t = \sum_{j=0}^{\infty} \beta^j M_{t+j}^{\eta}\, u(C_{t+j}/M_{t+j}).$$

In this preference specification, β is a discount factor, M_t is the number of members of the family, η is a parameter reflecting valuation of future membership, and the utility of per capita consumption, u(.), has a constant elasticity form:[1]

$$(9) \quad u(C_t) = \begin{cases} \frac{1}{1-\sigma}\; C_t^{1-\sigma} & \text{for } 0 < \sigma < 1 \text{ and } \sigma > 1 \\ \log(C_t) & \text{for } \sigma = 1. \end{cases}$$

In the most of the current paper, as in the bulk of the growth literature, we abstract from consideration of choice of labor supply, assuming that each population member supplies n hours, so that $N_t = n\, M_t$.

Transitional Dynamics. Growth in the basic neoclassical model can arise for two general reasons. First, there is steady state growth associated with growth in productivity and population. Second, there is transitional growth associated with movement from an initial capital stock toward the steady

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[1]See Barro and Becker [1989] for a detailed discussion of this type of dynastic utility function.

state growth path. For example, under Solow's [1956] assumption of a fixed savings rate with zero depreciation, then the dynamics of accumulation are given by

$$(10)\ K_{t+1} - K_t = sF(K_t,\ nM_tX_t).$$

Growth relative to the steady state path is then given by

$$(11)\ \gamma_X\gamma_N k_{t+1} - k_t = sF(k_t,\ n)$$

where $k_t = K_t/(M_tX_t)$. From any initial value of k_o, this difference equation converges monotonically to a unique stationary value satisfying $(\gamma_X\gamma_N-1)k^* = sF(k^*,n)$, as demonstrated in Solow [1956], but this general property leaves open the issue of the rapidity of this transitional growth.

Since along the steady state per capita output grows at rate γ_X, cross-country differences in growth rates can only be "explained" if we assume that they are the result of different rates of technical progress. It is now widely recognized that this explanation is vacuous. If the neoclassical model is to help us understand more than why consumption, investment and output move together along a growth path, the model's transitional dynamics have to play an important role in explaining cross-country growth differences.

In the sections below we refer to the fraction of growth explained by transitional dynamics which we define as $\Psi = [\gamma_Y/(\gamma_X\gamma_N) - 1]/(\gamma_Y-1)$, where γ_Y is the growth rate of aggregate output. This definition is a natural one: if the economy is at the steady state $\gamma_Y = \gamma_X\gamma_N$ and $\Psi = 0$ indicating that transitional dynamics play no role in the growth process; at the other extreme, if $\gamma_X=\gamma_N=1$, so that the steady state growth rate of output is zero and growth can only occur as a result of transitional dynamics, $\Psi = 1$.

It is worthwhile to note that the fraction of growth explained by transitional dynamics is different from the fraction of growth accounted for by factor movements in the growth accounting sense. The difference between these two concepts is clear along the steady state path: the fraction of growth accounted for movements in factors of production is $\gamma_K^{1-\alpha}\gamma_N^{\alpha}/\gamma_Y = \gamma_X^{-\alpha}$, which is less than 1, unless there is no technical progress ($\gamma_X=1$), and is always greater than zero, while the fraction of growth explained by transitional dynamics is zero.

III. Traditional Views of Transitional Dynamics

In this section, we discuss the conventional perspective on the quantitative importance of transitional dynamics. We begin by describing several key quantitative experiments with the neoclassical model that were performed in the 1960s which indicated that these dynamics could be very protracted. Then, we discuss the potential magnitude of transitional dynamics that is indicated by looking at cross country and within country economic growth. Finally, we consider this issue from the perspective of "growth accounting" that originates in the research of Solow [1957] and Denison [1962].

III.1 The Sato–Atkinson Experiments

If the neoclassical model is to be used as a description of actual growth experiences, then one is naturally led to ask what portion of observed growth is attributable to steady state mechanics—population and productivity—and what portion is attributable to transitional dynamics, i.e., growth relative to the steady state.

Two key quantitative experiments by R. Sato [1963] and Atkinson [1969] demonstrated that the neoclassical model's transitional dynamics may exhibit very slow adjustment toward the steady state path and hence be responsible for a significant fraction of the observed expansion in per capita output.

Working with the Cobb–Douglas production function and a fixed savings rate, Sato [1963] showed that there could plausibly be a very long adjustment period in response to a fiscal policy induced shift in the savings rate. Using parameters drawn from U.S. time series, Sato concluded that "for a 10 percent adjustment (in capital) 4 years must pass; for a 50 percent adjustment, 30 years; for a 70 percent adjustment, 50 years; and for a ninety percent adjustment, 100 years."[2]

Figure 2 provides our version of Sato's [1963] experiment. Rather than concentrate on a shift in the savings rate, we assume that the capital stock is such that output is 50 percent below the steady path in the initial period. We assume that α = 2/3, which is a conventional value for labor's share; that the savings rate is 12 percent; that the depreciation rate is 10 percent; that the growth rate of labor is 1.5 percent; and that the growth rate of labor augmenting technical change is 2 percent. (These parameter values conform to those employed by Sato [1963]). We study the transformed economy with $k_t=K_t/(M_tX_t)$; $y_t=Y_t/(M_tX_t)$; etc. Further, we express all variables as a percentage of steady state values.

In Figure 2, we see that the adjustment process is indeed very lengthy, with transitional dynamics that correspond reasonably closely to those described by Sato [1963] in the sentences quoted above, even though there are some differences in the details of our experiments[3].

Atkinson's [1969] experiments involved a model that admitted capital augmenting technical change, so that the asymptotic share of capital would be driven to zero and no steady state growth path existed. Atkinson showed that the model might never-the-less be consistent with the observed small

[2]Following Solow [1956], Ryuzo Sato worked with a model without depreciation and his results were critiqued by Kazuo Sato [1966], who showed that adoption of the saving specification $I_t = s\, Y_t$ resulted in the dramatically faster transition paths when depreciation was introduced. However, Kazuo Sato's results were much the same as Ryuzo Sato's when the saving specification was (7). For this reason, the basic lesson from the results of the two Sato's experiments was that plausible versions of the Solow model could generate transitional dynamics that were very protracted.

[3]His derivations were in continuous time and ours are in discrete time; we incorporate depreciation and use the savings function (7) rather than omitting depreciation.

movements in the share of capital over one hundred year periods. Thus, constancy of these factor shares alone could not be used to judge the adequacy of models of technical change and accumulation.

Taken together, the Sato and Atkinson experiments have been viewed as suggesting that the steady state need not be the full story about the growth of nations and, as well, that transitional dynamics could be a key component of observed growth experiences (see, for instance, Summers [1978], page 23).

III.2 The Convergence Implication

Even if the process of convergence is relatively slow, the neoclassical model does have the implication that convergence should ultimately occur and the Sato experiments suggest that one should be able to detect this process with several decades of economic data. That is, other things equal, countries which begin with a relatively low capital and, hence, low income, should initially grow faster. One specific device for testing this implication of the model is shown in Figure 3, which plots the level of real output per capita in 1950 versus the subsequent growth rate over the remainder of the postwar period for the countries included in the Summers and Heston [1984] data set. Contrary to the convergence prediction, we see in Figure 3 little tendency for a low initial level of income (in 1960) to be followed by high rates of expansion over the subsequent two decades.

This fact is often taken to be a strong refutation of the neoclassical model but we are skeptical about relying on it in a world with potential heterogeneity in production possibilities, preferences and public policies. The basis for our skepticism can be illustrated by using a version of the Solow [1956] model that incorporates heterogeneity by adding a country superscript j and specializing the production function to the Cobb-Douglas form. Solow's difference equation then takes the form:

$$(12)\ K_{j,t+1} - K_{jt} = s_j\ A_j\ K_{jt}^{1-\alpha}\ (N_{jt}X_t)^{\alpha}\ .$$

The implied dynamics of transformed capital are:

$$(13)\ \gamma_X\gamma_N\ k_{jt+1} - k_{jt} = s_j\ A_j\ k_{jt}^{1-\alpha}\ (n_{jt})^{\alpha}$$

with a stationary value $k_j^* = [(\gamma_X\gamma_N-1)/(s_jA_jn_j{}^{\alpha})]^{-1/\alpha}$.

In this simple application of the Solow model, there is potential heterogeneity in initial conditions ($k_{j,o}$) and terminal conditions (k_j^*). A country may be growing fast either because it has a low $k_{j,o}$ or because it has a high k_j^*. Thus, it is possible for levels and growth rates to be roughly uncorrelated as in Figure 3. That is, we have an identification problem of the same general form that arises when both the demand and supply curves shift, so that prices and quantities can easily become roughly uncorrelated.

III.3 Perspectives From Growth Accounting

Following the lead of Solow [1957] and Denison [1962], a basic macroeconomic accounting framework has been used to attempt to account for differences in economic growth across time and across countries. The net result of the early growth accounting studies was to (i) stress the difficulty of raising the growth rate of final output by raising the rate of physical capital accumulation, since a one percentage point change in the growth rate of capital translates to only a $(1-\alpha)$ percentage change in the growth rate of output; and (ii) to generally emphasize the importance of the "residual factor" in explaining growth (for example, Solow [1957] estimated that only one eighth of U.S. economic growth over 1909 through 1949 was due to physical capital accumulation). Since the transitional dynamics of the neoclassical model revolve around the accumulation of physical capital, these findings might suggest a minor role for this factor in the growth process.

But the growth accounting investigations that followed Solow [1957] proceeded to make this line of argument more tenuous. These investigations generally assign a much more important role to capital accumulation (a survey of these results can be found in Maddison [1987]). In a recent and comprehensive volume, Jorgenson, Gollop and Fraumeni [1987] conclude that "growth in capital input is the most important source of growth in value added, growth in labor input is the next most important source, and productivity growth is the least important." In particular, these authors estimated that capital input accounts for 46% of growth in aggregate output over 1948–1979, during which the average rate of growth per annum was 3.42%.

For this reason, we believe that one cannot understand the properties of the basic neoclassical model without undertaking a detailed quantitative evaluation of its properties when its parameters are restricted by empirical evidence.

IV. Transitional Dynamics of Quantities and Prices

The modern version of Sato's [1963] experiment that we wish to conduct involves consideration of the dynamic path arising with a particular specification of preferences over time. That is, we are interested in the character of outcomes when saving behavior is altered from the Solow [1956] form to that implied by optimal choices of consumption over time, as in Ramsey [1928], Cass [1965] and Koopmans [1965].

Since we are interested in considering solution paths that arise from intertemporal optimization, we must consider how the capital stock and its marginal value (shadow price) evolve through time, as is familiar from textbook presentations of optimal accumulation (see, e.g., Phelps [1966, essay 3] or Burmeister and Dobell [1970, chapter 11]). However, since we are working in discrete time, we are led to a system of difference equations in the capital stock and shadow price. Because preferences are concave and technology is convex in the models we consider, there is a unique competitive and optimal path for the economy. This path occurs when we select the unique, initial value of the shadow price for which the solution path satisfies the transversality condition and, hence, capital converges to the steady state path. In appendix A, we review the familiar numerical solution methods that we apply to produce our results.

IV.1 Perfect Foresight Transitional Dynamics

Our procedure in studying the transitional dynamics under perfect foresight is as follows. First, we restrict the production function to Cobb-Douglas form, $Y_t = A(K_t)^{1-\alpha}(nX_tM_t)^{\alpha}$. Then, we normalize the level parameter A to unity and choose the labor's share parameter α to be 2/3, which accords with the estimates reported in Maddison [1987, table 8] and is otherwise a conventional value. Second, we choose a constant value of per capita hours devoted to work, $n=.2$, a selection which accords with the post World War II U.S. experience.[4] Third, we select the depreciation rate $\delta=.10$ which is in the range reported by Maddison [1987, table 7]. Fourth, we require that the steady state real interest rate be 6.5% percent per annum, which corresponds to the annual average real return to equity for the post war U.S. Fifth, we set the growth rate of population to 1.4% per year, which is its average value for the U.S. in the period 1950–1980 (see Barro [1987], page 296). Given other parameters of the problem, this implies a value of the discount factor β.

