Value and Capital in the Equilibrium Business Cycle Program

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## I. INTRODUCTION

In the late 1920s and early 1930s, when Sir John Hicks was a young scholar at the London School of Economics, there was growing interest in the prospect of an equilibrium approach to the study of business cycles. In 1931, Hayek gave the LSE lectures that were to become *Prices and Production*, a first attempt to use Austrian capital theory to study aspects of economic fluctuations. Next, in one of his earliest published papers, "Equilibrium and the Cycle", written in 1932, Hicks made two key observations about the developing equilibrium approach.

First, it would require new analytical methods. The stationary state construct that was the main analytical engine of Austrian capital theory would have to be abandoned, since economic fluctuations clearly involved time variation inconsistent with the stationary state abstraction. In one of the major intellectual insights of twentieth century economics, Hicks recognized that the traditional equilibrium methods of Pareto [1896] and Walras [1877] could potentially be applied to an economic system outside of the stationary state by the simple device of time dating all commodities, treating physically identical goods at different dates as distinct commodities for analytical purposes.[1] Thus, Hicks provided an analytical direction for construction of a rigorous intertemporal equilibrium theory.

Second, Hicks argued that intertemporal equilibrium was the logical starting point for aggregate economic analysis, just as the equilibrium theory of Walras and Pareto was for other questions of resource allocation. Since an intertemporal equilibrium—defined by Hicks as a path along which

[1]Working independently, Lindahl [1929] also made this observation.

expectations were fulfilled—would be dynamic, with different economic quantities at different dates, its construction was a precondition to the definition and measurement of other components of economic activity. In Hicks' [1932a] words,

> "such a dynamic equilibrium is obviously still far from being a description of reality. It does nevertheless serve as a model of a perfectly working economic system, which is much more usable as a standard of comparison than is the model of a stationary equilibrium. Because of the ignorance of future changes of data (and still more of the consequences of changes of data—not only of future or present changes, but also of those that have already occurred in the past) such a perfect equilibrium is never attainable. A real economy is always in disequilibrium. The actual disequilibrium may be compared with an idealized state of dynamic equilibrium to give us a way of assessing the extent or degree of disequilibrium."

It is the objective of the modern equilibrium business cycle program, which aims at construction of quantitative dynamic models, to provide just this measurement of the state of disequilibrium in economic systems. Modern equilibrium theory is typically traced to the influential analysis of Lucas [1977], but an alternative perspective views its growth as an inevitable outcome of new general equilibrium tools developed from the insights of Hicks [1932a] and others.[2] No more compelling motivation for the program could be provided than that of Hicks [1932a]. The idea that agents exhaust all gains from trade—so that the role of intertemporal preferences

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[2]For an explicit statement of this view, see Lucas [1984].

and technologies is laid bare—is simply the natural starting point for aggregate analysis.

The subject of this conference, *Value and Capital,* contains two contributions that have influenced the design of the current study. First, at the microeconomic level, *Value and Capital* contains a persuasive treatment of modern consumer theory, building on Slutzky [1915] and Hicks and Allen [1934], with its logical decomposition of demand behavior into wealth and substitution effects. Though abstract, this decomposition has proved a fruitful empirical tool, since it suggests ways of developing experiments that isolate only one component. Even in macroeconomics, it is now standard for many discussions to be undertaken in terms of wealth and substitution effects, a process which has now even filtered down to undergraduate presentations.[3] Second, at the general equilibrium level, *Value and Capital* represents the detailed fleshing out of the analytical apparatus sketched in "Equilibrium and the Cycle." In particular, it systematically develops the intertemporal dimension to individual preferences and choices, as well as making clear the central role of capital accumulation in the structure of dynamic economics. The modern equilibrium approach to business cycles represents the application of these methods and their postwar general equilibrium descendants—along the path outlined by the contributions of Arrow [1951, 1953], Debreu [1959] and McKenzie [1954]—to the study of aggregate fluctuations.

In this paper, I will discuss a basic equilibrium business cycle model and relate results in this model to Hicks' work in *Value and Capital.* But the

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[3]See R.J. Barro's *Macroeconomics* [1984] for the initial treatment along these lines.

main contribution of this paper will be to outline an approach to future research in the equilibrium business cycle program that will be more in keeping with theoretical approach of *Value and Capital*. That is, the paper decomposes equilibrium variations in individual choice sequences into wealth and substitution components, within the context of a quantitative intertemporal equilibrium model. Although some might view this return to demand analysis as a retrogression in business cycle modeling,[4] my perspective is that it is an essential component to the process of understanding the structure and implications of dynamic economic models. In particular, the approach of this paper will enable us to make explicit quantitative statements about the magnitudes of the wealth and substitution elements that figure prominently in many discussions of macroeconomic issues.

The organization of the paper is as follows. In section II, a basic intertemporal specification of preferences is used to develop Hicksian concepts of wealth and substitution effects in a manner applicable to quantitative dynamic analysis. In section III, this specification of preferences is coupled with a capitalistic specification of intertemporal production possibilities. Then, in section IV, quantitative general equilibrium experiments are presented. The discussion focuses on how changes in a real disturbance—government absorption of output financed by lump sum taxes—influence the time profile of economic activity. Thus, we will be following in the tradition of real business cycle analysis in exploring how real shocks influence economic activity, but we choose to focus on a demand

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[4]Sargent [1982] urges macroeconomists to go 'beyond supply and demand curves' to preferences and technology. The equilibrium analysis of this paper does go through to this deeper level and, hence, does not conflict with Sargent's recommendation. However, it also provides the intermediate demand theoretic analysis missing in recent equilibrium business cycle modeling.

rather than a supply side shock. Our motivation for studying this topic stems from some applied research in macroeconomics which has noted that temporary, wartime increases in government purchases are associated with larger variations in real production activity than are permanent increases (Barro [1981] and Hall [1980]). These authors argue that intertemporal substitution effects arising from adjustments in intertemporal prices (i.e., interest rates) will lead to larger effort and production responses to temporary disturbances than to permanent ones. The Barro-Hall hypothesis is developed basically at the level of an individual worker-producer, rather than in a full general equilibrium. Recent efforts to model the real effects of government purchases in general equilibrium have shown that it has considerable explanatory power for the U.S. during World War II (Wynne [1987a]), but that temporary changes in purchases have smaller impacts than permanent ones (Baxter and King [1988], Ayigari, Christiano and Eichenbaum [1989])[5]. It thus appears that the intertemporal substitution responses discussed by Barro and Hall do not arise general equilibrium. But since the forgoing quantitative general equilibrium studies solve for quantities without considering the details of demand behavior, it is not possible to determine which substitution margins and wealth effects are dominant. Thus, the Barro-Hall hypothesis provides a natural motivation of and example for our effort to quantitatively measure the relative importance of various intertemporal substitution effects and wealth effects in general equilibrium. This decomposition is undertaken in section V, which provides some

[5]In the terminology of Baxter and King [1988], the displacement we consider is "basic spending" with no utility or productive consequence. Baxter and King also study changes in government spending that influence preferences or production technology.

interpretation of puzzling results obtained in the quantitative general analysis of section IV.

Throughout the main sections of the paper, the individual choice and comparative dynamics analysis is conducted in a certainty equivalence form, in keeping with the perspective of "Equilibrium and the Cycle." A final, concluding section briefly considers some extensions to the analysis, including alterations in preferences; alternative decompositions along the lines of Frisch [1932]; explicit uncertainty; and some discussion of the interaction of monetary and real phenomena in the modern equilibrium approach and in the latter chapters of *Value and Capital*.

## II. Preferences and Demand Theory

*Value and Capital* provides a powerful formal apparatus for the analysis of economic problems involving time, reducing these to decision problems of the same general form as those encountered in atemporal economies. Further, under perfect foresight, the competitive equilibrium of Hicksian initial date markets and sequential markets (as explored by Fisher [1908] and Hicks) could be seen to coincide. For this reason, many macroeconomic analyses now start by analyzing initial date markets and then provide a reinterpretation in terms of sequential markets. We follow that procedure here.

The standard equilibrium business cycle model is populated by a large number of identical agents possessing preferences over sequences of consumption and leisure, generally of the time separable form

$$\sum_{t=1}^{T} \beta^{t-1} u(c_t, L_t) \tag{1}$$

where $c_t$ is consumption at date t; $L_t$ is leisure at date t; and $\beta$ is a time discount factor. In some of our discussion below, we will study an infinite horizon version of these preferences ($T = \infty$). Throughout, we shall restrict momentary utility so that an increase in wealth raises the demand for both consumption and leisure.

Consider an individual operating in a Hicksian set of forward markets at date t=1. The individual under study has sequences of endowments of consumption goods and productive time, which are the basis for income flows in the intertemporal budget constraint

$$\sum_{t=1}^{T} [p_t c_t + p_t w_t L_t] \leq \sum_{t=1}^{T} [p_t e_t + p_t w_t]. \tag{2}$$

In this expression, the endowment of time per period is normalized to unity and $e_t$ is the endowment of goods at date t. The price $p_t$ is the forward market price of consumption; $w_t$ is the date t commodity wage rate.

Efficient choices of $c_t$ and $L_t$ require that the following conditions are satisfied,

$$\beta^{t-1} D_1 u(c_t, L_t) = \Lambda p_t \tag{3a}$$

$$\beta^{t-1} D_2 u(c_t, L_t) = \Lambda p_t w_t \tag{3b}$$

for all t = 1, 2, ... T as well as the intertemporal budget constraint (2).[6] The parameter $\Lambda$ is the marginal utility of consumption at date t=1 if the

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[6]By $D_i u(c_t, L_t)$, we mean the partial deviative of u with respect to its ith argument.

consumption good is numeraire, i.e., it is the multiplier on the lifetime budget constraint. This construction figures prominently in demand decompositions deriving from Frisch [1932].

## II.1 The Stationary State and Displacements

The starting point for our analysis will be a stationary state with quantities $c_t = c$ and $L_t = L$ for all t. (From the first order conditions, we can see that the stationary state prevails if the individual faces prices $p_t = p\beta^{t-1}$ and $w_t = w = D_2u(c,L)/D_1u(c,L)$.)[7]

Our objective will be to investigate how various displacements induce deviations of optimal consumption and leisure sequences from the stationary state levels. These displacements generally will imply perturbations in prices $(p_t - p\beta^{t-1})$, wages $(w_t - w)$ and endowments $(e_t - e)$ away from their stationary state values.

To make the discussion concrete in section IV below, our analysis will focus on government absorption of resources $g_t$, financed by lump sum taxation. Displacing $g_t$ from its stationary level g will have a direct implication for the wealth of private agents at given prices, altering it by the amount $-\sum_{t=1}^{T} \{p_t\ (g_t - g)\}$, since we assume for simplicity that there are no direct utility or productive consequences of the alteration in spending.[8] However,

[7]By focusing on a stationary state, we are not necessarily excluding growth associated with exogenous technical progress, since—under appropriate conditions on preferences and technology—growing economics with steady states can be transformed into stationary ones. For a recent discussion, see King, Plosser, and Rebelo [1988a].

[8]Following Baxter and King [1988], however, it is feasible to introduce such effects and to explore the implications of such alternative spending displacements.

in general equilibrium, the displacement also typically alters the sequence of prices and wages that individuals will face, inducing substitution responses. It is this feature that makes the analysis of these disturbances something that is hard to do outside of an explicit quantitative framework.

Although we focus on government purchases as a disturbance, the general strategy that is outlined here is applicable to a wide range of shifts that can be considered in equilibrium business cycle models, including shifts in technology; changes in tax rates; and various sorts of government spending shifts that alter private opportunities. The key requirement is that we must be able to express the implications of these shifts for consumer behavior in terms of changes in prices and endowments, which is a general characteristic of equilibrium models.

