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Abstract

This paper estimates the relative risk aversion coefficient by utilizing information in stochastic and deterministic time trends. The first order condition that equates the relative price and the contemporaneous marginal rate of substitution for two goods are used to derive a restriction that the relative price and consumption of two goods are cointegrated. The cointegrating vector involves curvature parameters, and these preference parameters are estimated with a cointegrating regression. This cointegration approach allows for borrowing constraints, aggregation over heterogeneous consumers, unknown preference shocks, and a general form of time-nonseparability. It also does not require the consumers to know the true stochastic law of motion of the economy. These factors have been pointed out as possible causes of empirical rejections of the Consumption-Based Asset Pricing Model by Hansen and Singleton (1982) with the Generalized Methods of Moments (*GMM*). We separate the elasticity of intertemporal substitution from the relative risk aversion coefficient by allowing for time-nonseparable preferences. Our estimates of the coefficient of the risk aversion are typically compatible with Hansen and Singleton's *GMM* estimates.

Key Words: Relative risk aversion, cointegration.

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1. Introduction

This paper estimates the relative risk aversion (*RRA*) coefficient by utilizing information in stochastic and deterministic time trends. Traditional empirical research has typically ignored information in trends by detrending prior to estimation or by first differencing variables. However, time trends may contain substantial information about the underlying structure of the economy that govern the secular and cyclical frequencies of economic time series simultaneously as emphasized by Singleton (1988) among others. To exploit information in trends, a restriction on trends of economic variables (the relative price and consumption of two goods) is derived from the first order condition that equates the relative price and the marginal rate of substitution of two goods. When these variables possess stochastic trends, this restriction implies that trends contain enough information to identify preference parameters including the *RRA* coefficient; these preference parameters can be estimated by a cointegrating regression.

Much work (see, e.g., Campbell and Shiller [1987] and Campbell [1987]) has utilized information in stochastic trends to estimate and test economic models, using standard econometric procedures for cointegrated systems developed by Engle and Granger (1987), Phillips and Durlauf (1986), and Phillips and Ouliaris (1988) among others. Econometric procedures used in this paper incorporate recent important improvements to standard econometric procedures for cointegrated systems.

There are four problems in the standard econometric procedures. First, the estimators of cointegrating vectors have nonstandard distributions. Many economic models imply cointegration, and structural parameters can be estimated as components of cointegrated vectors. The *OLS* consistently estimates a cointegrating vector even when regressors are not econometrically exogenous and when stationary measurement errors are present (see, e.g., Phillips and Durlauf [1986], Stock [1987], and Park and Phillips [1988]). However, the *OLS* estimators have nonstandard distributions, and inferences on structural parameters have been difficult for this reason. Recent work by Johansen (1988), Phillips (1988), Park (1988a), and Phillips and Hansen (1989) has solved this problem. The estimators developed by these authors have mixture of normal distributions. Hypothesis testing on structural parameters can be conducted by standard χ^2 tests.

Second, the standard econometric procedures take no cointegration as the null hypothesis when testing for cointegration (see, e.g., Engle and Granger [1987] and Phillips and Ouliaris [1988]). In applications, failures to reject the null of no cointegration are often interpreted as evidence against economic models which imply cointegration. However, these tests are known to have very low power in some cases and may fail to reject the null of no cointegration with high probability when the economic model tested is actually consistent with data. On the other hand, these tests may reject the null of no cointegration when the economic model tested is inconsistent with data for reasons that are remote from cointegration such as heteroskedasticity of the disturbance term. It is therefore much desirable to test the null of cointegration to control the probability of rejecting a valid economic model. Though this problem has been recognized in the literature, constructing tests with the null of cointegration has been difficult because of reasons suggested by Phillips and Ouliaris (1988). Recently, several authors have overcome these difficulties by constructing consistent tests for the null of stationarity (see Park and Choi [1988], Park [1988b], Park, Ouliaris, Choi [1988], and Fukushige and Hatanaka

[1989]).

Third, the standard econometric procedures do not utilize what we call the deterministic cointegration restriction in estimation and testing. Since most macro economic time series tend to drift (upward) over time, it is often reasonable to model such series as difference stationary process with nonzero drift. Such series have deterministic trends arising from drift as well as stochastic trends. Economic models often imply that the cointegrating vector eliminates both the deterministic and the stochastic trends. We can test this restriction, which is called the deterministic cointegration restriction, with the null of cointegration. This test arguably provides a useful diagnostic for detecting the absence of cointegration in small samples because it is easier to detect a deterministic trend than to detect a stochastic trend. Imposing this restriction on estimators will lead to more efficient estimators. Park (1988), West (1988) and Hansen (1989) developed procedures to impose the deterministic cointegration restriction on estimators. Park's (1988a) *Canonical Cointegrating Regression (CCR)* procedure can be used to test the deterministic cointegration restriction.

Fourth, the standard econometric procedures do not utilize information in the long run correlation of disturbance terms in the case of multiple regressions. Such information is utilized for sharper estimation in Johansen's (1988) maximum likelihood procedure and Park and Ogaki's (1989) *Seemingly Unrelated Canonical Cointegrating Regressions (SUCCR)*.

To incorporate all of these improvements, we employ Park's (1988a) *CCR* procedure for the single regression case and Park and Ogaki's (1989) *SUCCR* procedure for the multiple regressions case. We test the first order condition with Park, Ouliaris, and Choi (1988) and Park's (1988a) variable addition method. The variable addition testing procedure can be used in the framework of the *CCR* and the *SUCCR* to test the null of cointegration. The procedure does not require strong distributional assumptions, allows for a general from of serial correlation, and takes the whole region of a stable root as the null. Though our economic model implies linear cointegration relationship, the short-run dynamics implied by our model is highly nonlinear in the presence of time-nonseparable preferences. Because of this nonlinearity, Johansen's (1988) maximum likelihood procedure is not readily applicable to our model.

Hansen and Singleton (1982) showed how to estimate the *RRA* coefficient and test the Consumption-Based Asset Pricing Model (*C-CAPM*). They used the Generalized Methods of Moments (*GMM*) procedure proposed by Hansen (1982). The *C-CAPM* was rejected strongly by Hansen and Singleton (1982, 1984) when stock returns and Treasury Bill rates were used together. Their results are consistent with another form of empirical rejection of the *C-CAPM* by Mehra and Prescott (1985) that the observed equity risk premium has been too large for the *C-CAPM* to explain. Possible causes of the rejection of the *C-CAPM* have been pointed out. These include unknown preference shocks (e.g., Garber and King [1983]), time-nonseparable preferences (e.g., durability in the commodities usually labeled as nondurables and services as in Eichenbaum and Hansen [1987], Eichenbaum, Hansen, and Singleton [1988], habit formation as in Constantinides [1988], and both durability and habit formation as in Heaton [1988]), and borrowing constraints (see, e.g., Hayashi [1985] for a survey). The *GMM* approach also assumes that there are no measurement errors and the consumers know the true stochastic law of motion of the economy.

The cointegration approach allows for borrowing constraints, aggregation over heterogeneous consumers, unknown preference shocks, and a general form of time-nonseparability, any measurement errors that are stationary, and possibilities that consumers do not know the true stochastic law of motion of the economy. Hence as suggested by Hausman (1978), we can test the *C-CAPM* against the alternative of factors such as borrowing constraints by testing whether or not our estimate and the *GMM* estimate are the same. Our estimates of the coefficient of the risk aversion are typically compatible with Hansen and Singleton's (1982, 1984) *GMM* estimates. Thus the *C-CAPM* passes this specification test.[1] Our results are consistent with the view that the factors which we allow and the *GMM* does not allow such as borrowing constraints are not empirically important.

One of the important identifying assumptions in the cointegration approach is that at least one consumption series is difference stationary rather than trend stationary in the terminology of Nelson and Plosser (1982). Following Nelson and Plosser, a number of authors reported that the null hypothesis of difference stationarity cannot be rejected against the alternative of trend stationarity for most macroeconomic time series. However most of the tests used do not have much power against some trend stationary processes, and thus it is often difficult to discriminate between trend stationary and difference stationary processes (see, e.g., Evans and Saving [1981, 1984], Cochrane [1988], and Christiano and Eichenbaum [1989]). For this reason, each time series we use is tested for the null of difference stationarity and the null of trend stationarity. When neither of the hypotheses is rejected, we try both of the two alternative

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[1]Unfortunately, we cannot formalize this specification test because the joint distribution of the *CCR* estimator and the *GMM* estimator is not known. For this reason, we just compare point estimates and their standard errors in this paper.

specifications as a sensitivity analysis. When one of the consumption series is specified as trend stationary while the other consumption series is specified as difference stationary, we use the Seemingly Unrelated Canonical Cointegrating Regressions (*SUCCR*) procedure developed in Park and Ogaki (1989). When both consumption series are specified as trend stationary, the cointegration approach fails. Though trends, by themselves, do not contain enough information to identify the *RRA* coefficient in this case, the deterministic cointegration still contain useful information about preferences. Ogaki (1988, 1989a) estimated and tested the first order conditions utilizing the deterministic cointegrating restriction with the *GMM* procedure assuming that both consumption series are trend stationary.

The rest of this paper is organized as follows. In Section 2, the preferences of the representative consumer are specified, and then a restriction on trends of the relative price and consumption of two goods is deduced. We examine implications of the restriction in terms of stochastic and deterministic cointegration. We discuss intuition behind the cointegration approach and compare our approach with the *GMM* approach. Section 3 describes econometric procedures we use. In Section 4, we discuss test results for identifying assumptions made in terms of deterministic and stochastic trends. In Section 5, we present empirical results of cointegrating regressions. We compare our estimates for the *RRA* coefficient with *GMM* estimates in Section 6. Our conclusions are contained in Section 7.

2. *The Cointegration Approach*

In this section, we derive a restriction on trends of economic variables from an first order condition that equates the relative price and the marginal rate of substitution. We define notions of stochastic and deterministic cointegration and derive an implication of the restriction that the relative price and real consumption expenditures are stochastically and deterministically cointegrated. We discuss intuition and robustness of our results.

The Stationarity Restriction

The present paper employs the addilog utility function which was proposed and estimated by Houthakker (1960). The addilog utility function assumes that preferences are represented by the Constant Relative Risk Aversion (*CRRA*) form for each good and that goods are additively separable. Deaton and Wigley (1971), Deaton (1974), Ball (1984), and Miron (1986) estimated addilog utility functions, among others. Specifically, consider an economy with two goods. The number of the goods can be extended without any difficulty as long as the goods other than two goods considered here are additively separable from these two goods in preferences. Suppose that a representative consumer maximizes the lifetime utility function

$$(2.1) \qquad U = E_0\left[\sum_{t=0}^{\infty} \beta^t u(t)\right]$$

at period 0, where $E_t(\cdot)$ denotes expectations conditional on the information available at period t. The intra-period utility function is assumed to be of the addilog form

$$(2.2) \qquad u(t) = \sum_{i=1}^{n} \sigma_1(t) \frac{S_i(t)^{1-\alpha_1} - 1}{1-\alpha_i} ,$$

where $\alpha_i>0$ for $i=1,\ldots,n$. When $\alpha_i=1$, we interpret $S_i(t)^{1-\alpha_i}/(1-\alpha_i)$ to be $\log(S_i(t))$. Here $S_i(t)$ is service flow from consumption purchases of the ith good. Purchases of consumption goods and service flows are related by

$$(2.3) \qquad S_i(t) = A^i(L)C_i(t) = a_0^i C_i(t) + a_1^i C_i(t-1) + a_2^i C_i(t-2)+ \ldots$$

for $i=1,\ldots,n$, where $C_i(t)$ is the real consumption expenditure for the ith good at period t. We assume that the representative consumer is endowed with $C_i^*(t)$ units of the ith good at period t. We denote values of $S_i(t)$ obtained when $C_i(\tau)=C_i^*(\tau)$ for all $\tau\leq t$ by $S_i^*(t)$. We assume that $A^i(z)$ satisfies the condition that the life time utility U evaluated at $S_i(t)=S_i^*(t)$ is finite. This type of specification for time-nonseparability has been used by Hayashi (1985), Eichenbaum, Hansen, and Singleton (1988), Eichenbaum and Hansen (1987), and Heaton (1988) among others. Note that the purchase of one unit of the ith good at period t increases $S_i(\tau)$ by $a_{\tau-t}^i$ units for $\tau\geq t$. Preference shocks are allowed in equation (2.2) via the stochastic process $\{[\sigma_1(t),\ldots,\sigma_n(t)]' : -\infty<t<\infty\}$, which is assumed to be (strictly) stationary.

Let $P_i(t)$ be the price of the ith good. We take the first good as a numeraire for each period: $P_1(t)\equiv1$. Let $W^b(t)$ and $W^e(t)$ be the beginning of period wealth and the end of period wealth of the consumer at period t, respectively:

$$(2.4) \qquad W^e(t) = W^b(t) + \sum_{i=1}^{n} P_i(t)C_i^*(t) - \sum_{i=1}^{n} P_i(t)C_i(t)$$

We assume that there is only one asset in the economy in the present paper. The asset has the gross return rate of $R_t[W^e(t)]$ from period t to period $t+1$, which is a function of $W^e(t)$ to allow for borrowing constraints:

$$(2.5)\qquad W^b(t+1)=R_t[W^e(t)]W^e(t)$$

The function R_t is assumed to be in information available at t and its derivative, denoted R_t', is assumed to exist.

The consumer is assumed to maximize (2.1) subject to (2.4) and (2.5) and a boundary condition $\lim_{t\to\infty}\beta^t W^b(t)\geq 0$ with probability one. Assuming an interior solution, the first order necessary conditions include

$$(2.6)\qquad \partial U/\partial C_i(t) = \lambda(t)P_i(t)\{R_t + W^e(t)\ R_t'\}$$

for $t\geq 0$ and $i=1,..,n$, where $\lambda(t)$ is the Lagrange multiplier for the constraint (2.5) into which the lagged (2.4) is substituted. Since the first good is the numeraire, $P_i(t)=P_i(t)/P_1(t)$ is the relative price between the ith and the first goods, and by (2.6),

$$(2.7)\qquad P_i(t) = \frac{\partial U/\partial C_i(t)}{\partial U/\partial C_1(t)} = \frac{E_t[\sum_{\tau=0}^{\infty}\beta^\tau \partial u(t+\tau)/\partial C_i(t)]}{E_t[\sum_{\tau=0}^{\infty}\beta^\tau \partial u(t+\tau)/\partial C_1(t)]}$$

$$= \frac{E_t[\sum_{\tau=0}^{\infty}\beta^\tau \sigma_i(t+\tau)a_\tau^i(S_i(t+\tau))^{-\alpha_i}]}{E_t[\sum_{\tau=0}^{\infty}\beta^\tau \sigma_1(t+\tau)a_\tau^1(S_1(t+\tau))^{-\alpha_1}]}$$

We take $C_i(t)=C_i^*(t)$ for all i and t as an equilibrium condition. In an equilibrium, the first order condition (2.7) is satisfied with the equilibrium values of $S_i(t)$, $S_i^*(t)$. We derive a restriction from the first order condition (2.7), which is the foundation of the cointegration approach. Though (2.7) is derived from a specific intertemporal budget constraint (2.5), it should be obvious that (2.7) is not affected by the specification of intertemporal budget constraint.

We need the following assumption to insure that the ratio of $S_i(t)$ and $C_i(t)$ is stationary.

Assumption 1: The stochastic processes $\{C_i^*(t)/C_i^*(t-1): -\infty<t<\infty\}$ for $i=1,..,n$ are jointly stationary.

Under Assumption 1, the process $\{C_i^*(t+\tau)/C_i^*(t): -\infty<t<\infty\}$ is also stationary for any fixed integer τ because $C_i^*(t+\tau)/C_i^*(t) = \{C_i^*(t+\tau)/C_i^*(t+\tau-1)\}$ $\{C_i^*(t+\tau-1)/C_i^*(t+\tau-2)\}\cdots\{C_i^*(t+1)/C_i^*(t)\}$. It follows that the process $\{S_i^*(t+\tau)/C_i^*(t): -\infty<t<\infty\}$ is also stationary for any τ because the right hand side of

$$(2.8) \qquad S_i^*(t+\tau)/C_i^*(t) = a_0^i C_i^*(t+\tau)/C_i^*(t) + a_1^i C_i^*(t+\tau-1)/C_i^*(t) + a_2^i C_i^*(t+\tau-2)/C_i^*(t) + \ldots$$

is stationary. We also make an extra assumption that the growth rates of consumption are jointly stationary with the state variables on which the conditional expectations are based. Then $\{P_i(t)[C_1^*(t)]^{-\alpha_1}/[C_i^*(t)]^{-\alpha_i}: -\infty<t<\infty\}$ is stationary because the right hand side of

$$(2.9) \qquad \frac{P_i(t)[C_1^*(t)]^{-\alpha_1}}{[C_i^*(t)]^{-\alpha_i}} = \frac{E_t[\sum_{\tau=0}^{\infty}\beta^\tau \sigma_i(t+\tau) a_\tau^i \{S_i^*(t+\tau)/C_i^*(t)\}^{-\alpha_i}]}{E_t[\sum_{\tau=0}^{\infty}\beta^\tau \sigma_1(t+\tau) a_\tau^1 \{S_1^*(t+\tau)/C_1^*(t)\}^{-\alpha_1}]}$$

is stationary. Taking the natural log of the left hand side, we conclude that $p_i(t) - \alpha_1 c_1^*(t) + \alpha_i c_i^*(t)$ is stationary, where $p_i(t)=log(P_i(t))$,

$c_i^*(t)=log(C_i^*(t))$ for $i=1,\ldots,n$. We shall call this restriction the stationarity restriction.