A key determinant of the characteristics of solution paths is the preference parameter σ, which controls the intertemporal substitution of per capita consumption. In the baseline experiment, we set σ to unity and then we experiment with smaller elasticities of intertemporal substitution indicated by Hall [1988], raising σ to ten.[5]

Choice of initial conditions and of the growth rate of exogenous technical progress are obviously central determinants of solution paths. To choose the initial level of the capital stock we simply require that per capita output in the initial period, t=0, be one seventh of its steady state level: $F(k_o,1)/[F(k^*,1)(\gamma_X^{100})] =1/7$. This requirement allows us to compute

[4]King, Plosser and Rebelo [1988] discuss derivation of this number from the Household Survey published by the Bureau of Labor Statistics.

[5]This value of σ implies that if $\eta<1$, in order for the steady state real interest rate to be 6.5%, β has to be greater than one. Values of β greater than one are admissible since, in the optimizing model described in section II, the condition that is necessary for finiteness of utility is $\beta\ \gamma_X^{1-\sigma}\ \gamma_N^{\eta} <$ 1, not $\beta < 1$. See Kocherlakota [1988] for a general discussion of economies with $\beta > 1$.

the initial capital to labor in efficiency units ratio, k, as a function of its steady state value (which in turn is determined by the production function, the rate of steady state growth, the rate of time preference, and the depreciation rate). The initial capital stock, K_o is then given by $K_o = k_o n X_o M_o$.[6]

In all the parameterizations of the basic model described below we choose the growth rate of technical progress so that, if there were no transitional dynamics, the economy would experience half of the expansion in per capital output that occurred in the U.S. during the period 1870 to 1970. This yields a value of γ_X of 1.0114 which is the solution to the equation $\gamma_X^{100} = 1 + 6/2$.

Figure 4 provides basic information about the transitional dynamics of the neoclassical model when momentary utility is logarithmic (σ=1). Its six panels depict the variations in output, consumption, investment, share of output devoted to gross investment, growth rate of output, and real interest rate. All variables are expressed in per capita terms. Output, consumption and investment were deflated by X_t and their steady state value was normalized to one. Notable implications of these trajectories are as follows. First, consumption displays an increasing level and diminishing growth rate, as is familiar from analytical results with constant elasticity utility specifications. Second, there are three results that are less expected. The pace of convergence is very rapid, one half of the gap between the initial level of output and its stationary value is eliminated in about

[6]This calculation implicitly assumes that all transitional growth takes place within the first century. This is an approximation since, in effect, these dynamics are infinitely lived. However, since their effect after the initial one hundred years is negligible, this approximation is of no consequence.

six years: rates of economic growth are very rapid early on and then are sharply reduced. Investment displays a "hump shaped" trajectory, which would not be picked up by local approximations around the steady state. Finally, the implied value of the real interest rate at the beginning of the century of economic growth is very high, approximately 40% per year. This feature reflects the diminishing returns to reproducible factors that is a key feature of the neoclassical model. As we shall see, counterfactual implications for the marginal product of capital are a necessary implication of the model if its transitional dynamics are asked to explain major components of economic growth.

The pace of transitional dynamics can be slowed considerably if we reduce the intertemporal substitutability of consumption ($1/\sigma$). Figure 5 describes the transitional dynamics associated with a value of σ=10 which is among the lower estimates obtained by Hall [1988]. This decrease in the degree of intertemporal substitution changes the sign of the slope of the investment path, as would be predicted by the local dynamics of the model around the steady state. It also makes the growth process much more protracted—the half life is 24 years instead of 6 as we obtained when σ=1. But, when we reduce intertemporal substitutability and increase the duration of transitional dynamics, we also increase the interval over which there are very high levels of the real interest rate.

Taken together, these two experiments demonstrate that a plausible reparameterization of the neoclassical model simply shifts the key difficulty—diminishing marginal producivity—to another area. We will repeatedly encounter this theme as we proceed through this section.

The third parameterization, studied in Figure 6, modifies momentary utility to be of a Stone-Geary form: $u(C_t/M_t) = \log[(C_t/M_t) - \mathcal{C}]$, where $\mathcal{C}$

denotes the subsistence level of per capita consumption. With this specification of preferences the elasticity of intertemporal substitution is $[(C_t/M_t)-C]/(C_t/M_t)$ and thus is no longer constant.

In this model, there is an unstable steady state at the level of sustainable capital stock compatible with C. This low level steady state resembles somewhat the "poverty trap" familiar from the development literature. That is, despite the good investment opportunities the country does not invest because production is barely enough to attend to subsistence consumption and to the replacement of the depreciated capital stock. In the parameterization examined, we chose C to be 90% of production in period zero. The growth rate of output for this economy displays a "hump shaped" path which resembles the evolution of Japan after World War II (see Figure 11) as well as descriptive accounts of the growth process suggested by development economists. The reason for this pattern of evolution is that the elasticity of intertemporal substitution is variable, declining from an initial value of 1/60 to its steady state value of 1. Altering preferences to produce more protracted transitional dynamics generates a longer period with high real interest rates in initial stages of development than those associated with the baseline scenario.

Figure 7 displays the dynamics associated with a version of the basic model in which physical capital's share is 1/2, which we think is a plausible upper bound. Transitional dynamics are more persistent relative to the baseline model but real interest rates, though lower, are still high in the early stages. This parameterization makes clear that to generate protracted transitional dynamics that are consistent with moderate values for the real interest rate we need to postulate a share of capital that is close to one, so that the production function comes close to being constant returns in the factor that can be accumulated. But a capital share close to one is counterfactual, if we maintain that capital earns anything close to a competitive factor share.

The common result of the preceding experiments is that the real interest rate is very high in the early stages of development, if we require transitional dynamics to explain half of the growth in per capita output that occurred in the 1870–1970 period. In the next section, we explore the sensitivity of this result to the details of our experiment.

When technology is Cobb–Douglas there is a simple relation between the real interest rate and the capital–output ratio: $r_t = (1-\alpha)Y_t/K_t - \delta$. This relation shows that the behavior of the capital–output ratio is also problematic, the model predicts a significant increase of K_t/Y_t over time which contrasts with the small variation suggested by the data for this ratio (see, for example, Romer [1987]). Although the puzzling behavior of the real interest rate and the counterfactual behavior of the capital–output ratio are two sides of the same coin, we choose to emphasize the real interest rate implications for two reasons: (i) the information available about capital–labor ratios is restricted to few countries and short time periods; and (ii) there are substantial measurement problems associated with the capital stock data.

IV.2 The Real Interest Rate and Technology

One can extract valuable information about the behavior of the real interest rate in the neoclassical model without specifying preferences, simply by utilizing the implications of the production function for the level of output and for the marginal product of capital. In this section, we explore these implications. Since the computations discussed here are independent of the rate of population growth we treat population as constant throughout this section.

Our procedure is as follows. We require that there is a time invariant production function of the form, $Y_t = F(K_t, nX_t)$, where $\gamma_X = X_t/X_{t-1}$ is the rate of growth of technical change. In a steady state, we know that the marginal product of capital must satisfy $[D_1F(K_t, nX_t) - \delta] = r^*$, where r^* is the steady state rate of interest. This defines a steady state path of capital or, equivalently, a level of $K_t/(nX_t)$. With given values for X_o, γ_X, r^*, δ, then, we can determine the level of K_o that is compatible with output growing 7 fold over 100 years. When substituted into the marginal product schedule, the capital stock K_o implies a value of the initial real interest rate r_o. Throughout this section, we report results based solely on technology which are calculated in this manner.

Table 1A summarizes the predicted values for r_o under different hypotheses for the growth rate of exogenous technical progress. These hypotheses range from that displayed in the first row, in which all of growth is attributed to technical change and none to transition path dynamics, to that shown in the last row, in which all of the growth is attributed to transition path dynamics. Naturally, the value of the real interest rate in the beginning of the period is lower when a smaller fraction of growth is associated with transitional dynamics. Further, if all of the growth is attributed to technical change, so that there are no transitional dynamics, the rate of interest is the same in the beginning and in the end of the period.

The first column of Table 1A is devoted to the baseline model which has the technology we described in the last section. The computation of the real interest rate in the last line of this column (the case of no technical progress) is depicted in Figure 1.[7] Columns 2, 4 and 5 consider perturbations of this baseline scenario which involve different rates of depreciation, capital shares and terminal real interest rates. In Column 3 per capita hours worked is taken to be .36 in the beginning of the period and .2 at the end of the period, so as to reflect the decrease in hours devoted to market work occurred in the last century (see Maddison [1987], table A-9).

Table 1A makes clear that the tension that we identified in the last section carries over to a wide range of experiments with Cobb-Douglas technologies: transitional dynamics cannot account for a large fraction of the expansion in output without generating implausible values for the real interest rate in the beginning of the period. In order for all the output expansion to be associated with transitional dynamics the real interest rate one century ago should have been higher than 100%, unless we postulate an implausibly high share of capital in production.

Table 1B explores a variation of the baseline model in which the elasticities of substitution in production are different from the unitary elasticity implied by the Cobb-Douglas production function. All economies have a CES production function with elasticity of substitution ρ and the same terminal capital stock, $K_T = 100$. The remaining two parameters of the production function are chosen so that r_T is 6.5% and the share of capital in output at time T is 1/3. This ensures that at time T all the economies have

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[7]Looking first at Figure 1B we can find K^*, the steady state capital stock by searching for the value of K that has associated a marginal product of $r^* + \delta$. We can then use Figure 1A to determine K_o, the level of capital that implies that production is seven times smaller than at the steady state. Going back to Figure 1B we can find the marginal product of capital associated with K_o, which is roughy 800%.

the same capital stock, real interest rate, production and capital share but different elasticities of substitution.

With non-unitary elasticity of substitution in production the capital share is no longer constant over time when the economy is not following a steady state path. It decreases over time when $\rho < 1$ and it increases for $\rho > 1$. Table 2B indicates that varying the elasticity of factor substitution away from one moderates in some cases the predicted values for r_o—with no exogenous productivity growth the value of r_o associated with $\rho = .5$ is 111.6, roughly seven times smaller than that associated with Cobb-Douglas production.[8][9] However, the values of r_o continue to be extremely high in light of the historical evidence when the role of transition dynamics is significant. Furthermore, varying the elasticity of substitution generates implausible implications for the evaluation of the share of capital in production (for instance, with $\rho = .5$ the share of capital decreases by roughly 3 fold over the course of a century).

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[8]An estimate of the elasticity of substitution, $\hat{\rho} = .6$, is provided by Lucas [1967].

[9]One might expect that with elasticities of substitution lower than one the value of r_o would be higher than that associated with Cobb-Douglas production. This is not necessarily true as the last line of Table 1B shows—without technological progress, a decrease in the elasticity of substitution from .9 to .5 actually decreases r_o. When we lower ρ the value of K_o (associated with a seven fold increase in output) *increases*. If the marginal product schedule were independent of ρ this would lead to a decrease in the real interest rate. But the marginal product schedule is shifted by the decrease in ρ so that the value of r_o may increase, decrease, or remain the same depending on the combination of these two effects.