For the purpose of our discussion in the next two subsections, we will assume that a perfect foresight general equilibrium has been calculated and that we possess the change in lifetime utility (U) occasioned by a displacement to the model's exogenous variables as well as the changes in the equilibrium sequences of prices, wages, endowments, consumption and leisure. Our problem will be to decompose the variations in equilibrium consumption and leisure sequences into parts attributable to wealth effects and substitution effects.

### II.2 Demand Analysis as in *Value and Capital*

Viewing the preceding as a standard demand problem with (2T) choices, it is direct to use the Slutzky–Hicks–Allen approach, defining substitution effects as utility compensated. Consumption and leisure demand at date s depend on prices at all other dates, as well as on utility as a measure of real lifetime wealth.

$$c_s^d \; (\{p_t\}_1^T, \; \{p_t w_t\}_1^T; \; U) \tag{4a}$$

$$L_s^d \; (\{p_t\}_1^T, \; \{p_t w_t\}_1^T; \; U) \tag{4b}$$

In the standard treatment along these lines, the level of lifetime utility is implicitly determined by the budget constraint; here we view this level as delivered from a computed general equilibrium.

Locally, near a stationary point, we can find the alterations of demand arising from prices and utility using these standard methods (see, e.g., Intrilligator [1971] for a convenient review). That is, we can compute the derivatives of the Hicksian demand functions and use these to evaluate the near stationary state response of the representative household to alterations in prices and utility.

*Wealth Effects.* Hicksian wealth effects are then measured as the response of $c_s^d$ and $L_s^d$ to variations in lifetime welfare (U), interpreted as a measure of wealth in equilibrium. Since our displacements occur from an initial position in which market prices dictate $c_t = c$ for all t and $L_t = L$ for all t, it follows that these wealth effects will also involve shifts in $c_t$ and $L_t$ that will be uniform across periods.

*Substitution Effects.* The main computational difficulty in implementing the Hicksian approach lies in the richness of the price effects in (4a,b): to explore demand response at s, one must take into account price variations in all periods t = 1, 2,... T. Consider, for example, the influence of a change in the wage sequence from $w_t = w$ for all t to an alternative sequence $w_t$. To compute the implications of this change on leisure at date s, we must add up the changes in $L_s$ attributable to each $w_t$ for all dates t=1,2,...T. This process is facilitated by some properties of demand functions that arise with time separable preferences.[9]

The outcome of our demand analysis, then, will be a sequence (time path) of consumption and leisure responses to a sequence of deviations of price deviations from the stationary point values, either in the intertemporal price $(p_t - p\beta^{t-1})$ or in the wage rate $(w_t - w\beta^{t-1})$. In contrast to the wealth effect, such substitution effect sequences will not be constant over time, when the price displacements are not constant over time.

## III. Dynamic General Equilibrium

In this section, we lay out a basic equilibrium macroeconomic model and discuss the computation of equilibrium quantities and intertemporal prices. In view of our interest in studying effects of changes in government purchases on equilibrium outcomes, we will consider only a single exogenous variable for simplicity, but it is direct to expand the model to consider the range of displacements considered in King, Plosser and Rebelo [1988a,b] including a range of technical shifts and fiscal interventions.

The production technology of the economy for the single final good (y) is constant returns to scale in labor (N) and capital (k), i.e.,

$$y_t = F(k_t, N_t) \qquad (5)$$

[9]For a discussion of these properties, see Barro and King [1984]. Appendix B discusses the specific ways in which these properties are used in computing the decompositions discussed in the present paper.

with F() homogeneous of degree one. Resource constraints exist on goods and time, i.e.,

$$c_t + i_t + g_t = y_t \tag{6a}$$

$$N_t + L_t = 1 \tag{6b}$$

where $i_t$ denotes investment and $g_t$ denotes government spending. Finally capital accumulates according to

$$\gamma k_{t+1} = i_t - \delta k_t, \tag{7}$$

where $\delta$ is the rate of depreciation and $\gamma > 1$ is a parameter representing the gross growth rate of the economy.[10]

Equilibrium sequences for the infinite horizon economy will jointly solve the constraints (4-7) and the following efficiency conditions

$$D_1u(c_t, L_t) = \lambda_t \tag{8a}$$

$$D_2u(c_t, L_t) = \lambda_t D_2F(k_t, N_t) \tag{8b}$$

$$\beta^* \lambda_{t+1}[D_1F(k_{t+1}, N_{t+1}) + 1-\delta] = \lambda_t \gamma \tag{8c}$$

for t = 1, 2, ... and the transversality condition

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[10]That is, as discussed in footnote 5 above, we have tranformed the economy to a stationary one.

$$\lim_{t\to\infty} \beta^{t-1}\lambda_t k_{t+1} = 0. \tag{8d}$$

For the economies studied in this paper, there is a unique dynamic equilibrium, since the outcomes are just those of the standard neoclassical model of capital accumulation[11] subject modified to include some subtraction of resources via government.

### III.1 Computation of General Equilibrium

Quantitative evaluation of this economy requires: specification of the sequence of displacements $\{g_t\}_1^\infty$; specification of the functional forms and parametric values of preferences (u(c, L)) and technology ($\gamma$, $\delta$, and F(k, N)); and computation of solution sequences satisfying (4) – (8).

There are a variety of solution procedures that one could employ to compute this general equilibrium. In this paper, as in Kydland and Prescott [1982] and King, Plosser and Rebelo [1988a,b], the method is to linearly approximate the system near the stationary state.[12] Solving the resulting linear difference equation system, we obtain approximate solutions for percentage deviations from the steady state. These deviations will be denoted by a circumflex in the discussion below (e.g., $\hat{c}_t = (c_t - c)/c \simeq \log(c_t/c)$, where c is the stationary value of consumption).

Table 1 summarizes the parametric assumptions of the simulated economy. The preference specification is assumed to be a standard construct in the real business cycle literature, $u(c,L) = \theta \log(c) + (1-\theta) \log(L)$ with $\theta$ chosen

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[11]That is, it is the model of Solow [1956] as modified by Cass [1965] and Koopmans [1965].

[12]Appendix A provides a brief review.

so that stationary hours are N=.2.[13] This specification is one that has been much used in the real business cycle literature (see e.g., Prescott [1986] and King, Plosser and Rebelo [1988a]). Practically, it implies that there is unit wealth elasticity for both consumption and leisure as well as a unitary elasticity of substitution between these two goods at a particular date t. The production function is assumed to be Cobb–Douglas, i.e., $F(k,N)=k^{(1-\alpha)}N^{\alpha}$ with $0 < \alpha < 1$. The value of $\alpha = .58$ is chosen to equal the average value of labor's share in the post war United States (see King, Plosser and Rebelo [1988a] for a discussion of the details of this measurement).

### III.2 Equilibrium Prices and Wages

The solution yields sequences for the shadow prices $\{\lambda_t\}_1^{\infty}$, as well as for quantities, $\{c_t\}_1^{\infty}$, $\{L_t\}_1^{\infty}$, $\{N_t\}_1^{\infty}$, $\{k_t\}_1^{\infty}$ and $\{i_t\}_1^{\infty}$, with each expressed as a deviation from the stationary state level occasioned by the disturbance. In order to undertake our decomposition, we need the change in lifetime utility (U) and sequences of competitive prices $\{p_t\}_1^{\infty}$ and $\{w_t\}_1^{\infty}$. The lifetime utility variation is simply obtained by using the utility function (1) along with equilibrium changes in consumption and leisure.

We can readily construct $p_t$ and $w_t$ as follows, from the sequences $\{\lambda_t\}_1^{\infty}$, $\{k_t\}_1^{\infty}$ and $\{N_t\}_1^{\infty}$.

$$p_t = \beta^{t-1}\lambda_t \tag{9a}$$

$$w_t = D_2F(k_t,\ N_t) \tag{9b}$$

[13]Average weekly hours per capita are about 23 for the U.S. during the post war period. Taking a conservative view of total usable hours per week as 105, this leads to N = .23/105 = .2.

With these choices for prices and wages as well as $\Lambda = \lambda_1$, it is direct that the consumer's first order conditions will be satisfied at the solution values $\{c_t\}_1^\infty$ and $\{L_t\}_1^\infty$ since these satisfy both (9a,b) and (3a,b). The intertemporal budget constraint is also satisfied when $\Lambda = \lambda_1$, $e_1=[D_1F(k_1,N_1) + 1-\delta]k_1 - g_1$ and $e_t = -g_t$ for all $t \geq 1$.[14]

## IV. Temporary and Permanent Movements in Government Spending

In this section, we provide some initial perspective on an important applied problem in macroeconomics, namely how changes in the time profile of a disturbance—in our case, government spending—influence the character of intertemporal equilibrium outcomes.

### IV.1 Defining Temporary and Permanent Variations

In our analysis, we will consider changes in government spending that are differentiated by timing, specifically distinguishing between those that are strictly temporary—arising only in the initial period—and some that are more permanent. In particular, we will assume that a disturbance is more permanent than another disturbance when it has (i) the same magnitude displacement in an initial period (t=1); and (ii) at least as great an impact in every subsequent time period (t=2,3,...). Some alternatives to this definition are considered in section VI below; however, it is important to point out that this definition does not correspond to that used by Friedman [1957] in his celebrated analysis of consumption.

[14]For a demonstration, see appendix D.

In particular, we will assume that

$$g_t = g + (g_1 - g)\ \rho^{t-1} \tag{10}$$

where $0 < \rho < 1$. Given the our preceding discussion, the parameter $\rho$ can be used to index the extent of permanence in changes in government spending, since a higher value of $\rho$ raises the future path of government spending while holding its current level fixed. Starting from the stationary state in which $p_t = \beta^{t-1}$, the present value consequences of $g_1 \neq g$ are given by

$$\frac{1}{1-\beta\rho}\,(g_1 - g), \tag{11}$$

which is increasing in the parameter $\rho$. That is, when government purchases are more permanent ($\rho$ larger), there is a greater impact on private wealth at steady state prices.

If individuals are made poorer by an increase in the permanence of a disturbance, then it will necessarily be the case that consumption and leisure will fall by more when the disturbance is more permanent. This sort of response is what one would expect from the Friedman [1957] theory of the consumption function. However, with a greater decline in leisure when the disturbance is more permanent, it follows that labor input and commodity output will expand by a greater extent when the disturbance is more permanent.

However, this greater response of output to more permanent disturbances in spending—which translates into greater hours of work with more permanent spending—is opposite to hypotheses advanced by Barro [1981] and Hall [1980].

These authors argue that intertemporal substitution effects arising from adjustments in intertemporal prices (interest rates) will lead to larger effort responses in response to temporary disturbances. Their reasoning is that permanent changes should not exert much of an influence on interest rates and that temporary changes should. Thus, this intertemporal substitution hypothesis motivates our effort to quantitatively measure the relative importance of intertemporal substitution effects and wealth effects in general equilibrium.

In the remainder of this section below, we will study aspects of general equilibrium responses to temporary and permanent displacements. The "comparative dynamics" experiment is summarized in the following two questions. First, how will equilibrium quantities differ when there are alternative displacements from the stationary state? Second, what is the relative importance of substitution and wealth effects in bringing about these alternative paths? We answer the first of these in the next section and the second in section V.

### IV.2. Two Puzzles Arising in The Basic Equilibrium Experiment

Our equilibrium analysis will focus on comparing alternative displacements to private opportunities, indexed by alternative choices of $\rho$, $\rho = 0$ and $\rho = .9$. In conducting these comparisons, we replicate a puzzling result identified by Baxter and King [1988]: in contrast to the predictions of Barro and Hall, there is a larger magnitude output response in general equilibrium to the more permanent (higher $\rho$) government disturbance. We also find a second puzzle, that labor effort is proportionately more responsive than consumption in these basic equilibrium models. Our subsequent analysis then seeks to explain these two puzzling results using the microeconomic methods of *Value and Capital.*

Figure 1 shows the type of disturbances to government spending imposed in our temporary ($\rho$=0) and permanent ($\rho$=.9) cases. Notably, the impact at the initial date t=1 is the same in the two cases and is normalized to be a 3.33 percentage point variation in spending relative to its stationary level. (Since the stationary state level of government spending is .3 this implies that the steady state displacement is 1 percent of stationary output). In a procedure followed throughout the paper, this figure is constructed as follows: on the vertical axis a percent deviation from the stationary state is measured; on the horizontal axis the time unit (a year) is indicated, starting with a displacement at t=1 and continuing for 20 periods.