Stochastic and Deterministic Cointegration

We define notions of stochastic and deterministic cointegration in this subsection. When a scalar stochastic process is stationary after taking the first difference *and* the process has positive spectral density at frequency zero, the process is said to be *difference stationary*. The trend stationary process is also stationary after taking the first difference but has zero spectral density at frequency zero. A scalar difference stationary process $y(t)$ and a vector difference stationary process $X(t)$ are said to be cointegrated with a *normalized cointegrating vector* γ_x if $y(t)-\gamma_x'X(t)$ is stationary.

Let $X(t)$ be a k-dimensional difference stationary process:

$$X(t) - X(t-1) = \mu_x + \epsilon_x(t) \tag{2.10}$$

for $t\geq 1$ where μ_x is a k-dimensional vector of real numbers where $\epsilon_x(t)$ is stationary with mean zero. Then recursive substitution in (2.1) yields

$$X(t) = \mu_x t + X^0(t) \tag{2.11}$$

where $X^0(t)$ is

$$X^0(t) = X(0) + \sum_{\tau=1}^{t} \epsilon_x(t). \tag{2.12}$$

Relation (2.11) decomposes the difference stationary process $X(t)$ into deterministic trends arising from drift μ_x and the difference stationary process without drift, $X^0(t)$. We can further decompose the stochastic trend component, $X^0(t)$, into a random walk component and stationary stochastic process as in Beveridge and Nelson (1981), though such a further decomposition is not essential for the purpose of this paper. Suppose that $y(t)$ is a scalar difference stationary process with drift μ_y. Le us decompose $y(t)$ into a deterministic trend $\mu_y t$ and a difference stationary process without drift y_t^0 as in (2.12):

$$(2.13) \qquad y(t) = \mu_y t + y^0(t).$$

Difference stationary processes $y(t)$ and $X(t)$ are said to be *stochastically cointegrated* with a *normalized cointegrating vector* γ_x when there exists a *k*-dimensional vector γ_x such that $y^0(t) - \gamma_x' X^0(t)$ is stationary. In general, it is possible that $y(t)$ and $X(t)$ are stochastically cointegrated while they are not deterministically cointegrated. Stochastic cointegration only requires that stochastic trend components of the series are cointegrated. We may write $y^0(t) - \gamma_x' X^0(t) = \theta_c + \epsilon_c(t)$, assuming that $y^0(t) - \gamma_x' X^0(t)$ has mean θ_c. Here $\epsilon_c(t)$ is stationary with mean zero. Then by (2.12) and (2.13),

$$(2.14) \qquad y(t) = \theta_c + \mu_c t + \gamma_x' X(t) + \epsilon_c(t)$$

where

$$(2.15) \qquad \mu_c = \mu_y - \gamma_x' \mu_x \ .$$

Suppose that a vector γ_x^* satisfies

$$(2.16) \qquad \mu_y = \gamma_x^{*\prime} \mu_x.$$

Then $Y(t) - \gamma_x^{*\prime} X(t)$ does not possess any deterministic trend, and $Y(t)$ and

$X(t)$ are *cotrended* with a *normalized cotrending vector* γ_x^*. If $k>1$ and if one of the components of μ_x is nonzero, there are infinitely many cotrending vectors. Consider an extra restriction that the normalized cointegrating vector γ_x is a cotrending vector. This restriction, which we call the *deterministic cointegration restriction*, requires that the cointegrating vector eliminates both the stochastic and deterministic trends. In this case,

(2.17) $y(t) = \theta_c + \gamma_x' X(t) + \epsilon_c(t)$.

Let us consider a cointegrated system involving a trend stationary process $z(t)$:

(2.18) $z(t) = \theta_z + \mu_z t + \epsilon_z(t)$,

where $\epsilon_z(t)$ is stationary with zero mean and $\mu_z \neq 0$. Suppose that an economic model implies a restriction that $y(t) - \gamma_x' X(t) - \gamma_z z(t)$ is stationary. Since $y(t) - \gamma' X(t) - \gamma_z z(t) = -\gamma_z \theta_z + (\mu_y - \gamma_x' \mu_x - \gamma_z \mu_z)t + (y^0(t) - \gamma_x' X^0(t)) - \gamma_z \epsilon(t)$, this restriction implies that $y(t)$ and $X(t)$ are stochastically cointegrated with a normalized cointegrating vector γ_x and that $y(t)$ and $[X(t)', z(t)]'$ are cotrended with a cotrending vector $[\gamma_x', \gamma_z]'$:

(2.19) $\mu_y = \gamma_x' \mu_x + \gamma_z \mu_z$.

Implications of the Stationarity Restriction

In this subsection, implications of the stationarity restriction are discussed in terms of the notions of stochastic and deterministic cointegration defined above. The restriction on trend properties of the variables from the demand side is summarized by the stationarity restriction. For the supply side, we need to require that at least one of the endowment process is difference stationary for identification of preference parameters.

The Demand System with Two Goods

For now, we focus on the demand system with two goods.

First, we consider the case where both the log of the endowment of first good and that of the second good are difference stationary:

Assumption 2a: The process $\{c_i^*(t):t\geq 0\}$ is difference stationary for $i=1,2$.

Assumption 2b: The processes $\{c_1^*(t):t\geq 0\}$ and $\{c_2^*(t):t\geq 0\}$ are not stochastically cointegrated.

Let $C_i^m(t)$ be measured consumption and $\xi_i(t)=\{C_i^m(t)-C_i^*(t)\}/C_i^*(t)$ be the ratio of the measurement error and consumption.[2] We assume that $\xi_i(t)$ is stationary. Note that this assumption, together with Assumption 2, implies implies that the log of measurement error, $log[C(t)-C_i^m(t)]\ _i^*$is difference stationary with stochastic trends. Taking the log of both sides of $C_i^m(t)=[1+\xi_i(t)]C_i^*(t)$, we obtain $c_i^m(t)=c_i^*(t)+log[1+\xi_i(t)]$ where $c_i^m(t)=log[C_i^m(t)]$. It should be noted that $log[1+\xi_i(t)]$ is stationary. Then $c_i^m(t)$ is the sum of a difference stationary and stationary processes and therefore is difference stationary. Similarly, let $P_2^m(t)$ be the measured relative price and assume that $\xi_0(t)=\{P_2^m(t)-P_2(t)\}/P_2(t)$ is stationary. Since $p_i^m(t) - \alpha_1 c_1^m(t) + \alpha_i c_i^m(t) = \{p_i(t) - \alpha_1 c_1^*(t) + \alpha_i c_i^*(t)\} + + \{log[1+\xi_0(t)] - \alpha_1 log[1+\xi_1(t)] + \alpha_i log[1+\xi_2(t)\}$, the stationarity

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[2]We thank Adrian Pagan and Edward Prescott for helpful discussions about the formulation of measurement errors.

restriction implies that $p_2^m(t) - \alpha_1 c_1^m(t) + \alpha_2 c_2^m(t)$ is stationary. Let $y(t) = p_2^m(t)$ and $X(t) = [c_1^m(t),\ c_2^m(t)]'$, using the notation introduced above. Assumption 2a implies Assumption 1 in Section 2, and hence $y(t) - \gamma_x' X(t)$ is stationary with $\gamma_x = [\alpha_1,\ -\alpha_2]'$. Assumption 2b is equivalent to an assumption that there is no 2-dimensional vector γ_x such that $\gamma_x' X(t)$ is trend stationary. Assumption 2b requires that two endowment series possess different stochastic trends. Since $y(t) - \gamma_x' X(t)$ is stationary with $\gamma_x = [\alpha_1,\ -\alpha_2]'$, this implies that $y(t)$, which is the sum of a difference stationary $\gamma_x' X(t)$ and a stationary process, is difference stationary. Thus the stationarity restriction implies that (i) $p_2^m(t)$ is difference stationary, (ii) $p_2^m(t)$ and $[c_1^m(t),\ c_2^m(t)]'$ are stochastically cointegrated with a normalized cointegrating vector $[\alpha_1,\ -\alpha_2]'$, and (iii) the deterministic cointegration restriction is satisfied, under Assumption 2.

Second, we consider the case where the log of the endowment of the first good is difference stationary and that of the second good is trend stationary:

Assumption 3: The process $\{c_1^*(t): t \geq 0\}$ is difference stationary, and the process $\{c_2^*(t): t \geq 0\}$ is trend stationary.

Assumption 3 implies Assumption 1. In this case, we let $y(t) = p_2^m(t)$, $X(t) = c_1^m(t)$, and $z(t) = c_2^m(t)$ to apply the argument in the last subsection. The stationarity restriction implies that (i) $p_2^m(t)$ is difference stationary, (ii) $p_2^m(t)$ and $c_1^m(t)$ are stochastically cointegrated with a normalized cointegrating vector $\gamma_x = \alpha_1$, and (iii) $p_2^m(t)$ and $[c_1^m(t), c_2^m(t)]$ are cotrended with a normalized cotrending vector $[\gamma_x, \gamma_z]' = [\alpha_z,\ -\alpha_2]'$.

The Demand System with Three Goods

Let us consider the demand system with three goods. For simplicity, we only consider the case where the log of each of the three goods is difference stationary. Let $y_1(t)=p_2^m(t)$, $y_2(t)=p_3^m(t)$, $X_1(t)=[c_1^m(t),\ c_2^m(t)]'$, $X_2(t)=[c_1^m(t),\ c_3^m(t)]'$. The stationary restriction implies that $y_i(t)$ is stochastically and deterministically cointegrated with a normalized cointegrating vector γ_{xi} for i=1,2, where $\gamma_{x1}=[\alpha_1,\ \alpha_2]$ and $\gamma_{x2}=[\alpha_1,\ \alpha_3]$. Thus in this case, we have a system of cointegrating regressions and a cross-equation restriction that that the first component of γ_{x1} and that of γ_{x2} are the same.

Intuition and Robustness

We showed that the the first order condition (2.7) leads to restrictions that economic variables are cointegrated. As will be shown in the next section, these restrictions can be used to estimate some preference parameters and to test the first order condition. In this subsection, we discuss intuition behind these results and compare our approach with the *GMM* approach.

The Linear Expenditure System

First, we provide an intuitive explanation for the stationarity restriction by comparing the demand system with the addilog utility function with the linear expenditure system. We define any demand system with the property that the expenditure share of each good is stationary the *linear expenditure system*. For example, the addilog utility function with $\alpha_1=\alpha_2=1$ in (2.2) lead to a linear expenditure system. See Eichenbaum and Hansen [1988] and Ogaki (1989b) for examples of the linear expenditure system. The linear expenditure system has a property that $p_2(t)-c_1^*(t)+c_2^*(t)$ is stationary. To see this, let

$$(2.20) \quad P_i(t)C_i(t)/I(t) = \psi_i(t)$$

where $\psi_i(t)$ is a stationary random variable (i=1,2) and $I(t) = P_1(t)C_1(t) + P_2(t)C_2(t)$ is the total consumption expenditure on the two goods. Dividing (2.20) for i=2 by (2.20) for i=1 yields

$$(2.21) \quad [P_2(t)/P_1(t)]C_2(t)/C_1(t) = \psi_2(t)/\psi_1(t).$$

Since the right hand side of (2.21) is stationary, the left hand side of (2.21) is stationary. Therefore $p_2(t)-c_1(t)+c_2(t)$ is stationary.

In our empirical work, we typically found very strong evidence against this implication of the linear expenditure system that $p_2(t)$ and $[c_1(t), c_2(t)]'$ are cointegrated with a known normalized cointegrating vector [1, -1]'. We depart from the linear expenditure system, using the addilog utility function which allows nonhomotheticity. In the addilog utility function in equation (2.2), the curvature parameters, α_1 and α_2, govern income elasticities in the following sense. Let us consider the following situation. Suppose that a consumer with this utility function were able to trade the household capital stocks in an Arrow - Debreu market with no transaction costs. Fix all $S(\tau)$ (τ=0,1,...) except for $S(t)$, and increase income at t exogenously. It is easy to show that the income elasticities in this experiment for $S_1(t)$ and $S_2(t)$ are c/α_1 and c/α_2 respectively for some constant c which depends on t in general (see Ogaki [1989b]). Thus α_1/α_2 is the ratio of income elasticities of the second and the first goods. When $\alpha_1=\alpha_2=1$, the addilog utility function implies the linear expenditure system. When $\alpha_1=\alpha_2\neq1$, the addilog utility function represents homothetic preferences which are not the linear expenditure system. When $\alpha_1 \neq \alpha_2$, preferences that the addilog utility function represents are nonhomothetic.

To develop intuition for the stationarity restriction, imagine that the relative price $p_2(t)$ is stationary for simplicity. If the demand system for the two goods is the linear expenditure system, then $p_2(t)-c_1(t)+c_2(t)$ does not possess any time trends, and therefore $c_1(t)$ and $c_2(t)$ must grow at the same rate in the long run. The addilog utility function implies that $p_2(t)-\alpha_1 c_1(t)+\alpha_2 c_2(t)$ is stationary. When $\alpha_1 > \alpha_2$, the first good has a lower income elasticity than the second good and consumption for the first good can grow at a slower rate than the second good in the long run.

If at least one of the consumption series is difference stationary, then the parameters α_1 and α_2 are identified by information in trends and can be estimated by cointegrating regressions. This is because trends of the relative price and consumption contains information about income elasticities. For example, if consumption of food is growing at a slower rate than consumption of automobiles after correcting for the effect of the relative price, we can infer that food has a lower income elasticity than automobiles.

In our empirical work, we take a measure of nondurable consumption as the first good and interpret α_1 as the *RRA* coefficient for the nondurable consumption. We identify the *RRA* coefficient because it is harder for a consumer to tolerate riskiness of consumption of a necessary good than that of a luxury good. This identification of the *RRA* coefficient relies on the assumption of separability across the first and second goods in preferences. If nondurable consumption is not separable from the other goods, we cannot define the *RRA* coefficient for the nondurable consumption.

Robustness of the Cointegration Approach

One remarkable feature of cointegrating regressions is that structural parameters can be estimated consistently by the *OLS* without an assumption that regressors are econometrically exogenous. We utilize this property in the first step estimation for the *CCR* procedure. This property is important because few economic variables are econometrically exogenous in most of the stochastic and dynamic rational expectations equilibrium models. The *OLS* estimators are median biased, however, and we correct for endogeneity and serial correlations by the *CCR* procedure as we will describe.

We can allow for measurement errors without assuming that regressors are uncorrelated with measurement errors of the regressand. The only assumption we need is that the ratio of the measurement error and the true value is stationary for each variable as discussed above. In contrast, the *GMM* approach of Hansen and Singleton (1982) does not allow for measurement errors unless they are of very special form.

It should be noted that relation (2.7) equates the relative purchasing prices of consumption goods with the marginal rate of substitution for purchases of consumption goods. Relation (2.7) does *not* focus on the relation between user costs and the marginal rate of substitution for services. Borrowing constraints may contaminate the relation between user costs and the intraperiod marginal rate of substitution for service flows $[\{\partial u(t)/\partial C_2(t)\}/\{\partial u(t)/\partial C_1(t)\}$ in our notation]. The relation between the relative purchasing price $[p_2(t)]$ and the marginal rate of substitution for purchases of consumption $[\{\partial U(t)/\partial C_2(t)\}/\{\partial U(t)/\partial C_1(t)\}]$ is more robust. Also note that the stationarity restriction only involves the relative purchasing price and purchases of consumption. Consequently, the cointegration approach only requires data for purchasing prices and consumption of two goods and does not need data for user costs and service flows that are not directly observable.

The cointegration approach allows for aggregation over heterogeneous consumers as long as every consumer has the same curvature parameters, α_1 and α_2, and variance of the log of each consumption expenditure across consumers is stationary. Suppose that there are N consumers in the economy and that the consumption expenditures of each consumer satisfies the stationarity restriction. Let $C_i^j(t)$ be consumption on good i by consumer j and assume that we observe the equilibrium consumption $C_i^*(t) = (1/N)\sum_{j=1}^{N} C_i^j(t)$. Then $log(P_2(t)) - \alpha_1 (1/N)\sum_{j=1}^{N} log(C_1^j(t)) + \alpha_2 (1/N)\sum_{j=1}^{N} log(C_2^j(t))$ is stationary. This stationary process is different from observable $p_2(t) - \alpha_1 c_1^*(t) + \alpha_2 c_2^*(t)$ only by the difference between the log of average, $log(C_i^*(t))$, and the average of the log, $(1/N)\sum_{j=1}^{N} log(C_i^j(t))$. This difference is one half of variance of the log of consumption if we can approximate the distribution of consumption across the consumers by a log normal distribution. Hence the stationarity restriction is satisfied by the aggregated variables if the variance of the log of each consumption is stationary.