IV.3 Historical Evidence on Interest Rate Movements

The behavior of the real interest rate over the course of recent history is inconsistent with a major role for neoclassical transitional dynamics. Rough constancy of the real interest rate over time is one of the stylized facts of economic growth (see Kaldor [1961], Solow [1970], and Romer [1988] for discussions).[10] We present two tables that may aid the reader in thinking about the range of variation. The first provides information on the behavior of alternative rate of return constructs for the U.S. over 1926–1987 drawn from Ibbotson and Sinquefeld [1988]. Table 2 shows that there are major differences across returns on assets of varying risk, but that there are relatively minor differences across time. The second set of information is drawn from Homer [1963], providing long period evidence on movements in real interest rates, beginning with the 13th century. This table is provided since there is nothing necessary about the identification of a 100 year period of transitional growth with the last 100 years of U.S. history. This evidence needs to be interpreted with caution, the rates of return in Table 3 were constructed to be the closest possible analog to today's "prime rate", but are nominal rates and correspond to an extremely diverse set of assets. Nevertheless, it remains impossible to find the magnitude of interest rate variation that is suggested by the results of the last two sections.

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[10]This constancy led Cohen and Hickman [1987]—in their version of the neoclassical growth model—to postulate that entrepreneurs seek to earn a constant real rate of return rather than to maximize profit.

V. Robustness of the Results of the Basic Experiments

In this section, we consider whether our basic result—which suggests a modest role for the neoclassical model's transitional dynamics—is robust when we alter the basic model in several ways that are relatively standard in applied research in macroeconomics. First, we consider differentiating between production technologies for the production of physical capital and consumption goods. Second, we consider the introduction of investment adjustment costs, so that the marginal product of installed capital need not equal the real interest rate. Third, we consider a small open economy version of the neoclassical model.[11]

V.1 The Neoclassical Two Sector Model

One might think that the results of the experiments above are peculiar to the one-sector nature of the model. Figure 8 sheds light on this conjecture. It summarizes the adjustment path for a two-sector model in which both production functions are Cobb-Douglas with level parameters normalized to one. The labor share in the capital sector is taken to be .5. The labor share in the consumption sector was chosen so that, along the steady state path, the aggregate share of labor is 2/3 (this implies a labor share for the consumption industry of 72%). The remaining parameters coincide with those of the baseline model. The initial capital stock was chosen, as before, so

[11]Another common version of the neoclassical model involves making labor supply endogenous. We did not pursue this alteration of the model since the near-steady-state dynamics studied in King, Plosser and Rebelo (1988) indicate that, for standard preferences, when capital is below its steady state value, labor supply is greater than in the steady state, leading to higher values of the real interest rate than those for the exogenous labor supply models that we study.

that output increases by seven fold over the period considered.[12] The dynamics of this economy are remarkably similar to those of the comparable one sector model described in Figure 4. Separating out the capital sector and making its production function more linear in capital still generates implausible values for r_o. In order to obtain empirically plausible values for r_o, one has to postulate that the share of capital in the production function of the capital sector is close to one.

V.2 Investment Adjustment Costs

Costs of changing the capital stock are another potential avenue for eliminating the counterfactual implications for the behavior of the real interest rate. We consider below a version of the neoclassical model with adjustment costs similar to the one developed by Abel and Blanchard [1985]. To preview the results of this investigation, it is true that if one freely chooses the adjustment cost function, then one can overturn the implication for the beginning of period real interest rates. But there are then other undesirable implications. Moreover, we would like to employ adjustment cost functions that are empirically reasonable on other grounds. For this purpose, we draw on work by Hayashi [1982] that develops the connection between adjustment costs and Tobin's q—the ratio of stock market valuation of existing capital to its replacement cost. We conclude that one can only overturn the implication of implausibly high interest rates at the cost of generating counterfactual values for Tobin's q. That is, initial period q

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[12]The value of K_o was found by trial and error. It cannot be computed directly as in the one sector model since output, given by $Y_t = p_t I_t + C_t$ (p_t is the relative price of investment), depends on the allocation of factors of production between the consumption and capital sector.

falls well outside the range of values that have been estimated in the literature on empirical investment equations.

The introduction of adjustment costs requires that we alter the resource constraints of the neoclassical model as follows

$$(14) \qquad Y_t = C_t + Z_t[1 + h(Z_t/K_t)]$$

$$(15) \qquad K_{t+1} = Z_t + (1-\delta)K_t.$$

In the standard model, one unit of investment increases the capital stock by one additional unit. Now it is necessary to invest $1 + h(Z_t/K_t) + (Z_t/K_t)Dh(Z_t/K_t)$, where $Dh(.)$ denotes the derivative of $h(.)$.

The adjustment cost function $h(.)$ is assumed to be homogeneous of degree zero in Z and K. As Hayashi [1982] has shown, this makes the theory operational since it allows us to determine Tobin's marginal q by measuring average q. We assume that $h(\delta) = 0$ and $Dh(\delta) = 0$, so that the steady state capital stock is not affected by the introduction of adjustment costs. Without this assumption the adjustment costs economy would have a lower steady state capital stock than the comparable standard model. This would contribute to an increase in r_o. To make clear that our conclusions do not hinge on this effect, we chose to eliminate it.

Finally, we postulate that both the adjustment costs and the total cost of investing are increasing: $Dh(.) \geq 0$, and $2Dh(Z_t/K_t)) + (Z_t/K_t)D^2h(Z_t/K_t) > 0$, where $D^2h(.)$ denotes the second derivative of $h(.)$.

The value of Tobin's (marginal) q implied by this model is

$$(16) \qquad q_t = 1 + h(Z_t/K_t) + (Z_t/K_t)Dh(Z_t/K_t)$$

and the real interest rate is given by:

$$(17) \qquad r_t = [D_1F(k_t,n) + (Z_t/K_t)^2Dh(Z_t/K_t)]/q_{t-1} + (1-\delta)q_t/q_{t-1} - 1,$$

where $D_1F(.)$ denotes the partial derivative of $F(.)$ with respect to its first argument.

The consideration of adjustment costs introduces two conflicting effects on the real interest rate. First, the fact that the cost of increasing capital by an extra unit is now higher than one ($q_{t-1} = 1 + h(Z_{t-t}/K_{t-t}) + Dh(Z_{t-t}/K_{t-t})(Z_{t-t}/K_{t-t}) \geq 1$) lowers the real interest rate relative to the non-adjustment cost case. Second, the fact that an additional unit of capital lowers adjustment costs ($(Z_t/K_t)^2Dh(Z_t/K_t) \geq 0$) contributes to a higher value of the real interest rate. Equation (17) makes clear that low values of the real interest rate can only be obtained by introducing adjustment costs that imply large values of q.

Summers [1981] showed that when h(.) takes the functional form (18), the model described above predicts a linear relationship between Z_t/K_t and q_t.

$$(18) \qquad h(Z_t/K_t) = \frac{(b/2)(Z_t/K_t - a)^2}{Z_t/K_t} \qquad \text{when } Z_t/K_t \geq a$$

$$h(Z_t/K_t) = 0 \qquad \text{when } Z_t/K_t < a$$

Estimating this linear relation correcting q_t for the effects of taxation, Summers [1981] obtained the following estimates: b = 32.2 and a = .088. The requirement of no adjustment costs at the steady state implies that the steady state investment–capital ratio must be equal to a, so we set δ equal to $.088-(\gamma_X\gamma_N-1)$.

With these parameter values in hand we can study the model's implications for the behavior of real interest rate. Figure 9 summarizes the transitional dynamics of a version of the baseline model in which we introduced the form of adjustment costs described above. While the introduction of adjustment costs moderates the implications of the model for r_0, it does so by simultaneously generating implausibly high values for Tobin's q. The average value of q in the first five years of the simulation is 3.2. This value is well outside the range of values for q estimated in the investment literature (the highest value of q reported by Summers for the period 1933–1978 is barely above 2).

The conclusion that low values of r_0 can only be obtained by postulating empirically unacceptable adjustment costs is independent of the connection between adjustment costs and Tobin's q which we used to organize our discussion. To demonstrate the implausibility of the adjustment costs that underlie Figure 9 it is sufficient to cite the fact that they imply that—at time zero—the marginal adjustment costs associated with increasing installed capital by one unit are equal to 3.4 units of output.

V.3 Implications for A Small Open Economy

The numerical results reported so far have been interpreted using the neoclassical model as a model of a closed economy or alternatively of the world as a whole. Taken together, the versions of the model considered involved implausibly high real interest rates for the beginning of this century. Alternatively, one might view the neoclassical model as predicting how the real interest rates should be related to the level of development in the absence of international capital markets. Under this interpretation $r_o - r^*$ becomes the differential between the rate of return to capital in developed and underdeveloped countries predicted by the model. Assuming that the same technology is available in all parts of the world, the interest rate associated with poor countries is given by the last line of Table 1A. For the baseline model this interest rate is 798.5%, implying an interest rate differential between the U.S and these countries of 798.5%–6.5%=792% This differential is so large that it is hard to believe that investment flows from rich to poor countries would not take place, even taking into account such factors as political risk, transaction costs, etc.

In fact, in the standard open economy neoclassical view, capital flows would instantaneously equalize the rate of return in all countries so the process of adjustment would be instantaneous. Again, one might think that introducing adjustment costs would eliminate this unrealistic implication by creating a wedge between the marginal product of capital and the real rate of return to capital. In other words, making the cost of investment increasing in the rate of expansion of the capital stock might potentially smooth out the flow of investment from rich to poor countries so that the transition period might be very long. Table 4 summarizes the transition path of an economy with adjustment costs identical to the one that underlies Figure 9 but that can borrow and lend in the international capital market at the rate of 6.5% per year. The growth rates reported in this Table correspond to the case of no technical progress. They can be corrected for the presence of technical progress by computing $\gamma' = (1+\gamma)\gamma_X - 1$, where γ is the rate reported in the Table and γ' the corrected rate.

This Table shows that, even with adjustment costs that imply values for Tobin's q greater than 20, the model still predicts a fast process of convergence—the average growth rate of output in the first five years is 18% per annum. This leads us to conclude that it is not possible to attribute an important role to transitional dynamics in accounting for the expansion of per capita income observed in the last century. On the basis of the neoclassical model, we cannot reconcile the presence of (possibly imperfect) international capital markets, with the absence of a very rapid process of cross-country convergence.

VI. A Case Study: Economic Growth After World War II

The empirical power of economic theory is at times best tested by looking at the response to major events which put into the background other factors than those that one is primarily interested in investigating. For example, Cagan [1956], Sargent [1986] and many others have used interwar hyperinflations to study aspects of the dynamic relation between money and inflation; Barro [1981,1987], Ahmed [1986], Wynne [1987] and others have used wartime experiences to develop intertemporal substitution implications of equilibrium macroeconomic models. For economic growth, the post World War II experiences of developed countries appear to offer a similarly decisive field for evaluating aspects of the neoclassical model (see also Christiano [1989]).

Figures 10 and 11 are drawn from Robert Barro's *Macroeconomics*, which contains the first systematic investigation of the predictions of the neoclassical model for the post World War II experience. As Barro notes, there is a clear association between initial levels of output (in 1950) and the wartime positions of countries. That is, it is plausible that the winners (the U.S., the U.K.) lost less capital than occupied countries (Austria, Denmark and France) or the losers (Japan, Germany and Italy). It is also plausible that the losers of the war suffered the most severe decline in initial capital.

Broadly, these predictions are borne out for the levels of output per capita in 1950 as depicted in Figure 10. Further, Figure 11 shows that the countries with the lower initial levels of output subsequently display the higher growth rates, with reduction in cross-national dispersion of output levels at the end of the interval.

A plausible interpretation is that this Figure reflects the importance of transitional dynamics in economic growth. Under this interpretation, for example, Japan moved from about 1/5 of U.S. per capita output in 1950 to about 3/4 of the U.S. level in 1980 as a result of capital accumulation.