*Temporary Government Spending:* Figure 2 shows the sequence of quantities—each expressed as a percentage of its stationary state value—for output; consumption; investment; and labor input. The government purchase displacement underlying Figure 2 is one that is purely temporary ($\rho$=0), deviating from the stationary state only in the first of 20 time periods. During this initial period (time 1), output expands—via increased effort, since the capital stock is predetermined—but only by about .23 percent of its steady state value. Given that the disturbance is temporary, it is not surprising that consumption declines by a small amount about .27 percent of its steady state value.[15] Investment declines markedly, falling by –3.09 percent of its stationary value. Labor input is initially about .4 percent

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[15]For those who are used to thinking in "multipliers," the output multiplier $\Delta y/\Delta g$ is .25 in the temporary case. For consumption or investment, one must mulitply our "elasticities" by shares, leading to $\Delta c/\Delta y = (.5)(-.27) = -.13$ and $\Delta i/\Delta g = (.2)(-3.09) = -.60$.

higher than its stationary level, responding more sharply than either consumption or output, but less dramatically than investment.

In the subsequent periods (t=2,...20) shown in Figure 2, the transition path responses of the neoclassical model are dominant. That is, investment is high and consumption (of goods and leisure) is low as the effects of the shortfall in the capital stock are worked off.[16]

*More Permanent Shifts in Purchases*: When departures of government purchases from the stationary state level become more permanent, as in the four panels of Figure 3, there are altered sequences of quantities. Since government purchases are more permanent—leading to a higher total use of resources—it is not surprising that consumption declines by much more than in Figure 1, with the specific value being a fall of .76 percent of its stationary value. From the standpoint of the Barro–Hall view, though, the surprising feature is that output increases by .66 percent, which is over twice the response in the temporary case (.23 percent). Again, since the capital stock is predetermined, this larger output increase reflects a larger increase in work effort, which climbs to 1.14 percent above its stationary level. Another surprising feature is that investment exhibits a modest increase of .18 percent, although this feature depends on the exact value of $\rho$ that is chosen to represent the more permanent case.

The path of output reflects some additional allocation of time to productive activity and a decline in capacity stemming from reduced

[16]See King, Plosser and Rebelo (1988a) for some additional discussion of quantitative transition path responses, in models with fixed labor and with labor chosen by agents. As these authors indicate, the neoclassical mechanisms seem to provide only relatively weak propagation mechanisms for the effects of one time changes in variables such as that considered in this section.

investment. On net, output increases but not enough to offset the influence of increased spending on consumption. Again, a notable feature is that labor input responds more elastically than consumption, both on impact and along the transition path.

*Additional Comparative Dynamics Information*: Figures 4 and 5 display some additional information that is useful in evaluating the results of these experiments. First, two panels provide information on intertemporal price movements, either the forward market price movements which would obtain in initial date markets or in one period interest rates which would prevail in sequential one period credit markets. Second, two panels provide information on movements in the real wage rate and the deviation of leisure from the stationary state. The price movements illustrated in these figures are a key part of the market mechanism that brings about the preceding general equilibrium adjustments and, hence, are relevant background to our decomposition below.

Some useful information can be obtained by comparing Figures 4 and 5. Forward prices rise by a greater degree when the disturbance is more permanent than they do when the disturbance is strictly temporary. On the other hand, the real wage declines by more when the disturbance is more permanent.

The key puzzles that arise from the general equilibrium analysis of this section are as follows. First, why do permanent changes exert larger influences on output than temporary ones? Second, why does labor fluctuate more elastically than consumption? We now use the methods of *Value and Capital* to address these questions.

## V. Understanding the Comparative Dynamics

In developing an understanding of the economic factors leading to the differential comparative dynamics results, we proceed in two stages. First, we begin by thinking about the magnitude of wealth effects on the consumption and leisure decisions of the representative individual at fixed (stationary state) prices. Second, we study the decomposition of solution paths into wealth and substitution components.

### V.1 A Benchmark Analysis of Wealth Effects on Consumption and Leisure

What should one expect to happen if the size of government purchases is increased, reducing the resources available to the private sector? It is useful to begin our quantitative analysis with a discussion based on the permanent income perspective of Friedman [1957] and the concept of full income employed by labor economists (following Robbins [1930], Hicks [1932b], and Becker [1965]). In particular, if we change government spending and maintain all prices (intertemporal prices and the wage rate) at stationary state levels, we can express the intertemporal budget constraint as

$$c + wL \leq e + w - \frac{1-\beta}{1-\beta\rho}(g_1 - g),$$

where e is steady state nonwage income. This expression reorganizes the intertemporal constraint in terms of sustainable flows (annuity values) as in Friedman's work. The right-hand side is full income, i.e., the value of nonwage income plus the value of the endowment of time.

To begin, we must consider how full income differs from more standard constructions. This is easiest to do in the stationary state, where the ratio of full income to standard national income $y^f/y$ may be expressed as

$$\frac{y^f}{y} = \frac{e + w}{y} = \frac{e}{y} + \frac{1}{N}\frac{wN}{y} = [1 - \frac{g}{y} - \frac{i}{y} + \frac{1-N}{N}(\frac{wN}{y})].$$

with N=1-L being the fraction of time devoted to market work.

In deriving this expression, we have made use of the fact that nonwage income in the stationary state is profits less investment (i) and government spending (g), plus the fact that profits are output less wage payments.

A basic conclusion is that full income must be much larger than national income ($y^f/y >> 1$). Labor's share (wN/y) in the U.S. economy is generally estimated as about 2/3; we use the King, Plosser and Rebelo [1988a] estimate of .58. The ratio of leisure time to working time is variously estimated as between 4 and 2. King, Plosser and Rebelo [1988a] provide a stationary hours estimate of N = .2 , which implies (1-N)/N = 4. Kydland and Prescott [1982] use N = 1/3 which implies (1-N)/N = 2. Either of these numbers imply a value of nonmarket time in excess of national product. For the parameter values used in this study, it is more than double (4 x .58 = 2.32). The reason that this fact is of some importance to us is that it provides a benchmark for assessing how much "real income" is lost with a given magnitude increase in government purchases of commodities.

In particular, a given displacement of government spending away from its steady state value will exert an influence on full income that is proportionately much smaller than on national income. This displacement will be

$$- \frac{1-\beta}{1-\beta\rho} (\frac{y}{y^f}) [\frac{g_1-g}{y}].$$

Combining a consumption share (1 - g/y - i/y) = ( 1 - .3 - .21) = .49 and the value of nonmarket time, we find $y/y^f$ = .36.

With this basic information in hand, we can provide some quantitative background which will be useful in understanding the puzzles discussed above. Let us first focus on why labor is apparently more volatile than consumption and then consider issues that bear on the magnitude of wealth effects of government purchase disturbances.

*Consumption versus Leisure*: One reason that leisure is more volatile is that leisure is a bigger share of full expenditure than is consumption. We know from standard demand theory that share weighted income elasticities sum to unity. Thus, suppose that we start with a basic case in which there are unitary wealth (permanent income) elasticities for both consumption and leisure (as implied, for example, by the momentary utility function employed in our simulations). Then, both consumption and leisure respond in the same proportion to the government spending displacement (at steady state prices). But the fact that consumption is a relatively small portion of full income implies that there will be a relatively small expenditure burden on consumption in commodity terms, with most of the adjustment falling on leisure. In our case, the fraction of "consumption crowding out" would be less than a fifth of the change in g, since $(c/y^f) = (c/y)/(y^f/y) = .175$ implies that a one percent displacement to $(g_1 - g)/y$ will lead to at most a .175 percent reduction in consumption. This largest magnitude effect occurs when the change in government spending is fully permanent, i.e., $\rho = 1$ in Table 2. Thus, our steady state analysis suggests that most of the influence of this shock will be concentrated on leisure rather than consumption. But this line of reasoning should not affect <u>proportionate</u> changes in leisure. However, when we looked at Figures 2 and 3, we were concerned with proportionate changes in hours worked rather than leisure, a distinction which is of some importance.

*Leisure versus Work:* An increase in leisure by a given amount must necessarily lower hours worked by the amount. But percentage changes in leisure generally translate into much larger percentage changes in hours worked. Let $\hat{L}$ be the percentage change in leisure and $\hat{N}$ be the percentage change in hours worked. Then, it follows that

$$\hat{N} = -\frac{1-N}{N}\hat{L},$$

so that a one percent change in leisure yields a (minus) four percent change in hours worked when N = .2.

*Permanent versus Temporary*: As Table 2 makes clear, government spending movements that are purely temporary ($\rho$ = 0) have only about one tenth the wealth effect of changes that are relatively permanent ($\rho$ = .9). In our analysis, the time period is a year, so that a value of $\rho$ = .9 implies a half life of about 6.5 years.[17]

To summarize, some basic economic considerations dictate that (i) full income is much larger than national income, implying that most of full expenditure is on labor rather than leisure, and that the burden of wealth effects will thus fall on leisure rather than consumption; (ii) since leisure is larger than market work, changes in leisure give rise to greater than proportionate changes in hours; and (iii) changes in the persistence parameter ($\rho$) can exert order of magnitude effects on wealth at stationary prices.

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[17]It is also useful to note that there is an order of magnitude difference between the effects of $\rho$ = 0 and $\rho$ = .9 or between $\rho$ = .9 and $\rho$ = 1 as originally pointed out by Goodfriend [1987]. Thus, when $\rho$ is large, relatively small changes in $\rho$ have effects that are economically large.

### V.2 Decomposing the Effects of Temporary and Permanent Changes

We now want to use the method of *Value and Capital*—decomposition into wealth and substitution effects—to shed light on the discrepancy between the results of the basic model (in Figures 2 and 3) and the hypothesis advanced by Barro [1981] and Hall [1980]. Figures 6 and 7 show the decomposition of consumption and leisure sequences into a measure of the wealth effect, as well as substitution components attributable to wage and forward price adjustments. One general characteristic of these figures is that wealth and forward price substitutions are the same for consumption and leisure, which is a feature that will arise whenever (i) preferences over time are separable and (ii) momentary utility is such that demands for these two commodities are unit elastic. (See Barro and King [1984] for a discussion of the relationship between wealth and intertemporal substitution effects). However, there are distinct wage effects for consumption and leisure.

*Wealth Effects*. The temporary change in government purchases ($\rho$ = 0, previously illustrated in Figure 2) has a smaller wealth effect than its more permanent counterpart ($\rho$ = .9, previously illustrated in Figure 3) for direct reasons. As in Table 2, there is an order of magnitude larger wealth effect at stationary state prices and that remains true in Figures 6 and 7. When the disturbance is temporary, consumption and leisure each decline, but only by .0199 percent. When the disturbance is more permanent, consumption and leisure decline by a greater amount, .1142 percent. In comparing these general equilibrium wealth effects with those in Table 2, we see that they are uniformly larger. That is, the technology does not permit the full smoothing assumed in Table 2, so that there are larger declines in utility arising from a displacement of a given duration.

*Forward Price Effects*: It turns out that there is a larger component of intertemporal substitution of consumption and leisure induced by changes in intertemporal prices (in early periods, i.e., prior to ten years or so) when government purchases are more permanent ($\rho$ = .9) than when they are strictly temporary. (This reflects the more dramatic movements in near term forward prices displayed in Figure 5 when compared to those in Figure 4). Thus, with the specification of preferences used in this study, the Barro–Hall hypothesis that intertemporal substitution is larger with temporary movements in spending turns out not to be validated.