Garber and King (1983) pointed out that unknown preference shocks can explain empirical rejections of the *C-CAPM* by the *GMM* approach. The cointegration approach allows for preference shocks since the stationarity restriction is robust with respect to stationary unknown preference shocks. The cointegration approach does not allow for permanent preference shocks with a unit autoregressive root, however. At least for aggregate level of consumption such as nondurable goods, permanent preference shocks are unlikely compared with permanent shocks in productivity that have been considered in recent work on real business cycle models (see, e.g., King, Plosser, and Rebelo [1988] and references therein). Our consumption and income today may well be affected by permanent technological shocks that happened during the industrial revolution. It does not seem likely that our consumption on nondurable goods is affected by permanent shifts in preferences that happened during the industrial revolution.

Though permanent preference shocks are assumed away, we do allow for habit formation through time-nonseparability. Constantinides (1988) showed that adding habit persistence could help to explain asset market data. Durability of goods is a source of time-nonseparability as in Mankiw (1982) and Dunn and Singleton (1986), among others. Even goods that are usually labeled as nondurables may have durability (see, e.g., Hayashi [1985], Eichenbaum and Hansen [1987], and Eichenbaum, Hansen, and Singleton [1988]). Heaton (1988) investigated interactions of durability and habit formation for the *C-CAPM*.

When time-nonseparability is allowed in the *GMM* procedure as in Eichenbaum and Hansen (1987) and Eichenbaum, Hansen, and Singleton (1988), a particular form of time-nonseparability must be employed to obtain estimates of service flows. The form of time-nonseparability should be limited so that the number of free parameters is not too large. For example, Eichenbaum and Hansen (1987) and Eichenbaum, Hansen, and Singleton (1988) allowed for one month durability using monthly data for nondurables and services in the NIPA. They found a significant durability for nondurables plus services. Since nondurables in the NIPA include consumption measures that are durable for more than one month such as clothing, it may be reasonable to allow for longer terms of durability. However, to do so requires more free parameters to be estimated, which could cause problems.

A general form of time-nonseparability is allowed as in relation (2.3), but the cointegration approach does not require estimating the particular form of time-nonseparability, $A^{i}(L)$. This is because the stationarity restriction is in terms of purchases of consumption goods rather than in terms of unobservable service flows. Thus we can estimate the curvature parameter of the CRRA utility function without specifying any particular form of time-nonseparability as long as unit roots are present in the relevant time series.

Cochrane (1989) calculated utility losses caused by departures from the optimal consumption path in the *C-CAPM*. He concluded that utility losses that are associated with departures that are significant enough to cause rejections of tests of the *C-CAPM* are typically very small. If the representative consumer does not know the true stochastic law of motion of the economy and is in the process of learning, then such departures may be rational: the consumer's optimal path in this case will depart from the optimal path when the consumer knew the true law of motion. It should be noted that the *GMM* approach assumes that the consumer knows the true law of motion. The stationarity restriction does not require that the representative consumer knows the true law of motion. When the consumer is in the process of learning, the optimal probability measure to be used for the conditional expectation operator in relations (2.7) and (2.9) is the measure that maximizes entropy given available information (see El-Gamal [1989]). This coincides with Bayesian learning (see, e.g., Easley and Kiefer [1988] and references therein for Bayesian learning models) when the initial prior distribution of Bayesian is optimal. When random variables that generate available information are stationary, the right hand side of (2.9) is stationary when the optimal probability measure is used to form conditional expectations.

3. *Econometric Procedures*

In this section, we describe econometric procedures to estimate and test the economic model using stochastic deterministic cointegration. The first subsection treats the single regression case. The demand system with two goods under Assumption 2 leads to this case. The second subsection treats the multiple regressions case. The demand system with more than three goods or the demand system with two goods under Assumption 3 leads to this case.

Canonical Cointegrating Regressions

We consider the demand system with two goods under Assumption 2 in this subsection.

Estimation

Suppose that Assumptions 2a and 2b are satisfied, and let $y(t) = p_2^m(t)$ $X(t)=[c_1^m(t),c_2^m(t)]'$, and $\gamma_x = [\alpha_1, -\alpha_2]'$. Then $y(t)$ and $X(t)$ are stochastically and deterministically cointegrated with a normalized cointegrating vector γ_x. Since $y(t)$ and $X(t)$ are stochastically cointegrated, regression (2.14) can be used to estimate γ_x. Park and Phillips (1988) showed that the *OLS* estimator of (2.14) is consistent. Regression (2.14) ignores the deterministic cointegration, however. The deterministic cointegration implies that the true value of μ_c is zero. Hence we consider a regression (2.17), which leads to a more efficient estimator. We apply the *CCR* procedure developed by Park (1988a) to (2.17). We now briefly describe the *CCR* procedure.

Let $\epsilon(t) = [\epsilon_c(t), \epsilon_x(t)']'$. We assume that $\epsilon(t)$ is strictly stationary and ergodic with zero mean and finite positive definite covariance matrix Σ. We also assume that the partial sum process constructed from $\epsilon(t)$ satisfies the multivariate invariance principle and weakly converges to a vector Brownian motion with the positive definite covariance matrix Ω. We decompose Ω as $\Omega = \Sigma + \Lambda + \Lambda'$ where $\Sigma = E[\epsilon(t)\epsilon(t)']$ and $\Lambda = \sum_{j=1}^{\infty} E[\epsilon(0)\epsilon(j)']$; and we define $\Delta = \Sigma + \Lambda$. We partition Ω, Σ, Δ conformably with $\epsilon(t)$. Some sufficient conditions for $\epsilon(t)$ to satisfy the multivariate invariance principle are provided by Hall and Heyde (1980, theorem 5.5) and Phillips and Durlauf (1986). Let $\Delta_2 = [\Delta_{21}, \Delta_{22}]'$ and $\Omega_{11.2} = \Omega_{11} - \Omega_{12}\Omega_{22}^{-1}\Omega_{21}$.

The first step to obtain the *CCR* is to obtain consistent estimates of γ_x, Ω, Σ, Δ, and $\Omega_{11.2}$. Let $\hat{\gamma}_x$, $\hat{\Omega}$, $\hat{\Sigma}$, $\hat{\Delta}$, and $\hat{\Omega}_{11.2}$ denote consistent estimates of γ_x, Ω, Σ, Δ, and $\Omega_{11.2}$. To obtain these estimates, we apply the *OLS* to (2.10) and (2.17). From the residuals of these regressions, $\hat{\epsilon}(t) = [\hat{\epsilon}_c(t), \hat{\epsilon}_x(t)']'$, the sample counterparts of Ω and Δ are formed:

$$(3.1) \qquad \hat{\Omega} = (1/T)\sum_{t=1}^{T} \hat{\epsilon}(t)\hat{\epsilon}(t)' + (1/T)\sum_{\tau=1}^{\ell} w(\ell,\tau) \sum_{t=\tau+1}^{T} \{\hat{\epsilon}(t-\tau)\hat{\epsilon}(t)' + \hat{\epsilon}(t)\hat{\epsilon}(t-\tau)'\}$$

$$(3.2) \qquad \hat{\Delta} = (1/T)\sum_{t=1}^{T} \hat{\epsilon}(t)\hat{\epsilon}(t)' + (1/T)\sum_{\tau=1}^{\ell} w(\ell,\tau) \sum_{t=\tau+1}^{T} \hat{\epsilon}(t-\tau)\hat{\epsilon}(t)'$$

for some choice of the lag truncation number ℓ and the lag window $w(\ell,\tau)$. For the empirical results reported in the present paper, we employed the lag window of Parzen's estimates (see, e.g., Hannan [1970], p.279). Asymptotic theories indicate that we consistently estimate Ω and Δ by (3.1) and (3.2) as the lag truncation number is increased but provide little guidance for choosing the lag truncation number. In our empirical work, we checked sensitivity of our results with respect to the lag truncation number. For the results reported in the tables, we chose large enough lag truncation number that the test statistics settled down.

The second step is to run a regression with transformed data. Let $y^*(t) = y(t) - [\hat{\gamma}_x'\hat{\Delta}'_2\hat{\Sigma}^{-1} + (0, \hat{\Omega}_{12}\hat{\Omega}_{22}^{-1})]\hat{\epsilon}(t)$ and $X^*(t) = X(t)-\hat{\Delta}'_2\hat{\Sigma}^{-1}\hat{\epsilon}(t)$. Let $Z(t) = [1, X^*(t)]$. We regress $y^*(t)$ onto $Z(t)$ to get an *OLS* estimate, say $\hat{\phi}$, of the coefficient vector, say $\phi = [\theta_c, \gamma_x']'$. Then $\hat{\phi}$ has an asymptotic normal distribution $N(\phi, \Omega_{11.2}[\sum_{t=1}^{T} Z(t)\ Z(t)']^{-1})$ conditioned on the limit of $Z(t)$. (See Park [1988] for the precise meaning of this statement). We report square roots of diagonal elements of $\hat{\Omega}_{11.2}[\sum_{t=1}^{T} Z(t)\ Z(t)']^{-1}$ as standard errors of estimates. For the purpose of hypothesis testing and interval estimation, these standard errors may be interpreted just as the usual standard errors as explained below.

Since both α_1 and α_2 are different from zero, we can choose $c_1^*(t)$ or $c_2^*(t)$ as the regressand $y(t)$ rather than $p_2(t)$. Fist, consider the case where $c_1^*(t)$ is chosen as the regressand, and let $y(t)=c_1^*(t)$ and $X(t)=[p_2(t), c_2^*(t)]$. The stationarity restriction implies that $y(t)$ and $X(t)$ are cointegrated with a normalized cointegrating vector $\gamma_x = [1/\alpha_1, \alpha_2/\alpha_1]$. It is easy to show that Assumption 2b implies that the components of $X(t)$ are not stochastically cointegrated. Thus we can apply the *CCR* procedure to $y(t)$ and $X(t)$ with these definitions to estimate $\gamma_x=[1/\alpha_1, \alpha_2/\alpha_1]$. Consistent estimates of α_1 and α_2 are obtained from a *CCR* estimate of γ_x from the formulas $\alpha_1=1/\gamma_{X1}$ and $\alpha_2=\gamma_{X2}/\gamma_{X1}$. Asymptotic distributions of these estimators can be derived by a mean-value approximation (the delta method). Second, consider the case where $c_2^*(t)$ is chosen as the regressand, and let $y(t)=c_2^*(t)$ and $X(t)=[p_2(t), c_1^*(t)]$. In this case, $y(t)$ and $X(t)$ are cointegrated with a normalized cointegrating vector $\gamma_x = [-1/\alpha_2, \alpha_1/\alpha_2]$.

Testing

An important consequence of this result is that linear restrictions can be tested by χ^2 tests which are free from nuisance parameters. Let

$$(3.3)\qquad R\phi=r$$

be the null hypothesis, where R is a known matrix with q rows and full row rank q, and r is a known $q\times 1$ vector of real numbers. Partition R conformably with $\phi=[\theta_c,\gamma']'$. Assume that $R_{12} = R_{21} =0$. Let

$$(3.4)\qquad G_R = (R\hat{\phi} - r)'\{R\hat{\Omega}_{11.2}[\sum_{t=1}^{T}Z(t)\quad Z(t)']^{-1}R'\}^{-1}(R\hat{\phi} - r).$$

Then G_R asymptotically has an unconditional χ_q^2 distribution. This means that G_R tests for $q=1$ can be used to justify interpretations of the conditional standard errors: if an estimate $\hat{\phi}_i$ of the ith element of ϕ is away from a hypothetical value of ϕ_i by more than two standard errors, then the hypothesis that ϕ_i takes that value is rejected at the 5 per cent level.

We can use G_R statistics in a regression with spurious deterministic trends added to (2.17) to test for stochastic and deterministic cointegration. Consider a regression

$$(3.5)\qquad y(t) = \theta_c + \sum_{i=1}^{q} \eta_i t^i + \gamma_x' X(t) + \epsilon_c(t) \;.$$

We apply Park's (1988a) *CCR* procedure of to (3.5) using estimates of $\hat{\gamma}$, $\hat{\Omega}$, $\hat{\Sigma}$, $\hat{\Delta}$, and $\hat{\Omega}_{11.2}$ described above. Let $H(p,q)$ denote the G_R statistic to test the hypothesis $\eta_p=\eta_{p+1}= \dots =\eta_q=0$. Then $H(p,q)$ converges in distribution to a χ_{p-q}^2 random variable under the null of cointegration. In particular, the $H(0,1)$ statistic tests the hypothesis $\mu_c=0$ in (2.14) and can be used to test the deterministic cointegrating restriction. If $y(t)$ and $X(t)$ are not stochastically cointegrated, then $\epsilon_c(t)$ is difference stationary for any γ. In this case, (3.5) is a spurious regression and $H(1,q)$ statistics diverge in probability. Hence the $H(1,q)$ tests are consistent against the alternative of no stochastic cointegration.

Seemingly Unrelated Canonical Cointegrating Regressions

We consider the case of multiple regressions in this subsection. We mostly discuss this case in the context of the demand system with two goods under Assumption 3. It is simple to extend the analysis for the demand system with three goods.

Estimation

Now let us consider the case where Assumption 3 is satisfied. Let $y(t)=p_2^m(t)$, $X(t)=c_1^m(t)$, and $z(t)=c_2^m(t)$, using the notation introduced in the last section. Then since $y(t)$ and $X(t)$ are stochastically cointegrated, regression (2.14) can be used to estimate $\gamma_X=\alpha_1$ and μ_c. As shown in the last section, $y(t)$, $X(t)$, and $z(t)$ are deterministically cointegrated with a normalized cointegrating vector $\gamma=(\gamma_x,\ \gamma_z)'$, where $\gamma_z=\ -\alpha_2$. From relation (2.15) and the deterministic cointegration restriction (2.19), we obtain

$$(3.6)\qquad \gamma_z = \mu_c/\mu_z.$$

Since μ_z can be estimated from (2.18), the deterministic cointegration restriction can be used to identify γ_z.

Let us formulate (2.14) and (2.18) as a system of Seemingly Unrelated Regressions (*SUR*):

$$(3.7) \qquad Y = Z\phi + u$$

where $Y = [y(1),\ \ldots,\ y(T),\ z(1),\ \ldots,\ z(T)]'$, $\phi = [\theta_c,\ \mu_c,\ \gamma_x,\ \theta_z,\ \mu_z]'$, $u = [\epsilon_c(1),\ \ldots,\ \epsilon_c(T),\ \epsilon_z(1),\ \ldots,\ \epsilon_z(T)]'$, and

$$(3.8) \qquad Z = \begin{bmatrix} 1 & 1 & X(1) & 0 & 0 \\ 1 & 2 & X(2) & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & T & X(T) & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 2 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 1 & T \end{bmatrix}.$$

Park and Ogaki's (1989) seemingly unrelated canonical cointegrating regressions (*SUCCR*) procedure extended the *CCR* procedure to *SUR* systems. Though it is possible to obtain consistent estimates by applying the *CCR* procedure to each regression, the *SUCCR* procedure provides more efficient estimators by utilizing information in the long run correlations of the disturbances.

Let $\epsilon^*(t) = [\epsilon_c(t),\ \epsilon_z(t),\ \epsilon_x(t)]$. We define Ω^*, Σ^*, Δ^* from $\epsilon^*(t)$ as we defined Ω, Σ, Δ from $\epsilon(t)$ in the first subsection. Partition Ω^* and Δ^* so that the subscript *1* corresponds with $[\epsilon_c(t),\ \epsilon_z(t)]'$ and the subscript *2* corresponds with $\epsilon_x(t)$, so that $\Omega^*_{11.2}=\Omega^*_{11} - \Omega^*_{12}(\Omega^*_{22})^{-1}\Omega_{21}$ is a 2 by 2 matrix and $\Delta^*_2 = [\Delta^*_{21},\ \Delta^*_{22}]'$ is a 3 by 1 matrix. In the first step we apply the *OLS* to (2.10), (2.14), and (2.18) separately to obtain consistent estimates of γ_x, Ω^*, Σ^*, Δ^*, and $\Omega^*_{11.2}$. The sample counterparts of Ω^* and Δ^* are formed by (3.1) and (3.2) with $\hat{\epsilon}(t)$ replaced by $\hat{\epsilon}^*(t)=[\hat{\epsilon}_c(t),\ \hat{\epsilon}_z(t),\ \hat{\epsilon}_x(t)]'$. Let $\hat{\gamma}_x$,

$\hat{\Omega}^*$, $\hat{\Sigma}^*$, $\hat{\Delta}^*$, and $\hat{\Omega}^*_{11.2}$ denote these consistent estimates of γ_x, Ω^*, Σ^*, Δ^*, and $\Omega^*_{11.2}$. The second step is to run a regression with transformed data. Let

$$(3.9) \qquad \begin{bmatrix} y^*(t) \\ z^*(t) \end{bmatrix} = \begin{bmatrix} y(t) \\ z(t) \end{bmatrix} - \begin{bmatrix} \gamma_x \\ 0 \end{bmatrix} \hat{\Delta}_2^{*\prime} \hat{\Sigma}^{*-1} \hat{\epsilon}^*(t) - \hat{\Omega}^*_{12}(\hat{\Omega}^*_{22})^{-1} \hat{\epsilon}_x(t)$$

and $X^*(t) = X\ (t) - \hat{\Delta}_2^{*\prime} \hat{\Sigma}^{*-1} \hat{\epsilon}^*(t)$. The *SUCCR* system is

$$(3.10) \qquad Y^* = Z^* \phi + u^*,$$

where Y^* and Z^* are defined from $y^*(t)$, $z^*(t)$, and $X^*(t)$ as Y and Z were defined from $y(t)$, $z(t)$, and $X(t)$. Applying the *GLS* to (3.10), we obtain the *SUCCR* estimator

$$(3.11) \qquad \tilde{\phi} = [Z^{*\prime}(\hat{\Omega}^{*\,-1}_{11.2} \otimes I_T) Z^*]^{-1} Z^{*\prime}(\hat{\Omega}^{*\,-1}_{11.2} \otimes I_T) Y^*.$$

The asymptotic distribution of $\tilde{\phi}$ is (approximately) $N(\phi,\ [Z^{*\prime}(\Omega^{*\,-1}_{11.2} \otimes I_T) Z^*]^{-1})$. An consistent estimate of γ_z is $\tilde{\phi}_2/\tilde{\phi}_5$ in the light of (3.6), and its asymptotic distribution can be derived by the delta method.