To investigate whether the convergence suggested by Figures 10 and 11 can be the result of neoclassical transitional dynamics we study the implications of baseline model that underlies Figure 4 for the evolution of Japan. To accomplish this we assume that the U.S. was in 1950 following a steady state path. This allows us to use the familiar condition $[D_1F(K_t,nX_t) - \delta] = r^*$, where r^* is the steady state real interest rate (6.5%) to determine the steady state capital labor ratio, $k^* = K_t/(nX_t)$. The implied capital labor ratio for Japan in 1950 can then be computed using the fact that the Japanese per capita output in 1950 was 19% of that of the U.S.: $F(k_o^J,1)/F(k^*,1) = .19$. The capital stock for Japan in 1950 is then be given by $K_o^J = k_o^J(nX_o)$.[13]

Knowledge of the value of the Japanese capital stock in 1950 allows us to calculate the transitional path depicted in Figure 12. The most striking feature of this Figure is, as we would expect, the behavior of the real interest rate: the model implies that in order for the Japanese cathing-up to be a product of neoclassical transitional dynamics, the interest rate in Japan in 1950 should have been near 500%!

[13]Christiano [1989] also investigates the "reconstruction hypothesis" for the divergence post war development of Japan and the U.S. Christiano [1989, page 14] takes Japanese output in 1946 as about 47% below trend, extrapolating from pre World War II Japanese economic performance. By contrast, we take the U.S. as defining the steady state growth path and, then, find Japanese percapita output as 19% of U.S. percapita output. Christiano reports his initial condition as 12% of steady state capital; ours is .65%. Hence, Christiano's computations imply an initial interest rate of about 40%, while we find a much higher value.

Discussions of the convergence hypothesis, such as those of Baumol [1986] and DeLong [1988], suggest that a key element in the convergence process may be the embodied nature of technical progress. The idea is that countries who rebuilt their capital stock after the war were able to grow faster by virtue of their ability to invest in the new capital vintages. But altering the basic model of section II to view technological progress as embodied, along the lines of Solow [1959], does not mitigate the model's interest rate implications. As shown Appendix B, the resulting model is virtually observationally equivalent to the basic economy described in section II and in Figure 4, implying a value for the Japanese real interest rate in 1950 of 560%.

VII. Conclusion

The basic neoclassical model of capital accumulation, in its various versions, has been for three decades the central framework for most research that relates to the process of economic growth. Indeed, for this reason, it is frequently referred to as the "growth model."

A central feature of this model is its assumption of diminishing returns to the reproducible factor of production, physical capital. Under savings specifications as different of those of Solow [1956] and Cass [1965]—Koopmans [1965], diminishing returns to capital assures that there is a steady state growth path toward which the economy converges. The neoclassical model's transitional dynamics—the motion from a given capital stock to the steady state growth path—are well known to most economists in qualitative form and are shaped in important ways by diminishing returns to capital.

When we seek to use the neoclassical model's transitional dynamics to explain sustained cross-country differences in rates of economic growth, however, diminishing returns to capital turns out to induce major counterfactual implications.

On the one hand, when one starts from very low capital stocks, diminishing returns to capital induces intertemporal reallocations which mean that transitional dynamics are important only for very short periods, unless agents have little low intertemporal elasticity of substitution. Hence, it is difficult to use the neoclassical model to explain *sustained* differences in growth rates, with conventional assumptions about preferences. In this regard, we reached the opposite conclusion to that suggested by earlier research of Sato [1963] and Atkinson [1969], which has become part of the popular wisdom as indicated by Barro's [1987] textbook treatment of the economic growth process.

On the other hand, even if one makes agents very unwilling to substitute over time so as to deliver a sustained transitional period, interest rates or asset prices will dramatically display the implications of diminishing returns. In general, we found that in order for transitional dynamics to be important, the marginal product of capital has to be very high in the early stages of economic development. In simplest model of Solow [1956] with a Cobb-Douglas production function, for example, this marginal product translates directly into an implication for the real rate of return, implying that it is implausibly high relative to historical observations. Notably, in order for the Japanese convergence toward the U.S. income level in the post war era to be the result of transitional dynamics, the Japanese real interest rate would have been over 500% per year in 1950. In exploring some plausible alterations of the Solow model, we found that it was impossible to understand important components of economic growth in terms of transition dynamics without introducing some related implication that strongly contradicted historical experience. For example, introduction of adjustment costs simply shifts the marginal product implication from the interest rate to Tobin's "q", implying variations unlike anything observed.

Throughout the course of this research, we have received many suggestions from other researchers, most of which suggested that some straightforward modification of our setup would readily overcome the central message of this paper. We have tracked down many of these leads. But our conclusion remains unaltered: the transitional dynamics of the familiar model of capital accumulation cannot account for important parts of sustained cross country differences in rates of economic development.

We view our results as pointing to the use of models that do not rely on exogenous technical change—"endogenous growth" models such as those of Romer [1986] and Lucas [1988]—as the primary vehicle for research on the process of economic growth. But, more generally, our results suggest the value to a quantitative approach to evaluating the adequacy of alternative growth paradigms. The neoclassical model's qualitative properties are well understood by most economists, but we found surprising new implications about its properties as a growth model. In newer theoretical frameworks, with general properties as yet undocumented, the quantitative approach will also help us learn about which model predictions are robust and which are tightly dependent on aspects of economic structure.[14] In the process of quantitative evaluation, we thus will gain a sharper understanding of why models succeed or fail in explaining the pace and pattern of economic development.

[14]We apply this methodology to to some basic endogenous growth models in King and Rebelo [1988].

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APPENDIX A

Numerical Methods

This appendix describes the numerical method used to compute the transition paths discussed in the main text. For all the models considered in this paper characterizing the competitive equilibrium amounts to solving a two point boundary value problem, i.e. a system of difference equations with boundary conditions specified at two different points in time. We will use the basic neoclassical model of section 2 to illustrate the operation of the algorithm. The competitive equilibrium for that economy is characterized by a system of two first order difference equations:

$$\gamma_X \gamma_N \lambda_t = \lambda_{t+1} \beta^* [D_1 F(k_{t+1}, n) + (1-\delta)] \tag{A.1}$$

$$\gamma_X \gamma_N k_{t+1} = F(k_t, n) + (1-\delta) k_t - \lambda_t^{-1/\sigma} \tag{A.2}$$

where $\beta^* = \beta \gamma_N^{\eta} \gamma_X^{1-\sigma}$ is the discount factor modified for steady growth in consumption and population; λ_t is the current-valued Lagrange multiplier associated with the resource constraint; and k_t is the per capita capital stock deflated by X_t (i.e. $k_t = K_t/(M_t X_t)$). This system of difference equations has two boundary conditions, one at time zero (the initial value of k, $k_o = K_o/(X_o M_o)$) and the other at infinity (the transversality condition, $\lim_{t\to\infty} (\beta^*)^t \lambda_t k_{t+1} = 0$).

To solve this problem we employed a shooting method that relies on knowledge of the near-steady-state dynamics of this system of equations. By linearizing the system around the steady state it is possible to show that, depending of the value selected for λ_o three types of paths may arise: (i) paths along which the capital stocks always grows, eventually overshooting the steady state and continuing to grow at an accelerating rate; (ii) paths along which the capital stock decreases or increases initially and then decreases; (iii) one path along which the capital stock increases converging to the steady state. Paths type (i) and (ii) violate the transversality condition so only (iii) is the desired solution. We denote the value of λ_o associated with (iii) by λ_o^*. Paths type (i) occur for values of $\lambda_o > \lambda_o^*$ while paths type (ii) correspond to $\lambda_o < \lambda_o^*$. This suggests a simple algorithm to search for λ_o^*:

<u>Step 1</u>: find a value of λ_o that generates a path type (i); denote it by $\bar{\lambda}_o$

<u>Step 2</u>: find a value of λ_o that generates a path type (ii); denote it by $\underline{\lambda}_o$ ($\underline{\lambda}_o = 0$ will always work).

<u>Step 3</u>: Compute $\lambda_o = (\underline{\lambda}_o + \bar{\lambda}_o)/2$ and use it as initial condition to solve the system of difference equations . Set $\underline{\lambda}_o = \lambda_o$ if a path type (ii) is obtained and $\bar{\lambda}_o = \lambda_o$ otherwise. Repeat step 3 until $\underline{\lambda}_o - \bar{\lambda}_o$ is lower than a chosen tolerance error (usually the smallest number recognized by the computer as different from zero).

The number of iterations needed for convergence is given by the first integer j such that $j > \ln(\Delta/tol)/\ln 2$, where tol is the chosen tolerance and Δ the initial value of $\bar{\lambda}_o - \underline{\lambda}_o$.

This method is different from "simple" and "multiple" shooting which are the standard algorithms used to solve this type of problem. The advantage of both of these algorithms is that they require no knowledge of the dynamics of the system. A detailed discussion of these methods can be found in Roberts and Shipman (1972) and in Lipton et al (1982) but we provide here a brief description to contrast them with the shooting method that we employed.

The basic idea underlying simple shooting is that a system of equation such as (A.1) – (A.2) can be viewed as defining a function $Z_T = f(\lambda_o)$, where λ_o is an arbitrary guess and Z_T is the difference between the value of the boundary condition at T associated with λ_o and the desired value for that boundary condition. A numerical method for finding zeros of equations (e.g. Newton-Raphson) is then used to generate a new guess for λ_o with the process being repeated iteratively until $Z_T = 0$. In our example the second boundary condition is at infinity so it is usually approximated by choosing T to be a large number (say, 200 years). Z_T can be defined as $(\beta^*)^T\lambda_T k_{T+1} - 0$ or as $k_T - k_s$ since paths that satisfy the transversality condition converge to the steady state. Simple shooting does not usually work because arbitrary guesses for λ_o can generate paths for the capital stock along which k_t becomes negative leading to nonsensical complex values for k_T and λ_T. To avoid this it is often necessary to split the path into various parts and apply the method to each part (e.g. compute the path for the first five years, then use k_5 as an initial condition to compute the path for the following five years, etc.), a technique that is known as multiple shooting.

The numerical results that we obtained for the models described in section IV using multiple shooting were very similar to the paths computed with our shooting algorithm.

As a second check on the algorithm that we employed we also verified that the paths computed numerically for the one and two-sector models replicated the analytical solutions that can be obtained for the cases of 100% depreciation and logarithmic momentary utility (for a discussion of these closed forms see Radner [1966] and Long and Plosser [1983]).

APPENDIX B

Embodied Technical Progress

Tnis Appendix shows that modifying the model of section II to view technological progress as embodied, along the lines of Solow [1959], generates an economy that is basically observationally equivalent to the original model.

The technology of the Solow [1959] model translated to discrete time is comprised by the following equations:

$$(B.1) \qquad Y_{vt} = A\,(\gamma_E^v\, K_{vt})^{1-\alpha}\, N_{vt}^{\alpha}$$

$$(B.2) \qquad K_{v,t+1} = I_v\,(1-\delta)^{t-v}$$

$$(B.3) \qquad N_t = \sum_{v=o}^{t-1} N_{vt}$$

$$(B.4) \qquad Y_t = \sum_{v=o}^{t-1} Y_{vt}$$

$$(B.5) \qquad Y_t = C_t + I_t.$$

The first equation expresses the output at time t of a production technology of vintage v as a Cobb-Douglas function of the capital of that vintage in existence at time t and of the labor combined with that capital. The rate of embodied technical progress is denoted by γ_E. Equation (B.2) relates the stock of capital of vintage v existent at time t+1 to the original investment made in that vintage (I_v) and of the rate of depreciation. Equation (B.3) is the adding-up constraint on labor, (B.4) states that total output is the sum of the output produced by the various vintages and (B.5) that total output can be devoted to consumption or investment.