*Wage Effects*. For consumption (Figure 6), the wage effect is constant through time, which is a special property that derives from momentary utility which is additive and time separable preferences. In both the permanent and temporary cases, this substitution component is negative, since there declines in the wage rate. Once again, though, the magnitude of these declines are larger when the disturbance is more permanent. For consumption, all three components—wealth, wage, and intertemporal substitution components—lead consumption to decline more at date 1 when the disturbance is permanent than when it is strictly temporary.

For leisure (Figure 7), there is a time varying wage effect, which increases leisure in the early stages and lowers it in the latter stages of the time line. This variation in the wage partially offsets the negative effects on leisure arising from wealth and intertemporal substitution components due to consumption. However, leisure is still more responsive when the disturbance is more permanent than when it is strictly temporary.

### V.3 Understanding the Dynamic Equilibrium Puzzles

In the analysis of section IV, we found that (i) permanent disturbances to government purchases have a greater equilibrium effect on output than do temporary displacements; and (ii) labor moves more elastically than consumption in response to disturbances of either duration. To explain the origins of these puzzles, we now draw on the results of sections V.1 and V.2.

To explain the relative volatility of consumption and labor, it is essential to take into account the different starting points from which work and leisure are measured. Along the equilibrium paths developed above, leisure is always less sensitive (proportionately) than consumption. This characteristic arises because there is a wage effect that reinforces the wealth effects and intertemporal substitution effects for consumption, while the wage effect partially offsets the wealth and intertemporal substitution effects on leisure. But, even if the levels of effort and leisure move in an equal and opposite manner, the proportionate changes are quite different, as summarized by the formula $\hat{N} = [(1-N)/N]\ \hat{L}$. The smaller proportionate changes in leisure are translated into larger proportionate changes in effort, since the base points for computing these proportionate changes are quite different.

To explain the differences between permanent and temporary displacements, we find that the implicit assumption used by Barro and Hall—that intertemporal prices will not change very much with highly permanent changes in government purchases—is not borne out in our equilibrium analysis. Thus, more permanent changes have a larger intertemporal substitution components as well as wealth components, which work in reinforcing ways to lead to larger effects on labor input and output.

Thus, we find that our decompositions—along the Hicksian lines of *Value and Capital*—shed new light on the response of a dynamic economic system, i.e., in the comparative dynamics response to alternate changes in a real disturbance. This Hicksian tool should prove valuable in analysis of many other real disturbances in equilibrium macroeconomic models, including the technological shocks considered by Kydland and Prescott [1982] and Long and Plosser [1983]; tax rate changes as in Wynne [1987b] and Baxter and King [1988]; and—in open economy models—changes in the prices of products determined in a world market (Baxter [1988a]).

## VI. Extensions and Concluding Comments

This paper has involved analysis of a particular demand theoretic decomposition; of a particular definition of permanent and temporary components; of a particular class of preferences; of perfect foresight equilibrium; and of nonmonetary systems. Before closing, it seems useful to discuss each of these topics in turn.

*Alternative Demand Decompositions*: In dynamic analysis in labor economics, it is relatively common to use an alternative demand decomposition along Frisch [1932] lines, which treats the lifetime marginal utility parameter $\Lambda$ as the measure of wealth.[18] This alternative decomposition has the attractive advantage of being much easier to compute than the Hicksian decomposition used in the current paper. Mao [1989] has recently employed this decomposition in the context of an equilibrium business cycle model and it would be useful to undertake a systematic comparison of Frisch and Hicks decompositions.

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[18]For some alternative presentations, see Heckman [1974], MaCurdy [1981] and Browning, Deaton and Irish [1985].

*Alternative Experiments*: There are alternative comparative dynamics experiments which would aid us in thinking about the influences of the time profile of displacements. For example, Barro [1981] considers decomposing government spending into permanent and temporary components using definitions along Friedman [1956] lines. For the model at hand, such a decomposition does not alter the substantive conclusion about the relative implications of permanent and temporary components, but it does represent an alternative construction of some potential value. Another possibility would be to consider displacements that have the same wealth effect and, yet, have distinct dynamic patterns, as would be obtained—approximately—by scaling the size of the permanent displacement in Figure 1 by $(1-\beta)/(1-\beta\rho)$. Such experiments provide a potential means of focusing in on specific details of intertemporal and intratemporal substitution.[19]

*Nonseparabilities in Preferences*: Some recent dynamic macroeconomic models have focused on departures from time separable preferences that can be represented in the following form

$$U = \sum_{t=1}^{\infty} \beta^{t-1} u(c_t, L_t^*)$$

where $L^*$ is a leisure commodity that contains some "capitalistic" elements through the specification $L_t^* = \sum_{t=0}^{\infty} a_j L_{t-j}$ (see, for example, Sargent [1979] and Kydland and Prescott [1982]). Since preferences over the more fundamental commodities c and $L^*$ are time separable, it is possible to

[19]The value of this experiment was suggested to me independently by Mark Wynne and Neil Ericsson.

compute both Frisch and Hicks demand decompositions—albeit with some additional complications—in a manner parallel to that for the time separable preference case.

This altered class of preference specifications breaks the strong link between wealth and intertemporal substitution effects that is implied by time separable utility (Barro and King [1984]). However, it is not necessarily the case that this alters the predictions of quantitative neoclassical models about the relative responses to temporary and more permanent displacements. For example, some initial experiments with the specific functional specification and parameter selection chosen by Kydland and Prescott [1982] have maintained the inconsistency of the model with the Barro [1981] and Hall [1980] hypothesis.

*Uncertainty*: Much of the analysis of equilibrium macroeconomics to this point has been conducted in perfect foresight or certainty equivalence form. However, recent work by Greenwood, Hercowitz and Huffman [1988] and Mao [1989] has begun to produce alternate approximations that can be viewed as quantitative versions of the Brock and Mirman [1972] stochastic growth model. Within this context, Mao [1989] analyses demand behavior along Frischian lines, using a contingent commodities framework. But in stochastic systems, it is at present unclear how to best summarize the system responses or to compute the Hicksian decompositions.

*Money*: The latter chapters of *Value and Capital* and other contemporary components of Hicks' work focused on aspects of money and business fluctuations. To date, however, expected inflation wedges—which induce substitutions between real activities based on their monetary requirements—are the only coherent explanation of the real effects of monetary changes within equilibrium models. Such variations in expected inflation do not appear to have a quantitatively important influence on real activity, either within certainty equivalence analyses (Cooley and Hansen [1988]) or explicitly stochastic environments (Baxter [1988b]). An interesting potential application of the methods of this paper is to explore the private substitutions in equilibrium macroeconomic models which arise from inflation wedges.

However, even if the apparent cyclical influence of money originates in other aspects of economic structure, the equilibrium business cycle program is still first order business for macroeconomics. Paraphrasing "Equilibrium and the Cycle", one can only measure disequilibrium with a well specified dynamic equilibrium theory. It is my view that the decomposition methods described in this paper—in the tradition of *Value and Capital*—will assist us in understanding the structure of these dynamic models.

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Table 1

**A. Parameters of Equilibrium Business Cycle Model**

| Parameter | Definition | Value |
|---|---|---|
| $\beta$ | time discount factor | .96 |
| $\theta$ | utility weight | .83* |
| $\alpha$ | labor exponent | .58 |
| $s_g$ | share of government | .30 |
| $\gamma$ | gross growth rate | 1.016 |

**B. Implications for Shares and Ratios in Stationary State**

| Variable | Definition | Value |
|---|---|---|
| $s_c$ | share of consumption | .49 |
| $s_i$ | share of investment | .21 |
| N | average hours percapita | .20 |

* Chosen to make $N = .20$ satisfy $D_2u(c,L) = D_1u(c,L)D_2F(k,N)$.

Table 2:

Implications of Government Purchases
for Sustainable Flows at Steady State Prices:

| $\rho$ | $\hat{y}^f$ | $\Delta c/\Delta g$ | $w\Delta L/\Delta g$ |
|---|---|---|---|
| 0 | −0.0049 | −0.0081 | −0.0379 |
| 0.0500 | −0.0052 | −0.0085 | −0.0398 |
| 0.1000 | −0.0054 | −0.0089 | −0.0419 |
| 0.1500 | −0.0057 | −0.0094 | −0.0443 |
| 0.2000 | −0.0061 | −0.0100 | −0.0469 |
| 0.2500 | −0.0064 | −0.0106 | −0.0498 |
| 0.3000 | −0.0069 | −0.0113 | −0.0532 |
| 0.3500 | −0.0074 | −0.0121 | −0.0570 |
| 0.4000 | −0.0079 | −0.0130 | −0.0614 |
| 0.4500 | −0.0086 | −0.0141 | −0.0665 |
| 0.5000 | −0.0094 | −0.0154 | −0.0725 |
| 0.5500 | −0.0103 | −0.0170 | −0.0798 |
| 0.6000 | −0.0115 | −0.0189 | −0.0887 |
| 0.6500 | −0.0129 | −0.0212 | −0.0999 |
| 0.7000 | −0.0148 | −0.0243 | −0.1142 |
| 0.7500 | −0.0172 | −0.0284 | −0.1334 |
| 0.8000 | −0.0207 | −0.0341 | −0.1602 |
| 0.8500 | −0.0259 | −0.0427 | −0.2006 |
| 0.9000 | −0.0347 | −0.0571 | −0.2683 |
| 0.9500 | −0.0524 | −0.0861 | −0.4049 |
| 1.0000 | −0.1066 | −0.1753 | −0.8247 |

$\rho$ : parameter governing permanence of government purchase disturbance

$\hat{y}^f$: proportionate change in full income (equivalent to $\hat{c}$, the proportionate change in consumption, or $\hat{L}$, the proportionate change in leisure)

$\Delta c/\Delta g$: unit change in consumption (permanent) due to unit change in g

$w\Delta L/\Delta g$: value change in leisure (permanent) due to unit change in g