Since α_1 is different from zero, we can choose $c_1^*(t)$ as the regressand $y(t)$ rather than $p_2(t)$. Consider the case where $c_1^*(t)$ is chosen as the regressand, and let $y(t)=c_1^*(t)$, $X(t)=p_2(t)$, $z(t)=c_2^*(t)$. The stationarity restriction implies that $y(t)$ and $X(t)$ are stochastically cointegrated with a normalized cointegrating vector $\gamma = 1/\alpha_1$. In this case, $\gamma_z=\alpha_2/\alpha_1$. Thus we can apply the *SUCCR* procedure to $y(t)$, $X(t)$, and $z(t)$ with these definitions to estimate $\gamma_z=1/\alpha_1$ and $\gamma_z=\alpha_2/\alpha_1$. Consistent estimates of α_1 and α_2 are obtained from a *SUCCR* estimate of $\gamma=(\gamma_x,\ \gamma_z)'$ from the formulas

$\alpha_1 = 1/\gamma_x$ and $\alpha_2 = \gamma_z/\gamma_x$. Asymptotic distributions of these estimators can be derived by the delta method. It should be noted that $\alpha_2 = \mu_c/[\mu_z \gamma_x]$ and that the estimators of μ_c and μ_z converge at a faster rate than that for γ_x. Consequently, the sampling errors for estimates of μ_c and μ_z have no impact on the asymptotic distribution of the estimator of α_2. The distribution is derived as if estimates of μ_c and μ_z were known true values.

Testing

As in the first subsection, linear restrictions can be tested by χ^2 tests which are free from nuisance parameters. Consider a linear restriction (3.3), where R is a known matrix with q rows and full row rank q, and r is a known $q\times 1$ vector of real numbers. It should be noted that components of $\hat{\phi}$ and $\bar{\phi}$ converge at different rates: the estimators for μ_x and μ_c converge at a faster rate than those for γ_x converge. Hence we assume that each row of R does not pick up estimators with different convergence rates. Let

$$(3.14) \qquad G_R = (R\tilde{\phi} - r)'\{R[Z^{*\prime}(\hat{\Omega}^{*\,-1}_{11.2}\otimes I_T)Z^*]^{-1}R'\}^{-1}(R\tilde{\phi} - r)$$

for $\tilde{\phi}$ and

$$(3.15) \qquad G_R = (R\bar{\phi} - r)'\{R[Z^{*\prime}Z^*]^{-1}Z^{*\prime}(\Omega^*_{11.2}\otimes I_T)Z^*[Z^{*\prime}Z^*]^{-1}R'\}^{-1}(R\bar{\phi} - r)$$

for $\bar{\phi}$. Then each of these G_R statistics asymptotically has a χ^2_q distribution.

We can use G_R statistics for a *SUCCR* system with spurious deterministic trends added to (2.14) to test for stochastic cointegration. Consider a *SUCCR* system which consists of

$$(3.16)\quad y(t) = \theta_c + \sum_{i=1}^{q} \eta_i t^i + \gamma' X(t) + \epsilon_c(t) .$$

and (2.18). We apply the *SUCCR* procedure to this system. Let $H(p,q)$ denote the G_R statistic to test the hypothesis $\eta_p = \eta_{p+1} = \ldots = \eta_q = 0$. Then $H(p,q)$ converges in distribution to a χ^2_{p-q} random variable under the null of stochastic cointegration. As in the first subsection, the $H(1,q)$ tests are consistent against the alternative of no stochastic cointegration.

4. *Trend Properties of the Data*

In this section, we test empirical validity of Assumptions 2 and 3. In the first subsection, we explain the data used in this paper. In the second subsection, we report results of tests for difference stationarity and trend stationarity of time series of real consumption expenditures and relative prices. In the third subsection, we report results of tests for Assumption 2a.

The Data

We used seasonally adjusted monthly data in the NIPA. Three measures of nondurable consumption were alternatively used as the first good to obtain estimates of the relative risk aversion coefficient for each of three measures. These were nondurables plus services (NDS), nondurables (ND), and nondurables minus clothing (NDC). Durables consumption was used as the second good when NDS was taken as the first good; durables and services were alternatively used as the second good when ND was taken as the first good; and durables, services, clothing and shoes (clothing for short)were used alternatively as the second good when NDC was taken as the first good.

Seasonally adjusted monthly data for the NIPA was taken from the PCE magnetic tape of the NIPA prepared by the Bureau of Economic Analysis, U.S.

Department of Commerce. For $C_1^m(t)$ and $C_2^m(t)$ in the model, real per capita consumption expenditures were constructed by dividing personal consumption expenditures in constant 1982 dollars by the total population including armed forces overseas obtained from the CITIBASE.[3] The implicit deflator was used as the price for each consumption series. The implicit deflators for each series was constructed by dividing personal consumption expenditure in current dollars by that in constant 1982 dollars.

The sample period was from February 1959 to December 1986 unless otherwise noted. Hence each time series consists of three hundred thirty five observations. For the empirical results involving services series, we also used the sample period from January 1968 to December 1986 for the reason to be explained below. In this case, the sample size is two hundred twenty eight.

Tests for Difference and Trend Stationarity

Tests for Time Series of Consumption Expenditures

The stationarity restriction was derived under the assumption that $c_1^m(t)$ and $c_2^m(t)$ are stationary after first differencing (note that trend stationary processes also satisfy this requirement.) In figures 1-6, we plot the first differences of log of real per capita consumption for NDS, ND, NDC, durables, services, and clothing. These series show no apparent nonstationarity in these figures except for the series for services in figure 5. Variance of the series for services appears to increase substantially after 1968. Since 1967 is one of the benchmark years used to

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We incorporated the revision of population estimates reported in *Current Population Reports* (Series p-25, No.1036) issued in March 1989 by Bureau of the Census after the release of the version of the CITIBASE we used.

construct monthly consumption series (see, e.g., Byrnes, Donahoe, Hook, and Parker [1979, p.24]), this seems to be caused by nonstationary measurement errors that we do not allow. For this reason, we excluded the period from January 1959 to December 1967 from some of our empirical analyses.

Our next results are concerned with discrimination between trend stationarity and difference stationarity of consumption series. Let $\{X(t)\}$ be the process of interest. We are interested in whether $X(t)$ is difference stationary or trend stationary. Consider an *OLS* regression

$$(4.1) \qquad X(t) = \sum_{i=0}^{q} \hat{\eta}_i t^i + \hat{\epsilon}(t),$$

and define $\hat{\sigma}^2 = (1/T)\sum_{t=1}^{T} \hat{\epsilon}(t)^2$. Let $F(p,q)$ denote the standard Wald test statistic in regression (4.1) for the null hypothesis $\eta_{p+1}=\eta_{p+2}=,\ldots,=\eta_q=0$. Let $J(p,q) = (1/T)F(p,q)$ and $G(p,q) = (\hat{\sigma}^2/\hat{\Omega})F(p,q)$, where $\hat{\Omega}$ is defined by (3.1) for $\hat{\epsilon}(t)$ in (4.1). Then $J(1,q)$ converges in distribution to a nondegenerate random variable under the null hypothesis that $X(t)$ is difference stationary; $G(1,q)$, to a χ^2_{q-1} random variable under the null hypothesis that $X(t)$ is trend stationary (see Park and Choi [1988]). Hence $J(1,q)$ can be used to test the null of difference stationarity against the alternative of trend stationarity. We reject the null of difference stationarity when the $J(1,q)$ statistic is smaller than critical values tabulated by Park and Choi (1988). The $G(1,q)$ statistic can be can be used to test the null of trend stationarity against the alternative of difference stationarity. We reject the null of trend stationarity when the $G(1,q)$ statistic is larger than critical values. These tests are consistent.

For the null of difference stationarity, we also used Z_α and Z_t test statistics of Phillips and Perron (1988) and Ouliaris, Park, and Phillips

(1988) that correct Dickey and Fuller (1979) statistics for serial correlation. Critical values for Z_α and Z_t statistics are the same with those for Dickey and Fuller (1979) test statistics tabulated in Fuller (1976, p.371 and p.373) by construction. Consider an *OLS* regression

$$(4.2) \qquad X(t) = \hat{\eta}_0 + \hat{\eta}_1 t + \hat{\alpha} X(t-1) + \hat{\epsilon}(t).$$

The Z_α statistic modifies $T(\hat{\alpha}-1)$ and the Z_t statistic modifies the t statistic for the hypothesis that α is equal to one. These modifications for serial correlation involve estimation of the long run variance with the formula (3.1) for $\hat{\epsilon}(t)$ in (4.2). For reasons suggested by Park (1989), we focused on these single unit root tests rather than the joint tests for the null hypothesis that α is equal to one and η_1 is equal to zero that were analyzed by Dickey and Fuller (1981) among others.

A serious problem about the Z_α and Z_t statistics are size distortions in small samples. Simulations reported in Phillips and Perron (1988) showed that the size distortion problem could be substantial when the stationary component of the series is small relative to the random walk component in the sense of Cochrane (1988): the probability that Z_α and Z_t tests with nominal size 5 per cent reject the null hypothesis of difference stationarity may exceed 90 per cent when the null hypothesis is true (Also see Schwert [1987] for related simulation results). Simulations by Park and Choi (1988) showed that that the size distortion problem for $J(p,q)$ tests could be much less severe and could disappear at a much faster rate as the sample size increases than that for the Z_α and Z_t tests. The size distortion problem for the Z_α and Z_t tests seems to be related with the estimation of the long run variance. It should be noted that the $J(p,q)$

tests do not require the estimation of the long run variance.On the other hand, their simulations showed that the *J(p,q)* tests may have less power than the Z_α and Z_t tests in small samples.

Table 1 presents results of the *J(1,2)*, *J(1,3)*, ..., *J(1,6)*, Z_α and Z_t tests with the null of difference stationarity of log real per capita consumption expenditures. For services, we used the sample period of 1968:1-1986:12 as well as the sample period of 1959:1-1986:12 because of the suspected heteroskedasticity of the series mentioned above. We also used these two sample periods for the time series for NDS because suspected nonstationary measurement errors in services affect this series which is the sum of ND and services.

There was no evidence against difference stationarity of the time series examined according to the *J(1,q)* tests. On the other hand, there was evidence against difference stationarity for durables and clothing according to the Z_α test at the 5 per cent significance level. The Z_t tests showed similar evidence against difference stationarity of these series. For the results reported in table 1 for the Z_α and Z_t tests, we used the lag truncation number of 30 and the lag window of Parzen's estimates in estimating the long run variance as in (3.3). We also tried the lag truncation numbers of 10, 20, 40, 50, 60, 70, and 80. Our results for Z_α and Z_t tests were not very sensitive to the choice of the lag truncation number. For NDS, ND, NDC, and services, the Z_α and Z_t statistics were not significant at the 10 per cent level for any choice of the lag truncation number. For durables and clothing, the Z_α and Z_t statistics were not significantly negative at the 10 per cent level when the lag truncation number of 10 was used and were not significant at the 5 per cent level. However, they were always significantly negative at the 5 per cent level and sometimes at the 1 per cent level when the lag truncation numbers greater than 30 were used.

Two interpretations are possible for these conflicting results between the *J(p,q)* tests and the Z_α and Z_t tests for durables and clothing. A possibility is that the size distortion of Z_α and Z_t is the source of the problem. Since the time series for durables and clothing are not smooth, the random walk component of these series seem to be small relative to the stationary component (see Ogaki [1988] for diagnosis similar to that of Cochrane [1988] for the time series of durables consumption). This means that the Z_α and Z_t tests may not be reliable for these time series. Even if these series have nonzero random walk components and are difference stationary, these tests may reject the null of difference stationary with high probability. Another possibility is that lower power of *J(p,q)* tests in small samples is causing the problem when these series are trend stationary.

In table 2, we report results of the *G(1,2)*,..., *G(1,6)* tests for the null hypothesis of trend stationarity. The lag truncation number used for the results in table 2 was 80. When the sample period of 1959:1-1986:12 was used, the *G(1,2)* test rejected the null of trend stationarity in favor of the alternative of difference stationarity at the 5 per cent level for NDS, ND, NDC, services, and clothing. For these series, some of the *G(1,3)* and *G(1,4)* statistics were significantly large at the 10 per cent level. This evidence against trend stationarity is not compatible with our Z_α and Z_t test results. On the other hand, no test statistics were significant at the 10 per cent level for durables, which is compatible with our Z_a and Z_t test results. When the shorter sample period of 1968:1-1986:12 was used for NDS and services, no tests rejected the null hypothesis of trend stationarity.

We checked sensitivity of our results with respect to the choice of the lag truncation number. For NDS with the sample period of 1959:1-1968:12, the values of the *G(1,2)* statistic for the lag truncation numbers of 10, 20, 30, 40, 50, 60, 70, 80, 90, and 100 were 28.015 (0.000), 14.463 (0.000), 10.026 (0.002), 7.863 (0.005), 6.608 (0.010), 5.805 (0.016), 5.262 (0.022), 4.882 (0.027), 4.612 (0.032) and 4.418 (0.036), respectively, where the numbers in parentheses are probability values. Thus the *G(1,2)* statistic was stabilized enough to fall in the region between the critical value for the 5 per cent level and that for the 1 per cent level for the lag truncation numbers which are greater than 50. Statistical inferences for the results in table 2 at the 1 per cent and 5 per cent significance levels were robust to the choice of the lag truncation numbers of 60, 80, or 100 except for the following three cases: the *G(1,2)* statistic for ND was not significant at the 5 per cent level when the lag truncation number of 100 was used (the probability value was 0.058); the *G(1,2)* statistic for clothing was significant at the 1 per cent level when the lag truncation number of 60 was used; the *G(1,3)* statistic for clothing was not significant at the 5 per cent level when the lag truncation number of 100 was used (the probability value was 0.052).

We conclude this subsection by summarizing our results in tables 1 and 2. We did not find evidence against difference stationarity at the 10 per cent significance level for any of the consumption series we tested except for durables and clothing. For durables and clothing, we did not find evidence against difference stationarity for durables and clothing at the 10 per cent level in terms of the *J(1,q)* tests. We found evidence against trend stationarity for consumption series of NDS, ND, NDC, services, and clothing at the 5 per cent level but not for series of durables when we used the longer sample period. When the shorter sample period was used, we did not find evidence against either difference stationarity or trend stationarity for any of the consumption series we used.

In the light of these results, we employed the following specifications for the rest of the empirical results reported in the present paper. We employed the specification that the log of real consumption series is difference stationary for services when the longer sample period was used and for NDS, ND, NDC, and clothing. We tried both the specifications that log of consumption is difference stationary and that log of consumption is trend stationary for services when the shorter sample period is used and for durables.

Tests for Time series of Relative Prices

As shown in Section 2, the stationarity restriction implies that log of the relative price is difference stationary under either Assumption 2 or Assumption 3. We now test this implication of the model.

Table 3 presents results of the $J(1,2), \ldots, J(1,6)$ and Z_α and Z_t tests for log of relative prices. The lag truncation number used for the results of the Z_α and Z_t tests reported in table 3 was 30. No Z_α and Z_t test statistics showed evidence against the null hypothesis of difference stationary relative prices at the 10 per cent significance level. This conclusion was robust against the choice of the lag truncation numbers of 10, 20, 30, 40, 50, 60, 70, 80, 90, and 100. Results of the $J(1,q)$ tests were similar except for those for the relative price of durables and NDS when the sample period of 1968:1-1986:12 was used. The $J(1,2)$ test statistic was significantly small at the 5 per cent level in this case; the $J(1,4)$ and $J(1,6)$ statistics, at the 10 per cent level. Thus these $J(1,q)$

test for the relative price of NDS and durables showed evidence against our model for the shorter sample period. None of these statistics are significant at the 1 per cent level, however, and the *J(1,q)* tests did not reject the null hypothesis of the difference stationary relative price for the longer sample period at the 10 per cent level. Thus the rejection could be a manifestation of the size distortion problem of the *J(1,q)* tests that disappears as the sample size increases.