An efficient allocation of labor requires that its marginal product be equated across the different vintages. Solow [1959] showed that using this fact the vintage-specific capital stocks can be aggregated into a composite capital stock, J_t defined as:

$$J_t = \sum_{v=o}^{t-1} \gamma_E^v K_{v,t}. \tag{B.6}$$

The advantage of defining this composite capital good is that total output can be expressed as a function of N_t and J_t:

$$Y_t = A \; J_t^{1-\alpha} \; N_t^{\alpha}. \tag{B.7}$$

The law of motion for J_t (B.6) can also be expressed without reference to the vintage-specific capital stocks:

$$J_{t+1} = J_t \; (1-\delta) + \gamma_E^t \; I_t. \tag{B.8}$$

In the steady state capital grows at rate $\gamma_J = \gamma_N \; \gamma_E^{\;1/\alpha}$, where γ_N is the growth rate of population, while output, consumption and investment grow at rate $\gamma_Y = \gamma_N \; \gamma_E^{(1-\alpha)/\alpha}$.

It is easy to show, using the description of technology given by (B.5), (B.7) and (B.8), that at any point in time the real interest rate is given by:

$$r_t = [(1-\alpha) \; A \; (j_t)^{-\alpha} \; n^{\alpha} + (1-\delta)]/\gamma_E - 1. \tag{B.9}$$

where $j_t = J_t/(M_t\ \gamma_J^t)$, i.e. the per capita value of the composite capital stock detrended by its growth rate.

To study the model's implications for the Japanese real interest rate in 1950 we start by using (B.9) and the knowledge of the steady state real interest rate, r^*, to compute the steady state value of j_t, j^*. Next we use the fact that Japanese per capita output is 1950 was 19% of that of the U.S.:

$$\text{(B.10)} \qquad [A\ (J_o^J)^{1-\alpha}\ (N^J)^{\alpha}/M_o^J]/\ [A\ (J_o^{US})^{1-\alpha}\ (N^{US})^{\alpha}/M_o^{US}] = .19$$

Assuming that the number of hours worked per capita is the same in the two countries (in fact this number was higher in Japan so that this assumption biases the results toward finding a low interest rate), we can rewrite (B.10) in terms of j's as:

$$\text{(B.11)} \qquad j_o^J = j_o^{US}\ (.19)^{1/(1-\alpha)}.$$

Under the assumption that the U.S. was at the steady state in 1950, i.e., $j_o^{US} = j^*$, we can compute j_o^J an the associated real interest rate implied by (B.9). Using the parameter values employed in the main text, α=2/3, δ=.10, r_s=.065 and γ_N = 1.01 (the Japanese population grew at 1% in the post war period—see Barro [1987], page 296) and choosing γ_E so that the steady state growth rate of per capita output is 2% per year, the value of the Japanese interest rate implied by the model is 560%.

In terms of dynamics this model is almost identical to the baseline economy of section II. The system of Euler equations that governs the the competitive equilibrium for this economy is identical to (A.1), (A.2) with $(\gamma_X\gamma_N)$ replaced by γ_J and k_t replaced by j_t.

Table 1A

Behavior of Real Interest Rates in the Neoclassical Model

Real Interest Rate 100 years ago—Cobb–Douglas Production Function

Fraction of Growth Due to Transitional Dynamics Ψ	Growth Rate of Exogenous Technical Progress γ_x	Baseline Model (%)	Depreciation Rate $\delta = 0$ (%)	Depreciation Rate $\delta = .25$ (%)	Labor Supply $N_0 = .36$ (%)	Capital Share $\alpha_k = .5$ (%)	Capital Share $\alpha_k = .9$ (%)	End of Period Real Interest Rate $r_T = 4\%$ (%)	End of Period Real Interest Rate $r_T = 9\%$ (%)
0%	2%	6.5	6.5	6.5	43.5	6.5	6.5	4.0	9.0
25%	1.5%	16.7	10.5	26.0	76.6	11.0	6.9	12.7	20.8
50%	1%	40.5	19.9	71.5	153.7	18.9	7.6	32.9	48.2
75%	.5%	119.4	51.0	222.0	409.1	36.2	8.5	99.8	139.0
100%	0%	798.5	318.5	1518.5	2609.2	105.5	10.5	676.0	921.0

Table 1B

Real Interest Rate (r_o) and Capital Share (α_{ko}) 100 years ago

CES Production Function with Elasticity of Substitution ρ

Fraction of Growth Due to Transitional Dynamics Ψ	Growth Rate of Exogenous Technical Progress γ_x	$\rho = .9$ r_0	$\rho = .9$ α_{k0}	$\rho = .5$ r_0	$\rho = .5$ α_{k0}	$\rho = 1.1$ r_0	$\rho = 1.1$ α_{k0}	$\rho = 1.25$ r_0	$\rho = 1.25$ α_{k0}
0%	2%	6.5	33.3	6.5	33.3	6.5	33.3	6.5	33.3
25%	1.5%	17.6	35.1	23.7	47.6	16.0	31.9	15.0	30.0
50%	1%	41.5	37.4	46.9	61.9	39.7	29.9	38.6	25.4
75%	.5%	106.8	40.5	76.2	76.2	136.6	26.8	180.2	18.1
100%	0%	430.7	46.3	111.6	90.5	2193.2	20.4	2993474.3	1.6

TABLE 2

ANNUAL REAL RATES OF RETURN: SUMMARY STATISTICS

US Securities, 1926 – 1987

Series	*Average Real Rate of Return*	*Average Change in Real Rate of Return*	*Standard Error*
Common Stocks	6.65	0.0100	0.40
Small Stocks	8.80	–0.0024	1.07
Corporate Bonds	1.83	0.0019	0.17
US Treasury Bills	0.42	0.0001	0.04
Long Term Government Bonds	1.18	0.0004	0.23

Data Source: Ibbotson and Sinquefield [1988]. Units are percentage points. The first column reports the geometric average of returns. The last column reports Newey–West [1987] standard errors associated with the statistic reported in column 2.

Table 3
Historical Evidence on Long-Term Interest Rates

Date	England	France	Dutch Republic Holland—	Spanish Netherlands— Belgium	Germany	Sweden	Spain	Switzerland	Italy	United States	Canada
	Minimum Rates on Best Credits by Half Centuries*										
13th century											
1st half				8.00							
2nd half		14.00		8.00					6 5/8		
14th century											
1st half				8.00					4 7/8		
2nd half				8.00					5 1/4		
15th century											
1st half		10.00		8.00			4.00		6.00		
2nd half		10.00		8.00	5.00		4.00		5.00		
16th century											
1st half	10.00	8 1/3	8 1/3	4.00	4.00				4.00		
2nd half	10.00	8 1/3	6 1/6	4.00	4.00				4.00		
17th century											
1st half	8.00	8 1/3	5.00								
2nd half	4.00	5.00	3.00								
18th century											
1st half	3.05	5.00	3.00		4.00			5.00	4.00		
2nd half	3.13	5.00	2.50		4.00			4.00	4.00		
19th century											
1st half	3.26	4.06	4.53	4.53	3.84					4.55	
2nd half	2.47	3.03	2.93	2.71	3.53	3.68				3.23	
20th century											
1st half	2.79	3.06	3.20	2.94	3.64	3.04		3.31		2.31	2.93

* Lowest decennial average where available; otherwise lowerst reported rate. Lowest Yield or yields are underlined for each tme period. All yields in annual terms and expressed in percentage points.
Source: Homer: (1963)

Table 4
Transitional Dynamics, Open Economy with Adjustment Costs

Parameterizations		Time Period (each period has 5 years) 1	2	3	4	5	6	7	8	9	10
1) b = 32.2	Half life = 37										
	Average Growth Rate of Capital	64.1	22.6	12.5	8.1	5.7	4.3	3.3	2.6	2.0	1.6
	Average Growth Rate of Output	18.0	7.0	4.0	2.6	1.9	1.4	1.1	0.8	0.7	0.5
	Average Tobin's Q	22.4	8.3	5.0	3.6	2.8	2.4	2.0	1.8	1.7	1.5
2) b = 2.0	Half life = 9										
	Average Growth Rate of Capital	146.3	19.5	5.8	2.0	0.7	0.3	0.1	0.0	0.0	0.0
	Average Growth Rate of Output	35.0	6.1	1.9	0.7	0.2	0.1	0.0	0.0	0.0	0.0
	Average Tobin's Q	4.6	1.4	1.1	1.0	1.0	1.0	1.0	1.0	1.0	1.0

Each period has 5 years. All rates reported are the average over the five-year period in annual terms and expressed in percentage points.

FIGURE 1

NEOCLASSICAL PRODUCTION AND MARGINAL PRODUCT SCHEDULE

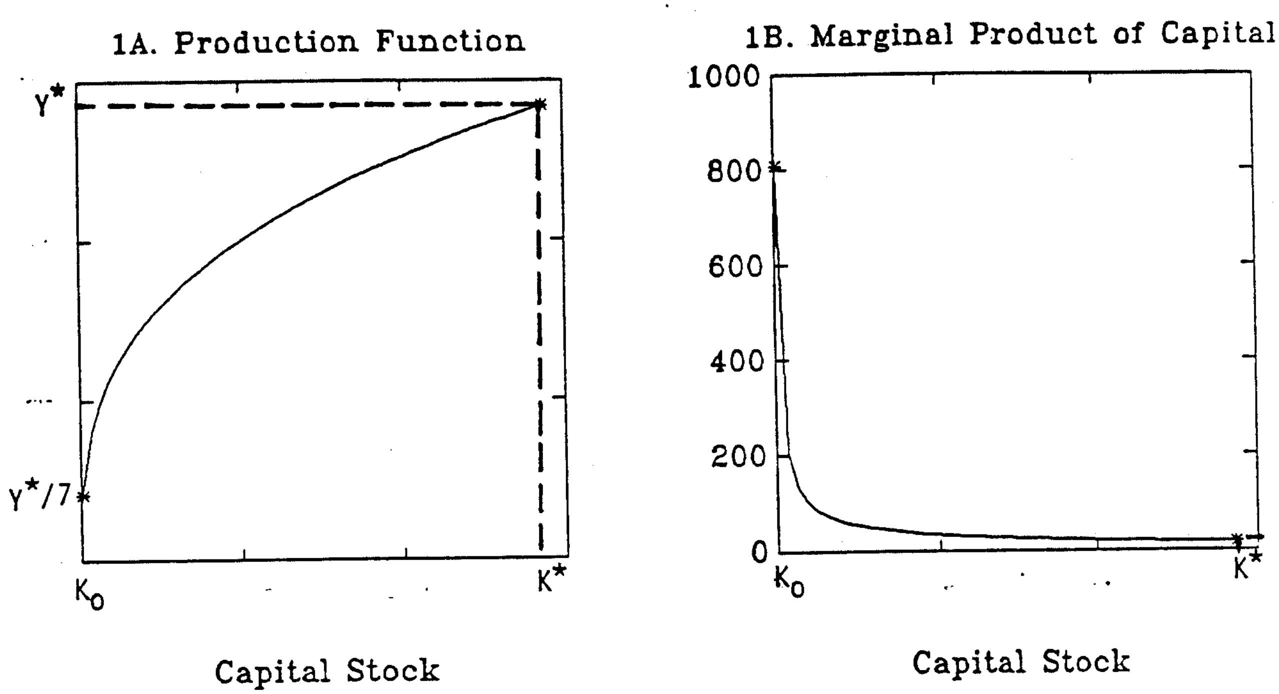


Figure 2:

Sato's Transitional Dynamics

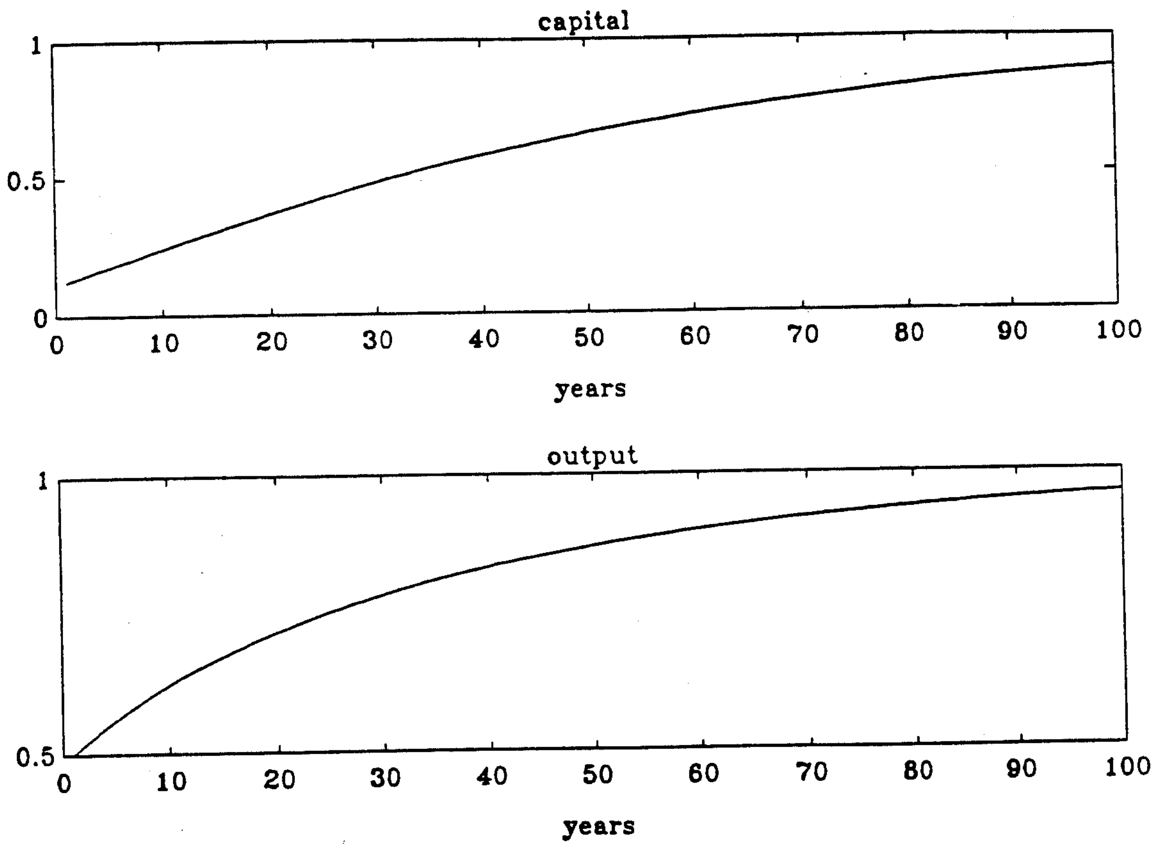


Figure 3

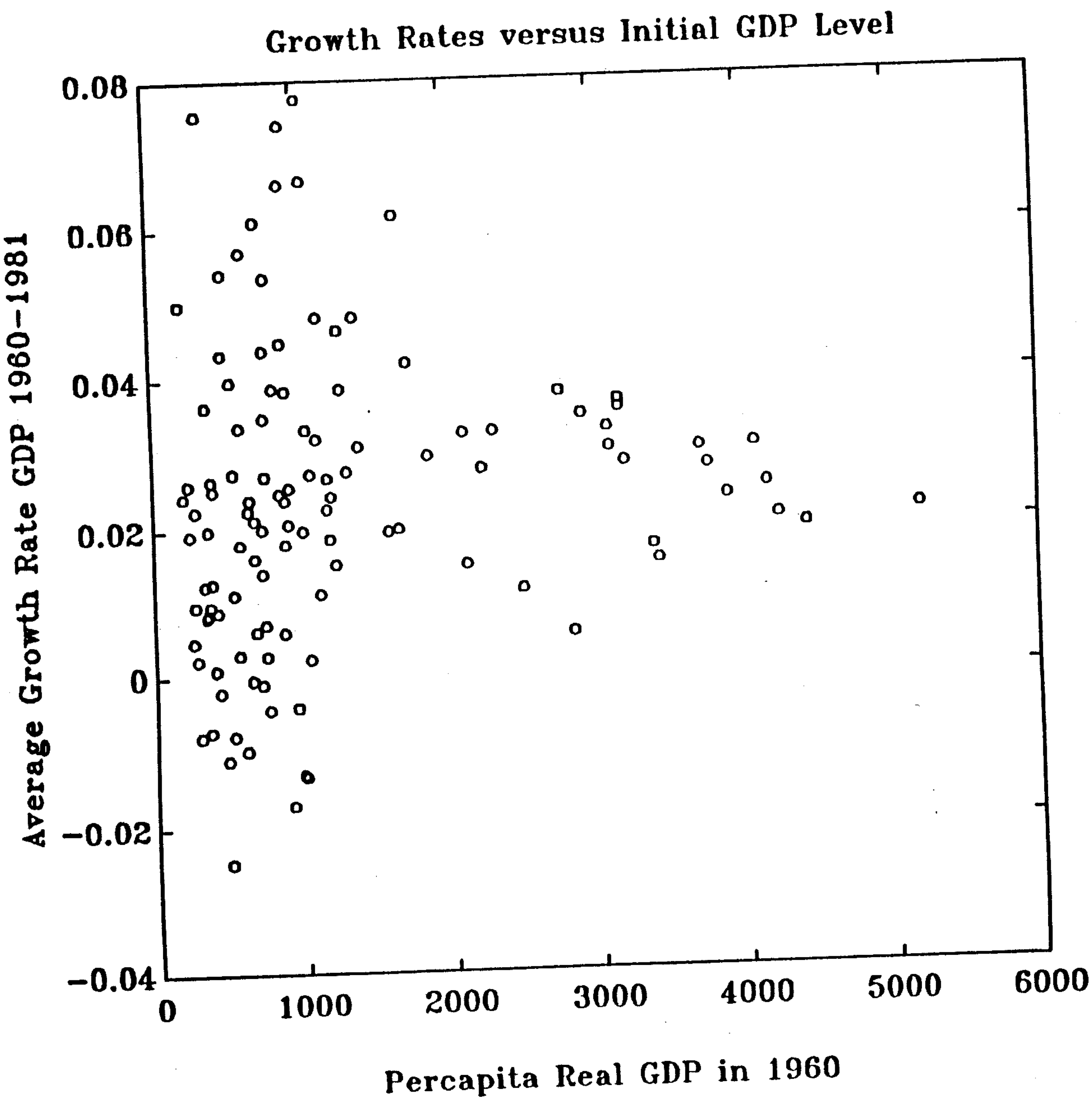

Growth Rates versus Initial GDP Level
Average Growth Rate GDP 1960-1981
Percapita Real GDP in 1960
0.08
0.06
0.04
0.02
0
-0.02
-0.04
0
1000
2000
3000
4000
5000
6000

FIGURE 4

BASELINE MODEL

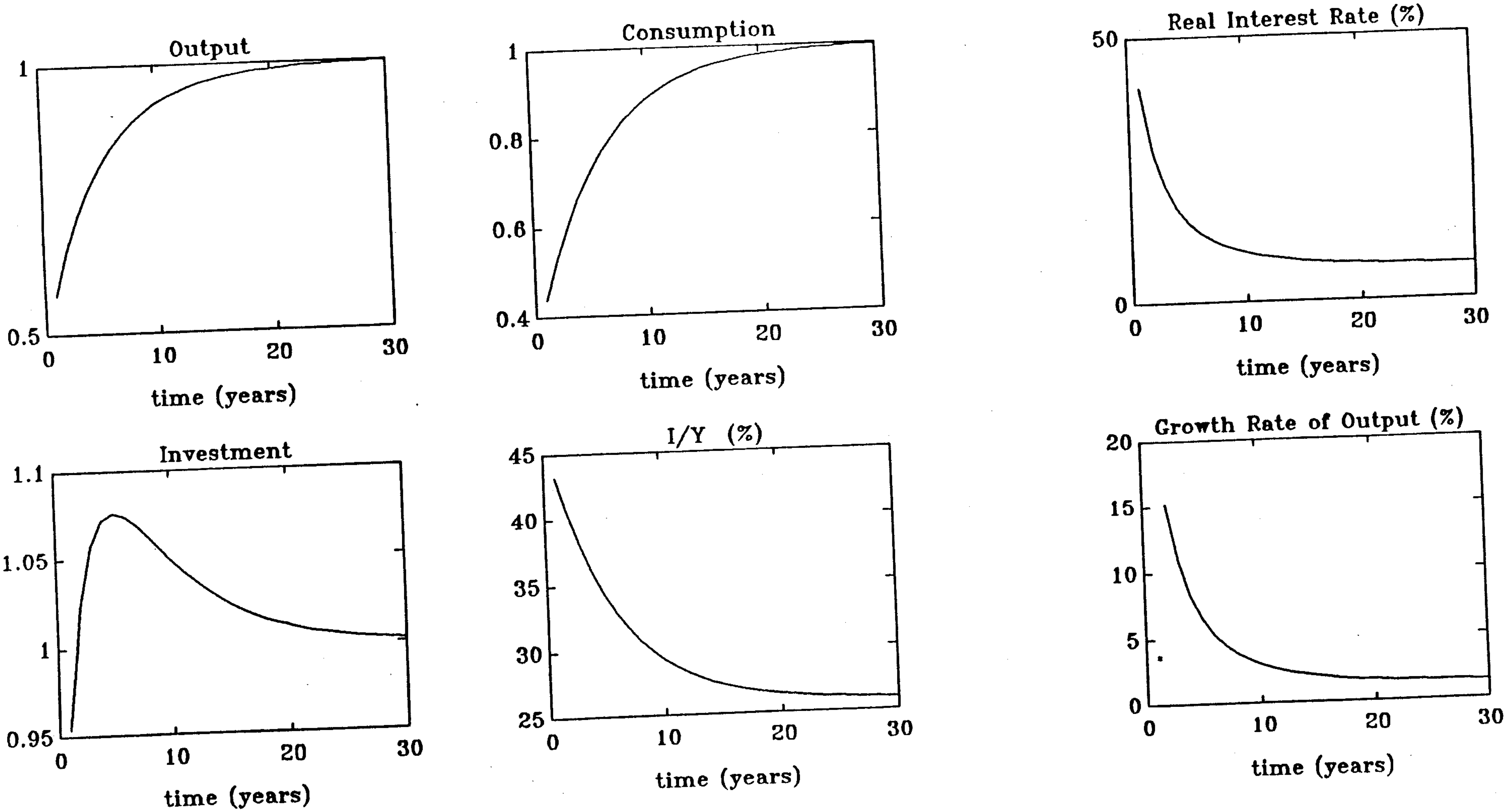


FIGURE 5

LOW INTERTEMPORAL SUBSTITUTION (sigma=10)