Figure 1

**Temporary and Permanent Displacements to Government Purchases**

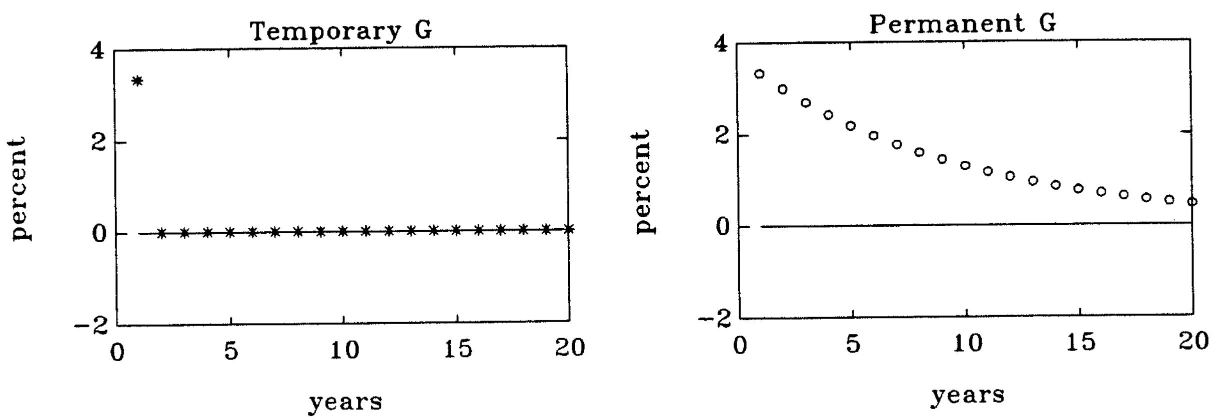


Figure 2

**Dynamic Responses to Temporary Government Purchases**

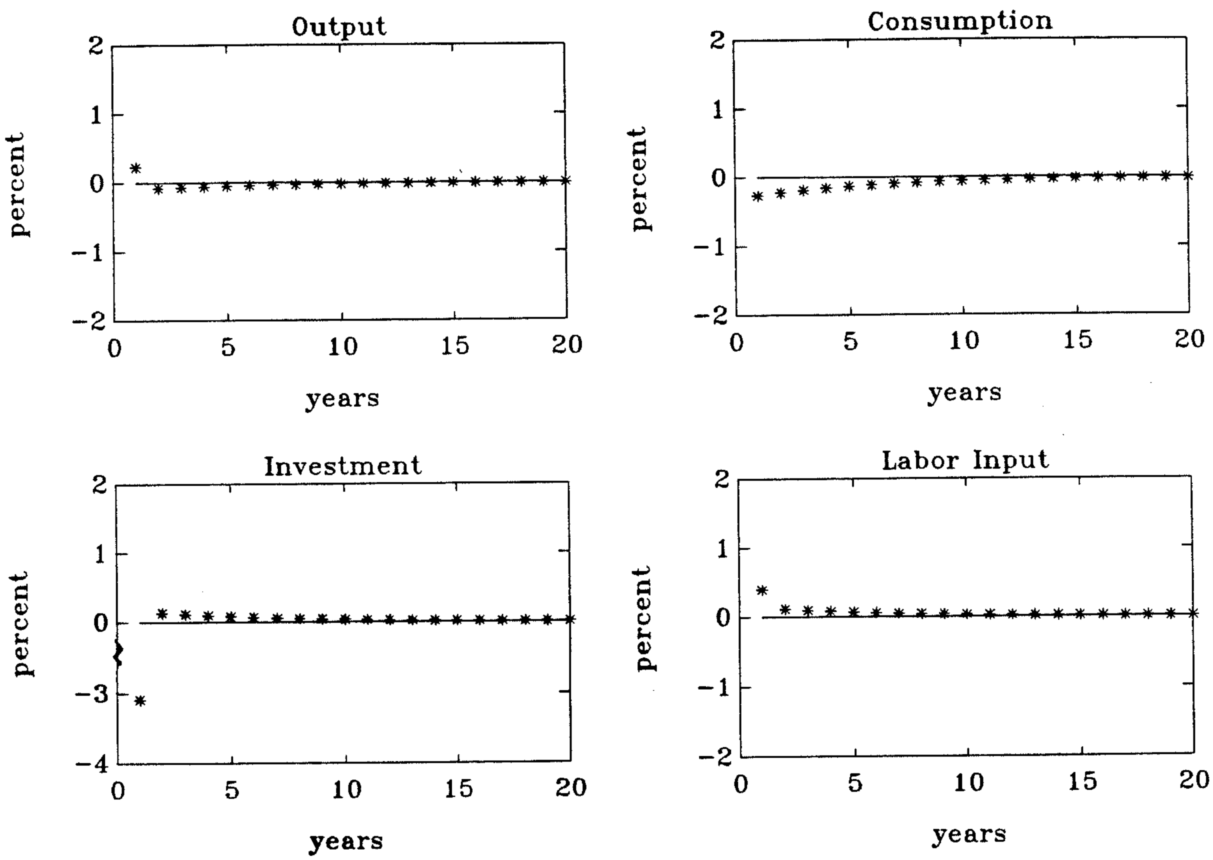


Figure 3

**Dynamic Responses to Permanent Government Purchases**

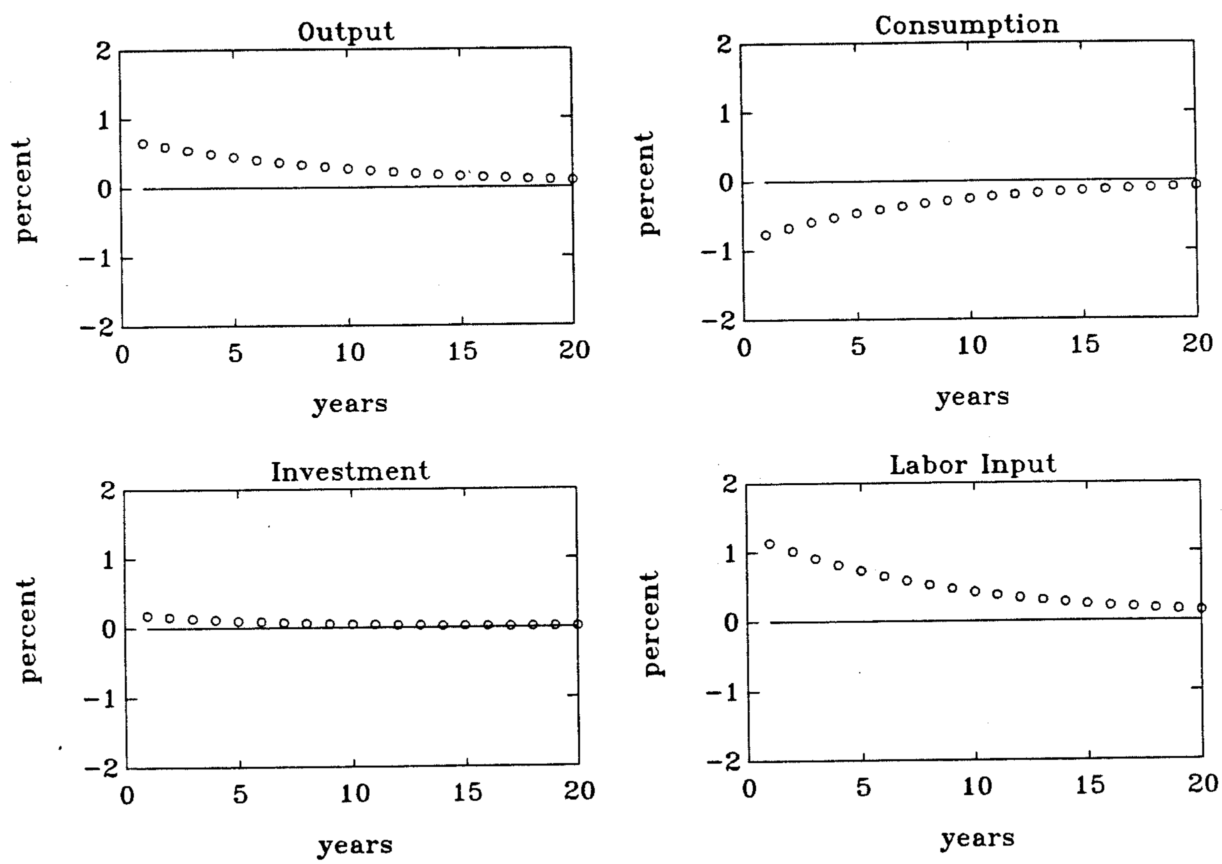


Figure 4

**Additional Implications of Temporary Government Purchases**

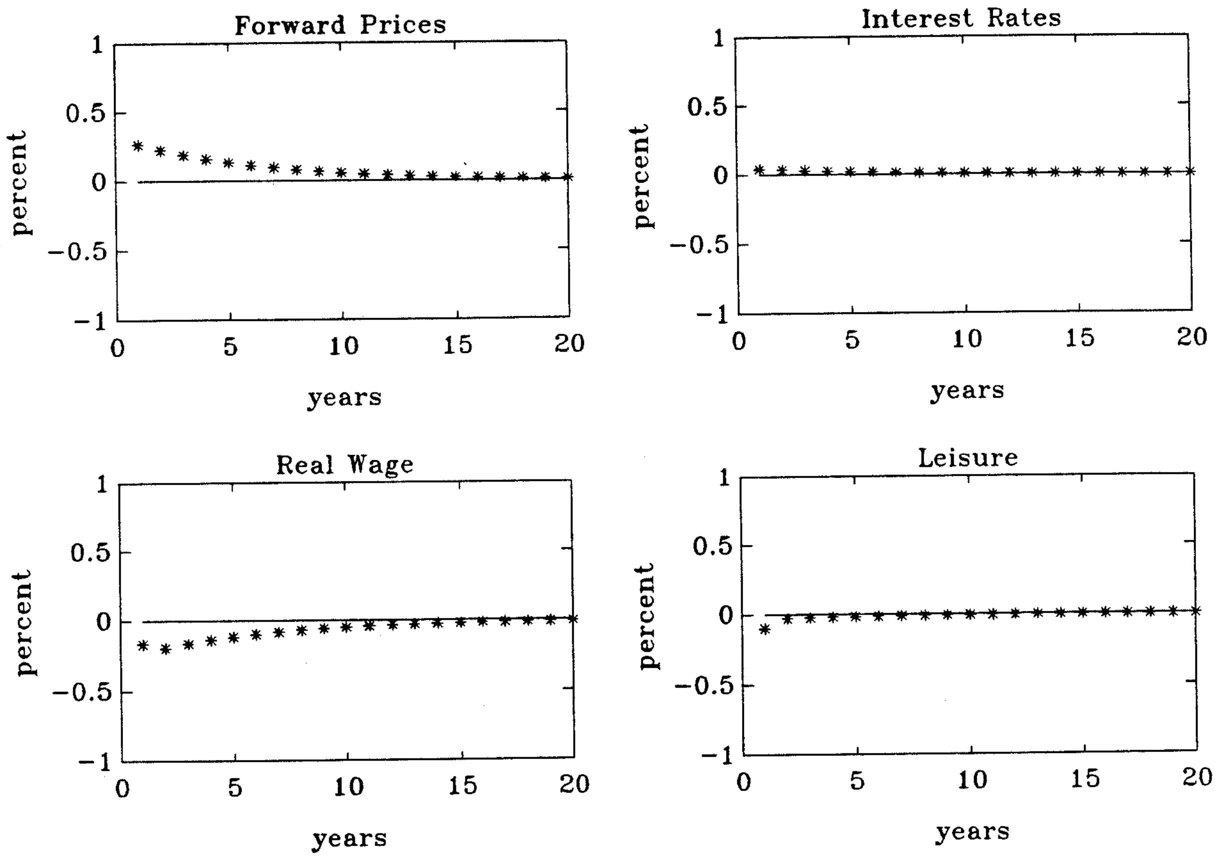


Figure 5

**Additional Implications of Permanent Government Purchases**

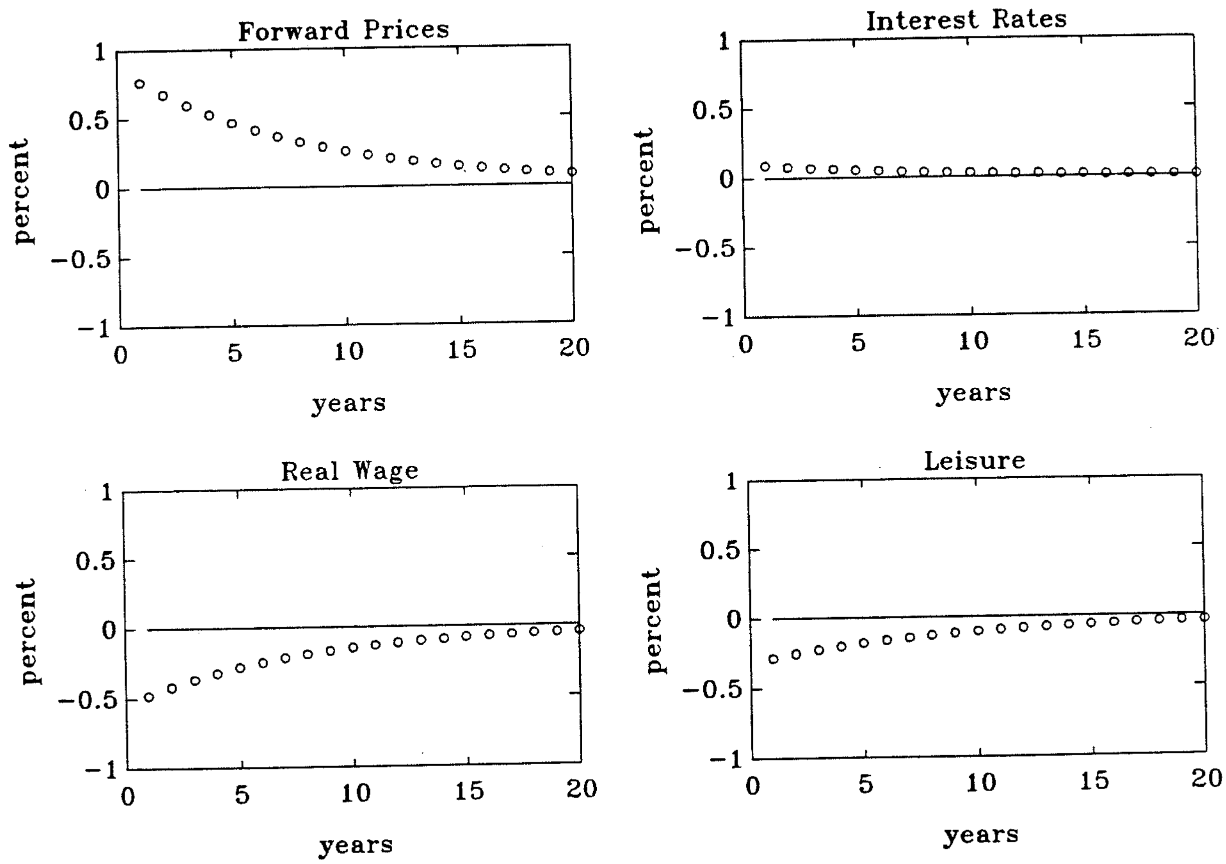
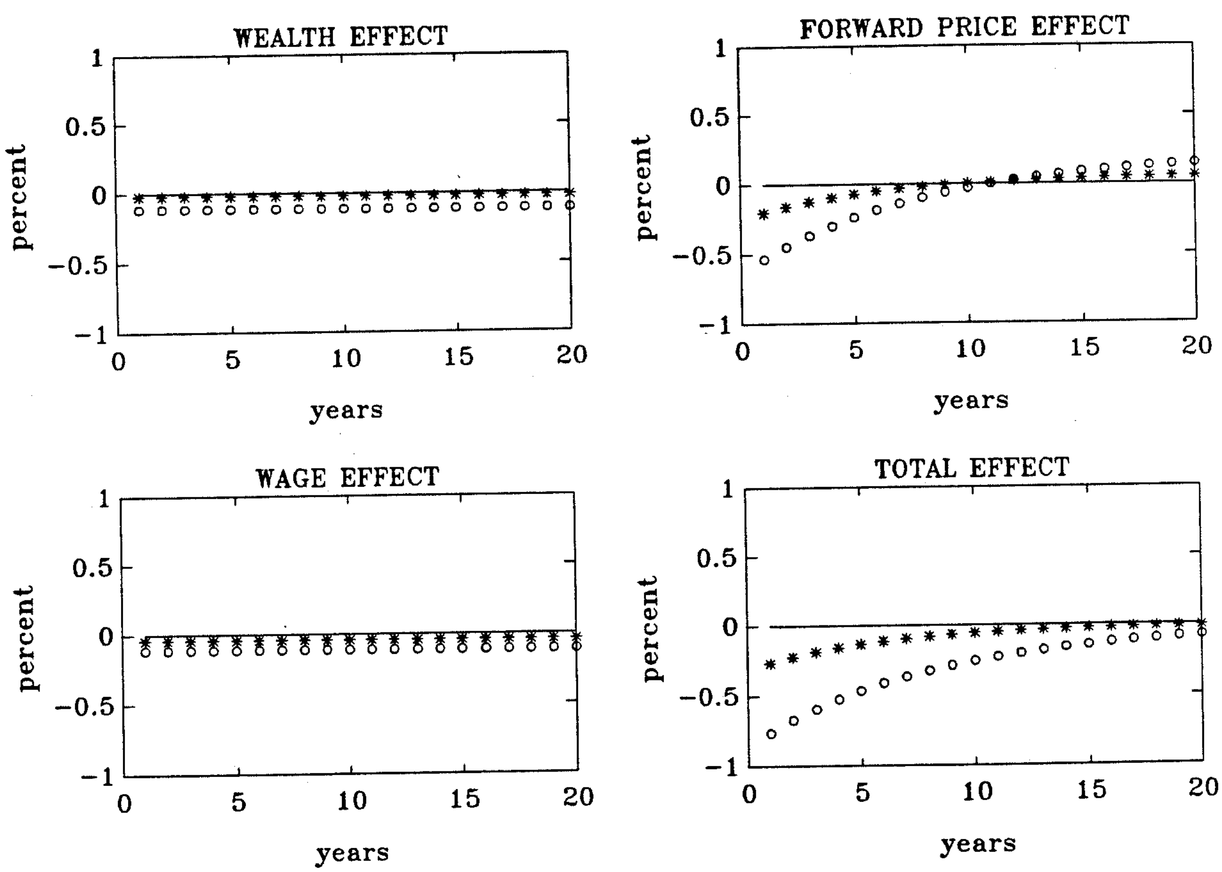


Figure 6

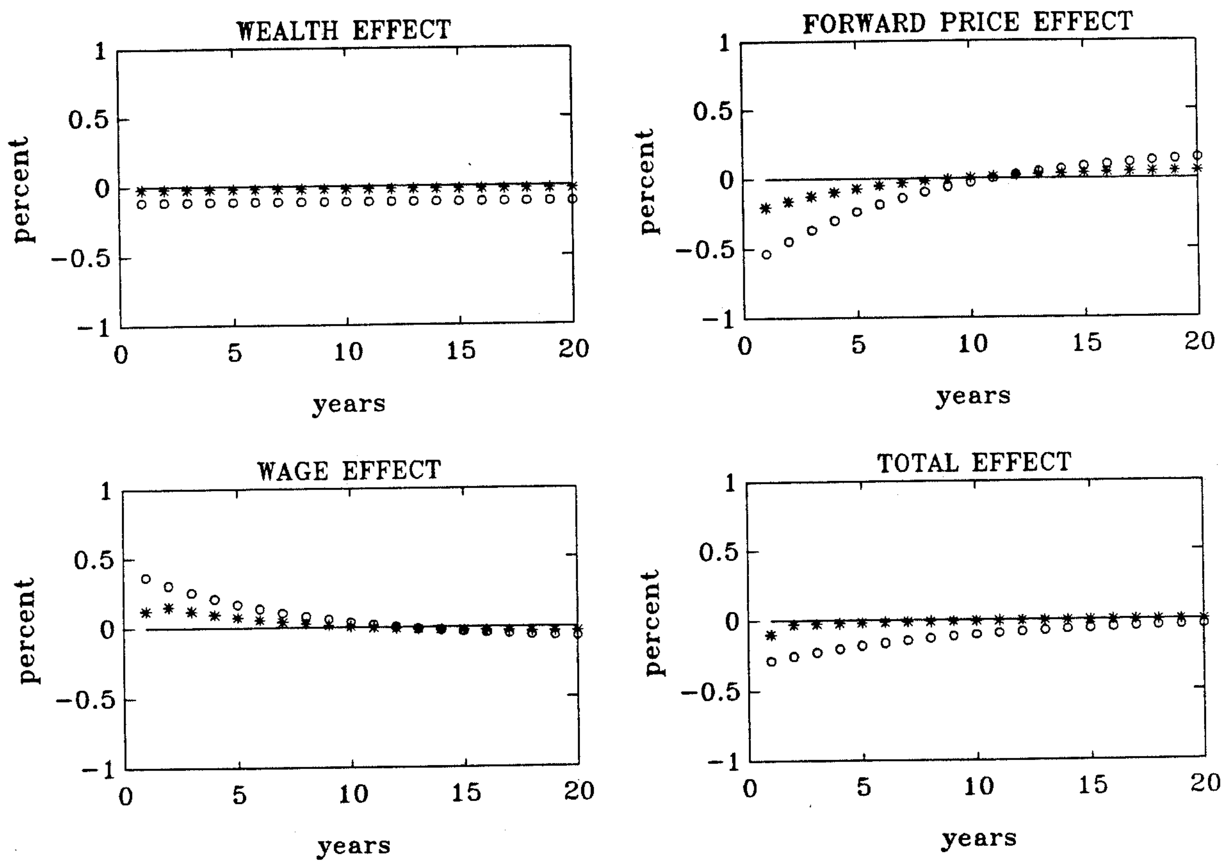
**Decomposition of Consumption**




* path is response to temporary government purchases
o path is response to permanent government purchases

Figure 7

**Decomposition of Leisure**




* path is response to temporary government purchases
o path is response to permanent government purchases

**Overview of Appendix Material:**

The objective of these appendices is to provide background on the computation of results reported in the text. The general approach is to take a log linear approximation near the stationary point of the system.

This approach is used in appendix A, which discusses the computation of general equilibrium following the methods outlined in King, Plosser and Rebelo [1987]. It is used in appendix B, which discusses the Hicksian decomposition of effects on demand as in conventional micro texts. Appendix C explains why the first order conditions outlined in the text and approximated in appendix A continue to hold in the presence of government purchases. Appendix D provides an outline of the method employed for calculating the discounted sums that appear in the decompositions.

There is one minor notational difference from the text—time is assumed to start at 0 rather than at one.

## Appendix A:

## Approximation Near a Stationary Point

This appendix briefly describes the computational strategy employed in approximating the dynamic general equilibrium, detailed in King, Plosser and Rebelo [1987]. The following equations restrict the dynamic behavior of the system:

(A–1a) $D_1u(c_t, L_t) = \lambda_t$

(A–1b) $D_2u(c_t, L_t) = \lambda_t D_2F(k_t, N_t)$