Table 4 presents results of the *G(1,2)*, ..., *G(1,6)* tests for the null of trend stationarity. The main purpose of table 4 is to give some ideas about small sample power of the *H(1,q)* tests as we discuss in the next section. The lag truncation number used for the results in this table was 80. When the sample period of 1959:1-1986:12 was used, either the *G(1,2)* or the *G(1,3)* tests rejected the null of trend stationarity in favor of the alternative of difference stationarity at the 5 per cent level for all the relative prices we tested. For this sample period, some of the *G(1,4)* and *G(1,5)* statistics were significantly large at the 10 per cent and 5 per cent levels.

We also used the shorter sample period of 1968:1-1986:12 for the relative prices involving NDS or services. The *G(1,2)* test rejected the null of trend stationarity for the relative price of services and ND and that of services and NDC. No tests rejected the null hypothesis of trend stationarity for the relative price of durables and NDS in favor of the alternative of difference stationarity. However, the *G(1,2)* statistic fell in the tail of "too good to be true" tail for the null of the trend stationarity at the 5 per cent level. This observation is compatible with our conjecture of the size distortion for the *J(1,2)* test for this relative price series with this sample period.

We checked sensitivity of our results with respect to the choice of the lag truncation number. For the relative price of durables and NDS with the sample period of 1959:1-1968:12, the values of *G(1,2)* for the lag truncation numbers of 10, 20, 30, 40, 50, 60, 70, 80, 90, and 100 were 25.844 (0.000), 13.909 (0.000), 10.014 (0.002), 8.144 (0.004), 7.091 (0.008), 6.440 (0.011), 6.016 (0.014), 5.732 (0.017), 5.540 (0.019) and 5.412 (0.020), respectively, where the numbers in parentheses are probability values. Thus *G(1,2)* statistic was stabilized enough to fall in the region between the critical value for the 5 per cent level and that for the 1 per cent level for the lag truncation numbers which are greater than 60. Statistical inferences for results in table 4 at the 1 per cent and 5 per cent significance levels were robust to the choice of the lag truncation numbers of 60, 80, or 100 except for the following cases: the *G(1,4)* statistic for the relative price of durables and NDS with the longer sample period, the *G(1,3)* statistic for the relative price of services and ND, the *G(1,3)* statistic for the relative price of services and NDC with the shorter sample period, and the *G(1,3)* and *G(1,4)* statistics for the relative price of clothing and NDC were marginally significant at the 5 per cent when the lag truncation number of 60 was used; the *G(1,4)* statistic with the longer sample period and the *G(1,3)* statistic with the shorter sample period for the relative price of services and ND were not significant at the 5 per cent level when the lag truncation number of 100 was used.

We conclude this subsection by summarizing empirical results in tables 3 and 4. We did not find evidence against difference stationarity for any of the relative prices we tested at the 10 per cent significance level except for the relative price of durables and NDS. We found evidence against trend stationarity for all the relative prices at the 5 per cent level when the longer sample period was used.

Tests for No Stochastic Cointegration

Suppose that Assumption 2a is satisfied. Then Assumption 2b (together with the stationarity restriction) implies that log of the relative price is difference stationary as in proposition 1. Thus rather strong evidence in favor of difference stationarity of most of the relative prices supports Assumption 2b. The difference stationarity of the relative price, however, does not imply Assumption 2b because the two consumption series may be stochastically cointegrated with a normalized cointegrating vector other than $[\alpha_1, -\alpha_2]'$. For this reason, we report results of tests for the null hypothesis of no stochastic cointegration in this subsection.

Consider an *OLS* regression

$$(4.3) \qquad y(t) = \hat{\theta} + \sum_{i=1}^{q} \hat{\eta}_i t^i + \hat{\gamma}' X(t) + \hat{\epsilon}(t).$$

where $y(t)$ and $X(t)$ are difference stationary processes, and let $F(p,q)$ denote the standard Wald test statistic in regression (8.3) for the null hypothesis $\eta_{p+1}=\eta_{p+2}=,\ldots,=\eta_q=0$. Define $I(p,q) = (1/T)F(p,q)$. Ouliaris, Park and Choi (1988) showed that $I(1,q)$ converges in distribution to a nondegenerate random variable under the null hypothesis that $y(t)$ and $X(t)$ are not stochastically cointegrated. We reject the null of no cointegration when $I(p,q)$ statistics are smaller than critical values of $I(p,q)$ test statistics tabulated by Park, Ouliaris, and Choi (1988). The $I(p,q)$ tests are consistent against the alternative of stochastic cointegration. The $I(p,q)$ tests basically apply the $J(p,q)$ tests to the residual of regression (4.3). Alternatively, we can apply the Z_α or Z_t tests to the residual as in

Phillips and Ouliaris (1988). We did not use these Z_{α} and Z_{t} tests because of the serious size distortion problem mentioned above.

Table 5 presents results of the *I(1,5)* test for the null of no cointegration of $c_1^m(t)$ and $c_2^m(t)$ for various choices of the first and second goods. For each pair of consumption goods, we can choose the first good as the regressand ($y(t)=c_1^m(t)$, $X(t)=c_2^m(t)$) or the second good as the regressand ($y(t)=c_2^m(t)$, $X(t)=c_1^m(t)$) for the *I(1,q)* tests. Hence we changed the choice of the regressand for each pair of goods. No *I(1,5)* test statistic was significantly small at the 10 per cent significance level. Thus the results in table 5 show no evidence against Assumption 2b.

5. Empirical Results of Cointegrating Regressions

In this section, we report results of cointegrating regressions. The first subsection presents results for the demand system with two goods under Assumption 2; the second subsection, for the demand system with two goods under Assumption 3; the third subsection, for the demand system with three goods.

The Demand System with Two Goods under Assumption 2

In this subsection, we assume that all the consumption series are difference stationary, so that Assumption 2 is satisfied for each pair of consumption series. Table 6 presents *CCR* results. For each pair of consumption series, we first chose the relative price, p_2, as the regressand. Table 6 reports estimates of α_1, α_2, and α_1/α_2, and the G_R test statistic for the null hypothesis $\alpha_1=\alpha_2=1$ from regression (3.2). The G_R statistic tests the null hypothesis that the demand system is the linear expenditure system as explained in Section 6. Table 6 also reports the *H(0,1)* test statistic for the deterministic cointegration restriction from regression (3.11) with q=1 and the $H(1,2)$, $H(1,3)$, and $H(1,4)$ test statistics for stochastic cointegration from regression (3.11) with q= 2, 3, and 4, respectively. When none of these $H(p,q)$ tests rejected the model at the 1 per cent significance level, we applied the CCR with c_1^m chosen as the regressand as reported in table 6. For this choice of the regressand, table 6 reports the G_R test statistic for the null hypothesis $1/\alpha_1=\alpha_2/\alpha_1=1$ that is a linear restriction on estimated parameters to test the linear expenditure system. When none of the $H(p,q)$ tests with c_1^m as the regressand rejected the model at the 1 per cent significance level, we applied the CCR with c_2^m chosen as the regressand as reported in table 6. For this choice of the regressand, the reported G_R statistic tests the null hypothesis $1/\alpha_2=\alpha_1/\alpha_2=1$. The lag truncation number used for the results in table 6 was 80. We will discuss sensitivity of the results with respect to the choice of the lag truncation number in the text.

The $G(1,q)$ test results reported in the last section provides some ideas about small sample power of the $H(1,q)$ tests. For example, the value $G(1,q)$ for p_2 will be close to the value of $H(1,q)$ in the regression with p_2 as the regressand if estimated α_1 and α_2 are close to zero. The reason why we do not report $H(1,5)$ and $H(1,6)$ in this paper is that the $G(1,5)$ and $G(1,6)$ test statistics were not significant for any variable at the 5 per cent level in the last section.

We found much evidence against the stationarity restriction for NDS and durables under Assumption 2. The deterministic cointegration restriction was rejected at the 1 per cent significance level by the $H(0,1)$ test with c_1^m chosen as the regressand for the longer sample period. In this case, we also found evidence against stochastic cointegration in terms of the $H(1,4)$ test. For the shorter sample period, the $H(0,1)$ test with p_2 chosen as the regressand rejected the deterministic cointegrating restriction at the 1 per cent level. Even more overwhelming evidence against the linear expenditure system was found by the G_R test. Most of the point estimates of α_1 and α_2 had theoretical incorrect negative sign for both sample periods (we do not report an estimate of α_1/α_2 when point estimates of either α_1 or α_2 was negative). Some of the point estimates of these parameters were significantly negative at the 1 per cent level.

We also found much evidence against the stationarity restriction when ND was used as the first good. In this case, we used either durables or services as the second good. When the p_2 was used for the longer sample period, the *H(0,1)* test rejected the deterministic cointegration restriction at the 1 per cent level for ND and durables and for ND and services. For the shorter sample period, the deterministic cointegration restriction for ND and services was rejected at the 1 per cent level. Most of the point estimates for α_1 and α_2 had theoretical correct positive sign for the models with ND, however. Even when the point estimate of α_1 was negative for the model with ND and durables, it was not significantly negative at the 5 per cent level. Also, there was no evidence against stochastic cointegration in terms of the *H(1,2)*, *H(1,3)*, and *H(1,4)* tests at the 5 per cent level. Evidence for stochastic trends found in the last section in terms of *G(1,2)*, *G(1,3)* and *G(1,4)* tests were removed in these cointegrating regressions. In these respects, the results for ND were more encouraging than those for NDS. We found overwhelming evidence against the linear expenditure system in terms of the G_R test for all the cases reported.

We found much evidence against the stationarity restriction for the models with NDC used as the first good and either durables, services or clothing chosen as the second good when the longer sample period was used:

the deterministic cointegration restriction was rejected at the 1 per cent level by the $H(0,1)$ test. Stochastic cointegration, however, was not rejected at the 1 per cent level, and all the point estimates had the theoretically correct positive sign for all the models. We found overwhelming evidence against the linear expenditure system in terms of the G_R test for NDC and durables and for NDC and services. We found much less evidence against the linear expenditure system for NDC and clothing in terms of the G_R test: the linear expenditure system was not rejected at the 1 per cent level by the G_R test.

We found little evidence against the stationarity restriction for the model with NDC and services when the shorter sample period was used: neither the deterministic cointegration restriction was rejected by the $H(0,1)$ test nor stochastic cointegration was rejected by the $H(1,2)$, $H(1,3)$, and $H(1,4)$ tests at the 5 per cent level for any choice of the regressand. All the point estimates for α_1 and α_2 have the theoretically correct positive sign. Again we found overwhelming evidence against the linear expenditure system for this model. When the longer sample period was used for this model, the deterministic cointegration was rejected at the 1 per cent level as we noted earlier. The point estimate of each of the parameters for the longer sample period, however, were not too different from the estimates of the parameter for the shorter sample period to be explained by the standard errors. The point estimates of α_1 in the model with NDC and durables and the model with NDC and clothing were also not very different from those in the model for NDC and services if we consider standard errors.

We checked sensitivity of our results in table 6 with respect to the choice of the lag truncation number. The lag truncation number of 80 was used for the results in table 6 because the $G(1,q)$ statistics in the last section were typically stabilized around the lag truncation number of 50 or 60. We tried the lag truncation numbers of 60 and 100. For all the cases where the $H(0,1)$ test rejected the deterministic cointegration restriction at the 1 per cent level in table 6, the test rejected the restriction at the 1 per cent level when the lag truncation number of 60 or 100 was used. For all the cases where the G_R statistic was significant at the 0.1 per cent in table 6, it was significant at the 0.1 per cent when the lag truncation number of 60 or 100 was used. The sign of the point estimate of α_1 or α_2 never changed when the lag truncation number of 60 or 100 was used. When a point estimate of α_1 was significantly negative at the 5 per cent level in table 6, it was significantly negative at the 5 per cent level when the lag truncation number of 60 or 100 was used. For the model with NDC and services when the shorter sample period was used, the $H(p,q)$ statistics were not significant at the 5 per cent level except for the following two cases: the value of $H(0,1)$ was 4.410 with the probability value 0.036 when c_1^m was used as the regressand and the lag truncation number of 60 was used; the value of $H(1,2)$ was 4.174 with the probability value 0.041 when c_2^m was used as the regressand and the lag truncation number of 100 was used.

We now examine results for the model with NDC and services when the shorter sample period was used in more detail. This is the only model that was not rejected at the 5 per cent level, and the other models were rejected at the 1 per cent level. Standard errors of each parameter indicate the source of finite sampling errors that caused differences in point estimates when different regressands were chosen. The point estimates for α_1 and α_2 had the lowest standard errors when p_2 was used as the regressand. These parameters are nonlinear functions of the estimated cointegrating vector when any of the other variables is used as the regressand. On the other hand, the point estimates for α_1/α_2 have the lowest standard error when c_2^m was chosen as the regressand. Again this parameter is a nonlinear function of the estimated cointegrating vector when any of the other variables is chosen as the regressand. Hence finite sampling errors caused by the linear approximation of nonlinear functions appears to be important source that caused the differences in point estimates.

Given that the linear approximation does not seem reliable (see Phillips and Park [1988] for related problems), inferences about a parameter of interest should be based on the regression for which the parameter is estimated linearly. The parameters α_1 and α_2 are estimated linearly in the regression with the regressand of $p_2(t)$. According to this regression, the *RRA* parameter for NDC, α_1, is likely to be between 0 and 2.7 and the curvature parameter for services, α_2, is likely to be between 0 and 0.6 with the 95% confidence level. The parameter α_1/α_2 is estimated linearly in the regression with the regressand of $c_2^m(t)$. According to this regression, the ratio of income elasticities for services and NDC is likely to be between 2.2 and 3.7 with the 95% confidence level. Hence there is evidence against the hypothesis that preferences for NDC and services are homothetic ($\alpha_1=\alpha_2$), which is weaker than the hypothesis of the linear expenditure system ($\alpha_1=\alpha_2=1$). Our estimates indicate that the income elasticity for services is greater than that for NDC.

We checked sensitivity of our results with respect to the choice of the data of population. For this purpose, we tried the civilian noninstitutional adult (age sixteen and over) population obtained from the CITIBASE to construct real per capita consumption series for the NDC-services model with the shorter sample period. We found little sensitivity for our results in tables 6 for this model.

In table 7 we compare estimates on which the deterministic cointegration restriction was imposed and those on which the restriction was not imposed for the model with NDC and services. We applied the *CCR* method to regression (2.14) to obtain the unrestricted estimates. Standard errors indicated that there was much gain in efficiency by imposing the restriction on the estimates. Imposing the restriction helped to reduce the dependency of point estimates on the choice of the regressand.

The Demand System with Two Goods under Assumption 3

In this subsection, we assume that the consumption series for durables and services are trend stationary. We did not find evidence against trend stationarity of consumption of durables at the 5 per cent level in the last section. We did not find evidence against trend stationarity of consumption of services at the 5 per cent level when we used the shorter sample period. We assume that consumption series for NDS, ND, and NDC are difference stationary, so that Assumption 3 is satisfied for each pair of consumption series.

Table 8 presents *SUCCR* results. For each pair of consumption series, we first chose the relative price, p_2, as the regressand. Table 8 reports estimates of α_1, α_2, and α_1/α_2, and the G_R test statistic for the null hypothesis α_1=1 and μ_c=-μ_z. Since $\alpha_2 = -\mu_c/\mu_z$ as in (3.14), the hypothesis μ_c=-μ_z is equivalent with the hypothesis α_2=1. Hence the G_R statistic tests the null hypothesis that the demand system is the linear expenditure system. Table 8 also reports the *H(1,2)*, *H(1,3)*, and *H(1,4)* test statistics for stochastic cointegration from the *SUCCR* system (3.12′) and (3.13) with q= 2, 3, and 4, respectively. When none of these *H(1,q)* tests rejected the model at the 1 per cent significance level, we applied the *SUCCR* with c_1^m chosen as the regressand as reported in table 8. (It happened that this was the case for all the results in table 8). With this choice of the regressand, the G_R statistic tests the hypothesis $1/\alpha_1=1$ and $\mu_c=\mu_z$, which is equivalent with the hypothesis $\alpha_1=\alpha_2=1$ in this case. The lag truncation number used for the results in table 8 was 80. We will discuss sensitivity of the results with respect to the choice of the lag truncation number in the text.