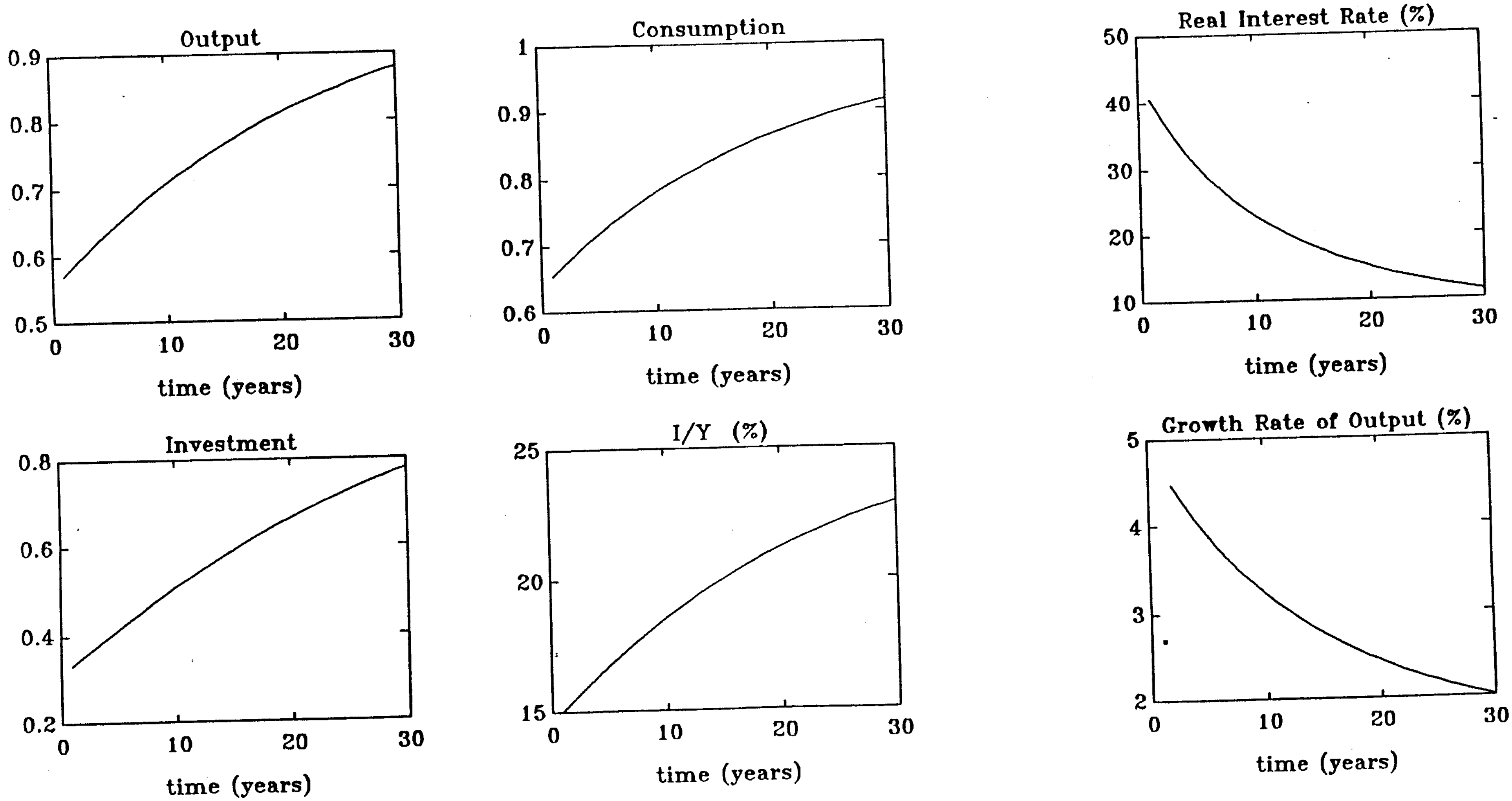


FIGURE 6

STONE-GEARY PREFERENCES

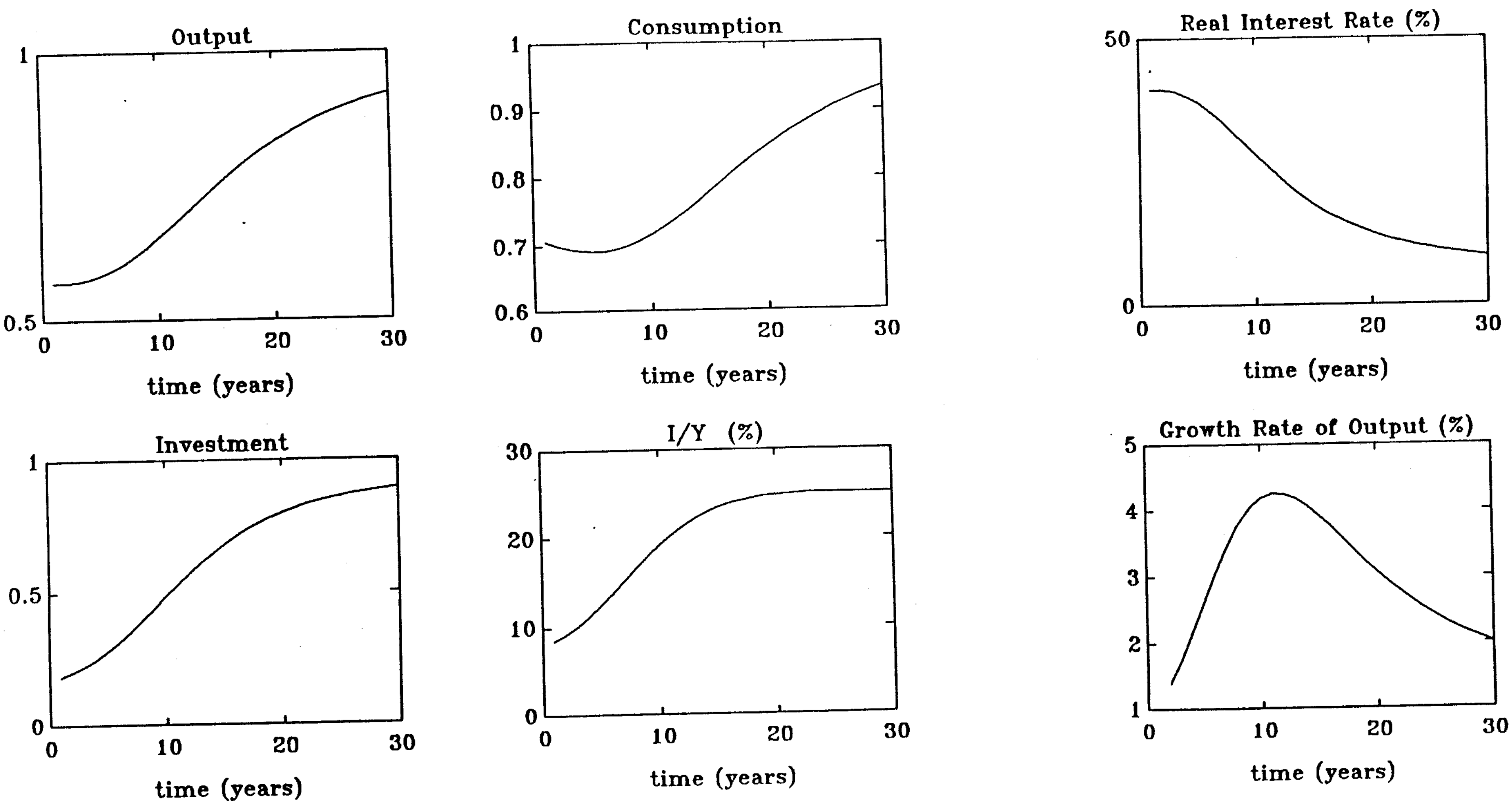


FIGURE 7

HIGH CAPITAL SHARE (50%)