(A–1c) $N_t + L_t = 1$

(A–1d) $\gamma\, k_{t+1} = F(k_t, N_t) + (1-\delta)k_t - c_t - g_t$

(A–1e) $\beta\, \lambda_{t+1}[D_1F(k_{t+1}, N_{t+1}) + 1-\delta] = \lambda_t\gamma$

along with the transversality condition $\lim_{T\to\infty} \beta^T\lambda_T k_{T+1} = 0$.

Adopting a "hat calculus" notation for proportionate deviations, i.e., $\hat{z}_t = (z_t-z)/z \simeq \log(z_t/z)$, approximation of the efficiency conditions (A–1a) and

(A–1b) around the stationary point yields the following

$$\text{(A–2a)} \qquad \xi_{cc}\,\hat{c}_t + \xi_{cL}\,\hat{L}_t = \hat{\lambda}_t$$

$$\text{(A–2b)} \qquad \xi_{Lc}\,\hat{c}_t + \xi_{LL}\,\hat{L}_t = \hat{\lambda}_t + \xi_{NK}\,\hat{K}_t + \xi_{KN}\,\hat{N}_t.$$

Approximation of the constraints (A–1c) and (A–1d) yields for the following

$$\text{(A–2c)} \qquad N\,\hat{N}_t + L\,\hat{L}_t = 0$$

$$\text{(A–2d)} \qquad s_K\,\hat{K}_t + s_N\,\hat{N}_t = s_c\,\hat{c}_t + s_g\,\hat{g}_t + s_i\,\phi\,(\hat{k}_{t+1}) + s_i(\phi-1)\,\hat{k}_t$$

Approximation of the efficiency condition for accumulation of capital yields

$$\text{(A–2e)} \qquad \eta_K\,\hat{k}_{t+1} + \eta_N\,\hat{N}_{t+1} + \hat{\lambda}_{t+1} = \hat{\lambda}_t.$$

In these expressions, the elasticities $(\xi,\eta)$ and ratios $(s_i,\ \phi,\ N,\ L)$ are defined as follows. First, $\xi_{cL}$, $\xi_{cc}$, $\xi_{Lc}$, and $\xi_{LL}$ are the elasticities of marginal utility—for example, $\xi_{cL}$ indicates the elasticity of the marginal utility of consumption with respect to leisure $(LD_2D_1u\ (c,L)/D_1u(c,\ L))$. Similarly, $\xi_{KN}$ and $\xi_{NN}$ are the elasticities of the marginal product of labor with respect to capital and labor, respectively. The elasticities of the gross marginal product of capital $(D_1F(k,N) + 1-\delta)$ with respect to its arguments are denoted $\eta_K$ and $\eta_N$. Second, ratios are indicated in natural notation. Ratios to national income/product are capital's share $(s_K)$ and labors's share $(s_N)$. Similarly, we define consumption $(s_c = c/y)$, government $s_g = (g/y)$ and investment $s_i = (i/y)$ shares on the expenditure side. N is the fraction of stationary state time spent in market work, $L = 1-N$ is the fraction spent in leisure. Finally, $\phi$ is the steady state ratio of $k_{t+1}/y_t$.