We found much evidence against the cointegration restriction for NDS and durables under Assumption 3 when the longer sample period was used. The deterministic cointegration restriction was rejected at the 1 per cent significance level by the *H(0,1)* test with c_1^m chosen as the regressand for the longer sample period. In this case, we also found evidence against stochastic cointegration in terms of the *H(1,2)* and *H(1,3)* tests at the 1 per cent level. When we used the shorter sample period for NDS and durables, we found little evidence against the stationarity restriction. The *H(1,q)* tests did not reject stochastic cointegration at the 5 per cent level. The point estimates of the parameters were positive and were not very different from those obtained with the longer sample period, considering their standard errors. Overwhelming evidence against the linear expenditure system was found by the G_R test for both the longer and shorter sample periods.

We also did not find evidence against the stationarity restriction in terms of the *H(1,q)* tests at the 5 per cent level when ND was used as the first good. When we used durables as the second good, only the longer sample period was used because the suspected nonstationary measurement errors should not affect ND and durables. When we used services as the second good, only the shorter sample period was used because the hypothesis of trend stationary consumption of services was rejected at the 5 per cent level for the longer sample period. All the point estimates for α_1 and α_2 had theoretical correct positive sign for these models with ND. We found overwhelming evidence against the linear expenditure system in terms of the G_R test.

When we used NDC as the first good, we used durables as the second good with the longer sample period or services as the second good with the shorter sample period. Though we found evidence against stochastic cointegration at the 5 per cent level in terms of the *H(1,2)* test for both cases, we did not reject the stationarity restriction at the 1 per cent level for either of the two cases. All the point estimates for α_1 and α_2 were positive. We found overwhelming evidence against the linear expenditure system in terms of the G_R test.

We checked sensitivity of our results in table 8 with respect to the choice of the lag truncation number. We tried the lag truncation numbers of 60 and 100. When the *H(1,q)* test rejected (did not reject) stochastic restriction at the 1 per cent level in table 8, the test rejected (did not reject) the restriction at the 1 per cent level when the lag truncation number of 60 or 100 was used (q=2,3,4) except for the following two cases. When the lag truncation number of 60 was used for NDC and durables, the value of the *H(1,2)* statistic was 6.784 with the probability value 0.009; when the lag truncation number of 100 was used for NDC and services the value of the *H(1,3)* statistic was 9.923 with the probability value 0.007. When the G_R statistic was significant at the 0.1 per cent in table 8, it was significant at the 0.1 per cent when the lag truncation number of 60 or 100 was used except for the following case: when the shorter sample period was used for NDS and durables, the lag truncation number of 60 yielded the G_R statistic of 10.906 with the probability value 0.004. The sign of the point estimate of α_1 or α_2 never changed when the lag truncation number of 60 or 100 was used.

In the *SUCCR* system, α_1 is estimated linearly by the regression with $p_2(t)$ as the regressand. Neither α_2 nor α_1/α_2 is estimated linearly by any regression in the *SUCCR* system. However, the regression with $p_2(t)$ as the regressand should be more reliable for estimation of α_2 because $\alpha_2 = \mu_c/\mu_z$ in this regression: the estimators for μ_c and μ_z converge at a faster rate than the estimators for the other parameters. For similar reasons, the regression with $c_1^m(t)$ as the regressand should be more reliable for estimation of α_1/α_2.

The *RRA* coefficient for NDS is likely to be between 0.22 and 0.77 with the 95 per cent confidence level; the curvature parameter for durables, α_2, is likely to be between 0.6 and 1.1; and the ratio of the income elasticities for durables and NDC, α_1/α_2, is likely to be between 0.8 and 0.9 according to our results for NDS and durables with the shorter sample period. Since we estimated α_1/α_2 to be significantly smaller than 1, there is evidence against the hypothesis that preferences for NDS and durables are homothetic that is weaker than the hypothesis of the linear expenditure system. Our estimates indicate that the income elasticity for NDS is slightly greater than that for durables.

We checked sensitivity of our results with respect to the data of population. For this purpose, we tried the civilian noninstitutional adult (age sixteen and over) population obtained from the CITIBASE to construct real per capita consumption series for the NDS-durable model and ND-services model with the shorter sample period. We found little sensitivity for our results in tables 8 for these models.

The Demand System with Three Goods

This subsection reports results for *SUCCR* estimation of the demand system for ND, services, and durables and the demand system for NDC, services, and durables. There are two purposes. First, we test the cross-equation restriction discussed in Section 2 for the demand system with three goods. This restriction requires that the *RRA* coefficient implied by the first order condition involving services and that implied by the first order condition involving durables be the same. Second, we impose the cross-equation restriction on estimates to obtain sharper estimates of the *RRA* coefficient.

Table 9 reports *SUCCR* results. We used the shorter sample period and prices as the regressands. The first two panels report results for ND; the second two panels, results for NDC. For each measure of nondurable consumption, the log of services consumption is assumed to be difference stationary in the first panel, and it is assumed to be trend stationary in the second panel. The log of ND consumption and that of NDC consumption are assumed to be difference stationary and the log of durables consumption is assumed to be trend stationary throughout the table. The G_R statistic tests the cross-equation restriction. For both NDC and ND, the restriction is accepted when the log of services is assumed to difference stationary and rejected decisively when the log of services consumption is assumed to be trend stationary.

6. *Comparisons with GMM estimates*

In this section, we compare our estimates of the *RRA* coefficient obtained by the cointegration approach with estimates of the *RRA* coefficient obtained by the *GMM* approach. The *GMM* approach of Hansen and Singleton

(1982) is closely related with our cointegration approach. In the model presented in Section 2, suppose that the preferences of the first good is time separable, so that $S_1(t) = C_1(t)$. In addition, assume that R_t does not depend on $W^e(t-1)$ and $\sigma(t)$ is constant over time. Thus there is no borrowing constraint or unknown preference shock. Then the asset pricing equation

$$(10.1) \qquad E_t\{R_{t+1}\beta[C_1^*(t+1)/C_1^*(t)]^{-\alpha_1}\}=1$$

must be satisfied in an equilibrium. Hansen and Singleton (1982) utilized asset pricing equation (10.1) to estimate α_1 and β using NDS or ND as the first good.

Tauchen (1986) showed that the *GMM* estimators could have reasonable small sample properties in simulations where the true parameter values were α_1=0.3 and β=0.97 while Kocherlakota (1988) showed that the *GMM* estimator could underestimate α_1 and β in small samples in simulations where α_1=13.7 and β=1.139. Kocherlakota also showed that Friend and Blume's (1975) technique could also underestimate α_1. Though the parameter values he used may be somewhat counterintuitive, these values are theoretically valid in the sense a well defined equilibrium exists. Kocherlakota argued that these large values of α_1 and β could explain empirical rejections of the *C-CAPM* with the *CRRA* utility function found by Hansen and Singleton (1982, 1984), Mehra and Prescott (1985), and Hansen and Jagannathan (1988) among others using Treasury Bills and stock market returns.

Our estimates are compatible with most of the *GMM* estimates reported in Hansen and Singleton (1982, 1984), taking standard errors into consideration. Our estimates of the *RRA* coefficient do not support a large value of the *RRA* coefficient that Kocherlakota (1988) proposed.

7. Conclusions

This paper proposed an approach to estimate the preference parameters and to test the first order condition based on stochastic and deterministic cointegration. The deterministic cointegration restriction played important roles in our empirical work. We obtained much sharper estimates by imposing the deterministic cointegration restriction. The test for the deterministic cointegration restriction with the null of cointegration provided strong evidence against many models.

We obtained favorable empirical results for the models when the log of nondurable consumption and the log of services consumption were assumed to be difference stationary and the log of durables consumption was assumed to be trend stationary. We found overwhelming evidence against the linear expenditure system for all the models we examined except for the model with NDC and clothing.

Our interval estimates with the 95 per cent confidence level of the *RRA* coefficient from our results for the models that were supported by the data are as follows. The *RRA* coefficient for NDS is estimated to be between 0.2 and 0.8 (see Table 8); that for ND, between 0 and 1.2 (see Table 10); that for NDC, between 0.4 and 1.3 (see Table 10).

We estimated the ratio of income elasticities for each pair of goods we used. Our estimates indicated evidence against the hypothesis of homothetic preferences that is weaker than the hypothesis of the linear expenditure system for the model with NDS and durables, the model with NDC and services, and the model with ND and services. The income elasticity for NDS is likely to be slightly greater than that for durables. The income elasticity for services is likely to be much greater than that for NDS and that for NDC.

REFERENCES

Ball, L. (1984): "Intertemporal Substitution and Constraints on Labor Supply: Evidence from Panel Data," manuscript, New York University.

Byrnes, J. C., G. F. Donahoe, M. W. Hook, and R. P. Parker (1979): "Monthly Estimates of Personal Income, Taxes, and Outlays," *Survey of Current Business*, November, 18-38.

Beveridge, S. and C. Nelson (1981): "A New Approach to Decomposition of Economic Time Series into Permanent and Transitory Components with Particular Attention to Measurement of the Business Cycle," *Journal of Monetary Economics*, 7, 151-174.

Campbell, J. Y. (1987): "Does Saving Anticipate Declining Labor Income? An Alternative Test of the Permanent Income Hypothesis," *Econometrica,* 55, 1249-1273.

Campbell, J. Y., and R. J. Shiller (1987): "Cointegration and Tests of Present Value Models," *Journal of Political Economy* 95, 1062-1088.

Christiano, L. J., and M. Eichenbaum (1989): "Unit Roots in Real GNP - Do We Know and Do We Care?" manuscript presented at the Carnegie-Rochester Public Policy Conference, April 21-22, 1989.

Cochrane, J. H. (1988): "How Big is the Random Walk in GNP?" *Journal of Political Economy*, 96, 893-920.

——— (1989): "The Sensitivity of Tests of the Intertemporal Allocation of Consumption to Near-Rational Alternatives," 79, 319-337.

Constantinides, G. M. (1987): "Habit Formation: A Resolution of the Equity Premium Puzzle," manuscript, University of Chicago.

Dickey, D. A. and W. A. Fuller (1979): "Distribution of the estimators for autoregressive time series with a unit root," *Journal of American Statistical Association*, 74, 427-31.

——— (1981): "Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root," *Econometrica*, 49, 1057-1072.

Dunn, K. B., and K. J. Singleton (1986): "Modeling the Term Structure of Interest Rates under Non-Separable Utility and Durability of Goods," *Journal of Financial Economics*, 17, 27-55.

Easley, D. and N. M. Kiefer (1988): "Controlling a Stochastic Process with Unknown Parameters," *Econometrica*, 56, 1045-1064.

Eichenbaum, M. S., and L. P. Hansen (1989): "Estimating Models with Intertemporal Substitution Using Aggregate Time Series Data," manuscript, Northwestern University and University of Chicago.

Eichenbaum, M. S., L. P. Hansen, and K. J. Singleton (1988): "A Time Series Analysis of Representative Agent Models of Consumption and Leisure Choice Under Uncertainty," *Quarterly Journal of Economics*, 103, 51-78.

El-Gamal, M. (1988): "Real Expectations: A Harmony Theoretic Approach to Decision Making Under Uncertainty," RCER Working Paper No. 176, University of Rochester.

Engle, R. F. and W. J. Granger (1987): "Co-Integration and Error Correction: Representation, Estimation, and Testing," *Econometrica*, 55, 251-276.

Evans, G. B. A. and N. E. Savin (1981): "Testing For Unit Roots: 1," *Econometrica*, 49, 753-779.

Evans, G. B. A. and N. E. Savin (1984) "Testing For Unit Roots: 2," *Econometrica*, 52, 1241-1269.

Finn, M. D., D. L. Hoffman, D. E. Schlagenhauf (1988): "Intertemporal Asset-Pricing Relationships in Barter and Monetary Economies: An Empirical Analysis," manuscript, Arizona State University.

Friend, I. and M. E. Blume (1975): "The Demand for Risky Assets," *American Economic Review* 65, 900-923.

Fukushige, M. and M. Hatanaka (1989): "A Test on the Attainability of Equilibrium Conditions," Discussion Paper 76, Faculty of Economics, Osaka University.

Fuller, W. A. (1976): *Introduction to Statistical Time Series*. New York: Wiley.

Garber, P. M., and R. G. King(1983): "Deep Structural Excavation? A Critique of Euler Equation Methods," NBER Technical Paper No. 31.

Hannan, E. J. (1970): *Multiple Time Series*, New York: Wiley.

Hansen, B. E. (1989): "Efficient Estimation of Cointegrating Vectors in the Presence of Deterministic Trends," manuscript, University of Rochester.

Hansen, L. P. (1982): "Large Sample Properties of Generalized Method of Moments Estimators," *Econometrica*, 50, 1029-1054.

Hansen, L. P. and R. Jagannathan (1986): "Restrictions on Intertemporal Marginal Rates of Substitution Implied by Asset Prices," manuscript.

Hansen, L. P., and K. J. Singleton (1982): "Generalized Instrumental Variable Estimation of Nonlinear Rational Expectations Models," *Econometrica*, 50, 1269-1286.

——— (1983): "Stochastic Consumption, Risk Aversion, and the Temporal Behavior of Asset Returns, " *Journal of Political Economy* 91, 249-265.

——— (1988): "Efficient Estimation of Linear Asset Pricing Models with

Moving Average Errors," manuscript.

Hayashi, F. (1985): "The Permanent Income Hypothesis and Consumption Durability: Analysis Based on Japanese Panel Data," *Quarterly Journal of Economics* 100, 1083-1115.

——— (1987): "Tests for Liquidity Constraints: A Critical Survey," in *Advances in Econometrics, Fifth World Congress*, Vol. 2., ed. by T. F. Bewley. Cambridge: Cambridge University Press.

Hausman, J. A. (1978): "Specification Tests in Econometrics," *Econometrica*, 46, 1251-1271.

Heaton, J. C. (1988): "Notes on an Empirical Investigation of Asset Pricing with Nonseparable Preference Specifications," manuscript.

Houthakker, H. S. (1960): "Additive Preferences," *Econometrica*, 28, 244-256.

Johansen, S. (1988): "Statistical Analysis of Cointegrating Vectors," *Journal of Economic Dynamics and Control* 12, 231-254.

Kahn, J. and M. Ogaki (1989): "A Chi-Square Test for a Unit Root," manuscript, University of Rochester.

King, R. G., C. I. Plosser, and S. T. Rebelo (1988): "Production, Growth and Business Cycles II. New Directions," *Journal of Monetary Economics* 21 309-341.

Kocherlakota, N. R. (1988): "On Tests of the Time and State Separable Utility-Based Asset Pricing Model," manuscript, Northwestern University.

Mankiw, N. G. (1982): "Hall's Consumption Hypothesis and Durable Goods" *Journal of Monetary Economics*, 10, 417-425.

Mehra, R. and E. C. Prescott (1985): "The Equity Premium: A Puzzle," *Journal of Monetary Economics*, 15, 145-161.

Miron, J. A. (1986): "Seasonal Fluctuations and the Life Cycle-Permanent Income Model of Consumption," *Journal of Political Economy*, 94, 1258-1279.

Nelson, C. R. and C. I. Plosser (1982): "Trends and Random Walks in Macroeconomic Time Series," *Journal of Monetary Economics*, 10, 139-162.

Ogaki, M. (1988): *Learning about Preferences from Time Trends*, Ph.D. dissertation, University of Chicago.

——— (1989a): "Information in Deterministic Trends about Preferences," manuscript, University of Rochester.

——— (1989b): "Engel's Law, the Linear Expenditure System, and Cointegration, manuscript, University of Rochester.

Ouliaris, S., J. Y. Park, and P. C. B. Phillips (1988): "Testing for a Unit Root in the Presence of a Maintained Trend," CAE Working Paper No. 88-32, Cornell University.

Park, J. Y. (1988a): "Canonical Cointegrating Regressions," CAE Working Paper No.88-29, Cornell University.

——— (1988b): "Testing for Unit Roots and Cointegration by Variable Addition," CAE Working Paper No.88-30, Cornell University.

——— (1989): "On the Joint Test of a Unit Root and Time Trend," manuscript, Cornell University.

Park, J. Y. and B. Choi (1988): "A New Approach to Testing for a Unit Root," CAE Working Paper No. 88-23, Cornell University.

Park, J. Y., and M. Ogaki (1989): "Seemingly Unrelated Canonical Cointegrating Regressions," manuscript, Cornell University and University of Rochester.

Park, J. Y., S. Ouliaris, and B. Choi (1988): "Spurious Regressions and Tests for Cointegration," CAE Working Paper No. 88-07, Cornell University.

Park, J. Y., and P. C. B. Phillips (1988): "Statistical Inferences in Regressions with Integrated Processes: Part 1," *Econometric Theory*, 4, 468-497.

Phillips, P. C. B., and S. N. Durlauf (1986): "Multiple Time Series Regression with Integrated Processes," *Review of Economic Studies*, 53, 473-495.

Phillips, P. C. B., and B. E. Hansen (1989): "Statistical Inference in Instrumental Variables Regression with I(1) Processes," Cowles Foundation Discussion Paper No. 869-R.

Phillips, P. C. B. and S. Ouliaris (1988): "Asymptotic Properties of Residual Based Tests for Cointegration," Cowles Foundation Discussion Paper No. 847-R, Yale University.

Phillips, P. C. B. and J. Y. Park (1988): "On the Formulation of Wald Tests of Nonlinear Restrictions," *Econometrica* 56, 1065-1083.