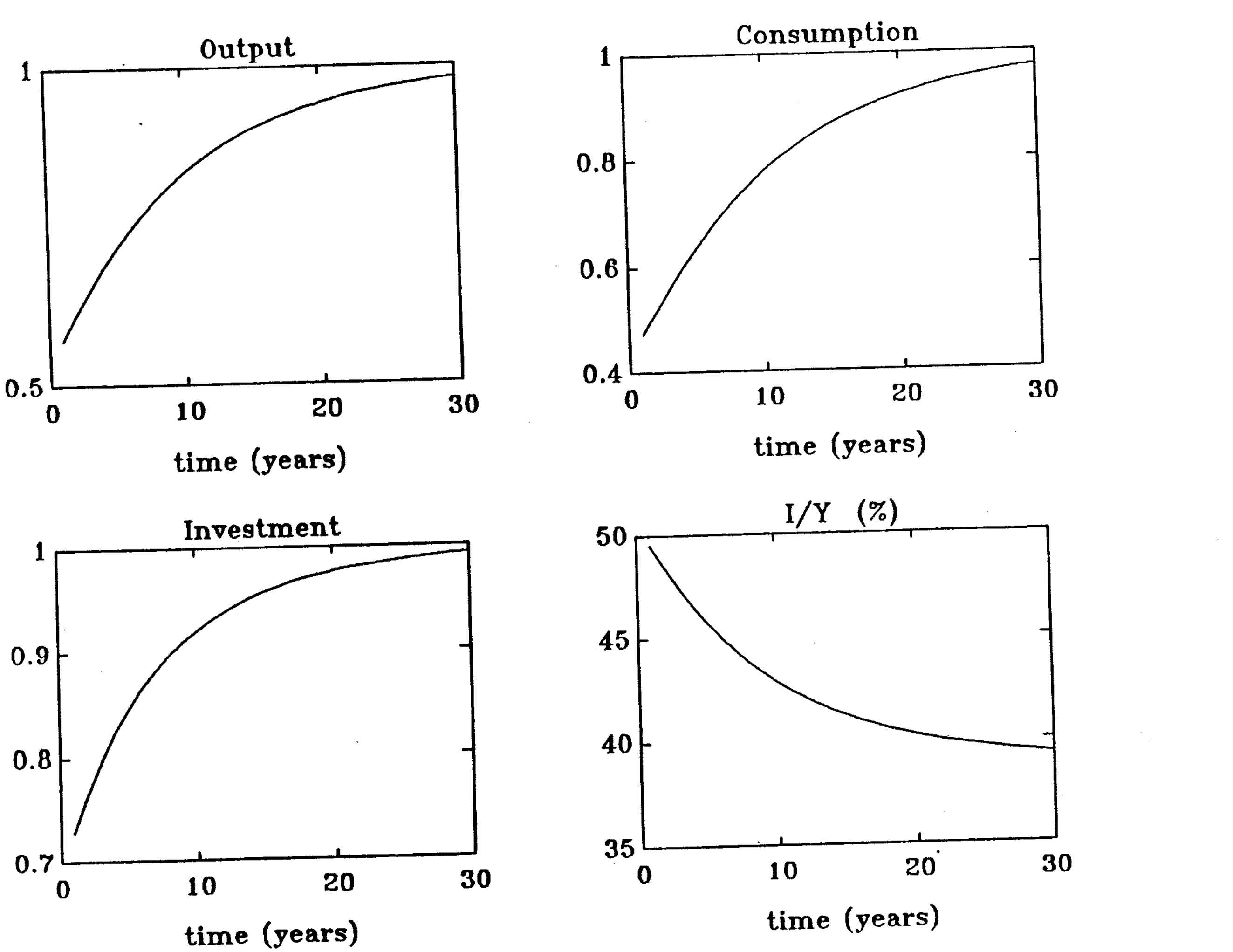


FIGURE 8

TWO-SECTOR MODEL

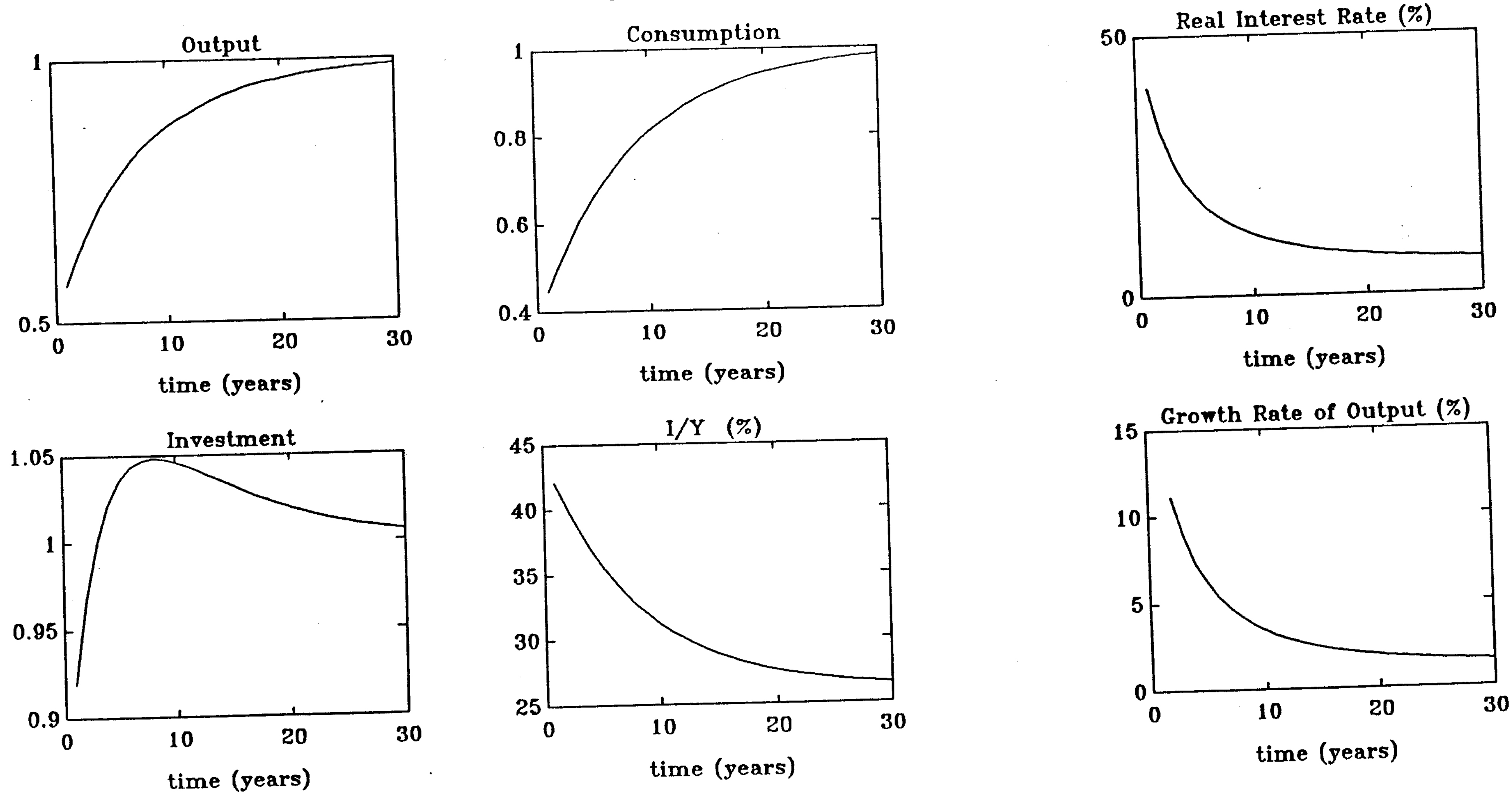


FIGURE 9

ADJUSTMENT COSTS MODEL

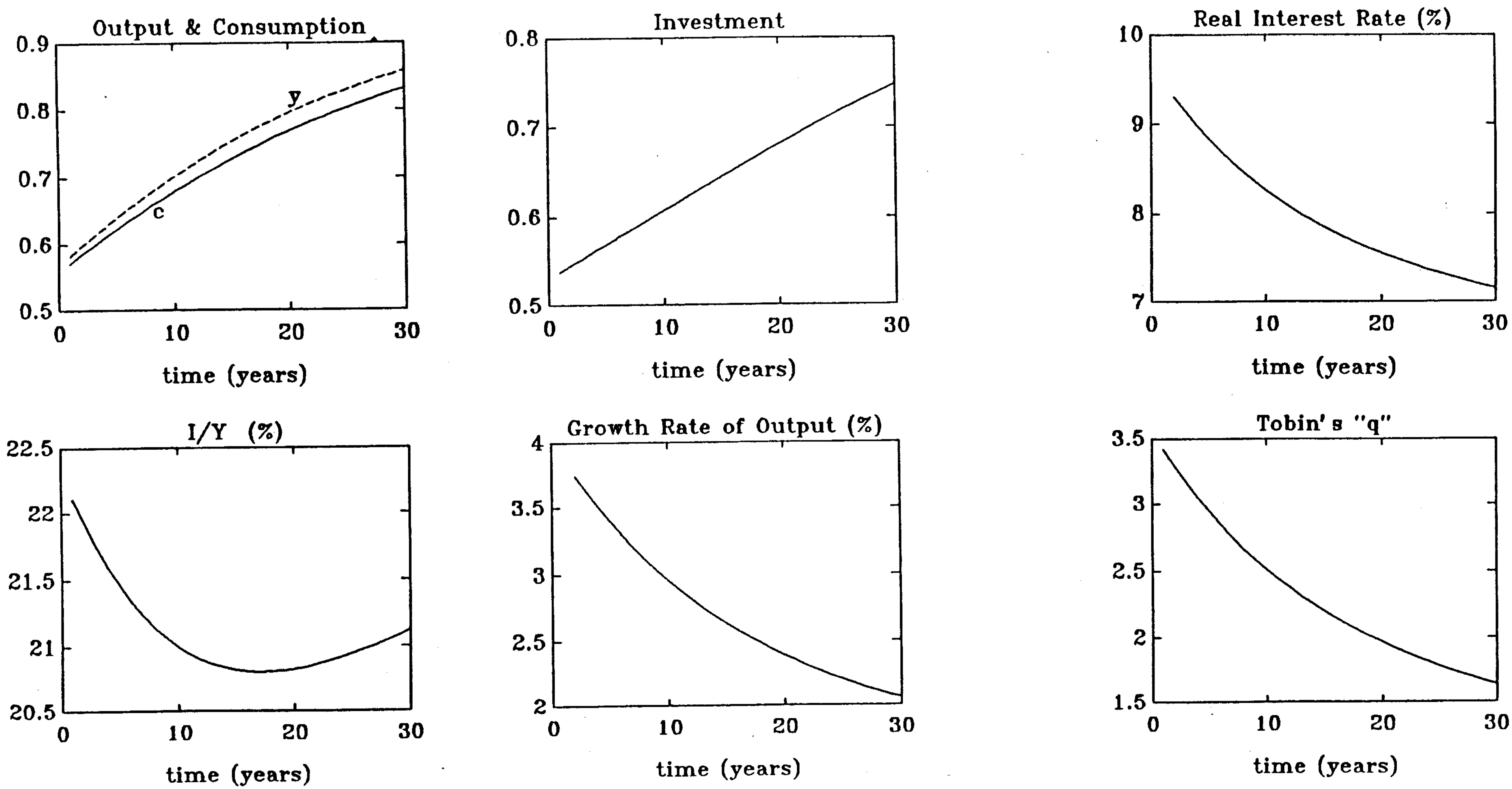


FIGURE 10

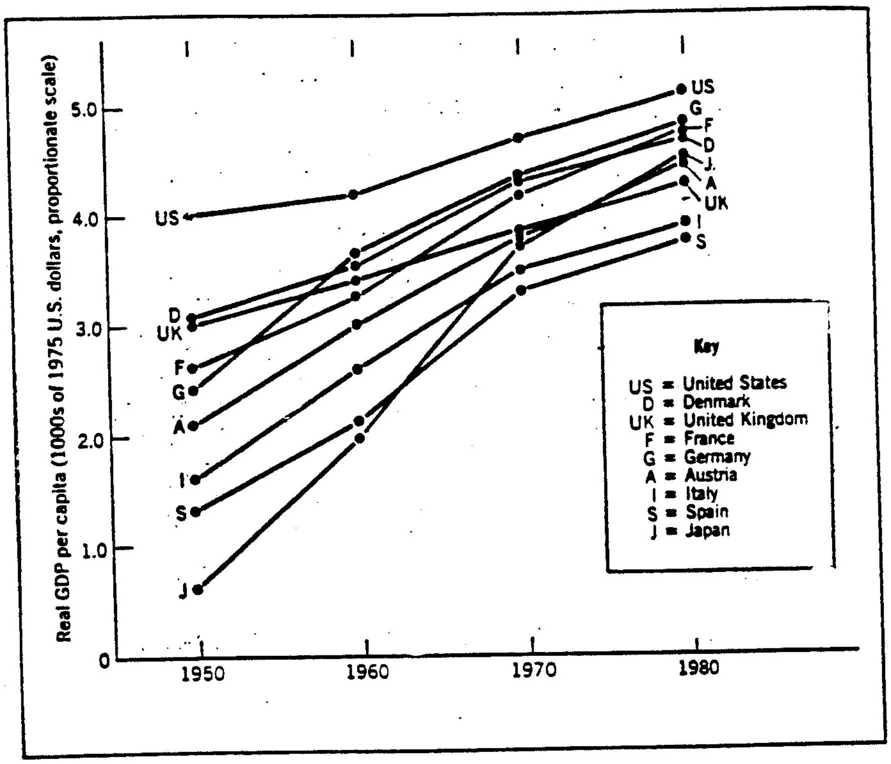
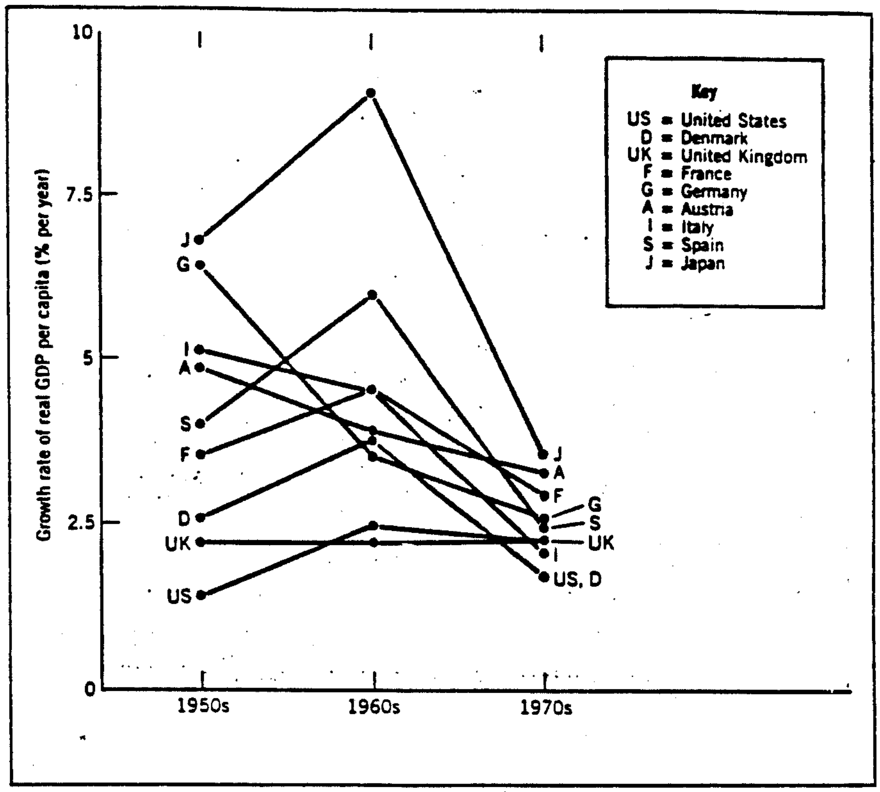


Figure 11.5 Levels of Output per Capita for Nine Industrialized Countries
The figure shows the convergence of output per person across the countries between 1950 and 1980.

Source: Barro (1987).

FIGURE 11



1.6 Growth Rates of Output per Capita for Nine Industrialized Countries

The figure shows the average growth rates of output per capita for the countries during three decades—the 1950s, 1960s, and 1970s.

Source: Barro (1987).

FIGURE 12

JAPAN versus U.S.