Specification of values for $N, \beta, \delta, s_K$ turns out to be sufficient to determine the other ratios, i.e. $s_c$, $s_i$ and $\phi$. Adding in substitution information (on the local elasticity of substitution of F between k and N as well as on the preference side) yields the $\xi$ and $\eta$ elasticities.

The expressions can be combined to produce a first order linear difference equation in $(\hat{\lambda}, \hat{k})$ that can be readily solved with one boundary condition being provided by $\hat{k}_0$ and the other by the transversality condition. Ultimately, as discussed in King, Plosser and Rebelo [1987], the outcomes may be expressed as a reduced form dynamic system of the form:

$$z_t = \pi \; s_t$$

$$s_{t+1} = M \; s_t$$

where $z_t$ is a vector that contains the deviations of the "flow variables" from stationary state values, $z_t = [\hat{c}_t \; \hat{L}_t \; \hat{N}_t \; \hat{w}_t \; \hat{\lambda}_t]'$, and $s_t$ is a vector of "state variable," $z_t = [\hat{k}_t \; \hat{g}_t]'$.

<u>Calculation of Utility Equivalents</u>

Given the sequences $\{\hat{c}_t\}_{t=0}^{\infty}$ and $\{\hat{L}_t\}_{t=0}^{\infty}$, our analysis requires us to compute those changes in consumption and effort which, when maintained over all periods, would yield the same deviation from stationary utility as that implied by $\{\hat{c}_t\}_{t=0}^{\infty}$ and $\{\hat{L}_t\}_{t=0}^{\infty}$.

With c and L constant over time, $U = \frac{1}{1-\beta} u(c,L)$, so

$$\hat{U} = \frac{(D_1 u)y}{u} \left[ (\frac{c}{y}) \hat{c} + \frac{wL}{y} \hat{L} \right]$$

$$= \frac{(D_1 u)y}{u} \left[ s_c \hat{c} + s_N (\frac{1-N}{N}) \hat{L} \right].$$

The utility perturbation attributable to deviations from the stationary state is

$$\hat{U} = \frac{(D_1 u)y}{u} (1-\beta) \sum_{t=0}^{\infty} \beta^t \left[ s_c \hat{c}_t + s_N (\frac{1-N}{N}) \hat{L}_t \right].$$

Equating these expressions yields one linear restriction on $\hat{c}$ and $\hat{N}$.

A second restriction derives from the requirement that the marginal rate of substitution equal the real wage rate,

$$D_2 u(c,L) = D_1 u(c,L) w,$$

which implies the following condition:

$$(\xi_{Lc} - \xi_{cc}) \hat{c} + (\xi_{LL} - \xi_{cL}) \hat{L} = 0.$$

Thus, calculation of utility equivalents may proceed with no additional information beyond that specified for computation of general equilibrium.

## Appendix B:

## Hicksian Wealth and Substitution Effects

This appendix discusses the calculation of wealth and substitution effects under the assumption that utility is time separable. The analysis builds on Barro and King [1984]. Throughout, we work with finite T and interpret infinite horizon results as those obtained when we drive T toward infinity.

We assume that individuals preferences for goods over time have the form

$$U = \sum_{t=0}^{T} \beta^t u(x_t)$$

where $x_t$ is a column vector (with $x_t = [c_t, L_t]'$ in the current application).

The intertemporal budget constraint takes the form

$$\sum_{t=0}^{T} p_t x_t = \sum_{t=0}^{T} p_t e_t,$$

where $p_t$ is a row vector of present value prices of goods of the same dimension of $x_t$; $e_t$ is an endowment vector at date t; and I is an initial wealth term.

Comparative statics results can be obtained by defining the complete decision vector $X' = [x_0', x_1', \ldots x_T']$, the complete price vector $P = [p_0, p_1, \ldots p_T]$ and the full endowment vector, $E' = [e_0', e_1', \ldots e_T']$. Then, the decision problem in standard form is

$$\max_{X} U(X), \text{ subject to } PX \leq I + PE,$$

where there are a total of $M = m(T+1)$ actions if there are m elements in x. Let H be the M x M Hessian matrix of the second partials of U. Then standard comparative statics results—see e.g., Intriligator [1971, Ch. VII]—lay out the effects of income changes on demand and the effects of compensated price changes on demand as

$$\frac{\partial X}{\partial I} = -\ \mu\ H^{-1}\ P'$$

$$\frac{\partial X}{\partial P}\Big|_{U} = \mu H^{-1}\ P'P\ H^{-1}\ \Lambda\ +\ \Lambda\ H^{-1}$$

where $\Lambda$ is the Lagrange multiplier attached to the constraint $I + PE - PX \geq 0$ and $\mu = -\ [P\ H^{-1}\ P']^{-1}$. Economically, as discussed in the text, $\Lambda$ corresponds to the lifetime marginal utility of wealth and $\mu = -\ \partial\Lambda/\partial I$. The total effect of price changes is given by

$$\frac{\partial X}{\partial P} = \frac{\partial X}{\partial P}\Big|_{U} - \frac{\partial X}{\partial I}(X-E)'.$$

It is computationally advantageous to note that the derivatives of the $\Lambda$ constant (Frisch) demands are

$$\frac{\partial X}{\partial P}\Big|_{\Lambda} = H^{-1}\ \Lambda$$

and, thus, one can write

$$\frac{\partial X}{\partial P}\Big|_U = \left(\frac{\Lambda}{\mu}\right) \left(\frac{\partial X}{\partial I}\right)\left(\frac{\partial X}{\partial I}\right)' + \frac{\partial X}{\partial P}\Big|_\Lambda .$$

This decomposition is important as it splits the utility compensated price effect into (a) into general a substitution effect—linked to $\partial X/\partial I$—and (b) to the specific substitution terms stressed in the Frischian analysis.

Implications of Time Separability

With utility time separable, the Hessian matrix H is block diagonal. The terms on the diagonal are $\beta^t h_t$, where $h_t$ is the Hessian matrix of the momentary utility u evaluated at date t quantities. Barro and King [1984] show that this implies important restrictions on demand behavior, which are translated into our terms as follows.

$$\frac{\partial x_t}{\partial I} = \mu\, \beta^{-t}\, h_t^{-1}\, p_t'$$

$$\frac{\partial x_t}{\partial p_t}\Big|_U = \left(\frac{\Lambda}{\mu}\right) \left(\frac{\partial x_t}{\partial I}\right) \left(\frac{\partial x_t}{\partial I}\right)' + \frac{\partial x_t}{\partial p_t}\Big|_\Lambda$$

$$\frac{\partial x_t}{\partial p_s}\Big|_U = \left(\frac{\Lambda}{\mu}\right) \left(\frac{\partial x_t}{\partial I}\right) \left(\frac{\partial x_s}{\partial I}\right)' \qquad \text{for } t \neq s.$$

where $\mu^{-1} = \left[\sum_{t=0}^{T} \beta^{-t}\, p_t\, h_t^{-1}\, p_t'\right]$ and $\frac{\partial x_t}{\partial p_t}\Big|_\Lambda = \beta^{-t}\, \Lambda\, h_t^{-1}$. In these expressions, all of the prices remain in present value form.

Evaluation Near A Stationary State

We will be evaluating changes in prices and wealth near a stationary point. For a stationary state in which $x_t = x$ for all t to obtain, it must be the case that intertemporal price vectors differ only by discounting, $p_t = \beta^t p$, where p is a column vector of (relative) prices. In our two good case, then, this implies $p_t = \beta^t p$ with $p = [1 \;\; w]$. Then, the preceding expressions can be written as

$$\frac{\partial x_t}{\partial I} = \mu\, h^{-1}\, p \equiv \frac{\partial x}{\partial I}$$

$$\frac{\partial x_t}{\partial p_t}\Big|_U = \frac{\Lambda}{\mu} \left(\frac{\partial x}{\partial I}\right) \left(\frac{\partial x}{\partial I}\right)' + \frac{\partial x_t}{\partial p_t}\Big|_\Lambda$$

$$\frac{\partial x_t}{\partial p_s}\Big|_U = \frac{\Lambda}{\mu} \left(\frac{\partial x}{\partial I}\right) \left(\frac{\partial x}{\partial I}\right)' \qquad \text{for s not equal to t}$$

In these expressions, $\mu^{-1} = B(\beta,T)p\, H^{-1}\, p'$ and $\frac{\partial x_t}{\partial p_t}\Big|_\Lambda = \beta^{-t}\, \Lambda\, h^{-1}$ for all t, where $B(\beta,T)$ is a T+1 period annuity factor, $B(\beta,T) = \left[\frac{1-\beta^{T+1}}{1-\beta}\right]$, in these expressions and below.

Evaluating The Effects of Price Changes

Our interest is in evaluating the effects of a sequence of price changes $\{dp_s\}_{s=0}^{T}$ on demands at date t. Using the preceding results, these are simply

$$\sum_{s=0}^{T} \frac{\partial x_t}{\partial p_s}\Big|_U dp_s' = \frac{\Lambda}{\mu} \left(\frac{\partial x}{\partial I}\right) \left(\frac{\partial x}{\partial I}\right)' \sum_{s=0}^{T} dp_s' + \left[ \frac{\partial x_t}{\partial p_t}\Big|_\Lambda dp_t' \right] .$$

The income effects of price changes on demands at t are just

$$-\frac{\partial X}{\partial I} = \sum_{s=0}^{T} (x_s - z_s)\, dp'_s \; .$$

<u>Conversion to Elasticities</u>

The results reported in the text involve expressions for elasticities near a stationary point. To convert to elasticities, it is convenient to define matrices with stationary quantities on the diagonal. In the current case, these are,

$$Q_p = \begin{bmatrix} 1 & 0 \\ 0 & w \end{bmatrix}$$

$$Q_x = \begin{bmatrix} c & 0 \\ 0 & L \end{bmatrix} .$$

In developing expressions below, it is useful to note that the derivatives of price changes involve discounting, i.e., are proportional to $\beta^{-t}$. On the other hand, proportionate price changes do not involve the effects of discounting.