Phillips, P. C. B. and P. Perron (1988): "Testing for a Unit Root in Time Series Regression," *Biometrica*, 75, 335-46.

Schwert, G. W. (1987): Tests for Unit Roots: A Monte Carlo Investigation, Working Paper GPB 87-01, University of Rochester.

Singleton, K. J. (1988): "Econometric Issues in the Analysis of Equilibrium Business Cycle Models," *Journal of Monetary Economics* 21, 361-386.

Stock, J. H. (1987): "Asymptotic Properties of Least Squares Estimators of Cointegrating Vectors," *Econometrica*, 55, 1035-1056.

Tauchen, G. (1986): "Statistical Properties of Generalized-Method-of-Moments Estimators of Structural Parameters Obtained from Financial Market Data," *Journal of Business and Economic Statistics*, 4, 397-425.

West, K. D. (1988): "Asymptotic Normality, When Regressors Have a Unit Root," *Econometrica*, 56, 1397-1417.

TABLE 1

TESTS FOR DIFFERENCE STATIONARITY OF CONSUMPTION

	$J(1,2)$	$J(1,3)$	$J(1,4)$	$J(1,5)$	$J(1,6)$	Z_α	Z_t
NDS 59:1-86:12	1.549	1.554	3.186	3.303	4.317	-5.770	-1.709
NDS 68:1-86:12	0.085	0.190	0.412	0.630	0.835	-15.059	-2.889
ND 59:1-86:12	0.714	0.788	1.662	1.826	2.286	-12.176	-2.494
NDC 59:1-86:12	1.542	1.598	3.396	3.580	4.119	-6.369	-1.790
Durables 59:1-86:12	0.129	0.184	0.749	0.776	0.978	-23.601†	-3.397*
Services 59:1-86:12	3.526	3.669	7.053	7.055	9.137	-3.307	-1.347
Services 68:1-86:12	0.709	0.871	1.448	2.041	2.451	-11.680	-2.767
Clothing 59:1-86:12	1.423	1.507	1.545	1.731	2.490	-25.774†	-3.679†

NOTE: The lag truncation number used for the Z_α and Z_t statistics reported in this table was 30. Critical values for the 1 per cent, 5 per cent, and 10 per cent significance levels are 0.000086, 0.0023, and 0.0093 for $J(1,2)$; 0.011, 0.055, and 0.12 for $J(1,3)$; 0.055, 0.16, and 0.29 for $J(1,4)$; 0.123, 0.295, and 0.452 for $J(1,5)$; 0.21, 0.43, 0.66 for $J(1,6)$; -29.5, -21.8, and -18.3 for Z_α; -3.96, -3.41, and -3.12 for Z_t. Critical values for $J(p,q)$ are from Park and Choi (1988) when they are reported, and were estimated using 500 observations and 10,000 iterations when they are not reported in Park and Choi (1988). Critical values for Z_α and Z_t are from Fuller (1976, p.371 and p.373).

*Significant at the 10 per cent significance level.

†Significant at the 5 per cent significance level.

TABLE 2

TESTS FOR TREND STATIONARITY OF CONSUMPTION

	$G(1,2)$	$G(1,3)$	$G(1,4)$	$G(1,5)$	$G(1,6)$
NDS 59:1-86:12	4.882 (0.027)	4.888 (0.087)	6.115 (0.106)	6.167 (0.187)	6.523 (0.259)
NDS 68:1-86:12	1.055 (0.304)	2.149 (0.342)	3.929 (0.269)	5.202 (0.267)	6.123 (0.294)
ND 59:1-86:12	3.874 (0.049)	4.100 (0.129)	5.808 (0.121)	6.011 (0.198)	6.472 (0.263)
NDC 59:1-86:12	4.860 (0.027)	4.929 (0.085)	6.190 (0.103)	6.263 (0.180)	6.447 (0.265)
Durables 59:1-86:12	1.681 (0.195)	2.293 (0.318)	6.322 (0.097)	6.451 (0.168)	7.299 (0.199)
Services 59:1-86:12	5.776 (0.016)	5.826 (0.054)	6.493 (0.090)	6.493 (0.165)	6.683 (0.245)
Services 68:1-86:12	3.344 (0.067)	3.753 (0.153)	4.768 (0.190)	5.410 (0.248)	5.724 (0.334)
Clothing 59:1-86:12	6.427 (0.011)	6.579 (0.037)	6.643 (0.084)	6.936 (0.139)	7.808 (0.167)

NOTE: Probability values are in parentheses. The lag truncation number used for the $G(p,q)$ statistics reported in this table was 80.

TABLE 3

TESTS FOR DIFFERENCE STATIONARITY OF RELATIVE PRICES

C_2	$J(1,2)$	$J(1,3)$	$J(1,4)$	$J(1,5)$	$J(1,6)$	Z_α	Z_t
			C_1: NDS				
Durables 59:1-86:12	1.164	1.862	2.386	2.763	3.285	-8.438	-2.363
Durables 68:1-86:12	0.00009†	0.259	0.273*	0.431	0.496*	-13.999	-2.671
			C_1: ND				
Durables 59:1-86:12	0.018	1.216	1.224	1.388	1.424	-10.845	-2.352
Services 59:1-86:12	0.454	2.587	3.015	3.080	3.237	-2.120	-0.603
Services 68:1-86:12	1.749	2.716	2.923	2.975	3.004	-2.440	-0.832
			C_1: NDC				
Durables 59:1-86:12	0.194	1.576	1.684	1.805	1.816	-9.865	-2.360
Services 59:1-86:12	0.077	1.332	2.050	2.072	2.211	-5.018	-1.207
Services 68:1-86:12	1.103	2.249	2.337	2.373	2.423	-2.813	-0.870
Clothing 59:1-86:12	1.908	2.585	12.126	12.393	14.354	-3.742	-1.689

NOTE: The lag truncation number used for the Z_α and Z_t statistics reported in this table was 30. See the first footnote of Table 1 for critical values of the test statistics reported in this table.

*Significant at the 10 per cent significance level.

†Significant at the 5 per cent significance level.

TABLE 4

TESTS FOR TREND STATIONARITY OF RELATIVE PRICES

C_2	$G(1,2)$	$G(1,3)$	$G(1,4)$	$G(1,5)$	$G(1,6)$
			C_1: NDS		
Durables 59:1-86:12	5.732 (0.017)	6.934 (0.031)	7.510 (0.057)	7.825 (0.098)	8.170 (0.147)
Durables 68:1-86:12	0.002 (0.968)	3.553 (0.169)	3.705 (0.295)	5.205 (0.267)	5.731 (0.333)
			C_1: ND		
Durables 59:1-86:12	0.209 (0.648)	6.675 (0.036)	6.694 (0.082)	7.069 (0.132)	7.145 (0.210)
Services 59:1-86:12	3.335 (0.068)	7.698 (0.021)	8.016 (0.046)	8.058 (0.089)	8.155 (0.148)
Services 68:1-86:12	4.865 (0.027)	5.589 (0.061)	5.697 (0.127)	5.722 (0.221)	5.736 (0.333)
			C_1: NDC		
Durables 59:1-86:12	1.777 (0.183)	6.695 (0.035)	6.867 (0.076)	7.043 (0.134)	7.058 (0.216)
Services 59:1-86:12	0.864 (0.353)	6.925 (0.031)	8.150 (0.043)	8.178 (0.085)	8.349 (0.138)
Services 68:1-86:12	4.387 (0.036)	5.789 (0.055)	5.857 (0.119)	5.884 (0.208)	5.919 (0.314)
Clothing 59:1-86:12	4.696 (0.030)	5.161 (0.076)	6.612 (0.085)	6.623 (0.157)	6.691 (0.245)

NOTE: The lag truncation number used for $G(p,q)$ statistics reported in this table was 80.

TABLE 5

TESTS FOR NO COINTEGRATION

Regressand	Regressor	Sample Period	$I(1,5)$
NDS	Durables	1959:1-1986:12	2.912
NDS	Durables	1968:1-1986:12	1.128
Durables	NDS	1959:1-1986:12	0.615
Durables	NDS	1968:1-1986:12	1.266
ND	Durables	1959:1-1986:12	1.215
Durables	ND	1959:1-1986:12	0.392
ND	Services	1959:1-1986:12	1.034
ND	Services	1968:1-1986:12	1.063
Services	ND	1959:1-1986:12	4.797
Services	ND	1968:1-1986:12	2.175
NDC	Durables	1959:1-1986:12	2.895
Durables	NDC	1959:1-1986:12	0.510
NDC	Services	1959:1-1986:12	0.590
NDC	Services	1968:1-1986:12	0.505
Services	NDC	1959:1-1986:12	1.796
Services	NDC	1968:1-1986:12	3.798
NDC	Clothing	1959:1-1986:12	4.215
Clothing	NDC	1959:1-1986:12	1.796

NOTE: Critical Values for $I(1,5)$ at the 5 per cent and 10 per cent significance levels are 0.251 and 0.384, respectively. These critical values are from Park, Ouliaris and Choi (1988).

TABLE 6

CANONICAL COINTEGRATING REGRESSION RESULTS

Regress- and	α_1^*	α_2^*	α_1/α_2^*	$G_R^\dagger$	$H(0,1)^\dagger$	$H(1,2)^\dagger$	$H(1,3)^\dagger$	$H(1,4)^\dagger$
				C_1: NDS, C_2: Durables, 1959:1-1986:12				
p_2^m	-0.843	-0.051	. . .	34.848	6.214	0.009	2.017	2.120
	(0.359)	(0.187)	. . .	(0.000)	(0.013)	(0.924)	(0.365)	(0.548)
c_1^m	-1.914	-0.504	. . .	250.902	53.326	0.059	0.898	14.584
	(0.376)	(0.185)	. . .	(0.000)	(0.000)	(0.808)	(0.638)	(0.002)
				C_1: NDS, C_2: Durables, 1968:1-1986:12				
p_2^m	-1.386	-0.144	. . .	337.760	11.875	3.412	4.266	4.321
	(0.134)	(0.063)	. . .	(0.000)	(0.001)	(0.065)	(0.119)	(0.229)
				C_1: ND, C_2: Durables, 1959:1-1986:12				
p_2^m	-0.485	0.162	. . .	87.380	15.503	2.931	2.947	3.194
	(0.658)	(0.221)	. . .	(0.000)	(0.000)	(0.087)	(0.229)	(0.363)
				C_1: ND, C_2: Services, 1959:1-1986:12				
p_2^m	0.453	0.040	11.456	163.139	7.838	0.831	1.099	1.100
	(0.649)	(0.315)	(75.873)	(0.000)	(0.005)	(0.362)	(0.577)	(0.777)
				C_1: ND, C_2: Services, 1968:1-1986:12				
p_2^m	1.023	0.099	10.299	83.541	2.667	0.170	0.572	3.239
	(0.740)	(0.315)	(25.667)	(0.000)	(0.102)	(0.680)	(0.751)	(0.356)
c_1^m	4.113	1.432	2.872	2053.731	10.025	0.912	1.196	1.224
	(0.876)	(0.379)	(0.214)	(0.000)	(0.002)	(0.340)	(0.550)	(0.747)

TABLE 6 - *Continued*

Regress-and	α_1^*	α_2^*	α_1/α_2^*	$G_R^\dagger$	$H(0,1)^\dagger$	$H(1,2)^\dagger$	$H(1,3)^\dagger$	$H(1,4)^\dagger$
			C_1: NDC, C_2: Durables, 1959:1-1986:12					
p_2^m	0.791 (0.446)	0.589 (0.123)	1.344 (0.495)	93.440 (0.000)	22.220 (0.000)	6.460 (0.011)	6.469 (0.039)	8.580 (0.035)
			C_1: NDC, C_2: Services, 1959:1-1986:12					
p_2^m	0.393 (0.372)	0.064 (0.149)	6.175 (9.093)	242.421 (0.000)	7.869 (0.005)	2.534 (0.111)	3.794 (0.150)	4.012 (0.260)
			C_1: NDC, C_2: Services, 1968:1-1986:12					
p_2^m	1.197 (0.742)	0.141 (0.194)	8.508 (7.947)	69.049 (0.000)	2.461 (0.117)	1.226 (0.268)	1.712 (0.425)	3.699 (0.296)
c_1^m	4.281 (1.181)	0.984 (0.318)	4.348 (0.557)	957.146 (0.000)	2.863 (0.091)	0.120 (0.729)	0.258 (0.879)	0.523 (0.914)
c_2^m	10.785 (7.302)	3.683 (2.700)	2.928 (0.349)	91.380 (0.000)	1.674 (0.196)	3.513 (0.061)	4.287 (0.117)	6.116 (0.106)
			C_1: NDC, C_2: Clothing, 1959:1-1986:12					
p_2^m	1.402 (0.178)	1.186 (0.064)	1.182 (0.101)	8.663 (0.013)	18.108 (0.000)	2.431 (0.119)	2.780 (0.249)	2.886 (0.409)

NOTE: The lag truncation number used for the results in this table was 80. The G_R statistic tests the null of the linear expenditure system.

*Standard errors are in parentheses.

†Probability values are in parentheses.

TABLE 7

CANONICAL COINTEGRATING REGRESSION RESULTS UNRESTRICTED AND RESTRICTED BY THE DETERMINISTIC COINTEGRATION RESTRICTION

Unrestricted	α_1*	α_2*	Restricted	α_1*	α_2*
		C_1: NDC, C_2: Services, 1968:1-1986:12			
p_2^m	2.306 (1.025)	1.796 (1.073)		1.197 (0.742)	0.141 (0.194)
c_1^m	4.956 (1.650)	2.558 (1.374)		4.281 (1.181)	0.984 (0.318)
c_2^m	26.845 (64.723)	10.495 (26.560)		10.785 (7.302)	3.683 (2.700)

NOTE: The lag truncation number used for the results in this table was 80.

*Standard errors are in parentheses.

TABLE 8

SEEMINGLY UNRELATED CANONICAL COINTEGRATING REGRESSION RESULTS

Regressand	α_1 *	α_2 *	α_1/α_2 *	G_R †	$H(1,2)$ †	$H(1,3)$ †	$H(1,4)$ †
			C_1: NDS, C_2: Durables, 1959:1-1986:12				
p_2^m	0.602 (0.138)	0.763 (0.087)	0.788 (0.181)	8.487 (0.014)	5.455 (0.020)	6.215 (0.045)	9.313 (0.025)
c_1^m	4.146 (2.215)	2.720 (1.453)	1.524 (0.160)	34.828 (0.000)	9.821 (0.002)	9.822 (0.007)	9.893 (0.019)
			C_1: NDS, C_2: Durables, 1968:1-1986:12				
p_2^m	0.493 (0.138)	0.863 (0.101)	0.571 (0.160)	15.469 (0.000)	0.311 (0.577)	4.566 (0.102)	5.826 (0.120)
c_1^m	3.591 (2.500)	2.496 (1.737)	1.439 (0.287)	16.144 (0.000)	3.116 (0.078)	3.275 (0.194)	3.285 (0.350)
			C_1: ND, C_2: Durables, 1959:1-1986:12				
p_2^m	0.550 (0.137)	0.570 (0.059)	0.966 (0.241)	29.967 (0.000)	1.013 (0.314)	5.170 (0.075)	5.971 (0.113)
c_1^m	8.699 (7.475)	3.307 (2.842)	2.630 (0.288)	120.712 (0.000)	3.423 (0.064)	3.548 (0.170)	3.869 (0.276)
			C_1: ND, C_2: Services, 1968:1-1986:12				
p_2^m	1.662 (0.223)	0.328 (0.131)	5.072 (0.682)	148.946 (0.000)	2.728 (0.099)	3.915 (0.141)	4.771 (0.189)
c_1^m	3.192 (0.364)	0.983 (0.112)	3.247 (0.251)	3751.575 (0.000)	1.976 (0.160)	4.831 (0.089)	5.933 (0.115)

TABLE 8 - *Continued*

Regressand	α_1*	α_2*	α_1/α_2*	G_R†	$H(1,2)$†	$H(1,3)$†	$H(1,4)$†
			C_1: NDC, C_2: Durables, 1959:1-1986:12				
p_2^m	0.692 (0.125)	0.640 (0.054)	1.082 (0.196)	24.456 (0.000)	0.532 (0.466)	5.363 (0.068)	5.851 (0.119)
c_1^m	3.440 (1.202)	1.372 (0.479)	2.507 (0.338)	74.713 (0.000)	5.581 (0.018)	5.640 (0.060)	7.763 (0.051)
			C_1: NDC, C_2: Services, 1968:1-1986:12				
p_2^m	1.494 (0.249)	0.181 (0.117)	8.257 (1.378)	115.915 (0.000)	4.477 (0.034)	5.411 (0.067)	6.350 (0.096)
c_1^m	3.912 (0.385)	0.818 (0.080)	4.784 (0.668)	6400.455 (0.000)	1.268 (0.260)	5.063 (0.080)	5.063 (0.167)

NOTE: The lag truncation number used for the results in this table was 80. The G_R statistic tests the null of the linear expenditure system.

*Standard errors are in parentheses.

†Probability values are in parentheses.

TABLE 9

SEEMINGLY UNRELATED CANONICAL COINTEGRATING REGRESSION RESULTS: RESTRICTED ESTIMATES FOR DEMAND SYSTEM WITH THREE GOODS

α_1*	α_2*	α_3*	α_1/α_2*	α_1/α_3*	G_R†
C_1: ND, C_2: Services, C_3: Durables, 1968:1-1986:12					
Diffrence stationary C_1 and C_2 and trend stationary C_3					
0.604	0.103	0.604	5.846	1.001	1.352
(0.267)	(0.132)	(0.078)	(9.441)	(0.339)	(0.245)
C_1: ND, C_2: Services, C_3: Durables, 1968:1-1986:12					
Diffrence stationary C_1 and trend stationary C_2 and C_3					
1.804	0.384	1.013	4.700	1.781	126.979
(0.225)	(0.131)	(0.116)	(1.407)	(0.203)	(0.000)
C_1: NDC, C_2: Services, C_3: Durables, 1968:1-1986:12					
Diffrence stationary C_1 and C_2 and trend stationary C_3					
0.906	-0.012	0.692	. . .	1.309	1.913
(0.220)	(0.104)	(0.054)	. . .	(0.291)	(0.167)
C_1: NDC, C_2: Services, C_3: Durables, 1968:1-1986:12					
Diffrence stationary C_1 and trend stationary C_2 and C_3					
1.264	0.133	0.828	9.522	1.526	63.317
(0.219)	(0.116)	(0.091)	(8.056)	(0.260)	(0.000)

NOTE: The lag truncation number used for the results in this table was 80. The G_R statistic tests the cross-equation restriction of the model.

*Standard errors are in parentheses.

†Probability values are in parentheses.

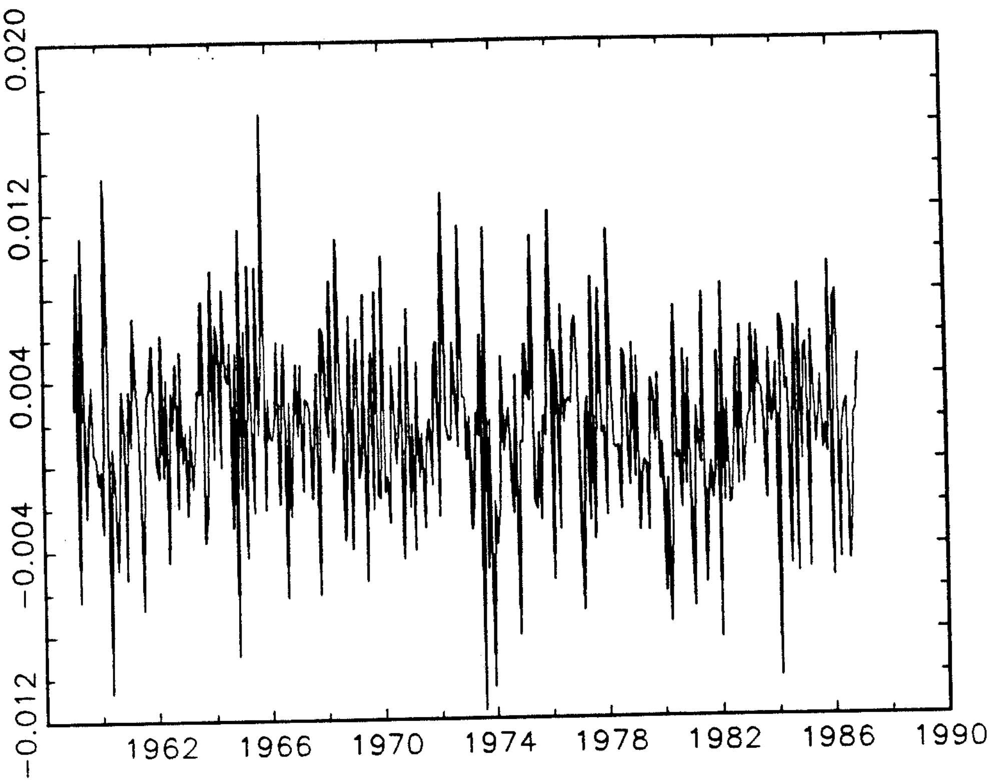

Fig. 1. The first difference of log real per capita consumption: NDS, 1959:1-1986:12

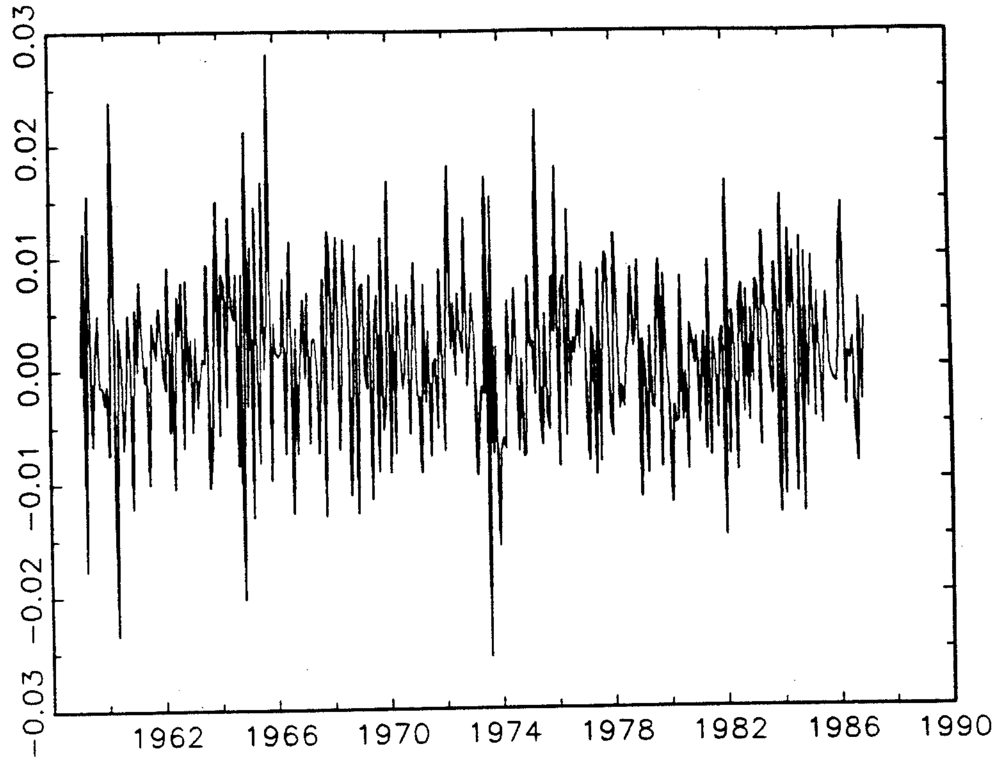

Fig. 2. The first difference of log real per capita consumption: ND, 1959:1-1986:12

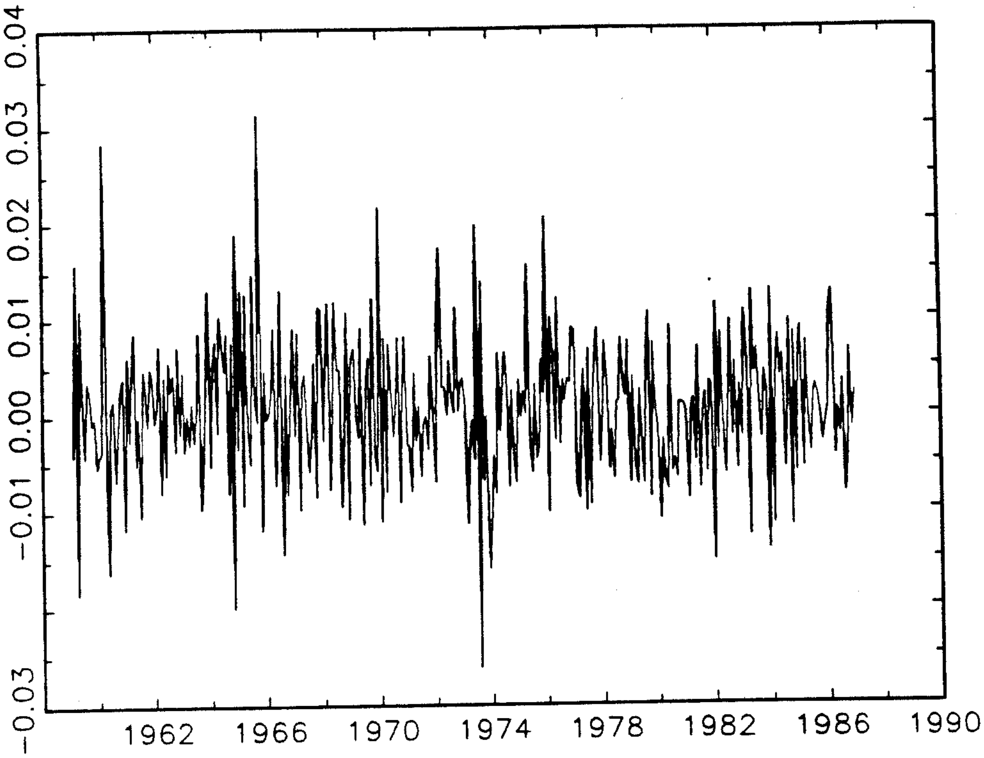

Fig. 3. The first difference of log real per capita consumption: NDC, 1959:1-1986:12

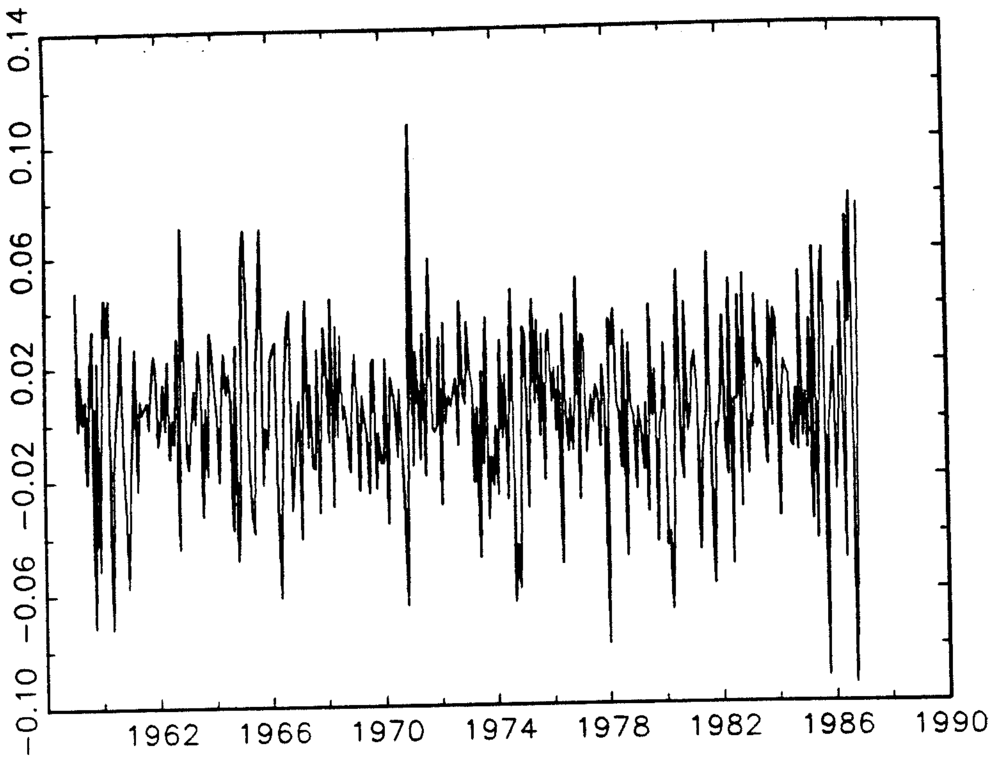

Fig. 4. The first difference of log real per capita consumption: Durables, 1959:1-1986:12

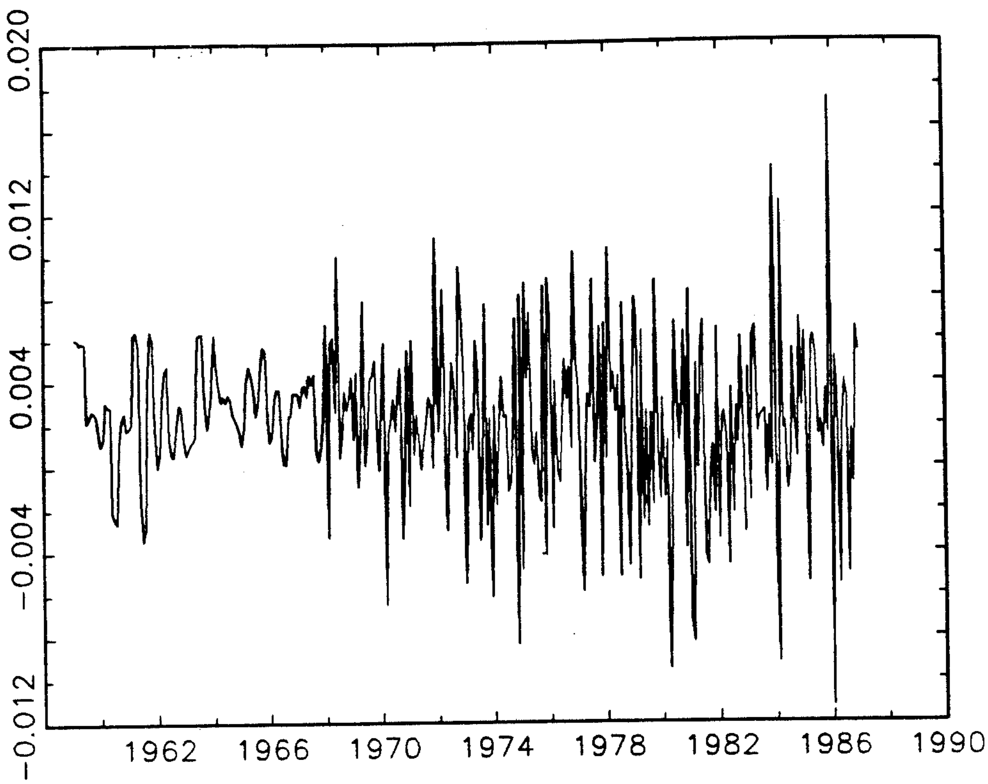

Fig. 5. The first difference of log real per capita consumption: Services, 1959:1-1986:12

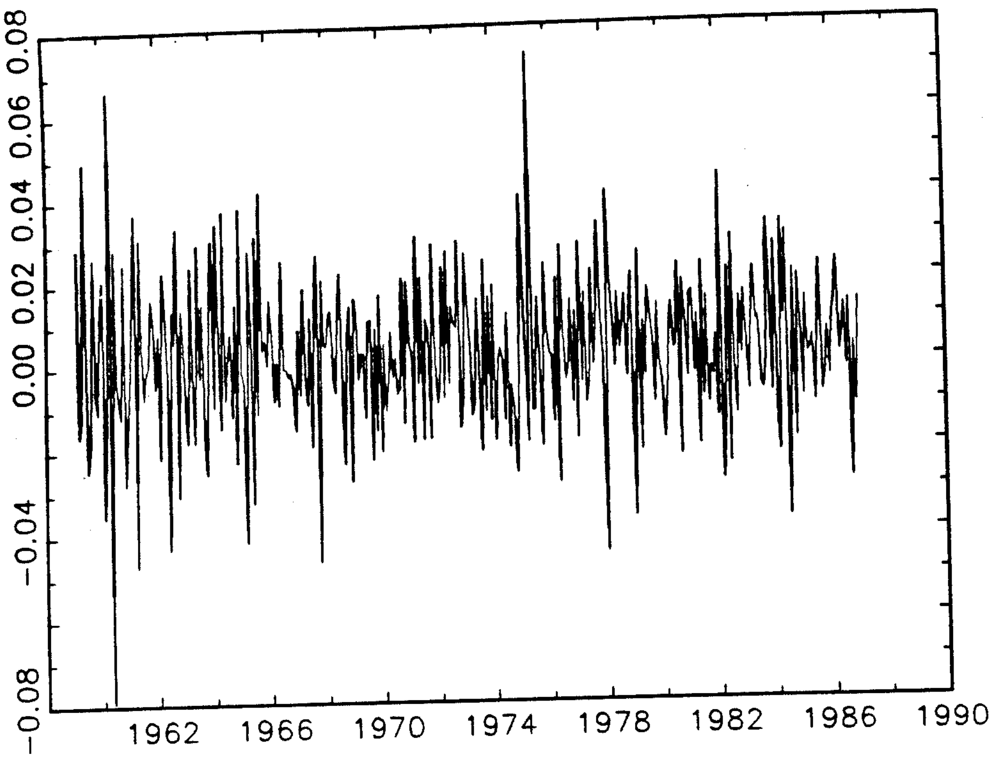

Fig. 6. The first difference of log real per capita consumption: Clothing, 1959:1-1986:12