Defining the vector of proportionate price changes near the stationary point, $\hat{p}_s = [\hat{p}_{s1}\ \hat{p}_{s2}\ \dots\ \hat{p}_{sm}]$, we can then express the full substitution influence of a sequence of price changes as follows. First, from our general expression above, we know that elasticities must satisfy

$$Q_x^{-1} \sum_{s=0}^{T} \frac{\partial x_t}{\partial p_s}\Big|_U Q_p\, Q_p^{-1}\, dp_s' = \sum_{s=0}^{T} \left[ Q_x^{-1} \frac{\partial x_t}{\partial p_s}\Big|_U Q_p\, \beta^s \right] \hat{p}_s'$$

$$= \frac{\Lambda}{\mu I} \left[ Q_x^{-1} \frac{\partial x}{\partial I}\, I \right] \left[ Q_x^{-1} \frac{\partial x}{\partial I}\, I \right]' \frac{Q_x\, Q_p}{I} \sum_{s=0}^{T} \beta^s\, \hat{p}_s'$$

$$+ \left[ Q_x^{-1} \frac{\partial x_t}{\partial p_t}\Big|_\Lambda Q_p\, \beta^t \right] \hat{p}_t'$$

where $[Q_x^{-1} \frac{\partial x}{\partial I} I]$ is a vector of income elasticities; $\frac{Q_x Q_p}{I}$ is a matrix with shares on its diagonal; and $[Q_x^{-1} \frac{\partial x_t}{\partial p_t}|_\Lambda\, Q_p\ \beta^t]$ is just the matrix of Frisch demand elasticities discussed earlier. Now, in terms of the economics of the problem, it makes sense to measure expenditure shares relative to the annuity value of income, represented as $I/B(\beta,T)$ so that the final version of our substitution expression becomes

$$(1/B(\beta,T))\ \frac{\Lambda}{\mu I} \left[ Q_x^{-1} \frac{\partial x}{\partial I}\, I \right] \left[ Q_x^{-1} \frac{\partial x}{\partial I}\, I \right]' Q_s \sum_{s=0}^{T} \beta^s\, \hat{p}_s'$$

$$+ \left[ Q_x^{-1} \frac{\partial x_t}{\partial p_t}\Big|_\Lambda Q_p\, \beta^t \right] \hat{p}$$

with $Q_s$ being a diagonal matrix with expenditure shares on the diagonal $Q_s = Q_x Q_p B(\beta,T)/I$.

<u>Implicit Specification of Preferences</u>

The results to this point use information on the nature of preferences near the stationary point—the Hessian matrix h and the level of $\Lambda$—as well as the global parameter $\beta$. By contrast, the approximation strategy outlined in appendix A above involves implicit specification of preferences, by which we mean that we proceed by specifying a near stationary state matrix of elasticities of marginal utility, potentially subject to some restrictions as discussed in King, Plosser and Rebelo [1987]. Let $\xi_{jk} = x_k D_k D_j u / D_j u$. We know that the first order conditions take the form $D_j u = \Lambda\, p_j/\beta^j$. Thus, it follows that the matrix of elasticities of marginal utilities,

$$\Xi = \begin{bmatrix} \xi_{11} & \xi_{12} \;\cdots & \xi_{1m} \\ \xi_{21} & \xi_{22} \;\cdots & \xi_{2m} \\ \xi_{m1} & \xi_{m2} \;\cdots & \xi_{mm} \end{bmatrix}$$

can be expressed as

$$\Xi \quad = \quad Q_p^{-1}\, h\, Q_x \,/\, \Lambda.$$

or $h = \Lambda\, Q_p\, \Xi\, Q_x^{-1}$.

From the first order conditions, it is direct to derive the result that

$$\Xi\ \nu = \left[ \frac{d\Lambda}{dI}\ \frac{I}{\Lambda} \right] \underline{1}$$

where $\nu = (\nu_1, \ldots, \nu_m)$ is a vector of m income elasticities ($\nu_i = \frac{\partial x_i}{\partial I} \frac{I}{x_i}$) and $\underline{1}$ is a column vector of m ones.

With this fact, it is then direct that $\nu = [\ \frac{d\Lambda}{dI}\ \frac{I}{\Lambda}\ ]\ \Xi^{-1}\ \underline{1}$, so that specification of $\Xi$ specifies income elasticities up to a scale factor. In turn, this scale factor can be determined by looking a stationary state income shares, since $[\sum_j p_j x_j] B(\beta,T) = I$ implies

$$\sum_{j=1}^{N} \left[ \frac{p_j x_j}{I} B(\beta,T) \right] \nu_i = \underline{1}' Q_s \nu = 1,$$

where $p_i x_j B(\beta,T)/I$ is the flow expenditure share of $x_j$ discussed above. As a consequence, information on steady state shares permits computation of $(d\Lambda/dI)\ (I/\Lambda) = -\mu I/\Lambda\ = 1/(\ \underline{1}' Q_s\ \Xi^{-1}\ \underline{1})$.

The bottom line of this analysis is that we can directly compute the substitution expression from information on shares ($Q_s$) and on elasticities ($\Xi$). The operational version of our substitution expression then is

$$(1/B(\beta,T))\ (-\ \frac{\Lambda}{\mu I} \left[ Q_x^{-1} \frac{\partial x}{\partial I} I \right] \left[ Q_x^{-1} \frac{\partial x}{\partial I} I \right]' Q_s \sum_{s=0}^{T} \beta^s \hat{p}_s'$$

$$+ \left[ Q_x^{-1} \frac{\partial x_t}{\partial p_t} \Big|_{\Lambda} Q_p\ \beta^t \right] \hat{p}$$

$$= (1/B(\beta,T))\ (-\underline{1}'Q_s\ \Xi^{-1}\ \underline{1})\ [\nu][\nu]'\ Q_s \sum_{s=0}^{T} \beta^s\ \hat{p}_s'$$

$$+\ [\Xi^{-1}\ \beta^t]\ \hat{p}$$

with $\nu = [\ \frac{d\Lambda}{dI}\ \frac{I}{\Lambda}\ ]\ \ \Xi^{-1}\ \underline{1} = \Xi^{-1}\ \underline{1}/(\ \underline{1}'Q_s\ \Xi^{-1}\ \underline{1})$.

Notice that the general substitution effect component of these expressions $(1/B(\beta,T))\ (-\underline{1}'Q_s\ \Xi^{-1}\ \underline{1})\ [\nu][\nu]'\ Q_s \sum_{s=0}^{T} \beta^s\ \hat{p}_s'$ is the same for each date t and the specific substitution effect for date t involves only date t price changes. Our computations exploit these features.

Computation of Income Elasticities

Following the general outline above, computation of income elasticities and $\mu/(\Lambda/I)$ proceeds as follows in the specific case studied in this paper. First, we form

$$\Xi = \begin{bmatrix} \xi_{cc} & \xi_{cL} \\ \xi_{Lc} & \xi_{LL} \end{bmatrix}$$

Then, $\Xi^{-1}\begin{bmatrix}1\\1\end{bmatrix}$ provides income elasticities up to the scalar $-\mu/(\Lambda/I)$. That scalar is provided by the requirement that share weighted elasticities sum to unity. The relevant shares are those in full income

$$c\ /\ [(y-i-g-wN)\ +\ w] \qquad\qquad wL/[(y-i-g-wN)\ +\ w]$$

or, with a little algebra, as

$$s_c/[1-s_i-s_g+\frac{1-N}{N}s_N] \qquad \text{and} \qquad \frac{1-N}{N}s_N/[1-s_i-s_g+\frac{1-N}{N}s_N]$$

so that no new parameters are required. Denoting the vector q as the vector of these shares, it follows that

$$-\frac{\mu}{(\Lambda/I)} = [q\Xi^{-1}1]^{-1}$$

and the income elasticities are thus

$$\begin{bmatrix} \frac{\partial c}{\partial I}/\frac{I}{c} \\ \\ \frac{\partial L}{\partial I}/\frac{I}{L} \end{bmatrix} = -\ [q\Xi^{-1}1]^{-1}\Xi^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

<u>Treatment of Income Effects of Price Changes in General Equilibrium</u>

Income effects of price changes may be expressed as

$$-\left[ Q_x^{-1} \frac{\partial x_t}{\partial I} I \right] \sum_{s=0}^{T} \left[ \frac{(x_s - e_s)\ Q_p\ \beta^s}{I} \right] \hat{p}_s'$$

Since we are studying a representative agent general equilibrium, we will ignore these effects, since $x_s = e_s$ element by element for all s. Standard arguments about market clearing imply that this is legitimate for changes in $\hat{p}_s$ that are proportional for all elements of the vector $p_s$—such as the effects of forward market prices (interest rates). In order to rationalize this assumption for relative price movements at date s, we must bring in the effects that these factor price movements have on profits, as discussed by Bailey [1971, pp. 106–108]. In particular by assuming that a wage rate change has no income effect—given production possibilities—we are using the fact that profits are just output net of payments to labor.

## Appendix C:

## The Intertemporal Budget Constraint

The sequence of resource constraints may be rewritten as

$$c_t + i_t + w_t L_t = w_t - g_t + [D_1 F(k_t, N_t)] k_t$$

using text equation (14b) and linear homogeneity of the production function. But since $\gamma k_{t+1} = i_t + (1-\delta) k_t$, it follows that

$$c_t + w_t L_t = w_t - g_t + [D_1 F(k_t, N_t) + 1-\delta] k_t - \gamma k_{t+1}$$

Multiplying through by $p_t = \beta^t \lambda_t / \lambda_0$, and summing over an arbitrary finite horizon, we obtain

$$\sum_{t=0}^{T} p_t [c_t + w_t L_t]$$

$$= \sum_{t=0}^{T} p_t [w_t - g_t]$$

$$+ \sum_{t=0}^{T} p_t [D_1 F(k_t, N_t) k_t + (1-\delta) k_t - \gamma k_{t+1}]$$

Using (13c), this final summation reduces to

$$[D_1F(k_0,\ N_0) + 1-\delta]k_0 + \frac{1}{\lambda_0}\ [\beta^T\lambda_T k_{T+1}]$$

The transversality condition (13d) insures that $\lim_{T\to\infty} \beta^T\lambda_T k_{T+1} = 0$ so that the intertemporal budget constraint just takes the form of (2) in the main text, with $e_0 = [D_1F(k_0,N_0) + 1-\delta]k_0 - g_0$ and $e_t = -g_t$ for all $t > 0$. Thus, our choice of $\Lambda = \lambda_0$ satisfies the intertemporal budget constraint.

## Appendix D:

## Evaluation of Discounted Sums

Analysis of the Hicksian substitution and wealth expressions require the evaluation of discounted sums. To take one example, the general substitution effect involves summing discounted forward price changes over all decision periods

$$\sum_{j=o}^{\infty} \beta^j \hat{p}_j .$$

To take another, the proportionate change in utility is expressible as

$$\hat{U} = (\frac{(D_1 u) y}{u}) \sum_{t=o}^{\infty} \beta^t [s_c \hat{c}_t + s_N (\frac{1-N}{N}) \hat{L}_t] (1-\beta^*)$$

Computation in each of these cases is facilitated by the fact that one can express the evolution of the "reduced form" system as $z_t = \pi s_t$ and $s_{t+1} = M s_t$, where $z_t = [\hat{c}_t \hat{L}_t \hat{N}_t \hat{w}_t \hat{\lambda}_t]'$, and $s_t$ is a vector of "state variable", $z_t = [\hat{k}_t \hat{g}_t]'$.

Let $P_M$ be the matrix of characteristic vectors of M and define $s_t^* = P_M^{-1} s_t = D_M s_t^*$, where $D_M$ is a diagonal matrix with the characteristic roots of M on the diagonal.

Further, let q be a vector that weights the relevant of elements of $z_t$ into expression that is to be discounted. For example, if $z_t$ = $[\hat{c}_t\ \hat{L}_t\ \hat{N}_t\ \hat{w}_t\ \hat{\lambda}_t]'$ then q would be [0 0 0 0 1] in the first example and $[s_c\ (s_N \frac{1-N}{N})\ 0\ 0\ 0]$ in the second.

Thus, the expression of interest may be developed as

$$\sum_{j=o}^{\infty} \beta^j\ q\pi s_j$$

$$= \sum_{j=o}^{\infty} \beta^j\ q\pi P_M\ s_j^*$$

$$= q\pi p[\sum_{j=o}^{\infty} \beta^j D_M^j]\ s_o^*$$

$$= q\pi P_M \begin{bmatrix} \frac{1}{1-d_1\beta} & 0 \\ 0 & \frac{1}{1-d_2\beta} \end{bmatrix} P_M^{-1} s_o$$

which permits rapid computation.